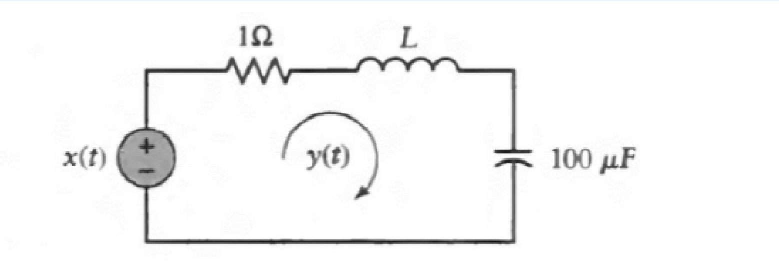
1.Consider a LCR circuit with input x(t) and output y(t).



Write the differential equation description for this system and find the frequency response.

Characterize the system as a filter.

Generate a plot of this filter. Use logarithmically spaced angular frequencies from 1 rad/s to 10^5 rad/s. Use logspace(d1, d2, N) to implement this. Assume the value of L to be

(i) L = 10 mH

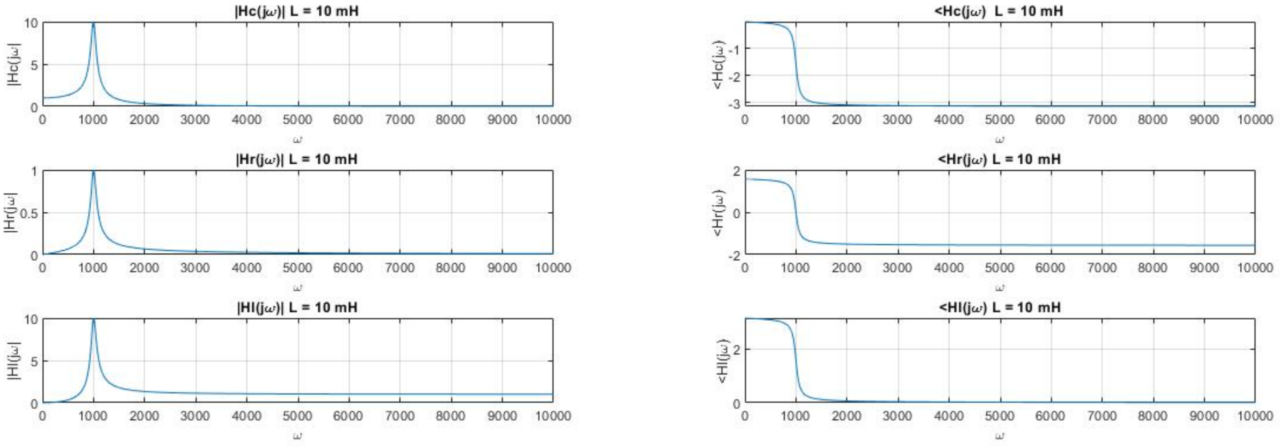
(ii) L = 4 mH

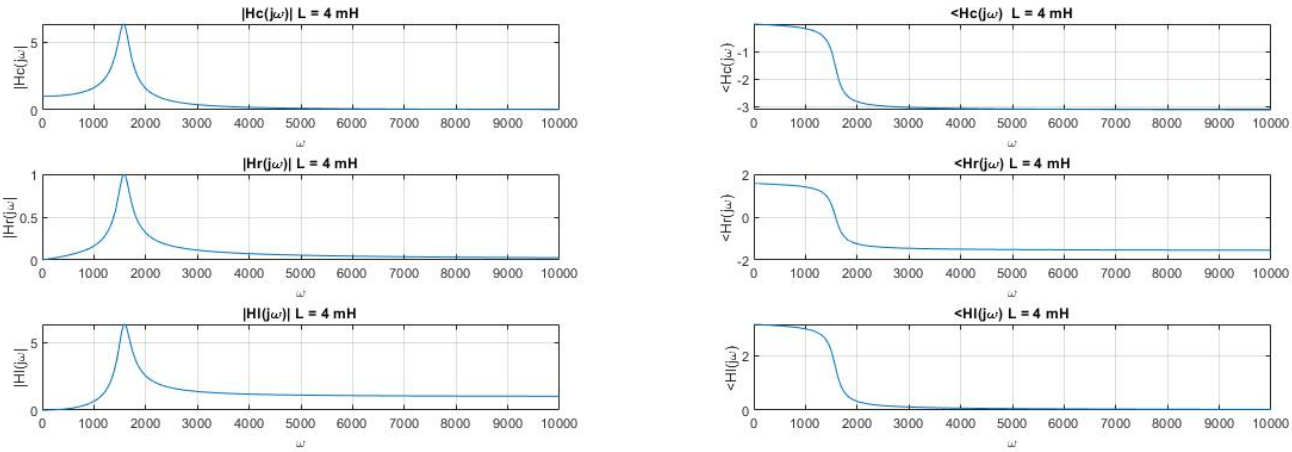
Determine and plot the output, using at least 99 harmonics in truncated FS expansion if the input is a square wave of fundamental period $T = 2\pi \times 10^{-3}$ s having pulse width

