

~~31/07/19~~

Charlie

Date

Page No.

m & n find G.C.D

1. Factorize m

$$m_1 \times m_2 \times \dots \times m_j$$

2. Factorize n

$$n_1 \times n_2 \times \dots \times n_j$$

3. Identify common factors

$$m = 36$$

$$n = 48$$

$$\text{factor } m = 2 \times 2 \times 3 \times \underline{3}$$

$$\text{factor } n = 2 \times 2 \times 2 \times \underline{3} \times 1$$

$$\text{Common} = 2 \times 2 \times 3$$

$$= 12$$

9 divisions

Euclid (m, n)

:
if

while m does not divide n

$$g = n \bmod m$$

$$n = m$$

$$m = g$$

end

return m

g

~~1|8|19~~

Lab

① GCD \rightarrow (a) factors
 \rightarrow (b) Euclidean

② Linear Search

```

int gcd (int a, int b)
{
    if (a == 0)
        return b;
    if (b == 0)
        return a;
    if (a > b)
        return gcd (a-b, b);
    else
        return gcd (a, b-a);
}

```

~~5/8/19~~

Insertion Sort

Insertion Sort (h)

1. for $j = 2$ to A.length
2. key = A[j]
3. $i \leftarrow j - 1$
4. while $i > 0$ and $A[i] > \text{key}$
5. $A[i+1] = A[i]$
6. $i = i - 1$ // end
7. $A[i+1] = \text{key}$.

1	2	3	4	5	6	7	8	9	10
27	19	41	0	5	6	8	3		

$$\text{key} = A[4] = 4$$

$$i' = 4 - 1 = 3$$

while $3 > 0$ and $0 > 4$.

$$A[4] = A[3]$$

$$A[3] = 0$$

Q 7 1 9 | 9 1 1 - 1 0 1 5 1 6 1 8 3

$$\text{middle } i = 3 - 1 = 2,$$

$$2 > 0 \text{ and } 7 > 4$$

$$A[3] = 7$$

1 2 1 7 1 7 1 9 | 1 0 1 5 | 0 8 1 3

$$i = 2 - 1 = 1$$

$$1 > 0 \text{ and } 2 > 0$$

$$A[2] = 4$$

1 2 4 1 7 1 9 | 1 1 0 1 5 1 6 | 8 1 3,

Worst Case

$$5 1 4 1 3 1 2 1 1 \quad 1 \rightarrow 1+1 = 2$$

$$2 \rightarrow 2+2 \leq 4$$

$$3 \rightarrow 3+3 = 6$$

$$O(1+2+3+\dots+n)$$

$$\cancel{\frac{O(n(n+1))}{2}}$$

$$= n^2 + n$$

$$= O(n^2)$$

average case complexity = worst case complexity.

Cast Best Case

$$T_n = C_1 n + C_2(n-1) + C_3(n-1) + C_4(n-1) + \underbrace{C_5 + C_6}_{\pm C_7(n-1)}$$

$$(c_1 + c_2 + c_3 + c_4 + c_y)n - (c_2 + c_3 + c_4 + c_y)$$

$$an - b$$

$\Omega(n)$

$\frac{8}{8} | 9$

Int

Algorithm

iterative

Recursive

① A()

```
int i;
for (i=1 to n)
{
    print ("Hi")
}
```

$$1 + n + n - 1$$

$$= 2n$$

$$= O(n)$$

② A()

```

int i;
for (i=1 to n)
    for (j=1 to n)
        print ("Statement")
    
```

i
 n
 n^2
 n^2

$$m^2 + n^2 + n + 1$$

$$2n^3 + n + 1$$

$$\approx O(n^3)$$

③ A()

```

i=1, S=0
while (S <= n)
    i++
    S = S + i
    print ("Statement")
    n = n - 1

```

~~loop~~ ~~break~~ ~~exit~~

$O(\sqrt{n})$

④ A()

```

i=1
for (i=1; i<n; i++)
    if (" ")

```

$O(\sqrt{n})$

(2)

A()
8

```

int i, j, k, n;
for (i = 1; i ≤ n; i++)
    for (j = 1; j ≤ i; j++)
        for (k = 1, k ≤ 100; k++)
            Op
    
```

for ("Statement")

<i>i</i>	<i>j</i> = 1	<i>j</i> = 2	<i>j</i> ≥ 3	<i>j</i> = n
<i>O(n²)</i>	<i>k</i> = 100	<i>k</i> = 200	<i>k</i> = 300	<i>k</i> = n × 100

$$100 + 200 + \dots + n \times 100$$

$$100(1 + 2 + 3 + \dots + n)$$

$$100 \left(\frac{n(n+1)}{2} \right)$$

$$50(n^2 + n)$$

$$O(n^2)$$

(6)

```

if (i = 1, i ≤ n; i++)
    for (j = 1, j ≤ i2; j++)
        for (k = 1, k ≤ n/2, k++)
            forint ('')
    
```

y

$$\begin{array}{l} i = 1 \\ j = 1^2 \\ k = \frac{n}{2} \end{array}$$

$$\begin{array}{l} i = 2 \\ j = 4 \\ k = \frac{4(n)}{2} \end{array}$$

$$\begin{array}{l} i = n \\ j = n^2 \\ k = n^2 \left(\frac{n}{2}\right) \end{array}$$

$$\frac{n}{2} (1^2 + 2^2 + 3^2 + \dots + n^2)$$

$$\frac{n}{2} - \left(\frac{n(n+1)(2n+1)}{6} \right)$$

$$\frac{n}{2} \left[\frac{n(n+1)(2n+1)}{6} \right] = O(n^3)$$

1 2 3 4 ... k

n()

for (i=1; i<n; i+=i*2)

