



Unit 6: Time & Work, Pipes and Cisterns, Time Speed and Distance, Problems on train crossings, Boats & streams

Time & Work:

Time and work problems develop when a given quantity of work must be completed in a specific length of time. There are different types of problems.

Group efficiency problems

Individual efficiency problems with people

These sorts of problems can be addressed utilising two approaches:

Fraction method or Unitary work Method

LCM Method

Points to Remember:

1) If a man can do a piece of work in n days, work done by him in one day = $\frac{1}{n}$ part of total work or he will finish $\frac{1}{n}$ th work in one day.

2) If a man completes $\frac{1}{n}$ th work in one day, he will complete the entire work in n days.

3) If A can complete a piece of work in X days and B can complete the same work in Y days,

Both A and B working together can finish the same work in $\frac{XY}{X+Y}$ days.

4) If A, B, C can do a piece of work in X , Y , and Z days respectively, if they work together they can do the same work in $\frac{XYZ}{XY+YZ+ZX}$ days.

5) A and B working together can finish a piece of work in X days, B and C working together can finish the same work in Y days and C and A in Z days. Then;

A, B and C working together will finish the work in $\frac{2XYZ}{XY+YZ+ZX}$ days.

A alone will finish the work in = $\frac{2XYZ}{XY+YZ-ZX}$ days.

B alone will finish the work in = $\frac{2XYZ}{YZ+ZX-XY}$ days.

C alone will finish the work in = $\frac{2XYZ}{ZX+XY-YZ}$ days.

6) If A is thrice as good as workman as B or A can work three times faster than B, the ratio of work done by A and B for the same duration of time will be = 3 : 1. And the ratio of time taken by A and B to finish the same amount of work will be = 1: 3.

7) A and B working together can do a work in X days. If A alone can do the same work in Y days, B alone can do the same work in $\frac{XY}{Y-X}$ days.

8) A can do a work in X days. If B is P times efficient than A, A and B working together can do the work in $\frac{X}{1+P}$ days.

9) There are two groups of workers with same efficiency. In one group M1 workers can do W1 work in D1 days or time. In the second group M2 workers can do W2 work in D2 days or time. Then;
 $M1 D1 W2 = M2 D2 W1$

10) There are two groups of workers with the same efficiency. In one group M1 workers can do W1 work in D1 time or days working T1 hours a day. In the second group, M2 workers can do W2 work in D2 time or days working T2 hours in a day. Then;
 $M1 D1 T1 W2 = M2 D2 T2 W1$

11) Wages are directly proportional to the work done by the individual and inversely proportional to the time taken by the individual.

Total wages = One person's one day's wage x Number of persons x Number of days.

A's share: B's share: C's share = (B's time x C's time) : (A's time x C's time) : (A's time x B's time).

Exercise:

1) Worker A completes a task in 8 days, and worker B completes the same task in 10 days. If both A and B work together, in how many days they will complete the task?

a) $\frac{41}{9}$ Days b) $\frac{40}{9}$ Days c) $\frac{40}{8}$ Days d) $\frac{40}{7}$ Days

Worker A completes the task in 8 days. So, in one day, he will complete $\frac{1}{8}$ part of the task.

So, A's one day work = $\frac{1}{8}$

Similarly, B's one day work = $\frac{1}{10}$

\therefore (A+B)'s one day work = $\frac{1}{8} + \frac{1}{10} = \frac{9}{40}$.

$\frac{9}{40}$ Of the task is completed in one day. So both will complete the whole task in $\frac{40}{9}$ Days.

OR:

Both will complete the whole task in $\frac{8 \times 10}{8 + 10} = \frac{40}{9}$ Days.

2) Vikas and Mohan working together can complete a work in 6 days. If Vikas alone completes the same work in 10 days, in how many days Mohan alone can complete the same work?

A) 13 days B) 14 days C) 15 days D) 16 days

Ans : C)

Vikas and Mohan together can complete the task in 6 days. So, in one day, they will complete $\frac{1}{6}$ part of the task.

Therefore, (Vikas + Mohan)'s one day work will be $= \frac{1}{6}$

Similarly, Vikas's one day work $= \frac{1}{10}$

Therefore, Mohan's one day work $= \frac{1}{6} - \frac{1}{10} = \frac{1}{15}$.

In one day Mohan completes the $\frac{1}{15}$ part of the work so he will complete the entire work in 15 days.

3) A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then the fraction of the work that is left is:

A) $\frac{3}{15}$ B) $\frac{5}{15}$ C) $\frac{7}{15}$ D) $\frac{8}{15}$

Ans: D)

A's 1 Days's Work $= \frac{1}{15}$

B's 1 Days's Work $= \frac{1}{20}$

(A + B)'s 1 Day's Work $= \left(\frac{1}{15} + \frac{1}{20} \right) = \frac{7}{60}$

(A + B)'s 4 Day's Work $= \left(\frac{7}{60} \times 4 \right) = \frac{7}{15}$

So Remaining Work $= 1 - \frac{7}{15} = \frac{8}{15}$.

4) A can do a work in 10 days and B can do the same work in 15 days. If they start working together but stop working after four days, find the fraction of the work that is left.

Ans : $\frac{1}{3}$

5) A can do a job in 12 days and B can do the same job in 10 days. With the help of C they can do the same job in 4 days. In how many days C alone can do this job?

A) 15 days B) 14 days C) 13 days D) 12 days

Ans: A)

$$\text{A's one day work} = \frac{1}{12}$$

$$\text{B's one day work} = \frac{1}{10}$$

$$\text{(A+B+C)'s one day work} = \frac{1}{4}$$

Therefore, C's one day work = [(A+B+C)'s one day work] – [(A+B)'s one day work]

$$\text{So, C's one day work} = \left[\left(\frac{1}{4} \right) - \left(\frac{1}{12} + \frac{1}{10} \right) \right] = \frac{1}{15}$$

So, C will complete the work in 15 days.

6) A, B, C can do a job in 10, 20 and 40 days respectively. In how many days A can complete the job if he is assisted by B and C on every third day?

A) 8 Days B) 7 Days C) 6 Days D) 9 Days

Ans: A)

$$\text{A's one day work} = \frac{1}{10}$$

$$\text{B's one day work} = \frac{1}{20}$$

$$\text{C's one day work} = \frac{1}{40}$$

(A+B+C)'s one day work =

$$\frac{1}{10} + \frac{1}{20} + \frac{1}{40} = \frac{7}{40}$$

Work done in three days will be the sum of A's two-day work and (A+B+C)'s one day work.

$$\text{A's two-day work} = \frac{1}{10} \times 2 = \frac{1}{5}$$

Therefore, the work is done in three days =

$$\frac{1}{5} + \frac{7}{40} = \frac{75}{200} = \frac{3}{8}$$

3/8 part of the job is done in 3 days.

$$\text{The entire job will be done by A in} = 3 \times \frac{8}{3} = 8 \text{ Days}$$

7) If 5 men can colour 50-meter long cloth in 5 days, in many days 4 men can color a 40-meter long cloth?

