PRACTICAL No:- 1. Limits & Continuity.

① $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

② $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

③ $\lim_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

④ $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

⑤ Examine the continuity of the following function at given points.

(i) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x \le \dfrac{\pi}{2} \\[2ex] \dfrac{\cos x}{\pi - 2x} & \text{for } \dfrac{\pi}{2} < x < \pi \end{cases} \quad \text{at } x = \frac{\pi}{2}$$

(ii) $$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\[1ex] x+3 & 3 \le x < 6 \\[1ex] \dfrac{x^2+9}{x+3} & 6 \le x < 9 \end{cases} \quad \text{at } x = 3 \text{ \& } x = 6$$

(6) Find value of k, so that the function $f(x)$ is continuous at the indicated point

(i) $f(x) = \frac{1-\cos 4x}{x^2}$, $x<0$; $= k$, $x=0$ } at $x=0$ ~~and~~

Doubt

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$; $= k$, $x = 0$ } at $x=0$

(iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \frac{\pi}{3}$; $= k$, $x = \pi/3$ } at $x = \frac{\pi}{3}$

(7) Discuss the continuity of the following function which of these functions have removable discontinuity? Redefine the function so as to remove the discontinuity.

(i) $f(x) = \frac{1-\cos 3x}{x \tan x}$, $x \neq 0$; $= 3$, $x=0$ } at $x=0$

(ii) $f(x) = \frac{(e^{3x}-1)\sin x^{0}}{x^2}$, $x \neq 0$; $= \frac{\pi}{60}$, $x = 0$ } at $x=0$

(8)

(9)

(8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x=0$ find $f(0)$

(9) If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \frac{\pi}{2}$ is continuous at $x = \pi/2$ find $f(\frac{\pi}{2})$

Solutions:-

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}\right] \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}}$

$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$

$$= \frac{2}{3\sqrt{3}}$$

$\rightarrow$ 2) $$\lim_{y \to 0} \left[\frac{(\sqrt{a+y} - \sqrt{a})}{y\sqrt{a+y}} \right]$$

$$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$\lim_{y \to 0} \frac{a + y - a}{y\sqrt{a+y} \cdot (\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y} \cdot (\sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0} \cdot (\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a} \cdot (\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\, 2\sqrt{a}}$$

$$= \frac{1}{2a}$$

3] $$\lim_{\cancel{x \to \pi/6}} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cosh \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6}{\pi - 6\left(\frac{6h+\pi}{6}\right)} - \sqrt{3}\sin h \cos \pi/6 + \cos h \sin \pi/6$$

$$\lim_{h\to 0} \frac{\cosh \cdot \frac{\sqrt{3}}{2} \cdot \sin \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{3}}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cos \frac{\sqrt{3}h}{2} - \sin \frac{h}{2} - \sin \frac{3h}{2} - \cos \frac{\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{-\sin \frac{4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{12h}$$

$$\frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3} \times 1$$

$$= \frac{1}{3}$$

4) $$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$$

By rationalizing Numerator and Denominator both i.e.

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)} \frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{8}{2}\frac{(\sqrt{x^2+3}+\sqrt{x^2+1}}{(\sqrt{x^2+5}+\sqrt{x^2-3}}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$$

After Applying limit
we got,

$= 4.$

5) $f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{-\cos 2(\pi/2)}}$

$\therefore f(\pi/2) = 0$

f at $x = \frac{\pi}{2}$ define

ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting Method.

$x - \frac{\pi}{2} = h$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \pi/2\right)}{\pi - 2\left(\frac{2x+\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot 0 \cdot \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

G.i) $\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2 \sin x \cdot \cos x}{\sqrt{2} \sin x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2 \cos x}{\sqrt{2}}$$

Ni.

$\frac{1}{\sqrt{2}}$ $\lim_{x \to \frac{\pi}{2}} = \cos x$

$\therefore$ L.H.L $\neq$ R.H.L

f is not continuous at $\frac{\pi}{2}$

ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $6 \le x < 9$

at $x = 3$ and $x = 6$

at $x - 3$

i) $f(3) = \frac{x^2-9}{x-3} = 6$

f at $x=3$ define.

ii) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} \cancel{f(x)} = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3}$

L.H.L = R.H.L

f l
for
f (

2. $\lim_{x \to 6^+}$

$\lim_{x \to 6^-}$

$\lim_{x \to 6^+}$

$\lim_{x \to 6^-}$

func

6.i) $f(x) =$

=

$\Rightarrow$ f is

$\lim_{x \to}$

$\lim_{x \to 0}$

2 lim

f is continuous at $x=3$
for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3.$$

2. $$\lim_{x\to 6^+} \frac{x^2-9}{x+3}$$

$$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$\lim_{x\to 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x\to 6^-} x+3 = 3+6 \quad 9.$$

= L.H.L ≠ R.H.L.
function is not ~~cont~~ continuous.

(ii) $$\left.\begin{aligned} f(x) &= \frac{1-\cos 4x}{x^2} && x<0 \\ &= K && x=0 \end{aligned}\right\} \text{ at } x=0$$

⇒ f is continuous at $x=0$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = K$$

$$2\lim_{x\to 0} \frac{\sin^2 x}{x^2} = K$$

Q5.

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$$

$$2(2)^2 = K$$

$$\therefore K = 8$$

ii) $$f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$$
$$= K \quad x = 0$$
$\Big\}$ at $x = 0$

Sol^n: $f(x) = (\sec^2 x)^{\cot^2 x}$

$$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{1/\tan^2 x}$$

We know that

$$\lim_{x \to 0} (1 + px)^{1/px} = e$$

$$\therefore = e$$

$$\therefore K = e.$$

iii) $$f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \frac{\pi}{3}$$
$$= K \quad x = \frac{\pi}{3}$$
$\Big\}$ at $x = \pi/3$

$$x - \frac{\pi}{3} = h$$

$$x = h + \frac{\pi}{3}$$

Where $h \to 0$.

$$f\left(\frac{\pi}{3}+h\right) = \frac{\sqrt{3} - \tan\left(\frac{\pi}{3}-h\right)}{\pi - 3\left(\frac{\pi}{3}+h\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3}+h\right)}{\pi - 3\left(\frac{\pi}{3}+h\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \tan\left(\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4\tan h}{-3(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \frac{1}{1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{1 - \sqrt{3}(0)}$$

$$= 4/3$$

'08.