? found ("Statement")

g

i=1 i=

$$2^{k+1} = n$$

$$k+1 = \log n$$

$$k = \log n$$

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Worst Case - Insertion Sort

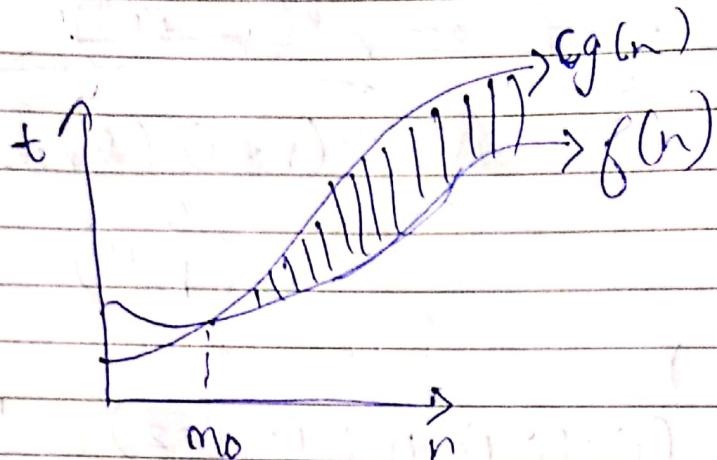
$$T(n) = C_1 n + C_2(n-1) + C_3(n-1) + C_4 \sum_{i=2}^n t_i + \\ C_5 \sum_{t=2}^n t_j - 1 + C_6 + \sum_{t=2}^n t_j - 1 + C_7(n-1)$$

$$= an^2 + bn + c$$

Complexity = $O(n^2)$

ASYMPTOTIC NOTATION

• Big O



$$f(n) = an + b$$

$$f(n) \leq C g(n)$$

after some $n \geq n_0$
 $C > 0, n_0 \geq 1$

$$3n+2 \leq \underline{2} g(n)$$

\Leftrightarrow

$$f(n) = O(g(n))$$

$$f(n) = \underline{3n+2}$$

$$f(n) \leq c g(n)$$

$$\underline{3n+2} \leq 4n$$

$$c = 4, n_0 = 2$$

$$f(n) = O(g(n))$$

$$= O(n)$$

Best Case

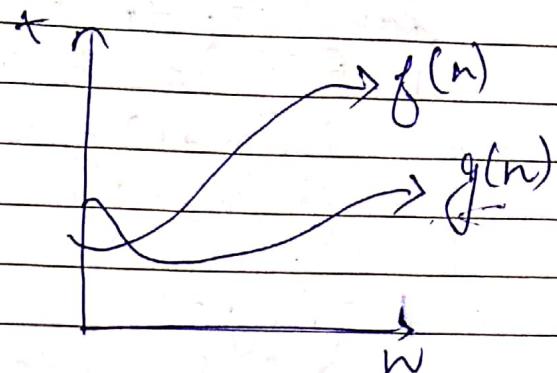
$$f(n) = 3n + 2$$

$$f(n) \geq c_0 g(n)$$

$$3n + 2 \geq c_0 g(n)$$

$$3n + 2 \geq c_1 g(n)$$

$$3n + 2 \geq 1(i)$$

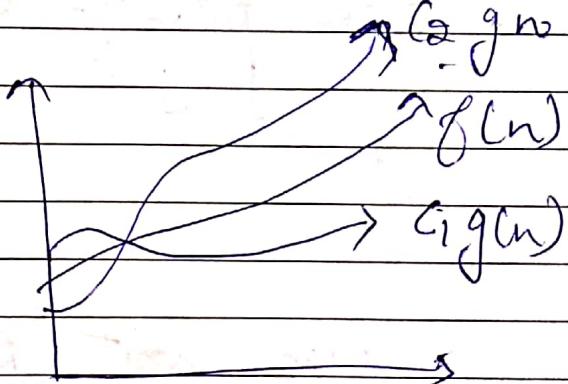


$$c = 1, n = 1$$

Average Case

$$c_0 g(n) \geq f(n) \geq c_1 g(n)$$

$$m \leq 3n + 2 \leq 4n$$



~~4/8/19~~

Space Complexity

$O(1) \rightarrow$ Insertion Sort

Merge Sort (OutPlace Algorithm)

