



**Parul University**  
**Faculty of Engineering and Technology**  
**Department of Applied Science & Humanities**  
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**Branch: CSE/ IT**

## **Unit-2: Averages and Progressions**

### **2.1 Average**

$$\text{Average} = \frac{\text{Sum of quantities}}{\text{Number of quantities}}$$

**Example-1:** A man purchased 5 toys at Rs.220 each, 6 toys at Rs. 250 each and 9 toys at Rs. 300 each. Calculate the average cost of 1 toy.

**Solution:** Price of 5 toys =  $220 \times 5 = 1100$

Price of 6 toys =  $250 \times 6 = 1500$

Price of 9 toys =  $300 \times 9 = 2700$

Total number of toys =  $5 + 6 + 9 = 20$

Average Price of 1 toy =  $\frac{1100+1500+2700}{20} = 260$ .

### **Shortcut method**

1. The Average of two or more groups taken together.

(a) If the number of quantities in two groups are  $n_1$  and  $n_2$  and their average is  $x$  and  $y$ , respectively, the combined average (average of all of them put together) is  $\frac{n_1x+n_2y}{n_1+n_2}$ .

(b) If the average of  $n_1$  quantities is  $x$ , and the average of  $n_2$  quantities out of them is  $y$ , the average of the remaining group (rest of the quantities) is  $\frac{n_1x-n_2y}{n_1-n_2}$ .

2. If  $x$  is the average of  $x_1, x_2, \dots, x_n$ , then

(a) The average of  $x_1 + a, x_2 + a, \dots, x_n + a$  is  $x + a$ .

(b) The average of  $x_1 - a, x_2 - a, \dots, x_n - a$  is  $x - a$ .

(c) The average of  $ax_1, ax_2, \dots, ax_n$  is  $ax$ , provided  $a \neq 0$ .

(d) The average of  $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \dots, \frac{x_n}{a}$  is  $\frac{x}{a}$  provided  $a \neq 0$ .

**Example-2:** The average weight of 24 students of section A of a class is 58 Kg, whereas the average weight of 26 students of section B of the same class is 60.5 Kg. Find out average weight of all the 50 students of the class.

**Solution:** Here,  $n_1 = 24, n_2 = 26, x = 58$  and  $y = 60.5$

$$\therefore \text{The average weight of all 50 students} = \frac{n_1x + n_2y}{n_1 + n_2}$$

$$= \frac{24 \times 58 + 26 \times 60.5}{24 + 26}$$

$$= \frac{1392 + 1573}{50}$$

$$= \frac{2965}{50} = 59.3 \text{ Kg}$$

**Example-3:** Average salary of all the 50 employees including 5 officers is Rs. 850. If the average salary of the officers is Rs. 2500, find the average salary of the remaining staff of the company.

**Solution:** Here,  $n_1 = 50, n_2 = 5, x = 850$  and  $y = 2500$

$$\therefore \text{The average salary of the remaining staff} = \frac{n_1x - n_2y}{n_1 - n_2}$$

$$= \frac{50 \times 850 - 5 \times 2500}{50 - 5}$$

$$= \frac{42500 - 12500}{45}$$

$$= \frac{30000}{45} = 667 \text{ Kg}$$

**Example-4:** The Average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

**Solution:** The new average =  $19+8=27$

**Example-5:** The average of  $x$  numbers is  $5x$ . If each number is multiplied by 8, find the average of a new set of numbers.

**Solution:** The average of a new set of numbers =  $ax = 8 \times 21 = 168$

### **Exercise**

1. A grocer has a sale of Rs 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs, 6500 ?

A) 4991

B) 5467

C) 5987

D) 6453

**Answer:** A) 4991

**Explanation:** Total sale for 5 months = Rs.  $(6435 + 6927 + 6855 + 7230 + 6562) = \text{Rs. } 34009$ .

Required sale = Rs.  $[(6500 \times 6) - 34009]$

= Rs.  $(39000 - 34009)$

= Rs. 4991.

2. The average of runs of a cricket player of 10 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4 ?

A) 76

B) 79

C) 85

D) 87

**Answer:** A) 76

**Explanation:** Average = total runs / no.of innings = 32

So, total = Average x no.of innings =  $32 \times 10 = 320$ .

Now increase in avg = 4runs. So, new avg =  $32+4 = 36$ runs

Total runs = new avg x new no. of innings =  $36 \times 11 = 396$

Runs made in the 11th inning =  $396 - 320 = 76$

3. A pupil's marks were wrongly entered as 83 instead of 63. Due to that the average marks for the class got increased by half. The number of pupils in the class is :

A) 45

B) 40

C) 39

D) 37

**Answer:** B) 40

**Explanation:** Let there be  $x$  pupils in the class.

Total increase in marks =  $(x * 1/2) = x/2$ .

$x/2 = (83 - 63) \Rightarrow x/2 = 20 \Rightarrow x = 40$ .

4. The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person ?

A) 70 kg

B) 75 kg

C) 80 kg

D) 85 kg

**Answer:** D) 85 kg

**Explanation:** Total weight increased =  $(8 \times 2.5) \text{ kg} = 20 \text{ kg}$ .

Weight of new person =  $(65 + 20) \text{ kg} = 85 \text{ kg}$ .

