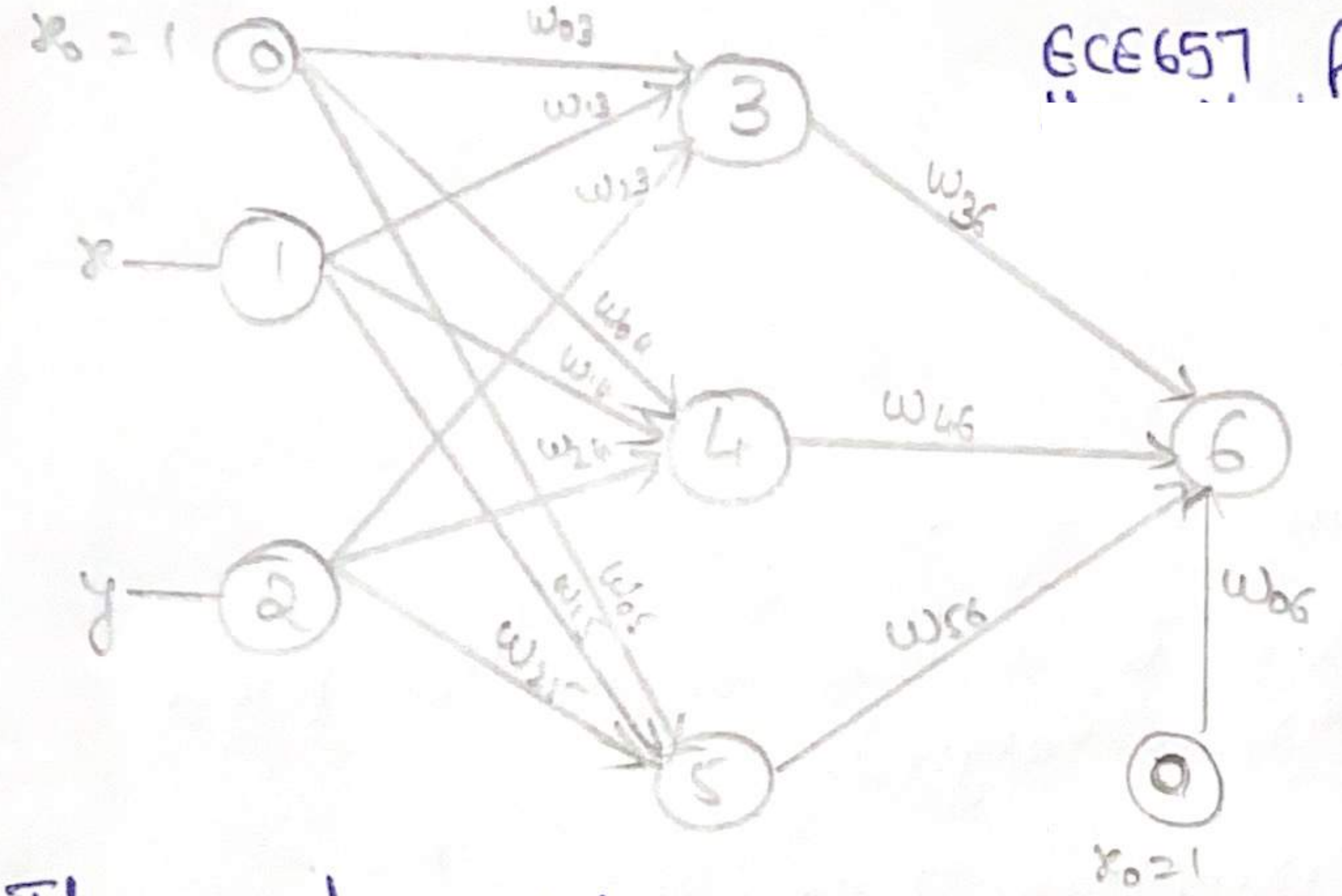