$\Theta(\text{merge}(A, B, p, q, r))$

$m_1 = q - p + 1$

$m_2 = r - q$

Let $L[1..n+1]$ and $R[1..n+1]$ be new arrays

for ($i = 1$ to n)

$L[i] = A[p+i-1]$

for ($j=1$ to n_2)

$$R[j] = A[q+j]$$

$$L[n_1+1] = \infty$$

$$R[n_2+1] = \infty$$

$$i=1, j=1$$

for ($k=p$ to q)

if ($L[i] \leq R[j]$)

$$A[k] = L[i];$$

$$i = i + 1;$$

$$\text{else } A[k] = R[j]$$

$$j = j + 1$$

q.

merge-Sort (A, p, q)

{

if $p < q$

$$q' = \lfloor (p+q)/2 \rfloor \rightarrow \text{floor value}$$

merge-Sort (A, p, q')

merge-Sort (A, q'+1, q), q'

merge (A, P, q, q')

q'

Space Complexity $\log n$

~~2/8/19~~

Labs - Me-2

① Insertion Sort

Execution time \Rightarrow include <time.h>
clock_t begin, end;
begin = clock(),
end = clock();

point (begin - end)

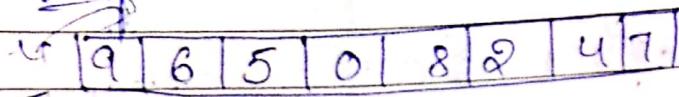
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Insertion Sort \rightarrow n Best
 n^2 Average
 n^2 Worst

Merge sort \rightarrow $n \log n$
 $n \log n$
 $n \log n$

Quick sort \rightarrow $n \log n$
 $n \log n$
 n^2

Quick Sort \Rightarrow



Partition (A, P, x)

$$x = A[g]$$

$$i = p-1$$

for ($j = p$ to $g-1$)
 $i \leftarrow i + 1$ if ($A[j] \leq x$)

$$i = i + 1$$

exchange $A[i]$ with $A[j]$

g

endony = $P(i+1)$ with $A[g]$

switch $i+1$

g

9 6 5 0 8 2 4 7

(1) 6 9 5 0 8 2 4 7 \Rightarrow pivot

(2) 6 5 9 0 8 2 4 7

(3) 6 5 0 9 8 2 4 7

(4) 6 5 0 2 8 9 4 7

(5) 6 5 0 8 4 9 2 7

(6) 6 5 0 2 4 7 8 9

Quicksort (A, P, q)

\downarrow
if ($P < q$)
 \downarrow

$q = \text{Partition } (A, B, q)$,
 Quicksort ($A, B, q - 1$),
 Quicksort ($A, q + 1, B$),

5	6	1	3	2	4
1	5	5	3	2	4
1	3	5	6	2	4
1	3	2	6	5	4
1	3	2	4	5	5

$\downarrow \in C(1, 6)$

\downarrow
P(1, 6)

\downarrow
Q(1, 3)

\downarrow
Q(5, 6)

\downarrow
P(1, 3)

\downarrow
Q(1, 1)

\downarrow
P(5, 6) Q(5, 5)

≈ 2

19/8/19

Quick Sort v/s merge sort

~~20/8/19~~ Heap Sort

Height	1	2	3	4
Nodes	3	7	15	31

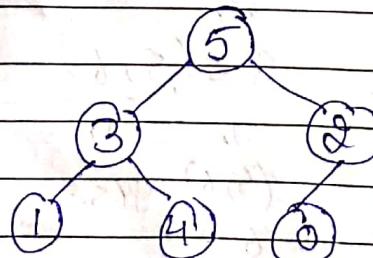
$$\text{Nodes } n = (2^{h+1} - 1)$$

$$h = \lfloor \log n \rfloor$$

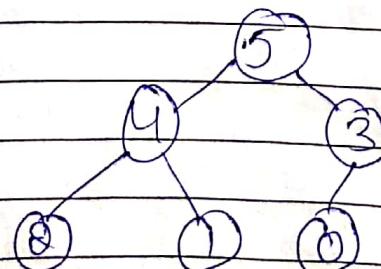
[5 | 3 | 2 | 1 | 4 | 0]

$$\text{left child} = 2i^{\circ}$$

$$\text{right child} = 2i^{\circ} + 1$$



[5 | 4 | 3 | 2 | 1 | 0]



max-heapify (A, i^*)