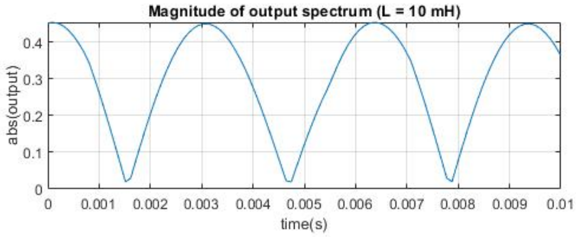
$T0 = (\pi/2) \times 10^{-3}$s.

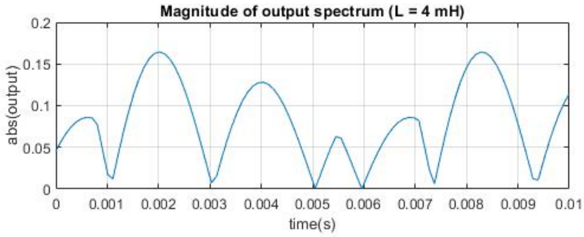
```
%Initial conditions
R=1;
C=100*10^-6;
L=10*10^-3;%Use 4*10^-3 for L=4 mH
w=logspace(0,4,1000);

%Part (a)
%Frequency response
Hc=1./(1+R*C*j*w+L*C*(j*w).^2);
Hr=(R*C*j*w./(1+R*C*j*w+L*C*(j*w).^2));
Hl=((L*C*(j*w).^2)./(1+R*C*j*w+L*C*(j*w).^2));

%Display
figure(1)
subplot(3,1,1)
plot(w,abs(Hc));
grid on
title('|Hc(j\omega)|L=10 mH'); xlabel('\omega'); ylabel('|Hc(j\omega|');

figure(1);
subplot(3,1,2);
plot(w,abs(Hr));
grid on;
title('|Hr(j\omega)|L=10 mH'); xlabel('\omega'); ylabel('|Hr(j\omega|');

figure(1);
subplot(3,1,3);
plot(w,abs(Hl));
 grid on;
 title('|Hl(j\omega)|L=10 mH'); xlabel('\omega');  ylabel('|Hl(j\omega|')  ;

 figure(2);
 subplot(3,1,1);
 plot(w,angle(Hc));
grid on; title('<Hc(j\omega) L=10 mH') xlabel('\omega') ylabel('<Hc(j\omega)');

 figure(2);
 subplot(3,1,2);
 plot(w,angle(Hr));
grid on title('<Hr(j\omega) L=10 mH') xlabel('\omega');
ylabel('<Hr(j\omega)');
figure(2) subplot(3,1,3) plot(w,angle(Hl));
```

