

AlgoLab AS 2014

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1 ACM

1.1 Even Pairs

Keywords: Scanline

```
#include <iostream>
#include <vector>

using namespace std;

int main(void)
{
    int n;
    cin >> n;

    vector<int> elems(n);
    for(int i = 0; i < n; i++) {
        cin >> elems.at(i);
    }

    int even = 0;
    for(int start = 0; start < n; start++) {
        for(int end = start; end < n; end++) {
            int sum = 0;
            for(int i = start; i <= end; i++) {
                sum += elems.at(i);
            }

            if(sum % 2 == 0) {
                even++;
            }
        }
    }

    cout << even << endl;

    return 0;
}
```

```
#include <iostream>
#include <vector>

using namespace std;

int main () {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int n;
    cin >> n;

    int nr_odd_input = 0;
    int even = 1;
    int odd = 0;
    int total_pairs = 0;
    short next_number;
    for (int i = 0; i < n; i++) {
        cin >> next_number;
        if (next_number == 1) {
            nr_odd_input++;
        }
        if ((nr_odd_input & 0x1) == 0) {
            //cout << "even: " << even << endl;
            total_pairs += even++;
        } else {
            //cout << "odd: " << odd << endl;
            total_pairs += odd++;
        }
    }

    cout << total_pairs << endl;
    return 0;
}
```

1.2 Build The Sum

Keywords:

```
#include <iostream>
#include <vector>

using namespace std;

int main(void)
{
    int n;
    cin >> n; // ignore this

    while(cin >> n) {
        float sum = 0.0;
    }
}
```

```

        for(int i=0; i < n; i++) {
            float v = 0.0;
            cin >> v;
            sum += v;
        }
        cout << sum << endl;
    }

    return 0;
}

```

```

#include <iostream>
#include <vector>

using namespace std;

int main () {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int m;
        cin >> m;

        double sum = 0, c = 0, summand;
        for (int i = 0; i < m; i++) {
            cin >> summand;
            double y = summand - c;
            double t = sum + y;
            c = (t - sum) - y;
            sum = t;
        }

        cout << sum << endl;
    }

    return 0;
}

```

1.3 Shelves

Keywords:

```

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int N;
    cin >> N;

    for(int i = 0; i < N; i++) {
        int length, large_shelve_length, small_shelve_length;
        cin >> length;
        cin >> small_shelve_length;
        cin >> large_shelve_length;

        int opt_small_shelves = 0;
        int opt_large_shelves = 0;
        int opt_wall_left = length;

        int sqrt_len = (int)sqrt(length);
        if(large_shelve_length > sqrt_len) {
            for(int large_shelves = min(sqrt_len, (int)(length / large_shelve_length)); large_shelves >= 0; large_shelves--) {
                int wall_left_for_small = (length - (large_shelves * large_shelve_length));
                int wall_left = wall_left_for_small % small_shelve_length;
                if(wall_left < opt_wall_left) {
                    opt_wall_left = wall_left;
                    opt_small_shelves = (int)(wall_left_for_small / small_shelve_length);
                    opt_large_shelves = large_shelves;
                }

                if(opt_wall_left == 0) {
                    break;
                }
            }
        } else {
            int max_small = min(sqrt_len, (int)(length / small_shelve_length));
            for(int small_shelves = 0; small_shelves <= max_small; small_shelves++) {
                int wall_left_for_large = (length - (small_shelves * small_shelve_length));
                int wall_left = wall_left_for_large % large_shelve_length;
                if(wall_left < opt_wall_left) {

```

```

        opt_wall_left = wall_left;
        opt_small_shelves = small_shelves;
        opt_large_shelves = (int)(wall_left_for_large / large_shelve_length);
    }

    if(opt_wall_left == 0) {
        break;
    }
}

cout << opt_small_shelves << " " << opt_large_shelves << " " << opt_wall_left << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        unsigned int total_length, small_length, large_length;
        cin >> total_length >> small_length >> large_length;

        unsigned int best_nr_small_shelves = 0, best_nr_large_shelves = 0, best_uncovered_wall = total_length;
        int min_large_shelves = 0;
        if (((long long)small_length)*((long long)large_length) < (long long)numeric_limits<int>::max() && small_length*
            large_length <= total_length) {
            min_large_shelves = total_length / large_length - small_length;
        }

        for (int nr_large_shelves = total_length / large_length; nr_large_shelves >= min_large_shelves; nr_large_shelves--) {
            unsigned int uncovered_wall_large = total_length - (large_length*nr_large_shelves);
            unsigned int nr_small_shelves = uncovered_wall_large / small_length;
            unsigned int uncovered_wall = uncovered_wall_large - (small_length*nr_small_shelves);
            if (uncovered_wall < best_uncovered_wall) {
                best_uncovered_wall = uncovered_wall;
                best_nr_large_shelves = nr_large_shelves;
                best_nr_small_shelves = nr_small_shelves;
                if (best_uncovered_wall == 0) {
                    break;
                }
            }
        }

        cout << best_nr_small_shelves << " " << best_nr_large_shelves << " " << best_uncovered_wall << endl;
    }

    return 0;
}

```

1.4 Checking Change

Keywords: Dynamic Programming

```

#include <iostream>
#include <vector>
#include <cmath>
#include <climits>

using namespace std;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int n;
    cin >> n;

    for(int i = 0; i < n; i++) {
        int c, m;
        cin >> c;
        cin >> m;

        vector<int> coins(c);
        for(int j = 0; j < c; j++) {
            cin >> coins.at(j);
        }

        vector<int> ret(m);
        int max_ret = 0;
    }
}

```

```

    for(int j = 0; j < m; j++) {
        int ret_val;
        cin >> ret_val;

        ret.at(j) = ret_val;
        if(ret_val > max_ret) {
            max_ret = ret_val;
        }
    }

    vector<int> table(max_ret + 1);
    table.at(0) = 0;

    for(int v = 1; v <= max_ret; v++) {
        table.at(v) = INT_MAX;
        for(auto coin : coins) {
            if(v - coin >= 0 && coin <= v) {
                if(table.at(v-coin) != INT_MAX) {
                    table.at(v) = min(table.at(v), table.at(v-coin) + 1);
                }
            }
        }
    }

    for(auto r : ret) {
        if(table.at(r) == INT_MAX) {
            cout << "not possible" << endl;
        } else {
            cout << table.at(r) << endl;
        }
    }
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_coins, nr_test_values;
        cin >> nr_coins >> nr_test_values;

        vector<int> coins(nr_coins);
        for (int i = 0; i < nr_coins; i++) {
            cin >> coins[i];
        }

        int max_test_value = 0;
        vector<int> test_values(nr_test_values);
        for (int test_value_id = 0; test_value_id < nr_test_values; test_value_id++) {
            cin >> test_values[test_value_id];
            if (test_values[test_value_id] > max_test_value) {
                max_test_value = test_values[test_value_id];
            }
        }

        vector<int> nr_coins_for_values(max_test_value + 1, -1);
        nr_coins_for_values[0] = 0;
        for (int value = 0; value <= max_test_value; value++) {
            int nr_coins_for_value = nr_coins_for_values[value];
            if (nr_coins_for_value != -1) {
                for (int coin_id = 0; coin_id < nr_coins; coin_id++) {
                    int new_value = value + coins[coin_id];
                    if (new_value <= max_test_value) {
                        if (nr_coins_for_values[new_value] == -1) {
                            nr_coins_for_values[new_value] = nr_coins_for_value + 1;
                        }
                        else {
                            nr_coins_for_values[new_value] = min(nr_coins_for_values[new_value], nr_coins_for_value + 1);
                        }
                    }
                }
            }
        }

        for (int test_value_id = 0; test_value_id < nr_test_values; test_value_id++) {
            int nr_coins_for_value = nr_coins_for_values[test_values[test_value_id]];
            if (nr_coins_for_value == -1) {
                cout << "not possible" << endl;
            }
            else {
                cout << nr_coins_for_value << endl;
            }
        }
    }
}

```

```

    }
}
}

return 0;
}

```

1.5 Even Matrices

Keywords:

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int n;
        cin >> n;
        vector<vector<bool>> > matrix(n, vector<bool>(n, 0));
        for (int i = 0; i < n; i++) {
            vector<bool>& row = matrix[i];
            for (int j = 0; j < n; j++) {
                short input;
                cin >> input;
                row[j] = (input == 1);
            }
        }

        int even = 0;
        for (int row_i = 0; row_i < n; row_i++) {
            vector<bool> previous(n, 0);
            for (int row_j = row_i; row_j < n; row_j++) {
                for (int i = 0; i < n; i++) {
                    previous[i] = previous[i] != matrix[row_j][i];
                    //cout << matrix[row_j][i] << " ";
                }
                //cout << endl;
                int nr_odd_input = 0;
                int e = 1;
                int o = 0;
                for (int i = 0; i < n; i++) {
                    if (previous[i] == 1) {
                        nr_odd_input++;
                    }
                    if ((nr_odd_input & 0x1) == 0) {
                        //cout << "even: " << even << endl;
                        even += e++;
                    }
                    else {
                        //cout << "odd: " << odd << endl;
                        even += o++;
                    }
                }
            }
        }
        cout << even << endl;
    }

    return 0;
}

```

1.6 Race Tracks

Keywords: BFS (Graph)

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>

using namespace std;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;

```



```

cin >> test_cases;

for(int i = 0; i < test_cases; i++) {
    int width_x, height_y;
    cin >> width_x;
    cin >> height_y;

    int start_x, start_y;
    cin >> start_x;
    cin >> start_y;

    int end_x, end_y;
    cin >> end_x;
    cin >> end_y;

    int obstacle_count;
    cin >> obstacle_count;
    vector<vector<bool>> obstacles(width_x, vector<bool>(height_y, false));
    for(int j = 0; j < obstacle_count; j++) {
        int x1, x2, y1, y2;
        cin >> x1;
        cin >> y1;
        cin >> x2;
        cin >> y2;

        for(int k = x1; k <= x2; k++) {
            for(int l = y1; l <= y2; l++) {
                obstacles.at(k).at(l) = true;
            }
        }
    }

    // check
    /*cout << "width: " << width_x << " & height: " << height_y << endl;
    cout << "start: x=" << start_x << ", y=" << start_y << endl;
    cout << "end: x=" << end_x << ", y=" << end_y << endl;
    cout << "obstacles:" << endl;
    for(int y = 0; y < height_y; y++) {
        for(int x = 0; x < width_x; x++) {
            if(obstacles.at(x).at(y)) {
                cout << "X";
            } else if(start_x == x && start_y == y) {
                cout << "S";
            } else if(end_x == x && end_y == y) {
                cout << "E";
            } else {
                cout << "0";
            }
        }
        cout << endl;
    }
    cout << "-----" << endl << endl;*/

    // algo start
    //      x , y , v_x, v_y, depth
    queue<tuple<int, int, int, int, int>> next_tiles;
    next_tiles.push(make_tuple(start_x, start_y, 0, 0, 0));
    bool found = false;
    vector<vector<vector<vector<bool>>>> visited(width_x, vector<vector<vector<bool>>>(height_y, vector<vector<bool>>>(7, vector<bool>(7, false))));
    while(!next_tiles.empty()) {
        auto tile = next_tiles.front();
        next_tiles.pop();
        int x = get<0>(tile);
        int y = get<1>(tile);
        int v_x = get<2>(tile);
        int v_y = get<3>(tile);
        int depth = get<4>(tile);

        if(x < 0 || x >= width_x ||
           y < 0 || y >= height_y ||
           v_x < -3 || v_x > 3 ||
           v_y < -3 || v_y > 3 ||
           obstacles.at(x).at(y) ||
           visited.at(x).at(y).at(v_x + 3).at(v_y + 3)) {
            //cout << "ignore: x=" << x << ", y=" << y << ", v_x=" << v_x << ", v_y=" << v_y << ", depth=" << depth
            << endl;
            continue;
        }

        /*for(int i_y = 0; i_y < height_y; i_y++) {
            for(int i_x = 0; i_x < width_x; i_x++) {
                if(x == i_x && y == i_y) {
                    cout << "P";
                } else if(obstacles.at(i_x).at(i_y)) {
                    cout << "X";
                } else if(start_x == i_x && start_y == i_y) {
                    cout << "S";
                } else if(end_x == i_x && end_y == i_y) {
                    cout << "E";
                } else {
                    cout << "0";
                }
            }
            cout << endl;
        }
    }
}

```

```

        cout << endl << endl;*/

//cout << "jump: x=" << x << ", y=" << y << ", v_x=" << v_x << ", v_y=" << v_y << ", depth=" << depth << endl
;

if(x == end_x && y == end_y) {
    cout << "Optimal solution takes " << depth << " hops." << endl;
    found = true;
    break;
}

visited.at(x).at(y).at(v_x + 3).at(v_y + 3) = true;

next_tiles.push(make_tuple(x + v_x - 1, y + v_y - 1, v_x - 1, v_y - 1, depth + 1));
next_tiles.push(make_tuple(x + v_x - 1, y + v_y, v_x - 1, v_y, depth + 1));
next_tiles.push(make_tuple(x + v_x - 1, y + v_y + 1, v_x - 1, v_y + 1, depth + 1));

next_tiles.push(make_tuple(x + v_x, y + v_y - 1, v_x, v_y - 1, depth + 1));
next_tiles.push(make_tuple(x + v_x, y + v_y, v_x, v_y, depth + 1));
next_tiles.push(make_tuple(x + v_x, y + v_y + 1, v_x, v_y + 1, depth + 1));

next_tiles.push(make_tuple(x + v_x + 1, y + v_y - 1, v_x + 1, v_y - 1, depth + 1));
next_tiles.push(make_tuple(x + v_x + 1, y + v_y, v_x + 1, v_y, depth + 1));
next_tiles.push(make_tuple(x + v_x + 1, y + v_y + 1, v_x + 1, v_y + 1, depth + 1));

}

if(!found) {
    cout << "No solution." << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>

using namespace std;

class State {
public:
    short x, y, speed_x, speed_y;
    int hops;

    State(short x, short y, short speed_x, short speed_y, int hops)
        :x(x), y(y), speed_x(speed_x), speed_y(speed_y), hops(hops) {}
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int width, height, start_x, start_y, end_x, end_y, nr_obstacles;
        cin >> width >> height >> start_x >> start_y >> end_x >> end_y >> nr_obstacles;

        vector<vector<vector<vector<bool>>>>> race_track(width, vector<vector<vector<bool>>>(height, vector<vector<bool>>(7, vector<bool>(7, false))));

        for (int obstacle_id = 0; obstacle_id < nr_obstacles; obstacle_id++) {
            int obstacle_x_start, obstacle_y_start, obstacle_x_end, obstacle_y_end;
            cin >> obstacle_x_start >> obstacle_y_start >> obstacle_x_end >> obstacle_y_end;

            vector<vector<bool>> obstacle_indicator(7, vector<bool>(7, true));
            for (int x = obstacle_x_start; x <= obstacle_x_end; x++) {
                for (int y = obstacle_y_start; y <= obstacle_y_end; y++) {
                    race_track[x][y] = obstacle_indicator;
                }
            }
        }

        int min_hops = -1;
        if (start_x == end_x && start_y == end_y) {
            min_hops = 0;
        }
        else {
            queue<State> states;
            states.push(State(start_x, start_y, 3, 3, 0));
            race_track[start_x][start_y][3][3] = true;
            //cout << "start: " << start_x << " " << start_y << " " << end_x << " " << end_y << endl;
            //cout << "width: " << width << " height: " << height << endl;
            while (!states.empty()) {
                State current_state = states.front();
                states.pop();
                //cout << current_state.x << " " << current_state.y << " " << current_state.speed_x << " " << current_state.speed_y << " " << current_state.hops << endl;
                for (int new_speed_x = max(current_state.speed_x - 1, 0); new_speed_x <= min(current_state.speed_x + 1, 6); new_speed_x++) {

```

```
for (int new_speed_y = max(current_state.speed_y - 1, 0); new_speed_y <= min(current_state.speed_y + 1, 6);
    new_speed_y++) {
    int new_x = current_state.x + new_speed_x - 3;
    int new_y = current_state.y + new_speed_y - 3;
    //cout << "Checking " << new_x << " " << new_y << " " << new_speed_x << " " << new_speed_y << " " <<
        current_state.hops+1 << endl;
    //cout << " " << (new_x >= 0) << " " << (new_x < width) << " " << (new_y >= 0) << " " << (new_y < height) <<
        endl;
    if (new_x >= 0 && new_x < width && new_y >= 0 && new_y < height) {
        if (!race_track[new_x][new_y][new_speed_x][new_speed_y]) {
            if (new_x == end_x && new_y == end_y) {
                min_hops = current_state.hops + 1;
                goto solution_found;
            }
            else {
                race_track[new_x][new_y][new_speed_x][new_speed_y] = true;
                //cout << "Adding " << new_x << " " << new_y << " " << new_speed_x << " " << new_speed_y << " " <<
                    current_state.hops+1 << endl;
                states.push(State(new_x, new_y, new_speed_x, new_speed_y, current_state.hops + 1));
            }
        }
    }
}
}
}
}
}
solution_found:
if (min_hops == -1) {
    cout << "No solution." << endl;
}
else {
    cout << "Optimal solution takes " << min_hops << " hops." << endl;
}
}
return 0;
}
```

1.7 Boats

Keywords: Custom compare, Class with compare, Uses class, Greedy

```
#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>

using namespace std;

class Data {
public:
    int length, ring_pos, min_pos;

    Data(int length, int ring_pos) : length(length), ring_pos(ring_pos), min_pos(ring_pos) {}

    bool operator< (const Data& other) const {
        if(min_pos == other.min_pos) {
            return ring_pos > other.ring_pos;
        }
        return min_pos > other.min_pos;
    }
};

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int N;
        cin >> N;

        priority_queue<Data> boats;
        for(int j = 0; j < N; j++) {
            int l, p;
            cin >> l;
            cin >> p;
            boats.push(Data(l, p));
        }

        int last_end = boats.top().ring_pos;
        boats.pop();
        int counter = 1;
        while(!boats.empty()) {
            auto boat = boats.top();
            boats.pop();
```

```

        if(boat.min_pos - boat.length >= last_end) {
            counter++;
            last_end = boat.min_pos;
        } else if(last_end <= boat.ring_pos) {
            // boat overlaps with a previous boat, we have to move it
            boat.min_pos = last_end + boat.length;
            if(boat.min_pos >= boat.ring_pos) {
                boats.push(boat);
            }
        }
    }

    cout << counter << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>

using namespace std;

class Boat {
public:
    int length, ring_position, min_end_position;
    Boat(int length, int ring_position)
        :length(length), ring_position(ring_position) {
        this->min_end_position = ring_position;
    }
};

class CompareBoat {
public:
    bool operator()(const Boat& lhs, const Boat& rhs) {
        if (lhs.min_end_position == rhs.min_end_position) {
            return lhs.ring_position > rhs.ring_position;
        }
        return lhs.min_end_position > rhs.min_end_position;
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_wizard_boats;
        cin >> nr_wizard_boats;

        priority_queue<Boat, vector<Boat>, CompareBoat> boats;
        for (int wizard_boat_id = 0; wizard_boat_id < nr_wizard_boats; wizard_boat_id++) {
            int boat_length, ring_position;
            cin >> boat_length >> ring_position;
            boats.push(Boat(boat_length, ring_position));
        }

        int current_end = numeric_limits<int>::min();
        int max_boats = 0;
        while (!boats.empty()) {
            Boat b = boats.top();
            boats.pop();
            //cout << "Check boat " << b.min_end_position << " " << b.ring_position << " " << b.length << endl;
            if (b.min_end_position - b.length >= current_end) {
                current_end = b.min_end_position;
                max_boats++;
            }
            else {
                if (current_end <= b.ring_position) {
                    b.min_end_position = current_end + b.length;
                    boats.push(b);
                }
            }
        }
        cout << max_boats << endl;
    }

    return 0;
}

```

1.8 Aliens

Keywords: Custom compare, Compare function, Compare struct, Uses class, Scanline

```

#include <iostream>
#include <vector>

```

```

#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>

using namespace std;

class Alien {
public:
    int from;
    int to;
    bool dominated;
    Alien(int from, int to) : from(from), to(to), dominated(false) {}
};

bool cmp(Alien left, Alien right) {
    if(left.from == right.from) {
        return left.to > right.to;
    } else {
        return left.from < right.from;
    }
}

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int alien_count, human_count;
        cin >> alien_count >> human_count;

        vector<Alien> aliens;

        for(int j = 0; j < alien_count; j++) {
            int p, q;
            cin >> p >> q;

            aliens.push_back(Alien(p, q));
        }

        sort(aliens.begin(), aliens.end(), cmp);

        /*for(auto a : aliens) {
            cout << "alien: [" << a.from << ", " << a.to << "]" << endl;
        }
        cout << endl << endl;
        */

        int last_human_attacked = 0;
        int last_alien_end = 0;
        int count = 0;
        bool not_all_humans = false;
        bool id = false;
        Alien last = Alien(0, 0);

        for(int cur = 0; cur < alien_count; cur++) {
            Alien &cur_alien = aliens.at(cur);

            if(cur_alien.from > last_human_attacked + 1) {
                not_all_humans = true;
                break;
            } else {
                last_human_attacked = max(cur_alien.to, last_human_attacked);
            }

            if(!id && last.from == cur_alien.from && last.to == cur_alien.to && cur_alien.to != 0) {
                count--;
                id = true;
            } else if(cur_alien.to > last_alien_end) {
                count++;
                last = cur_alien;
                last_alien_end = cur_alien.to;
                id = false;
            }
        }

        if(not_all_humans || last_human_attacked != human_count) {
            //cout << "!0" << " last = " << last_human_attacked << endl;
            cout << "0" << endl;
        } else {
            cout << count << endl;
        }
    }

    return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>

```

```

#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

struct RangeComparator {
    bool operator() (const pair<int, int>& lhs, const pair<int, int>& rhs) {
        return (lhs.first == rhs.first) ? lhs.second > rhs.second : lhs.first < rhs.first;
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_alien, nr_humans;
        cin >> nr_alien >> nr_humans;

        multiset<pair<int, int>, RangeComparator> ranges;
        for (int alien_id = 0; alien_id < nr_alien; alien_id++) {
            int first_wounded, last_wounded;
            cin >> first_wounded >> last_wounded;
            if (first_wounded != 0) {
                ranges.insert(make_pair(first_wounded, last_wounded));
            }
        }

        pair<int, int> last_pair = make_pair(0, 0);
        int nr_superior = 0;
        bool gap_found = false;
        bool found_ident = false;

        //cout << endl;
        for (multiset<pair<int, int> >::iterator it = ranges.begin(); it != ranges.end(); ++it) {
            //cout << (*it).first << " " << (*it).second << endl;
            if ((*it).second > last_pair.second) {
                if ((*it).first > last_pair.second + 1) {
                    //gap in ranges found no superior alien
                    gap_found = true;
                    break;
                }
                last_pair = (*it);
                found_ident = false;
                nr_superior++;
            }
            else if ((*it).second == last_pair.second && (*it).first == last_pair.first && !found_ident) {
                nr_superior--;
                found_ident = true;
            }
        }
        if (!gap_found && last_pair.second == nr_humans) {
            cout << nr_superior << endl;
        }
        else {
            cout << 0 << endl;
        }
    }

    return 0;
}

```

1.9 Next Path

Keywords: Custom compare, Compare struct, BFS (Graph)

```

#include <iostream>
#include <algorithm>
#include <vector>
#include <queue>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        // clean up
        best_distance.clear();

        // get graph properties
        int vertex_count, edge_count;
        cin >> vertex_count >> edge_count;
    }
}

```

```

// get start and target vertices
int start_vertex, target_vertex;
cin >> start_vertex >> target_vertex;
start_vertex -= 1; // input starts at 1
target_vertex -= 1; // input starts at 1

// read in edges
vector<vector<int> >> edges(vertex_count, vector<int>());
for(int edge_index = 0; edge_index < edge_count; edge_index++) {
    int from, to;
    cin >> from >> to;
    from -= 1; // input starts at 1
    to -= 1; // input starts at 1

    // create edge
    edges.at(from).push_back(to);
}

// keeps track of the second best solution
int second_best = -1;

// queue for vertices to visit
// pair.first: length so far
// pair.second: next vertex
priority_queue<pair<int, int>, vector<pair<int, int> >, PairCompare> next_moves;
next_moves.push(make_pair(0, start_vertex));

// keep track how many times a vertex was reached
vector<int> visited_counters(vertex_count, 0);
visited_counters.at(start_vertex) = 1;

// "The Algorithm" TM
while(!next_moves.empty()) {
    pair<int, int> cur_move = next_moves.top();
    next_moves.pop();

    // extract information from current vertex we sit on
    int vertex = cur_move.second;
    int length = cur_move.first;

    // iterate over all neighbors
    for(int neighbor_index = 0; neighbor_index < edges.at(vertex).size(); neighbor_index++) {
        int neighbor_vertex = edges.at(vertex).at(neighbor_index);

        if(visited_counters.at(neighbor_vertex) < 2) {
            // ^- only visit neighbor if it wasn't visited already twice
            // v- update visited counter
            visited_counters.at(neighbor_vertex)++;

            if(neighbor_vertex == target_vertex && visited_counters.at(neighbor_vertex) == 2) {
                // ok, so we found our target vertex and it was already visited twice (i.e. we visit it right now
                // for the second time)
                // we can abort early :-D
                second_best = length + 1;
                goto _solution; // haha
            } else {
                // visit neighbor
                next_moves.push(make_pair(length + 1, neighbor_vertex));
            }
        }
    }
}

_solution:
if(second_best == -1) {
    cout << "no" << endl;
} else {
    cout << second_best << endl;
}

}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

struct PairComparator {
    bool operator() (const pair<int, int>& lhs, const pair<int, int>& rhs) {
        return lhs.first > rhs.first;
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

```

```

int nr_test_cases;
cin >> nr_test_cases;

for (int test_case = 0; test_case < nr_test_cases; test_case++) {
    int nr_vertices, nr_edges;
    cin >> nr_vertices >> nr_edges;

    int start_v, end_v;
    cin >> start_v >> end_v;
    start_v--;
    end_v--;

    vector<vector<int>> > graph(nr_vertices, vector<int>());
    for (int edge_id = 0; edge_id < nr_edges; edge_id++) {
        int edge_source, edge_target;
        cin >> edge_source >> edge_target;
        graph[edge_source - 1].push_back(edge_target - 1);
    }

    //cout << endl;
    int solution = -1;
    priority_queue<pair<int, int>, vector<pair<int, int>>, PairComparator> moves;
    moves.push(make_pair(0, start_v));
    vector<int> reachable(nr_vertices, 0);
    reachable[start_v] = 1;
    while (!moves.empty()) {
        pair<int, int> move = moves.top();
        //cout << move.first << " " << move.second << endl;
        moves.pop();
        int v = move.second;
        int length = move.first;
        for (unsigned int edge_id = 0; edge_id < graph[v].size(); edge_id++) {
            int reachable_v = graph[v][edge_id];
            if (reachable[reachable_v] < 2) {
                reachable[reachable_v]++;
                if (reachable_v == end_v && reachable[reachable_v] == 2) {
                    solution = length + 1;
                    goto solution_found;
                }
            }
            else {
                moves.push(make_pair(length + 1, reachable_v));
            }
        }
    }
    solution_found:
    if (solution == -1) {
        cout << "no" << endl;
    }
    else {
        cout << solution << endl;
    }
}

return 0;
}

```


2 Dynamic Programming

2.1 Longest Path

Keywords: BFS (Graph)

```
#include <iostream>
#include <vector>
#include <stack>

using namespace std;

int main(void) {
    int testcases;
    cin >> testcases;

    for(int t = 0; t < testcases; t++) {
        int vertices;
        cin >> vertices;

        vector<vector<int>> graph(vertices, vector<int>());
        for(int i = 0; i < vertices - 1; i++) {
            int from, to;
            cin >> from >> to;

            graph.at(from).push_back(to);
            graph.at(to).push_back(from);
        }

        int longest = 0;
        int longest_v = -1;

        int i = 0; // random start
        stack<int> tovisit;
        vector<bool> visited(vertices, false);
        int longest_from_i = 0;

        tovisit.push(i);
        int prev = -1;
        while(!tovisit.empty()) {
            int next = tovisit.top();
            tovisit.pop();

            if(next == -1) {
                if(longest < longest_from_i) {
                    longest = longest_from_i;
                    longest_v = prev;
                    //cout << "longest_v: " << longest_v << endl;
                }

                longest_from_i--;
                //cout << " ... " << endl;
                continue;
            }

            if(!visited.at(next)) {
                prev = next;

                longest_from_i++;
                //cout << "visiting: " << next << ", depth: " << longest_from_i << endl;
                visited.at(next) = true;
                tovisit.push(-1);
                for(auto add_v : graph.at(next)) {
                    tovisit.push(add_v);
                }
            }
        }

        // -----

        stack<int> tovisit2;
        vector<bool> visited2(vertices, false);
        int longest_from_i2 = 0;

        tovisit2.push(longest_v);

        while(!tovisit2.empty()) {
            int next = tovisit2.top();
            tovisit2.pop();

            if(next == -1) {
                longest = max(longest, longest_from_i2);
                longest_from_i2--;
                //cout << " ... " << endl;
                continue;
            }

            if(!visited2.at(next)) {
                longest_from_i2++;
                //cout << "visiting: " << next << ", depth: " << longest_from_i2 << endl;
                visited2.at(next) = true;
                tovisit2.push(-1);
                for(auto add_v : graph.at(next)) {
                    tovisit2.push(add_v);
                }
            }
        }
    }
}
```

```

    }
}

cout << longest << endl;

/*int longest = 0;
for(int i = 0; i < vertices; i++) {
    if(graph.at(i).size() != 1) {
        continue;
    }

    stack<int> tovisit;
    vector<bool> visited(vertices, false);
    int longest_from_i = 0;

    tovisit.push(i);

    while(!tovisit.empty()) {
        int next = tovisit.top();
        tovisit.pop();

        if(next == -1) {
            longest = max(longest, longest_from_i);
            longest_from_i--;
            //cout << " ... " << endl;
            continue;
        }

        if(!visited.at(next)) {
            longest_from_i++;
            //cout << "visiting: " << next << ", depth: " << longest_from_i << endl;
            visited.at(next) = true;
            tovisit.push(-1);
            for(auto add_v : graph.at(next)) {
                tovisit.push(add_v);
            }
        }
    }

    cout << longest << endl;*/
}
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_vertices;
        cin >> nr_vertices;
        vector<set<int>> vertices(nr_vertices, set<int>());
        for (int i = 0; i < nr_vertices - 1; i++) {
            int v1, v2;
            cin >> v1 >> v2;
            vertices[v1].insert(v2);
            vertices[v2].insert(v1);
        }
        int longest_path = -1;
        queue<int> leafs;
        for (int i = 0; i < nr_vertices; i++) {
            if (vertices[i].size() == 1) {
                leafs.push(i);
            }
        }

        vector<int> path_length(nr_vertices, 0);
        while (!leafs.empty()) {
            int leaf = leafs.front();
            leafs.pop();
            if (vertices[leaf].size() == 1) {
                int parent = *(vertices[leaf].begin());
                //cout << leaf << " " << parent << " " << path_length[leaf] << endl;
                vertices[parent].erase(leaf);
                if (vertices[parent].size() == 1) {
                    leafs.push(parent);
                }
                longest_path = max(longest_path, path_length[parent] + path_length[leaf] + 1);
                path_length[parent] = max(path_length[parent], path_length[leaf] + 1);
            }
        }
    }
}

```

```

    cout << (longest_path + 1) << endl;
}

return 0;
}

```

2.2 Light Pattern

Keywords: Scanline

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>

using namespace std;

void debug_state(vector<bool> state) {
    for(int i = 0; i < state.size(); i++) {
        if(state.at(i)) {
            cout << "[X]";
        } else {
            cout << "[ ]";
        }
    }

    cout << endl << endl;
}

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int bulb_count, bulbs_in_lightpattern, pattern;
        cin >> bulb_count >> bulbs_in_lightpattern >> pattern;

        // read initial state
        vector<bool> state(bulb_count);
        for(int j = 0; j < bulb_count; j++) {
            int v;
            cin >> v;
            state.at(j) = v == 1;
        }

        //debug_state(state);
        //cout << "T0" << endl;

        // calc target state
        vector<bool> target_state(bulbs_in_lightpattern);
        for(int j = bulbs_in_lightpattern - 1; j >= 0; j--) {
            target_state.at(j) = pattern & 0x1;
            pattern = pattern >> 1;
        }

        //debug_state(target_state);

        int group_count = (int)bulb_count / bulbs_in_lightpattern;
        int swapping_all = 1;
        int single_switching = 0;
        for(int group = group_count; group > 0; group--) {
            int from = group * bulbs_in_lightpattern - bulbs_in_lightpattern;
            //cout << "working on group: " << group << " from: " << from << " to: " << from + bulbs_in_lightpattern - 1
            << endl;
            int local_swapping_all = 0;
            int local_single_switching = 0;
            for(int pos = 0; pos < bulbs_in_lightpattern; pos++) {
                if(state.at(from + pos) != target_state.at(pos)) {
                    local_single_switching++;
                } else {
                    local_swapping_all++;
                }
            }
            //cout << "\t current all: " << local_swapping_all << ", current single: " << local_single_switching << endl;

            int tmp_swapped = min(
                // already swapped till at least previously visited block
                min(
                    swapping_all + local_swapping_all, // how much do we need to changes in case we are already swapped
                    swapping_all + 2 + local_single_switching // we swap again, but only till current block and do normal
                    changes
                ),
                // not swapped yet
                min(
                    single_switching + 1 + local_swapping_all, // we swap till current block and do swapping changes
                    single_switching + 1 + local_single_switching // we swap till the next block and do normal changes in
                    current block
                )
            );
        }
    }
}

```

```

    )
    );

    int tmp_single_change = min(
        // not swapped yet
        min(
            single_switching + local_single_switching, // we just change each bulb by itself
            single_switching + 2 + local_swapping_all // swap, change swapped, swap back
        ),
        // swapped already
        min(
            swapping_all + 1 + local_single_switching, // swap back, do normal changes
            swapping_all + 1 + local_swapping_all
        )
    );

    swapping_all = tmp_swapped;
    single_switching = tmp_single_change;