7) i) $f(x) = 1 - \cos 3x \quad x \neq 0$
$ = 9 \quad x = 0$ } at $x = 0$

$$f(x) = \frac{1 - \cos 3x}{x \tan x}$$

$$\lim_{x \to 0} \frac{2 \sin^2 3/_2 x}{x \tan x}$$

$$\lim_{x \to 0} 2 \frac{\frac{\sin^2 3x/2}{x^2} \times x^2}{\frac{x \tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x \to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$\lim_{x \to 0} f(x) = \frac{9}{2} \quad 9 = f(0).$$

$\therefore f$ is not continuous at $x = 0$

Redifine function

$$f(x) = \begin{cases} \dfrac{1 - \cos 3x}{x \tan x} & x \neq 0 \\[2ex] \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0).$

f has removable discontinuity at $x = 0$

vii) $f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}$ $x \neq 0$
$= \frac{\pi}{6}$ $x = 0$ } at $x = 0$

$\Rightarrow \lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x\to 0} 3 \cdot \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi x}{180}\right)$

$3 \lim_{x\to 0} \frac{e^{x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \cdot \log e \frac{\pi}{180} = \frac{\pi}{6} = f(0)$

$\therefore f$ is continuous at $x = 0$

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x \neq 0$

is continuous at $x = 0$

$\therefore$ Given,
f is continuous at $x = 0$
$\lim_{x\to 0} f(x) = f(0)$
$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

7

$$= \lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x} \right)^2$$

Multiply with 2 on Num and Denominator

$$= 1 + 2 \times \frac{1}{4}$$

$$= \frac{3}{2}$$

$$= f(0)$$

9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ $\quad x \neq \frac{\pi}{2}$

~~$f(0)$ is continuous at $x = \frac{\pi}{2}$~~

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2 - 1 - \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})}$$

$\lim_{x \to \frac{\pi}{2}}$

$= \lim_{x \to \frac{\pi}{2}}$

$= \lim_{x \to \frac{\pi}{2}}$

$\lim_{x \to \frac{\pi}{2}}$

6/12/19

$$\lim_{x \to \frac{\pi}{2}} \frac{1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 + \sin x}{1 - \sin x) (1 + \sin x) (\sqrt{2} + \sqrt{1 + \sin x})}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 + \sin x}{(1 - \sin x) (1 + \sin x) (\sqrt{2} + \sqrt{1 + \sin x})}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{1}{(1 - \sin x) (\sqrt{2} + \sqrt{1 + \sin x})}$$

$$= \frac{1}{2 (\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2 (2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

6/12/19

PRACTICAL No. 2

DERIVATIVES.

(1) Show that the following function are defined from R to R are differentiable

Q i) $\cot x$

$\Rightarrow$ $f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x - a) \cdot \tan x \cdot \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \cdot \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan(a - a - h) - (1 + \tan a + \tan(a+h))}{h \cdot \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$$

$$= \cancel{-1 \times \frac{1 + \tan^2 a}{\tan^2 a}}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

C.E.

$= -\operatorname{cosec}^2 a$

$Df(a) = -\operatorname{cos}^2 a$

f is differentiable $\forall a \in R$

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a) \cdot \sin x \cdot \sin a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \cdot \sin a \cdot \sin(a+h)}$$

$$= \frac{-1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a}$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cdot \cos x \cdot \cos a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \cdot \cos a \cdot \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a}{2}\right) \sin(-h/2) \times -1/2}{\cos \cdot a \cdot \cos(a+h) \times -h/2}$$

$$= \frac{-1}{2} \times \frac{-2\sin(2a + 0/2)}{\cos a \cdot \cos(a+0)}$$

$$= \cancel{\frac{-1}{2} \times -2} \times \frac{\sin a}{\cos a \cdot \cos a}$$

$= \tan a \cdot \sec a$.

② If $f(x) = 4x+1 \;;\; x \le 2$ at $x=2$ then find fun
$\quad\quad = x^2+5 \quad x>0$

is differentiable or not.

⇒ LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)}$$

$$= 4$$

$$Df(2^-) = 4$$

RHD:

$$Df(2^+) \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 2+2$$

$$= 4$$

$$Df(2^+) = 4$$

L.H.D = R.H.D

f is differentiable at $x=2$

③ If $f(x) = 4x + 7$; $x < 3$
$= x^2 + 3x + 1$, $x \geq 3$
find f is differentiable at $x = 3$ then or not.

$\Rightarrow$ RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 \cdot 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{x - 3}$$

$$= 3 + 6$$

$$= 9$$

$$Df(3^+) = 9$$

<u>L.H.D</u>

$$Df(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4(x - 3)}{x - 3}$$

$$= 4$$

$Df(3^-) = 4$

$LHD \neq RHD$

$\therefore f$ is not differentiable at $x = 3$

(4) If $f(x) = 8x - 5 \; ; x \leq 2$
$= 3x^2 - 4x + 7 \; ; x > 2$ at $x = 2$ then

find f is differentiable or not.

$\Rightarrow$ $f(2) = 8 \times 2 - 5 = 11$

RHD

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$= 3 \times 2 + 2$

$= 8$

$Df(2^+) = 8$

LHD

Df

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LHD :-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 \cdot 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$$= 8$$

$$Df(2^-) = 8$$

LHD = RHD

$\therefore f$ is differentiable at $x = 3$ //

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20/12/19

PRACTICAL 3

Application of Derivative.