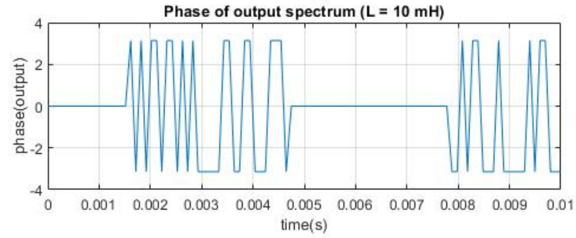
```
grid on;
title('<Hl(j\omega) L=10 mH');
xlabel('\omega');
ylabel('<Hl(j\omega)');

%Part (b);
t=linspace(0,10^-2);
y=zeros(1,length(t));
T=2*pi*10^-3 w_0=2*pi/T;
for k=-99:99 a_k=2*sin(k*pi/4)*(1i*w_0)/(R*w_0*k*1j+1/C+L*(k*j*w_0)^2);
y=y+(1/(2*pi))*a_k*exp(1j*k*w_0*t);
end figure(3);
subplot(2,1,1);
plot(t,abs(y));
title('Magnitude of output spectrum (L=10 mH)');
xlabel('time(s)');ylabel('abs(output)');
grid on;
subplot(2,1,2);
plot(t,angle(y));
title('Phase of output spectrum (L=10 mH)');
xlabel('time(s)');
ylabel('phase(output)');
grid on;
```

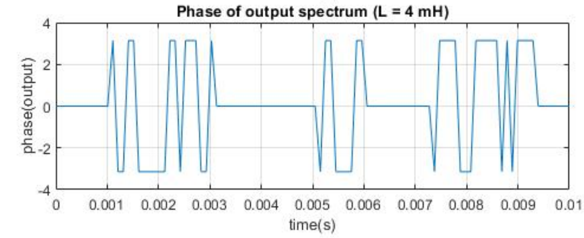












For LCR circuit

$$V_R + V_C + V_L = V_S$$

Now the above equation can be written as

$$V_C(t) + RC\frac{dV_C}{dt} + LC\frac{d^2V_C}{dt} = V_S(t)$$

$$RC\frac{dV_R(t)}{dt} + LC\frac{d^2V_R(t)}{dt} + V_R(t) = \frac{d\,V_S(t)}{dt}(RC)$$

$$V_L(t) + RC\frac{dV_L(t)}{dt} + LC\frac{d^2V_L(t)}{dt} = LC\frac{d^2V_S(t)}{dt}$$

Assuming initial rest and LTI, in order to determine its frequency response $H(j\omega)$ we put $V_S(t) = e^{j\omega t}$.

So, $$H_C(j\omega) = \frac{1}{1 + j\omega(RC) + (j\omega)^2 LC}$$

$$H_R(j\omega) = \frac{RC(j\omega)}{1 + (j\omega)RC + (j\omega)^2 LC}$$

$$H_L(j\omega) = \frac{LC(j\omega)^2}{1 + (j\omega)RC + (j\omega)^2 LC}$$

$$x(t) = \begin{cases} 1 & |t| < \left(\frac{\pi}{2} \times 10^{-3}\right)/2 \\ 0 & \left(\frac{\pi}{2} \times 10^{-3}\right)/2 < |t| < \frac{T}{2} \end{cases}$$

where $T = 2\pi \times 10^{-3}$

$\& \; x(t+T) = x(T)$

$$x(t) \xleftrightarrow{\mathcal{F}} \sum_{k=-\infty}^{\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$$

$$T_1 = \left(\frac{\pi}{2} \times 10^{-3}\right)/2, \; \omega_0$$

Frequency response of current is $\frac{H_R(j\omega)}{R}$

Output has frequency response.

$$\frac{H_R(j\omega)}{R} X(j\omega) \Rightarrow$$

$$\sum_{k=-\infty}^{\infty} \frac{2 \sin(k\pi/4)}{k} \frac{Cj\omega}{1 + j\omega RC - \omega^2 LC} \delta(\omega - k\omega_0)$$

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} \frac{2 \sin k(\pi/4)}{k^2} \frac{C(j\omega)}{(1 + j\omega RC - \omega^2 LC)} \delta(\omega - k\omega_0) \right] e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \left[\sum_{k=-\infty}^{\infty} \frac{2 \sin(k\pi/4) \; j\omega_0}{[kR\omega_0 j + 1 - \omega_0^2 LK^2]} e^{j\omega_0 kt} \right]$$