    //cout << "\tall: " << swapping_all << ", single: " << single_switching << endl;
}

cout << min(swapping_all, single_switching) << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_lights, nr_lights_for_pattern, pattern;
        cin >> nr_lights >> nr_lights_for_pattern >> pattern;

        int nr_patterns = nr_lights / nr_lights_for_pattern;
        vector<int> pattern_changes(nr_patterns);
        for (int i = 0; i < nr_patterns; i++) {
            int tmp_pattern = 0;
            for (int j = 0; j < nr_lights_for_pattern; j++) {
                int light;
                cin >> light;
                tmp_pattern <= 1;
                tmp_pattern |= light;
            }
            int pattern_xor = tmp_pattern ^ pattern;
            int changes = 0;
            for (int j = 0; j < nr_lights_for_pattern; j++) {
                if ((pattern_xor & (0x1 << j)) != 0) {
                    changes++;
                }
            }
            pattern_changes[i] = changes;
        }

        int seconds_swapped = 1;
        int seconds_not_swapped = 0;
        for (int i = nr_patterns - 1; i >= 0; i--) {
            int changes = pattern_changes[i];
            int swapped_changes = nr_lights_for_pattern - changes;
            int tmp_seconds_not_swapped = min(
                min(seconds_not_swapped + changes, seconds_not_swapped + 2 + swapped_changes),
                min(seconds_swapped + 1 + changes, seconds_swapped + 1 + swapped_changes));

            seconds_swapped = min(
                min(seconds_swapped + swapped_changes, seconds_swapped + 2 + changes),
                min(seconds_not_swapped + 1 + changes, seconds_not_swapped + 1 + swapped_changes));
            seconds_not_swapped = tmp_seconds_not_swapped;
            //cout << seconds_not_swapped << " " << seconds_swapped << endl;
        }

        cout << min(seconds_not_swapped, seconds_swapped) << endl;
    }

    return 0;
}

```

2.3 Burning Coins

Keywords: Dynamic Programming

```
#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>

using namespace std;

int at_least(vector<vector<int>> &table, vector<int> &values, int left, int right)
{
    // check first if we know the value already
    if(left == right) {
        // only one coin on the table, we can choose so it's ours
        return values[left];
    }

    if(table[left][right] > 0) {
        // already calculated value, reuse
        return table[left][right];
    }

    // calculate easy cases, fast paths
    int delta = right - left;
    if(delta == 1) {
        // case: left_coin, right_coin
        int best_of_both = max(values[left], values[right]);
        table[left][right] = best_of_both;
        return best_of_both;
    } else if(delta == 2) {
        // case: left_coin, middle_coin, right_coin
        // we have to take middle coin into account and resolve all three coins
        int left_coin = values[left];
        int middle_coin = values[left+1];
        int right_coin = values[right];

        int resulting_value = -2;
        if(left_coin > right_coin) {
            // we take the left coin, now we have to take into account that opposite
            // side will take the best of middle/right coin
            resulting_value = left_coin;
            if(middle_coin > right_coin) {
                resulting_value += right_coin;
            } else {
                resulting_value += middle_coin;
            }
        } else {
            // we take right coin, again take middle coin into account
            resulting_value = right_coin;
            if(middle_coin > left_coin) {
                resulting_value += left_coin;
            } else {
                resulting_value += middle_coin;
            }
        }

        table[left][right] = resulting_value;
        return resulting_value;
    }

    /*
     * now decide what happens in general
     */

    // assume we took the left coin
    int we_left_other_left = at_least(table, values, left+2, right); // opposite side takes also the next left one
    int we_left_other_right = at_least(table, values, left+1, right-1); // opposite side takes the right one

    // assume we took the right coin
    int we_right_other_left = we_left_other_right; //at_least(table, values, left+1, right-1);
    int we_right_other_right = at_least(table, values, left, right-2); // opposite side takes the right one too

    // find the minimum value we're guaranteed to get
    int we_get_at_least = max(
        values[left] + min(we_left_other_left, we_left_other_right),
        values[right] + min(we_right_other_left, we_right_other_right)
    );
    table[left][right] = we_get_at_least;
    return we_get_at_least;
}

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int coin_amount;
```

```

    cin >> coin_amount;
    vector<int> coins;
    for(int j = 0; j < coin_amount; j++) {
        int value;
        cin >> value;
        coins.push_back(value);
    }

    vector<vector<int> > table(coin_amount + 1, vector<int>(coin_amount + 1, -1));
    cout << at_least(table, coins, 0, coin_amount - 1) << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_coins;
        cin >> nr_coins;
        vector<int> coins(nr_coins);
        for (int i = 0; i < nr_coins; i++) {
            cin >> coins[i];
        }

        vector<vector<int> > ta(nr_coins + 1, vector<int>(nr_coins + 1, 0));
        vector<vector<int> > tb(nr_coins + 1, vector<int>(nr_coins + 1, 0));

        for (int l = 1; l <= nr_coins; l++) {
            for (int i = 0; i < nr_coins; i++) {
                ta[i][l] = max(coins[i] + tb[i + 1][l - 1], coins[i + 1 - l] + tb[i][l - 1]);
                tb[i][l] = min(ta[i + 1][l - 1], ta[i][l - 1]);
            }
        }

        cout << ta[0][nr_coins] << endl;
    }

    return 0;
}

```

2.4 Poker Chips

Keywords: Dynamic Programming

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>

#define EMPTY_CELL -1

using namespace std;

int round(int stack_count, vector<int>& stack_sizes, vector<vector<int> >& chips, vector<vector<vector<vector<vector<int>
> > > > & table, vector<int>& conf) {
    //cout << "at: " << conf.at(0) << " " << conf.at(1) << " " << conf.at(2) << " " << conf.at(3) << " " << conf.at(4) <<
    endl;
    // only calculate unknown cell values
    if(table[conf.at(0)][conf.at(1)][conf.at(2)][conf.at(3)][conf.at(4)] == EMPTY_CELL) {
        int max_value = EMPTY_CELL;

        // go over possible sets of poker chip stacks
        for(int set_index = 1; set_index < 32; set_index++) {
            int color = EMPTY_CELL;
            bool valid = true;
            int removed = 0;
            vector<int> new_conf(5);

            // go over all stacks and decide if it is part of the set, if so do calculation
            for(int stack_index = 0; stack_index < 5; stack_index++) {
                if((set_index >> stack_index) & 1) { // stack is in the current set
                    if(conf.at(stack_index) == 0) {
                        // can't remove a chip from current stack

```

```

        valid = false;
        break;
    }

    if(color == EMPTY_CELL) {
        // set current color
        color = chips.at(stack_index).at(conf.at(stack_index) - 1);
    } else if(color != chips.at(stack_index).at(conf.at(stack_index) - 1)) {
        // color top of current stack not the same as we selected
        valid = false;
        break;
    }

    removed++;
    new_conf.at(stack_index) = conf.at(stack_index) - 1;
} else {
    new_conf.at(stack_index) = conf.at(stack_index);
}
}

if(valid) {
    int points = 0;
    if(removed > 1) {
        points = pow(2, removed - 2);
    }

    int val = points + round(stack_count, stack_sizes, chips, table, new_conf);
    max_value = max(max_value, val);
}
}

table[conf.at(0)][conf.at(1)][conf.at(2)][conf.at(3)][conf.at(4)] = max_value;
}

return table[conf.at(0)][conf.at(1)][conf.at(2)][conf.at(3)][conf.at(4)];
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_count;
    cin >> test_count;

    for(int test_index = 0; test_index < test_count; test_index++) {
        int stack_count;
        cin >> stack_count;

        // read in size of each stack
        vector<int> stack_sizes(5, 0);
        vector<int> conf(5, 0);
        for(int stack_index = 0; stack_index < stack_count; stack_index++) {
            int chip_amount;
            cin >> chip_amount;

            stack_sizes.at(stack_index) = chip_amount;
            conf.at(stack_index) = chip_amount;
        }

        // contains the chips of each stack at the start
        vector<vector<int> > > chips;
        chips.reserve(stack_count);

        // read in chips in stack
        for(int stack_index = 0; stack_index < stack_count; stack_index++) {
            chips.push_back(vector<int>(stack_sizes[stack_index], -1));
            for(int chip_index = 0; chip_index < stack_sizes[stack_index]; chip_index++) {
                cin >> chips.at(stack_index).at(chip_index);
            }
        }

        // create DP table, initial value is EMPTY_CELL
        vector<vector<vector<vector<vector<int> > > > > table(stack_sizes[0] + 1,
            vector<vector<vector<vector<int> > > > >(stack_sizes[1] + 1,
                vector<vector<vector<int> > > >(stack_sizes[2] + 1,
                    vector<vector<int> > >(stack_sizes[3] + 1,
                        vector<int>(stack_sizes[4] + 1, EMPTY_CELL))))));

        // initialise table
        table[0][0][0][0][0] = 0;

        // play the game
        cout << round(stack_count, stack_sizes, chips, table, conf) << endl;
    }
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

```

```

int max_stacks = 5;

vector<int> stack_size;
vector<vector<int>> > stacks;

vector<vector<vector<vector<vector<int>>>>> > maximum;

int getMaximum(int first, int second, int third, int fourth, int fifth) {
    //cout << first << " " << second << " " << third << " " << fourth << " " << fifth << endl;
    if (maximum[first][second][third][fourth][fifth] == -1) {
        for (int i = 0; i < 2; i++) {
            if (first - i < 0) continue;
            for (int j = 0; j < 2; j++) {
                if (second - j < 0) continue;
                for (int k = 0; k < 2; k++) {
                    if (third - k < 0) continue;
                    for (int l = 0; l < 2; l++) {
                        if (fourth - l < 0) continue;
                        for (int m = 0; m < 2; m++) {
                            if (fifth - m < 0) continue;
                            int chips_to_remove = i + j + k + l + m;
                            if (chips_to_remove == 0) continue;

                            int chip = -1;
                            if (i == 1) {
                                //cout << "Setting chip to " << stacks[0][first-1] << endl;
                                chip = stacks[0][first - 1];
                            }
                            if (j == 1) {
                                if (chip == -1) {
                                    chip = stacks[1][second - 1];
                                }
                                else {
                                    //cout << "Compare to " << stacks[1][second-1] << endl;
                                    if (chip != stacks[1][second - 1]) continue;
                                }
                            }
                            if (k == 1) {
                                if (chip == -1) {
                                    chip = stacks[2][third - 1];
                                }
                                else {
                                    if (chip != stacks[2][third - 1]) continue;
                                }
                            }
                            if (l == 1) {
                                if (chip == -1) {
                                    chip = stacks[3][fourth - 1];
                                }
                                else {
                                    if (chip != stacks[3][fourth - 1]) continue;
                                }
                            }
                            if (m == 1) {
                                if (chip == -1) {
                                    chip = stacks[4][fifth - 1];
                                }
                                else {
                                    if (chip != stacks[4][fifth - 1]) continue;
                                }
                            }
                            //cout << "Maximum for: " << first-i << " " << second-j << " " << third-k << " " << fourth-l << " " <<
                                fifth-m << endl;
                            int newMax = getMaximum(first - i, second - j, third - k, fourth - l, fifth - m);
                            if (chips_to_remove > 1) {
                                //cout << "Updataing for: " << first << " " << second << " " << third << " " << fourth << " " << fifth <<
                                    endl;
                                //cout << "By: " << (1 << (chips_to_remove-2)) << endl;
                                newMax += 1 << (chips_to_remove - 2);
                            }
                            maximum[first][second][third][fourth][fifth] = max(maximum[first][second][third][fourth][fifth], newMax);
                        }
                    }
                }
            }
        }
    }
    return maximum[first][second][third][fourth][fifth];
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_stacks;
        cin >> nr_stacks;
        stack_size = vector<int>(max_stacks);
        for (int i = 0; i < nr_stacks; i++) {
            cin >> stack_size[i];
        }
        for (int i = nr_stacks; i < max_stacks; i++) {

```



```

    stack_size[i] = 0;
}

stacks = vector<vector<int>>>(max_stacks);
for (int i = 0; i < nr_stacks; i++) {
    stacks[i] = vector<int>(stack_size[i]);
    for (int j = 0; j < stack_size[i]; j++) {
        cin >> stacks[i][j];
    }
}

for (int i = nr_stacks; i < max_stacks; i++) {
    stacks[i] = vector<int>();
}

maximum = vector<vector<vector<vector<vector<int>>>>>>>(
    stack_size[0] + 1,
    vector<vector<vector<vector<int>>>>>(
        stack_size[1] + 1,
        vector<vector<vector<int>>>>(
            stack_size[2] + 1,
            vector<vector<int>>>(
                stack_size[3] + 1,
                vector<int>(stack_size[4] + 1, -1))))));
maximum[0][0][0][0][0] = 0;
cout << getMaximum(stack_size[0], stack_size[1], stack_size[2], stack_size[3], stack_size[4]) << endl;
}

return 0;
}

```

3 BGL Introduction

3.1 Building a Graph

Keywords: Shortest path, Spanning tree, Graph with edge weight map

```
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, property<edge_weight_t, int> > Graph;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        //read graph
        int nr_vertices, nr_edges;
        cin >> nr_vertices >> nr_edges;

        Graph g(nr_vertices);
        property_map<Graph, edge_weight_t>::type weight_map = get(edge_weight, g);
        for (int i = 0; i < nr_edges; i++) {
            int v1, v2, weight;
            cin >> v1 >> v2 >> weight;
            bool success;
            Edge e;
            tie(e, success) = add_edge(v1, v2, g);
            weight_map[e] = weight;
        }

        vector<Edge> spanning_tree;
        kruskal_minimum_spanning_tree(g, back_inserter(spanning_tree));

        int total_spanning_tree_edge_weight = 0;
        for (vector<Edge>::iterator it = spanning_tree.begin(); it != spanning_tree.end(); ++it) {
            total_spanning_tree_edge_weight += get(weight_map, *it);
        }

        vector<int> distances(num_vertices(g));
        dijkstra_shortest_paths(g, 0, distance_map(&distances[0]));
        int max_distance = 0;
        for (int i = 0; i < distances.size(); i++) {
            max_distance = max(max_distance, distances[i]);
        }

        cout << total_spanning_tree_edge_weight << " " << max_distance << endl;
    }

    return 0;
}
```

3.2 Ant Challenge

Keywords: Spanning tree, Shortest path, Graph with edge index map

```
#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, property<edge_index_t, int> > Graph;
```

```

typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int vertices, edges, species, start, end;
        cin >> vertices;
        cin >> edges;
        cin >> species;
        cin >> start;
        cin >> end;

        Graph graph(vertices);
        property_map<Graph, edge_index_t>::type indices = get(edge_index, graph);

        vector<vector<int> > > weights(species, vector<int>(edges, -1));

        for(int j = 0; j < edges; j++) {
            int from, to;
            cin >> from;
            cin >> to;

            // add edge
            bool success;
            Edge edge;
            tie(edge, success) = add_edge(from, to, graph);
            indices[edge] = j;

            for(int k = 0; k < species; k++) {
                int species_weight;
                cin >> species_weight;

                // set weight
                weights[k][j] = species_weight;
            }
        }

        // no reason to know where the hives are...
        int ignore;
        for(int k = 0; k < species; k++) {
            cin >> ignore;
        }

        // find minimum spanning for each species
        vector<int> spanning_tree_weights(edges, numeric_limits<int>::max());
        for(int k = 0; k < species; k++) {
            vector<Edge> spanning_tree;
            kruskal_minimum_spanning_tree(graph, back_inserter(spanning_tree), weight_map(make_iterator_property_map(
                weights[k].begin(), indices)));

            for(vector<Edge>::iterator spanning_tree_edge = spanning_tree.begin(); spanning_tree_edge != spanning_tree.
                end(); ++spanning_tree_edge) {
                if(weights[k][indices[*spanning_tree_edge]] < spanning_tree_weights[indices[*spanning_tree_edge]]) {
                    spanning_tree_weights[indices[*spanning_tree_edge]] = weights[k][indices[*spanning_tree_edge]];
                }
            }
        }

        // now we have a minimal spanning tree we can use to find the shortest path from start to end
        vector<Vertex> predecessors(num_vertices(graph));
        vector<int> distances(num_vertices(graph));
        dijkstra_shortest_paths(graph, start, predecessor_map(&predecessors[0]).distance_map(&distances[0]).weight_map(
            make_iterator_property_map(&spanning_tree_weights[0], indices)));

        // ok, now we can read out the path length
        cout << distances[end] << endl;
    }

    return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>

using namespace std;
using namespace boost;

```

```

typedef adjacency_list<vecS, vecS, undirectedS, no_property, property<edge_index_t, int> > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

/*
typedef adjacency_list<vecS, vecS, undirectedS, no_property, property<edge_weight_t, vector<int> > > Graph;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;

struct get_weight {
    int species_id;
    get_weight(int species_id) : species_id(species_id) {}

    typedef int result_type;
    int operator()(vector<int> x) const {return x[0];}
};

class record_edges : public dijkstra_visitor<>
{
public:
    record_edges(map<int, Edge>& edges)
    : edges(edges) {}

    template <class Edge, class Graph>
    void edge_relaxed(Edge e, Graph& g) {
        cout << "bla" << endl;
        edges[target(e, g)] = e;
    }
protected:
    map<int, Edge>& edges;
};

struct VertexInformation
{
    typedef boost::vertex_property_type type;
};

record_edges use_edges_visitor(use_edges);
dijkstra_shortest_paths(g, hives[i], weight_map(make_transform_value_property_map(get_weight(i), get(edge_weight, g))).
    visitor(use_edges_visitor));
*/

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_vertices, nr_edges, nr_species, start_edge, end_edge;
        cin >> nr_vertices >> nr_edges >> nr_species >> start_edge >> end_edge;

        Graph g(nr_vertices);
        property_map<Graph, edge_index_t::type ind = get(edge_index, g);
        vector<vector<int> > graph_species_weights(nr_species, vector<int>(nr_edges));
        for (int i = 0; i < nr_edges; i++) {
            int v1, v2;
            vector<int> weights(nr_species);
            cin >> v1 >> v2;
            for (int j = 0; j < nr_species; j++) {
                cin >> graph_species_weights[j][i];
            }
            bool success;
            Edge e;
            tie(e, success) = add_edge(v1, v2, g);
            ind[e] = i;
        }

        vector<int> hives(nr_species);
        for (int i = 0; i < nr_species; i++) {
            cin >> hives[i];
        }

        vector<int> edge_weights(nr_edges, numeric_limits<int>::max());
        for (int i = 0; i < nr_species; i++) {
            vector<Edge> spanning_tree;
            kruskal_minimum_spanning_tree(g, back_inserter(spanning_tree), weight_map(make_iterator_property_map(
                graph_species_weights[i].begin(), ind)));
            for (vector<Edge>::iterator ei = spanning_tree.begin(); ei != spanning_tree.end(); ++ei) {
                edge_weights[ind[*ei]] = min(edge_weights[ind[*ei]], graph_species_weights[i][ind[*ei]]);
            }
        }

        vector<int> distances(num_vertices(g), numeric_limits<int>::max());
        dijkstra_shortest_paths(g, start_edge, distance_map(&distances[0]).weight_map(make_iterator_property_map(&
            edge_weights[0], ind)));

        cout << distances[end_edge] << endl;
    }

    return 0;
}

```

3.3 Important Bridges

Keywords: Articulation points, Biconnected components

```
#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/biconnected_components.hpp>

#define ISLAND 1
#define BRIDGE 2

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        // read test case
        int islands, bridges;
        cin >> islands;
        cin >> bridges;

        // create graph for test case
        Graph graph(islands + bridges);

        // keep track which vertex is what
        vector<int> vertex_type(islands + bridges, -1);

        // keep track of bridges through island pairs
        vector<pair<int, int>> bridge_to_island(bridges, make_pair(-1, -1));

        // set vertex properties
        for(int bridge_vertex = 0; bridge_vertex < bridges; bridge_vertex++) {
            assert(vertex_type.at(bridge_vertex) == -1);
            vertex_type.at(bridge_vertex) = BRIDGE;
        }

        for(int island_vertex = bridges + 1; island_vertex < bridges+islands; island_vertex++) {
            assert(vertex_type.at(island_vertex) == -1);
            vertex_type.at(island_vertex) = ISLAND;
        }

        // read in bridges
        for(int k = 0; k < bridges; k++) {
            int from_island, to_island;
            cin >> from_island;
            cin >> to_island;

            // each bridge is also a vertices and connects both islands (also vertices) through edges
            // calculate vertices indexes first
            int from_island_index = bridges + from_island - 1; // input starts at 1, we start at 0
            int to_island_index = bridges + to_island - 1; // input starts at 1, we start at 0
            int bridge_index = k;

            // add edges:
            //      bridge
            //      /   \
            // from island to island
            bool success;
            Edge edge;
            tie(edge, success) = add_edge(from_island_index, bridge_index, graph);
            tie(edge, success) = add_edge(to_island_index, bridge_index, graph);

            // keep track of islands the bridge connects
            bridge_to_island.at(bridge_index) = make_pair(from_island, to_island);
        }

        // find the important bridges
        vector<Vertex> vertices;
        articulation_points(graph, back_inserter(vertices));

        set<pair<int, int>> out_bridges;
        for(vector<Vertex>::iterator v2 = vertices.begin(); v2 != vertices.end(); ++v2) {
            //cout << "type: " << vertex_type[*v2] << " edge nr: " << *v2 << endl;
            // only if articulation point is a bridge, we're interested because it's an important bridge!
            if(vertex_type[*v2] == BRIDGE) {
                pair<int, int> islands_connected_by_important_bridge = bridge_to_island.at(*v2);
```

```

        out_bridges.insert(make_pair(min(islands_connected_by_important_bridge.first,
            islands_connected_by_important_bridge.second), max(islands_connected_by_important_bridge.first,
            islands_connected_by_important_bridge.second)));
    }
}

/** print out the edges, sorted order
set<pair<int, int>> out_bridges;
for(vector<Vertex>::iterator v2 = vertices.begin(); v2 != vertices.end(); ++v2) {
    //cout << "city: " << *v2 << endl;
    //sort(island_to_bridge[*v2].begin(), island_to_bridge[*v2].end());
    for(vector<pair<int, int>>::iterator bridge_islands = island_to_bridge[*v2].begin(); bridge_islands !=
        island_to_bridge[*v2].end(); ++bridge_islands) {
        //cout << (*bridge_islands).first << " " << (*bridge_islands).second << endl;
        out_bridges.insert(make_pair(min((*bridge_islands).first, (*bridge_islands).second), max((*bridge_islands)
            ).first, (*bridge_islands).second)));
    }
}*/

cout << out_bridges.size() << endl;
for(pair<int, int> bout : out_bridges) {
    cout << bout.first << " " << bout.second << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/biconnected_components.hpp>

using namespace std;
using namespace boost;

struct edge_component_t {
    enum
    {
        num = 555
    };
    typedef edge_property_tag kind;
} edge_component;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, property<edge_component_t, size_t> > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef property_map<Graph, vertex_index_t>::type IndexMap;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_vertices, nr_edges;
        cin >> nr_vertices >> nr_edges;

        Graph g(nr_vertices + 1);
        for (int i = 0; i < nr_edges; i++) {
            int v1, v2;
            cin >> v1 >> v2;
            bool success;
            Edge e;
            tie(e, success) = add_edge(v1, v2, g);
        }

        property_map<Graph, edge_component_t::type component = get(edge_component, g);
        size_t nr_components = biconnected_components(g, component);

        vector<int> component_edges(nr_components, -1);

        graph_traits<Graph>::edge_iterator ei, ei_end;
        for (tie(ei, ei_end) = edges(g); ei != ei_end; ++ei) {
            size_t edge_component_id = component[*ei];
            if (component_edges[edge_component_id] == -1) {
                component_edges[edge_component_id] = 1;
            }
            else {
                component_edges[edge_component_id] = -2;
            }
        }
    }
}

```

```

}

vector<pair<int, int> > bridges = vector<pair<int, int> >();
IndexMap index = get(vertex_index, g);
for (tie(ei, ei_end) = edges(g); ei != ei_end; ++ei) {
    size_t edge_component_id = component[*ei];
    if (component_edges[edge_component_id] == 1) {
        int source_ind = index[source(*ei, g)];
        int target_ind = index[target(*ei, g)];
        bridges.push_back(make_pair(min(source_ind, target_ind), max(source_ind, target_ind)));
    }
}
sort(bridges.begin(), bridges.end());
cout << bridges.size() << endl;
for (vector<pair<int, int> >::iterator b_it = bridges.begin(); b_it != bridges.end(); ++b_it) {
    cout << b_it->first << " " << b_it->second << endl;
}
}

return 0;
}

```

3.4 Shy Programmers

Keywords: Planarity

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        // read test case
        int employee_count, friendship_count;
        cin >> employee_count;
        cin >> friendship_count;

        // create graph for test case
        Graph graph(employee_count + 1);

        // add personal door edge
        for(int k = 0; k < employee_count; k++) {
            bool success;
            Edge edge;
            tie(edge, success) = add_edge(k, employee_count + 1, graph);
        }

        // add friendship connections
        for(int k = 0; k < friendship_count; k++) {
            int friend_A, friend_B;
            cin >> friend_A;
            cin >> friend_B;

            bool success;
            Edge edge;
            tie(edge, success) = add_edge(friend_A, friend_B, graph);
        }

        if(boyer_myrvold_planarity_test(graph)) {
            cout << "yes" << endl;
        } else {
            cout << "no" << endl;
        }
    }

    return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_vertices, nr_edges;
        cin >> nr_vertices >> nr_edges;

        Graph g(nr_vertices + 1);
        for (int i = 0; i < nr_edges; i++) {
            int v1, v2;
            cin >> v1 >> v2;
            bool success;
            Edge e;
            tie(e, success) = add_edge(v1, v2, g);
        }

        for (int i = 0; i < nr_vertices; i++) {
            bool success;
            Edge e;
            tie(e, success) = add_edge(i, nr_vertices, g);
        }

        if (boyer_myrvold_planarity_test(g)) {
            cout << "yes" << endl;
        }
        else {
            cout << "no" << endl;
        }
    }

    return 0;
}

```

3.5 Fluid Borders

Keywords: Planarity

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {

```



```

int meta_blob_count;
cin >> meta_blob_count;

// create graph
Graph graph(meta_blob_count);

// read in election results
vector<vector<int>> > election_results(meta_blob_count, vector<int>(meta_blob_count - 1, -1));
for(int voting_meta_blob = 0; voting_meta_blob < meta_blob_count; voting_meta_blob++) {
    for(int vote_place_index = 0; vote_place_index < meta_blob_count - 1; vote_place_index++) {
        // read in the meta-blob number that is at 'vote_place_index' place in the election of meta-blob '
        // voting_meta_blob'
        cin >> election_results.at(voting_meta_blob).at(vote_place_index);
    }
}

int t = 0; // needed as result later
for(; t < meta_blob_count - 1; t++) {
    for(int voting_meta_blob = 0; voting_meta_blob < meta_blob_count; voting_meta_blob++) {
        // add edge for current place under check 't'
        Edge edge;
        tie(edge, tuples::ignore) = add_edge(voting_meta_blob, election_results.at(voting_meta_blob).at(t), graph);
    }

    // check if it's possible
    if(!boyer_myrvold_planarity_test(graph)) {
        break;
    }
}

cout << t << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_vertices;
        cin >> nr_vertices;

        vector<vector<int>> > election_outcomes(nr_vertices, vector<int>(nr_vertices - 1));
        for (int i = 0; i < nr_vertices; i++) {
            for (int j = 0; j < nr_vertices - 1; j++) {
                cin >> election_outcomes[i][j];
            }
        }

        int t = 0;
        Graph g(nr_vertices);
        for (int i = 0; i < nr_vertices - 1; i++) {
            for (int j = 0; j < nr_vertices; j++) {
                bool success;
                Edge e;
                tie(e, success) = add_edge(j, election_outcomes[j][i], g);
            }
            if (boyer_myrvold_planarity_test(g)) {
                t++;
            }
            else {
                break;
            }
        }

        cout << t << endl;
    }
}

```

```
} return 0;
```

4 Flows and Matchings

4.1 Buddies Selection

Keywords: Matching, Match size

```
#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/max_cardinality_matching.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        // read test case
        int student_count, characteristic_count, min_solution;
        cin >> student_count >> characteristic_count >> min_solution;

        // create graph for test case
        Graph graph(student_count);

        // create a set of characteristics for each student from input
        map<int, set<string>> student_to_characs;
        for(int student_i = 0; student_i < student_count; student_i++) {
            set<string> charac_set;

            for(int j = 0; j < characteristic_count; j++) {
                string charac_value;
                cin >> charac_value;

                //cout << "\tbla: " << charac_value << endl;

                charac_set.insert(charac_value);
            }

            student_to_characs[student_i] = charac_set;
        }

        for(int k = 0; k < student_count; k++) {
            for(int p = k + 1; p < student_count; p++) {
                vector<string> intersec;

                set_intersection(student_to_characs.at(k).begin(), student_to_characs.at(k).end(),
                                student_to_characs.at(p).begin(), student_to_characs.at(p).end(),
                                back_inserter(intersec));

                //cout << "size A: " << student_to_characs.at(k).size() << endl;
                //cout << "size B: " << student_to_characs.at(p).size() << endl;
                //cout << "intersec size: " << intersec.size() << endl;
                if(intersec.size() > min_solution) {
                    //cout << "added" << endl;
                    // more in common than minimum given, add edge between students
                    bool success;
                    Edge edge;
                    tie(edge, success) = add_edge(k, p, graph);
                }
            }
        }

        // find matching
        vector<Vertex> mate(student_count);
        checked_edmonds_maximum_cardinality_matching(graph, &mate[0]);

        //cout << "size: " << matching_size(graph, &mate[0]) << endl;
        //cout << "half: " << (int)(student_count / 2) << endl;

        if(matching_size(graph, &mate[0]) == (int)(student_count / 2)) {
            cout << "not optimal" << endl;
        } else {
            cout << "optimal" << endl;
        }
    }
}
```

```

    return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_students, nr_char, min_common_char;
        cin >> nr_students >> nr_char >> min_common_char;

        map<string, int> char_map = map<string, int>();
        vector<vector<int>> > student_chars(nr_students, vector<int>(nr_char));
        int next_char_id = 0;
        for (int i = 0; i < nr_students; i++) {
            for (int j = 0; j < nr_char; j++) {
                string charact;
                cin >> charact;
                map<string, int>::iterator it = char_map.find(charact);
                int char_id;
                if (it == char_map.end()) {
                    char_id = next_char_id;
                    char_map[charact] = next_char_id++;
                }
                else {
                    char_id = it->second;
                }
                student_chars[i][j] = char_id;
            }
        }
        for (int i = 0; i < nr_students; i++) {
            sort(student_chars[i].begin(), student_chars[i].end());
        }

        Graph g(nr_students);
        for (int i = 0; i < nr_students; i++) {
            vector<int>& char_a = student_chars[i];
            for (int j = i + 1; j < nr_students; j++) {
                vector<int>& char_b = student_chars[j];
                int common = 0;
                vector<int>::iterator a_it = char_a.begin();
                vector<int>::iterator b_it = char_b.begin();
                while (a_it != char_a.end() && b_it != char_b.end()) {
                    if (*a_it == *b_it) {
                        common++;
                        ++a_it;
                        ++b_it;
                    }
                    else if (*a_it < *b_it) {
                        ++a_it;
                    }
                    else {
                        ++b_it;
                    }
                }
                if (common > min_common_char) {
                    bool success;
                    Edge e;
                    tie(e, success) = add_edge(i, j, g);
                }
            }
        }

        vector<Vertex> mate(nr_students);
        edmonds_maximum_cardinality_matching(g, &mate[0]);

        bool success = true;
    }
}

```

```

    for (vector<Vertex>::iterator v_it = mate.begin(); v_it != mate.end(); ++v_it) {
        if (*v_it == graph_traits<Graph>::null_vertex()) {
            success = false;
            break;
        }
    }
    if (success) {
        cout << "not optimal" << endl;
    }
    else {
        cout << "optimal" << endl;
    }
}

return 0;
}

```

4.2 Satellites

Keywords: Matching, DFS (Graph), Matching with DFS

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/max_cardinality_matching.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

// code given
typedef graph_traits<Graph>::out_edge_iterator OutEdgeIt;

void DFS(int u, Graph &G, vector<bool> &visited, vector<Vertex> &mate) {
    OutEdgeIt ebeg, eend;
    visited[u] = true;
    for (tie(ebeg, eend) = out_edges(u, G); ebeg != eend; ++ebeg) {
        const int v = target(*ebeg, G);
        // v not vis.  && left to right with Non-Matching edges
        //                right to left with Matching edges
        if (!visited[v] && ((v == mate[u]) != (u < v))) {
            DFS(v, G, visited, mate);
        }
    }
}

// END code given

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        // read test case
        int ground_count, satellite_count, link_count;
        cin >> ground_count >> satellite_count >> link_count;

        // create bipartit graph
        Graph graph(ground_count + satellite_count);

        // read links and create edges for them
        for(int li = 0; li < link_count; li++) {
            int ground_index, satellite_index;
            cin >> ground_index >> satellite_index;

            bool success;
            Edge edge;
            tie(edge, success) = add_edge(ground_index, satellite_index + ground_count, graph);
        }

        // find maximum matching
        vector<Vertex> mate(ground_count + satellite_count);
        checked_edmonds_maximum_cardinality_matching(graph, &mate[0]);

        // keep track of visited vertices
        vector<bool> visited(ground_count + satellite_count);
    }
}

```

```

// code given
// starting points for VC
vector<int> startpoints;
//cout << "Matching:" << endl;
for (int i = 0; i < ground_count + satellite_count; ++i) {
    // output the matching
    if (mate[i] == graph_traits<Graph>::null_vertex() && i < ground_count) {
        startpoints.push_back(i);
    }
}

// run depth first visit
for (int i = 0; i < startpoints.size(); ++i) {
    DFS(startpoints[i], graph, visited, mate);
}

