Answer 2:

The authors in the paper "The Capacity of the Hopfield Associative Memory" discuss associative memory and techniques for rigorously analyzing it. Traditional measures of associative memory performance are its memory capacity and content - addressability. Memory capacity refers to the maximum number of associated pattern pairs that can be stored and correctly retrieved. Content-addressability is the ability of the network to retrieve the correctly stored pattern. The primary focus of the research paper is to address the memory capacity of the Hopfield network.

We consider an associative structure based upon a neural net. In Hopfield, The model neurons we consider are simple bistable elements each being capable of assuming two values: - 1 (off) and + 1 (on). The state of each neuron then represents one bit of information, and the state of the system as a whole is described by a binary n-tuple if there are n neurons in the system. The equation for state change is given by:

$$x_i' = \operatorname{sgn}\left\{ \sum_{j=1}^{n} T_{ij} x_j \right\} = \begin{cases} +1, & \text{if } \Sigma T_{ij} x_j \geq 0 \\ -1, & \text{if } \Sigma T_{ij} x_j < 0 \end{cases}$$

Where T_{ij} is the connection matrix, it is assumed to have zero diagonal elements. x_j is the input probe. There are two modes of changing x. In *synchronous* operation, all the n neurons are simultaneously updated whereas in *asynchronous* operation the components of the current state of x is updated one at a time.

Connection matrix: Let 'm' be the total set of memories. For each memory x^{α} , an n x n matrix can be formed by: $T_{\alpha} = x^{(\alpha)}(x^{(\alpha)})^T - I_n$ The connection matrix is given by:

$$T = \sum_{\alpha=1}^{m} T_{\alpha}$$

From this equation (right-hand side) it is assumed that once T is calculated, all other information about x^{α} is forgotten when a new memory is added or to be learnt.

$$T = \sum_{\alpha=1}^{m} (x^{(\alpha)})((x^{(\alpha)})^T - I_n).$$

Alternate Connection matrix: In this part authors discuss complex matrices and assumes that it might give better results. To construct these matrices fundamental memories are taken as eigenvectors of the connection matrix which have positive eigenvalues. Let the memories x(U) be eigenvectors of T with positive eigenvalues Then $\lambda^{(\alpha)}$ $\operatorname{sgn}\left((Tx^{(\alpha)})i\right) = \operatorname{sgn}\left(\lambda^{(\alpha)} x_i^{(\alpha)}\right) = x_i^{(\alpha)}$

With the help of an example, the authors have shown two possible problems associated with Hopfield's retrieval algorithm.

1. If we begin with a probe x, the resulting fixed point may not be one of the memories
2. If it is, it may not be the nearest memory

The memory can never be "recalled" by the algorithm if the memory is not a fixed point. Hopfield investigated the model with asynchronous dynamics and demonstrated that associative recall of chosen data was quite feasible with a measure of error correction.

In **Stability** we come across 3 possibilities of the convergence for the asynchronous case:
1. Every transition which is actually a change in a component is a change in the right direction.
2. With high enough probability(not 1), a random step is in the right direction makes the probe with high probability come very close to the center of the fundamental memory.
3. The components can change back and forth, but will at least get better with each change. So after finite changes, the system reaches the fixed point which is mostly the correct memory.

Further, from Hopfield's symmetric connection matrix, we understand that it always reaches a fixed point in the asynchronous model. This stable point is referred to as energy minimum or correlation maximum point.
The energy function of the system is then given by: $-\sum_i \sum_j T_{ij} x_i x_j$

Thus, there is a domain or basin of attraction around each fundamental memory where each probe will reach the fundamental memory at the center as a stable or a fixed point, in both the synchronous and asynchronous models.

While discussing the **Concept of asymptotic memory** authors have described two concepts - They defined capacity as the rate of growth, the output will be with probability approaching 1 instead of 0 if we increase the capacity by a fraction 1+ ε with ε > 0.

1. Higher the probability every one of the 'm' fundamental memories may be fixed
2. With a higher probability almost every memory is good, but not necessarily every memory. We can make a few wrong moves but still get close enough to the fundamental memory so that we then have direct convergence. The derivation of capacity in as mentioned in capacity heuristics states that maximum n / (2 log n) memories can be stored in the Hopfield matrix.

The authors have also mentioned about **three lemmas** in section VII, Lemma A, B and C.
Lemma A – It represents a uniform estimate for the probability that the summation of N independent and '1 random variable' takes on a particular integer value not too far from the mean sum, which is valid for the large-deviation theorem.
Lemma B – Uses Lemma A to get a known uniform asymptotic expression for the cumulative distribution of a summation of N independent and 1 random variables, valid for the same large deviations as Lemma A. The strong form of the large-deviation central limit theorem is given as Lemma B'
Lemma C – Is known as Bonferroni's inequality.

In the **Big Theorem** author conveys that direct attraction fails for almost every vector in the sphere of radius pn. It could be strengthened

From the paper's conclusion and section X we can infer about the memory capacity of the Hopfield model for direct and non-direct convergence as mentioned below:

SN.	Direct Convergence	Non direct Convergence
1	For any 0 < p < l/2, if $\frac{(1-2\rho)^2}{2}\frac{n}{\log n}$ with high probability, the unique fundamental memory is to be recovered by direct convergence to the fundamental memory, except for a vanishingly small fraction of the fundamental memories	For any 0 < p <l/2 and ε > 0, if $m=(1-\epsilon)\frac{n}{2\log n}$ we expect that almost all the pn-sphere around almost all the m fundamental memories are ultimately attracted to the correct fundamental memory
2	$\frac{(1-2\rho)^2}{4}\frac{n}{\log n}$ if, in the above scenario, no fundamental memory can be exceptional	$m=(1-\epsilon)\frac{n}{4\log n}$ If we try, then with high probability almost all the pn-sphere around all the fundamental memories is attracted
3	if 0 <= p < l/2, p given, where some wrong moves are permitted (although two steps suffice) $\frac{n}{2\log n}$ we can have as above a small fraction of exceptional fundamental memories	$m=(1+\epsilon)\frac{n}{2\log n}$ If we try then a vanishingly small fraction of the fundamental memories themselves are even fixed points.
4	$\frac{n}{4\log n}$ if as above some wrong moves are permitted (although two synchronous moves suffice) but no fundamental memory can be exceptional	$m=(1+\epsilon)\frac{n}{4\log n}$ If we try, then with probability near 1 there is at least one fundamental memory not fixed.