A) 3 Days B) 4 Days C) 5 Days D) 6 Days

Ans: C)

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$5 \times 5 \times 40 = 4 \times D_2 \times 50$$

$$D_2 = \frac{1000}{200} = 5 \text{ Days}$$

8) If 4 men can finish 4 times of a work in 4 days, in how many days 6 men can finish the 6 times of same work ?

a) 3 days

- b) 4 days
- c) 5 days
- d) 6 days

Ans: B)

$$M_1D_1W_2 = M_2D_2W_1$$

Let the work be X

Work done by 4 men, $W_1 = 4X$

Work done by 6 men, $W_2 = 6X$

$$4 \times 4 \times 6X = 6 \times D_2 \times 4X$$

$$96X = D_2 \times 24X$$

$$D_2 = 4 \text{ days}$$

9) A can do a piece of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it?

- A) 8 Hrs B) 10 Hrs C) 12 Hrs D) 24 Hr

Ans : 12 Hrs

$$A's \text{ 1 Hour Work} = \frac{1}{4}; (B + C)'s \text{ 1 Hour's Work} = \frac{1}{3}; (A + C)'s \text{ 1 Hour's Work} = \frac{1}{2}$$

$$(A + B + C)'s \text{ 1 Hour's Work} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

$$\begin{aligned} \text{So B's 1 Hour's Work} &= (A + B + C)'s \text{ 1 Hour's Work} - (A + C)'s \text{ 1 Hour's Work} \\ &= \frac{7}{12} - \frac{1}{2} = \frac{1}{12} \end{aligned}$$

So B Alone will take 12 Hrs to do the work.

10) A alone can do a piece of work in 6 days and B alone in 8 days. A and B undertook to do it for Rs. 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?

- A) Rs. 375 B) Rs. 400 C) Rs. 600 D) Rs. 800

Ans: B)

$$\begin{aligned} C's \text{ 1 Day's Work} &= (A + B + C)'s \text{ 1 Day's Work} - (A + B)'s \text{ 1 Day's Work} \\ &= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8} \right) = \frac{1}{24} \end{aligned}$$

$$A's \text{ Wages} : B's \text{ Wages} : C's \text{ Wages} = \frac{1}{16} : \frac{1}{8} : \frac{1}{24}$$

$$C's \text{ Share (For 3 Days)} = 3 \times \frac{1}{24} \times 3200 = 800 \text{ ₹}$$

11) If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be:

- A) 4 Days B) 5 Days C) 6 Days D) 7 Days

Ans: A)

Let 1 Man 1 Day's Work = x, 1 Boy 1 Day's Work = y

$$\text{Then } 6x + 8y = \frac{1}{10} \text{ \& } 26x + 48y = \frac{1}{2}$$

$$\text{Solving these two equations } x = \frac{1}{100} \text{ \& } y = \frac{1}{200}$$

$$\text{So (15 Men + 20 Boy)'s 1 Day's Work} = \frac{15}{100} + \frac{20}{200} = \frac{1}{4}$$

\therefore 15 Men and 20 Boys Can do the work in 4 Days.

12) A can do a job in 30 days. B alone can do the same job in 20 days. If A starts the work and joined by B after 10 days, in how many days the job will be done?

A) 15 days B) 16 days C) 17 days D) 18 days

Ans: D

$$\text{A's one day work} = \frac{1}{30}$$

$$\text{A's ten-day work} = 10 \times \frac{1}{30} = \frac{1}{3}$$

$$\text{So the remaining work would be } = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{B's one day work} = \frac{1}{20}$$

$$\text{A and B's one day work} = \frac{1}{30} + \frac{1}{20} = \frac{1}{12}$$

$\frac{1}{12}$ of the job will be done by them in one day.

So, the remaining job $\frac{2}{3}$ will be done in $\frac{2}{3} \times 12 = 8$ days

Therefore, the total number of days required to do the job would be $= 10 + 8 = 18$ days.

13) Time taken by A to finish a piece of work is twice the time taken B and thrice the time taken by C. If all three of them work together, it takes them 2 days to complete the entire work. How much work was done by B alone?

A) 2 days B) 6 days C) 3 days D) 5 days

Ans: B)

Time taken by A = x days, Time taken by B = x/2 days, Time Taken by C = x/3 days

$$\Rightarrow \left\{ \frac{1}{x} + \frac{2}{x} + \frac{3}{x} \right\} = \frac{1}{2}$$

$$\Rightarrow \frac{6}{x} = \frac{1}{2}$$

$$\Rightarrow x = 12$$

Time taken by B = $x/2 = 12/2 = 6$ days.

14) Dev completed the school project in 20 days. How many days will Arun take to complete the same work if he is 25% more efficient than Dev?

A) 5 Days B) 10 Days C) 12 Days D) 16 Days

Ans : D)

Let the days taken by Arun to complete the work be x.

The ratio of time taken by Arun and Dev = 125:100 = 5:4.

So 5:4 :: 20:x

$$\Rightarrow x = \{(4 \times 20) / 5\}$$

$$\Rightarrow x = 16$$

15) A can do a work in 12 days. B is 60% more efficient than A. In how many days will B complete the work?

Ans: $7\frac{1}{2}$ Days

16) P is thrice as good a workman as Q and, therefore, is able to finish a job in 60 days less than Q. Working together, they can do it in:

Let time taken by P = x days

Then, time taken by Q = 3x days

$$\therefore 3x - x = 60$$

$$\Rightarrow 2x = 60$$

$$\Rightarrow x = 30$$

$$\therefore (P+Q)'s \text{ 1 day's work} = (1/30) + (1/90) = (3+1)/90 = 4/90$$

$$\therefore \text{required time} = 22\frac{1}{2} \text{ Days.}$$

17) 25 men can do a work in 5 days. Find the % amount of work done by 5 men in 10 days.

Answer:

The total work done by 25 men in 5 days = $25 * 5 = 125$ units.

The work done by 5 men in 10 days = 50 units

The percentage of the work done = $50 / 125 * 100 = 40\%$

18) 60 men can do a work in 50 days. Find the % amount of work done by 30 men in 10 days.

Answer: 10%

19) Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio between the capacity of a man and a woman?

A) 4:3 B) 5:3 C) None of these D) 3:4

Ans: A)

20 women can complete the work in **16** days

So, **1** women can complete the work = **(20×16)** days = **320** days

1 women's one day work = **1/320**

16 men can complete the work in **15** days

1 man can complete the work = **15×16= 240** days

1 man's one day work = **1/240**

Ratio of capacity of man and woman,

$$= (1/320) : (1/240)$$

$$= 4:3$$

20) 10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?

A) 3 Days B) 5 Days C) 7 Days D) 9 Days

Ans: C)

Pipes and Cisterns:

A pipe is connected to a tank or cistern. It is used to fill or empty the tank; accordingly, it is called an inlet or an outlet.

Inlet: A pipe which is connected to fill a tank is known as an inlet.

Outlet: A pipe which is connected to empty a tank is known as an outlet.

Problems on pipes and cisterns are similar to problems on time and work. In pipes and cistern problems, the amount of work done is the part of the tank of filled or emptied. And, the time taken to do a piece of work is the time take to fill or empty a tank completely or to a desired level.