Q1 Find the intervals in which functions is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$ (ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$ (iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q2 Find the intervals in which function is concave upwards and concave downwards.

i) $y = 3x^2 - 2x^3$ (ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$ (iv) $y = 69 - 24x - 9x^2 + 2x^3$

(v) $y = 2x^3 + x^2 - 20x + 4$

Solutions:-

i) $f(x) = x^3 - 5x - 11$

$\rightarrow f'(x) = 3x^2 - 5$

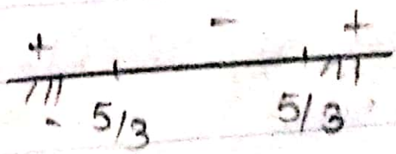
f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm\sqrt{\frac{5}{3}}$



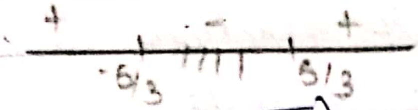
$x \in \left(-\infty, \sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < \pm\sqrt{\frac{5}{3}}$



$x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

f is increasing iff $f(x) > 0$

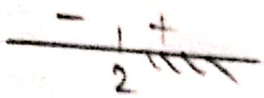
$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x > 2$

$x \in (2, \infty)$



∴ f is decreasing iff $f(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x < 2$

$x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$

$+ \quad - \quad +$

$-2 \qquad 10/6$

$\therefore x \in (-\infty, -2) \cup \left(\frac{10}{6}, \infty\right)$

f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x - 10)(x+2) < 0$

$+ \quad - \quad +$

$-2 \qquad 10/6$

$x \in \left(-2, \frac{10}{6}\right)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

$f(x)$ is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$+ \quad -3 \quad 3 \quad +$

$x \in (-3, 3)$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

i.e. $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

f is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$x \in (-\infty, -1) \cup (4, \infty)$

f is decreasing iff $f'(x) < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x+1)(x-4) < 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (1, 4)$

Q2

i) $y = 3x^2 - 2x^3$

Let,

$f(x) = y = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1-2x)$

$f''(x)$ is concave upwards iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1 - 2x > 0$

$-2x > -1$

$2x < -1$

$x < \frac{1}{2}$

$x \in (-\infty ; \frac{1}{2})$

$f''(x)$ is concave downwards iff

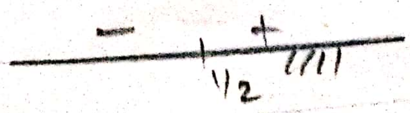
$f''(x) < 0$

$6(1-2x) < 0$

$1 - 2x < 0$

$x > 1/2$

$x \in (\frac{1}{2}, \infty)$



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7 (ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\rightarrow$ Let $f(x) = y$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

f is concave upward iff $f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$\cancel{x^2 - 3x + 2)}$

$x^2 - x - 2x + 2 > 0$

$(x-2)(x-1) > 0$

$\therefore x \in (-\infty, 1) \cup (2, \infty)$

f is concave downward iff $f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - x - 2x + 2 < 0$

$(x-2)(x-1) < 0$

$\therefore x \in (1, 2)$

(iii) $y = x^3 - 27x + 5$

$\Rightarrow$ let $f(x) = y$

$\therefore f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave upward iff $f''(x)$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

(iv)

$\Rightarrow$

$f''(x)$ is concave downward iff: $f''(x) <$

$6x < 0$

$x < 0$

$\therefore x \in (-\infty, 0)$

$- \quad | \quad +$ (0)

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

$\Rightarrow$ let $f(x) = y$

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$= 12x - 18$

$= 6(2x - 3)$

$f''(x)$ is concave upward iff $f''(x) > 0$

$6 \cdot (2x - 3) > 0$

$2x - 3 > 0$

$x > 3/2$

$- \quad | \quad +$ ($3/2$)

$x \in (3/2, \infty)$

$f''(x)$ is concave downward iff $f''(x) < 0$

$6(2x - 3) < 0$

$2x - \cancel{3} < 0$

$x < 3/2$

$- \quad | \quad +$ ($3/2$)

$\therefore x \in (-\infty, 3/2)$

N3) $y = 2x^3 + x^2 - 20x + 4$

$\rightarrow$ let $f(x) = y$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$= 2(6x+1)$

$f''(x)$ is concave upward iff $f''(x) > 0$

$2(6x+1) > 0$

$6x + 1 > 0$

$x > -1/6$

$-x < 1/6$

$\therefore \cancel{x \in (-\infty, 1/6)}$ $x \in (-1/6, \infty)$

$f''(x)$ is concave downward iff $f''(x) < 0$

$2(6x+1) < 0$

$6x + 1 < 0$

$x < -1/6$

$x > 1/6$

$\therefore x \in (1/6, \infty)$

$x \in (-\infty, -1/6)$

20/12/19

PRACTICAL NO - 4

Newton's method.

Q1 find the minimum & maximum value of the following function

(i) $f(x) = x^2 + \frac{16}{x^2}$

$\Rightarrow f'(x) = 2x + \frac{x^2 - 32x}{x^4}$

$= \frac{2x^5 + 32x}{x^3}$

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{24}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^2 + \frac{16}{(2)^2}$

$= 4 + 4$

$= 8$

$f(-2) = 2 + \frac{96}{16}$

$8 > 0$

$\therefore f$ has minimum value at $x = -2$ at $x = 2$

$\therefore$ function reaches minimum value

at $x = -2$

(2) $f(x) = 3 - 5x^3 + 3x^5$

$\Rightarrow f'(x) = -15x^2 + 15x^4$

consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$-15x^2(1 - x^2) = 0$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum valu at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''($

$\therefore f$

f ha

(3) $f(x)$

$f'($

con

$\therefore$

f

$\therefore f''(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$

f has the minimum value 1 at $x = 1$

(3) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider,

$f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ and $x = 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$\therefore f$ has maximum value at $x = 0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 24 + 1$
$= 9 - 12$
$= -3$

$\therefore$ f has minimum value as 1 at $x = 0$

f has minimum value as -3 at $x = 2$

(4) $f(x) = 2x^3 - 3x^2 - 12x - 1$
$f'(x) = 6x^2 - 6x - 12$
$= 6(x^2 - x - 2)$

consider,

$f'(x) = 0$
$6(x^2 - x - 2) = 0$
$x^2 - x - 2 = 0$
$(x+1) \cdot (x-2) = 0$
$x = -1$ & $x = 2$

$f''(x) = 12x - 6$
$= 6(x-1)$

$f''(-1) = \cancel{12(-1)} - 6$
$= -12 - 6$
$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$
$f(-1) = 2(-1)^3 - 3 \cdot (-1)^2 - 12(-1) + 1$

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$= -2 - 3 + 12 + 1$
$= 8$

$f''(2) = 12 \cdot (2) - 6$
$= 24 - 6$
$= 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) - 1$
$= 16 - 12 - 24 - 1$
$= 16 - 37$
$= -27$

$\therefore f$ has minimum value as -27 at $x = 2$

f has maximum value as 8 at $x = -1$

Q.2 Find the root of the equation correct up to 4 decimal place.