// END code given

// collect unmarked
vector<int> out_ground_ids;
vector<int> out_satellite_ids;
for (int index = 0; index < ground_count + satellite_count; index++) {
    if (index < ground_count && !visited[index]) {
        //cout << "ground ID: " << index << endl;
        out_ground_ids.push_back(index);
    } else if (index >= ground_count && visited[index]) {
        //cout << "satellite ID: " << index - ground_count << endl;
        out_satellite_ids.push_back(index - ground_count);
    }
}

// NOTE: abstand am ende kein Problem! So kompliziertes spaces einfügen nicht nötig :)
cout << out_ground_ids.size() << " " << out_satellite_ids.size() << endl;
bool first = true;
for (int og : out_ground_ids) {
    if (!first) {
        cout << " ";
    } else {
        first = false;
    }

    cout << og;
}

if (!first) {
    cout << " ";
}

first = true;
for (int os : out_satellite_ids) {
    if (!first) {
        cout << " ";
    } else {
        first = false;
    }

    cout << os;
}

cout << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS> Graph;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::out_edge_iterator OutEdgeIt;
const Vertex NULL_VERTEX = graph_traits<Graph>::null_vertex();

void DFS(int u, Graph &G, vector<bool> &visited, vector<Vertex> &mate) {
    OutEdgeIt ebegin, eend;
    visited[u] = true;
    for (tie(ebegin, eend) = out_edges(u, G); ebegin != eend; ++ebegin) {

```

```

    const int v = target(*ebeg, G);
    // v not vis.  && left to right with Non-Matching edges
    //           right to left with Matching edges
    if (!visited[v] && ((v == mate[u]) != (u < v))) {
        DFS(v, G, visited, mate);
    }
}
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_ground, nr_sat, nr_links;
        cin >> nr_ground >> nr_sat >> nr_links;

        int nr_vertices = nr_ground + nr_sat;
        Graph g(nr_vertices);
        for (int i = 0; i < nr_links; i++) {
            int ground_id, sat_id;
            cin >> ground_id >> sat_id;
            add_edge(ground_id, nr_ground + sat_id, g);
        }

        // maximum matching
        vector<Vertex> mate(nr_vertices);
        edmonds_maximum_cardinality_matching(g, &mate[0]);

        // starting points for VC
        vector<int> startpoints;
        //cout << "Matching:" << endl;
        for (int i = 0; i < nr_vertices; ++i) {
            // output the matching
            if (mate[i] != NULL_VERTEX && i < nr_ground){
                //cout << i << " - " << mate[i] << endl;
                // make unmatched L vertices the root vertices of DFS,
                // i.e. "mark as visited"
            }
            else if (mate[i] == NULL_VERTEX && i < nr_ground) {
                startpoints.push_back(i);
            }
        }

        // run depth first visit
        vector<bool> visited(nr_vertices + 1);
        for (int i = 0; i < startpoints.size(); ++i) {
            DFS(startpoints[i], g, visited, mate);
        }

        // output Minimum vertex cover
        //cout << "Minimum Vertex Cover:" << endl;
        vector<int> monitored_ground;
        vector<int> monitored_station;
        for (int i = 0; i < nr_vertices; ++i) {
            if (visited[i] == 0 && i < nr_ground) {
                monitored_ground.push_back(i);
            }
            else if (visited[i] > 0 && i >= nr_ground) {
                monitored_station.push_back(i - nr_ground);
            }
        }

        cout << monitored_ground.size() << " " << monitored_station.size() << endl;
        for (vector<int>::iterator it = monitored_ground.begin(); it != monitored_ground.end(); ++it) {
            cout << *it << " ";
        }
        for (vector<int>::iterator it = monitored_station.begin(); it != monitored_station.end(); ++it) {
            cout << *it << " ";
        }
        cout << endl;
    }

    return 0;
}

```

4.3 Coin Tossing

Keywords: Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Custom add edge function, Max-flow

```

#include <iostream>
#include <algorithm>
#include <vector>

#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <boost/graph/edmonds_karp_max_flow.hpp>
#include <boost/tuple/tuple.hpp>

using namespace std;

```

```

using namespace boost;

typedef adjacency_list_traits<vecS, vecS, directedS> Traits;
typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, long,
        property<edge_residual_capacity_t, long,
            property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;

// Custom add_edge, also creates reverse edges with corresponding capacities.
void addEdge(int u, int v, long c, EdgeCapacityMap &capacity, ReverseEdgeMap &rev_edge, Graph &G) {
    Edge e, reverseE;
    tie(e, tuples::ignore) = add_edge(u, v, G);
    tie(reverseE, tuples::ignore) = add_edge(v, u, G);
    capacity[e] = c;
    capacity[reverseE] = 0;
    rev_edge[e] = reverseE;
    rev_edge[reverseE] = e;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int players, rounds;
        cin >> players >> rounds;

        // create graph and additional stuff
        Graph graph(players + rounds + 2);
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, graph);
        ResidualCapacityMap res_capacity = get(edge_residual_capacity, graph);

        // source and target vertex
        int source = players + rounds;
        int target = players + rounds + 1;

        for(int roundIndex = 0; roundIndex < rounds; roundIndex++) {
            int playerA, playerB, outcome;
            cin >> playerA >> playerB >> outcome;

            int roundVertex = players + roundIndex;
            // source -> round vertex
            addEdge(source, roundVertex, 1, capacity, rev_edge, graph);

            // round vertex -> player vertex representing possible wins
            if(outcome == 0) {
                // both might have won
                addEdge(roundVertex, playerA, 1, capacity, rev_edge, graph);
                addEdge(roundVertex, playerB, 1, capacity, rev_edge, graph);
            } else if(outcome == 1) {
                // player A won
                addEdge(roundVertex, playerA, 1, capacity, rev_edge, graph);
            } else if(outcome == 2) {
                // player B won
                addEdge(roundVertex, playerB, 1, capacity, rev_edge, graph);
            }
        }

        int totalPoints = 0;
        for(int playerIndex = 0; playerIndex < players; playerIndex++) {
            int points;
            cin >> points;

            totalPoints += points;

            // player vertex -> target
            addEdge(playerIndex, target, points, capacity, rev_edge, graph);
        }

        long flowValue = push_relabel_max_flow(graph, source, target);

        if(flowValue == totalPoints && totalPoints == rounds) {
            cout << "yes" << endl;
        } else {
            cout << "no" << endl;
        }
    }

    return 0;
}

```

```

#include <iostream>
#include <vector>

```



```

#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_players, nr_rounds;
        cin >> nr_players >> nr_rounds;

        vector<int> winners;
        vector<pair<int, int> > undecided;
        for (int i = 0; i < nr_rounds; i++) {
            int player_a, player_b, c;
            cin >> player_a >> player_b >> c;
            if (c == 0) {
                undecided.push_back(make_pair(player_a, player_b));
            }
            else if (c == 1) {
                winners.push_back(player_a);
            }
            else if (c == 2) {
                winners.push_back(player_b);
            }
        }

        vector<int> points(nr_players);
        for (int i = 0; i < nr_players; i++) {
            cin >> points[i];
        }

        //Remove certain results
        for (vector<int>::iterator it = winners.begin(); it != winners.end(); ++it) {
            points[*it]--;
        }

        int total_points_needed = 0;
        bool possible = true;
        for (int i = 0; i < nr_players; i++) {
            if (points[i] < 0) {
                possible = false;
                break;
            }
            total_points_needed += points[i];
        }
        if (total_points_needed != undecided.size()) {
            possible = false;
        }

        if (possible) {
            int nr_vertices = nr_players + undecided.size() + 2;
            int source = nr_vertices - 2;
            int sink = nr_vertices - 1;
            Graph g(nr_vertices);
            EdgeCapacityMap capacity = get(edge_capacity, g);
            ReverseEdgeMap rev_edge = get(edge_reverse, g);

            for (int i = 0; i < undecided.size(); i++) {
                int edge_id = nr_players + i;
                Edge e, rev_e;
                bool success;

                //add middle edges
                tie(e, success) = add_edge(edge_id, undecided[i].first, g);
                tie(rev_e, success) = add_edge(undecided[i].first, edge_id, g);
            }
        }
    }
}

```

```

    capacity[e] = 1;
    capacity[rev_e] = 0;
    rev_edge[e] = rev_e;
    rev_edge[rev_e] = e;

    tie(e, success) = add_edge(edge_id, undecided[i].second, g);
    tie(rev_e, success) = add_edge(undecided[i].second, edge_id, g);
    capacity[e] = 1;
    capacity[rev_e] = 0;
    rev_edge[e] = rev_e;
    rev_edge[rev_e] = e;

    //add source edge
    tie(e, success) = add_edge(source, edge_id, g);
    tie(rev_e, success) = add_edge(edge_id, source, g);
    capacity[e] = 1;
    capacity[rev_e] = 0;
    rev_edge[e] = rev_e;
    rev_edge[rev_e] = e;
}

int total_need = 0;
for (int i = 0; i < nr_players; i++) {
    if (points[i] <= 0) {
        continue;
    }
    Edge e, rev_e;
    bool success;

    tie(e, success) = add_edge(i, sink, g);
    tie(rev_e, success) = add_edge(sink, i, g);
    capacity[e] = points[i];
    capacity[rev_e] = 0;
    rev_edge[e] = rev_e;
    rev_edge[rev_e] = e;
}

possible = (total_points_needed == push_relabel_max_flow(g, source, sink));
}
if (possible) {
    cout << "yes" << endl;
}
else {
    cout << "no" << endl;
}
}
return 0;
}

```

4.4 Kingdom Defence

Keywords: Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Max-flow, Custom add edge function inlined by hand

```

#include <iostream>
#include <vector>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <algorithm>

#include <boost/tuple/tuple.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;
typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, long,
    property<edge_residual_capacity_t, long,
    property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef graph_traits<Graph>::edge_descriptor Edge;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_count;
    cin >> test_count;
    for (int test = 0; test < test_count; test++) {
        int location_count, path_count;
        cin >> location_count >> path_count;

        // create graph
        Graph graph(location_count + 2); // don't forget source and sink
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, graph);
        ResidualCapacityMap res_capacity = get(edge_residual_capacity, graph);
    }
}

```

```

// define source and sink vertex
int source = location_count;
int sink = location_count + 1;

// holds locations, first element of pair is the amount of soldier it has and
// the second the amount it needs
vector<pair<int, int> > locations(location_count);
for(int location_index = 0; location_index < location_count; location_index++) {
    int has_soldiers, needs_soldiers;
    cin >> has_soldiers >> needs_soldiers;

    locations.at(location_index) = make_pair(has_soldiers, needs_soldiers);
}

// read in paths for soldiers
for(int path_index = 0; path_index < path_count; path_index++) {
    int from, to, min_soldiers, max_soldiers;

    cin >> from >> to >> min_soldiers >> max_soldiers;

    // modify how many soldiers a location needs
    // it needs more soldiers as some of them must move on to the next city
    locations.at(from).second += min_soldiers;
    // needs less soldiers, as it will for sure get some from the current path!
    locations.at(to).second -= min_soldiers;

    // add edge
    Edge edge, r_edge;
    tie(edge, tuples::ignore) = add_edge(from, to, graph);
    tie(r_edge, tuples::ignore) = add_edge(to, from, graph);
    // we already basically moved the minimum amount of soldiers above by modifying the locations
    // we therefore are interested in the rest that might move over the path and can compare the resulting
    // flow with the expected sum of soldiers
    capacity[edge] = max_soldiers - min_soldiers;
    capacity[r_edge] = 0;
    rev_edge[edge] = r_edge;
    rev_edge[r_edge] = edge;
}

// add edges from the source to the city, with its "have soldiers" weights
for(int location_index = 0; location_index < location_count; location_index++) {
    Edge edge, r_edge;
    tie(edge, tuples::ignore) = add_edge(source, location_index, graph);
    tie(r_edge, tuples::ignore) = add_edge(location_index, source, graph);
    capacity[edge] = locations.at(location_index).first;
    capacity[r_edge] = 0;
    rev_edge[edge] = r_edge;
    rev_edge[r_edge] = edge;
}

// add edges from the city to the sink with the city's "needs soldiers" weights
// also keep track how much we need in total
int need_total = 0;
for(int location_index = 0; location_index < location_count; location_index++) {
    int needs = locations.at(location_index).second;
    if(needs <= 0) {
        continue;
    }

    Edge edge, r_edge;
    tie(edge, tuples::ignore) = add_edge(location_index, sink, graph);
    tie(r_edge, tuples::ignore) = add_edge(sink, location_index, graph);
    capacity[edge] = needs;
    capacity[r_edge] = 0;
    rev_edge[edge] = r_edge;
    rev_edge[r_edge] = edge;

    need_total += needs;
}

// do max flow
int max = push_relabel_max_flow(graph, source, sink);

// check if it corresponds to the expected amount (at least)
if(max >= need_total) {
    cout << "yes" << endl;
} else {
    cout << "no" << endl;
}
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

```

```

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_locations, nr_paths;
        cin >> nr_locations >> nr_paths;

        Graph g(nr_locations + 2);
        int source = nr_locations;
        int sink = nr_locations + 1;
        EdgeCapacityMap capacity = get(edge_capacity, g);
        ReverseEdgeMap rev_edge = get(edge_reverse, g);

        //first -> nr_stationed, second -> nr_needed
        vector<pair<int, int> > locations(nr_locations);
        for (int i = 0; i < nr_locations; i++) {
            int nr_stationed, nr_needed;
            cin >> nr_stationed >> nr_needed;
            locations[i] = make_pair(nr_stationed, nr_needed);
        }

        for (int i = 0; i < nr_paths; i++) {
            int from, to, min, max; //from, to can be equal!
            cin >> from >> to >> min >> max;
            locations[from].second += min;
            locations[to].second -= min;

            Edge e, rev_e;
            bool success;

            tie(e, success) = add_edge(from, to, g);
            tie(rev_e, success) = add_edge(to, from, g);
            capacity[e] = max - min;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
        }

        for (int i = 0; i < nr_locations; i++) {
            Edge e, rev_e;
            bool success;

            tie(e, success) = add_edge(source, i, g);
            tie(rev_e, success) = add_edge(i, source, g);
            capacity[e] = locations[i].first;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
        }

        int total_need = 0;
        for (int i = 0; i < nr_locations; i++) {
            if (locations[i].second < 0) {
                locations[i].second = 0;
                continue;
            }
            Edge e, rev_e;
            bool success;

            tie(e, success) = add_edge(i, sink, g);
            tie(rev_e, success) = add_edge(sink, i, g);
            capacity[e] = locations[i].second;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
            total_need += locations[i].second;
        }

        int max = push_relabel_max_flow(g, source, sink);
    }
}

```

```

    if (max >= total_need) {
        cout << "yes" << endl;
    }
    else {
        cout << "no" << endl;
    }
}

return 0;
}

```

4.5 The Great Game

Keywords: Dynamic Programming

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

using namespace std;

int get_min_moves(int start, vector<vector<int>> &trans, vector<int> &min_fields, vector<int> &max_fields, int field_count);

int get_max_moves(int start, vector<vector<int>> &trans, vector<int> &min_fields, vector<int> &max_fields, int field_count) {
    // if we reached the end, nothing can be doen
    if(start == field_count - 1) {
        return 0;
    }

    // only calculate if value unknown
    if(max_fields.at(start) == -1) {
        // keeps track of the maximum amount of steps needed to reach the end from the given starting point
        int cur_max_found = -1;

        // iterate over all edges one can follow at the 'start' position
        for(vector<int>::iterator iter = trans.at(start).begin();
            iter != trans.at(start).end();
            ++iter) {
            // now we search for the largest minimum amount of steps needed to reach the end
            int possible_max = get_min_moves(*iter, trans, min_fields, max_fields, field_count);
            if(possible_max > cur_max_found) {
                cur_max_found = possible_max;
            }
        }

        // update value
        max_fields.at(start) = cur_max_found + 1; // we still have to take the edge we followed
    }

    // return solution
    return max_fields.at(start);
}

int get_min_moves(int start, vector<vector<int>> &trans, vector<int> &min_fields, vector<int> &max_fields, int field_count) {
    // if we reached the end, no more moves needed
    if(start == field_count - 1) {
        return 0;
    }

    // only calculate if we don't know the solution yet
    if(min_fields.at(start) == -1) {
        // search for the minimum amount of moves to win from the current starting point
        int cur_min_found = numeric_limits<int>::max();

        // iterate over all possible next moves, i.e. the edges leaving the starting position
        for(vector<int>::iterator iter = trans.at(start).begin();
            iter != trans.at(start).end();
            ++iter) {
            // 'iter' refers now to the edge we can follow, i.e. the next position we reach

            // now we have to assume that our opponent will be in our way and make our life complicated.
            // search for the maximum of moves from the next point we can reach to the finish line.
            int possible_min = get_max_moves(*iter, trans, min_fields, max_fields, field_count);
            if(possible_min < cur_min_found) {
                cur_min_found = possible_min;
            }
        }

        min_fields.at(start) = cur_min_found + 1; // we still have to use the edge, so add one
    }

    // return solution
    return min_fields.at(start);
}

```

```

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        // read in basic information
        int position_count, transition_count;
        cin >> position_count >> transition_count;

        int start_red, start_black;
        cin >> start_red >> start_black;
        start_red -= 1; start_black -= 1; // let it start with 0, not 1

        // read in the possible transitions
        vector<vector<int>> > trans(position_count, vector<int>());
        for(int transition_index = 0; transition_index < transition_count; transition_index++) {
            int from, to;
            cin >> from >> to;
            from -= 1; to -= 1; // let it start at 0

            // create transition, it's directed!
            trans.at(from).push_back(to);
        }

        vector<int> min_fields(position_count, -1);
        vector<int> max_fields(position_count, -1);

        int red_min = get_min_moves(start_red, trans, min_fields, max_fields, position_count);
        int black_min = get_min_moves(start_black, trans, min_fields, max_fields, position_count);

        // now we calculate the minimum amount of games each of the players does
        int min_sherlock = -1;
        int min_moriarty = -1;

        // check if the steps needed to win with the red meeple is even ...
        if(red_min % 2 == 0) {
            // ... ok it is even. Now we have to find out how many game steps were needed to
            // move the red meeple to the target position, as every second move sherlock moves the
            // black and not the red meeple
            min_sherlock = ((red_min - 2) / 2) * 4 + 4;
        } else {
            min_sherlock = ((red_min - 1) / 2) * 4 + 1;
        }

        if(black_min % 2 == 0) {
            min_moriarty = ((black_min - 2) / 2) * 4 + 3;
        } else {
            min_moriarty = ((black_min - 1) / 2) * 4 + 2;
        }

        if(min_sherlock < min_moriarty) {
            cout << 0 << endl;
        } else {
            cout << 1 << endl;
        }
    }

    return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

using namespace std;

int last_position;
vector<vector<int>> > field;
vector<int> min_field;
vector<int> max_field;

int getMax(int pos);

int getMin(int pos) {
    if (pos == last_position) {
        return 0;
    }
    if (min_field[pos] == -1) {
        int cur_min = numeric_limits<int>::max();
        for (vector<int>::iterator it = field[pos].begin(); it != field[pos].end(); ++it) {
            int new_min = getMax(*it);
            if (new_min < cur_min) {
                cur_min = new_min;
            }
        }
    }
}

```

```

    }
}
min_field[pos] = cur_min + 1;
}
return min_field[pos];
}

int getMax(int pos) {
    if (pos == last_position) {
        return 0;
    }
    if (max_field[pos] == -1) {
        int cur_max = -1;
        for (vector<int>::iterator it = field[pos].begin(); it != field[pos].end(); ++it) {
            int new_max = getMin(*it);
            if (new_max > cur_max) {
                cur_max = new_max;
            }
        }
        max_field[pos] = cur_max + 1;
    }
    return max_field[pos];
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_positions, nr_transitions;
        cin >> nr_positions >> nr_transitions;
        last_position = nr_positions - 1;

        int r_start, b_start;
        cin >> r_start >> b_start;
        //adjust starting position
        r_start--;
        b_start--;

        field = vector<vector<int>>(nr_positions, vector<int>());
        for (int i = 0; i < nr_transitions; i++) {
            int from, to;
            cin >> from >> to;
            from--;
            to--;
            field[from].push_back(to);
        }

        min_field = vector<int>(nr_positions, -1);
        max_field = vector<int>(nr_positions, -1);

        int r_min = getMin(r_start);
        int b_min = getMin(b_start);

        if (r_min % 2 == 1) {
            r_min = ((r_min - 1) / 2) * 4 + 1;
        }
        else {
            r_min = ((r_min - 2) / 2) * 4 + 4;
        }

        if (b_min % 2 == 1) {
            b_min = ((b_min - 1) / 2) * 4 + 2;
        }
        else {
            b_min = ((b_min - 2) / 2) * 4 + 3;
        }
        if (r_min < b_min) {
            cout << 0 << endl;
        }
        else {
            cout << 1 << endl;
        }
    }

    return 0;
}

```

4.6 Surveillance Photographs

Keywords: Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Custom add edge function, Max-flow

```

#include <iostream>
#include <algorithm>
#include <vector>

#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

```

```

#include <boost/graph/edmonds_karp_max_flow.hpp>
#include <boost/tuple/tuple.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list_traits<vecS, vecS, directedS> Traits;
typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, long,
        property<edge_residual_capacity_t, long,
            property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;

// Custom add_edge, also creates reverse edges with corresponding capacities.
void addEdge(int u, int v, long c, EdgeCapacityMap &capacity, ReverseEdgeMap &rev_edge, Graph &G) {
    Edge e, reverseE;
    tie(e, tuples::ignore) = add_edge(u, v, G);
    tie(reverseE, tuples::ignore) = add_edge(v, u, G);
    capacity[e] = c;
    capacity[reverseE] = 0;
    rev_edge[e] = reverseE;
    rev_edge[reverseE] = e;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        int intersection_count, street_count, station_count, photograph_count;
        cin >> intersection_count >> street_count >> station_count >> photograph_count;

        // create graph and additional stuff
        // we need a graph that can hold a sink, a source and twice the cities map
        Graph graph(2 * intersection_count + 2);
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, graph);
        // not used: ResidualCapacityMap res_capacity = get(edge_residual_capacity, graph);
        int source = 2 * intersection_count;
        int sink = 2 * intersection_count + 1;

        // read in where police stations are located and connect them to source/sink
        for(int station_index = 0; station_index < station_count; station_index++) {
            int station_loc;
            cin >> station_loc;

            // source to police station with weight 1 as each station has only one officer
            addEdge(source, station_loc, 1, capacity, rev_edge, graph);

            // police station to sink with weight 1 as each station can only hold one photograph,
            // ATTENTION here police station is in the second set!
            addEdge(intersection_count + station_loc, sink, 1, capacity, rev_edge, graph);
        }

        // read where photographs are stored and connect this location from our first set (where policemen reach the
        // location)
        // to our second set (where policemen can only use a street once to get back to a station)
        for(int photo_index = 0; photo_index < photograph_count; photo_index++) {
            int photo_loc;
            cin >> photo_loc;

            addEdge(photo_loc, intersection_count + photo_loc, 1, capacity, rev_edge, graph);
        }

        // read where the streets are
        for(int street_index = 0; street_index < street_count; street_index++) {
            int from, to;
            cin >> from >> to;

            // first add street with unbound traffic to the first set, as all policemen are free to move multiple times
            // through the same street without a photograph
            addEdge(from, to, numeric_limits<int>::max(), capacity, rev_edge, graph);

            // now we add the same street, but it can be used only once as now it is used by policemen
            // with photographs
            addEdge(from + intersection_count, to + intersection_count, 1, capacity, rev_edge, graph);
        }

        long flowValue = push_relabel_max_flow(graph, source, sink);

        cout << flowValue << endl;
    }

    return 0;
}

```



```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_intersections, nr_streets, nr_police_stations, nr_photographs;
        cin >> nr_intersections >> nr_streets >> nr_police_stations >> nr_photographs;

        Graph g(nr_intersections * 2 + 2);
        int source = nr_intersections * 2;
        int sink = nr_intersections * 2 + 1;
        EdgeCapacityMap capacity = get(edge_capacity, g);
        ReverseEdgeMap rev_edge = get(edge_reverse, g);

        for (int i = 0; i < nr_police_stations; i++) {
            int station_location;
            cin >> station_location;

            Edge e, rev_e;
            bool success;

            tie(e, success) = add_edge(source, station_location, g);
            tie(rev_e, success) = add_edge(station_location, source, g);
            capacity[e] = 1;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;

            tie(e, success) = add_edge(station_location + nr_intersections, sink, g);
            tie(rev_e, success) = add_edge(sink, station_location + nr_intersections, g);
            capacity[e] = 1;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
        }

        for (int i = 0; i < nr_photographs; i++) {
            int photograph_location;
            cin >> photograph_location;

            Edge e, rev_e;
            bool success;

            tie(e, success) = add_edge(photograph_location, photograph_location + nr_intersections, g);
            tie(rev_e, success) = add_edge(photograph_location + nr_intersections, photograph_location, g);
            capacity[e] = 1;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
        }

        vector<pair<int, int> > streets(nr_streets);
        for (int i = 0; i < nr_streets; i++) {
            int from, to;
            cin >> from >> to;

            Edge e, rev_e;
            bool success;

            tie(e, success) = add_edge(from, to, g);

```

```

    tie(rev_e, success) = add_edge(to, from, g);
    capacity[e] = nr_police_stations;
    capacity[rev_e] = 0;
    rev_edge[e] = rev_e;
    rev_edge[rev_e] = e;

    tie(e, success) = add_edge(from + nr_intersections, to + nr_intersections, g);
    tie(rev_e, success) = add_edge(to + nr_intersections, from + nr_intersections, g);
    capacity[e] = 1;
    capacity[rev_e] = 0;
    rev_edge[e] = rev_e;
    rev_edge[rev_e] = e;
}

int max = push_relabel_max_flow(g, source, sink);
cout << max << endl;
}

return 0;
}

```

5 CGAL Introduction

5.1 Hit?

Keywords: CGAL Segment, CGAL Ray, CGAL do_intersect

```
#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <iostream>
#include <stdexcept>

typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Segment_2 Segment;
typedef K::Ray_2 Ray;

using namespace std;

int main()
{
    while(true) {
        int obsticale_count;
        cin >> obsticale_count;

        // kill switch for application
        if(obsticale_count == 0) {
            break;
        }

        // get the laser
        double ray_start_x, ray_start_y, ray_other_x, ray_other_y;
        cin >> ray_start_x >> ray_start_y >> ray_other_x >> ray_other_y;
        Ray laser_ray = Ray(Point(ray_start_x, ray_start_y), Point(ray_other_x, ray_other_y));

        // read in obstacles and check if we hit them with the ray
        bool hit = false;
        for(int i = 0; i < obsticale_count; i++) {
            // read in and create obstacle
            double obstacle_start_x, obstacle_start_y, obstacle_end_x, obstacle_end_y;
            cin >> obstacle_start_x >> obstacle_start_y >> obstacle_end_x >> obstacle_end_y;

            // we know we hit something, read input but do nothing with it :)
            if(hit) {
                continue;
            }

            Segment obstacle = Segment(Point(obstacle_start_x, obstacle_start_y), Point(obstacle_end_x, obstacle_end_y));

            // check if we hit it
            if(CGAL::do_intersect(laser_ray, obstacle)) {
                hit = true;
            }
        }

        if(hit) {
            cout << "yes" << endl;
        } else {
            cout << "no" << endl;
        }
    }
}
```

```
#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Segment_2 Segment;
typedef K::Ray_2 Ray;

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int l;
    for (cin >> l; l != 0; cin >> l) {
        long long x, y, a, b;
        cin >> x >> y >> a >> b;
        Ray ray(Point(x, y), Point(a, b));

        bool found_intersect = false;
        for (int i = 0; i < l; i++) {
            long long r, s, t, u;
            cin >> r >> s >> t >> u;
            if (CGAL::do_intersect(ray, Segment(Point(r, s), Point(t, u)))) {
                for (i++; i < l; i++) {
```

```

        cin >> r >> s >> t >> u;
    }
    found_intersect = true;
}
}
if (found_intersect) {
    cout << "yes" << endl;
}
else {
    cout << "no" << endl;
}
}
return 0;
}

```

5.2 Antenna

Keywords: floor_to_double, ceil_to_double, Minimal Circle, CGAL sqrt

```

#include <iostream>
#include <cmath>

#include <CGAL/Exact_predicates_exact_constructions_kernel_with_sqrt.h>
#include <CGAL/Min_circle_2.h>
#include <CGAL/Min_circle_2_traits_2.h>
#include <CGAL/number_utils.h>

// typedefs
typedef CGAL::Exact_predicates_exact_constructions_kernel_with_sqrt K; // IMPORTANT!!!! OTHERWISE NO SQRT!!!!
typedef CGAL::Min_circle_2_traits_2<K> Traits;
typedef CGAL::Min_circle_2<Traits> Min_circle;
typedef K::Point_2 Point;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int people_count;
        cin >> people_count;

        // kill switch for application
        if(people_count == 0) {
            break;
        }

        // collect all positions of people living at
        vector<Point> peoples(people_count);
        for(int i = 0; i < people_count; i++) {
            double person_x, person_y;
            cin >> person_x >> person_y;

            peoples.at(i) = Point(person_x, person_y);
        }

        // create the circle covering all people with minimal surface
        Min_circle min_circle(peoples.begin(), peoples.end(), true);

        // get radius
        cout << ceil_to_double(sqrt(min_circle.circle().squared_radius())) << endl;
    }
}

```

```

#include <CGAL/Exact_predicates_exact_constructions_kernel_with_sqrt.h>
#include <CGAL/Min_circle_2.h>
#include <CGAL/Min_circle_2_traits_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>

```

```

#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_exact_constructions_kernel_with_sqrt K;
typedef CGAL::Min_circle_2_traits_2<K> Traits;
typedef CGAL::Min_circle_2<Traits> Min_circle;
typedef K::Point_2 Point;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int n;
    for (cin >> n; n != 0; cin >> n) {
        vector<Point> points(n);
        for (int i = 0; i < n; i++) {
            double x, y;
            cin >> x >> y;
            points[i] = Point(x, y);
        }
        Min_circle mc(points.begin(), points.end(), true);
        cout << ceil_to_double(sqrt(mc.circle().squared_radius())) << endl;
    }

    return 0;
}

```

5.3 First Hit

Keywords: CGAL randomise, CGAL intersection

```

#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <iostream>
#include <stdexcept>

typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Segment_2 Segment;
typedef K::Ray_2 Ray;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

inline void laser_obstacle_segment(Segment &lo_segment, CGAL::Object intersec_obj) {
    if (const Point* op = CGAL::object_cast<Point>(&intersec_obj)) {
        lo_segment = Segment(lo_segment.source(), *op);
    } else if (const Segment* os = CGAL::object_cast<Segment>(&intersec_obj)) {
        // ray hits a segment, three possibilities
        if (CGAL::collinear_are_ordered_along_line(lo_segment.source(), (*os).source(), (*os).target())) {
            // order is: laser source -> start of segment -> end of segment
            lo_segment = Segment(lo_segment.source(), (*os).source());
        } else {
            lo_segment = Segment(lo_segment.source(), (*os).target());
        }
    } else {
        throw runtime_error("Bad Wolf");
    }
}

```

```

    }
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    std::cout << std::setiosflags(std::ios::fixed) << std::setprecision(0);
    //cout << fixed << setprecision(0);

    while(true) {
        int obsticale_count;
        cin >> obsticale_count;

        // kill switch for application
        if(obsticale_count == 0) {
            break;
        }

        // get the laser
        double ray_start_x, ray_start_y, ray_other_x, ray_other_y;
        cin >> ray_start_x >> ray_start_y >> ray_other_x >> ray_other_y;
        Ray laser_ray = Ray(Point(ray_start_x, ray_start_y), Point(ray_other_x, ray_other_y));

        vector<Segment> obsticle_segments(obsticale_count);

        // read in and create obsticle list
        for(int i = 0; i < obsticale_count; i++) {
            double obsticle_start_x, obsticle_start_y, obsticle_end_x, obsticle_end_y;
            cin >> obsticle_start_x >> obsticle_start_y >> obsticle_end_x >> obsticle_end_y;

            obsticle_segments[i] = Segment(Point(obsticle_start_x, obsticle_start_y), Point(obsticle_end_x, obsticle_end_y));
        }

        random_shuffle(obsticle_segments.begin(), obsticle_segments.end());

        // segment that starts at the source of the laser and ends, after our algo is done, at the point where the laser hits
        // an
        // obsticle.

        // search for one intersection between the laser ray and an obsticle, create a segment that starts at the laser
        // source and ends
        // at the point where the laser hits the obsticle
        bool hit_found = false;
        Segment lo_segment(laser_ray.source(), laser_ray.source());
        int obst_index = 0; // used to jump over already checked segments in the second for-loop

        for(; obst_index < obsticale_count; ++obst_index) {
            if(do_intersect(obsticle_segments[obst_index], laser_ray)) {
                hit_found = true;
                laser_obsticale_segment(lo_segment, intersection(obsticle_segments[obst_index], laser_ray));
                break;
            }
        }

        // check if we hit something
        if(!hit_found) {
            cout << "no" << endl;
            continue;
        }

        for(; obst_index < obsticale_count; ++obst_index) {
            if(do_intersect(lo_segment, obsticle_segments[obst_index])) {
                laser_obsticale_segment(lo_segment, intersection(obsticle_segments[obst_index], laser_ray));
            }
        }

        cout << floor_to_double(lo_segment.target().x()) << " " << floor_to_double(lo_segment.target().y()) << endl;
    }
}

```

```

#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Segment_2 Segment;
typedef K::Ray_2 Ray;

using namespace std;

double floor_to_double(const K::FT& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x) {