Points to remember:

1) If an inlet connected to a tank fills it in X hours, part of the tank filled in one hour is = $1/X$

2) If an outlet connected to a tank empties it in Y hours, part of the tank emptied in one hour is = $1/Y$

3) An inlet can fill a tank in X hours and an outlet can empty the same tank in Y hours. If both the pipes are opened at the same time and $Y > X$, the net part of the tank filled in one hour is given by;

$$= \left(\frac{1}{X} - \frac{1}{Y} \right)$$

If X is greater than Y, more water is flowing out of the tank than flowing into the tank. And, the net part of the tank emptied in one hour is given by;

$$= \left(\frac{1}{Y} - \frac{1}{X} \right)$$

4) An inlet can fill a tank in X hours and another inlet can fill the same tank in Y hours. If both the inlets are opened at the same time, the net part of the tank filled in one hour is given by;

$$= \left(\frac{1}{X} + \frac{1}{Y} \right)$$

In a similar way, If an outlet can empty a tank in X hours and another outlet can empty the same tank in Y hours, the part of the tank emptied in one hour when both the pipes start working together is given by;

$$= \left(\frac{1}{X} + \frac{1}{Y} \right)$$

5) Three inlets A, B, and C can fill a tank in X, Y and Z hours respectively. If all the inlets are opened together, the time taken to fill the tank is given by;

$$= \left(\frac{X+Y+Z}{XY+YZ+ZX} \right) \text{ hours}$$

6) Two pipes can fill a tank in X and Y hours respectively and an outlet can empty the same tank in Z hours. If all the pipes are opened together, part of the tank filled in one hour is given by;

$$= \frac{1}{X} + \frac{1}{Y} - \frac{1}{Z}$$

7) A pipe can fill a tank in X hours but due to a leak in the bottom, it can be filled in Y hours. The time taken by the leak to empty the tank is given by;

$$\frac{XY}{Y-X}$$

8) An inlet A is X times faster than inlet B and takes Y minutes less than the inlet B, time taken to fill a tank when both the pipes are opened together is given by;

$$\frac{XY}{(X-1)^2}$$

Question Bank:

1) A pipe can fill a tank in 6 hours and another pipe can empty the tank in 12 hours. If both the pipes are opened at the same time, the tank can be filled in

- a) 10 hours
- b) 12 hours
- c) 14 hours
- d) 16 hours

Ans: B)

Part of the tank filled in one hour = $\frac{1}{6}$

Part of the tank emptied in one hour = $\frac{1}{12}$

Net part of the tank filled in one hour;

$$= \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$$

$\frac{1}{12}$ Part of the tank can be filled in one hour.

∴ The tank will be filled completely in 12 hours.

OR

Apply formula = $\frac{XY}{Y-X}$

X = 6 hours and Y = 12 hours

$$\therefore \frac{6 \times 12}{12-6} = 12 \text{ hours.}$$

2) A tank can be filled in 10 hours. After a leak in its bottom, it takes 12 hours to fill the tank. Find the time taken by the leak to empty the full tank?

- a) 45 hours
- b) 60 hours

- c) 50 hours
d) 55 hours
Ans: B)

Apply: $\frac{XY}{Y-X}$

3) Three pipes A, B and C can fill a cistern in 8 minutes, 12 minutes and 16 minutes respectively. What is the time taken by three pipes to fill the cistern when they are opened together?

- a) 3.7 minutes
b) 4 minutes
c) 4.5 minutes
d) 5 minutes

Ans: A)

4) Two pipes can fill a tank in 6 hours and 8 hours respectively. A third pipe can empty the same tank in 12 hours. If all the pipes start working together, how long it will take to fill the tank?

- a) 4 hours
b) 4.5 hours
c) 4.8 hours
d) 5.2 hours

Ans: C)

Part of the tank filled by two pipes in one hour = $\frac{1}{6} + \frac{1}{8}$

Part of the tank emptied by the third pipe in one hour = $\frac{1}{12}$

\therefore Net part of the tank filled in one hour = $\frac{1}{6} + \frac{1}{8} - \frac{1}{12}$

= $\frac{4 + 3 - 2}{24} = \frac{5}{24}$

$\frac{5}{24}$ Part of tank can be filled in one hour

\therefore The whole tank will be filled in $\frac{24}{5} = 4.8$ hours

5) Two pipes can fill a tank in 10 and 14 minutes respectively. A third pipe can empty the tank at the rate of 10 liters/minute. If all the pipes working together can fill the empty tank in 8 minutes, what is the capacity of the tank?

- A) 210 liters
B) 215.4 liters
C) 220 liters
D) 225.4 liters

Ans: B)

Let the capacity of the tank is X liters.

Part of the tank filled by two pipes in one minute = $\frac{1}{10} + \frac{1}{14}$

10 liters is emptied in 1 minute

X liters will be emptied in X/10 minutes

In X/10 minutes the whole tank will be emptied.

In one minute 10/X part of the tank will be emptied.

As per question;

$$\frac{1}{10} + \frac{1}{14} - \frac{10}{X} = \frac{1}{8}$$

$$\frac{1}{10} + \frac{1}{14} - \frac{1}{8} = \frac{10}{X}$$

$$\frac{112 + 80 - 140}{1120} = \frac{10}{X}$$

$$\frac{52}{1120} = \frac{10}{X}$$

$$52X = 11200$$

$$X = \frac{11200}{52} = 215.4 \text{ liters}$$

6) A cistern can be filled by an inlet in 6 hours and can be emptied by an outlet in 8 hours. If the inlet and outlet are opened together, in what time the cistern can be filled?

A) 24 hours

B) 26 hours

C) 20 hours

D) 18 hours

Ans: A)

Apply: $\frac{XY}{Y-X}$

7) Pipe A can fill a tank in 12 minutes whereas pipe A along with pipe B can fill the same tank in 8 minutes. In what time pipe B alone can fill the tank?

A) 24 minutes

B) 20 minutes

C) 25 minutes

D) 22 minutes

Ans: A)

Part of the tank filled by pipe A in one minute = $\frac{1}{12}$

Part of the tank filled by A+B in one minute = $\frac{1}{8}$

Part of the tank filled by B alone = $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

∴ Pipe B will fill the whole tank in 24 minutes.

Solution 2:

Apply formula; = $\frac{XY}{Y+X}$

X= 12 minutes

Y=?

As per question;

$$\frac{12*Y}{Y+12} = 8$$

$$12Y = 8Y + 96$$

$$4Y = 96$$

$$Y = 24 \text{ minutes}$$

8) Two pipes working together can fill a fish tank in 12 minutes. If one pipe fills the fish tank 10 minutes faster than the second pipe, in what time the second pipe alone can fill the fish tank?

A) 20 minutes

B) 25 minutes

C) 30 minutes

D) 35 minutes

Ans: C)

Let the first pipe fill the reservoir in X minutes

So, the second pipe will fill the reservoir in (X+10) minutes

As per question;

$$\frac{1}{X} + \frac{1}{X+10} = \frac{1}{12}$$

$$\frac{X+10+X}{X(X+10)} = \frac{1}{12}$$

$$12X + 120 + 12X = X^2 + 10X$$

$$X^2 + 10X - 24X - 120 = 0$$

$$X^2 - 14X - 120 = 0$$

$$X^2 - 20X + 6X - 120 = 0$$

$$X(X-20) + 6(X-20) = 0$$

$$(X+6)(X-20) = 0$$

$$X = 20$$

∴ Second pipe will fill the reservoir in 20 + 10 = 30 minutes.