$$l = 2i^*$$

$$r = 2i^* + 1$$

if ($l \leq A.\text{heapsize}$ and $A[l] > A[i^*]$)
 largest = l ,

else largest = i^*

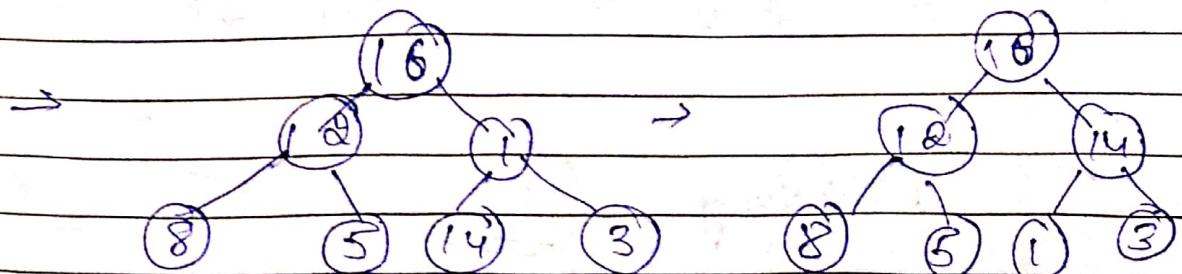
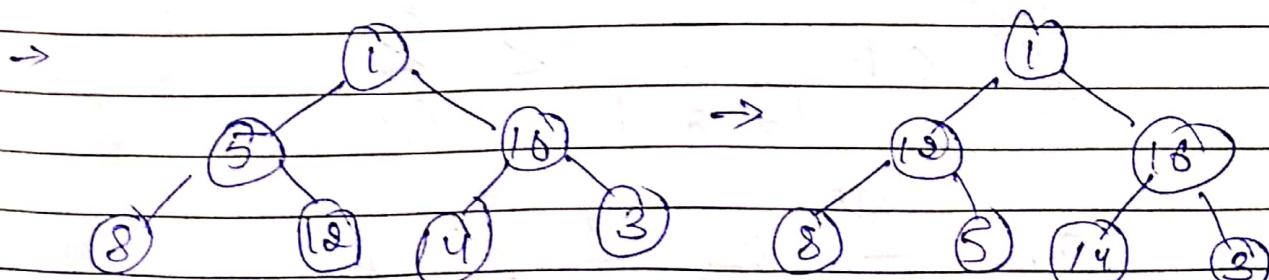
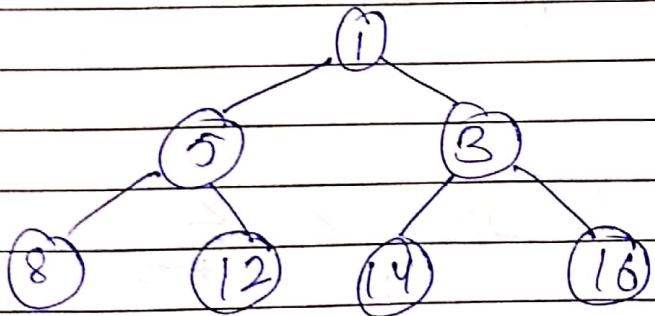
if ($r \leq A.\text{heapsize}$ and $A[r] > A[\text{largest}]$)
 largest = r ,

if (largest $\neq i^*$)

exchange $A[i^*]$ with $A[\text{largest}]$

max-heapify ($A, \text{largest}$),

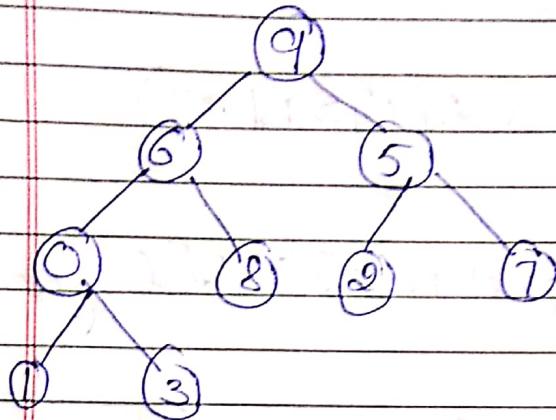
no. of leaf nodes = $\binom{n}{2} + 1$



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9 6 5 0 8 2 7 1 3



Build Maxheap(A)

A heap size is A.length
for ($i = \lfloor A.length / 2 \rfloor$ down to 1)
 max_heapify(A, i)

$$\sum_{h=0}^{\log n} O(h) ch \left\lceil \frac{n}{2^{h+1}} \right\rceil < \sum_{h=0}^{\infty} \frac{n}{2^h}$$

$$\sum_{h=0}^{\log n} ch \left\lceil \frac{n}{2^{h+1}} \right\rceil < \sum_{h=0}^{\infty} \frac{n}{2^h}$$

$$\frac{cn}{2} \sum_{h=0}^{\log n} \frac{1}{2^h} < \sum_{h=0}^{\infty} \frac{c}{2^h}$$