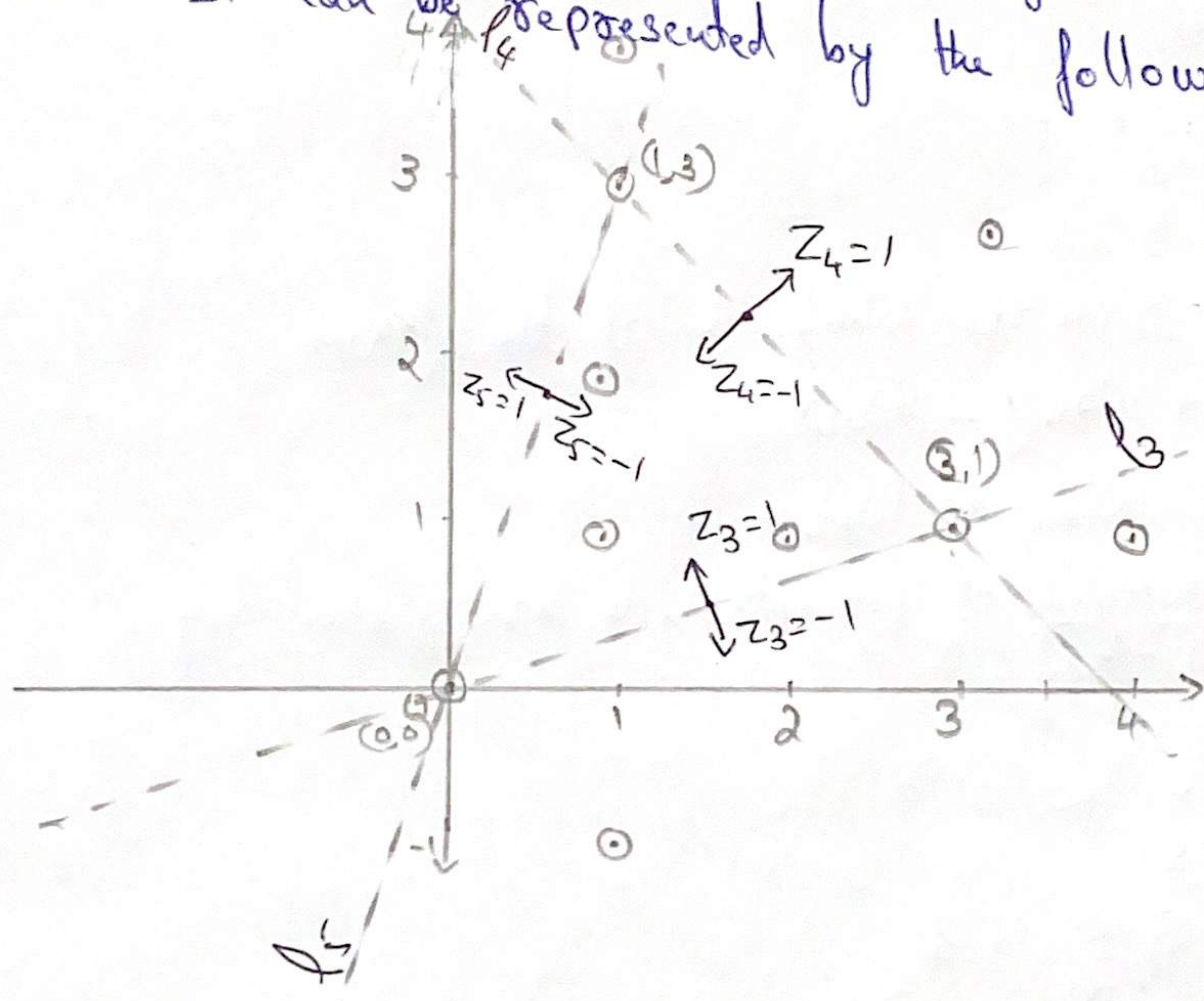