9) 20 buckets can fill a tank when the capacity of each bucket is 12 liters. If the capacity of each bucket is 10 liters, find the number of buckets required to fill the tank.

A) 30 buckets

B) 34 buckets

C) 24 buckets

D) 27 buckets

Ans: C)

Capacity of each bucket = 12 liters

20 buckets can fill the tank. So, capacity of tank = 20 * 12 = 240 liters

New capacity of bucket = 10 liters

So, 10 liters can be poured into the tank by one bucket

$$240 \text{ liters will be poured by } \frac{1}{10} * 240 = 24 \text{ buckets}$$

10) Pipe A can fill the tank 3 times faster in comparison to pipe B. It takes 36 minutes for pipe A and B to fill the tank together. How much time will pipe B alone take to fill the tank?

A) 100 minutes

- B) 124 minutes
- C) 134 minutes
- D) 144 minutes

Answer: (D) 144 minutes

Let the time taken by pipe B be x minutes.

So, the time taken by pipe A = $x/3$ minutes.

Thus, $1/x + 3/x = 1/36$

$$\Rightarrow 4/x = 1/36$$

$$\Rightarrow x = 4 \times 36$$

$$\therefore x = 144 \text{ minutes.}$$

11) If two pipes can fill a tank in 24 and 20 minutes respectively and another pipe can empty 3 gallons of water per minute from that tank. When all the three pipes are working together, it takes 15 minutes to fill the tank. What is the capacity of the tank?

- A) 100 gallons
- B) 150 gallons
- C) 125 gallons
- D) 120 gallons

Answer: (D) 120 gallons Solution:

$$\text{Work done by the outlet pipe in 1 minute} = \{1/15 - (1/24) + (1/20)\} = 1/15 - 11/120 = - (1/40)$$

Here, the negative sign indicates the negative work done that is the loss of water from the outlet
The capacity of $1/40$ part = 3 gallons

So, Capacity of whole tank = $40 \times 3 = 120$ gallons.

12) It takes two pipes A and B, running together, to fill a tank in 6 minutes. It takes A 5 minutes less than B to fill the tank, then what will be the time taken by B alone to fill the tank?

- A) 10 minutes
- B) 15 minutes
- C) 20 minutes
- D) 25 minutes

Answer: (B) 15 minutes Solution:

Let the time taken by pipe A to fill the tank be x minutes.

Time is taken by pipe B to fill the tank = $x+5$ minutes.

$$\text{So, } 1/x + 1/(x+5) = 1/6$$

$$\Rightarrow x = 10$$

Thus, time taken by B alone to fill the tank is $10+5$, i.e., 15 minutes.

13) It takes 6 hours for three pipes, X, Y and Z to fill a tank. When the three worked together for 2 hours, Z was closed and, X and Y filled the remaining tank in 7 hours. How many hours would it take Z alone to fill the tank?

- A) 15 hours
- B) 23 hours
- C) 12 hours
- D) 14 hours

Answer: (4) 14 hours Solution:

Part of the tank which was filled in 2 hours = $\frac{2}{6} = \frac{1}{3}$.

The part of the tank remaining to be filled = $1 - (\frac{1}{3}) = (\frac{2}{3})$.

Work done by X and Y together in 7 hours = $\frac{2}{3}$.

Work done by X and Y together in 1 hour = $[(\frac{2}{3}) / 7] = \frac{2}{21}$

Work done by Z in 1 hour = $\{[(X+Y+Z)'s\ 1\ hour's\ work] - [(X+Y)'s\ 1\ hour's\ work]\}$

= $(\frac{1}{6}) - (\frac{2}{21}) = \frac{1}{14}$

Therefore, it would take Z alone 14 hours to fill in the tank.

14) Two pipes can fill a small tank in 7 hr and 8 hr respectively. A leakage was found when two pipes are opened simultaneously and due to leakage it took 16 min more to fill it up. How much time the leakage will take to empty the full tank?

A) 50 Hrs B) 56 Hrs C) 58 Hrs D) 60 Hrs

Ans: B)

Works done by two pipes in 1 hours = $(\frac{1}{7}) + (\frac{1}{8})$
= $\frac{15}{56}$.

Time taken by the pipes to fill the tank = $\frac{56}{15}$ hrs
= 3 hrs 44 min.

Due to leakage time taken = 3 hr 44 min + 16 min = 4 hrs.

Work done by (two pipes+leak) in 1 hours = $\frac{1}{4}$.

Work done by the leak in 1 hour = $(\frac{15}{56}) - (\frac{1}{4}) = \frac{1}{56}$ hours.

So leak will empty the full as term in 56 hours.

15) There are three Taps A, B and C in a tank. They can fill the tank in 10 hrs, 20 hrs and 25 hrs respectively. At first, all of them are opened simultaneously. Then after 2 hours, tap C is closed and A and B are kept running. After the 4th hour, tap B is also closed. The remaining work is done by Tap A alone. Find the percentage of the work done by Tap A.

A) 62% B) 70% C) 72% D) 80%

Ans: C)

Work by tap A in percentage = $\frac{100}{10} = 10\%$.

Work by tap B in percentage = $\frac{100}{20} = 5\%$.

Work by tap C in percentage = $\frac{100}{25} = 4\%$.

When all of them are opened for 2 hours $A+B+C = 19\%$, that is work in one hour.

Therefore, work done by tap (A+B+C) in 2 hours = $19 \times 2 = 38\%$ of the work.

After 2 hours, tap C is closed, Therefore, work done by (A+B) in 2 hours = $15 \times 2 = 30\%$ of the work.

Total work done till now = work done by tap $\{(A+B+C)+(A+B)\}$ total work done till now = $38\%+30\%$.

Total work done till now = 68% .

After the 4th hour, tap B is also closed.

The last 32% of the work will be done by A alone.

Hence, tap A does 40% (first 4 hours) + $32\% = 72\%$.

Speed, Time and Distance:

Points to remember:

1.) **Speed:** The rate at which an object is moving is known as its speed. It is the distance traveled per unit time e.g. an object moving at 20 m/s shows that the object covers a distance of 20 meters every second.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The formula for speed: **Speed** = $\frac{\text{Distance}}{\text{Time}}$

$$\text{Therefore, Time} = \frac{\text{Distance}}{\text{Speed}}$$

And, **Distance** = Speed x Time.

2.) A man covers a certain distance D1 km at a speed of S1 km/hr and, D2 km at a speed of S2 km/hr, his average speed during the whole journey is given by:

$$\text{Average speed} = \frac{S1S2(D1+D2)}{S1D2+S2D1}$$

Average speed = $\frac{S1S2(D1+D2)}{S1D2+S2D1}$ km/hr

3.) A man travels from P to Q at a speed of S1 km/hr and returns from Q to P at S2 km/hr, his average speed during the whole journey is given by:

$$\text{Average speed} = \frac{2S1S2}{S1+S2}$$

4.) Two men A and B start travelling at the same time from points P and Q towards each other and after crossing each other A takes time T1 to reach Q and B takes time T2 to reach P,:

$$\frac{A's \text{ speed}}{B's \text{ speed}} = \frac{\sqrt{T2}}{\sqrt{T1}}$$

5.) If a man travels at $\frac{a}{b}$ of his usual or original speed, the change in time taken to cover the same distance is given by:

$$\text{Change in time} = \left(\frac{b}{a} - 1\right) * \text{usual or original time.}$$

6.) A man covers a distance D in time T1 with speed S1. When he travels at speed S2 and covers the same distance D in time T2;

$$\frac{\text{Product of speed}}{D} = \frac{S1}{T2} = \frac{S2}{T1} = \frac{\text{Difference of speed}}{\text{Difference of time}}$$

You can equate any two of the above relations to find the unknown values.