Heapifying $\underline{O(n)}$

Space = ~~$\Theta(n)$~~ $\log n$

23/8/19

Tute

Complexity for Recursive Program \Rightarrow
Back substitution

Recursion tree

Master's Theorem

BACK SUBSTITUTION

$$T(n) = 1 + T(n-1) \quad \text{--- (1)}$$

$$T(n-1) = 1 + T(n-2) \quad \text{--- (2)}$$

$$T(n-2) = 1 + T(n-3) \quad \text{--- (3)}$$

$$T(n-3) = 1 + T(n-4) \quad \text{--- (4)}$$

$$\begin{aligned} T(n) &= 1 + 1 + T(n-2) \\ &= 2 + T(n-2) \\ &= 2 + 1 + T(n-3) \\ &= 3 + T(n-3) \end{aligned}$$

for k recursion

$$T(n) = k + T(n-k)$$

$$n-k = 1 \quad k = n-1$$

Complexity =

$$\begin{aligned} T(n) &= n - 1 + n - 1 + 1 \\ &= n \end{aligned} \quad O(n)$$

$$T(n) = n + T(n-1)$$

$$\log_{10} 1 = 1 \\ \log_{10} 2 = 2 \\ \log_{10} 3 = 3$$

$$T(m-1) = m-1 + T(n-2)$$

$$T(n-2) = n-2 + T(n-3)$$

$$T_n = m + n-1 + T(n-2)$$

$$T_n = n + n-1 + n-2 + T(n-3)$$

$$T_n = n + n-1 + n-2 + n-3 + T(n-4)$$

$$T_n = n + n-1 + n-2 + n-3 + \dots + n-k + T(n-(k+1))$$

$$n-k-1 = 1$$

$$n-k = 2$$

$$k = n-2$$

$$\frac{n - n+2}{2} + \frac{n - n+2+1}{2}$$

$$\frac{n(n+1)}{2}$$

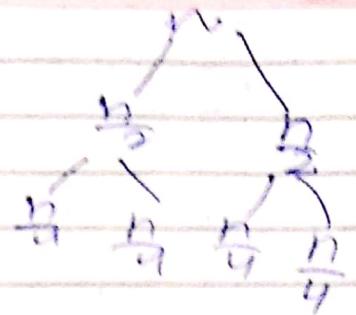
$$\Theta(n^2)$$

Recursion Tree

$$T(n) = 2T\left(\frac{n}{2}\right) + C$$

$$C(2^0 + 2^1 + 2^2 + \dots + 2^{\log n})$$

$$C \times \log n = O(n \log n)$$



~~20/8/19~~

Counting Recurrences

~~30/8/19~~

Tute

Master Theorem

$$T(n) = aT\left(\frac{n}{b}\right) + O(n^k \log^p n)$$

$a \geq 1$, $b > 1$, $k \geq 0$ and p is real no.

▷ if $a > b^k$, then $T(n) = O(n^{\log_b a})$

▷ if $a = b^k$

(a) if $p > -1$, then $T(n) = O(n^{\log_b a} \log^{p+1} n)$

(b) if $p = -1$, then $T(n) = O(n^{\log_b a} \log n)$

(c) if $p < -1$, then $T(n) = O(n^{\log_b a})$

▷ if $a < b^k$

(a) if $p \geq 0$, then $T(n) = O(n^k \log^p n)$

(b) if $p < 0$, then $T(n) = O(n^k)$

~~Note~~
~~Algo's part~~

$$1. T(n) = 2T\left(\frac{n}{2}\right) + n \log n$$

$$2. T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

$$3. T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$$

$$4. T(m) = \sqrt{2}T\left(\frac{m}{2}\right) + \log m$$

$$5. T(n) = 4T\left(\frac{n}{3}\right) + n^2$$

$$6. T(n) = 16T\left(\frac{n}{4}\right) + n$$

$$7. T(n) = 2T\left(\frac{n}{4}\right) + n^{0.5}$$

~~T(m)~~

① $T(n) = 2T\left(\frac{n}{2}\right) + n \log n$

$$a=2, b=2, k=1, \beta=1$$

$$a = b^k \Rightarrow 2 = 2^1$$

$$T(n) = O(n^{\log_2 2} \log^2 n)$$

$$= O(n \log^2 n)$$

Randomized
Quick Sort

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$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

$$a=2, b=2, k=2, \beta=0$$

$$2 < 2^2$$

$$\begin{aligned} T(n) &= O(n^2 \log^0 n) \\ &= O(n^2) \end{aligned}$$

$$T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$$

$$a=6, b=3, k=2, \beta=1$$

$$6 < 3^2$$

$$T(n) = O(n^2 \log n)$$

98% 10

29/19

798

Radius Sheet :-

98, 40, 1, 500, 67
0 1 2 3 4

10 J802
800

(2)

$$(98/1) = 98 \cdot 1.10 = 8$$

exp. 21 (98/1.) % 10

101111178

- 91178

Recursion
Successive
dynamic.

Divide & conquer
Randomized
Charlie

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Dynamic Programming

$$f(n) = f(n-1) + f(n-2)$$

0 1 1 2 3 5 ...

int fib (int n)
if ($n \leq 1$)

return n

return fib(n-1) + fib(n-2)

$$\begin{aligned} T(n) &= T(n-1) + T(n-2) + O(1) \\ &= O(2^n) \end{aligned}$$

Memoization

Tabulation

Dynamic Programming \rightarrow It is an algorithmic paradigm that solves a given complex problem by breaking it into subproblems and storing the result of subproblems to avoid computing the same results again and again.