```

```

double a = ceil(CGAL::to_double(x));
while (a < x) a += 1;
while (a - 1 >= x) a -= 1;
return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << setiosflags(ios::fixed) << setprecision(0);

    int l;
    for (cin >> l; l != 0; cin >> l) {
        double x, y, a, b;
        cin >> x >> y >> a >> b;
        Point origin(x, y);
        Ray ray(origin, Point(a, b));

        bool found_intersect = false;
        K::FT nearest_dist;
        Point nearest_point;
        for (int i = 0; i < l; i++) {
            double r, s, t, u;
            cin >> r >> s >> t >> u;
            Segment seg(Point(r, s), Point(t, u));
            if (CGAL::do_intersect(ray, seg)) {
                auto o = CGAL::intersection(ray, seg);
                K::FT new_dist;
                Point new_point;
                if (const Point* op = boost::get<Point>(&*o)) {
                    new_dist = Segment(origin, *op).squared_length();
                    new_point = *op;
                }
                else if (const Segment* os = boost::get<Segment>(&*o)) {
                    if (CGAL::collinear_are_ordered_along_line(origin, os->source(), os->target())) {
                        new_dist = Segment(origin, os->source()).squared_length();
                        new_point = os->source();
                    }
                    else if (CGAL::collinear_are_ordered_along_line(origin, os->target(), os->source())) {
                        new_dist = Segment(origin, os->target()).squared_length();
                        new_point = os->target();
                    }
                    else {
                        //segment going through ray origin point -> distance is 0
                        new_dist = 0.0;
                        new_point = origin;
                    }
                }
            }
            else {
                throw runtime_error("strange segment intersection");
            }
            if (!found_intersect) {
                nearest_dist = new_dist;
                nearest_point = new_point;
                found_intersect = true;
            }
            else {
                if (new_dist < nearest_dist) {
                    nearest_dist = new_dist;
                    nearest_point = new_point;
                }
            }
        }
        if (found_intersect) {
            cout << floor_to_double(nearest_point.x()) << " " << floor_to_double(nearest_point.y()) << endl;
        }
        else {
            cout << "no" << endl;
        }
    }

    return 0;
}

```

5.4 Almost Antenna

Keywords: CGAL qrt, Minimal Circle, floor_to_double, ceil_to_double

```

#include <iostream>
#include <cmath>
#include <unordered_set>

#include <CGAL/Exact_predicates_exact_constructions_kernel_with_sqrt.h>
#include <CGAL/Min_circle_2.h>
#include <CGAL/Min_circle_2_traits_2.h>
#include <CGAL/number_utils.h>

// typedefs
typedef CGAL::Exact_predicates_exact_constructions_kernel_with_sqrt K; // IMPORTANT!!!! OTHERWISE NO SQRT!!!!
typedef CGAL::Min_circle_2_traits_2<K> Traits;

```

```

typedef CGAL::Min_circle_2<Traits>      Min_circle;
typedef K::Point_2 Point;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int people_count;
        cin >> people_count;

        // kill switch for application
        if(people_count == 0) {
            break;
        }

        // collect all positions of people living at
        vector<Point> peoples(people_count);
        for(int i = 0; i < people_count; i++) {
            double person_x, person_y;
            cin >> person_x >> person_y;

            peoples.at(i) = Point(person_x, person_y);
        }

        // create the circle covering all people with minimal surface
        Min_circle min_circle(peoples.begin(), peoples.end(), true);

        // now we remove the support points, calculate the radius for the resulting min circles and choose the smallest one
        // as the solution
        K::FT min_rad = sqrt(min_circle.circle().squared_radius());
        for(auto iter = min_circle.support_points_begin(); iter != min_circle.support_points_end(); iter++) {
            for(int i = 0; i < people_count; i++) {
                if(peoples.at(i) == *iter) {
                    int add = i > 0 ? -1 : 1;
                    peoples.at(i) = peoples.at(i + add);
                }

                Min_circle almost_circ(peoples.begin(), peoples.end(), true);
                K::FT almost_rad = sqrt(almost_circ.circle().squared_radius());
                if(almost_rad < min_rad) {
                    min_rad = almost_rad;
                }

                peoples.at(i) = *iter;
            }
            break;
        }
    }

    // get radius
    cout << ceil_to_double(min_rad) << endl;
}

```

```

#include <CGAL/Exact_predicates_exact_constructions_kernel_with_sqrt.h>
#include <CGAL/Min_circle_2.h>
#include <CGAL/Min_circle_2_traits_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_exact_constructions_kernel_with_sqrt K;
typedef CGAL::Min_circle_2_traits_2<K> Traits;
typedef CGAL::Min_circle_2<Traits> Min_circle;
typedef K::Point_2 Point;

using namespace std;

```



```

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int n;
    for (cin >> n; n != 0; cin >> n) {
        vector<Point> points(n);
        for (int i = 0; i < n; i++) {
            double x, y;
            cin >> x >> y;
            points[i] = Point(x, y);
        }
        Min_circle mc(points.begin(), points.end(), true);
        double min_radius = ceil_to_double(sqrt(mc.circle().squared_radius()));
        for (auto sp_it = mc.support_points_begin(); sp_it != mc.support_points_end(); ++sp_it) {
            Point support_point = *sp_it;
            for (int i = 0; i < n; i++) {
                if (points[i] == support_point) {
                    /*Min_circle almost_mc(points.begin(), points.begin()+i, true);
                    if (i+1 < n) {
                        almost_mc.insert(points.begin()+i+1, points.end());
                    }
                    double new_radius = ceil_to_double(sqrt(almost_mc.circle().squared_radius()));
                    if (new_radius < min_radius) {
                        min_radius = new_radius;
                    }*/
                    if (i == 0) {
                        if (n > 1) {
                            points[i] = points[i + 1];
                        }
                        else {
                            points[i] = Point(0, 0);
                        }
                    }
                    else {
                        points[i] = points[i - 1];
                    }
                    Min_circle almost_mc(points.begin(), points.end(), true);
                    points[i] = support_point;
                    double new_radius = ceil_to_double(sqrt(almost_mc.circle().squared_radius()));
                    if (new_radius < min_radius) {
                        min_radius = new_radius;
                    }
                    break;
                }
            }
        }
        cout << min_radius << endl;
    }

    return 0;
}

```

6 Proximity Structures

6.1 Graypes

Keywords: setprecision, Delaunay Triangulation, Finite edge iteration

```
#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2<K>   Triangulation;
typedef Triangulation::Edge_iterator   Edge_iterator;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int graype_count;
        cin >> graype_count;

        // kill switch for application
        if(graype_count == 0) {
            break;
        }

        // collect graype locations
        vector<K::Point_2> graype_locs;
        graype_locs.reserve(graype_count);

        for(int i = 0; i < graype_count; i++) {
            double graype_x, graype_y;
            cin >> graype_x >> graype_y;

            graype_locs.push_back(K::Point_2(graype_x, graype_y));
        }

        // construct triangulation
        Triangulation triang;
        triang.insert(graype_locs.begin(), graype_locs.end());

        // go trough apes, and search for shortest edge
        K::FT min_time;
        bool first = true;
        for (Edge_iterator edge = triang.finite_edges_begin(); edge != triang.finite_edges_end(); ++edge) {
            K::FT edge_time = triang.segment(edge).squared_length();
            if(edge_time < min_time || first) {
                first = false;
                min_time = edge_time;
            }
        }

        // calculate time to run
        cout << ceil(sqrt(CGAL::to_double(min_time)) / 2.0 * 100.0) << endl;
    }
}
```

```
#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef CGAL::Delaunay_triangulation_2<K>   Triangulation;
typedef Triangulation::Edge_iterator   Edge_iterator;
typedef Triangulation::Point   Point;
```

```

typedef Triangulation::Segment Segment;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int n;
    for (cin >> n; n != 0; cin >> n) {
        vector<Point> points;
        points.reserve(n);
        for (int i = 0; i < n; i++) {
            double x, y;
            cin >> x >> y;
            points.push_back(Point(x, y));
        }

        Triangulation t;
        t.insert(points.begin(), points.end());
        Segment shortest = t.segment(*(t.finite_edges_begin()));
        for (Edge_iterator e = t.finite_edges_begin(); e != t.finite_edges_end(); ++e) {
            Segment s = t.segment(*e);
            if (s.squared_length() < shortest.squared_length()) {
                shortest = s;
            }
        }

        cout << ceil(sqrt(CGAL::to_double(shortest.squared_length()))) / 2.0*100.0 << endl;
    }

    return 0;
}

```

6.2 Bistro

Keywords: Delaunay Triangulation

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2<K> Triangulation;
typedef Triangulation::Edge_iterator Edge_iterator;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int existing_count;
        cin >> existing_count;

        // kill switch for application
    }
}

```

```

    if(existing_count == 0) {
        break;
    }

    // collect existing restaurants
    vector<K::Point_2> existing_locs;
    existing_locs.reserve(existing_count);

    for(int i=0; i < existing_count; i++) {
        double loc_x, loc_y;
        cin >> loc_x >> loc_y;

        existing_locs.push_back(K::Point_2(loc_x, loc_y));
    }

    // construct triangulation
    Triangulation triang;
    triang.insert(existing_locs.begin(), existing_locs.end());

    // go through possible location
    int possible_count;
    cin >> possible_count;

    for(int i = 0; i < possible_count; i++) {
        int possible_x, possible_y;
        cin >> possible_x >> possible_y;

        K::Point_2 possible_point = K::Point_2(possible_x, possible_y);

        // find nearest vertex and by that the nearest point
        K::Point_2 nearest = triang.nearest_vertex(possible_point)->point();
        cout << CGAL::to_double(CGAL::squared_distance(nearest, possible_point)) << endl;
    }
}
}

```

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef CGAL::Delaunay_triangulation_2<K> Triangulation;
typedef Triangulation::Edge_iterator Edge_iterator;
typedef Triangulation::Point Point;
typedef Triangulation::Segment Segment;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int n;
    for (cin >> n; n != 0; cin >> n) {
        vector<Point> points;
        points.reserve(n);
        for (int i = 0; i < n; i++) {
            double x, y;
            cin >> x >> y;
            points.push_back(Point(x, y));
        }

        Triangulation t;
        t.insert(points.begin(), points.end());
        int m;
        cin >> m;

        for (int i = 0; i < m; i++) {
            double x, y;

```

```

    cin >> x >> y;
    Point loc(x, y);
    K::FT squared_dist = CGAL::squared_distance(t.nearest_vertex(loc)->point(), loc);
    cout << CGAL::to_double(squared_dist) << endl;
}
}

return 0;
}

```

6.3 H1N1

Keywords: Delaunay Triangulation, CGAL Triangulation with DFS, Finite face iteration

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2<K> Triangulation;
typedef Triangulation::Edge_iterator Edge_iterator;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

bool DFS(Triangulation &triang, double min_dist_for_edge, Triangulation::Face_handle &start, map<Triangulation::Face_handle, int> &visitor_map, int cur_iter) {
    // each infected person's point is connected to three edges, otherwise we wouldn't get triangle
    for(int edge_i = 0; edge_i < 3; edge_i++) {
        // get the segment representing the edge
        Triangulation::Segment edge_segment = triang.segment(start, edge_i);

        // check if we are still far away from both endpoints of the segment while passing through
        double distance = CGAL::to_double(edge_segment.squared_length());
        if(distance >= min_dist_for_edge) {
            // get the neighboring face
            Triangulation::Face_handle neighbor_face_h = start->neighbor(edge_i);

            // check that we didn't visit it already
            if(visitor_map[neighbor_face_h] == cur_iter) {
                continue; // use other edge
            }

            // check if neighboring face is infinite, i.e. we found a way out
            if(triang.is_infinite(neighbor_face_h)) {
                return true;
            }

            // mark as visited by current iteration
            visitor_map[neighbor_face_h] = cur_iter;

            // recursion, DFS
            if(DFS(triang, min_dist_for_edge, neighbor_face_h, visitor_map, cur_iter)) {
                return true;
            }
        }
    }

    return false;
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int infected_count;
        cin >> infected_count;

        // kill switch for application
        if(infected_count == 0) {
            break;
        }

        // read in infected people's location

```

```

vector<K::Point_2> infected_locs;
infected_locs.reserve(infected_count);
for(int i = 0; i < infected_count; i++) {
    double infected_x, infected_y;
    cin >> infected_x >> infected_y;

    infected_locs.push_back(K::Point_2(infected_x, infected_y));
}

// create triangulation with the infected people
Triangulation triang;
triang.insert(infected_locs.begin(), infected_locs.end());

// prepare a map mapping a face handle to an integer, the integer represents the DFS step in which the face was
// visited already (to not visit multiple time the same face)
map<Triangulation::Face_handle, int> visitor_map;
for (Triangulation::Face_iterator it = triang.finite_faces_begin(); it != triang.finite_faces_end(); it++) {
    visitor_map[it] = -1;
}

// go trough people trying to escape
int escapee_count;
cin >> escapee_count;
for(int i = 0; i < escapee_count; i++) {
    // read location of escapee and the minimum distance expected
    double escapee_x, escapee_y, min_dist;
    cin >> escapee_x >> escapee_y >> min_dist;

    K::Point_2 escapee_loc(escapee_x, escapee_y);

    // get nearest vertex's point in the triangulation
    Triangulation::Point nearest_vertex_point = triang.nearest_vertex(escapee_loc)->point();

    // check that we not already violate the distance condition
    if(CGAL::to_double(CGAL::squared_distance(escapee_loc, nearest_vertex_point)) < min_dist) {
        cout << "n";
        continue; // jump over the rest, we already know the escapee is near an infected person
    }

    // find the face handle of our escapee
    Triangulation::Face_handle face_h = triang.locate(escapee_loc);

    // update for the face handle that we visited it in the current round
    visitor_map[face_h] = i;
    if(triang.is_infinite(face_h) || // face is already outside, escapee can escape, no DFS needed
        DFS(triang, min_dist * 4.0, face_h, visitor_map, i)) { // use DFS to find a way out, multiply by 4.0 (=
        2.0^2 (not squared distances)) as we must be min_dist away from one end of an edge and min_dist
        from the other, multiply by 2^2
        cout << "y";
    } else {
        cout << "n";
    }
}

cout << endl;
}
}

```

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_face_base_with_info_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef CGAL::Triangulation_vertex_base_2<K> Vb;
typedef CGAL::Triangulation_face_base_with_info_2<int, K> Fb;
typedef CGAL::Triangulation_data_structure_2<Vb, Fb> Tds;
typedef CGAL::Delaunay_triangulation_2<K, Tds> Triangulation;

typedef Triangulation::Edge_iterator Edge_iterator;
typedef Triangulation::Face_handle Face_handle;
typedef Triangulation::Face_iterator Face_iterator;
typedef Triangulation::All_faces_iterator All_faces_iterator;
typedef Triangulation::Point Point;
typedef Triangulation::Segment Segment;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

```

```

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

bool find_exit(Triangulation& t, Face_handle& face, K::FT& min_segment_squared_length, int current_id) {
    for (int i = 0; i < 3; i++) {
        Segment s = t.segment(face, i);
        if (s.squared_length() >= min_segment_squared_length) {
            Face_handle neighbor_face = face->neighbor(i);
            if (neighbor_face->info() == current_id) {
                continue;
            }
            if (t.is_infinite(neighbor_face)) {
                return true;
            }
            neighbor_face->info() = current_id;
            if (find_exit(t, neighbor_face, min_segment_squared_length, current_id)) {
                return true;
            }
        }
    }
    return false;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int n;
    for (cin >> n; n != 0; cin >> n) {
        vector<Point> points;
        points.reserve(n);
        for (int i = 0; i < n; i++) {
            double x, y;
            cin >> x >> y;
            points.push_back(Point(x, y));
        }

        Triangulation t;
        t.insert(points.begin(), points.end());
        for (All_faces_iterator f = t.all_faces_begin(); f != t.all_faces_end(); ++f) {
            f->info() = 0;
        }

        int m;
        cin >> m;

        for (int i = 0; i < m; i++) {
            double x, y;
            K::FT distance, min_segment_squared_length;
            cin >> x >> y >> distance;
            Point loc(x, y);
            K::FT squared_dist = CGAL::squared_distance(t.nearest_vertex(loc)->point(), loc);
            Face_handle face = t.locate(loc);
            min_segment_squared_length = distance*4.0;

            face->info() = i + 1;
            if (squared_dist >= distance && (t.is_infinite(face) || find_exit(t, face, min_segment_squared_length, i + 1))) {
                cout << 'y';
            }
            else {
                cout << 'n';
            }
        }
        cout << endl;
    }

    return 0;
}

```

6.4 Germs

Keywords: Delaunay Triangulation, Finite vertices iteration, Finite edge iteration, Iteration over std::map

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2<K> Triangulation;
typedef Triangulation::Edge_iterator Edge_iterator;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));

```

```

    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

double hours(double distance) {
    double x = sqrt(distance) - 0.5; // first sqrt to get the distance ...
    double result;
    if (x <= 0.0) {
        result = 0;
    } else {
        result = ceil(sqrt(x)); // second sqrt because  $t^2 + 0.5 = x \Rightarrow t = \sqrt{x - 0.5}$ ;
    }
    return result;
}

int main() {
    while(true) {
        int bacteria_count;
        cin >> bacteria_count;

        // kill switch for application
        if(bacteria_count == 0) {
            break;
        }

        // read in boundaries of the dish
        double left_border, right_border, bottom_border, top_border;
        cin >> left_border >> bottom_border >> right_border >> top_border;

        // collect bacteria's center information
        vector<K::Point_2> bacteria_centers;
        bacteria_centers.reserve(bacteria_count);
        for(int i = 0; i < bacteria_count; i++) {
            double bacteria_x, bacteria_y;
            cin >> bacteria_x >> bacteria_y;

            bacteria_centers.push_back(K::Point_2(bacteria_x, bacteria_y));
        }

        // create triangulation
        Triangulation triang;
        triang.insert(bacteria_centers.begin(), bacteria_centers.end());

        // keep track of the distances for each bacteria
        map<Triangulation::Point, double> distances;
        //distances.reserve(bacteria_count);

        // calculate initial distance: distance between the bacteria and the nearest dish boundary
        for(Triangulation::Finite_vertices_iterator vertex_iter = triang.finite_vertices_begin(); vertex_iter != triang.finite_vertices_end(); ++vertex_iter) {
            Triangulation::Point vertex = vertex_iter->point();
            distances[vertex] = min(
                min(vertex.x() - left_border, right_border - vertex.x()), // left/right minimum
                min(vertex.y() - bottom_border, top_border - vertex.y()) // top/bottom minimum
            );

            distances[vertex] *= distances[vertex]; // square distance as we work with squared ones
        }

        // compute distance to other two neighbours and update distance if it is smaller
        for(Triangulation::Finite_edges_iterator edge_iter = triang.finite_edges_begin(); edge_iter != triang.finite_edges_end(); ++edge_iter) {
            Triangulation::Vertex_handle vertex1 = edge_iter->first->vertex(triang.cw(edge_iter->second));
            Triangulation::Vertex_handle vertex2 = edge_iter->first->vertex(triang.ccw(edge_iter->second));

            Triangulation::Point vertex1_point = vertex1->point();
            Triangulation::Point vertex2_point = vertex2->point();

            // calculate distance of the points of both vertex and half them (divide by 4 as distance is squared and  $4 = 2^2$ )
            double vertex_distance = CGAL::to_double(CGAL::squared_distance(vertex1_point, vertex2_point)) / 4;

            // update distances to minimum
            distances[vertex1_point] = min(distances[vertex1_point], vertex_distance);
            distances[vertex2_point] = min(distances[vertex2_point], vertex_distance);
        }

        // now we know the minimum distance for each bacteria to another one or the borders of the dish

        // extract distances into a vector and sort it
        vector<double> only_distances;
        only_distances.reserve(bacteria_count);
        for(map<Triangulation::Point, double>::iterator iter = distances.begin(); iter != distances.end(); ++iter) {
            only_distances.push_back(iter->second);
        }
    }
}

```



```

// sort distances
sort(only_distances.begin(), only_distances.end());

// print out information
cout << hours(only_distances[0]) << " " << hours(only_distances[bacteria_count/2]) << " " << hours(only_distances
[bacteria_count - 1]) << endl;
}
}

```

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_face_base_with_info_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef CGAL::Triangulation_vertex_base_with_info_2<K::FT, K> Vb;
typedef CGAL::Triangulation_face_base_2<K> Fb;
typedef CGAL::Triangulation_data_structure_2<Vb, Fb> Tds;
typedef CGAL::Delaunay_triangulation_2<K, Tds> Triangulation;

typedef Triangulation::Edge_iterator Edge_iterator;
typedef Triangulation::Face_handle Face_handle;
typedef Triangulation::Face_iterator Face_iterator;
typedef Triangulation::All_faces_iterator All_faces_iterator;
typedef Triangulation::Finite_vertices_iterator Finite_vertices_iterator;
typedef Triangulation::Finite_edges_iterator Finite_edges_iterator;
typedef Triangulation::Point Point;
typedef Triangulation::Segment Segment;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

double distance_to_time(K::FT distance) {
    double tmp = sqrt(CGAL::to_double(distance)) - 0.5;
    return (tmp > 0.0) ? ceil(sqrt(tmp)) : 0.0;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int n;
    for (cin >> n; n != 0; cin >> n) {
        double l, b, r, t;
        cin >> l >> b >> r >> t;

        vector<Point> points;
        points.reserve(n);
        for (int i = 0; i < n; i++) {
            double x, y;
            cin >> x >> y;
            points.push_back(Point(x, y));
        }
        Triangulation bac;
        bac.insert(points.begin(), points.end());

        for (Finite_vertices_iterator it = bac.finite_vertices_begin(); it != bac.finite_vertices_end(); ++it) {
            Point& p = it->point();
            it->info() = min(min(p.x() - l, r - p.x()), min(p.y() - b, t - p.y()));
            it->info() *= it->info();
        }

        for (Finite_edges_iterator it = bac.finite_edges_begin(); it != bac.finite_edges_end(); ++it) {
            Triangulation::Vertex_handle v1 = it->first->vertex((it->second + 1) % 3);
            Triangulation::Vertex_handle v2 = it->first->vertex((it->second + 2) % 3);
            K::FT d = CGAL::squared_distance(v1->point(), v2->point()) / 4;
            v1->info() = min(v1->info(), d);
            v2->info() = min(v2->info(), d);
        }
    }
}

```

```

vector<K::FT> bac_expand_distances;
bac_expand_distances.reserve(n);
for (Finite_vertices_iterator it = bac.finite_vertices_begin(); it != bac.finite_vertices_end(); ++it) {
    bac_expand_distances.push_back(it->info());
}
sort(bac_expand_distances.begin(), bac_expand_distances.end());
cout << distance_to_time(bac_expand_distances[0]) << " " << distance_to_time(bac_expand_distances[n / 2]) << " " <<
    distance_to_time(bac_expand_distances[n - 1]) << endl;
}

return 0;
}

```

6.5 Hiking Maps

Keywords: Delaunay Triangulation, CGAL turn function, CGAL right_turn, Scanline

```

#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_face_base_with_info_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_exact_constructions_kernel K;

typedef CGAL::Triangulation_vertex_base_2<K> Vb;
typedef CGAL::Triangulation_face_base_with_info_2<int, K> Fb;
typedef CGAL::Triangulation_data_structure_2<Vb, Fb> Tds;
typedef CGAL::Delaunay_triangulation_2<K, Tds> Triangulation;

typedef Triangulation::Edge_iterator Edge_iterator;
typedef Triangulation::Face_handle Face_handle;
typedef Triangulation::Face_iterator Face_iterator;
typedef Triangulation::All_faces_iterator All_faces_iterator;
typedef Triangulation::Finite_vertices_iterator Finite_vertices_iterator;
typedef Triangulation::Finite_edges_iterator Finite_edges_iterator;
typedef Triangulation::Point Point;
typedef Triangulation::Segment Segment;
typedef Triangulation::Line_face_circulator Line_face_circulator;
typedef K::Line_2 Line;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

inline bool containsSegment(const vector<Point>& triangle, const Point& s1, const Point& s2) {
    return !CGAL::right_turn(triangle[1], triangle[0], s1) &&
        !CGAL::right_turn(triangle[3], triangle[2], s1) &&
        !CGAL::right_turn(triangle[5], triangle[4], s1) &&
        !CGAL::right_turn(triangle[1], triangle[0], s2) &&
        !CGAL::right_turn(triangle[3], triangle[2], s2) &&
        !CGAL::right_turn(triangle[5], triangle[4], s2);
}

void print_array(vector<int>& a) {
    for (int i = 0; i < a.size(); i++) {
        cout << a[i] << " ";
    }
    cout << endl;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int nr_testcases;
    cin >> nr_testcases;
    for (int testcase = 0; testcase < nr_testcases; ++testcase) {
        int m, n;

```

```

cin >> m >> n;

vector<Point> legs;
legs.reserve(m);
for (int i = 0; i < m; i++) {
    int x, y;
    cin >> x >> y;
    legs.push_back(Point(x, y));
}

vector<vector<Point> > triangles;
triangles.reserve(n);
for (int i = 0; i < n; i++) {
    vector<Point> triangle;
    triangle.reserve(6);

    int x0, y0, x1, y1, x2, y2, x3, y3, x4, y4, x5, y5;
    cin >> x0 >> y0 >> x1 >> y1 >> x2 >> y2 >> x3 >> y3 >> x4 >> y4 >> x5 >> y5;
    Point p0(x0, y0), p1(x1, y1), p2(x2, y2), p3(x3, y3), p4(x4, y4), p5(x5, y5);

    if (CGAL::left_turn(p2, p1, p0)) {
        triangle.push_back(p0);
        triangle.push_back(p1);
    }
    else {
        triangle.push_back(p1);
        triangle.push_back(p0);
    }
    if (CGAL::left_turn(p4, p3, p2)) {
        triangle.push_back(p2);
        triangle.push_back(p3);
    }
    else {
        triangle.push_back(p3);
        triangle.push_back(p2);
    }
    if (CGAL::left_turn(p0, p5, p4)) {
        triangle.push_back(p4);
        triangle.push_back(p5);
    }
    else {
        triangle.push_back(p5);
        triangle.push_back(p4);
    }
    triangles.push_back(triangle);
}

vector<int> leg_to_triangle(m - 1, -1);
vector<int> triangle_contributions(n, 0);
int found_legs = 0;
int bound_max = 0;
int i = 0;
while (found_legs < m - 1 && i < n) {
    for (int j = 0; j < m - 1; j++) {
        if (containsSegment(triangles[i], legs[j], legs[j + 1])) {
            if (leg_to_triangle[j] == -1) {
                found_legs++;
            }
            else {
                triangle_contributions[leg_to_triangle[j]]--;
                //cout << "remove leg " << j << " covered by " << leg_to_triangle[j] << endl;
            }
            leg_to_triangle[j] = i;
            //cout << "leg " << j << " covered by " << i << endl;
            triangle_contributions[i]++;
        }
    }
    bound_max = i;
    i++;
}
int bound_min = 0;
while (triangle_contributions[bound_min] == 0) bound_min++;
int min_range = bound_max - bound_min;

//cout << "min: " << bound_min << " max: " << bound_max << endl;
while (i < n) {
    for (int j = 0; j < m - 1; j++) {
        if (containsSegment(triangles[i], legs[j], legs[j + 1])) {
            triangle_contributions[leg_to_triangle[j]]--;
            leg_to_triangle[j] = i;
            triangle_contributions[i]++;
        }
    }
    bound_max = i;
    while (triangle_contributions[bound_min] == 0) bound_min++;
    min_range = min(min_range, bound_max - bound_min);
    //cout << "min: " << bound_min << " max: " << bound_max << endl;
    i++;
}

cout << min_range + 1 << endl;
}

return 0;
}

```

7 Linear/Quadratic Programming

7.1 What is the Maximum?

Keywords: Quadratic Program, `ceil_to_double`, `floor_to_double`

```
#include <iostream>
#include <cassert>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

using namespace std;

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

const int var_x = 0; const int X = var_x;
const int var_y = 1; const int Y = var_y;
const int var_z = 2; const int Z = var_z;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {

    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int p, a, b;
        cin >> p;

        // kill switch for application
        if(p == 0) {
            break;
        }

        cin >> a >> b;

        if(p == 1) {
            // v- set finite lower bound (x, y >= 0), defaults to 0
            // v- no upper bound
            Program lp(CGAL::SMALLER, true, 0, false, 0);

            // x + y <= 4
            lp.set_a(var_x, 0, 1);
            lp.set_a(var_y, 0, 1);
            lp.set_b(0, 4);

            // 4x + 2y <= ab
            lp.set_a(var_x, 1, 4);
            lp.set_a(var_y, 1, 2);
            lp.set_b(1, a * b);

            // -x + y <= 1
            lp.set_a(var_x, 2, -1);
            lp.set_a(var_y, 2, 1);
            lp.set_b(2, 1);

            // maximize: by -ax^2 so we have to minimize -by + ax^2
            lp.set_c(var_y, -b);
            lp.set_d(var_x, var_x, 2*a);

            // solve it
            Solution s = CGAL::solve_nonnegative_quadratic_program(lp, ET());
            assert(s.solves_quadratic_program(lp));

            if(s.is_unbounded()) {
                cout << "unbounded" << endl;
            } else if(s.is_infeasible()) {
```

```

    cout << "no" << endl;
} else {
    cout << floor_to_double(-s.objective_value()) << endl;
}
} else {
    // here we have an upper bound for the variables, but no lower bound
    Program lp(CGAL::LARGER, false, 0, false, 0);

    // bounds by benji
    // z >= 0
    lp.set_u(X, true, 0);
    lp.set_u(Y, true, 0);
    lp.set_l(Z, true, 0);

    // x + y >= -4
    lp.set_a(var_x, 0, 1);
    lp.set_a(var_y, 0, 1);
    lp.set_b(0, -4);

    // 4x + 2y + z^2 => -ab => we substitute z^2 by just z (don't forget to do the same for the minimized formula
    // dingsi)
    // i.e. we get 4x + 2y + z >= -ab
    lp.set_a(var_x, 1, 4);
    lp.set_a(var_y, 1, 2);
    lp.set_a(var_z, 1, 1);
    lp.set_b(1, -(a*b));

    // -x + y >= -11
    lp.set_a(var_x, 2, -1);
    lp.set_a(var_y, 2, 1);
    lp.set_b(2, -1);

    // minimize ax^2 + by + z^4 => after our substitution: ax^2 + by + z^2
    lp.set_d(var_x, var_x, 2*a);
    lp.set_d(var_z, var_z, 2 * 1);
    lp.set_c(var_y, b);

    // solve it
    Solution s = CGAL::solve_quadratic_program(lp, ET());
    assert(s.solves_quadratic_program(lp));

    if(s.is_unbounded()) {
        cout << "unbounded" << endl;
    } else if(s.is_infeasible()) {
        cout << "no" << endl;
    } else {
        cout << ceil_to_double(s.objective_value()) << endl;
    }
}
}
}
}

```

7.2 Diets

Keywords: Quadratic Program, Non-negative quadratic program

```

#include <iostream>
#include <cassert>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

using namespace std;

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

const int var_x = 0; const int X = var_x;
const int var_y = 1; const int Y = var_y;
const int var_z = 2; const int Z = var_z;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
}

```

```

    return a;
}

int main() {

    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int nutrient_count, food_count;
        cin >> nutrient_count >> food_count;

        // kill switch for application
        if(nutrient_count == 0 && food_count == 0) {
            break;
        }

        // create linear programming application
        Program lp(CGAL::SMALLER, true, 0, false, 0);

        // read in nutrients
        vector<int> nutrient_min;
        vector<int> nutrient_max;
        nutrient_min.reserve(nutrient_count);
        nutrient_max.reserve(nutrient_count);
        for(int i = 0; i < nutrient_count; i++) {
            int min_amount, max_amount;
            cin >> nutrient_min[i] >> nutrient_max[i];
        }

        // read in foods and amount of nutrients
        vector<int> prices;
        vector<vector<int>> > nutrient_amount(food_count, vector<int>(nutrient_count));
        prices.reserve(food_count);
        for(int i = 0; i < food_count; i++) {
            cin >> prices[i];

            for(int j = 0; j < nutrient_count; j++) {
                cin >> nutrient_amount[i][j];
            }
        }

        // vars: first food, then nutrients

        // set objective
        for(int i = 0; i < food_count; i++) {
            // we want to pay the minimum
            // results in the sum of food_var * food_price
            lp.set_c(i, prices[i]);
        }

        // set inequalities
        int eq_counter = 0;
        for(int nutrient_index = 0; nutrient_index < nutrient_count; nutrient_index++) {
            for(int food_index = 0; food_index < food_count; food_index++) {
                lp.set_a(food_index, eq_counter, nutrient_amount[food_index][nutrient_index]); // A := sum over: food * amount of
                // nutrition
            }

            lp.set_b(eq_counter, nutrient_max[nutrient_index]); // A <= maximum needed
            eq_counter++;
        }

        for(int nutrient_index = 0; nutrient_index < nutrient_count; nutrient_index++) {
            for(int food_index = 0; food_index < food_count; food_index++) {
                lp.set_a(food_index, eq_counter, nutrient_amount[food_index][nutrient_index]); // B := sum over: food * amount of
                // nutrition
            }

            // B >= minimum needed
            lp.set_b(eq_counter, nutrient_min[nutrient_index]);
            lp.set_r(eq_counter, CGAL::LARGER);
            eq_counter++;
        }

        // find solution and print out
        Solution s = CGAL::solve_nonnegative_quadratic_program(lp, ET());

        if(s.is_optimal()) {
            cout << floor_to_double(s.objective_value()) << endl;
        } else {
            cout << "No such diet." << endl;
        }
    }
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>

```

```

#include <utility>
#include <cmath>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int n, m;
    for (cin >> n >> m; n != 0; cin >> n >> m) {
        Program lp(CGAL::SMALLER, true, 0, false, 0);

        for (int i = 0; i < n; i++) {
            int min_nut, max_nut;
            cin >> min_nut >> max_nut;
            lp.set_r(i, CGAL::LARGER);
            lp.set_b(i, min_nut);
            lp.set_r(i + n, CGAL::SMALLER);
            lp.set_b(i + n, max_nut);
        }

        for (int i = 0; i < m; i++) {
            int price;
            cin >> price;
            lp.set_c(i, price); //add price to function to minimize
            for (int j = 0; j < n; j++) {
                int nut_amount;
                cin >> nut_amount;
                lp.set_a(i, j, nut_amount);
                lp.set_a(i, j + n, nut_amount);
            }
        }

        Solution s = CGAL::solve_linear_program(lp, ET());
        assert(s.solves_linear_program(lp));

        if (s.is_infeasible() || s.is_unbounded()) {
            cout << "No such diet." << endl;
        }
        else {
            cout << floor_to_double(s.objective_value()) << endl;
        }
    }

    return 0;
}