7.) A body covers a certain distance at a speed of S1 km/hr without stoppage and with stoppage it covers the same distance at S2 km/hr, the stoppage time per hour is given by:

$$\left(\frac{S_1 - S_2}{S_1} \right) \text{ hrs or } \left(\frac{\text{Difference of speeds}}{\text{speed without stoppages}} \right) \text{ hrs}$$

If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover same distance is $\frac{1}{a} : \frac{1}{b}$.

→ To convert from km / hour to m / sec, we multiply by $5 / 18$. So, 1 km / hour = $5 / 18$ m / sec.

→ To convert from m / sec to km / hour, we multiply by $18 / 5$. So, 1 m / sec = $18 / 5$ km / Hour = 3.6 km / hour.

Questions:

1) A running man crosses a bridge of length 500 meters in 4 minutes. At what speed he is running?

- A) 8.5 km/hr
- B) 7.5 km/hr
- C) 9.5 km/hr
- D) 6.5 km/hr

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Distance = 500 meters

Time = 4 minutes → $4 \times 60 = 240$ seconds

$$\text{Speed} = \frac{500}{240} = \frac{25}{12} \text{ m/s}$$

We need answer in km/hr:

$$\text{Speed in km/hr} = \frac{25}{12} * \frac{18}{5} = \frac{90}{12} \rightarrow \frac{30}{4} = 7.5 \text{ km/hr.}$$

2) A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?

- A) 3.6
- B) 7.2
- C) 8.4
- D) 10

Ans : B)

3) A car running at a speed of 140 km/hr reached its destination in 2 hours. If the car wants to reach at its destination in 1 hour, at what speed it needs to travel?

- A) 300 km/hr
- B) 280 km/hr
- C) 250 km/hr
- D) 240 km/hr

Ans: B)

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\begin{aligned} \text{Distance to be covered} &= \text{Speed} \times \text{Time} \\ &= 140 * 2 = 280 \text{ km} \end{aligned}$$

Time = 1 hour

Required Speed = $\frac{280}{1} = 280 \text{ km/hr.}$

4) A jogger is running at a speed of 15 km/hr. In what time he will cross a track of length 400 meters?

A) 96 sec

B) 100 sec

C) 104 sec

D) 110 sec

Ans: A)

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Distance = 400

Speed = 15 km/hr or $15 * \frac{5}{18} = \frac{75}{18} \rightarrow \frac{25}{6} \text{ m/s}$

Required Time = $\frac{400}{25/6} \rightarrow \frac{400 * 6}{25} = \frac{2400}{25} = 96 \text{ seconds}$

5) An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in $1\frac{2}{3}$ hours, it must travel at a speed of:

A) 300 km/hr

B) 360 km/hr

C) 600 km/hr

D) 720 km/hr

Ans: D)

Distance = $(240 \times 5) = 1200 \text{ km.}$

Speed = Distance/Time

Speed = $1200 / (5/3) \text{ km/hr.}$

Required Speed = $1200 \times \frac{3}{5} = 720 \text{ Km/Hr.}$

6) A cyclist moving at a speed of 20 km/hr crosses a bridge in 2 minutes. What is the length of the bridge?

A) 555.5 m

B) 444.4 m

C) 777.7 m

D) 666.6 m

Ans: D)

The length of the bridge is equal to the distance covered by the cyclist at a speed of 20 km/hr in 2 minutes.

So, Distance = Speed * Time

Speed = 20 km/hr

Speed in m/s = $20 * \frac{5}{18} = \frac{100}{18} = \frac{50}{9} \text{ m/s}$

Time = 2 minutes $\rightarrow 2 * 60 = 120 \text{ seconds}$

Required distance = $\frac{50}{9} * 120 = \frac{6000}{9} = 666.6 \text{ meters.}$

7) If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is:

- A) 50 km
- B) 56 km
- C) 70 km
- D) 80 km

Ans: A)

Let the actual distance travelled be x km.

Then,

$$\frac{x}{10} = \frac{x+20}{14}$$

$$\rightarrow x = 50 \text{ km.}$$

8) Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

- A) 9 B) 10 C) 20 D) 12

Ans: B)

Due to stoppages, it covers 9 km less.

$$\text{Time taken to cover 9 km} = \left(\frac{9}{54} \times 60 \right) \text{ min} = 10 \text{ min.}$$

9) A man complete a journey in 10 hours. He travels first half of the journey at the rate of 21 km/hr and second half at the rate of 24 km/hr. Find the total journey in km.

- A) 220 km
- B) 224 km
- C) 230 km
- D) 234 km

Ans: B)

$$\frac{\left(\frac{1}{2}\right)x}{21} + \frac{\left(\frac{1}{2}\right)x}{24} = 10$$

$$\text{So } x = 224 \text{ Km.}$$

10) In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is:

- A) 1 hour
- B) 2 hours
- C) 3 hours
- D) 4 hours

Ans: A)

Let The Duration of the flight be X hrs.

$$\frac{600}{x} - \frac{600}{x + (\frac{1}{2})} = 200$$

So X = 1 Hrs.

11) In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is:

A) 6kmph B) 5kmph C) 9kmph D) 8kmph

Ans: B)

Let Abhay's speed be x km/hr.

Then, $(30/x) - (30/2x) = 3$

$\Rightarrow 6x = 30$

$\Rightarrow x = 5 \text{ km/hr.}$

12) A man takes 6 hours 35 mins in walking to a certain place and riding back. He would have taken 2 hours less by riding both ways. What would be the time he would take to walk both ways?

A) 8 hrs 35 min B) 8 hrs 45 min C) 9 hrs 15 min D) 8 hrs 25 min

Ans: A)

Walk one way + Ride one way = 6 hrs 35 min.

Ride both ways = 4 hrs 35 min i.e 2 hours less

hence to Ride one way he would take 2 hrs 17 min 30 sec.

Hence to walk one way he would take 6hrs 35min - 2 hrs 17 min 30 sec = 4 hrs 17 min 30 sec.

Thus to walk both ways he would take 8 hrs 35 min.

13) Ram and Shyam travel the same distance at the speeds of 10 kmph and 15 kmph respectively. If Ram takes 30 min longer than Shyam, then the distance travelled is

A) 12km B) 20km C) 15km D) 24km

Ans: C)

Let's assume the distance traveled by them be 'd' km.

As per the question, Time taken by Ram = Time taken by Shyam + 30 minutes (or 0.5 hours).

We know, Time = Distance/Speed.

So, $d/10 = d/15 + 0.5$.

Multiplying both sides of the equation by 30 (LCM of 10 and 15).

$3d = 2d + 15$.

$d = 15$.

Therefore, the distance travelled by both Ram and Shyam is 15 km.

14) A man covers a distance of 110 km between two cities in 10 hours. He traveled partly on foot at 9 km/hr and partly on a bicycle at 15 km/hr. Find the distance traveled on foot.