① $f(x) = x^3 - 3x^2 - 55x + 9.5$

$x_0 = 0$ — (Given).

$f'(x) = 3x^2 - 6x - 55$

By newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{9.5}{55}$$

$\therefore x_1 = 0.1727$.

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$= \underline{0.1712}$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.4167 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 + \frac{0.0011}{55.9393}$$

$= 0.1712$ //

The roo

2. $f(x) = x^3$ –

$f'(x) = 3x$

$f(2) = 2^3$

$= -$

$f(3) = 3^3 -$

$= 6$

let $x_0 = 3$

By Newto

$x_{n+1} =$

$x_1 = x_0$

$\therefore 3$

$= 2.$

$f(x_1) = ($

$= 2$

$= 0$

$f'(x_1) =$

$=$

$=$

$\therefore x_2 = x_1$

The root of the equation is 0.1712.

(2). $f(x) = x^3 - 4x - 9 \; - [2,3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= -9$

$f(3) = 3^3 - 4(2) - 9$

$= 6$

Let $x_0 = 3$ be initial approximation.

By Newton's method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.5960$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$= 2.7392 - \frac{0.5910}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 19.8386 - 10.8284 - 9$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - 0.0056$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= \cancel{21.8943 - 4}$
$= 17.8943$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 2.7015 + \frac{0.0901}{17.8943}$

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$= 2.7015 + 0.0050$
$= 2.7065$

(8) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

→ $f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17.$
$= 1 - 1.8 - 10 + 17.$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2$

Let $x_0 = 2$ be initial approximation.
By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2 - \frac{2.2}{5.2}$$

$= 2 - 0.4230$
$= 1.577$

$f(x_1) = (1.577)^3 - 1.8 \cdot (1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$

21

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$
$$= 7.4608 - 5.6772 - 10$$
$$= -8.2164.$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 1.577 + \frac{0.6755}{8.2164}$$
$$= 1.577 + 0.0822$$
$$= 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592)$$
$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$
$$= -7.7143.$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.6592 + \frac{0.0204}{7.7143}$$
$$= 1.6592 + 0.0026$$
$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 0.00004.$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$
$$= 8.2847 - 5.9824 - 10$$

$= -7.6$

$\therefore x_4 =$

$= 1$

$= 1.6$

$\therefore$ The ro

$= -7.6977$

$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.6618 + \frac{0.0004}{-7.6977}$

$= 1.6618$

$\therefore$ The root of the equation is 1.6618.

Practical No. 5

Integration.

(1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$\Rightarrow \quad I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

Comparing with $\therefore \int \frac{dx}{\sqrt{x^2-a^2}} \quad = \quad x^2 = (x+1)^2$

$I = \log|x+\sqrt{x^2-a^2}|+c$

$= \log|x+1+\sqrt{(x+1)^2-(2)^2}|+c$

ii) $\int (4e^{3x}+1)\,dx$

$I = \int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x + c$

iii) $\int (2x^2 - 3\sin x + \sqrt{5})\,dx$

$I = \int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$= 2\int x^2\,dx - 3\int \sin x\,dx + 5\int \sqrt{x}\,dx$

$= \frac{2}{3} x^3\,dx + 3\cos x + \frac{5\sqrt{2}}{3} x^{3/2} + c$

$= \frac{2}{3} x^3 + 3\cos x + \frac{10}{3} x^{3/2} + c$

iv] $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$\Rightarrow \quad I = \int \frac{x^3+3x+4\,dx}{\sqrt{x}}$

$I = \int \frac{x^3+3x+4\,dx}{\sqrt{x}}$

$= \int \frac{x^3+3x+4\,dx}{\sqrt{x}}$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right) dx$

$= \int (x^3 \cdot x^{-1/2} + 3x \cdot x^{-1/2} + 4x^{-1/2})\,dx$

$= \int x^{5/2}\,dx + 3\int x^{-1/2}\,dx + 4\int x^{-1/2}\,dx$

$= \frac{2}{7} x^{7/2} + 3\cdot\frac{2}{3} x^{3/2} + 4x^{-1/2}\cdot 2 + c$

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8\sqrt{x} + c$

v] $\int t^7 \sin(2t^4)\,dt$

$I = \int t^4 \sin(2t^4)\,dt$

let $t^4 = x$

$4t^3\,dt = dx$

$I = \frac{1}{4}\int 4t^3 \cdot t^4 (\sin(2t^4))\,dt$

$= \frac{1}{4}\int x \cdot \sin(2x)\,dx$

$= \frac{1}{4}$

$= \frac{1}{4}$

$= \frac{1}{4}$

$= -\frac{1}{8}$

$= -\frac{1}{8} +$

(6) $\int \sqrt{x}\,(x$

$I = \int \sqrt{x}$

$= \int (\sqrt{}$

$= \int ($

$= \frac{2}{7}$

(7) $\int \frac{1}{x^3}$

$\Rightarrow I = \int$

let

x^2

$\frac{-2}{x^3}$

$I =$

$=$

$=$

$= \frac{1}{4} [x \int \sin 2x - \int [\int \sin 2x \cdot \frac{d}{dx} x]$

$= \frac{1}{4} [-x \frac{\cos 2x}{2} + \frac{1}{2} \int \cos 2x \cdot 1]$

$= \frac{1}{4} [-\frac{\cos 2x}{2} + \frac{1}{4} \sin 2x] + c$

$= -\frac{1}{8} \cdot x \cos 2x + \frac{1}{16} \sin 2x + c$

$= \frac{-11}{8} + 4 \cos (2+4) + \frac{1}{16} \sin (2+4) + c.$

⑥ $\int \sqrt{x} (x^2 - 1) dx$

$I = \int \sqrt{x} (x^2 - 1) dx$

$= \int (\sqrt{x} \cdot x^2 - \sqrt{x}) dx$

$= \int (x^{5/2} - \sqrt{x}) dx$

$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + c$

⑦ $\int \frac{1}{x^3} \sin (\frac{1}{x^2}) dx$

$\Rightarrow I = \int \frac{1}{x^3} \sin (\frac{1}{x^2}) dx$

let $\frac{1}{x^2} = t$

$x^{-2} = t,$

$\frac{-2}{x^3} dx = dt.$

$I = -\frac{1}{2} \int \frac{-2}{x^3} \sin (\frac{1}{x^2}) dx$

$= -\frac{1}{2} \int \sin t$

$= -\frac{1}{2} (-\cos t) + c$

$= \frac{1}{2} \cos t + c$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$

(8) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$\Rightarrow I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

let ~~co~~ $\sin x = t$

$\cos x \, dx = dt$

$I = \int \frac{dt}{\sqrt[3]{t^2}}$

$= \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3} \cdot dt$

$= 3t^{1/3} + c$

$= 3(\sin x)^{1/3} + c$

$= 3 \cdot \sqrt[3]{\sin x} + c$

(9) $\int e^{\cos^2 x} \cdot \sin^2 x \, dx$

$\Rightarrow I = \int e^{\cos^2 x} \sin^2 x \, dx$

let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$I = \int -\sin 2x \, e^{\cos^2 x} dx$