```

7.3 Portfolios

Keywords: Quadratic Program

```

#include <iostream>
#include <cassert>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type

```

```

#ifndef CGAL_USE_GMP
#include <CGAL/Gmpzf.h>
typedef CGAL::Gmpzf ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

using namespace std;

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {

    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int asset_count, people_count;
        cin >> asset_count >> people_count;

        // kill switch for application
        if(asset_count == 0 && people_count == 0) {
            break;
        }

        // containers for asset properties
        vector<int> asset_cost(asset_count);
        vector<int> asset_expected_return(asset_count);

        // read in asset properties
        for(int asset_index = 0; asset_index < asset_count; asset_index++) {
            cin >> asset_cost.at(asset_index) >> asset_expected_return.at(asset_index);
        }

        // read and store covariance
        vector<vector<int>> > covariance(asset_count, vector<int>(asset_count));
        for(int assetA = 0; assetA < asset_count; assetA++) {
            for(int assetB = 0; assetB < asset_count; assetB++) {
                cin >> covariance.at(assetA).at(assetB);
            }
        }

        // go through each investor and calculate for him/her the result
        for(int person_index = 0; person_index < people_count; person_index++) {
            int max_cost, min_return, max_variance;
            cin >> max_cost >> min_return >> max_variance;

            // by default, we have a nonnegative QP with  $Ax \geq b$ 
            Program qp (CGAL::LARGER, true, 0, false, 0);

            // equation counter
            int eq_counter = 0;

            // add inequation for expected return
            for(int asset_index = 0; asset_index < asset_count; asset_index++) {
                // sum of each asset amount times its expected return ...
                qp.set_a(asset_index, eq_counter, asset_expected_return[asset_index]);
            }
            // ...  $\geq$  investor's expected minimal return
            qp.set_b(eq_counter, min_return);
            eq_counter++;

            // add inequation for max cost for the investor
            for(int asset_index = 0; asset_index < asset_count; asset_index++) {
                // sum of each asset's cost times how many of them we buy ...
                qp.set_a(asset_index, eq_counter, asset_cost[asset_index]);
            }
            // ...  $\leq$  investor's maximum cost
            qp.set_b(eq_counter, max_cost);
            qp.set_r(eq_counter, CGAL::SMALLER);
            eq_counter++;

            // objective function
            for(int assetA = 0; assetA < asset_count; assetA++) {
                for(int assetB = 0; assetB < asset_count; assetB++) {
                    qp.set_d(assetA, assetB, 2 * covariance[assetA][assetB]);
                }
            }
        }
    }
}

```



```

    }
}

// calculate
Solution s = CGAL::solve_nonnegative_quadratic_program(qp, ET());
assert(s.solves_quadratic_program(qp));

// Output
if (s.is_optimal() && s.objective_value() <= max_variance) {
    cout << "Yes." << endl;
} else {
    cout << "No." << endl;
}
}
}
}
}
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    //n = Total assets
    //m = number of people
    int n, m;
    for (cin >> n >> m; n != 0; cin >> n >> m) {
        Program qp(CGAL::LARGER, true, 0, false, 0);
        qp.set_r(0, CGAL::SMALLER); //cost
        qp.set_r(1, CGAL::LARGER); //expectet return

        //read cost and expected return of assets
        for (int i = 0; i < n; i++) {
            int cost, expected_return;
            cin >> cost >> expected_return;
            qp.set_a(i, 0, cost);
            qp.set_a(i, 1, expected_return);
        }

        //read covariance matrix
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                int covar;
                cin >> covar;
                if (j <= i) {
                    qp.set_d(i, j, covar * 2);
                }
            }
        }

        for (int i = 0; i < m; i++) {
            int C, R, V;
            cin >> C >> R >> V;

```

```

qp.set_b(0, C);
qp.set_b(1, R);

Solution s = CGAL::solve_nonnegative_quadratic_program(qp, ET());
assert(s.solves_quadratic_program(qp));

if (s.is_infeasible() || s.is_unbounded() || s.objective_value() > V) {
    cout << "No." << endl;
}
else {
    cout << "Yes." << endl;
}
}
}

return 0;
}

```

7.4 Inball

Keywords: Quadratic Program, Quadratic Program: Maximize

```

#include <iostream>
#include <cassert>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

using namespace std;

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {

    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int inequality_count;
        cin >> inequality_count;

        // kill switch for application
        if(inequality_count == 0) {
            break;
        }

        int dimension;
        cin >> dimension;

        // create linear programming application
        Program lp(CGAL::SMALLER, false, 0, false, 0);

        // set our target of maximizing the radius, i.e. we take the negative of the radius and minimize the whole thing
        int var_radius = dimension; // everything below 'dimension' is a variable for the dimensions (?)
        lp.set_c(var_radius, -1); // maximize by minimizing the negative

        // add constraints from input
        for(int constraint_index = 0; constraint_index < inequality_count; constraint_index++) {
            int distance = 0;

            for(int dim_var_index = 0; dim_var_index < dimension; dim_var_index++) {
                int a_i;
                cin >> a_i;

                lp.set_a(dim_var_index, constraint_index, a_i);
            }
        }
    }
}

```

```

        lp.set_a(dim_var_index, constraint_index + inequality_count, a_i);

        distance += a_i * a_i;
    }

    lp.set_a(var_radius, constraint_index, sqrt(distance));

    int b_i;
    cin >> b_i;
    lp.set_b(constraint_index, b_i);
    lp.set_b(constraint_index + inequality_count, b_i);
}

// find solution and print out
Solution s = CGAL::solve_linear_program(lp, ET());
assert(s.solves_linear_program(lp));
if(s.is_optimal()) {
    cout << -1 * ceil_to_double(s.objective_value()) << endl;
} else if (s.is_unbounded()) {
    cout << "inf" << endl;
} else {
    cout << "none" << endl;
}
}
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int n;
    for (cin >> n; n != 0; cin >> n) {
        int dimensions;
        cin >> dimensions;

        Program lp(CGAL::LARGER, false, 0, false, 0);
        for (int i = 0; i < n; ++i) {
            int sum = 0;
            for (int j = 0; j < dimensions; ++j) {
                int ai;
                cin >> ai;
                lp.set_a(j, i, ai);
                sum += pow(ai, 2);
            }
            lp.set_a(dimensions, i, -static_cast<long>(sqrt(sum)));

            int bi;
            cin >> bi;
            lp.set_b(i, -bi);
        }
        lp.set_l(dimensions, true, 0);
        lp.set_c(dimensions, -1);
    }
}

```

```

Solution s = CGAL::solve_linear_program(lp, ET());
assert(s.solves_linear_program(lp));
if (s.status() == CGAL::QP_OPTIMAL){
    cout << -1 * ceil_to_double(s.objective_value()) << endl;
}
else if (s.status() == CGAL::QP_UNBOUNDED) {
    cout << "inf" << endl;
}
else {
    cout << "none" << endl;
}
}
return 0;
}

```

7.5 Collisions

Keywords: Point set, Delaunay Triangulation, Finite edge iteration, Triangulation with info()

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <CGAL/Point_set_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef CGAL::Triangulation_vertex_base_with_info_2<int, K> Vb;
typedef CGAL::Triangulation_face_base_2<K> Fb;
typedef CGAL::Triangulation_data_structure_2<Vb, Fb> Tds;
typedef CGAL::Delaunay_triangulation_2<K, Tds> Triangulation;

typedef Triangulation::Edge_iterator Edge_iterator;
typedef Triangulation::Face_handle Face_handle;
typedef Triangulation::Face_iterator Face_iterator;
typedef Triangulation::All_faces_iterator All_faces_iterator;
typedef Triangulation::Finite_vertices_iterator Finite_vertices_iterator;
typedef Triangulation::Finite_edges_iterator Finite_edges_iterator;
typedef Triangulation::Point Point;
typedef Triangulation::Segment Segment;
typedef Triangulation::Vertex_handle Vertex_handle;

using namespace std;

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;
    for(int test = 0; test < test_count; test++) {
        int airplane_count;
        cin >> airplane_count;

        // read minimum distance
        K::FT input_dist;
        cin >> input_dist;

        K::FT min_dist = input_dist * input_dist;

        // read in airplane location
        vector<pair<Point, int>> airplanes;
        airplanes.reserve(airplane_count);
        for(int airplane_index = 0; airplane_index < airplane_count; airplane_index++) {
            int x, y;
            cin >> x >> y;

            airplanes.push_back(make_pair(Point(x, y), airplane_index));
        }

        // keeps track of planes on collision course
        vector<bool> collisions(airplane_count, false);

        // triangulate planes
        Triangulation airplane_triang;
        airplane_triang.insert(airplanes.begin(), airplanes.end());

        // iterate through all edges
        for(Finite_edges_iterator edge_iter = airplane_triang.finite_edges_begin();
            edge_iter != airplane_triang.finite_edges_end();
            ++edge_iter) {
            // get the airplanes
            Vertex_handle plane1_vertex = edge_iter->first->vertex((edge_iter->second + 1) % 3);
            Vertex_handle plane2_vertex = edge_iter->first->vertex((edge_iter->second + 2) % 3);

            Point plane1 = plane1_vertex->point();
            Point plane2 = plane2_vertex->point();
            //cout << "plane1: " << plane1 << ", plane2: " << plane2 << endl;

            int plane1_index = plane1_vertex->info();

```

```

int plane2_index = plane2_vertex->info();
//cout << "\tindex: plane1: " << plane1_index << ", plane2: " << plane2_index << endl;

// check if distance is not violated
K::FT plane_dist = CGAL::squared_distance(plane1, plane2);
//cout << "\tdist: " << plane_dist << endl;

if(plane_dist < min_dist) {
    collisions.at(plane1_index) = true;
    collisions.at(plane2_index) = true;
}
}

// calculate how many airplanes have a plane not far enough away
int planes_in_danger = 0;
for(int plane_index = 0; plane_index < airplane_count; plane_index++) {
    if(collisions.at(plane_index)) {
        planes_in_danger++;
    }
}

cout << planes_in_danger << endl;
}

return 0;
}

```

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_face_base_with_info_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef CGAL::Triangulation_vertex_base_with_info_2<bool, K> Vb;
typedef CGAL::Triangulation_face_base_2<K> Fb;
typedef CGAL::Triangulation_data_structure_2<Vb, Fb> Tds;
typedef CGAL::Delaunay_triangulation_2<K, Tds> Triangulation;

typedef Triangulation::Edge_iterator Edge_iterator;
typedef Triangulation::Face_handle Face_handle;
typedef Triangulation::Face_iterator Face_iterator;
typedef Triangulation::All_faces_iterator All_faces_iterator;
typedef Triangulation::Finite_vertices_iterator Finite_vertices_iterator;
typedef Triangulation::Finite_edges_iterator Finite_edges_iterator;
typedef Triangulation::Point Point;
typedef Triangulation::Segment Segment;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int nr_testcases;
    cin >> nr_testcases;
    for (int testcase = 0; testcase < nr_testcases; testcase++) {
        int nr_planes;
        K::FT min_distance;
        cin >> nr_planes >> min_distance;
        K::FT min_distance_squared = min_distance*min_distance;

        vector<Point> points;
        points.reserve(nr_planes);
        for (int i = 0; i < nr_planes; i++) {
            int x, y;
            cin >> x >> y;
            points.push_back(Point(x, y));
        }
    }
}

```

```

}

Triangulation t;
t.insert(points.begin(), points.end());

for (Finite_vertices_iterator it = t.finite_vertices_begin(); it != t.finite_vertices_end(); ++it) {
    it->info() = false;
}

for (Finite_edges_iterator it = t.finite_edges_begin(); it != t.finite_edges_end(); ++it) {
    Triangulation::Vertex_handle v1 = it->first->vertex((it->second + 1) % 3);
    Triangulation::Vertex_handle v2 = it->first->vertex((it->second + 2) % 3);
    K::FT d = CGAL::squared_distance(v1->point(), v2->point());
    if (d < min_distance_squared) {
        v1->info() = true;
        v2->info() = true;
    }
}

int colliding_planes = 0;
for (Finite_vertices_iterator it = t.finite_vertices_begin(); it != t.finite_vertices_end(); ++it) {
    if (it->info()) {
        colliding_planes++;
    }
}

cout << colliding_planes << endl;
}

return 0;
}

```

8 Exam Preparation

8.1 TheeV

Keywords: CGAL, Minimal Circle, Custom compare, Compare function, Binary search

```
#include <iostream>
#include <cmath>
#include <unordered_set>

#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <CGAL/Min_circle_2.h>
#include <CGAL/Min_circle_2_traits_2.h>
#include <CGAL/number_utils.h>

// typedefs
typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef CGAL::Min_circle_2_traits_2<K> Traits;
typedef CGAL::Min_circle_2<Traits> Min_circle;
typedef K::Point_2 Point;

using namespace std;

// global vars, needed by multiple functions
Point capital;
vector<Point> cities;
vector<K::FT> distances;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a+1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

bool city_order(Point cityA, Point cityB) {
    return squared_distance(cityA, capital) > squared_distance(cityB, capital);
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;

    for(int t = 0; t < test_count; t++) {
        int city_count;
        cin >> city_count;

        // check for early termination
        if(city_count <= 2) {
            int ignore;

            // read in not needed information
            cin >> ignore >> ignore;
            if(city_count == 2) {
                cin >> ignore >> ignore;
            }

            // put in each of the cities one antenna with radius 0, done.
            cout << 0 << endl;
            continue;
        }

        // read in capital city
        double capital_x, capital_y;
        cin >> capital_x >> capital_y;
        capital = Point(capital_x, capital_y);

        // create vector containing all non-capital cities
        cities = vector<Point>(city_count - 1); // capital not part of it

        // read in city coordinates
        for(int city_index = 1; city_index < city_count; city_index++) { // capital already read, so start with 1, i.e.
            // second city
            double city_x, city_y;
            cin >> city_x >> city_y;

            cities.at(city_index - 1) = Point(city_x, city_y);
        }

        // sort cities by descending distance from the capital
```

```

sort(cities.begin(), cities.end(), city_order);

// create vector with all distances for each city from the capital
distances = vector<K::FT>(city_count - 1);
for(int i = 0; i < city_count - 1; i++) {
    distances.at(i) = squared_distance(cities.at(i), capital);
}

// initiate radius temporary information
K::FT radius1 = distances[1]; // currently the first antenna has maximal radius to contain in the furthest city
K::FT old_radius1 = radius1;

K::FT radius2 = 0.0; // second antenno has no radius yet
K::FT old_radius2 = 0.0;

// create min circle for all cities
Min_circle min_circ_antenna2(cities.begin(), ++cities.begin(), true); // antenna 2 contains only the city farthest
away from first anttena as a start
Traits::Circle circ2;
int cur_index = 1;

// make first antenna's radius smaller and second antenna's radius larger to find an optimum
while(radius2 < radius1) { // iterate as long as the second antenna is still smaller as the first one
    // move current radius to old radius
    old_radius1 = radius1;
    old_radius2 = radius2;

    // add next outmost city to antenna 2's reach
    min_circ_antenna2.insert(cities[cur_index]);

    // get the circle of the min circ to get information about the radius
    circ2 = min_circ_antenna2.circle();

    // update radius
    radius1 = distances[cur_index + 1]; // first antenna only has to reach one city less as the one after that is now
    covered by the second antenna
    radius2 = circ2.squared_radius();

    // update index, maybe we add another city to second antenna's reach
    cur_index++;
}

// we updates all four radius variables, now extract the optimum
K::FT result = min( // we're interested in the minimum radius needed
    max(radius1, radius2), // we must take the maximum of both, as both antenna have to have the same radius!
    max(old_radius1, old_radius2) // same reason as before
);
cout << ceil_to_double(result) << endl;
}
}

```

```

#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <CGAL/Min_circle_2.h>
#include <CGAL/Min_circle_2_traits_2.h>
#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef CGAL::Min_circle_2_traits_2<K> Traits;
typedef CGAL::Min_circle_2<Traits> Min_circle;
typedef K::Point_2 Point;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);
    int nr_testcases;
    cin >> nr_testcases;
    for (int testcase = 0; testcase < nr_testcases; testcase++) {
        int n;

```



```

cin >> n;
vector<Point> points(n);
vector<K::FT> distances(n);
for (int i = 0; i < n; i++) {
    int x, y;
    cin >> x >> y;
    points[i] = Point(x, y);
    distances[i] = CGAL::squared_distance(points[0], points[i]);
}
vector<K::FT> distances_sorted = distances;
sort(distances_sorted.begin(), distances_sorted.end());

int min_bound = 0;
int max_bound = distances_sorted.size() - 1;
while (min_bound != max_bound) {
    int half = (min_bound + max_bound) / 2;
    K::FT squared_dist = distances_sorted[half];
    //cout << "min = " << min_bound << " max = " << max_bound << " half = " << half << " squared_dist = " <<
    squared_dist << endl;

    vector<Point> uncovered_points;
    for (int i = 0; i < n; i++) {
        if (distances[i] > squared_dist) {
            //cout << "adding point" << points[i] << endl;
            uncovered_points.push_back(points[i]);
        }
    }
    //cout << uncovered_points.size() << endl;
    Min_circle mc(uncovered_points.begin(), uncovered_points.end(), true);
    //cout << "valid = " << mc.is_valid() << endl;
    K::FT second_rad_squared = mc.circle().squared_radius();
    //cout << "second_rad_squared = " << second_rad_squared << endl;
    if (second_rad_squared > squared_dist) {
        //cout << "increase lower bound" << endl;
        if (second_rad_squared < distances_sorted[half + 1]) {
            max_bound = min_bound = half;
        }
        else {
            min_bound = half + 1;
        }
    }
    else {
        //cout << "decrease upper bound" << endl;
        max_bound = half;
    }
}

K::FT squared_dist = distances_sorted[min_bound];
vector<Point> uncovered_points;
for (int i = 0; i < n; i++) {
    if (distances[i] > squared_dist) {
        uncovered_points.push_back(points[i]);
    }
}
Min_circle mc(uncovered_points.begin(), uncovered_points.end(), true);
K::FT second_rad_squared = mc.circle().squared_radius();
cout << ceil_to_double(max(squared_dist, second_rad_squared)) << endl;
}

return 0;
}

```

8.2 Algocoön Group

Keywords: BGL, Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Max-flow

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list_traits<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, int,
    property<edge_residual_capacity_t, int,
    property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;

typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;

```

```

typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_count;
    cin >> test_count;

    for(int test_index = 0; test_index < test_count; test_index++) {
        int figure_count, limb_count;
        cin >> figure_count >> limb_count;

        // create graph
        Graph graph(figure_count);

        // get graph's properties
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap reverse_edge_map = get(edge_reverse, graph);
        ResidualCapacityMap res_capacity = get(edge_residual_capacity, graph);

        // read in limbs
        for(int limb_index = 0; limb_index < limb_count; limb_index++) {
            int from, to, cost;
            cin >> from >> to >> cost;

            Edge edge;
            Edge rev_edge;
            tie(edge, tuples::ignore) = add_edge(from, to, graph);
            tie(rev_edge, tuples::ignore) = add_edge(to, from, graph);
            capacity[edge] = cost;
            capacity[rev_edge] = 0;
            reverse_edge_map[edge] = rev_edge;
            reverse_edge_map[rev_edge] = edge;
        }

        // find best source and sink
        int best_source = -1;
        int best_sink = -1;
        int best_value = numeric_limits<int>::max();

        // attention: start at 1, otherwise assertion because sink == source
        for(int figure_index = 1; figure_index < figure_count; figure_index++) {
            // search for best sink
            int max_flow = push_relabel_max_flow(graph, 0, figure_index);
            if(max_flow < best_value) {
                best_value = max_flow;
                best_source = 0;
                best_sink = figure_index;
            }

            // search for best source
            max_flow = push_relabel_max_flow(graph, figure_index, 0);
            if(max_flow < best_value) {
                best_value = max_flow;
                best_source = figure_index;
                best_sink = 0;
            }
        }

        // rerun for found best sink and source
        push_relabel_max_flow(graph, best_source, best_sink);

        std::queue<int> Q;
        Q.push(best_source);

        vector<bool> visited(figure_count, false);
        visited.at(best_source) = true;

        while(!Q.empty()) {
            const int figure = Q.front();
            Q.pop();
            graph_traits<Graph>::out_edge_iterator out_iter, out_end;
            for(tie(out_iter, out_end) = out_edges(figure, graph); out_iter != out_end; ++out_iter) {
                const int edge_end_v = target(*out_iter, graph);

                if(res_capacity[*out_iter] == 0 || visited[edge_end_v]) {
                    continue;
                }

                visited[edge_end_v] = true;
                Q.push(edge_end_v);
            }
        }

        cout << best_value << endl << count(visited.begin(), visited.end(), true);
        for(int i = 0; i < figure_count; i++) {
            if(visited[i]) {
                cout << " " << i;
            }
        }
        cout << endl;
    }
}

```

```

    return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <boost/graph/connected_components.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualMap;

inline void add_flow_edge(int start, int end, int c, EdgeCapacityMap& capacity, ReverseEdgeMap& rev_edge, Graph& g) {
    Edge e, rev_e;
    bool success;
    tie(e, success) = add_edge(start, end, g);
    if (success) {
        tie(rev_e, success) = add_edge(end, start, g);
        capacity[e] = c;
        capacity[rev_e] = 0;
        rev_edge[e] = rev_e;
        rev_edge[rev_e] = e;
    }
    else {
        capacity[e] += c;
    }
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_figures, nr_limbs;
        cin >> nr_figures >> nr_limbs;

        Graph g(nr_figures);

        EdgeCapacityMap capacity = get(edge_capacity, g);
        ReverseEdgeMap rev_edge = get(edge_reverse, g);
        ResidualMap residual_map = get(edge_residual_capacity, g);

        for (int i = 0; i < nr_limbs; i++) {
            int from, to, cost;
            cin >> from >> to >> cost;
            add_flow_edge(from, to, cost, capacity, rev_edge, g);
        }

        int best_sink;
        int best_source;
        int min_flow = numeric_limits<int>::max();
        for (int i = 1; i < nr_figures; i++) {
            int flow = push_relabel_max_flow(g, 0, i);
            if (flow < min_flow) {
                min_flow = flow;
                best_source = 0;
                best_sink = i;
            }
        }
        flow = push_relabel_max_flow(g, i, 0);
        if (flow < min_flow) {
            min_flow = flow;
            best_source = i;
            best_sink = 0;
        }
    }

    min_flow = push_relabel_max_flow(g, best_source, best_sink);

    vector<int> vis(nr_figures, false);
    vis[best_source] = true;
    std::queue<int> to_visit;
    to_visit.push(best_source);

```

```

while (!to_visit.empty()) {
    const int u = to_visit.front();
    to_visit.pop();

    graph_traits<Graph>::out_edge_iterator ebegin, eend;
    for (tie(ebegin, eend) = out_edges(u, g); ebegin != eend; ++ebegin) {
        const int v = target(*ebegin, g);
        if (residual_map[*ebegin] == 0 || vis[v]) {
            continue;
        }
        vis[v] = true;
        to_visit.push(v);
    }
}

cout << min_flow << endl;
cout << count(vis.begin(), vis.end(), true);
for (int i = 0; i < nr_figures; ++i) {
    if (vis[i]) {
        cout << " " << i;
    }
}
cout << endl;
}

return EXIT_SUCCESS;
}

```

8.3 Monkey Island

Keywords: BGL, Strong component, Edge iteration, Uses class, Custom compare, Compare function, Greedy, DFS (Graph)

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/strong_components.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_iterator EdgeIterator;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int i = 0; i < test_cases; i++) {
        int location_count, road_count;
        cin >> location_count >> road_count;

        // create graph
        Graph graph(location_count);

        // read in directed roads and create edges for them
        for(int j = 0; j < road_count; j++) {
            // read vertices information
            int from, to;
            cin >> from >> to;

            // locations start at 1, inside our graph at 0
            from -= 1;
            to -= 1;

            // create edge
            add_edge(from, to, graph);
        }

        // read in costs for building police station at a vertex
        vector<int> costs(location_count);
        for(int j = 0; j < location_count; j++) {
            cin >> costs.at(j);
            //cout << "-> " << costs.at(j) << endl;;
        }

        // retrieve strong components (maximum set of vertices for which we have a path from any pair in this set).
        // each vertex is labeled with an int representing the set the vertex belongs to
        // (to which strong component set)
        int scc_count = strong_components(graph, &scc[0]);
    }
}

```

```

// records which strong component set has an incoming edge from an other strong component set
vector<bool> incoming(scc_count, false);

// iterate over all graph edges
EdgeIterator edge_iter, edge_end;
for(tie(edge_iter, edge_end) = edges(graph); edge_iter != edge_end; ++edge_iter) {
    // get vertices connected by the edge
    int from_vertex = source(*edge_iter, graph);
    int to_vertex = target(*edge_iter, graph);

    // check that both vertices are not part of the same strong component
    if(scc[from_vertex] != scc[to_vertex]) {
        // ok, so both vertices don't have edges in both directions.
        // furthermore we have a *directed* edge from the strong component set of from_vertex to the one of
        // to_vertex, record that we have an incoming
        // edge.
        incoming[scc[to_vertex]] = true;
    }
}

// so, now we know which strong component set has incoming edges from other sets. Therefore we don't have to
// build a police station inside a strong component
// set that has an incoming edge! Reason: we can build one in the strong component set that has an edge to the
// other set and with that we reach all vertices in
// both strong component sets.

// keeps track of minimum costs needed
vector<int> min_costs(scc_count, numeric_limits<int>::max());

// iterate over all locations
for(int location_index = 0; location_index < location_count; location_index++) {
    if(!incoming[scc[location_index]]) {
        // the strong component set the location belongs to has *no* incoming edge.

        // assign the minimum cost for the strong component set the current location belongs to is defined
        // by the smaller number from the set {already known minimum, cost to build in the current location a
        // police station}
        //cout << "min_costs = " << min(min_costs[scc[location_index]], costs[location_index]) << " ( " <<
        //min_costs[scc[location_index]] << " or " << costs[location_index] << " )" << endl;
        min_costs[scc[location_index]] = min(min_costs[scc[location_index]], costs[location_index]);
    }
}

// sum up the costs for all police stations, jumping over strong component sets with incoming edges!
int cost_sum = 0;
for(int scc_index = 0; scc_index < scc_count; scc_index++) {
    if(!incoming[scc_index]) {
        //cout << "==" << cost_sum << " + " << min_costs[scc_index] << " = ";
        cost_sum += min_costs[scc_index];
        //cout << cost_sum << endl;
    }
}

// out with result
cout << cost_sum << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <algorithm>
#include <iomanip>
#include <queue>
#include <limits>
#include <stack>

using namespace std;

class Location {
public:
    int cost, id;
    bool visited;
    vector<Location*> roads;

    Location(int id)
        : cost(0), id(id), visited(false), roads(vector<Location*>()) {}
};

bool sortLocations(const Location* lhs, const Location* rhs) {
    return lhs->cost < rhs->cost;
}

int main(void) {
    int cases;
    cin >> cases;

    for (int c = 0; c < cases; c++) {
        int nr_locations;
        cin >> nr_locations;
        int nr_roads;
        cin >> nr_roads;
    }
}

```

```

vector<Location*> locations(nr_locations, NULL);
for (int i = 0; i < nr_locations; i++) {
    locations[i] = new Location(i + 1);
}
for (int i = 0; i < nr_roads; i++) {
    int from, to;
    cin >> from >> to;
    locations[from - 1]->roads.push_back(locations[to - 1]);
}
for (int i = 0; i < nr_locations; i++) {
    int cost;
    cin >> cost;
    locations[i]->cost = cost;
}

sort(locations.begin(), locations.end(), sortLocations);

int cost = 0;
for (int i = 0; i < nr_locations; i++) {
    Location* currentLoc = locations[i];
    //cout << "Location " << currentLoc->id << " with cost " << currentLoc->cost << endl;
    if (!currentLoc->visited) {
        cost += currentLoc->cost;
        int tmpCost = currentLoc->cost;
        stack<Location*> locToVisit;
        locToVisit.push(currentLoc);
        vector<bool> vis(nr_locations, false);
        while (!locToVisit.empty()) {
            Location* nextLocation = locToVisit.top();
            locToVisit.pop();
            if (!vis[nextLocation->id]) {
                if (nextLocation->visited) {
                    if (nextLocation->cost > 0) {
                        //cout << "Reducing cost by " << nextLocation->cost << endl;
                        cost -= nextLocation->cost;
                        nextLocation->cost = 0;
                    }
                }
                else {
                    nextLocation->visited = true;
                    nextLocation->cost = 0;
                }
                vis[nextLocation->id] = true;
                for (unsigned int j = 0; j < nextLocation->roads.size(); j++) {
                    locToVisit.push(nextLocation->roads[j]);
                }
            }
        }
        currentLoc->cost = tmpCost;
    }
}

cout << cost << endl;
for (int i = 0; i < nr_locations; i++) {
    delete locations[i];
}
}
}

```

8.4 Odd Route

Keywords: ACM, BFS (Graph), Custom BFS

```

#include <iostream>
#include <algorithm>
#include <vector>
#include <queue>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        // get graph properties
        int vertex_count, edge_count;
        cin >> vertex_count >> edge_count;

        // get start and target vertices
        int start_vertex, target_vertex;
        cin >> start_vertex >> target_vertex;

        // read in vertices
        vector<Vertex> vertices(vertex_count);
        for(int edge_index = 0; edge_index < edge_count; edge_index++) {
            int from, to, weight;
            cin >> from >> to >> weight;

            vertices.at(from).edges.push_back(make_pair(to, weight));

```

```

}

priority_queue<NextMove, vector<NextMove>, MoveCompare> to_visit;
to_visit.push(NextMove(start_vertex, 0, 0));

int shortest_weight_sum = numeric_limits<int>::max();

while(!to_visit.empty()) {
    NextMove cur_move = to_visit.top();
    to_visit.pop();

    if(cur_move.is_odd_weight()) {
        if(cur_move.is_odd_length()) {
            // odd weight, odd length
            if(vertices.at(cur_move.current_vertex).visited_odd_edges_odd_weight) {
                // next edge was already visited with odd weight and odd length, do not revisit
                continue;
            } else {
                if(cur_move.current_vertex == target_vertex) {
                    // found the end with searched configuration, update shortest weight
                    shortest_weight_sum = cur_move.weight_sum;
                    break;
                }

                // visited it now with the configuration
                vertices[cur_move.current_vertex].visited_odd_edges_odd_weight = true;
            }
        } else {
            // odd weight, even length
            if(vertices.at(cur_move.current_vertex).visited_even_edges_odd_weight) {
                // already visited with that configuration
                continue;
            } else {
                vertices.at(cur_move.current_vertex).visited_even_edges_odd_weight = true;
            }
        }
    } else {
        if(cur_move.is_odd_length()) {
            // even weight, odd length
            if(vertices.at(cur_move.current_vertex).visited_odd_edges_even_weight) {
                // already visited with that configuration
                continue;
            } else {
                vertices.at(cur_move.current_vertex).visited_odd_edges_even_weight = true;
            }
        } else {
            // even weight, even length
            if(vertices.at(cur_move.current_vertex).visited_even_edges_even_weight) {
                // already visited with that configuration
                continue;
            } else {
                vertices.at(cur_move.current_vertex).visited_even_edges_even_weight = true;
            }
        }
    }

    // ok, if we reach this point, we found an edge that we didn't visit in the current configuration,
    // have to visit it with current configuration
    vector<pair<int, int> >& edges = vertices.at(cur_move.current_vertex).edges;
    for(int next_edge = 0; next_edge < edges.size(); next_edge++) {
        to_visit.push(NextMove(edges.at(next_edge).first, // use the next vertex that can be reached by current
                                vertex
                                cur_move.weight_sum + edges.at(next_edge).second, // add weight of the edge we would follow
                                cur_move.edges_length + 1)); // we use an edge more, wow!
    }
}

if(shortest_weight_sum == numeric_limits<int>::max()) {
    cout << "no" << endl;
} else {
    cout << shortest_weight_sum - 1 << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>

using namespace std;

class Vertex {
public:
    bool v_even_edges_even_weight;
    bool v_even_edges_odd_weight;
    bool v_odd_edges_even_weight;
    bool v_odd_edges_odd_weight;

    vector<pair<int, int> > edges;

```

```

Vertex() {
    v_even_edges_even_weight = false;
    v_even_edges_odd_weight = false;
    v_odd_edges_even_weight = false;
    v_odd_edges_odd_weight = false;
}
};

class Move {
public:
    int current_vertex;
    int weight_sum;
    int edges_sum;

    Move(int current_vertex, int weight_sum, int edges_sum)
        : current_vertex(current_vertex), weight_sum(weight_sum), edges_sum(edges_sum)
    { }

    bool is_odd_weight() { return weight_sum % 2; }
    bool is_odd_edges() { return edges_sum % 2; }
};

struct MoveComparator {
    bool operator() (const Move& lhs, const Move& rhs) {
        return lhs.weight_sum > rhs.weight_sum;
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_vertices, nr_edges;
        cin >> nr_vertices >> nr_edges;
        vector<Vertex> vertices(nr_vertices);

        int start, end;
        cin >> start >> end;

        for (int i = 0; i < nr_edges; i++) {
            int from, to, weight;
            cin >> from >> to >> weight;
            vertices[from].edges.push_back(make_pair(to, weight));
        }

        priority_queue<Move, vector<Move>, MoveComparator> to_visit;
        to_visit.push(Move(start, 0, 0));
        int shortest_weight_sum = -1;
        while (!to_visit.empty()) {
            Move cur_move = to_visit.top();
            to_visit.pop();
            if (cur_move.is_odd_weight()) {
                if (cur_move.is_odd_edges()) {
                    if (vertices[cur_move.current_vertex].v_odd_edges_odd_weight) {
                        continue;
                    }
                }
                else {
                    if (cur_move.current_vertex == end) {
                        shortest_weight_sum = cur_move.weight_sum;
                        break;
                    }
                    vertices[cur_move.current_vertex].v_odd_edges_odd_weight = true;
                }
            }
            else {
                if (vertices[cur_move.current_vertex].v_even_edges_odd_weight) {
                    continue;
                }
                else {
                    vertices[cur_move.current_vertex].v_even_edges_odd_weight = true;
                }
            }
        }
        else {
            if (cur_move.is_odd_edges()) {
                if (vertices[cur_move.current_vertex].v_odd_edges_even_weight) {
                    continue;
                }
                else {
                    vertices[cur_move.current_vertex].v_odd_edges_even_weight = true;
                }
            }
            else {
                if (vertices[cur_move.current_vertex].v_even_edges_even_weight) {
                    continue;
                }
                else {
                    vertices[cur_move.current_vertex].v_even_edges_even_weight = true;
                }
            }
        }
    }
    vector<pair<int, int> >& edges = vertices[cur_move.current_vertex].edges;
}