ECE657 Assignment-1 Question-2

To calculate weights of the perceptron such that it will form a triangle with vertices (x, y) at $(0,0)$ $(1,3)$ and $(3,1)$

The question states that each perceptron acts as a linear separator. Thus 3 lines are formed by the perceptrons 3, 4 and 5. It can be represented by the following diagram:



The weights of the first layer can be calculated by comparing the line equations with the equation of weighted inputs. The line equations are:

Perceptron 3:

Slope $= \frac{1-0}{3-0} = 1/3$

$\Rightarrow y = 1/3 x$

$\therefore l_3 = y - 1/3 x$ which is of the form

$w_{13}x + w_{23}y$ (There is no bias as the line passes through origin)

Thus the weights of perceptron 3 are given by

$$w_3 = \begin{bmatrix} 0 \\ -1/3 \\ 1 \end{bmatrix}$$

Perceptron 4:

Slope $= \frac{3-1}{1-3} = -1$

$y-3 = -1(x-1)$

$\Rightarrow l_4 = y + x - 4 = 0$ which is of the form

$w_{14}x + w_{24}y + w_{04}x_0$.

$\Rightarrow w_{14} = 1 \quad w_{24} = 1 \quad w_{04} = -1$

Thus the weights of perceptron 4 are given by

$$w_4 = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$$

Perceptron 5:

Slope $= \frac{3-0}{1-0} = 3$

$\Rightarrow y = 3x$

$l_5 = y - 3x$ which is of form $w_{15}x + w_{25}y$

$\Rightarrow w_{15} = -3 \quad w_{25} = 1 \quad w_{05} = 0$ (bias is zero since line passes through origin)

The weights of perceptron 5 are $w_5 = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

Calculating W_{36}, W_{46}, W_{56}, W_{06} +

The activation function is given by $Sgn(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$
The output from perceptron 3, 4 and 5 are passed to perceptron 6. The possible outputs of each perceptron after activation function is either 1 or -1

Since there are 3 i/p to 'perceptron 6' with values either (-1, 1), there are 8 possible outcomes (2^3) which would define all the possible inputs to 'perceptron 6'. This method is selected be cause it allows the model to be trained on all possible inputs which would be better than trial and error where there are chances of bias.

Let Z_3, Z_4 and Z_5 be the outputs from perceptron 3, 4 and 5. The truth table is given by

x_0	Z_3	Z_4	Z_5	Z
1	-1	-1	-1	-1
1	1	-1	-1	1
1	-1	1	-1	-1
1	1	1	-1	-1
1	-1	-1	1	-1
1	1	-1	1	-1
1	-1	1	1	-1
1	1	1	1	-1

– There are no points that will satisfy this condition

Z is the final output and Z = 1 implies that the point lies inside the triangle

Weight Initialisation:

let $W_{36} = W_{46} = W_{56} = 0.5$

bias weight $W_{06} = -1$

learning rate $\eta = 0.1$

Epoc 1:

Step 1: $x = [1\ \ -1\ \ -1\ \ -1]$ $\quad w = \begin{bmatrix} -1 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$ $\quad z = -1$

$$\hat{z} = \mathrm{Sgn}\left([1\ -1\ -1\ -1]\begin{bmatrix} -1 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}\right)$$

$$= \mathrm{Sgn}(-2.5)$$

$$\hat{z} = -1$$

$z = \hat{z}$ $\therefore$ no weight change required

Step 2: $x = [1\ \ 1\ \ -1\ \ -1]$ $\quad w = \begin{bmatrix} -1 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$ $\quad z = 1$

$$\hat{z} = \mathrm{Sgn}\left([1\ 1\ -1\ -1]\begin{bmatrix} -1 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}\right)$$

$$\hat{z} = \mathrm{Sgn}(-1.5)$$

$\hat{z} = -1$ but $z = 1$

$$\hat{z} \neq z$$

$$w^{new} = w^{old} + \eta(z - \hat{z})x$$

$$= \begin{bmatrix} -1 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} + 0.1 \times 2 \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$w^{new} = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}$$

step 3:

$x = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn(xw)$

$= Sgn\left(\begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}\right)$

$= Sgn(-1.5)$

$\hat{z} = -1$ $\quad \therefore z = \hat{z}$, no weight change required.

step 4:

$x = \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left(\begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}\right)$

$\hat{z} = Sgn(-0.1)$

$= -1$ $\quad \therefore z = \hat{z}$, no weight change required

step 5:

$x = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left(\begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}\right)$

$= Sgn(-1.5)$

$\hat{z} = -1$ $\quad z = \hat{z}$, no weight change required.

step 6:

$x = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left(\begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}\right)$

$\hat{z} = Sgn(-1.5)$

$= -1$

$z = \hat{z}$ no change required

~~Test 7~~: Step 7:

$x = [1 \quad 1 \quad 1 \quad 1]$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left([1 \quad 1 \quad 1 \quad 1]\begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix}\right)$

$= Sgn(0.5)$

$= 1$

$z \neq \hat{z}$

$w^{new} = \begin{bmatrix} -0.8 \\ 0.7 \\ 0.3 \\ 0.3 \end{bmatrix} + 0.1 \times -2 \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$w^{new} = \begin{bmatrix} -1.0 \\ 0.5 \\ 0.1 \\ 0.1 \end{bmatrix}$

Epoc 2:

Step 1:

$x = [1 \quad -1 \quad -1 \quad -1]$ $\quad w = \begin{bmatrix} -1.0 \\ 0.5 \\ 0.1 \\ 0.1 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left([1 \quad -1 \quad -1 \quad -1]\begin{bmatrix} -1.0 \\ 0.5 \\ 0.1 \\ 0.1 \end{bmatrix}\right)$

$\hat{z} = Sgn(-1.7)$

$= -1$ $\quad z = \hat{z}$ $\therefore$ no weight change required.

