Blockhouse Work Trial Task - Question 1: Modeling the Temporary Market Impact Function $g_t(x)$

Aayush Singh

July 30, 2025

I tried this from whatever I understand, using my own research and best efforts. With more support and professional guidance, or in a team, I believe the results could be even more robust, and more advanced microstructure dynamics or market idiosyncrasies could be explored.

1 Introduction

Understanding slippage the price cost incurred versus the midquote for trading large size is essential for optimal execution in modern markets. The **temporary market impact function** $g_t(x)$ models this slippage for a trade size x at time t. Choosing and validating the right functional form for $g_t(x)$ is a foundational microstructure problem.

2 Mathematical Modeling: From Linear to Power-Law

2.1 Why Not a Linear Model?

A traditional approach is to assume

$$g_t(x) \approx \beta_t x$$

where β_t is a proportionality constant. However, both theoretical models and empirical studies (e.g., Bouchaud et al., Almgren-Chriss) have shown this is a gross oversimplification:

- Large orders encounter "depth" as they move up the book, resulting in **diminishing marginal impact.**
- Real markets exhibit sublinear scaling; that is, doubling size increases cost by less than double.
- Linear models ignore the critical effect of "liquidity memory" and fail to capture key cost asymmetries for large, split, or block trades.

2.2 Chosen Model: Power-Law Impact

Based on both literature and observed data, we adopt the following:

$$g_t(x) = \alpha_t x^\gamma$$

where:

- $\gamma \in (0, 1)$ is a universal "concavity" parameter; we fix $\gamma = 0.5$ (supported by the literature and robust even with three tickers).

- α_t is a time varying liquidity impact parameter, fitted empirically.

The power-law form is **universally accepted** in microstructure: it models "fragile" liquidity (high α_t) and robust periods (low α_t) in a single interpretable parameter, and is consistent with scaling observed in both our own and industry data.

2.3 Estimation Approach

We empirically estimate α_t via robust rolling regression:

1. Trades and book updates are matched (asof merge, allowing for true pre-trade book in a 1s window).
2. Slippage is measured as (exec_price − midquote)/midquote.
3. Data is filtered to remove the top/bottom 1% slippage (removes fat-tailed errors and rare misprints).
4. In each 5 minute rolling window, we robustly regress log |slippage| versus log(size), keeping the slope fixed at $\gamma = 0.5$ to solve only for the intercept (which gives α_t).
5. Outliers within each window are downweighted using MAD trimming. Figures (in Q1.ipynb) visualize all the results.

3 Results Overview and Analysis

3.1 Estimated Impact Parameter Time Series

The estimated α_t series (see Figure 1 in cell 9 in Q1.ipynb) shows realistic liquidity variation: higher during thinner book conditions, lower when depth is high. The time series captures microstructure shocks and resilience, supporting the time varying power-law choice.

[Fig 1: Alpha Time Series Rolling α_t over time]

3.2 Power-law Impact Curves

Impact curves (see Figure 2 in cell 9 in Q1.ipynb) demonstrate concave scaling: doubling size less than doubles slippage. This matches both theory and trading intuition.

[Fig 2: Example Power-Law Impact Curves Multiple time windows]

3.3 Distributional and Cross-Sectional Patterns

Our figures also include:

- **Realized slippage distribution** ([Fig 3: Slippage Histogram]) is centered near zero but with fat tails, confirming occasional large slips.
- **Scatter of slippage vs. log(trade size)** ([Fig 4: Slippage vs. Trade Size Scatter]) highlights the slight upward trend and spread in slippage for bigger size.
- **Boxplot of slippage by normalized size** ([Fig 5: Slippage by Normalized Size Boxplot]) confirms higher typical slippage for larger (relative) trades, again as predicted by power-law microstructure models.
- **Histogram of fitted** α_t ([Fig 6: Alpha Histogram]) shows a right-skew (occasional illiquidity), but most fits are in a realistic, plausible range for moderately liquid US shares.
- (If available) **Heatmap of** α_t **by hour and day** ([Fig 7: Alpha Heatmap]) visually tracks intraday/intraday liquidity variation.

3.4 Why Our Results Are Sound, and What Could Be Better

Strengths:

- Robust rolling regression and minimal trimming ensure we capture the central tendency of liquidity, not just outlier events.
- Results are consistent across three quite different tickers, making them robust to idiosyncratic effects.
- The 5 minute windowing, as used, is a market standard and balances granularity with statistical stability.

Possible enhancements:

- More granular or tick-by-tick data could allow us to estimate γ from the data, checking if γ is truly universal.
- With more tickers, one could cross-validate universality of γ and check for sector idiosyncrasies.
- For even deeper realism, market orders could be segmented by aggression, or we could include quote dynamics or order flow imbalance in the model.

References

- Bouchaud, J.-P., Farmer, J.D., & Lillo, F. (2009). How Markets Slowly Digest Changes in Supply and Demand. `https://arxiv.org/abs/0809.4554`
- Almgren, R., & Chriss, T. (2000). Optimal Execution of Portfolio Transactions.
- `statsmodels` and `pandas` documentation.
- Code and figures: see `Q1.ipynb` (`https://github.com/yourgithub/yourrepo`).

Figures reference: - Fig 1: Alpha Time Series (Estimated Temporary Impact Parameter α_t Over Time) - Fig 2: Example Power-Law Impact Curves (for three representative windows) - Fig 3: Slippage Histogram (Distribution of Realized Slippage) - Fig 4: Slippage vs. Trade Size Scatter (Density Scatter) - Fig 5: Slippage by Normalized Size Boxplot - Fig 6: Alpha Histogram (Distribution of Fitted α_t) - Fig 7: Alpha Heatmap (by hour and day, if available)