$= -\int e^t dt$

$= e^t + c$

Resubstituting $t = \cos^2 x$

$= -e^{\cos^2 x} + c$

(4) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

$I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

Let $x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x) dx = dt$

$3(x^2 - 2x) dx = dt$

$(x^2 - 2x) dx = \frac{dt}{3}$

$I = \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + c$ Resubstituting $t = x^3 - 3x^2 + 1$

$I = \frac{1}{3} \log(x^3 - 3x^2 + 1) + c.$

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PRACTICAL NO - 06

Topic :- Application of Integration and Numerical Integration

Q.1 Find the length of following curve.

1. $x = t - \sin t$, $y = 1 - \cos t$ $\quad t \in [0, 2\pi]$
2. $y = \sqrt{4 - x^2}$ $\quad x \in [-2, 2]$
3. $y = x^{3/2}$ $\quad [0, 4]$
4. $x = 3\sin t$, $y = 3\cos t$ $\quad t \in [0, 2\pi)$
5. $x = \frac{1}{6} y^3 + \frac{1}{2y}$ $\quad y \in [1, 2]$

Q.2 Using Simpson's Rule Solve the following.

1. $\int_0^2 e^{x^2} dx$ with $n = 4$
2. $\int_0^4 x^2 dx$ with $n = 4$
3. $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

Solution :-

(i) $x = t - \sin t$ $\qquad y = 1 - \cos t$

$$L = \int_0^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$\frac{dy}{dt} = \sin t$ $\qquad \frac{dx}{dt} = 1 - \cos t$

$$L = \int_0^{2\pi} \sqrt{(\sin t)^2 + (1 - \cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{\sin^2 t + 1 + \cos^2 t - 2\cos t}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \sqrt{2} \int_0^{2\pi} \sqrt{1 - \cos t} \cdot dt$$

$$= \sqrt{2} \int_0^{2\pi} \sqrt{2\sin^2\left(\frac{t}{2}\right)} dt$$

$$\cancel{= \sqrt{2} \cdot \sqrt{2}}$$

$$= 2 \times 2 \left[-\cos\left(\frac{t}{2}\right) \right]_0^{2\pi}$$

$$= 4[-1 - 1]$$

$$= 8$$

$x \in [-2, 2]$

(2) $y = \sqrt{4 - x^2}$

$$L = \int_0^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1(-2x)}{2\sqrt{a - x}}$$

$$= \frac{-x}{\sqrt{4 - x}}$$

$$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x^2}}\right)^2}\, dx$$

$$L = \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4 - x^2}}$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= 2\int_{-2}^{2} \sqrt{\frac{4}{4 - x^2}}\, dx$$

$$= 2\int_{-2}^{2} \frac{1}{\sqrt{(2)^2-(x)^2}}\,dx$$

$$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= 2\pi$$

(3) $y = x^{3/2}$ $\quad x \in [0, 4]$ (5)

$$L = \int_0^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$$

$$L = \int_0^4 \sqrt{1+\frac{9}{4}x}\,dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\,dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{1}{9}\left[(4+9x)^{3/2}\right]_0^9$$

$$= \frac{1}{27}\left[40^{3/2} - 8\right]$$

(4) $x = 3\sin t$ $\quad y = 3\cos t$ $\quad t \in [0, 2\pi]$

$$L = \int_0^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt$$

$$\frac{dy}{dt} = -3\sin t \quad \frac{dx}{dt} = 3\cos t$$

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$L = \int_0^{2\pi} \sqrt{(-3\sin^2 t) + (3\cos t)^2}$

$L = 3\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\ dt$

$= 3\int_0^{2\pi} \sqrt{1}\ dt$

$= 3[t]_0^{2\pi}$

$= 6\pi$

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$

$L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$

$\frac{dx}{dy} = \frac{3}{6}y^2 - \frac{1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}y^2 - \frac{1}{2y^2}\right)^2}\ dy$

$= \int_1^2 \sqrt{1 + \left(\frac{y^4 - 1}{2y^2}\right)^2}\ dy$

$= \int_1^2 \sqrt{\frac{(y^4-1)^2 + 4 \times y^4 \times 1}{4y^4}}\ dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\ dy$

$\int_1^2 \frac{y^4+1}{2y^2}\ dy$

$= \int_1^2 \frac{y^2}{2} dy + \int_1^2 \frac{1}{2y^2} dy$

$= \frac{1}{2}\left[\frac{y^3}{3}\right]_1^2 + \frac{1}{2}\left[\frac{-1}{y}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{3}\right] + \frac{1}{2}\left[\frac{1}{2} - 1\right]$

$= \frac{7}{6} - \frac{1}{4}$

$= \frac{14-3}{12}$

$= \frac{11}{12}$ //

Q. 2

① $\int_0^2 e^{x^2} dx$ with $n = 4$

$a = 0,\ b = 2,\ n = 4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

x	0	0.5	1	1.5	2
y	1	1.2540	2.7182	9.4877	54.5981

By Simpson's Rule

$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182)\right]$

$= 17.3535$

② $\int_0^4 x^2 dx$

$a = 0,\ b$

$h = \frac{b-a}{n}$

x 0 1

y 0 1

By Simpson's

$\int_0^4 x^2 dx =$

③ $\int_0^{\pi/} \sqrt{\sin}$

$a = 0$

$h = \pi$

x 0

y 0

By Sim

$\int_0^{\pi/3} \sqrt{\sin x}$

② $\int_0^4 x^2\,dx$ with $n=4$

$a=0$, $b=4$, $n=4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

By simpson's Rule,

$\int_0^4 x^2\,dx = \frac{1}{3}[(0+16)+4(1+9)+2(4)]$

$= \frac{1}{3}(16+40+8)$

$= \frac{1}{3}64$

$= \frac{64}{3}$

③ $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n=6$

$a=0$, $b=\frac{\pi}{3}$, $n=6$

$h = \frac{\frac{\pi}{3}-0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752	0.9306