For truncated version cho choose k suitably

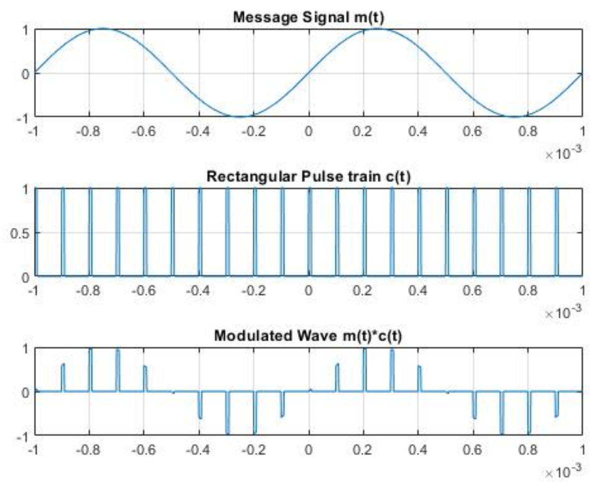
2. Natural sampling involves the multiplication of your message signal m (t) with rectangular pulse c (t) . The pulse repetition frequency of the train is ω_s, the duration of each rectangular pulse is T_0 . The fundamental period of pulse train is T .

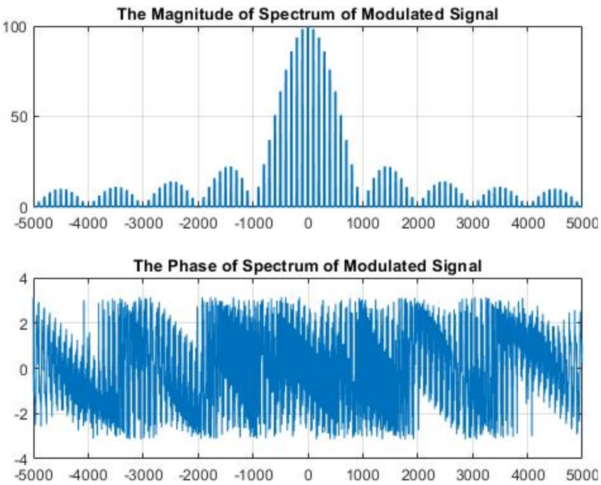
(i) Name the type of modulation performed in this problem .
(ii) Generate and display the modulated wave for a sinusoidal wave, given the following specification :

Modulation frequency : 1 kHz
Pulse repetition frequency (1/T) = 10 kHz
Pulse duration T0 = 10 μs

(iii) Compute and display the spectrum of the modulated wave . Specify the requirements for which you can recover the original signal without distortion using a low pass filter .

```
%Plotting rectangular pulse
t=-0.5:1/100:0.5;
u1=[zeros(1,250),ones(1,751)];
u2=[zeros(1,751),ones(1,250)];
u=u1-u2;

%Displaying rectangular pulse train
Ts=1e-6;
t=-0.001:Ts:0.001-Ts;
modf=1000;%Modulation frequency
m=sin(modf*2*pi*t);%Message Signal
fT=1e4;%Pulse Repetition Frequency
periodT=1/fT;%Pulse Time Period
pulseDur=1e-5;%Pulse Duration
sampPerPulse=floor(pulseDur/Ts);
sampPerT=floor(periodT/Ts);
num=floor(length(m)/sampPerT);
r=[];
for i=1:num
r=[r ones(1,sampPerPulse) zeros(1,sampPerT-sampPerPulse)];
end

figure (1);
subplot(3,1,1)
plot(t,m)
grid on
title("Message Signal m(t)")

figure (1);
subplot(3,1,2);
plot(t,r)
grid on
title("Rectangular Pulse train c(t)")

figure (1)
subplot(3,1,3);
plot(t,r.*m)
grid on
title("Modulated Wave m(t)*c(t)")

n=length(r.*m);
MS=fftshift(fft(r.*m));
figure(2);
subplot(2,1,1);
plot((-n/2:n/2-1)*fT/n,abs(MS));
title("The Magnitude of Spectrum of Modulated Signal")
grid on
figure(2);
subplot(2,1,2);
plot((-n/2:n/2-1)*fT/n,angle(MS));
title("The Phase of Spectrum of Modulated Signal")
grid on
```

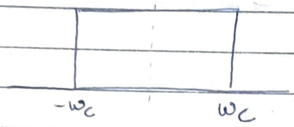




a) Pulse Amplitude Modulation is being performed in the problem .