Conditions :-

1. Overlapping subproblem
2. Optimal substructure

Given problem is said to have optimal substructure property if an optimal solution of given problem can be obtained by using solution of its subproblems.

Chain matrix multiplication

$$\begin{bmatrix}
 1 & 5 & 9 & 7 & 3 & 4 \\
 2 & 1 & 9 & 7 & 2 & 6 \\
 9 & 5 & 2 & 2 & 3 & 5 \\
 6 & 6 & 1 & 3 & 1 & 7
 \end{bmatrix}_{4 \times 6} \cdot
 \begin{bmatrix}
 5 & 1 & 3 \\
 9 & 5 & 1 \\
 8 & 7 & 6 \\
 9 & 6 & 8 \\
 8 & 1 & 3 \\
 2 & 2 & 9
 \end{bmatrix}_{6 \times 3}$$

= $\boxed{\quad}$
 $\boxed{\quad}$
 $\boxed{\quad}$

No. of multiplication ~~1000000000~~ = 72

A_1	A_2	A_3	A_4	A_5
4×10	10×3	3×12	12×20	20×7
$P_1 P_2$	$P_2 P_3$	$P_3 P_4$	$P_4 P_5$	$P_5 P_6$
$P_0 P_1$	$P_1 P_2$	$P_2 P_3$	$P_3 P_4$	$P_4 P_5$
4×3		3×12		7×20
		40×12	120×20	
$\frac{12}{4}$			4×20	20×7
$\frac{24}{48}$				
120	10			
144	12			
960				

A_1	A_2	A_3	A_4	A_5
4×10	10×3	3×12	12×20	20×7
120				
144	4×3	3×12		
960		4×12	12×20	
560			4×20	20×7
1784				
		1784		

Goal: Find optimal way to multiply these matrices to perform fewest multiplication.

Dynamic Programming

Step 1: Check if the problem have optimal substructure and overlapping subproblem.

Step 2: Define a recursive step formula.

$$M[i, j] = \begin{cases} 0 & \text{if } i \geq j \\ M[i, k] + M[k+1, j] + P_{i, k} P_k P_j \\ \min_{i \leq k < j} & \end{cases}$$

Step 3: Note the optimal solution

$A_1 A_2$	$A_1 A_2 A_3$	$A_1 A_2 A_3 A_4$
$A_2 A_3$	$A_2 A_3 A_4$	$A_2 A_3 A_4 A_5$
$A_3 A_4$	$A_3 A_4 A_5$	$A_1 A_2 A_3 A_4 A_5$
$A_1 A_2 A_3 A_4 A_5$		

i/j →	1	2	3	4	5
1	0	120	264	1080	1344
2	x	0	360	1320	1350
3	x	x	0	720	1140
4	x	x	x	0	1680
5	x	x	x	x	0

$$i=1, j=2$$

$$\begin{aligned}
 M[1, 2] &= M[1, 1] + M[2, 2] + P_0 P_1 P_2 \\
 &= 0 + 0 + 120 \\
 &= 120
 \end{aligned}$$

$$i=2, j=3$$

$$\begin{aligned}
 M[2, 3] &= M[2, 2] + M[3, 3] + P_1 P_2 P_3 \\
 k=2 &= 0 + 0 + 360 \\
 &= 360
 \end{aligned}$$

$$\begin{aligned}
 M[1, 3] &= M[1, 2] + M[2, 3] + P_0 P_2 P_3 \\
 \textcircled{k=2} &= 120 + 360 + 144 \\
 &= 624 \\
 b=1 &= M[1, 1] + M[2, 3] + P_0 P_1 P_3 \\
 &= 0 + 360 + 480 = 840
 \end{aligned}$$

$$\begin{aligned}
 M[3, 4] &= M[3, 3] + M[4, 4] + P_2 P_3 P_4 \\
 k=3 &= 0 + 0 + 720 \\
 &= 720
 \end{aligned}$$

$$\begin{aligned}
 M[2, 4] &= M[2, 3] + M[3, 4] + P_1 P_3 P_4 \\
 k=3 &= 360 + 0
 \end{aligned}$$

$$\begin{array}{ll}
 P_0 = 4 & P_3 = 12 \\
 P_1 = 10 & P_4 = 20 \\
 P_2 = 3 & P_5 = 7
 \end{array}$$