By Simpson's Rule,

$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}[(y_0+y_6) + 4(y_1+y_5+y_5)+2(y_2+y_4)]$

$$= \frac{\pi}{54}\Big[(0+0.9306) + 4(0.4167 + 0.7071 + 0.8752) + 2(0.5848 + 0.5017)\Big]$$

$$= \frac{\pi}{54} \times 11.6996$$

$$= 0.6806$$

$$\int_0^{\pi/3} \sqrt{\sin x} = 0.6806$$

PRACTICAL No - 07.

Topic : Differential Equation.

Q Solve the following differential equation

1) $x\frac{dy}{dx} + y = e^x$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x} y = 2x$

6) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

7] $\frac{dy}{dx} = \sin^2 (x-y+1)$

8] $\frac{dy}{dx} = \frac{2y + 3y - 1}{6x + 9y + 6}$

Solution!-

1) $x\frac{dy}{dx} + y = e^x$

$\therefore \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{1}{x}$; $Q(x) = \frac{e^x}{x}$

I.F $= e^{\int 1/x\, dx}$

$= e^{\log x}$

I.F $= x$

$y\,(I.F) = \int Q(x)\,(I.F)\,dx$

$y\,(x) = \int \frac{e^x}{x} \cdot x\, dx$

$yx = \int e^x\, dx$

$xy = e^x + c$

2] $e^x \frac{dy}{dx} + 2e^x y = 1$

$e^x \left(\frac{dy}{dx} + 2y\right) = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2$; $Q(x) = \frac{1}{e^x}$

I.F $\cancel{= e^{\int 2\, dx}}$

$= e^{2x}$

$y\,(I.F) = \int Q(x)\cdot(I.F)\, dx$

$ye^{2x} = \int \frac{1}{e^{2x}}\, e^{2x}\, dx$

$ye^{2x} = \int e^{-x} \cdot e^{e^x}\, dx$

$ye^{2x} = \int e^x\, dx$

$ye^{2x} = e^x + c$

(3). $x \frac{dy}{dx} = \frac{\cos}{x}$

$x \frac{dy}{dx} + 2y =$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y$

I.F $= e^{\int 2/x\, d}$

$= e^{2 \log}$

$= e^{\log x^2}$

I.F $= x^2$

$\therefore y\,(I.F) =$

$y\,(x^2) =$

$x^2 y = \sin$

4) $x \frac{dy}{dx} + 3y$

$\frac{dy}{dx} + \left(\frac{3}{x}\right.$

Comparing

P(

I.F $= e$

$= e$

$= e$

$= x$

$y\,(I.F) =$

$y\,(x^3)$

(3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = \frac{\cos x}{x^2}$

$I.F = e^{\int 2/x\, dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$I.F = x^2$

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$y(x^2) = \int \frac{\cos x}{x^2}(x^2)\,dx$

$x^2 y = \sin x + c$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1}$. $Q(x) = \frac{\sin x}{x^3}$

$I.F = e^{\int 3/x\, dx}$

$= e^{3\log x}$

$= e^{\log x^3}$

$= x^3$

$y(I.F) = \int Q(x)(I.F)\,dx$

$y(x^3) = \int \frac{\sin x}{x^3}(x^3)\,dx$

$$x^3 y = -\cos x + C$$

5) $$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

Comparing with $\frac{dy}{dx} + P(x)\,y = Q(x)$

$\therefore P(x) = 2 \qquad Q(x) = \frac{2x}{e^{2x}}$

$$I.F = e^{\int 2\,dx}$$
$$= e^{2x}$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx$$

$$y\,(e^{2x}) = \int \frac{2x}{e^{2x}}\,(e^{2x})\,dx$$

$$y e^{2x} = \frac{2x^2}{2} + C$$

$$y e^{2x} = x^2 + C$$

6) $$\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$$

$$\sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$$

$$\frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 y}{\tan y}\,dy$$

$$\int \frac{\sec^2 x}{\tan x}\,dx = -\int \frac{\sec^2 y}{\tan y}\,dy$$

$$\therefore \log|\tan x| = -\log|\tan y| + c$$

$\log|\tan x| + \log|\tan y| = c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = 1 - \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$1 - \sin^2 v = \frac{dv}{dx}$

$dx = \frac{dv}{1-\sin^2 v}$

$\int dx = \int \sec^2 v \, dv$

$x = \tan v + c$

Put $v = x + y - 1$

$x = \tan(x+y-1) + c$

8) $\frac{dy}{dx} = \frac{2y+3y-1}{6x+9y+6}$

$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$

Put $2x + 3y = v$

18.

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int \frac{(v+1+1)}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\int 1 + \frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}(v + \log(v+1)) = x + c$$

But $v = 2x + 3y$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c$$

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PRACTICAL No :- 08

Topic :- Euler's Method.

① $\frac{dy}{dx} = y + e^x - 2$ $\quad y(0) = 2, h = 0.5$, find $y(2)$

② $\frac{dy}{dx} = 1 + y^2$ $\quad y(0) = 0, h = 0.2$ find $y(1)$

③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1, h = 0.2$ find $y(1)$

④ $\frac{dy}{dx} = 3x^2 + 1$ $\quad y(1) = 2$, find $y(2)$
for $h = 0.5$, $h = 0.25$

⑤ $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y(1) = 1$ Find $y(1.2)$ with $h = 0.2$

Solution :-

① $\frac{dy}{dx} = y + e^x - 2$

$f(x, y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

$\therefore$ By Euler's formula
$y(2) = 9.2831$

② $\frac{dy}{dx} = 1+y^2$

$f(x,y) = 1+y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula.