5. The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers :

A) 4

B) 8

C) 12

D) 16

**Answer:** B) 8

**Explanation:** Let the numbers be  $x, x + 2, x + 4, x + 6$  and  $x + 8$ .

Then  $[x + (x + 2) + (x + 4) + (x + 6) + (x + 8)] / 5 = 61$ .

or  $5x + 20 = 305$  or  $x = 57$ . So, required difference =  $(57 + 8) - 57 = 8$

**Sequence:** Succession of numbers arranged in a definite order forming a definite pattern is known as sequence.

**Series:** If  $a_1, a_2, a_3, a_4, \dots, a_n, \dots$  is a sequence, then the expression

$a_1 + a_2 + a_3 + a_4 + \dots + a_n + \dots$  is a series.

A series is finite or infinite according to as the number of terms in the corresponding sequence is finite or infinite.

**Progressions:** Those sequences whose terms follow certain patterns are called progressions. Arithmetic Progression (A.P.): A sequence is called an Arithmetic Progression if the difference between two consecutive terms is always same.

i.e.,  $a_{n+1} - a_n = \text{constant} = d$  for all  $n \in N$

The constant difference, generally denoted by 'd' is called the common difference.

$a_n$  is called the  $n^{\text{th}}$  or last term of an A.P.

$$a_n = l = a + (n - 1)d$$

- (i) Three consecutive terms in an A.P are taken as  $a - d, a, a + d$ .
- (ii) Four consecutive terms in an A.P taken as  $a - 3d, a - d, a + d, a + 3d$ .

Note : If each term of an A.P. is (increased/ decreased) by K then A.M. is also (increased/ decreased) by K.

If each term of an A.P. is (multiplied/Divided) by K, then A.M is also (multiplied/Divided) by same number K.

**Rule 1.** Let  $a$  be the first term and  $d$  be the common difference of an A.P. Then its  $n^{\text{th}}$  term is  $a + (n - 1)d$  i.e.,  $a_n = a + (n - 1)d$ .

**Rule 2.** The sum  $S_n$  of  $n$  terms of an A.P. with first term is ' $a$ ' and common difference is ' $d$ ' is  $S_n = \frac{n}{2}[2a + (n - 1)d]$  or  $S_n = \frac{n}{2}[a + l]$ , where  $l = \text{last term} = a + (n - 1)d$ .

### Geometric Progression :

A sequence of non-zero numbers is called a Geometric Progression (abbreviated as G.P.) if the ratio of a term and the term preceding to it is always same. The constant ratio is called the common ratio ( $r$ ) of the G.P.

In other words, a sequence  $a_1, a_2, a_3, \dots, a_n$  is called a Geometric Progression if

$$\frac{a_{n+1}}{a_n} = \text{Constant} \forall n \in N$$

Three numbers in G.P is taken as  $a, ar, ar^2$  or  $\frac{a}{r}, a, ar$

Geometric Series : If  $a_1, a_2, a_3, a_4, \dots, a_n, \dots$  is a G.P., then the expression

$a_1 + a_2 + a_3 + a_4 + \dots + a_n + \dots$  is called a geometric series.

Rule 3. The  $n^{\text{th}}$  term of a G.P. with first term  $a$  and common ratio  $r$  is given by

$$a_n = ar^{n-1}.$$

Rule 4. The sum of  $n$  terms of a G.P. with first term ' $a$ ' and common ratio ' $r$ ' is given by

$$S_n = a \left( \frac{1 - r^n}{1 - r} \right) \text{ for } r < 1$$

$$S_n = a \left( \frac{r^n - 1}{r - 1} \right) \text{ for } r > 1$$

In fact these two are exactly identical. The only thing which must be noted is that the above formulas do not hold for  $r = 1$ , the sum of  $n$  terms of the G.P. is  $S_n = na$ . where  $r = 1$ .

Rule 5. The sum of an infinite G.P. with 1st term is ' $a$ ' and common ratio is  $r$  ( $-1 < r < 1$  i.e.,  $|r| < 1$ ) is given by

$$S_{\infty} = \frac{a}{1 - r}$$

Rule 6. Three non-zero numbers  $a, b, c$  are in G.P. if  $b^2 = ac$  or  $b = \sqrt{ac}$ .

Here,  $b$  is known as the Geometric Mean of  $a$  and  $c$ .

Harmonic Progression: If  $a, b, c, d$ , are in H.P. then,  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$  will form an A.P. and then we can apply all rules of A.P.

$$n^{\text{th}} \text{ term of HP} = \frac{1}{n^{\text{th}} \text{ term of AP}}$$

Note: No term of HP can be zero.