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$$i=2, j=4$$

$$\begin{aligned}
 M[2,4] &= M[2,2] + M[3,4] + P_1 P_2 P_4 \\
 k=2 &= 0 + 720 + 600 \times 10 \times 12 \times 20 \\
 &= 1820
 \end{aligned}$$

$$\begin{aligned}
 k=3 &= M[2,3] + M[4,4] + P_1 P_3 P_4 \\
 &= 360 + 0 + 10 \times 12 \times 20 \\
 &= 360 + 2400 \\
 &= 2760
 \end{aligned}$$

$$\begin{aligned}
 M[3,5] &= M[3,3] + M[4,5] + P_2 P_3 P_5 \\
 k=3 &= 0 + 1680 + 3 \times 12 \times 9 \\
 &= 1680 + 252 \\
 &= 1932
 \end{aligned}$$

$$\begin{aligned}
 k=4 &\Rightarrow M[3,4] + M[5,5] + P_2 P_4 P_5 \\
 &= 720 + 0 + 3 \times 20 \times 7 \\
 &= 720 + 420 \\
 &= 1140
 \end{aligned}$$

$$\begin{aligned}
 M[1,4] &= M[1,1] + M[2,4] + P_0 P_1 P_4 \\
 k=1 &= 0 + 1320 + 4 \times 10 \times 20 \\
 &= 1320 + 800 \\
 &= 2120
 \end{aligned}$$

$$\begin{aligned}
 M[1,4] &= M[1,2] + M[3,4] + P_0 P_2 P_4 \\
 k=2 &= 120 + 720 + 4 \times 3 \times 20 \\
 &= 840 + 240 \\
 &= 1080
 \end{aligned}$$

$$m[1,4] = m[1,3] + m[4,4] + P_0 P_3 P_4$$

$k=3$

$$\begin{aligned}
 &= 264 + 0 + 4 \times 12 \times 20 \\
 &= 264 + 960 \\
 &= 1224
 \end{aligned}$$

$$\begin{array}{l}
 m[2,5] = m[2,2] + m[3,5] + \cancel{P_0} P_2 P_5 \\
 \boxed{k=2}
 \end{array}$$

$$\begin{aligned}
 &= 0 + 1140 + 10 \times 3 \times 7 \\
 &= 1140 + 210 = 1350
 \end{aligned}$$

$$\begin{aligned}
 k=3 &= m[2,3] + m[4,5] + P_1 P_3 P_5 \\
 &= 360 + 1680 + 10 \times 12 \times 7 \\
 &= 2040 + 840 \\
 &= 2880
 \end{aligned}$$

$$\begin{aligned}
 k=4 &= m[2,4] + m[5,5] + P_1 P_4 P_5 \\
 &= 1320 + 0 + 10 \times 20 \times 7 \\
 &= 1320 + 1400 \\
 &= 2720
 \end{aligned}$$

$$\begin{aligned}
 m[1,5] &= m[1,1] + m[2,5] + P_0 P_1 P_5 \\
 k=1 &= 0 + 1350 + 4 \times 10 \times 7 \\
 &= 1350 + 280 \\
 &= 1630
 \end{aligned}$$

$$\begin{array}{l}
 m[1,5] = m[1,2] + m[3,5] + P_0 P_2 P_5 \\
 \boxed{k=2}
 \end{array}$$

$$\begin{aligned}
 &= 120 + 1140 + 4 \times 3 \times 7 \\
 &= 1260 + 84 \\
 &= 1344
 \end{aligned}$$

1140
 120
 1260
 12

119
 84.
 Charlie
 Date
 29
 FBB No. 6

26
 42
 72

$$M[1,5] = M[1,3] + M[4,5] + P_0 P_3 P_5$$

$$\begin{aligned}
 R=3 &= 264 + 1680 + 4 \times 12 \times 7 \\
 T &= 1944 + 336 \\
 &= 2280 \\
 \cancel{1944} &
 \end{aligned}$$

$$M[1,5] = M[1,4] + M[3,5] + P_0 P_4 P_5$$

$$\begin{aligned}
 R=4 &= 1080 + 9 + 4 \times 20 \times 7 \\
 &= 1080 + 560 \\
 &= 1640
 \end{aligned}$$

$$(A_1 : A_2) (A_3 : A_4 : A_5)$$

$$\begin{matrix} 120 \\ (4,3) \end{matrix}$$

$$\begin{matrix} 720 \\ (3,20) \end{matrix}$$

$$\begin{matrix} 420 \\ (3,1) \end{matrix}$$

$$120$$

$$720$$

$$420$$

$$84$$

$$\underline{1344}$$

4/9/19

Charlie

Date

Page No.

LCS \rightarrow Longest common subsequence

S1 abcdefg

S2 abx dg

a, b, d, g, ab, dg, ~~abd~~, ~~abt~~,

a, b, d, g, ag, bg, bd, ab, dg, ~~abd~~, abd

$\text{LCS}("AXYT", "AYZX")$

$$\text{LCS}[i][j] = \text{LCS}[i-1][j-1] + 1$$

$$\text{LCS}[i][j] = (\text{LCS}[i-1][j], \text{LCS}[i][j-1])$$

$\text{LCS}("AXYT", "AYZX")$

$\text{LCS}("AXY", "AYZX")$

$\text{LCS}("AXYT", "AYZ")$

$\text{LCS}(AX, AYZX)$

$\text{LCS}(AXY, AYZ)$

$\text{LCS}(AXY, AYZ)$

$\text{LCS}(AXY, AY)$

LCS

	G	T	A	B
G	0	0	0	0
T	0	1	1	1
A	0	1	1	1
B	0	1	1	1

G T A B

~~G | 9 | 9~~

Tute

Function chain Matrix (P)