A) 60 km

B) 92 km

C) 94 km

D) 96 km

Ans: A)

Let the distance travelled on foot =

The distance travelled on bicycle would be = $(90 - X)$ km

Total time taken to cover the distance = 10 hrs

Time taken to cover the 110 km would be equal to the sum of the time taken to cover the distances partly on foot and partly on bicycle.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{So, } \frac{x}{9} + \frac{110-x}{15} = 10$$

$$5X + 3(110 - X) = 45 \times 10$$

$$5X + 330 - 3X = 450$$

$$2X = 450 - 330$$

$$2X = 120$$

$$X = \frac{120}{2} = 60 \text{ km.}$$

15) Two guns were fired from the same place at an interval of 10 minutes and 30 seconds, but a person in the train approaching the place hears the second shot 10 minutes after the first. The speed of the train (in km/hr), supposing that speed travels at 330 metres per second, is:

A) 42.4kmph

B) 52.4kmph

C) 59.4kmph

D) 49.4kmph

Ans: C)

Let the speed of the train be x m/sec.

Then distance traveled by train in 10 min = Distance traveled by sound in 30 sec.

$$\Rightarrow x \times 10 \times 60 = 330 \times 30$$

$$\Rightarrow x = (330 \times 30) / (10 \times 60) = 16.5$$

$$\therefore \text{Speed of the train} = 16.5 \text{ m/sec} = 16.5 \times 18/5 = 59.4 \text{ km/hr.}$$

Aptitude Problems on Trains Concepts and Formulas

Points to remember:

Keep same units for all values mentioned in the problem i.e. as per the units of the given answers convert kilometre per hour (km/hr) to meters per second (m/s) and vice versa.

1) The distance traveled by a train to cross a pole or person is equal to the length of the train.

2) The distance traveled by train when it crosses a platform is equal to the sum of the length of the train and length of the platform.

3) When two trains are travelling in opposite directions at speeds V_1 m/s and V_2 m/s then their relative speed is the sum of their individual speeds $(V_1 + V_2)$ m/s.

4) Two trains are travelling in the same direction at V_1 m/s and V_2 m/s where $V_1 > V_2$ then their relative speed will be equal to the difference between their individual speeds $(V_1 - V_2)$ m/s.

5) When two trains of length X meters and Y meters are moving in opposite direction at V_1 m/s and V_2 m/s then the time taken by the trains to cross each other is $= \frac{X+Y}{V_1+V_2}$

6) When two trains of length X meters and Y meters are moving in same direction at V_1 and V_2

where $V_1 > V_2$ then the time taken by the faster train to cross the slower train $= \frac{X+Y}{V_1-V_2}$

7) When two trains X and Y start moving towards each other at the same time from points A and B and after crossing each other the train X reaches point B in a seconds and train Y reaches points A in b seconds, then

Train X speed: Train Y speed $= b^{1/2} : a^{1/2}$

Problems:

1) A train moving at speed of 90 km/hr crosses a pole in 7 seconds. Find the length of the train.

A) 150 m

B) 165 m

C) 175 m

D) 170 m

Ans: C)

Length of the train is equal to the distance covered by train to cross the pole.

Distance = Speed x Time

Distance = $25 * 7 = 175$ meters.

2) A train of length 200 meters crosses a man running at 10 km/hr in the same direction in 10 seconds. What is the speed of the train?

A) 72 km/hr

B) 95 km/hr

C) 85 km/hr

D) 82 km/hr

Ans: D)

When the train and man are moving in same direction then relative speed will be the difference between their individual speeds.

Relative Speed $= \frac{200}{10}$

We will convert it into Km/hr

$\frac{200}{10} * \frac{18}{5} = 72$ km/hr

Now, let the speed of the train is X km/hr. So, the relative speed, 72 km/hr = X km/hr - 10 km/hr

$X - 10 = 72$

$X = 72 + 10$

$X = 82$ km/hr.

3) A train moving at 50 km/hr crosses a bridge in 45 seconds. The length of train is 150 meters. Find the length of the bridge.

A) 525 m

B) 545 m

C) 575 m

D) 500 m

Ans: C)

The distance covered by train when it crosses the bridge is equal to the sum of length of the train and length of the bridge.

$$\text{Speed of train in m/s} = 50 \times \frac{5}{18} = \frac{250}{18} = \frac{125}{9} \text{ m/s}$$

Time = 40 seconds

Let the length of the bridge is X.

$$\frac{\text{length of bridge} + \text{length of train}}{\text{time taken to cross the bridge}} = \text{speed of train}$$

$$9(X+150) = 45 \times 125$$

$$9X + 450 = 5625$$

$$9X = 5625 - 450$$

$$9X = 5175$$

$$X = \frac{5175}{9} = 575 \text{ m.}$$

4) A train is moving at 120 km/hr. The length of the train is 150 meters. How long it will take to cross a platform of length 100 meters?

A) 10 seconds

B) 7.5 seconds

C) 20 Seconds

D) 25 seconds

Ans: B)

$$\text{Speed of train in m/s} = 120 \times \frac{5}{18} = \frac{50}{3} \text{ m/sec.}$$

Distance covered to cross the platform is equal to the sum of length of the train and length of the platform.

$$\text{So, distance} = 150 + 100 = 250 \text{ meters}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$= 250 \times \frac{3}{50}$$

$$= 7.5 \text{ seconds.}$$

5) A train of length 100 meters is moving at a speed of 70 km/hr. In what time it will cross a man who is walking at 10 km/hr in the same direction?

A) 5 seconds

B) 6 seconds

C) 8 seconds

D) 7 seconds

Ans: B)

Both the train and man are moving so we will find the relative speed of the train. They are moving in the same direction, so the relative speed = (speed of train - speed of man)

Relative Speed = (70-10) = 60 km/hr

Relative Speed in m/s = $60 \times \frac{5}{18} = \frac{50}{3}$ m/sec.

Distance covered to cross the man = length of the train (100 meters)

Time = $\frac{\text{Distance covered}}{\text{Speed}}$

$$\text{Time} = \frac{100}{\frac{50}{3}} = 6 \text{ Sec.}$$

6) Two trains running in opposite direction cross a man standing on the platform in 36 seconds and 26 seconds respectively. The trains cross each other in 30 seconds. What is the ratio of their speeds?

A) 2/4 B) 4/6 C) 3/9 D) 4/8

Ans: B)

Let the speeds of the two trains be x m/sec and y m/sec respectively.

Then, length of the first train = $27x$ metres,

And length of the second train = $17y$ metres.

\therefore Time = Total Distance / Relative Speed

$$\therefore 30 = \frac{27x + 17y}{x + y}$$

$$\therefore x / y = 4 / 6.$$

7) A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 seconds and 10 seconds respectively. The length of the train (in metres)

A) 45 m

B) 50 m

C) 54 m

D) 72 m

$$2\text{kmph} = (2 \times 5/18) \text{ m/sec} = 5/9 \text{ m/sec.}$$

$$4\text{kmph} = (4 \times 5/18) \text{ m/sec} = 10/9 \text{ m/sec.}$$

Let the length of the train be x metres and its speed be y m/sec.

$$\text{Then, } x / (y - 5/9) = 9 \text{ and } x / (y - 10/9) = 10$$

$$\therefore 9y - 5 = x \text{ and } 10(9y - 10) = 9x$$

$$\Rightarrow 9y - x = 5 \text{ and } 90y - 9x = 100.$$

On solving, we get $x = 50$

\therefore Length of the train is 50m.