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

By Euler's formula,
$y(1) = 1.2942$

③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

(4) $\frac{dy}{dx} = 3x^2 + 1$ $\quad y_0 = 2,\ x_0 = 1,\ h = 0$

for $h = 0.5$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

$\therefore$ By Euler's Formula

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1817	8.9048
4	2	8.9048		

By Euler's Formula

$y(2) = 8.9048$ //

13

(5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y_0 = 1$, $x_0 = 1$, $h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

∴ By Euler's formula,

$y(1.2) = 1.6$,,

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Pratical No: 9

Limits & Partial order derivatives

Q1 Evaluate the following limits.

i) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit:

$$= \frac{(-4)^3-3(-4)+(-1)^2-1}{-4(-1)+5}$$

$$= \frac{-64+3+1-1}{4+5}$$

$$= \frac{-61}{9}$$

ii) $$\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit.

$$\frac{(0+1)((2)^2+0-4(2))}{2+0}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{-4}{2}$$

$$= -2$$

iii) $$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$$

At (1,1,1) Denominator = 0

$$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2z^2}{x^3 - x^2y^2}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{(\cancel{x-yz})(x+yz)}{x^2(\cancel{x^2-y^2})}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

On applying limit

$$= \frac{1+1(1)}{(1)^2}$$

$$= 2$$

Q.2

i) $f(x,y) = xye^{x^2+y^2}$

$$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{dx}(xye^{x^2+y^2})$$

$$= ye^{x^2+y^2}(2x)$$

$$\therefore f_x = 2xye^{x^2+y^2}$$

$$f_y = \frac{\partial}{\partial y} f(x,y)$$

$$= \frac{\partial}{dy}(xye^{x^2+y^2})$$

$$= xe^{x^2+y^2}(2y)$$

$$f_y = 2yxe^{x^2+y^2}$$

ii) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$\therefore f_y = -e^x \sin y$

iii) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3 y^2 - 3x^2 y + y^3 + 1)$

$f_x = 3x^2 y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3 y^2 - 3x^2 y + y^3 + 1)$

$\therefore f_y = 2x^3 y - 3x^2 + 3y^2$

Q.3)

i) $f(x,y) = \dfrac{2x}{1+y^2}$

$$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \cancel{\frac{\partial}{\partial x}}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

Q4)

i) $f(x,y)$

$f_x =$

$f_y =$

f_x

f_{yy}

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

(ii)

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \quad \text{—} (1)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right) \quad \text{—} (2)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial y} \frac{(-x^2 y - 2xy^2 + 2x^2 y)}{x^4}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{dx}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{dx}(2y - x) - (2y - x)\frac{\partial}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{— (4)}$$

from (3) & (4);

$f_{xy} = f_{yx}$

2) $f(x, y) = x^3 + 3x^2 y^2 - \log(x^2 + 1)$

$$f_x = \frac{\partial}{dx}(x^3 + 3x^2 y^2 - \log(x^2 + 1))$$

$$f_y = \frac{\partial}{dy}(x^3 + 3x^2 y^2 - \log(x^2 + 1))$$

$$f_x = 3x^2 + 6xy^2 - \frac{2x}{x^2 + 1} \qquad f_y = 0 + 6x^2 y - 0$$

$$= 6x^2 y$$

$$f_{xx} = 6x + 6y^2 - \left(x^2 + 1\frac{\partial}{dx} 2x - 2x \frac{\partial(x^2+1)}{\partial x}\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2 + 1) - 4x^2}{(x^2 + 1)^2}\right)$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2 y)$$

$$= 6x^2 \quad \text{— (2)}$$

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$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$ — (3)

$f_{yx} = \frac{\partial}{\partial x}(6x^2 y)$

$= 12xy$ — (4)

from (3) & (4)

$f_{xy} = f_{yx}$

$f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — (1)

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) \cancel{+ e^{x+y}})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — (2)

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$ — (3)

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$

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$$f_{yx} = \frac{\partial}{\partial x}\left(x \cdot \cos(xy) + e^{x+y}\right)$$

$$= -x^2 \sin(xy) + \cos(xy) + e^{x+y} \quad \text{—— (4)}$$

$\therefore$ from (3) & (4)

$f_{xy} \neq f_{yx}$

Q.3)

1) $f(x, y) = \sqrt{x^2+y^2}$ at $(1, 1)$

$\longrightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x) \qquad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{x}{\sqrt{x^2+y^2}} \qquad = \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$$

$$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

Q2 $f(x,y) = 1 - x + y \sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y \cos x$ $\qquad$ $f_y = 0 - 0 + \sin x$

f_x at $(\pi/2, 0) = -1 + 0$ $\qquad$ f_y at $(\pi/2, 0) = \sin \pi/2$

$= -1$ $\qquad$ $= 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y - 0)$

$= 1 - \cancel{\pi/2} - x + \cancel{\pi/2} + y$

$= 1 - x + y$

Q3 $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad$ $f_y = 0 + 1/y$

f_x at $(1,1) = 1$ $\qquad$ f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$ //

11/11/2020

PRACTICAL NO:- 10

Directional Derivative, Gradient vector & maxima minima, & Tanget & normal vectors.

Q.1

1) $f(x,y) = x + 2y - 3$ $a = (1,-1)$, $u = 3i - j$

$\Rightarrow$ Here,

$u = 3i - j$ is not a unit vector.

$\bar{u} = 3i - j$

$|u| = \sqrt{10}$

$\therefore$ Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3i - j)$

$= \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now,

$f(a + hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$= f\left(1 + \frac{3u}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$

$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}}, \frac{-2h}{\sqrt{10}}$

$= -4 + \frac{h}{\sqrt{10}}$

$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

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$$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} - (-4)}{h}$$

$$= \lim_{h \to 0} \frac{4}{4\sqrt{10}}$$

$$= \frac{1}{\sqrt{10}}$$

③ $f(x, y) = y^2 - 4x + 1$; $a = (3, 4)$, $u = i + 5j$

⇒ Here,

$u = i + 5j$ is not a unit vector

$\bar{u} = i + 5j$

$|\bar{u}| = \sqrt{26}$

∴ Unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(i + 5j)$

$$= \frac{1}{\sqrt{26}}(1, 5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

Now

$$f(a + hu) = f\left((3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, \frac{4 + 5h}{\sqrt{26}}\right)$$

$$= \left(\frac{4 + 5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{26h^2}{26} + \frac{36h}{26} + 5$$

Q5.

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

3) $f(x,y) = 2x + 3y \qquad a = (1,2)\ ;\ u = 3i + 4j$

Here,

$u = 3i + 4j$ is not a unit vector.