Some Special Sequences:

- 1) The sum of first  $n$  natural numbers,

$$\sum n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

2) The sum of squares of first n natural numbers

$$\sum n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3) The sum of cubes of first n natural numbers

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

Exercise:

1. Show that the series 1,3,5,7,9, ...in Arithmetic progression
2. Find the  $n^{\text{th}}$  term and  $19^{\text{th}}$  term of the sequence 5,2, -1, -4, ...
3. Find the sum of the series .5 + .51 + .52 + ... to 100 terms.
4. Find the sum of 20 terms of an A.P. whose first term is 3 and last term is 57.
5. Find the sum of 8 terms and n terms of the sequence 9, -3, 1,  $-\frac{1}{3}$ , ...
6. Find the 100th of the sequence  $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$

### Exercise

1. Determine 25th term of an A.P. whose 9th term is -6 and common difference is  $\frac{5}{4}$ .

- (a) 16                      (b) 18                      (c) 12                      (d) 14

**Answer:** (d) 14

**Explanation:**  $d = \frac{5}{4}, a_9 = -6$

$$a_n = a + (n-1)d \Rightarrow a_9 = a + 8d \Rightarrow -6 = a + 8\left(\frac{5}{4}\right)$$

$$\Rightarrow a = -16$$

$$\text{The } 25^{\text{th}} \text{ term } a_{25} = a + 24d$$

$$\Rightarrow a_{25} = -16 + 24\left(\frac{5}{4}\right) = 14$$

2. Which term of the A.P. 5, 13, 21, ... is 181?

- (a)  $21^{\text{st}}$                       (b)  $22^{\text{nd}}$                       (c)  $23^{\text{rd}}$                       (d)  $24^{\text{th}}$

**Answer:** (c)  $23^{\text{rd}}$

**Explanation:** Here, first term  $a = 5$

Common difference  $d = 8$

Let, 181 be the  $n^{\text{th}}$ , i.e.,  $a_n = 181$

$$\therefore 181 = 5 + (n-1)8 \text{ or, } 176 = (n-1)8$$

$$\therefore n-1 = 176 \div 8 = 22$$

$$\therefore n = 23$$

Hence, 181 is the  $23^{\text{rd}}$  term.

3. Determine  $k$  so that  $2/3$ ,  $k$  and  $5k/8$  are the three consecutive terms of an A.P.

- (a)  $16/33$       (b)  $14/33$       (c)  $12/33$       (d)  $18/33$

**Answer:** (a)  $16/33$

**Explanation:**  $\frac{2}{3}, k, \frac{5}{8}k$  be 3 terms of A.P.

Common difference between two consecutive terms is equal.

$$k - \frac{2}{3} = \frac{5}{8}k - k \Rightarrow 2k - \frac{5}{8}k = \frac{2}{3} \Rightarrow k = \frac{16}{33}$$

4. If the 12th term of an A.P. is -13 and the sum of the first four terms is 24, then what is the sum of the first 10 terms?

- (a) 0      (b) 2      (c) 1      (d) 4

**Answer:** (a) 0

**Explanation:**  $a_n = a + (n - 1)d \Rightarrow a_{12} = a + 11d$

$$\Rightarrow -13 = a + 11d \text{ ---(1)}$$

$$S_n = \frac{n}{2} [2a + (n - 1)d] \Rightarrow S_4 = \frac{4}{2} [2a + 3d]$$

$$\Rightarrow 12 = 2a + 3d \text{ ---(2)}$$

Solving \_\_\_(1) and \_\_\_(2)

$$a = 9, d = -2$$

$$S_{10} = \frac{10}{2} [2a + 9d] = 0$$

5. How many terms are there in an A.P. whose first and fifth terms are -14 and 2, respectively and the sum of terms is 40?

- (a) 15      (b) 5      (c) 10      (d) 20

**Answer:** (c)

**Explanation:** Here,  $a = -14$

Let,  $d$  be the common difference

$$a_5 = 2 \Rightarrow a + 4d = 2 \Rightarrow -14 + 4d = 2$$

$$\therefore d = 4$$

Let 40 be the sum of  $n$  terms of this A.P.

$$S_n = \frac{n}{2} [2a + (n - 1)d] \Rightarrow 40 = \frac{n}{2} [2(-14) + (n - 1)4]$$

$$\Rightarrow (n + 2)(n - 10) = 0$$

$$\therefore n = 10 \text{ or, } -2. \text{ But } n \neq -2.$$

Hence, the required number of terms are 10.

6. Find the seventh term of the series  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

- (a)  $\frac{1}{120}$       (b)  $\frac{1}{128}$       (c)  $\frac{1}{144}$       (d)  $\frac{1}{121}$

**Answer:** (b)



7. How many terms are there in the G.P. 1, 2, 4, 8,..., 4096?

- (a) 14                      (b) 13                      (c) 12                      (d) 15

**Answer: (b)**

8. Find the sum of the series  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \infty$ .

- (a) 1                      (b) 12                      (c) 15                      (d) 13

**Answer: (a)**

9. What term of progression 18, -12, 8,... is  $\frac{512}{729}$ ?

- (a) 15                      (b) 18                      (c) 9                      (d) 12

**Answer: (c)**

10. The 3<sup>rd</sup> term of G.P. is the square of the first term. If the 2<sup>nd</sup> term is 8, determine the 6<sup>th</sup> term.

- (a) 136                      (b) 132                      (c) 128                      (d) 124

**Answer: (c)**