8) A train M leaves Meerut at 5 a.m and reaches Delhi at 9 a.m Another train leaves Delhi at 7 a.m and reaches Meerut at 10.30 a.m At what time do the two trains cross each other?

- A) 7:56 a.m.
- B) 7:36 a.m.
- C) 6:36 a.m.
- D) 6:56 a.m.

Train 1 travel time = 4 hours.

Train 2 travel time is $3.5 = 7/2$ hours.

We know, speed = Distance/time.

Let the distance b/w Meerut & Delhi is 28 Km (LCM of 4 & 72).

Speed of 1st train = $28/4 = 7$ km/h.

Speed of 2nd train = $28/3.5 = 8$ km/hr.

Relative speed = Train 1 speed + Train 2 speed.

Relative speed = $7+8 = 15$ km.

Train 1 travels for 2 hours before train 2 starts.

Remaining distance = $28 - (28/2) = 14$ km.

Time is taken to cross each other Time = $14/15$ hours or 56 minutes.

Total time taken = 2 hours + 56 minutes

∴ They cross each other at 7:56 a.m.

9) A train moving at 108 km/hr crosses a platform in 30 seconds. Then it crosses a man running at 12 km/hr in the same direction of train in 9 seconds. What is the length of train and platform?

- A) 220 & 600
- B) 200 & 620
- C) 240 & 660
- D) 250 & 640

Ans: C)

Let the length of train X meters and length of platform Y meters.

Relative Speed (Speed of train relative to man) = $108 - 12 = 96$ km/hr

Relative Speed in m/sec = $96 \times \frac{5}{18} = \frac{80}{3}$

The distance covered by train to cross the man is equal to its length.

So, it is = X meters

So, $X = \text{Relative Speed} \times \text{Time (time taken to cross the man)}$

∴ $X = \frac{80}{3} \times 9 = 240$ meters .

Speed of train in m/s = $108 \times \frac{5}{18}$

Time taken to cross the platform = $\frac{\text{Distance covered (length of train + length of platform)}}{\text{Speed of train}}$

So, $30 = \frac{240+Y}{30}$

$900 = 240 + y$

∴ $Y = 900 - 240 = 660$ meters.

10) Two stations P and Q are 160 km apart on a straight track. A train starts running from station P at 8 a.m. at a speed of 30 km/hr towards station Q. Another train starts from station Q at 9 a.m. at a speed of 35 km/hr towards station P. At what time they will meet?

- A) 10 a.m. B) 11 a.m. C) 12 a.m. D) 1 p.m.

Ans: B)

Let the trains meet X hours after the first train starts from station P at 8 a.m.

Distance covered by train starting from station P = speed * time

So, distance = $30 * X = 30X$ km

Similarly, Distance covered by trains starting from Q = $35 * (X - 1)$ km, as it starts running 1 hour later than the first train.

The distance between the two stations = 160 km

The sum of the distances travelled by both trains to meet each other will be equal to the distance between two stations.

So, $30X + 35 * (X - 1) = 160$

$30X + 35X - 35 = 160$

$65X = 160 + 35$

$65X = 195$

$X = \frac{195}{65} = 3 \text{ hours}$

It means they will meet 3 hours after the first train starts at 8 a.m. So, they will meet at $8 + 3 = 11$ a.m.

11) Two trains are moving towards each other with speeds 40 km/hr and 45 km/hr from different stations P and Q. When they meet the second train from station Q has covered 20 km more distance than the first train which starts from station P. What is the distance between the two stations?

A) 300 km B) 320 km C) 340 km D) 360 km

Ans: C)

The distance between the stations is equal to the sum of distance covered by each train.

Let the distance covered by first distance = X

So, the distance covered by second train = $X + 20$

When the two trains starts from two different stations at the same time towards each other, they take same time to meet each other.

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{So, } \frac{X}{40} = \frac{X+20}{45}$$

$$45X = 40X + 800$$

$$45X - 40X = 800$$

$$5X = 800$$

$$X = \frac{195}{65} = 160 \text{ km}$$

Then distance covered by second train = $160 + 20 = 180$ km

So, the distance between stations P and Q = $160 + 180 = 340$ km.

Boats and Streams Concepts and Formulas

General terms:

- 1) **Still water:** The water of a river or any other water body which is not flowing is known as still water.
- 2) **Stream:** It is the flowing water of a river which is moving at a certain speed.
- 3) **Upstream:** The boat or a swimmer moving against the stream is known as moving upstream i.e. against the flow of water.
- 4) **Downstream:** The boat or a swimmer moving along the stream is known as moving downstream i.e. along the flow of water.

Points to remember:

1) If the speed of the boat or swimmer is X km/hr and the speed of the stream is Y km/hr, The speed of the boat or swimmer in the direction of the stream is known as speed downstream. It is given by;

Speed downstream = $(X+Y)$ km/hr

And, the speed of the boat or swimmer against the stream is known as speed upstream. It is given by;

Speed upstream = $(X-Y)$ km/hr.

2) Speed of man or boat in still water is given by = $(\text{speed downstream} + \text{speed upstream}) / 2$.

3) Speed of the stream is given by = $(\text{speed downstream} - \text{speed upstream}) / 2$.

4) A man can row at a speed of X km/hr in still water. If the speed of the stream is Y km/hr and the man rows the same distance up and down the stream, his average speed for the entire journey is given by;

$$= \frac{\text{speed upstream} * \text{speed downstream}}{\text{speed of man in still water}}$$

$$= \frac{(X-Y) * (X+Y)}{X} \text{ km/hr.}$$

5) A man can row a boat in still water at X km/hr. If the stream is flowing at Y km/hr it takes him t hours more to row upstream than to row downstream to cover the same distance. The distance is given by;

$$\text{Distance} = \frac{(X^2 - Y^2)t}{2Y}$$

6) A man can swim in still water at X km/hr. If the stream is flowing at Y km/hr it takes him t hours to reach a place and return back to the starting point. The distance between the place and the starting point is given by;

$$\text{Distance} = \frac{(X^2 - Y^2)t}{2X}$$

7) A boat or swimmer covers a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream is Y km/hr, the speed of boat or man in still water is given by;

$$= Y \left(\frac{t_2 + t_1}{t_2 - t_1} \right) \text{ km/hr.}$$

8) A boat or swimmer takes K times as long to move upstream as to move downstream to cover a certain distance. If the speed of the stream is Y km/hr, the speed of the boat or man in still water is given by;

$$= Y \left(\frac{K+1}{K-1} \right) \text{ km/hr.}$$

Problems:

1) The speed of a boat in still water is 5km/hr. If the speed of the boat against the stream is 3 km/hr, what is the speed of the stream?

Let the speed of stream = X km/hr

Speed of boat = 5 km/hr

Speed upstream = 3km/hr

Apply formula: Speed upstream = speed of boat - speed of stream

$$\therefore 3 = 5 - X$$

$$X = 5 - 3 = 2 \text{ km/hr.}$$

2) A boat can travel with a speed of 13 km/hr in still water. If the speed of the stream is 4 km/hr, find the time taken by the boat to go 68 km downstream.