```



```

        for (int i = 0; i < edges.size(); i++) {
            to_visit.push(Move(edges[i].first, cur_move.weight_sum + edges[i].second, cur_move.edges_sum + 1));
        }
    }

    if (shortest_weight_sum == -1) {
        cout << "no" << endl;
    }
    else {
        cout << shortest_weight_sum << endl;
    }
}

return 0;
}

```

8.5 Divisor Distance

Keywords: Prime Sieve, Next Prime, ACM

```

#include <iostream>
#include <algorithm>
#include <vector>
#include <queue>
#include <bitset>

using namespace std;

#define MAX_PRIM 10000000
bitset<MAX_PRIM> is_prime;

void prime_sieve() {
    is_prime.set(); // sets all to true
    // 0, 1 not prime
    is_prime.reset(0);
    is_prime.reset(1);

    for(int prime = 2; prime * prime <= MAX_PRIM; prime++) {
        if(is_prime.test(prime)) {
            // is a prime number, so all multiples of it are not
            for(int multiple = prime + prime; multiple < MAX_PRIM; multiple += prime) {
                is_prime.reset(multiple); // set multiple to false, as it is not prime
            }
        }
    }
}

int get_next_prime(int cur) {
    for(int i = cur + 1; i * i < MAX_PRIM; i++) {
        if(is_prime.test(i)) {
            return i;
        }
    }

    return 0;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    prime_sieve();

    for(int test = 0; test < test_cases; test++) {
        int max_nr, vertices_pair_count;
        cin >> max_nr >> vertices_pair_count;

        for(int vertices_pair_index = 0; vertices_pair_index < vertices_pair_count; vertices_pair_index++) {
            int from, to;
            cin >> from >> to;

            int length = 0;
            int a_factor = 2;
            int b_factor = 2;

            // divide 'from' and 'to' with prime numbers as long as they do not match
            while(from != to) {
                // always lower the current "maximum" of 'from' and 'to'
                if(from > to) {
                    if(is_prime.test(from)) {
                        from = 1;
                    } else {
                        // find a divisor for current 'from'
                        while(from % a_factor != 0) {
                            a_factor = get_next_prime(a_factor);
                        }
                    }

                    // found a divisor for 'from', " " remove " " it
                    from = from / a_factor;
                }
            }
        }
    }
}

```

```

    } else {
        // following code works the same as for 'from' and 'a_factor'
        if(is_prime.test(to)) {
            to = 1;
        } else {
            while(to % b_factor != 0) {
                b_factor = get_next_prime(b_factor);
            }

            to = to / b_factor;
        }
    }

    length++;
}

cout << length << endl;
}
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <algorithm>
#include <iomanip>
#include <queue>
#include <limits>
#include <cmath>

using namespace std;

int main(void) {
    ios_base::sync_with_stdio(false);

    const int max_n = 10000000;
    const int sqrt_max_n = sqrt(max_n) + 1;
    vector<bool> is_prime(max_n, true);
    vector<int> primes;

    //cout << "Calculating primes" << endl;
    is_prime[0] = false;
    is_prime[1] = false;
    for (int i = 2; i <= sqrt_max_n; i++) {
        if (is_prime[i]) {
            for (int j = 2 * i; j <= max_n; j += i) {
                is_prime[j] = false;
            }
        }
    }

    //cout << "Adding primes to vector" << endl;
    for (int i = 0; i <= max_n; i++) {
        if (is_prime[i]) {
            primes.push_back(i);
        }
    }

    int tests;
    cin >> tests;

    //cout << "Starting tests" << endl;
    for (int test = 0; test < tests; test++) {
        int n, cases;
        cin >> n >> cases;
        for (int c = 0; c < cases; c++) {
            int v1, v2, hops = 0;
            cin >> v1 >> v2;
            while (v1 != v2) {
                if (v1 > v2) {
                    //cout << "Changing " << v1 << " to " << largest_div[v1] << endl;
                    if (is_prime[v1]) {
                        v1 = 1;
                    }
                    else {
                        for (unsigned int j = 0; j < primes.size(); j++) {
                            if (v1 % primes[j] == 0) {
                                v1 = v1 / primes[j];
                                break;
                            }
                        }
                    }
                    //v1 = largest_div[v1];
                    hops++;
                }
                else {
                    //cout << "Changing " << v2 << " to " << largest_div[v2] << endl;
                    if (is_prime[v2]) {
                        v2 = 1;
                    }
                    else {
                        for (unsigned int j = 0; j < primes.size(); j++) {
                            if (v2 % primes[j] == 0) {

```

```

        v2 = v2 / primes[j];
        break;
    }
}
}
//v2 = largest_div[v2];
hops++;
}
}
cout << hops << endl;
}
}
}
}

```

8.6 Portfolios Revisited

Keywords: CGAL, Quadratic Program, Non-negative quadratic program, Exponential bound search, Binary search

```

#include <iostream>
#include <cassert>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpzf.h>
typedef CGAL::Gmpzf ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

using namespace std;

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

bool feasible(int asset_count, int max_cost, int max_covar, int expected_return, vector<int>& asset_costs, vector<int>& asset_returns, vector<vector<int>>& asset_covar) {
    // use quadratic programming to check if given parameters result in a feasible result, i.e. given parameters result in a possible solution

    // create QP program: lower bound is 0 for amount of assets bought and generally we're interested in <= inequations
    Program qp(CGAL::SMALLER, true, 0, false, 0);

    // restrict total cost
    for(int asset_index = 0; asset_index < asset_count; asset_index++) {
        qp.set_a(asset_index, 0, asset_costs.at(asset_index)); // sum of all asset's cost that were bought must be ...
    }

    qp.set_b(0, max_cost); // ... <= maximum cost investor is ready to pay

    // restrict how much expected return we expect, be careful as our general operation is '<=', we have to switch it here, as we want at least some amount of
    // expected return
    for(int asset_index = 0; asset_index < asset_count; asset_index++) {
        qp.set_a(asset_index, 1, asset_returns[asset_index]); // sum of all asset's expected return that were bought must be ...
    }

    qp.set_r(1, CGAL::LARGER); // ... >= ...
    qp.set_b(1, expected_return); // the expected return

    // set the objective function:
    // total portfolio variance must be not too large
    for(int assetA_index = 0; assetA_index < asset_count; assetA_index++) {
        for(int assetB_index = 0; assetB_index <= assetA_index; assetB_index++) { // careful: '<= assetA_index'!
            qp.set_d(assetA_index, assetB_index, 2 * asset_covar.at(assetA_index).at(assetB_index)); // careful: don't forget the 2 * because of how the matrix works...
        }
    }

    // find solution
    Solution sol = CGAL::solve_nonnegative_quadratic_program(qp, ET());
    return sol.is_optimal() && sol.objective_value() <= max_covar;
}

```

```

int main() {

    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    while(true) {
        int asset_count, friend_count;
        cin >> asset_count >> friend_count;

        // kill switch
        if(asset_count == 0 && friend_count == 0) {
            break;
        }

        // collect asset costs and risk factors
        vector<int> asset_costs(asset_count);
        vector<int> asset_returns(asset_count);
        for(int asset_index = 0; asset_index < asset_count; asset_index++) {
            cin >> asset_costs.at(asset_index) >> asset_returns.at(asset_index);
        }

        // collect covariance between two assets
        vector<vector<int> > > asset_covar(asset_count, vector<int>(asset_count));
        for(int assetA_index = 0; assetA_index < asset_count; assetA_index++) {
            for(int assetB_index = 0; assetB_index < asset_count; assetB_index++) {
                cin >> asset_covar.at(assetA_index).at(assetB_index);
            }
        }

        // go through investors and calculate maximum expected outcome for given conditions
        for(int investor_index = 0; investor_index < friend_count; investor_index++) {
            int max_cost, max_covar;
            cin >> max_cost >> max_covar;

            // search for upper bound for return, so we don't have to search through every possible combination
            int r = 1;
            while(feasible(asset_count, max_cost, max_covar, r, asset_costs, asset_returns, asset_covar)) {
                r *= 2;
            }

            // now we know in which range ([r/2, r]) to search for the maximum
            // use binary search to find it
            int ok = r / 2;
            int low = r / 2;
            int high = r;
            while(low <= high) {
                r = (high + low) / 2 + low;
                if(feasible(asset_count, max_cost, max_covar, r, asset_costs, asset_returns, asset_covar)) {
                    ok = r;
                    low = r + 1;
                } else {
                    high = r - 1;
                }
            }

            cout << ok << endl;
        }
    }
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

// program and solution types
typedef CGAL::Quadratic_program<int> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

```

```

}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

bool is_possible(Program& qp, int expected_return, int max_variance) {
    qp.set_b(1, expected_return);

    Solution s = CGAL::solve_nonnegative_quadratic_program(qp, ET());
    assert(s.solves_quadratic_program(qp));
    return !(s.is_infeasible() || s.is_unbounded() || s.objective_value() > max_variance);
}

int main() {

    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int nr_assets, nr_friends;
    for (cin >> nr_assets >> nr_friends; nr_assets != 0 && nr_friends != 0; cin >> nr_assets >> nr_friends) {
        Program qp(CGAL::LARGER, true, 0, false, 0);
        qp.set_r(0, CGAL::SMALLER); //cost
        qp.set_r(1, CGAL::LARGER); //expectet return

        for (int i = 0; i < nr_assets; ++i) {
            int asset_cost, asset_return;
            cin >> asset_cost >> asset_return;
            qp.set_a(i, 0, asset_cost);
            qp.set_a(i, 1, asset_return);
        }

        for (int i = 0; i < nr_assets; ++i) {
            for (int j = 0; j < nr_assets; ++j) {
                int asset_covariance;
                cin >> asset_covariance;
                if (j <= i) {
                    qp.set_d(i, j, asset_covariance * 2);
                }
            }
        }

        for (int friend_id = 0; friend_id < nr_friends; ++friend_id) {
            int max_invest, max_variance;
            cin >> max_invest >> max_variance;
            qp.set_b(0, max_invest);

            int expected_return = 1;
            while (is_possible(qp, expected_return, max_variance)) {
                expected_return *= 2;
            }
            int ok = expected_return / 2;
            int min_bound = expected_return / 2;
            int max_bound = expected_return;
            while (min_bound <= max_bound) {
                int middle = (max_bound - min_bound) / 2 + min_bound;
                if (is_possible(qp, middle, max_variance)) {
                    ok = middle;
                    min_bound = middle + 1;
                }
                else {
                    max_bound = middle - 1;
                }
            }
            cout << ok << endl;
        }
    }
    return 0;
}

```

8.7 Tetris

Keywords: BGL, Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Max-flow

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;

```

```

using namespace boost;

typedef adjacency_list_traits<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, unsigned int,
    property<edge_residual_capacity_t, unsigned int,
    property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;

typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test_index = 0; test_index < test_cases; test_index++) {
        int game_width, brick_count;
        cin >> game_width >> brick_count;

        // define index of source and sink vertex
        int source_index = game_width + 1;
        int sink_index = game_width;

        // create graph, don't forget we need a source and sink too
        Graph graph(game_width * 2 + 1);

        // get graph's properties
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, graph);
        // not used: ResidualCapacityMap res_capacity = get(edge_residual_capacity, graph);

        // create edges for splits, as only one can exist at any location. So add edges with weight 1
        for(int game_location = 0; game_location < game_width; game_location++) {
            bool new_edge;
            Edge edge, reverse_edge;
            tie(edge, new_edge) = add_edge(game_location, game_width + game_location + 1, graph);
            tie(reverse_edge, new_edge) = add_edge(game_width + game_location + 1, game_location, graph);

            capacity[edge] = 1;
            capacity[reverse_edge] = 0;
            rev_edge[edge] = reverse_edge;
            rev_edge[reverse_edge] = edge;
        }

        // read in widths of blocks
        for(int brick_index = 0; brick_index < brick_count; brick_index++) {
            int start, end;
            cin >> start >> end;

            // make sure we get expected order
            if(start > end) {
                int tmp = end;
                end = start;
                start = tmp;
            }

            // check if we stay inside the game field
            if(end > game_width) {
                continue;
            }

            // add edge for brick
            bool new_edge;
            Edge edge, reverse_edge;
            tie(edge, new_edge) = add_edge(start + game_width + 1, end, graph);
            tie(reverse_edge, new_edge) = add_edge(end, start + game_width + 1, graph);

            capacity[edge] = 1;
            capacity[reverse_edge] = 0;
            rev_edge[edge] = reverse_edge;
            rev_edge[reverse_edge] = edge;
        }

        long max_flow = push_relabel_max_flow(graph, source_index, sink_index);
        cout << max_flow << endl;
    }

    return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>

```

```

#include <queue>
#include <set>
#include <utility>
#include <cmath>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    //cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int width, nr_bricks;
        cin >> width >> nr_bricks;

        Graph g(width * 2 + 1);
        int source = width + 1;
        int sink = width;
        EdgeCapacityMap capacity = get(edge_capacity, g);
        ReverseEdgeMap rev_edge = get(edge_reverse, g);

        Edge e, rev_e;
        bool success;
        for (int i = 1; i < width; i++) {
            tie(e, success) = add_edge(i, width + i + 1, g);
            tie(rev_e, success) = add_edge(width + i + 1, i, g);
            capacity[e] = 1;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
        }

        for (int i = 0; i < nr_bricks; i++) {
            int first_end, second_end;
            cin >> first_end >> second_end;
            if (first_end > second_end) {
                int tmp = first_end;
                first_end = second_end;
                second_end = tmp;
            }
            tie(e, success) = add_edge(width + 1 + first_end, second_end, g);
            tie(rev_e, success) = add_edge(second_end, width + 1 + first_end, g);
            capacity[e] = 1;
            capacity[rev_e] = 0;
            rev_edge[e] = rev_e;
            rev_edge[rev_e] = e;
        }

        int flow = push_relabel_max_flow(g, source, sink);
        cout << flow << endl;
    }

    return 0;
}

```

8.8 Stamp Exhibition

Keywords: CGAL, Quadratic Program

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

```

```

#include <CGAL/basic.h>
#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpzf ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Segment_2 Segment;

typedef CGAL::Quadratic_program<ET> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

using namespace CGAL;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    //cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;

    for (int test = 0; test < test_count; test++) {
        int lamp_count, stamp_count, wall_count;
        cin >> lamp_count >> stamp_count >> wall_count;

        // read in lamps
        vector<Point> lamps;
        lamps.reserve(lamp_count);

        for(int lamp_index = 0; lamp_index < lamp_count; lamp_index++) {
            int lamp_x, lamp_y;
            cin >> lamp_x >> lamp_y;

            lamps.push_back(Point(lamp_x, lamp_y));
        }

        // collect lamps and the maximum light they are allowed to get
        vector<Point> stamps;
        stamps.reserve(stamp_count);

        vector<int> stamp_max_light;
        stamp_max_light.reserve(stamp_count);

        for(int stamp_index = 0; stamp_index < stamp_count; stamp_index++) {
            int stamp_x, stamp_y, max_light;
            cin >> stamp_x >> stamp_y >> max_light;

            stamps.push_back(Point(stamp_x, stamp_y));
            stamp_max_light.push_back(max_light);
        }

        // read in walls
        vector<Segment> walls;
        walls.reserve(wall_count);
        for(int wall_index = 0; wall_index < wall_count; wall_index++) {
            int wall_start_x, wall_start_y, wall_end_x, wall_end_y;
            cin >> wall_start_x >> wall_start_y >> wall_end_x >> wall_end_y;

            walls.push_back(Segment(Point(wall_start_x, wall_start_y), Point(wall_end_x, wall_end_y)));
        }

        // Create quadratic programming instance
        Program qp(CGAL::SMALLER, true, 1, true, 4096);
        for(int stamp_index = 0; stamp_index < stamp_count; stamp_index++) {
            for(int lamp_index = 0; lamp_index < lamp_count; lamp_index++) {
                Segment light_to_stamp(lamps.at(lamp_index), stamps.at(stamp_index));

                // check if a wall blocks the light beam from lamp to stamp
                bool light_blocked = false;
                for(int wall_index = 0; wall_index < wall_count; wall_index++) {

```



```

        if(do_intersect(light_to_stamp, walls.at(wall_index))) {
            light_blocked = true;
            break;
        }
    }

    if(!light_blocked) {
        double d = CGAL::to_double(1 / light_to_stamp.squared_length());
        qp.set_a(lamp_index, stamp_index, d);
        qp.set_a(lamp_index, stamp_count + stamp_index, d);
    }
}

qp.set_b(stamp_index, stamp_max_light.at(stamp_index));
qp.set_b(stamp_count + stamp_index, 1);
qp.set_r(stamp_count + stamp_index, CGAL::LARGER);
}

// get solution
Solution sol = solve_linear_program(qp, ET());
if(sol.is_infeasible()) {
    cout << "no" << endl;
} else {
    cout << "yes" << endl;
}
}

return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <CGAL/basic.h>
#include <CGAL/Exact_predicates_exact_constructions_kernel.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpzf ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

typedef CGAL::Exact_predicates_exact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Segment_2 Segment;

typedef CGAL::Quadratic_program<ET> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

using namespace CGAL;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    //cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_lights, nr_stamps, nr_walls;
        cin >> nr_lights >> nr_stamps >> nr_walls;

        vector<Point> lights;
        lights.reserve(nr_lights);
        for (int i = 0; i < nr_lights; i++) {
            int light_x, light_y;
            cin >> light_x >> light_y;

```

```

    lights.push_back(Point(light_x, light_y));
}
vector<Point> stamps;
vector<int> stamps_max_int;
stamps.reserve(nr_stamps);
stamps_max_int.reserve(nr_stamps);
for (int i = 0; i < nr_stamps; i++) {
    int stamp_x, stamp_y, stamp_max_light_intensity;
    cin >> stamp_x >> stamp_y >> stamp_max_light_intensity;
    stamps.push_back(Point(stamp_x, stamp_y));
    stamps_max_int.push_back(stamp_max_light_intensity);
}
vector<Segment> walls;
walls.reserve(nr_walls);
for (int i = 0; i < nr_walls; i++) {
    int wall_start_x, wall_start_y, wall_end_x, wall_end_y;
    cin >> wall_start_x >> wall_start_y >> wall_end_x >> wall_end_y;
    walls.push_back(Segment(Point(wall_start_x, wall_start_y), Point(wall_end_x, wall_end_y)));
}

Program qp(CGAL::SMALLER, true, 1, true, 4096);
for (int i = 0; i < nr_stamps; i++) {
    for (int j = 0; j < nr_lights; j++) {
        Segment light_to_stamp(lights[j], stamps[i]);
        bool blocked = false;
        for (int k = 0; k < nr_walls; k++) {
            if (do_intersect(light_to_stamp, walls[k])) {
                blocked = true;
                break;
            }
        }
        if (!blocked) {
            double d = CGAL::to_double(1 / light_to_stamp.squared_length());
            qp.set_a(j, i, d);
            qp.set_a(j, nr_stamps + i, d);
        }
    }
    qp.set_b(i, stamps_max_int[i]);
    qp.set_b(nr_stamps + i, 1);
    qp.set_r(nr_stamps + i, CGAL::LARGER);
}
Solution sol = solve_linear_program(qp, ET());
assert(sol.solves_linear_program(qp));
if (sol.is_infeasible()) {
    cout << "no" << endl;
}
else {
    cout << "yes" << endl;
}
}
return 0;
}

```

8.9 Placing Knights

Keywords: BGL, Matching

```

#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <limits>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/max_cardinality_matching.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        int width;
        cin >> width;

        // 2D vector indicating if the field is not a hole
        vector<bool> can_occupy(width * width);
    }
}

```

```

int hole_count = 0;
for(int row = 0; row < width; row++) {
    for(int column = 0; column < width; column++) {
        int indicator;
        cin >> indicator;

        if(indicator == 1) {
            can_occupy.at(row + column * width) = true;
        } else {
            hole_count++;
            can_occupy.at(row + column * width) = false;
        }
    }
}

// create graph for game field
Graph graph(width * width);

// create edges
for(int row = 0; row < width; row++) {
    for(int column = 0; column < width; column++) {
        int field_index = row + column * width;

        if(!can_occupy.at(field_index)) {
            continue;
        }

        // i = row, j = column
        // [i - 1, j - 2]
        if(row - 1 >= 0 && column - 2 >= 0 && can_occupy.at((row - 1) + (column - 2) * width)) {
            add_edge(field_index, (row - 1) + (column - 2) * width, graph);
        }

        // [i - 1, j + 2]
        if(row - 1 >= 0 && column + 2 < width && can_occupy.at((row - 1) + (column + 2) * width)) {
            add_edge(field_index, (row - 1) + (column + 2) * width, graph);
        }

        // [i + 1, j - 2]
        if(row + 1 < width && column - 2 >= 0 && can_occupy.at((row + 1) + (column - 2) * width)) {
            add_edge(field_index, (row + 1) + (column - 2) * width, graph);
        }

        // [i + 1, j + 2]
        if(row + 1 < width && column + 2 < width && can_occupy.at((row + 1) + (column + 2) * width)) {
            add_edge(field_index, (row + 1) + (column + 2) * width, graph);
        }

        // [i - 2, j - 1]
        if(row - 2 >= 0 && column - 1 >= 0 && can_occupy.at((row - 2) + (column - 1) * width)) {
            add_edge(field_index, (row - 2) + (column - 1) * width, graph);
        }

        // [i - 2, j + 1]
        if(row - 2 >= 0 && column + 1 < width && can_occupy.at((row - 2) + (column + 1) * width)) {
            add_edge(field_index, (row - 2) + (column + 1) * width, graph);
        }

        // [i + 2, j - 1]
        if(row + 2 < width && column - 1 >= 0 && can_occupy.at((row + 2) + (column - 1) * width)) {
            add_edge(field_index, (row + 2) + (column - 1) * width, graph);
        }

        // [i + 2, j + 1]
        if(row + 2 < width && column + 1 < width && can_occupy.at((row + 2) + (column + 1) * width)) {
            add_edge(field_index, (row + 2) + (column + 1) * width, graph);
        }
    }
}

// max cardinality
vector<Vertex> mate(width * width);
checked_edmonds_maximum_cardinality_matching(graph, &mate[0]);
int matches = matching_size(graph, &mate[0]);

//cout << "-> " << matches << endl;

cout << ((width * width) - hole_count) - matches << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <map>
#include <string>

#include <boost/config.hpp>

```

```

#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS> Graph;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    vector<pair<int, int> > moves;
    moves.reserve(8);
    moves.push_back(make_pair(-1, -2));
    moves.push_back(make_pair(-1, 2));
    moves.push_back(make_pair(1, -2));
    moves.push_back(make_pair(1, 2));
    moves.push_back(make_pair(-2, -1));
    moves.push_back(make_pair(-2, 1));
    moves.push_back(make_pair(2, -1));
    moves.push_back(make_pair(2, 1));

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int board_side_length;
        cin >> board_side_length;

        vector<vector<int> > board(board_side_length, vector<int>(board_side_length, -1));
        int vertex_id = 0;
        for (int i = 0; i < board_side_length; ++i) {
            for (int j = 0; j < board_side_length; ++j) {
                int field_status;
                cin >> field_status;
                if (field_status == 1) {
                    //add_vertex(g);
                    board[i][j] = vertex_id++;
                }
            }
        }
        int nr_vertices = vertex_id;
        Graph g(nr_vertices);

        for (int i = 0; i < board_side_length; ++i) {
            for (int j = 0; j < board_side_length; ++j) {
                if (board[i][j] != -1) {
                    for (int k = 0; k < moves.size(); ++k) {
                        int x_cord = i + moves[k].first;
                        int y_cord = j + moves[k].second;
                        if (x_cord >= 0 && x_cord < board_side_length && y_cord >= 0 && y_cord < board_side_length && board[x_cord][y_cord] != -1) {
                            add_edge(board[i][j], board[x_cord][y_cord], g);
                        }
                    }
                }
            }
        }

        vector<Vertex> mate(nr_vertices);
        edmonds_maximum_cardinality_matching(g, &mate[0]);
        int m_size = matching_size(g, &mate[0]);
        cout << m_size << endl;
        cout << nr_vertices - m_size << endl;
    }

    return 0;
}

```

8.10 Beach Bar

Keywords: ACM, Scanline

```

#include <iostream>
#include <vector>
#include <algorithm>
#include <limits>
#include <utility>
#include <set>
#include <limits>

using namespace std;

#define MAX_BAR_RANGE 200

int main() {

```

```

cin.sync_with_stdio(false);
cout.sync_with_stdio(false);

int test_count;
cin >> test_count;

for(int test = 0; test < test_count; test++) {
    int parasols_count;
    cin >> parasols_count;

    // read in parasol locations
    vector<int> parasols_locs(parasols_count);
    for(int parasol_index = 0; parasol_index < parasols_count; parasol_index++) {
        cin >> parasols_locs.at(parasol_index);
    }

    // first sort parasols, so we can start from the one furthest away in negative direction
    sort(parasols_locs.begin(), parasols_locs.end());

    // keep track of the maximum amount of parasols
    int max_parasols = 0;
    int cur_parasol_count = 0;

    // keep track of 'parasols_locs' indexes resulting in a ranges in which we can build a bar
    vector<pair<int, int>> bar_ranges;

    // keeps track of current start/stop index
    int start_index = 0;
    int stop_index = 0;

    bar_ranges.push_back(make_pair(start_index, stop_index));

    // iterate through parasols and collect bar ranges
    for(int parasol_index = 0; parasol_index < parasols_count; parasol_index++) {
        // if the current parasol is too far away from the previous one (defined by 'start_index'), remove
        // parasols as long as there are more between the original 'start_index' and current one until we're
        // back in the range of a bar
        for(; parasols_locs.at(parasol_index) - parasols_locs.at(start_index) > MAX_BAR_RANGE; start_index++) {
            cur_parasol_count--;
        }

        stop_index = parasol_index; // new "last" parasol in range of the bar
        cur_parasol_count++; // as we added the current parasol, increase the amount

        if(max_parasols < cur_parasol_count) {
            // we found a range with more parasols as before!
            max_parasols = cur_parasol_count;

            // as we found a better range, remove the previously found ones
            bar_ranges.clear();
            // ... and add the better one
            bar_ranges.push_back(make_pair(start_index, stop_index));
        } else if(max_parasols == cur_parasol_count) {
            // found another location for the bar with equal amount of reached parasols
            // we add it to our list of possible locations
            bar_ranges.push_back(make_pair(start_index, stop_index));
        }
    }

    // so, now we have a list of parisol ranges where we can build a bar in the middle and reach maximum
    // amount of customers. Next we have to find the one bar with the minimum distance.

    set<int> bar_locs;
    int min_distance_found = numeric_limits<int>::max();

    for(int range_index = 0; range_index < bar_ranges.size(); range_index++) {
        // stupid name, but basically 'added_range / 2' is the location of the bar :-)
        int added_range = parasols_locs[bar_ranges.at(range_index).first] +
            parasols_locs[bar_ranges.at(range_index).second];

        int diff = (parasols_locs[bar_ranges.at(range_index).second] -
            parasols_locs[bar_ranges.at(range_index).first] + 1) / 2; // +1 because of rounding

        if(diff > min_distance_found) {
            // we found something with a better minimal distance, so ignore this bar!
            continue;
        } else if(diff < min_distance_found) {
            // oh, we found a better minimum, replace everything
            min_distance_found = diff;
            bar_locs.clear();
        }

        // now add the current bar to our bar location collection, as we know it is the best one
        // or as good as the best we not till now
        if(added_range % 2 == 0) {
            // even distance
            bar_locs.insert(added_range / 2);
        } else {
            bar_locs.insert((added_range - 1) / 2);
            bar_locs.insert((added_range - 1) / 2 + 1);
        }
    }

    // output basic information
    cout << max_parasols << " " << min_distance_found << endl;
}

```

```

// output bar location in order: it is, we use a set!

set<int>::iterator loc_iter = bar_locs.begin();
cout << *loc_iter++;
for(; loc_iter != bar_locs.end(); ++loc_iter) {
    cout << " " << *loc_iter;
}

cout << endl;
}
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <stack>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr parasols;
        cin >> nr parasols;
        vector<int> parasols;
        parasols.reserve(nr parasols);
        for (int i = 0; i < nr parasols; i++) {
            int location;
            cin >> location;
            parasols.push_back(location);
        }
        sort(parasols.begin(), parasols.end());

        queue<int> covered_parasols;
        int best_nr_covered = 0;
        int best_min_distance = 0;
        set<int> best_locations;

        for (int i = 0; i < nr parasols; i++) {
            int cur = parasols[i];
            covered_parasols.push(cur);
            while (covered_parasols.front() < cur - 200) {
                covered_parasols.pop();
            }
            int min_distance = ceil((cur - covered_parasols.front()) / 2.0);
            if (covered_parasols.size() > best_nr_covered) {
                best_nr_covered = covered_parasols.size();
                best_min_distance = min_distance;
                best_locations.clear();
                //add location
                int added = cur + covered_parasols.front();
                if (added % 2 == 0)
                    best_locations.insert(added / 2);
                else {
                    best_locations.insert((added - 1) / 2);
                    best_locations.insert((added - 1) / 2 + 1);
                }
            }
            else if (covered_parasols.size() == best_nr_covered) {
                if (min_distance < best_min_distance) {
                    best_min_distance = min_distance;
                    best_locations.clear();
                    //add location
                    int added = cur + covered_parasols.front();
                    if (added % 2 == 0)
                        best_locations.insert(added / 2);
                    else {
                        best_locations.insert((added - 1) / 2);
                        best_locations.insert((added - 1) / 2 + 1);
                    }
                }
                else if (min_distance == best_min_distance) {
                    //add location
                    int added = cur + covered_parasols.front();
                    if (added % 2 == 0)
                        best_locations.insert(added / 2);
                    else {
                        best_locations.insert((added - 1) / 2);
                        best_locations.insert((added - 1) / 2 + 1);
                    }
                }
            }
        }
    }
}

```

```

    cout << best_nr_covered << " " << best_min_distance << endl;
    set<int>::iterator it = best_locations.begin();
    cout << *it;
    for (++it; it != best_locations.end(); ++it) {
        cout << " " << *it;
    }
    cout << endl
}

return 0;
}

```

8.11 Light the Stage

Keywords: CGAL, Point set, Triangulation with info()

```

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/basic.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <CGAL/Point_set_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef K::Point_2 Point;
typedef K::Circle_2 Circle;

typedef CGAL::Triangulation_vertex_base_with_info_2<int, K> Vb;
typedef CGAL::Triangulation_data_structure_2<Vb> Tds;
typedef CGAL::Point_set_2<K, Tds> PSet;
typedef CGAL::Point_set_2<K, Tds>::Vertex_handle Vertex_handle;

using namespace std;

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;

    for(int test = 0; test < test_count; test++) {
        int participant_count, lamp_count;
        cin >> participant_count >> lamp_count;

        // read in participants
        vector<K::Point_2> participant_locations(participant_count);
        vector<int> participant_radi(participant_count, -1);
        for(int participant_index = 0; participant_index < participant_count; participant_index++) {
            double p_x, p_y, p_r;
            cin >> p_x >> p_y >> p_r;

            participant_locations.at(participant_index) = K::Point_2(p_x, p_y);
            participant_radi.at(participant_index) = p_r;
        }

        // read in lamp information
        int lamp_height;
        cin >> lamp_height;

        // make it a vector of a pair. The moment we feed it in to Point_set_2 we can access the second
        // member of the pair by accessing ->info() :-)
        vector<pair<K::Point_2, int>> lamp_locations(lamp_count);
        for(int lamp_index = 0; lamp_index < lamp_count; lamp_index++) {
            double l_x, l_y;
            cin >> l_x >> l_y;

            lamp_locations.at(lamp_index) = make_pair(K::Point_2(l_x, l_y), lamp_index);
        }

        // create a triangulation of the lamps using Point_set_2
        PSet lamps_triangu(lamp_locations.begin(), lamp_locations.end());

        // contains the ID of the light that hit a participant first, or MAX if he survived till the end
        vector<int> min_light_hit(participant_count, numeric_limits<int>::max());

        // keeps track of the max light that hit someone
        int max_light_hit = -1;

        // now we iterate over each person and find out which lamp hits each person first, i.e. how
        // long they survive
        for(int participant_index = 0; participant_index < participant_count; participant_index++) {
            K::Point_2 participant_point = participant_locations.at(participant_index); // we need it more than once :-)
            // get nearest lamp
            Vertex_handle lamp_vertex = lamps_triangu.nearest_neighbor(participant_point);

            /* Geometry lesson:
             * We know that the light is a cone with given height and it's a 90 degree one:
             *
             *           x <---- lamp point
             *           /\