Let $2\omega_m$ be the bandwidth of spectrum of $m(t)$.

$H(\omega)$ is the ideal low pass filter



$\Rightarrow \omega_M < \omega_c < \omega_s - \omega_M$ & $\omega_s > 2\omega_m$

where ω_s is the frequency of ~~C(ω)~~ $c(t)$.

$c(t) \overset{\mathcal{F}}{\longleftrightarrow} C(\omega)$.

$$C(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \,\delta(\omega - k\omega_s).$$

3. Consider a signal x (t) = sin (40 π t) + 0.5 cos (120 π t) is sampled at a frequency xxxx Hz, where xxxx is last 4 digits of your roll no . Give the spectrum of the sampled signal . From the samples reconstruct a continuous - time signal using the following filters :

(a) Zero - order hold filter

(b) First - order hold filter

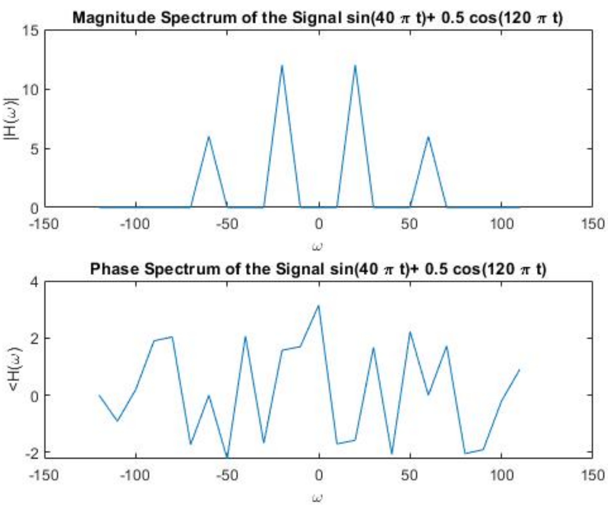
(c) Ideal low - pass filter (sinc)

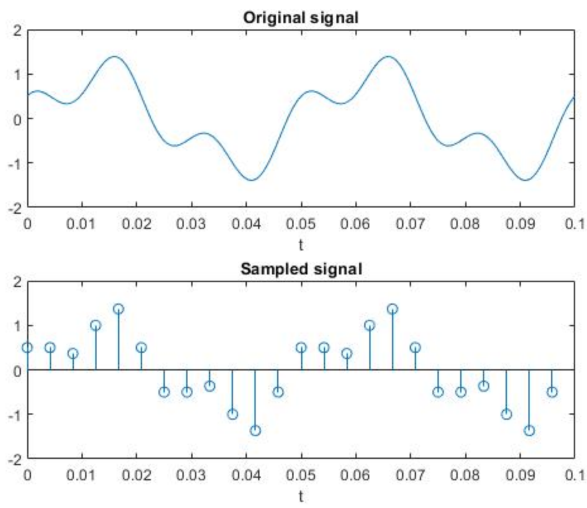
Explain your output if you multiply your sampling frequency by 2.