Step 2:

$x = [1 \quad 1 \quad -1 \quad -1]$ $\quad w = \begin{bmatrix} -1.0 \\ 0.5 \\ 0.1 \\ 0.1 \end{bmatrix}$ $\quad z = 1$

$\hat{z} = Sgn\left([1 \quad 1 \quad -1 \quad -1]\begin{bmatrix} -1.0 \\ 0.5 \\ 0.1 \\ 0.1 \end{bmatrix}\right)$

$= Sgn(-0.7)$

$\hat{z} = -1 \neq z$ $\quad \therefore w^{new} = \begin{bmatrix} -1.0 \\ 0.5 \\ 0.1 \\ 0.1 \end{bmatrix} + 0.1 \times 2 \times \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

$w^{new} = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$

Step 3:

$x = [1 \ -1 \ 1 \ -1]$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ 0.1 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left([1 \ -1 \ 1 \ -1]\begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ 0.1 \end{bmatrix}\right)$

$\hat{z} = Sgn(-1.5) = -1 = z$

$\therefore$ no weight change required.

Step 4:

$x = [1 \ 1 \ 1 \ -1]$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn(-0.1)$

$\hat{z} = -1 = z$ $\therefore$ no weight change required.

Step 5:

$x = [1 \ -1 \ -1 \ 1]$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left([1 \ -1 \ -1 \ 1]\begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}\right)$

$= Sgn(-1.5)$

$= -1 = z$ $\therefore$ no weight change required.

Step 6:

$x = [1 \ 1 \ -1 \ 1]$ $\quad w = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$ $\quad z = -1$

$\hat{z} = Sgn\left([1 \ 1 \ -1 \ 1]\begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}\right)$

$\hat{z} = Sgn(-0.1)$

$= -1 = \hat{z}$ $\therefore$ no weight change required

Step 7.

$x = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ $w = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$ $\hat{z} = -1$

$\hat{z} = Sgn\left(\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}\right)$

$= Sgn(-0.3)$

$\hat{z} = -1 = z$ ∴ no change required

Checking Solution on Random data points.

$P_1(1,1)$. This point lies inside the triangle so output should be one

$w_3 = \begin{bmatrix} 0 \\ -1/3 \\ 1 \end{bmatrix}$ $w_4 = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$ $w_5 = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$ $w_6 = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$

$O_3 = Sgn(w_{03} + w_{13}x + w_{23}y)$

$= Sgn(0 + -1/3 \times 1 + 1)$

$O_3 = Sgn(2/3) = 1$

$O_4 = Sgn(w_{04} + w_{14}x + w_{24}y)$

$= Sgn(-4 + 1 + 1)$

$= Sgn(-2) = -1$

$O_5 = Sgn(w_{05} + w_{15}x + w_{25}y)$

$= Sgn(0 - 3 + 1) = Sgn(-2) = -1$

$O_6 = Sgn(w_{06} + w_{36}O_3 + w_{46}O_4 + w_{56}O_5)$

$O_6 = Sgn(-0.8 + 1 \times 0.7 + 1 \times 0.1 + 1 \times 0.1) = Sgn(0.1)$

$O_6 = 1$. Thus this model classifies this point inside the triangle

Point (3,3):

This point lies outside the triangle so the output should be -1.

$O_3 = Sgn(W_{03} + W_{13}x + W_{23}y)$

$= Sgn(0 + 3\times -1/3 + 3\times 1)$

$= Sgn(2)$

$O_3 = 1$

$W_3 = \begin{bmatrix} 0 \\ -1/3 \\ 1 \end{bmatrix}$

$O_4 = Sgn(W_{04} + W_{14}x + W_{24}y)$

$= Sgn(-4 + 1\times 3 + 1\times 3)$

$= Sgn(2) = 1$

$W_4 = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$

$O_5 = Sgn(W_{05} + W_{15}x + W_{25}y)$

$= Sgn(0 - 3\times 3 + 3\times 1)$

$= Sgn(-6)$

$= -1$

$W_5 = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

$O_6 = Sgn(W_{06} + W_{36}O_3 + W_{46}O_4 + W_{56}O_5)$

$= Sgn(-0.8 + 0.7\times 1 - 0.1\times 1 - 0.1\times -1)$

$= Sgn(-0.1)$

$O_6 = -1$

$W_6 = \begin{bmatrix} -0.8 \\ 0.7 \\ -0.1 \\ -0.1 \end{bmatrix}$

The predicted o/p shows that the point lies outside the triangle which meets the target output.