```

```

*           / | \
*          g  h  g
*         /  |  \
*        -----
*           ^- 'h' and the bottom have a 90 degree, therefore the triangle on the left/right (h, g and half
           of the bottom)
* have a 90 degree and two 45 degrees, i.e. we can find out what the length of the bottom is (twice the bottom
           part of the triangle)
* which is in this case 2 * 'h' (bottom is of 2 * 'h' length).
*/

// calculate the distance from our participant that still could be hit by light to "kill" the
// participant.
// do it ^2 (squared) because we compare it to a squared distance
double max_dist_to_kill = pow(participant_radi.at(participant_index) + lamp_height, 2);

// if the nearest lamp is already too far away, we don't have to search for other lamps,
// as they will even farther away
if(max_dist_to_kill <= squared_distance(lamp_vertex->point(), participant_point)) {
    continue;
}

// iterate over all lamps in the search for one that hits the participant first
for(int check_lamp_index = 0; check_lamp_index < lamp_count; check_lamp_index++) {
    if(max_dist_to_kill > squared_distance(lamp_locations.at(check_lamp_index).first, participant_point)) {
        // found such a lamp, stop searching as we only interested in the minimum!
        min_light_hit.at(participant_index) = lamp_locations.at(check_lamp_index).second;
        max_light_hit = max(max_light_hit, lamp_locations.at(check_lamp_index).second);
        break;
    }
}

/* nice idea, but too slow...

// create a circle in which every light source would kill the current participant
K::Circle_2 circle_of_death = K::Circle_2(participant_point, max_dist_to_kill);

// create a vector of vertex handles that represent our lights that are inside the circle_of_death
vector<Vertex_handle> lights_hitting_participant;

// so, micro optimizing stuff: don't do the rest of the algorithm if the lamp ID is higher than the
// already found one
if(min_light_hit.at(participant_index) < lamp_vertex->info()) {
    continue;
}

// collect the lights "of death" :-)
lamps_triang.range_search(circle_of_death, back_inserter(lights_hitting_participant));

// iterate over the bad lights and write down the smallest light hitting
for(vector<Vertex_handle>::iterator iter = lights_hitting_participant.begin();
    iter != lights_hitting_participant.end();
    ++iter) {
    int lamp_id = (*iter)->info();

    if(circle_of_death.has_on_boundary((*iter)->point())) {
        // point is on boundary, this is not a hit according to the exercise
        continue;
    }

    min_light_hit.at(participant_index) = min(min_light_hit.at(participant_index), lamp_id);
    max_light_hit = max(max_light_hit, lamp_id);
}
*/
}

// now we have to extract for each light who survived the longest
vector<int> rank_list;

// first we search for participants which were not hit by a light
for(int participant_index = 0; participant_index < participant_count; participant_index++) {
    if(min_light_hit.at(participant_index) == numeric_limits<int>::max()) {
        rank_list.push_back(participant_index);
    }
}

// if all participants were hit, we have to search for the ones that were hit last
if(rank_list.size() <= 0) {
    for(int participant_index = 0; participant_index < participant_count; participant_index++) {
        if(min_light_hit.at(participant_index) == max_light_hit) {
            rank_list.push_back(participant_index);
        }
    }
}

// print out rank in sorted order
sort(rank_list.begin(), rank_list.end());
for(int i = 0; i < rank_list.size(); i++) {
    cout << rank_list.at(i) << " ";
}
cout << endl;
}
}

```

```
#include <iostream>
```



```

#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/basic.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <CGAL/Point_set_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef K::Point_2 Point;
typedef K::Circle_2 Circle;

typedef CGAL::Triangulation_vertex_base_with_info_2<int, K> Vb;
typedef CGAL::Triangulation_data_structure_2<Vb> Tds;
typedef CGAL::Point_set_2<K, Tds> PSet;
typedef CGAL::Point_set_2<K, Tds>::Vertex_handle Vertex_handle;

using namespace std;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

class CompareVertex {
public:
    bool operator()(const Vertex_handle& lhs, const Vertex_handle& rhs) {
        return lhs->info() < rhs->info();
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_players, nr_lights;
        cin >> nr_players >> nr_lights;

        vector<Point> players;
        vector<K::FT> players_r;
        players.reserve(nr_players);
        players_r.reserve(nr_players);
        for (int i = 0; i < nr_players; i++) {
            int player_x, player_y, player_r;
            cin >> player_x >> player_y >> player_r;
            players.push_back(Point(player_x, player_y));
            players_r.push_back(player_r);
        }

        K::FT light_r;
        cin >> light_r;
        vector<pair<Point, int>> lights;
        lights.reserve(nr_lights);
        for (int i = 0; i < nr_lights; i++) {
            int light_x, light_y;
            cin >> light_x >> light_y;
            lights.push_back(make_pair(Point(light_x, light_y), i));
        }

        PSet pset(lights.begin(), lights.end());
        vector<int> min_light_hit(nr_players, numeric_limits<int>::max());
        for (int i = 0; i < nr_players; i++) {
            Point player = players[i];
            Vertex_handle light = pset.nearest_neighbor(player);
            double max_dist = pow(players_r[i] + light_r, 2);

            if (max_dist <= squared_distance(light->point(), player)) {
                continue;
            }

            for (int j = 0; j < nr_lights; j++) {

```

```

        if (max_dist > squared_distance(lights[j].first, player)) {
            min_light_hit[i] = lights[j].second;
            break;
        }
    }
}

int max_light_hit = *max_element(min_light_hit.begin(), min_light_hit.end());
for (int i = 0; i < nr_players; i++) {
    if (min_light_hit[i] == max_light_hit) {
        cout << i << " ";
    }
}
cout << endl;

//Binary search would probably work...

//Too slow
/*PSet pset(lights.begin(), lights.end());
vector<int> player_death_time(nr_players, -1);
for (int i = 0; i < nr_players; i++) {
    K::FT circle_squared_dist = pow(players_r[i] + light_r, 2);
    Circle c(players[i], circle_squared_dist);
    vector<Vertex_handle> result;
    pset.range_search(c, back_inserter(result));
    //cout << "player: " << i << " found: " << result.size() << endl;
    sort(result.begin(), result.end(), CompareVertex());
    for (int j = 0; j < result.size(); ++j) {
        int death_time = result[j]->info();
        if (player_death_time[i] == -1 || death_time < player_death_time[i]) {
            if (squared_distance(result[j]->point(), players[i]) < circle_squared_dist) {
                player_death_time[i] = death_time;
                break;
            }
        }
    }
}

int death_max = 0;
for (int i = 0; i < nr_players; i++) {
    if (player_death_time[i] > death_max) {
        death_max = player_death_time[i];
    }
    if (player_death_time[i] == -1) {
        death_max = -1;
        break;
    }
}
for (int i = 0; i < nr_players; i++) {
    if (player_death_time[i] == death_max) {
        cout << i << " ";
    }
}
cout << endl;*/
}

return 0;
}

```

8.12 Search Snippets

Keywords: ACM, Scanline, Custom compare, Compare in class

```

#include <iostream>
#include <algorithm>
#include <vector>
#include <queue>
#include <bitset>
#include <set>

using namespace std;

class PairCompare
{
public:
    bool operator() (pair<int, int>& lhs, pair<int, int>& rhs) {
        return lhs.first > rhs.first;
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        int word_count;
        cin >> word_count;

        // read in how often each word occurs
    }
}

```

```

vector<int> occurs(word_count, 0);
for(int word_index = 0; word_index < word_count; word_index++) {
    cin >> occurs.at(word_index);
}

// read in where it occurs
priority_queue<pair<int, int>, vector<pair<int, int> >, PairCompare> locs;
for(int word_index = 0; word_index < word_count; word_index++) {
    for(int i = 0; i < occurs.at(word_index); i++) {
        int l;
        cin >> l;
        locs.push(make_pair(l, word_index));
    }
}

// find first range with all words
vector<int> first_set_loc(word_count, -1);
set<pair<int, int> > words;
int words_found = 0;
int last_loc = numeric_limits<int>::min();
while(!locs.empty() && words_found != word_count) {
    pair<int, int> cur = locs.top();
    locs.pop();

    if(first_set_loc.at(cur.second) == -1) {
        words_found++;
    } else {
        words.erase(make_pair(first_set_loc.at(cur.second), cur.second));
    }

    first_set_loc.at(cur.second) = cur.first;
    words.insert(cur);
    last_loc = cur.first;
}

//cout << "words_found: " << words_found << endl;
//for(int x = 0; x < word_count; x++) {
//    cout << "\tword " << x << ", loc: " << first_set_loc.at(x) << endl;
//    locs.push(make_pair(first_set_loc.at(x), x));
//}
//cout << "min: " << words.begin()->first << ", max: " << last_loc << endl;

int min_dist = last_loc - words.begin()->first;

while(!locs.empty()) {
    pair<int, int> cur = locs.top();
    locs.pop();

    // modify location
    words.erase(make_pair(first_set_loc.at(cur.second), cur.second));
    words.insert(cur);
    first_set_loc.at(cur.second) = cur.first;

    // check range
    int d = cur.first - words.begin()->first;
    min_dist = min(min_dist, d);
}

cout << min_dist + 1 << endl;
}

return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#define MOD_NUM 100003

using namespace std;

struct PairComparator {
    bool operator() (const pair<int, int>& lhs, const pair<int, int>& rhs) {
        return lhs.first > rhs.first;
    }
};

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    //cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {

```

```

int nr_words;
cin >> nr_words;

vector<int> nr_word_occurrences;
nr_word_occurrences.reserve(nr_words);
for (int i = 0; i < nr_words; ++i) {
    int nr_word_occurrence;
    cin >> nr_word_occurrence;
    nr_word_occurrences.push_back(nr_word_occurrence);
}

priority_queue<pair<int, int>, vector<pair<int, int> >, PairComparator> words;
for (int i = 0; i < nr_words; ++i) {
    for (int j = 0; j < nr_word_occurrences[i]; ++j) {
        int word_occurrence;
        cin >> word_occurrence;
        words.push(make_pair(word_occurrence, i));
    }
}

//find first valid range
set<pair<int, int> > max_word_occurrences_sorted;
vector<int> max_word_occurrences(nr_words, -1);
int found_words = 0;
int bound_max = 0;
while (found_words < nr_words) {
    pair<int, int> word = words.top();
    words.pop();

    if (max_word_occurrences[word.second] == -1) {
        //new word
        found_words++;
    }
    else {
        //already inserted word
        max_word_occurrences_sorted.erase(make_pair(max_word_occurrences[word.second], word.second));
    }
    max_word_occurrences[word.second] = word.first;
    max_word_occurrences_sorted.insert(word);
    bound_max = word.first;
}

int min_range = bound_max - max_word_occurrences_sorted.begin()->first;
while (!words.empty()) {
    pair<int, int> word = words.top();
    words.pop();

    max_word_occurrences_sorted.erase(make_pair(max_word_occurrences[word.second], word.second));
    max_word_occurrences[word.second] = word.first;
    max_word_occurrences_sorted.insert(word);
    bound_max = word.first;

    min_range = min(min_range, bound_max - max_word_occurrences_sorted.begin()->first);
}

cout << min_range+1 << endl;
}

return 0;
}

```

8.13 Radiation Therapy

Keywords: CGAL, Quadratic Program, Exponential bound search, Binary search

```

#include <iostream>
#include <cassert>
#include <tuple>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

using namespace std;

// program and solution types
typedef CGAL::Quadratic_program<ET> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

```

```

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

bool can_solve(int dimension, vector<tuple<int, int, int> >& healthy, vector<tuple<int, int, int> >& cancer) {
    // use EQUAL, as we set it manually if we wish => or <=
    Program qp(CGAL::EQUAL, false, 0, false, 0);

    // add healthy cell inequations
    for(int cell = 0; cell < healthy.size(); cell++) {
        int var_index = 0;
        for(int x_dim = 0; x_dim <= dimension; x_dim++) {
            for(int y_dim = 0; y_dim <= dimension; y_dim++) {
                for(int z_dim = 0; z_dim <= dimension; z_dim++) {
                    if(x_dim + y_dim + z_dim <= dimension) {
                        double value = pow(get<0>(healthy.at(cell)), x_dim) *
                            pow(get<1>(healthy.at(cell)), y_dim) *
                            pow(get<2>(healthy.at(cell)), z_dim);

                        qp.set_a(var_index, cell, value);
                        //cout << "value H: " << value << endl;

                        var_index++;
                    } else {
                        break;
                    }
                }
            }
        }

        qp.set_r(cell, CGAL::LARGER);
        qp.set_b(cell, 1);
    }

    // add cancer cell inequations
    for(int cell = 0; cell < cancer.size(); cell++) {
        int var_index = 0;
        for(int x_dim = 0; x_dim <= dimension; x_dim++) {
            for(int y_dim = 0; y_dim <= dimension; y_dim++) {
                for(int z_dim = 0; z_dim <= dimension; z_dim++) {
                    if(x_dim + y_dim + z_dim <= dimension) {
                        double value = pow(get<0>(cancer.at(cell)), x_dim) *
                            pow(get<1>(cancer.at(cell)), y_dim) *
                            pow(get<2>(cancer.at(cell)), z_dim);

                        qp.set_a(var_index, cell + healthy.size(), value);

                        //cout << "value C: " << value << endl;

                        var_index++;
                    } else {
                        break;
                    }
                }
            }
        }

        qp.set_r(cell + healthy.size(), CGAL::SMALLER);
        qp.set_b(cell + healthy.size(), -1);
    }

    // solve
    CGAL::Quadratic_program_options options;
    options.set_pricing_strategy(CGAL::QP_BLAND); // Bland's rule to avoid cycling...
    Solution s = CGAL::solve_linear_program(qp, ET(), options);

    //cout << s << endl;

    return !s.is_infeasible();
}

int main() {

    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;
    for(int test = 0; test < test_count; test++) {
        //cout << endl << endl;
        int healthy_count, cancer_count;
        cin >> healthy_count >> cancer_count;

        // read in healthy cell location
        vector<tuple<int, int, int> > healthy(healthy_count);
        for(int healthy_index = 0; healthy_index < healthy_count; healthy_index++) {
            int x, y, z;
            cin >> x >> y >> z;

```

```

    healthy.at(healthy_index) = make_tuple(x, y, z);
}

// read in cancer cell location
vector<tuple<int, int, int> > cancer(cancer_count);
for(int cancer_index = 0; cancer_index < cancer_count; cancer_index++) {
    int x, y, z;
    cin >> x >> y >> z;

    cancer.at(cancer_index) = make_tuple(x, y, z);
}

// search upper lower
int exp_value = 1;
bool exp_found = false;
do {
    exp_value = exp_value * 2;
    exp_found = true;
    //cout << "inc: for: " << exp_value << " is: " << can_solve(exp_value, healthy, cancer) << endl;
} while(exp_value <= 30 && !can_solve(exp_value, healthy, cancer));

// do binary search for the right dimension
int min_d = 0;
int max_d = 30;
if(exp_found) {
    min_d = exp_value / 2 - 1;
    max_d = exp_value > 30 ? 30 : exp_value;
}

//cout << "min set to " << min_d << " and max to " << max_d << endl;

bool last_worked = false;
while(min_d < max_d) {
    int mid_d = (min_d + max_d) / 2;
    //cout << "binary search from: " << min_d << " to " << max_d << ", checking: " << mid_d << endl;
    if(can_solve(mid_d, healthy, cancer)) {
        // [min_d, mid_d] contains the solution
        max_d = mid_d;
        last_worked = true;
        //cout << "\tOK" << endl;
    } else {
        // [mid_d + 1, max_d] contains the solution
        min_d = mid_d + 1;
        last_worked = false;
        //cout << "\tBAD!" << endl;
    }
}

if(min_d == 30 && !last_worked) {
    cout << "Impossible!" << endl;
} else {
    cout << min_d << endl;
}
}
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <CGAL/basic.h>
#include <CGAL/QP_models.h>
#include <CGAL/QP_functions.h>

// choose exact integral type
#ifdef CGAL_USE_GMP
#include <CGAL/Gmpz.h>
typedef CGAL::Gmpz ET;
#else
#include <CGAL/MP_Float.h>
typedef CGAL::MP_Float ET;
#endif

// program and solution types
typedef CGAL::Quadratic_program<ET> Program;
typedef CGAL::Quadratic_program_solution<ET> Solution;

using namespace std;

int floor_to_double(const CGAL::Quotient<ET>& x) {
    double a = floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

int ceil_to_double(const CGAL::Quotient<ET>& x) {
    double a = ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
}

```

```

    return a;
}

bool check_degree(vector<tuple<int, int, int> > cells, int nr_healthy, int degree) {
    //cout << "check with: " << degree << " ";
    Program qp(CGAL::LARGER, false, 0, false, 0);
    for (int i = 0; i < cells.size(); i++) {
        int id = 0;
        tuple<int, int, int>& cell = cells[i];
        for (int x = 0; x <= degree; x++) {
            for (int y = 0; y <= degree; y++) {
                for (int z = 0; z <= degree; z++) {
                    if (x + y + z <= degree) {
                        qp.set_a(id, i, pow(get<0>(cell), x)*pow(get<1>(cell), y)*pow(get<2>(cell), z));
                        id++;
                    }
                    else {
                        break;
                    }
                }
            }
        }
        if (i < nr_healthy) {
            qp.set_b(i, 1);
            qp.set_r(i, CGAL::LARGER);
        }
        else {
            qp.set_b(i, -1);
            qp.set_r(i, CGAL::SMALLER);
        }
    }
    CGAL::Quadratic_program_options options;
    options.set_pricing_strategy(CGAL::QP_BLAND); // Bland's rule to avoid cycling...
    Solution s = CGAL::solve_linear_program(qp, ET(), options);
    //cout << (s.is_optimal() == 1 ? "true" : "false") << endl;
    return !s.is_infeasible();
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_healthy, nr_tumor;
        cin >> nr_healthy >> nr_tumor;
        int nr_cells = nr_healthy + nr_tumor;

        vector<tuple<int, int, int> > cells;
        cells.reserve(nr_cells);
        for (int i = 0; i < nr_cells; i++) {
            int cell_x, cell_y, cell_z;
            cin >> cell_x >> cell_y >> cell_z;
            cells.push_back(make_tuple(cell_x, cell_y, cell_z));
        }

        int exp_value = 0;
        while (exp_value < 30 && !check_degree(cells, nr_healthy, exp_value)) {
            exp_value = exp_value == 0 ? 1 : exp_value * 2;
        }

        int min_bound = exp_value == 0 ? 0 : exp_value / 2 + 1;
        int max_bound = exp_value > 30 ? 30 : exp_value;

        //int min_bound = 0;
        //int max_bound = 30;

        bool last_check = false;
        while (min_bound < max_bound) {
            int test_degree = (min_bound + max_bound) / 2;
            //int test_degree = (min_bound*3 + max_bound) / 4;
            if (check_degree(cells, nr_healthy, test_degree)) {
                max_bound = test_degree;
                last_check = true;
            }
            else {
                min_bound = test_degree + 1;
                last_check = false;
            }
        }

        if (min_bound == 30 && !last_check) {
            cout << "Impossible!" << endl;
        }
        else {
            cout << min_bound << endl;
        }
    }

    return 0;
}

```

8.14 Island Hopping

Keywords: ACM, Dynamic Programming

```
#include <iostream>
#include <algorithm>
#include <vector>
#include <queue>
#include <cassert>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        int island_count, attack_radius, attacker_strength_start;
        cin >> island_count >> attack_radius >> attacker_strength_start;

        // read in the defense strength
        vector<int> defense(island_count + 1, 0);
        for(int island_index = 1; island_index < island_count; island_index++) {
            cin >> defense.at(island_index);
        }

        assert(defense.at(0) == 0);
        assert(defense.at(island_count) == 0);

        // keeps track of the path length
        vector<int> path_length(island_count + 1, numeric_limits<int>::max()); // first island is the attacker's island,
        // this is just always -inf
        path_length.at(0) = 0;

        // our DP table containing the resulting fighter strength left at an island
        vector<int> table(island_count + 1, 0);
        table.at(0) = attacker_strength_start;

        // DP algo
        for(int cur_island = 0; cur_island < island_count; cur_island++) {
            if(table.at(cur_island) <= 0) {
                continue;
            }

            for(int next_island = cur_island + 1;
                next_island <= cur_island + attack_radius && next_island <= island_count;
                next_island++) {

                // calculate how many attackers are left if we send the people from island 'cur_island' to
                // island 'next_island'
                int attackers_left = table.at(cur_island) - defense.at(next_island);

                // if this is a greater number than the one already in our DP table, we found a better way
                if(attackers_left > table.at(next_island)) {
                    table.at(next_island) = attackers_left;
                    // don't forget to keep track of the path
                    path_length.at(next_island) = path_length.at(cur_island) + 1;
                } else if(attackers_left == table.at(next_island)) {
                    // maybe we found an other, better path with the same results
                    path_length.at(next_island) = min(path_length.at(cur_island) + 1, path_length.at(next_island));
                }
            }
        }

        if(table.at(island_count) <= 0) {
            cout << "safe" << endl;
        } else {
            cout << path_length.at(island_count) - 1 << " " << table.at(island_count) << endl;
        }
    }

    return 0;
}
```

```
#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <stack>

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
```



```

cin >> nr_test_cases;

for (int test_case = 0; test_case < nr_test_cases; test_case++) {
    int nr_islands, attack_radius, initial_strength;
    cin >> nr_islands >> attack_radius >> initial_strength;

    vector<int> defence_strengths;
    defence_strengths.reserve(nr_islands + 1);
    defence_strengths.push_back(0);
    for (int i = 1; i < nr_islands; i++) {
        int defence_strength;
        cin >> defence_strength;
        defence_strengths.push_back(defence_strength);
    }
    defence_strengths.push_back(0);

    vector<int> max_attack_forces(nr_islands + 1, 0);
    vector<int> min_islands(nr_islands + 1, 0);
    max_attack_forces[0] = initial_strength;
    for (int i = 0; i < nr_islands; i++) {
        int current_attack_force = max_attack_forces[i];
        if (current_attack_force > 0) {
            for (int j = 1; j <= attack_radius; j++) {
                if (i + j <= nr_islands) {
                    int new_attack_force = current_attack_force - defence_strengths[i + j];
                    if (new_attack_force > max_attack_forces[i + j]) {

                        max_attack_forces[i + j] = new_attack_force;
                        min_islands[i + j] = min_islands[i] + 1;
                    }
                    else if (new_attack_force == max_attack_forces[i + j]) {
                        min_islands[i + j] = min(min_islands[i + j], min_islands[i] + 1);
                    }
                }
            }
            else {
                break;
            }
        }
    }

    if (max_attack_forces[nr_islands] > 0) {
        cout << min_islands[nr_islands] - 1 << " " << max_attack_forces[nr_islands] << endl;
    }
    else {
        cout << "safe" << endl;
    }
}

return 0;
}

```

8.15 Sweepers

Keywords: BGL, Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Max-flow, Euler circuit, Components

```

#include <iostream>
#include <algorithm>
#include <vector>

#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <boost/graph/edmonds_karp_max_flow.hpp>
#include <boost/tuple/tuple.hpp>
#include <boost/graph/connected_components.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list_traits<vecS, vecS, directedS> Traits;
typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, long,
        property<edge_residual_capacity_t, long,
            property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> SimpleGraph;
typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;

// Custom add_edge, also creates reverse edges with corresponding capacities.
void addEdge(int u, int v, long c, EdgeCapacityMap &capacity, ReverseEdgeMap &rev_edge, Graph &G) {
    Edge e, reverseE;
    tie(e, tuples::ignore) = add_edge(u, v, G);
    tie(reverseE, tuples::ignore) = add_edge(v, u, G);
    capacity[e] = c;
    capacity[reverseE] = 0;
    rev_edge[e] = reverseE;
    rev_edge[reverseE] = e;
}

```

```

}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        int room_count, corridor_count, sweeper_count;
        cin >> room_count >> corridor_count >> sweeper_count;

        int door_count = sweeper_count;

        // create graph for flow analysis
        Graph flow_graph(room_count + 2);
        EdgeCapacityMap capacity = get(edge_capacity, flow_graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, flow_graph);
        //ResidualCapacityMap res_capacity = get(edge_residual_capacity, flow_graph);

        // create graph with just the rooms, corridors
        SimpleGraph castle_graph(room_count);

        // define source and sink
        int source = room_count;
        int sink = room_count + 1;

        // keep track of vertices degrees
        vector<int> degrees(room_count, 0);
        vector<int> non_modified_degrees(room_count, 0);

        // read in location of sweepers
        vector<int> sweepers(room_count, 0);
        for(int sweeper_index = 0; sweeper_index < sweeper_count; sweeper_index++) {
            int location;
            cin >> location;

            // source -> sweeper location
            addEdge(source, location, 1, capacity, rev_edge, flow_graph);

            degrees.at(location)++;
            sweepers.at(location)++;
        }

        // read in location of doors
        for(int door_index = 0; door_index < door_count; door_index++) {
            int location;
            cin >> location;

            // door location -> sink
            addEdge(location, sink, 1, capacity, rev_edge, flow_graph);

            degrees.at(location)--;
        }

        // read in corridors
        for(int corridor_index = 0; corridor_index < corridor_count; corridor_index++) {
            int from, to;
            cin >> from >> to;

            addEdge(from, to, 1, capacity, rev_edge, flow_graph);
            addEdge(to, from, 1, capacity, rev_edge, flow_graph);

            add_edge(from, to, castle_graph);

            degrees.at(from)++;
            degrees.at(to)++;
            non_modified_degrees.at(from)++;
            non_modified_degrees.at(to)++;
        }

        // so now we have a basic graph, check if all sweepers can reach a door.
        // do this by doing a flow and check if flow result same as sweepers count
        int sweepers_exited = push_relabel_max_flow(flow_graph, source, sink);

        //cout << "sweepers exited: " << sweepers_exited << ", total sweepers: " << sweeper_count << endl;
        if(sweepers_exited != sweeper_count) {
            cout << "no" << endl;
            continue;
        }

        // check for euler circuit
        bool has_even_degree = true;
        for(int vertex = 0; vertex < room_count; vertex++) {
            if(degrees.at(vertex) < 0 || degrees.at(vertex) % 2 != 0) {
                has_even_degree = false;
                break;
            }
        }

        //cout << "even: " << has_even_degree << endl;
        if(!has_even_degree) {
            cout << "no" << endl;
            continue;
        }
    }
}

```

```

// make sure every edge, i.e. corridor, was visited at least once
// first get the connected component information
vector<int> component(room_count);
connected_components(castle_graph, &component[0]);
vector<int> cleaned_by_sweepers(room_count, 0);
int max_component_nr = 0;
for(int room_index = 0; room_index < room_count; room_index++) {
    // we're interested that every component is cleaned, so we use this as the index. Keep track of
    // how many components we have
    int component_nr = component.at(room_index);
    max_component_nr = max(max_component_nr, component_nr);

    // if the room has no corridors attached, we assume it is cleaned (only corridors need cleaning,
    // no corridor, no cleaning needed).
    if(non_modified_degrees.at(room_index) == 0) {
        cleaned_by_sweepers.at(component_nr) = 1;
    } else {
        // the degree of the room might be 0, then there is no sweeper added to the component. If never one
        // is added, component not cleaned.
        cleaned_by_sweepers.at(component_nr) += sweepers.at(room_index);
    }
}

// make sure every component is cleaned, i.e. contains some sweepers. Other requirements make sure
// the component is cleaned if it contains sweepers
bool components_cleaned = true;
for(int i = 0; i <= max_component_nr; i++) {
    if(cleaned_by_sweepers.at(i) == 0) {
        components_cleaned = false;
        break;
    }
}

if(components_cleaned) {
    cout << "yes" << endl;
} else {
    cout << "no" << endl;
}
}

return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <stack>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <boost/graph/connected_components.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> SimpleGraph;

inline void add_flow_edge(int start, int end, int c, EdgeCapacityMap& capacity, ReverseEdgeMap& rev_edge, Graph& g) {
    Edge e, rev_e;
    bool success;
    tie(e, success) = add_edge(start, end, g);
    if (success) {
        tie(rev_e, success) = add_edge(end, start, g);
        capacity[e] = c;
        capacity[rev_e] = 0;
        rev_edge[e] = rev_e;
        rev_edge[rev_e] = e;
    }
    else {
        capacity[e] += c;
    }
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
}

```

```

int nr_test_cases;
cin >> nr_test_cases;

for (int test_case = 0; test_case < nr_test_cases; test_case++) {
    int nr_rooms, nr_corridors, nr_sweepers;
    cin >> nr_rooms >> nr_corridors >> nr_sweepers;

    //cout << endl;

    Graph g(nr_rooms + 2);
    SimpleGraph components_g(nr_rooms);
    int source = nr_rooms;
    int sink = nr_rooms + 1;

    EdgeCapacityMap capacity = get(edge_capacity, g);
    ReverseEdgeMap rev_edge = get(edge_reverse, g);

    vector<int> starting_locations(nr_rooms, 0);
    for (int i = 0; i < nr_sweepers; i++) {
        int starting_location;
        cin >> starting_location;

        add_flow_edge(source, starting_location, 1, capacity, rev_edge, g);

        starting_locations[starting_location]++;
    }

    vector<int> outside_doors(nr_rooms, 0);
    for (int i = 0; i < nr_sweepers; i++) {
        int outside_door;
        cin >> outside_door;

        add_flow_edge(outside_door, sink, 1, capacity, rev_edge, g);

        outside_doors[outside_door]++;
    }

    vector<int> vertex_degrees(nr_rooms, 0);
    for (int i = 0; i < nr_corridors; i++) {
        int end_one, end_two;
        cin >> end_one >> end_two;

        vertex_degrees[end_one]++;
        vertex_degrees[end_two]++;

        add_flow_edge(end_one, end_two, 1, capacity, rev_edge, g);
        add_flow_edge(end_two, end_one, 1, capacity, rev_edge, g);

        add_edge(end_one, end_two, components_g);
    }

    //check if all vertices have an even degree
    bool found_uneven = false;
    for (int i = 0; i < nr_rooms; i++) {
        if ((vertex_degrees[i] + outside_doors[i] + starting_locations[i]) % 2 == 1) {
            found_uneven = true;
            break;
        }
    }
    if (found_uneven) {
        //cout << "uneven degree found" << endl;
        cout << "no" << endl;
        continue;
    }

    //check if every component of the graph has at least one assigned sweeper
    vector<int> comp(nr_rooms);
    int num = connected_components(components_g, &comp[0]);
    vector<int> cleaned(num, 0);
    for (int i = 0; i < nr_rooms; i++) {
        if (vertex_degrees[i] == 0) {
            cleaned[comp[i]] = 1;
        }
        else {
            cleaned[comp[i]] += starting_locations[i];
        }
    }
    bool uncleaned_comp = false;
    for (int i = 0; i < num; i++) {
        if (cleaned[i] == 0) {
            uncleaned_comp = true;
            break;
        }
    }

    //cout << "no_sweeper: " << no_sweeper << endl;
    if (uncleaned_comp) {
        //cout << "component without sweeper" << endl;
        cout << "no" << endl;
        continue;
    }

    //check if all sweepers can flee
    int escaped_sweepers = push_relabel_max_flow(g, source, sink);
    if (escaped_sweepers < nr_sweepers) {
        //cout << "max flow to low" << endl;
    }
}

```

```

        cout << "no" << endl;
        continue;
    }

    cout << "yes" << endl;
}

return 0;
}

```

8.16 Clues

Keywords: BGL, CGAL, BGL with CGAL, CGAL with BGL, Point set, Triangulation with info(), Bipartite, Components

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/graph_traits.hpp>
#include <boost/graph/bipartite.hpp>
#include <boost/graph/connected_components.hpp>

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/basic.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <CGAL/Point_set_2.h>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::out_edge_iterator Edge_iterator;

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef K::Point_2 Point;
typedef K::Circle_2 Circle;

typedef CGAL::Triangulation_vertex_base_with_info_2<int, K> Vb;
typedef CGAL::Triangulation_data_structure_2<Vb> Tds;
typedef CGAL::Point_set_2<K, Tds> PSet;
typedef CGAL::Point_set_2<K, Tds>::Vertex_handle Vertex_handle;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;

    for(int test = 0; test < test_count; test++) {
        int station_count, clue_count;
        K::FT range;
        cin >> station_count >> clue_count >> range;

        range = pow(range, 2); // work with squared distances

        // read in station location
        // pair.first: the point where the station is located
        // pair.second: the index of the station
        vector<pair<Point, int>> stations(station_count);
        for(int station_index = 0; station_index < station_count; station_index++) {
            int x, y;
            cin >> x >> y;

            stations.at(station_index) = make_pair(Point(x, y), station_index);
        }

        // read in clue location
        vector<pair<Point, Point>> clues(clue_count);
        for(int clue_index = 0; clue_index < clue_count; clue_index++) {
            // read in from where to where we would like to go
            int start_x, start_y, target_x, target_y;
            cin >> start_x >> start_y >> target_x >> target_y;

            clues.at(clue_index) = make_pair(Point(start_x, start_y), Point(target_x, target_y));
        }

        // create a graph that can be used to check for graph properties
        Graph graph(station_count);

        // calculate the point set so we can find the nearest one for a given point and all stations
    }
}

```

```

// inside a circle of given radius
PSet station_triangu;
station_triangu.insert(stations.begin(), stations.end());

// go through stations, search for stations that are inside the radius and process
// them.
for(int stationA_index = 0; stationA_index < station_count; stationA_index++) {
    Point stationA = stations.at(stationA_index).first;
    Circle stationA_area = Circle(stationA, range);

    // collect all stations inside the circle
    vector<Vertex_handle> stations_in_range;
    station_triangu.range_search(stationA_area, back_inserter(stations_in_range));

    // go through found stations and make sure that each pair is at least 'range' away from each other
    // as long as it's not our stationA one of the pair's stations
    for(int i = 0; i < stations_in_range.size(); i++) {
        for(int j = i + 1; j < stations_in_range.size(); j++) {
            int some_station_1 = stations_in_range.at(i)->info();
            int some_station_2 = stations_in_range.at(j)->info();

            if(some_station_1 != stationA_index && some_station_2 != stationA_index) {
                if(CGAL::squared_distance(stations_in_range.at(i)->point(), stations_in_range.at(j)->point()) <= range) {
                    // another pair of stations in reachable station, conflict!
                    //cout << "blub" << endl;
                    goto not_bipartit;
                }
            }
        }
    }

    // add edges
    for(int i = 0; i < stations_in_range.size(); i++) {
        int to = stations_in_range.at(i)->info();
        if(to != stationA_index) {
            add_edge(stationA_index, to, graph);
        }
    }
}

// check if graph is not bipartit, if so, we can stop
if(!is_bipartite(graph)) {
    //cout << "no blub" << endl;
    not_bipartit:
    for(int clue_index = 0; clue_index < clue_count; clue_index++) {
        cout << "n";
    }
    cout << endl;
} else {
    // now we have to check which of the clues can be received
    // get connected components information, so we can find out if start and end of a clue is in the same
    // component and therefore reachable
    vector<int> in_component(station_count);
    connected_components(graph, &in_component[0]);

    for(int clue_index = 0; clue_index < clue_count; clue_index++) {
        // get where the clue starts and should end
        Point start = clues.at(clue_index).first;
        Point target = clues.at(clue_index).second;