Speed downstream = (13 + 4) km/hr = 17 km/hr.

$$\text{Time taken to travel 68 km downstream} = \frac{68}{17} = 4 \text{ hrs.}$$

3) A man rows downstream at 20 km/hr and rows upstream at 15 km/hr. At what speed he can row in still water?

Apply formula: Speed in still water = (speed downstream + speed upstream) / 2.

Speed downstream = 20 km/hr

Speed upstream = 15 km/hr

$$\therefore \text{required speed} = [(20 + 15) / 2] \text{ km/hr}$$

$$= 35 / 2 = 17.5 \text{ km/hr.}$$

4) In one hour, a boat goes 11 km/hr along the stream and 5 km/hr against the stream. The speed of the boat in still water (in km/hr) is:

Answer: 8 km/hr.

5) A man swims 12 km downstream and 10 km upstream. If he takes 2 hours each time, what is the speed of the stream?

$$\text{Speed downstream} = \frac{\text{Distance}}{\text{Time}} = \frac{12}{2} = 6 \text{ km/hr}$$

$$\text{Speed upstream} = \frac{\text{Distance}}{\text{Time}} = \frac{10}{2} = 5 \text{ km/hr}$$

Apply formula: Speed of stream = (speed downstream - speed upstream) / 2.

$$X = 6 \text{ km/hr}$$

$$Y = 5 \text{ km/hr}$$

$$\therefore \text{required speed} = [(6 - 5) / 2] \text{ km/hr}$$

$$= 1/2 = 0.5 \text{ km/hr.}$$

6) A boat covers 800 meters in 600 seconds against the stream and returns downstream in 5 minutes. What is the speed of the boat in still water?

$$\text{Speed upstream} = \frac{\text{Distance}}{\text{Time}} = \frac{800}{600} = \frac{8}{6} \text{ m/s}$$

$$\text{Speed downstream} = \frac{\text{Distance}}{\text{Time}} = \frac{800}{5 \times 60} = \frac{8}{3} \text{ m/s}$$

Apply formula: Speed in still water = (speed downstream + speed upstream) / 2.

$$X = \frac{8}{3} \text{ m/s}, \quad Y = \frac{8}{6} \text{ m/s}$$

$$\therefore \text{Speed in still water} = \left(\frac{8}{3} + \frac{8}{6} \right) / 2$$

$$= 4 / 2 = 2 \text{ m/s.}$$

7) A man can row a boat at a speed of 20 km/hr in still water. If the speed of the stream is 5 km/hr, in what time he can row a distance of 75 km downstream?

Speed of boat = 20 km/hr

Speed of stream = 5 km/hr

\therefore Speed downstream = 20 + 5 = 25 km/hr

$$\text{Required Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{75}{25} = 3 \text{ hours}$$

8) A man swimming in a river which is flowing at $3\frac{1}{2}$ km/hr finds that in a given time he can swim twice as far downstream as he can swim upstream. What will be his speed in still water?

Let the man swims at X km/hr in still water.

As per the question, he covers twice the distance downstream as he covers upstream in a given time.

Distance = Speed * Time

\therefore Speed downstream * Time = 2 (Speed upstream * Time)

Time is same in both the cases.

$$\therefore x + 3\frac{1}{2} = 2 * \left(x - 3\frac{1}{2} \right)$$

$$X = 10.5 \text{ km/hr.}$$

9) A man's speed with the current is 15 km/hr and the speed of the current is 2.5 km/hr. The man's speed against the current is: ?

Man's rate in still water = (15-2.5) km/hr = 12.5 km/hr.

Man's rate against the current = (12.5-2.5) km/hr = 10 km/hr.

10) The speed of a boat in still water is 15 km/hr and the rate of current is 3 km/hr. The distance travelled downstream in 12 minutes is:

Speed downstream = (15+3) km/hr = 18 km/hr .

Distance travelled = 18 × (12/60) km = 3.6 km.

11) Rahul can row a certain distance downstream in 6 hour and return the same distance in 9 hour. If the speed of Rahul in still water is 12 km/hr, find the speed of the stream.

Let the speed of Rahul be u and that of stream be v and the distance travelled be d .

$$(d/u) + v = 6 \text{ and } (d/u) - v = 9.$$

$$\text{Given } u = 12 \text{ km/hr } 6u+6v = d \text{ and } d = 9u-9v.$$

$$\text{So, } 6u+6v = 9u-9v,$$

$$3u = 15v, \text{ so } u = 5v.$$

$$v = 12/5 = 2.4 \text{ km/hr.}$$

12) A speedboat, whose speed in 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. What is the speed of the stream in km/hr?

Let the speed of the stream be x km/hr.

$$\text{Upstream Speed} = 15 + x.$$

$$\text{Downstream Speed} = 15 - x$$

$$\text{So, } \{30 / (15+x)\} + \{30 / (15-x)\} = 4 \frac{1}{2} \text{ (4 hours 30 minutes)}$$

$$\Rightarrow \{900 / (225-x^2)\} = 9/2$$

$$\Rightarrow 9x^2 = 225$$

$$\Rightarrow x = 5 \text{ km/hr.}$$

13) A boat is moving 2 km against the current of the stream in 1 hour and moves 1 km in the direction of the current in 10 minutes. How long will it take the boat to go 5 km in stationary water?

$$\text{Downstream} = (1/10 \times 60) = 6 \text{ km/hr Upstream} = 2 \text{ km/hr}$$

$$\text{Speed in still water} = \frac{1}{2} (6+2) = 4 \text{ km/hr}$$

$$\text{So, the time is taken by the boat to go 5km in stationary water} = 5/4 \text{ hrs} = 1 \frac{1}{4} \text{ hrs} = 1 \text{ hr } 15 \text{ minutes.}$$

14) A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?

Let the man's rate upstream be x kmph and that downstream be y kmph.

$$\text{Then, distance covered upstream in 8 hrs 48 min} = \text{Distance covered downstream in 4 hrs.}$$

$$\Rightarrow X \times 8 \frac{4}{5} = y \times 4$$

$$\Rightarrow (44 x/5) = 4y$$

$$\Rightarrow y = (11/5) x$$

$$\therefore \text{Required ratio} = (y+x)/2 : (y-x)/2$$

$$= ((16x/5) \times (1/2)) : ((6x/5) \times (1/2))$$

$$= (8/5) : (3/5)$$

$$= 8:3$$

15) A man can row 9 and $\frac{1}{3}$ km/hr in still water and finds that it takes him thrice as much time to row up than as to row down the same distance in the river. The speed of the current is?

$$\text{Let speed upstream be } x \text{ kmph Then speed downstream} = 3x \text{ km/hr.}$$

$$\text{Speed in still water} = \frac{1}{2}(3x+x) \text{ km/hr} = 2x \text{ km/hr.}$$

$$\therefore 2x = 283$$

$$\Rightarrow x = \frac{14}{3} \text{ So Speed upstream} = \frac{14}{3} \text{ km/hr.}$$

Speed downstream = 14 km/hr.

$$\text{Hence speed of the current} = \frac{1}{2} \left(14 - \frac{14}{3} \right) \text{ km/hr} = \frac{14}{3} \text{ km/hr} = 4\frac{2}{3} \text{ km/hr.}$$