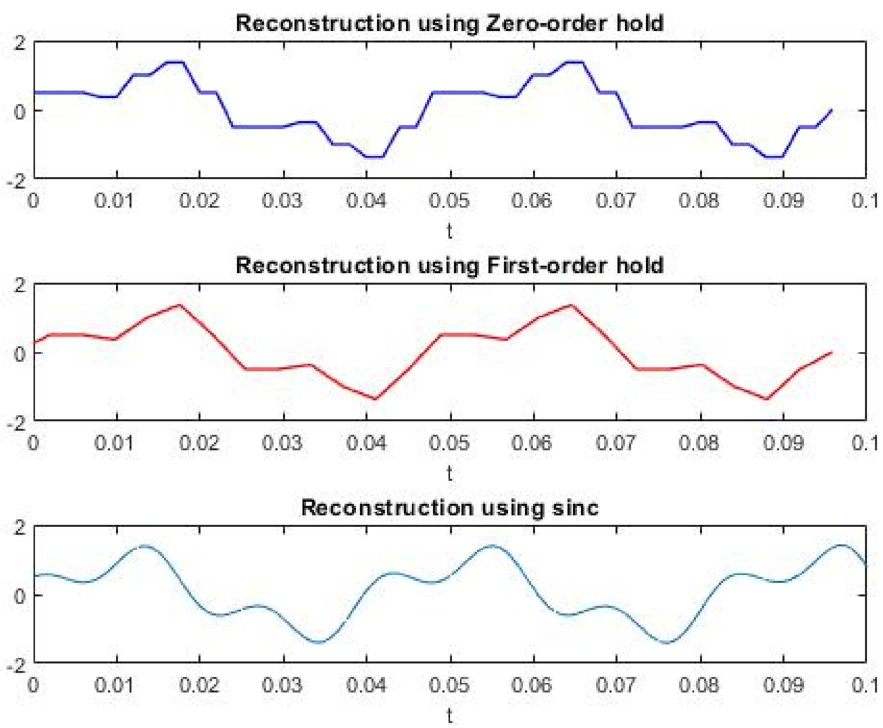
```
fs=240;%Sampling rate
Ts=1/(fs);%Sampling period
td=0:Ts:0.1-Ts;
tc=0:1e-4:0.1-1e-4;

%Signal
x=sin(2*pi*20*td)+0.5*cos(2*pi*60*td);

%Spectrum
S3_shift=fftshift(fft(x));
n=length(x);
fshift=(-n/2:n/2-1)*(fs/n);

%Plot
figure(1);
```

```
subplot(2,1,1)
plot(fshift,abs(S3_shift));
title("Magnitude Spectrum of the Signal sin(40 \pi t)+ 0.5 cos(120 \pi t)");
xlabel('\omega');
ylabel('|H(\omega)|');

figure(1);
subplot(2,1,2)
plot(fshift,angle(S3_shift));
title("Phase Spectrum of the Signal sin(40 \pi t)+ 0.5 cos(120 \pi t)");
xlabel('\omega');
ylabel('<H(\omega)');

%Plotting original signal
xc=sin(2*pi*20*tc)+0.5*cos(2*pi*60*tc);
figure(2);
subplot(2,1,1);
plot(tc,xc);
title("Original signal");
xlabel("t");


%Plotting sampled signal
figure(2);
subplot(2,1,2);
stem(td,x)
title("Sampled signal")
xlabel("t")

%Reconstruction
xu=[x;zeros(size(x))];
xu=xu(:).';% upsampled version of x

h_zoh=[1 1];
h_foh=[1 2 1]/2;

y_zoh=conv(xu,h_zoh);% zero-order hold
y_foh=conv(xu,h_foh);% delayed first order hold

xr=zeros(size(tc));%initialization
sinc_train=zeros(N,length(tc));%initialization
for n=0:N-1
%unless we define our sinc with a value in zero it will introduce NaN which
%lead to a small error
sinc_train(n+1,:)=sin(pi*(tc-n*Ts)/Ts)./(pi*(tc-n*Ts)/Ts);%sinc train
%sinc_train(n+1,:)=sinc((tc-n*Ts)/Ts);%sinc train
current_sinc=xd(n+1)*sinc_train(n+1,:);%a sinc scaled by the sample value
xr=xr+current_sinc;%generation of the reconstructed signal summing the sinc scaled
end

%Displaying
t_zoh=linspace(0,0.1-Ts,length(y_zoh));
figure(3);
subplot(3,1,1)
plot(t_zoh,y_zoh,'b','linewidth',1)
title("Reconstruction using Zero-order hold");
xlabel("t")

t_foh=linspace(0,0.1-Ts,length(y_foh));
figure(3);
subplot(3,1,2);
plot(t_foh,y_foh,'r','linewidth',1)
title("Reconstruction using First-order hold");
```

```
xlabel("t")

figure(3);
subplot(3,1,3);
tr=linspace(0,0.1,length(xr));
plot(tr,xr);
title("Reconstruction using sinc")
xlabel("t")
hold on
```







By multiplying my sampling frequency by 2. we ~~so~~ get more closer samples and the reconstruction of continuous signal is better for zero & first order hold.