$\bar{u} = 3i + 4j$

$|\bar{u}| = \sqrt{25} = 5$

Unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3i + 4j)$

$$= \frac{1}{5}(3,4)$$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

Now,

$$f(a+hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$$

$$= f\left(1 + \frac{3h}{5},\ 2 + \frac{4h}{5}\right)$$

$$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$= 2$

$= 8 +$

$D_u f(a)$

Q.2

i) $f(x,y) =$

$f_x = y$

$f_y = x$

$\nabla f(x,$

$\nabla f(x$

$= (1\ ($

$= (1,$

ii) $f(x,y$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= 8 + \frac{18h}{5}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{8 + \frac{18h}{5} - 8}{h}$

$= \lim_{h \to 0} \frac{18h}{5h}$

$= \frac{18}{5}$

$f(x, y) = x^y + y^x$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x, y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, xy^{x-1} + x^y \log x)$

$\nabla f(x, y)$ at $(1, 1)$

$= (1(1)^0 + 1^1 \log(1), 1(1)^{1-1} + 1^1 \log 1)$

$= (1, 1)$

$f(x, y) = \tan^{-1} x \cdot y^2$, $a = (1, -1)$

$f_x = y^2 \left(\frac{1}{1+x^2} \right) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$\nabla f(x,y)$ at $(1,-1)$

$= \left(\frac{(-1)^2}{1+(1)^2} ; 2(-1) \tan^{-1}(1)\right)$

$= \frac{1}{2}, \frac{-2\pi}{4}\Big)$

$= \left(\frac{1}{2}, \frac{\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$
$f_y = xz - e^{x+y+z}$
$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy$

$\nabla f(x,y,z)$ at $(+1,-1,0)$

$= \cancel{-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+}}$

$= \cancel{(0-1, 0-1, -1-1)}$

$= (-1, -1, -2)$

Q3)

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f(x,y) = x^2 \cos y + e^{xy} - 2$

$f_x = 2x \cos y + y e^{xy}$

$f_y = -x^2 \sin y + x e^{xy}$

$(x_0, y_0) = (1,0)$

f_x at $(1,0) = 2(1)\cos(0) + 0$
$= 2$

f_y at $(1,0) = -(1)^2 \sin 0 + 1 (e)^{(0)}$
$= 1$

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0 \longrightarrow$ Equation of Tangent.

Now,

For equation of Normal,

$bx + ay + d = 0$

$x + 2y + d = 0$ At $(1,0)$

$(1) + 2(0) + d = 0$

$1 + d = 0$

$d = -1$

$x + 2y - 1 = 0 \longrightarrow$ Equation of Normal.

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$ $\quad f_x$ at $(2,-2) = 2(2) - 2$

$= 2x - 2$ $\quad = 2$

$f_y = 0 + 2y - 0 + 3 + 0$ $\quad f_y$ at $(2,-2) = 2(-2) + 3$

$= 2y + 3$ $\quad = -1$

∴ Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0$ → Equation of Tangent.

For Equation at Normal.

$bx + ay + d = 0$

$-x + 2y + d = 0$

$-(2) + 2(-2) + d = 0$ at $(2,-2)$

$-2 - 4 + d = 0$

$d = 6$

$-x + 2y + 6 = 0$ → Equation of Normal.

Q.4.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z - 0$ $\quad$ ∴ f_x at $(2,1,0) = 2(2) + 0$

$= 2x + z$ $\quad = 4$

$f_y = -2z + 3 + 0 - 0$ $\quad$ ∴ f_y at $(2,1,0) = -2(0) + 3$

$= -2z + 3$ $\quad = 3$

$f_z = 0 - 2y + 0 + x - 0$ $\quad f_z$ at $(2,1,0) = -2(1)+2$
$= -2y + x$ $\quad = 0$

Equation of tangent,

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0$ — Equation of tangent

Equation of normal,

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$ — Equation of normal.

ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$f(x,y,z) = 3xyz - x - y + z + 4$.

$f_x = 3yz - 1 - 0 + 0 + 0$ $\quad f_x$ at $(1,-1,2) = 3(-1)(2) - 1$
$= 3yz - 1$ $\quad = -7$

$f_y = 3xz - 0 - 1 + 0 + 0$ $\quad f_y$ at $(1,-1,2) = 3(1)(2) - 1$
$= 3xz - 1$ $\quad = 5$

$f_z = 3xy - 0 - 0 + 1 - 0$ $\quad f_z$ at $(1,-1,2) = 3(1)(-1)+1$
$= 3xy + 1$ $\quad = -2$

Equation of tangent,

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$-7(x-1) + 5(y+1) + (-2)(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$ — Equation of tangent.

$-7x + 5y - 2z + 16 = 0$

Equation of normals

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$ — Equation of normal.

Q9

1) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\therefore f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$ — (1)

$f_y = 2y - 3x + 0 - 4$

$= 2y - 3x - 4$ — (2)

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — (3)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (4)

Multiplying (3) by (2) and substracting (4) from (3)

$4x - \cancel{2y} = -4$

$+\cancel{2y} - 3x = 4$

$7x = 0$

$x = 0$

Substituting value of x in (3)

$2(0) - y = -2$

$-y = -2$

$y = 2$

$\therefore$ Critical points are $(0,2)$

Now,

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$= 3 > 0$

Here, $r > 0$ and $rt - s^2 > 0$

f has minimum at $(0,2)$

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

) $f(x,y) = 2x^4 + 3x^2 y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

Now

$f_x = 0$

$8x^3 + 6xy = 0$

$4x^2 + 6xy = 0$ —(1)

$f_y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$ —(2)

Multiplying in (1) by 3 and (2) by 4

Subtracting (2) from (1)

$12x^2 + 18y = 0$

$12x^2 - 8y = 0$

$24y = 0$

$y = 0$ —(3)

Substituting $y = 0$ in ②

$x = 0$ — ④

Critical points are $(0, 0)$

Now,

$r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -84x^2 - 12y$

At $(0, 0)$

$r = 24(0)^2 + 6(0)$

$= \underline{\underline{0}}$

$rt - s^2 = 84(0)^2 + 12(0) = 0$

$r = 0$ & $rt - s^2 = 0$

$\therefore$ Nothing can be said.

07/09/20