Step 1: Initialization loop for ($i \rightarrow 1$ to n)
 Set $mmul[i, i] \leftarrow 0$ // $i = j$

Step 2: loop \rightarrow length of Subsequence
 for length $\leftarrow 2$ to n

{ for $i \leftarrow 1$ to n -length + 1
 Set $j \leftarrow i + \text{length} - 1$
 set $mmul[i, j] \leftarrow \infty$

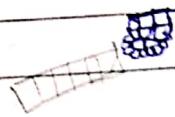
for $r \leftarrow i$ to $j-1$

{ set $q \leftarrow mmul[i, k] + mmul[k+1, j]$
 $+ P[i-1] \times P[k] \times P[j]$

if ($q < \text{mmul}[i, j]$) then

Set $\text{mmul}[i, j] \leftarrow q$
Set $s[i, j] \leftarrow k$

Step 3: return $\text{mmul}[i, j] \{ s[i, j]$



chain matrix

heap sort (algo + example + analysis)

Diff b/w Quick Sort and Randomized Quick Sort

11/9/19

A_1 A_2 A_3 A^4
 5×4 4×6 6×9 2×7

	1	2	3	4		1	2	3	4
1	0	10	86			1	0	1	1
2		0	48	104		2		0	2
3			0	84		3		0	3
4				0		4			0

0/1 Knapsack Problem

$W \rightarrow$ maximum capacity.

Total wt. ≤ 7

wt Val

1 1

3 4

4 5

5 7

wt	0	1	2	3	4	5	6	7
----	---	---	---	---	---	---	---	---

(1)	1	0	1	1	1	1	1	1
-----	---	---	---	---	---	---	---	---

(2)	2	0	1	1	4	5	5	5
-----	---	---	---	---	---	---	---	---

(3)	3	0	1	1	4	5	6	6
-----	---	---	---	---	---	---	---	---

(4)	4	0	1	1	4	5	7	8
-----	---	---	---	---	---	---	---	---

(5)	5	0	1	1	4	5	7	8
-----	---	---	---	---	---	---	---	---

Max value = 8

~~11/0/19~~

wt value

1 1

3 4

4 5

5 7

wt	0	1	2	3	4	5	6	7
----	---	---	---	---	---	---	---	---

0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---

(1)	1	0	1	1	1	1	1	1
-----	---	---	---	---	---	---	---	---

(2)	2	0	1	1	4	5	5	5
-----	---	---	---	---	---	---	---	---

(3)	3	0	1	1	4	5	6	6
-----	---	---	---	---	---	---	---	---

(4)	4	0	1	1	4	5	7	8
-----	---	---	---	---	---	---	---	---

(5)	5	0	1	1	4	5	7	8
-----	---	---	---	---	---	---	---	---

Algorithm \Rightarrow

Step 1: loop, initialize

for $w \leftarrow 0$ to w
Set $K_p[0, w] \leftarrow 0$

Step 2: loop

for $(i \leftarrow 1 \text{ to } n)$
Set $K_p[i, 0] \leftarrow 0$

Step 3: loop, checking if another part
of solution or not

for $(i \leftarrow 1 \text{ to } n)$

for $w \leftarrow 0$ to w

If $(w_i \leq w)$ then item i can be part of
solution.

If $(v_i + K_p[i-1, w-w_i] > K_p[i-1, w])$ then
Set $K_p[i, w] \leftarrow v_i + K_p[i-1, w-w_i]$

else

Set $K_p[i, w] \leftarrow K_p[i-1, w]$

else

Set $K_p[i, w] \leftarrow K_p[i-1, w] \rightarrow w_i > w$

Step 4: Finished.

(5|3|2|1|4|)

heap size - 6

fun (6 / 2 \rightarrow 1)

fun (3 \rightarrow 1)

i = 3

l = 2i° = 6°

m = 7°

, 3

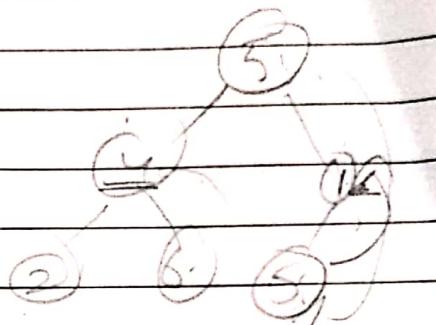
$$\frac{6}{2} + 1 = 2$$

5	4	<u>1</u>	2	6	8
0	1	2	3	4	5

largest = 2

$$l = 2 \times 1^0 + 1 \\ = 5$$

$$l = 2 \times 1 + 2 \\ = 2$$



2 5 < 6 arr[s] > 1

$$(5) > 1 \quad i=5$$

largest = 2 = 5

(7 < 6)

arr[6, 5]

largest = 5

$$l = 2 \times 6 = 12$$