        // check if start and target of the clue are near each other and communicate directly
        if(range >= CGAL::squared_distance(start, target)) {
            cout << "y";
            continue;
        }

        // get nearest vertex to the start and check if we can reach it
        Vertex_handle near_start_station = station_triangu.nearest_vertex(start);
        if(range < CGAL::squared_distance(start, near_start_station->point())) {
            cout << "n"; // next station from start too far away
            continue;
        }

        // now the same for the end station/clue location
        Vertex_handle near_end_station = station_triangu.nearest_vertex(target);
        if(range < CGAL::squared_distance(target, near_end_station->point())) {
            cout << "n";
            continue;
        }

        // last part is to check if both are in the same component
        if(in_component.at(near_start_station->info()) == in_component.at(near_end_station->info())) {
            cout << "y";
        } else {
            cout << "n";
        }
    }

    cout << endl;
}

return 0;
}

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/graph_traits.hpp>
#include <boost/graph/bipartite.hpp>
#include <boost/graph/connected_components.hpp>

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/basic.h>
#include <CGAL/squared_distance_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <CGAL/Point_set_2.h>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::vertex_descriptor Vertex;
typedef graph_traits<Graph>::edge_descriptor Edge;

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;

typedef K::Point_2 Point;
typedef K::Circle_2 Circle;

typedef CGAL::Triangulation_vertex_base_with_info_2<int, K> Vb;
typedef CGAL::Triangulation_data_structure_2<Vb> Tds;
typedef CGAL::Point_set_2<K, Tds> PSet;
typedef CGAL::Point_set_2<K, Tds>::Vertex_handle Vertex_handle;

double floor_to_double(const K::FT& x)
{
    double a = std::floor(CGAL::to_double(x));
    while (a > x) a -= 1;
    while (a + 1 <= x) a += 1;
    return a;
}

double ceil_to_double(const K::FT& x)
{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a - 1 >= x) a -= 1;
    return a;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_stations, nr_clues, max_radius;
        cin >> nr_stations >> nr_clues >> max_radius;

        vector<pair<Point, int>> stations;
        stations.reserve(nr_stations);
        for (int i = 0; i < nr_stations; ++i) {
            int station_x, station_y;
            cin >> station_x >> station_y;
            stations.push_back(make_pair(Point(station_x, station_y), i));
        }

        vector<pair<Point, Point>> clues;
        clues.reserve(nr_clues);
        for (int i = 0; i < nr_clues; ++i) {
            int from_x, from_y, to_x, to_y;
            cin >> from_x >> from_y >> to_x >> to_y;
            clues.push_back(make_pair(Point(from_x, from_y), Point(to_x, to_y)));
        }

        PSet pset(stations.begin(), stations.end());
        K::FT max_radius_squared = pow(max_radius, 2);
        Graph g(nr_stations);
        for (int i = 0; i < nr_stations; ++i) {
            Circle c(stations[i].first, max_radius_squared);
            vector<Vertex_handle> result;
            pset.range_search(c, back_inserter(result));

            for (int j = 0; j < result.size(); ++j) {
                for (int k = j + 1; k < result.size(); ++k) {
                    if (result[j]->info() != i && result[k]->info() != i) {
                        if (CGAL::squared_distance(result[j]->point(), result[k]->point()) <= max_radius_squared) {
                            goto not_bipartite;
                        }
                    }
                }
            }
        }
    }
}

```

```

    }
  }
}
for (int j = 0; j < result.size(); ++j) {
  int to = result[j]->info();
  if (to != i) {
    add_edge(i, to, g);
  }
}
}
if (!is_bipartite(g)) {
  not_bipartite:
  for (int i = 0; i < nr_clues; i++) {
    cout << "n";
  }
  cout << endl;
}
else {
  vector<int> component(nr_stations);
  connected_components(g, &component[0]);

  for (int i = 0; i < nr_clues; i++) {
    if (CGAL::squared_distance(clues[i].first, clues[i].second) <= max_radius_squared) {
      cout << "y";
      continue;
    }
    Vertex_handle start_p = pset.nearest_vertex(clues[i].first);
    if (CGAL::squared_distance(start_p->point(), clues[i].first) > max_radius_squared) {
      cout << "n";
      continue;
    }
    Vertex_handle end_p = pset.nearest_vertex(clues[i].second);
    if (CGAL::squared_distance(end_p->point(), clues[i].second) > max_radius_squared) {
      cout << "n";
      continue;
    }
    int start = start_p->info();
    int end = end_p->info();
    if (component[start] == component[end]) {
      cout << "y";
    }
    else {
      cout << "n";
    }
  }
  cout << endl;
}
}
return 0;
}

```

8.17 Radiation 2

Keywords: CGAL, Delaunay Triangulation, Point set, Triangulation with info()

```

//
// ONLY FIRST TWO TEST CASES!
//

#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/Delaunay_triangulation_2.h>
#include <CGAL/Triangulation_vertex_base_with_info_2.h>
#include <CGAL/Point_set_2.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2<K> Triangulation;
typedef Triangulation::Edge_iterator Edge_iterator;

typedef K::Point_2 Point;
typedef K::Circle_2 Circle;

typedef CGAL::Triangulation_vertex_base_with_info_2<int, K> Vb;
typedef CGAL::Triangulation_data_structure_2<Vb> Tds;
typedef CGAL::Point_set_2<K, Tds> PSet;
typedef CGAL::Point_set_2<K, Tds>::Vertex_handle PSVertex_handle;

typedef CGAL::Delaunay_triangulation_2<K> Triangulation;
typedef CGAL::Delaunay_triangulation_2<K>::Vertex_handle TriangVertex_handle;

using namespace std;

// from slides, fun!
double floor_to_double(const K::FT& x)
{
  double a = std::floor(CGAL::to_double(x));
  while (a > x) a -= 1;
  while (a+1 <= x) a += 1;
  return a;
}

double ceil_to_double(const K::FT& x)

```



```

{
    double a = std::ceil(CGAL::to_double(x));
    while (a < x) a += 1;
    while (a-1 >= x) a -= 1;
    return a;
}

int main() {
    // some basic setup stuff
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);
    cout << fixed << setprecision(0);

    int test_count;
    cin >> test_count;
    for(int test = 0; test < test_count; test++) {
        int healthy_count, cancer_count;
        cin >> healthy_count >> cancer_count;

        // read in healthy and cancer cells
        vector<Point> cancer_cells;
        cancer_cells.reserve(cancer_count);

        vector<Point> healthy_cells;
        healthy_cells.reserve(healthy_count);

        for(int healthy_index = 0; healthy_index < healthy_count; healthy_index++) {
            double x, y;
            cin >> x >> y;

            healthy_cells.push_back(Point(x, y));
        }

        for(int cancer_index = 0; cancer_index < cancer_count; cancer_index++) {
            double x, y;
            cin >> x >> y;

            cancer_cells.push_back(Point(x, y));
        }

        // create a point set of cancer cells, this allows us to find all cells inside a circle quickly (I hope!)
        PSet cancer_triang(cancer_cells.begin(), cancer_cells.end());

        // create a triangulation of healthy cells to find quickly the nearest one to a cancer point
        Triangulation healthy_triang(healthy_cells.begin(), healthy_cells.end());

        // iterate over all cancer cells to search for the best radius
        unsigned long max_cells_killed = 0;
        for(int cancer_index = 0; cancer_index < cancer_count; cancer_index++) {
            // get the point of the cancer cell
            Point cancer_cell = cancer_cells.at(cancer_index);

            // get the nearest healthy cell, defining by that the radius
            TriangVertex_handle next_healthy = healthy_triang.nearest_vertex(cancer_cell);

            // create the circle which will be radiated
            Circle radiation_region = Circle(cancer_cell, CGAL::squared_distance(cancer_cell, next_healthy->point()));

            // get the cancer cells that would be eliminated
            vector<PSVertex_handle> killed_cancer_cells;
            cancer_triang.range_search(radiation_region, back_inserter(killed_cancer_cells));

            max_cells_killed = max(max_cells_killed, killed_cancer_cells.size());
        }

        cout << max_cells_killed << endl;
    }
}

```

8.18 Knights

Keywords: BGL, Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Max-flow

```

#include <iostream>
#include <vector>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <algorithm>

#include <boost/tuple/tuple.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;
typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, long,
    property<edge_residual_capacity_t, long,
    property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

```

```

typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef graph_traits<Graph>::edge_descriptor Edge;

void addEdge(int from, int to, int weight, Graph& graph, EdgeCapacityMap& capacity, ReverseEdgeMap& rev_edge) {
    Edge edge, r_edge;
    tie(edge, tuples::ignore) = add_edge(from, to, graph);
    tie(r_edge, tuples::ignore) = add_edge(to, from, graph);
    capacity[edge] = weight;
    capacity[r_edge] = 0;
    rev_edge[edge] = r_edge;
    rev_edge[r_edge] = edge;
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_count;
    cin >> test_count;
    for(int test = 0; test < test_count; test++) {
        int width, height, knight_count;
        cin >> width >> height >> knight_count;

        // create graph
        Graph graph(2 * width * height + 2 + knight_count); // don't forget source and sink
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, graph);
        ResidualCapacityMap res_capacity = get(edge_residual_capacity, graph);

        /* Index
        * [0 .. knight_count - 1]: knights
        * [knight_count .. knight_count + width * height - 1]: intersections of set 1 with edges to set 2
        * [knight_count + width * height .. knight_count + 2 * width * height - 1]: intersections of set 2 with edges to set
        * 1
        */

        // source / sink
        int source = 2 * width * height + knight_count;
        int sink = source + 1;

        // read in knight position
        for(int knight_index = 0; knight_index < knight_count; knight_index++) {
            int x, y;
            cin >> x >> y;

            // source -> sink
            addEdge(source, knight_index, 1, graph, capacity, rev_edge);
            //cout << "source -> " << knight_index << endl;

            // knight -> starting position in first intersection set
            addEdge(knight_index, knight_count + x + y * width, 1, graph, capacity, rev_edge);
            //cout << knight_index << " -> " << knight_count + x + y * width << endl;
        }

        // add missing edges
        for(int edge_x = 0; edge_x < width; edge_x++) {
            for(int edge_y = 0; edge_y < height; edge_y++) {
                int intersection_offset = edge_x + edge_y * width;

                // edge from first intersection set to second one
                addEdge(knight_count + intersection_offset, knight_count + width * height + intersection_offset, 1, graph,
                    capacity, rev_edge);
                //cout << "first set to second set: " << intersection_offset << " -> " << intersection_offset << endl;

                // edge from second intersection set to all in set one that can be reached by a path of length 1 in the cave
                if(edge_x - 1 >= 0) {
                    addEdge(knight_count + width * height + intersection_offset, knight_count + edge_x - 1 + edge_y * width, 1,
                        graph, capacity, rev_edge);
                    //cout << "A second set to first set: " << intersection_offset << " -> " << edge_x - 1 + edge_y * width << endl;
                }

                if(edge_x + 1 < width) {
                    addEdge(knight_count + width * height + intersection_offset, knight_count + edge_x + 1 + edge_y * width, 1,
                        graph, capacity, rev_edge);
                    //cout << "B second set to first set: " << intersection_offset << " -> " << edge_x + 1 + edge_y * width << endl;
                }

                if(edge_y - 1 >= 0) {
                    addEdge(knight_count + width * height + intersection_offset, knight_count + edge_x + (edge_y - 1) * width, 1,
                        graph, capacity, rev_edge);
                    //cout << "C second set to first set: " << intersection_offset << " -> " << edge_x + (edge_y - 1) * width <<
                        endl;
                }

                if(edge_y + 1 < height) {
                    addEdge(knight_count + width * height + intersection_offset, knight_count + edge_x + (edge_y + 1) * width, 1,
                        graph, capacity, rev_edge);
                    //cout << "D second set to first set: " << intersection_offset << " -> " << edge_x + (edge_y + 1) * width <<
                        endl;
                }

                // add sink edge to second intersection set
                if(edge_x == 0 || edge_x + 1 == width || edge_y == 0 || edge_y + 1 == height) {
                    addEdge(knight_count + width * height + intersection_offset, sink, 1, graph, capacity, rev_edge);
                }
            }
        }
    }
}

```

```

        //cout << knight_count + intersection_offset << " -> sink" << endl;
    }
}

// do max flow
int max = push_relabel_max_flow(graph, source, sink);

cout << max << endl;

}

return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/kruskal_min_spanning_tree.hpp>
#include <boost/property_map/transform_value_property_map.hpp>
#include <boost/graph/boyer_myrvold_planar_test.hpp>
#include <boost/graph/max_cardinality_matching.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

inline void add_flow_edge(int start, int end, int c, EdgeCapacityMap& capacity, ReverseEdgeMap& rev_edge, Graph& g) {
    Edge e, rev_e;
    bool success;
    tie(e, success) = add_edge(start, end, g);
    if (success) {
        tie(rev_e, success) = add_edge(end, start, g);
        capacity[e] = c;
        capacity[rev_e] = 0;
        rev_edge[e] = rev_e;
        rev_edge[rev_e] = e;
    }
    else {
        capacity[e] += c;
    }
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    //cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int x_dim, y_dim, nr_knights;
        cin >> x_dim >> y_dim >> nr_knights;
        int nr_intersections = x_dim*y_dim;
        int nr_nodes = nr_intersections * 2 + 2;
        int source = nr_intersections * 2;
        int sink = nr_intersections * 2 + 1;

        Graph g(nr_nodes);
        EdgeCapacityMap capacity = get(edge_capacity, g);
        ReverseEdgeMap rev_edge = get(edge_reverse, g);

        for (int i = 0; i < x_dim; i++) {
            for (int j = 0; j < y_dim; j++) {
                int edge_id = j*x_dim + i;

                //add "only go through intersection once constraint"
                add_flow_edge(edge_id, edge_id + nr_intersections, 1, capacity, rev_edge, g);

                //add horizontal edges
                if (i == 0) {
                    add_flow_edge(edge_id + nr_intersections, sink, 1, capacity, rev_edge, g);
                }
            }
        }
    }
}

```

```

    else {
        add_flow_edge(edge_id + nr_intersections, edge_id-1, 1, capacity, rev_edge, g);
    }

    if (i == x_dim - 1) {
        add_flow_edge(edge_id + nr_intersections, sink, 1, capacity, rev_edge, g);
    }
    else {
        add_flow_edge(edge_id + nr_intersections, edge_id + 1, 1, capacity, rev_edge, g);
    }

    //add vertical edges
    if (j == 0) {
        add_flow_edge(edge_id + nr_intersections, sink, 1, capacity, rev_edge, g);
    }
    else {
        add_flow_edge(edge_id + nr_intersections, edge_id - x_dim, 1, capacity, rev_edge, g);
    }

    if (j == y_dim - 1) {
        add_flow_edge(edge_id + nr_intersections, sink, 1, capacity, rev_edge, g);
    }
    else {
        add_flow_edge(edge_id + nr_intersections, edge_id + x_dim, 1, capacity, rev_edge, g);
    }
}
}

for (int i = 0; i < nr_knights; i++) {
    int knight_x, knight_y;
    cin >> knight_x >> knight_y;

    int edge_id = knight_y*x_dim + knight_x;

    add_flow_edge(source, edge_id, 1, capacity, rev_edge, g);
}

int flow = push_relabel_max_flow(g, source, sink);
cout << flow << endl;
}

return 0;
}

```

8.19 Tight Words

Keywords: ACM, Dynamic Programming

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#define MOD_NUM 100003

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_count;
    cin >> test_count;

    for(int test = 0; test < test_count; test++) {
        int max_word, max_length;
        cin >> max_word >> max_length;

        if(max_length > 0) {
            vector<vector<int>> > table(max_word + 1, vector<int>(max_length + 1, 0));

            // init for length 1
            for(int word = 0; word <= max_word; word++) {
                table.at(word).at(1) = 1;
            }

            // go through length of the word
            for(int length = 2; length <= max_length; length++) {
                // go through each additional word
                for(int word = 0; word <= max_word; word++) {
                    // add current word to a string ending with the previous word
                    if(word - 1 >= 0) { // only if there is a "previous word" of course
                        table.at(word).at(length) += table.at(word - 1).at(length - 1);
                    }

                    // add current word to a string ending with the same word
                    table.at(word).at(length) += table.at(word).at(length - 1);
                }
            }
        }
    }
}

```

```

        // add current word to a string ending with the next word
        if(word + 1 <= max_word) {
            table.at(word).at(length) += table.at(word + 1).at(length - 1);
        }

        table.at(word).at(length) = table.at(word).at(length) % MOD_NUM;
    }
}

// now we have to count together all the words possible (it can end with any of the words)
int solution = 0;
for(int word = 0; word <= max_word; word++) {
    solution += table.at(word).at(max_length);
}

cout << solution % MOD_NUM << endl;
} else {
    cout << "1" << endl;
}
}

return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>

#define MOD_NUM 100003

using namespace std;

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    //cout << fixed << setprecision(0);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_letters, word_length;
        cin >> nr_letters >> word_length;

        if (word_length > 0) {
            vector<vector<int>> > dp(word_length + 1, vector<int>(nr_letters + 1, 0));

            for (int i = 0; i <= nr_letters; i++) {
                dp[i][i] = 1;
            }

            for (int i = 2; i <= word_length; i++) {
                for (int j = 0; j <= nr_letters; j++) {
                    if (j - 1 >= 0) {
                        //cout << "adding1: " << dp[i - 1][j - 1] << endl;
                        dp[i][j] += dp[i - 1][j - 1];
                    }
                    //cout << "adding2: " << dp[i - 1][j] << endl;
                    dp[i][j] += dp[i - 1][j];
                    if (j + 1 <= nr_letters) {
                        //cout << "adding3: " << dp[i - 1][j + 1] << endl;
                        dp[i][j] += dp[i - 1][j + 1];
                    }
                    dp[i][j] = dp[i][j] % MOD_NUM;
                    //cout << dp[i][j] << " ";
                }
                //cout << endl;
            }
            int solution = 0;
            for (int i = 0; i <= nr_letters; i++) {
                solution += dp[word_length][i];
            }
            solution = solution % MOD_NUM;
            cout << solution << endl;
        }
        else {
            cout << 1 << endl;
        }
    }

    return 0;
}

```

8.20 Cantonal Courier

Keywords: BGL, Graph with edge capacity, Graph with residual capacity, Graph with reverse edges, Max-flow

```
#include <iostream>
#include <vector>
#include <queue>
#include <tuple>
#include <cmath>
#include <climits>
#include <algorithm>
#include <climits>
#include <string>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list_traits<vecS, vecS, directedS> Traits;
typedef adjacency_list<vecS, vecS, directedS, no_property,
    property<edge_capacity_t, int,
    property<edge_residual_capacity_t, int,
    property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;

typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_residual_capacity_t>::type ResidualCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

// Custom add_edge, also creates reverse edges with corresponding capacities.
void addEdge(int u, int v, int c, EdgeCapacityMap &capacity, ReverseEdgeMap &rev_edge, Graph &G) {
    Edge e, reverseE;
    tie(e, tuples::ignore) = add_edge(u, v, G);
    tie(reverseE, tuples::ignore) = add_edge(v, u, G);
    capacity[e] = c;
    capacity[reverseE] = 0;
    rev_edge[e] = reverseE;
    rev_edge[reverseE] = e;
}

int main(void)
{
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int test_cases;
    cin >> test_cases;

    for(int test = 0; test < test_cases; test++) {
        int zone_count, job_count;
        cin >> zone_count >> job_count;

        // create graph for flow analysis
        Graph graph(zone_count + job_count + 2);

        // helper variables to find
        int zone_offset = 0;
        int job_offset = zone_count;
        int source = zone_count + job_count;
        int sink = source + 1;

        // get graph's properties
        EdgeCapacityMap capacity = get(edge_capacity, graph);
        ReverseEdgeMap rev_edge = get(edge_reverse, graph);

        // read in the cost for a zone ticket
        for(int zone_index = 0; zone_index < zone_count; zone_index++) {
            int cost;
            cin >> cost;

            addEdge(zone_offset + zone_index, sink, cost, capacity, rev_edge, graph);
        }

        // read in job rewards
        int total_reward = 0;
        for(int job_index = 0; job_index < job_count; job_index++) {
            int reward;
            cin >> reward;

            total_reward += reward;

            addEdge(source, job_offset + job_index, reward, capacity, rev_edge, graph);
        }

        // read in needed zones
        for(int job_index = 0; job_index < job_count; job_index++) {
            int zones_needed_count;
            cin >> zones_needed_count;
            for(int col = 0; col < zones_needed_count; col++) {
                int zone;
```

```

        cin >> zone;
        zone--;

        addEdge(job_offset + job_index, zone_offset + zone, numeric_limits<int>::max(), capacity, rev_edge, graph);
    }
}

// calculate flow which represents the amount of money we have to spent
int we_pay = push_relabel_max_flow(graph, source, sink);

cout << total_reward - we_pay << endl;

}

return 0;
}

```

```

#include <iostream>
#include <cassert>
#include <vector>
#include <limits>
#include <algorithm>
#include <queue>
#include <set>
#include <utility>
#include <cmath>
#include <stack>

#include <boost/config.hpp>
#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/push_relabel_max_flow.hpp>
#include <boost/graph/connected_components.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, directedS> Traits;

typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_capacity_t, long, property<
    edge_residual_capacity_t, long, property<edge_reverse_t, Traits::edge_descriptor> > > > Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

typedef property_map<Graph, edge_capacity_t>::type EdgeCapacityMap;
typedef property_map<Graph, edge_reverse_t>::type ReverseEdgeMap;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> SimpleGraph;

inline void add_flow_edge(int start, int end, int c, EdgeCapacityMap& capacity, ReverseEdgeMap& rev_edge, Graph& g) {
    Edge e, rev_e;
    bool success;
    tie(e, success) = add_edge(start, end, g);
    if (success) {
        tie(rev_e, success) = add_edge(end, start, g);
        capacity[e] = c;
        capacity[rev_e] = 0;
        rev_edge[e] = rev_e;
        rev_edge[rev_e] = e;
    }
    else {
        capacity[e] += c;
    }
}

int main() {
    cin.sync_with_stdio(false);
    cout.sync_with_stdio(false);

    int nr_test_cases;
    cin >> nr_test_cases;

    for (int test_case = 0; test_case < nr_test_cases; test_case++) {
        int nr_zones, nr_jobs;
        cin >> nr_zones >> nr_jobs;

        int nr_vertices = nr_zones + nr_jobs + 2;
        Graph g(nr_vertices);
        int source = nr_vertices - 2;
        int sink = nr_vertices - 1;

        EdgeCapacityMap capacity = get(edge_capacity, g);
        ReverseEdgeMap rev_edge = get(edge_reverse, g);

        for (int i = 0; i < nr_zones; ++i) {
            int ticket_cost;
            cin >> ticket_cost;
            add_flow_edge(nr_jobs + i, sink, ticket_cost, capacity, rev_edge, g);
        }
        int sum_job_reward = 0;
        for (int i = 0; i < nr_jobs; ++i) {
            int job_reward;
            cin >> job_reward;
            add_flow_edge(source, i, job_reward, capacity, rev_edge, g);
            sum_job_reward += job_reward;
        }
    }
}

```

```

    }
    for (int i = 0; i < nr_jobs; ++i) {
        int nr_tickets_for_job;
        cin >> nr_tickets_for_job;
        for (int j = 0; j < nr_tickets_for_job; ++j) {
            int ticket_id;
            cin >> ticket_id;
            ticket_id--;
            add_flow_edge(i, nr_jobs + ticket_id, numeric_limits<int>::max(), capacity, rev_edge, g);
        }
    }
    int cost = push_relabel_max_flow(g, source, sink);

    cout << sum_job_reward - cost << endl;
}

return 0;
}

```

9 Useful Snippets and Stuff

9.1 General remarks

- around 10'000'000 operations/iterations per second (ACM slide below claims its less, but oh well)

Asymptotic Running Time

- **Rule of Thumb:** Processor can do 1M operations per second, Timelimit is 3 seconds.
- $n < 1M$: Algorithm should be $O(n)$
- $n < 100K$: Algorithm should be $O(n \log n)$
- $n < 1K$: Algorithm should be $O(n^2)$
- $n < 100$: Algorithm should be $O(n^3)$
- $n < 50$: Algorithm should be $O(n^4)$
- $n < 20$: Algorithm should be $O(n^5)$ or $O(2^n)$
- $n < 10$: Algorithm should be $O(n^6)$ or $O(n!)$

9.2 Custom Sorting

```

// sorting
// ATTENTION: be careful, each container has its own order!
#include <vector>
#include <algorithm>

/*
 * #1: Define custom compare function
 */
struct Edge {
    int from, to, weight;
};

bool compare(const Edge& lhs, const Edge& rhs) {
    return lhs.weight > rhs.weight;
}

// ...
vector<Edge> edges;
// ...
sort(edges.begin(), edges.end(), compare);

/* #2: Define the '<' operator for a struct/class
 * Generally, for some type T define 'bool operator<(T other) const {}'
 */
struct Edge {
    int from, to, weight;

    bool operator<(Edge other) const {
        return weight > other.weight;
    }
};

// ...
vector<Edge> edges;
// ...
sort(edges.begin(), edges.end());

/*
 * #3: Define 'operator()'
 */
struct Edge {
    int from, to, weight;
};

```



```

class Compares {
    int ref_weight;

public:
    Compares(const int& weight) : ref_weight(weight) {}

    bool operator()(const Edge& lhs, const Edge& rhs) const {
        return lhs.weight > rhs.weight && lhs.weight >= ref_weight;
    }
};

// ...
Compares comp(100);
// ...
vector<Edge> edges;
// ...
sort(edges.begin(), edges.end(), comp);

```

9.3 CMake Configuration

Add to CMakeLists.txt:

```

project( some-project_ )

# ^^ start of the CMakeLists.txt ...

# enable C++11
add_definitions("-std=c++11")

# enable all warnings
set(CMAKE_CXX_FLAGS_DEBUG "${CMAKE_CXX_FLAGS_DEBUG} -Wall")
set(CMAKE_CXX_FLAGS_RELEASE "${CMAKE_CXX_FLAGS_RELEASE} -Wall")

# enable debug always
set(CMAKE_BUILD_TYPE "Debug")

# additional information regarding the makefile creation, not really useful I think
set(CMAKE_VERBOSE_MAKEFILE ON)

# ... rest of the CMakeLists.txt ...
cmake_minimum_required(VERSION 2.6.2)

```

Debugging can also be enabled by calling `cmake -DCMAKE_BUILD_TYPE=Debug`, however this adds only the `-g` flag!

9.4 CMake and CGAL

```

# Step 0: IMPORTANT: first, create the C++ source code file

# Step 1: call CGAL CMake script
cgal_create_cmake_script

# Step 2: modify CMakeLists.txt if needed (adding C++11 support, debugging, etc.)
vim CMakeLists.txt
# or
nano CMakeLists.txt
# or ...

# Step 3: call CMake, don't forget the dot
cmake .

# Step 4: from now on always enough to call make
make

# Step 5: execute application

```

9.5 BGL

URL for normal graph functions: https://judge.inf.ethz.ch/doc/boost/libs/graph/doc/graph_concepts.html. Also good starting point is the “A Quick Tour of the Boost Graph Library” (see TOC).

```

#include <climits>
#include <iostream>
#include <vector>

#include <boost/graph/adjacency_list.hpp>
#include <boost/tuple/tuple.hpp> // tuples::ignore

// BGL algo specific includes ...
#include <boost/graph/dijkstra_shortest_paths.hpp>
#include <boost/graph/strong_components.hpp>
// .. end

using namespace std;
using namespace boost;

// Directed graph with int weights on edges.
typedef adjacency_list<vecS, vecS, directedS, no_property, property<edge_weight_t, int> > Graph;

```

```

// ... or one with multiple properties for some fancy algorithm
typedef adjacency_list<vecS, vecS, directedS, property<vertex_name_t, string,
    property<vertex_distance_t, int> > > Graph;

// and don't forget:
typedef property_map<Graph, vertex_name_t>::type NameMap;

// Edge type (edge descriptor in BGL speak).
typedef graph_traits<Graph>::edge_descriptor Edge;

// Edge iterator.
typedef graph_traits<Graph>::edge_iterator EdgeIterator;

// Out Edge iterator (directed graph)
typedef graph_traits<Graph>::out_edge_iterator OutEdgeIterator;

// Map edge -> weight.
typedef property_map<Graph, edge_weight_t>::type WeightMap;

// ...

void main() {
    // create graph with 'n' vertices
    Graph graph(n);

    // accessing a property map
    NameMap name_map = get(vertex_name, graph_instance);

    // iterate over all outgoing (directed graph) edges
    OutEdgeIterator out_edge_iterator, out_edge_end;
    for(tie(out_edge_iterator, out_edge_end) = out_edges(some_vertices, graph);
        out_edge_iterator != out_edge_end;
        ++out_edge_iterator) {
        // get the other end's vertex
        int other_vertex = target(*out_edge_iterator, graph);
    }
}

```

9.6 CGAL: Linear/Quadratic Programming

```
typedef CGAL::Quadratic_program<IT> P;  
typedef CGAL::Quadratic_program_solution<ET> S;
```

- The input type **IT**
 - ▶ Typically `int` or `double` (check in the manual)
- The exact type **ET**
 - ▶ Requirement: input type can be converted to exact type
 - ▶ `int` / `double` (not recommended)
 - ▶ GMP
 - ★ `CGAL::Gmpz` integral numbers
 - ★ `CGAL::Gmpq` rational numbers
 - ★ `CGAL::Gmpzf` “floating point” numbers
 - ▶ `CGAL::MP_Float` “floating point” numbers
- The solution type `CGAL::Quotient<ET>`

Multiplication in `CGAL::MP_Float` has complexity $\Theta(n^2)$, multiplication for the GMP number types uses a faster algorithm (depends on the magnitude), asymptotic runtime $\approx n^{1.5}$

| | |
|--|---|
| QuadraticProgram $\min x^T D x + c^T x + c_0$ $\text{s.t. } Ax \preceq b$ $\ell \leq x \leq u$ | NonnegativeQuadraticProgram $\min x^T D x + c^T x + c_0$ $\text{s.t. } Ax \preceq b$ $x \geq 0$ |
| LinearProgram $\min c^T x + c_0$ $\text{s.t. } Ax \preceq b$ $\ell \leq x \leq u$ | NonnegativeLinearProgram $\min c^T x + c_0$ $\text{s.t. } Ax \preceq b$ $x \geq 0$ |

The solvers

`solve_nonnegative_{linear/quadratic}_program()`
will completely ignore any manually set lower or upper bounds ℓ or u .

Debugging:

- Check the dimensions: `lp.get_n()`, `lp.get_m()`
- `CGAL::print_linear_program(std::cerr, lp, "lp");`

Change the pivot rule

- Bland's pivot rule avoids cycling (but it is slower...)

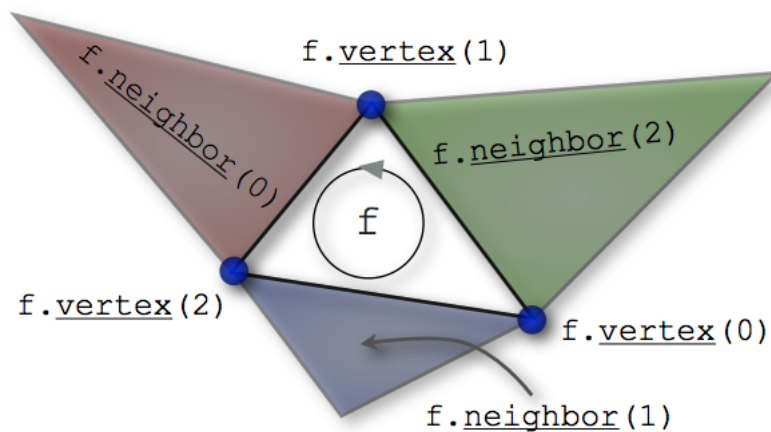
```
CGAL::Quadratic_program_options options;  
options.set_pricing_strategy(CGAL::QP_BLAND);  
Solution s = CGAL::SOLVER(program, ET(), options);
```

BUG - Don't Forget!

There is a bug in the
assignment operator= for
Solution objects...

9.7 CGAL: Approximation (Triangulation)

Nex image 'f' is a 'face handle'



```
...  
Triangulation::Edge e;  
...  
// get the vertices of e  
Triangulation::Vertex_handle v1 = e.first->vertex((e.second + 1) % 3);  
Triangulation::Vertex_handle v2 = e.first->vertex((e.second + 2) % 3);  
std::cout << "e = " << v1->point() << " <-> " << v2->point() << std::endl;  
...
```

9.8 Matching

- Undirected graph $G = (V, E)$
- Is a subset $M \subseteq E$ of edges
- Each pair of edges of the matching set $(e_1, e_2 \in M)$ don't have a common vertices $v \in V$
- Or based on the vertices: Every pair of vertices $v_1, v_2 \in V$ aren't the start or end point of an edge $e \in M$
- **Maximal Matching:** If it's not possible to add another edge $e \in E$ M to M and M being a matching
- **Maximum Matching/Maximum Cardinality Matching:** A maximal matching with the largest amount of edges. There might exists multiple maximum matchings.

9.9 Vertex Cover

- Undirected graph $G = (V, E)$
- A vertex cover is a subset $V' \subseteq V$ such that for each edge $(u, v) =: e \in E$ $u \in V'$ and/or $v \in V'$
- **Minimal Vertex Cover:** Generally NP-complete. Is the smallest possible vertex cover.

9.10 König's Theorem

- only for bipartite graphs
- The size of a maximum matching (maximum cardinality matching) is equal to the minimal vertex cover

Algorithm:

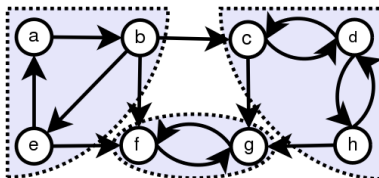
1. Let $G = (V, E)$ be a undirected bipartite graph, i.e. $L \cup R = V \wedge L \cap R = \emptyset$
2. Calculate the Maximum Matching (Maximum Cardinality Matching), resulting in a matching $M \subseteq E$
3. Mark all vertices $v \in L$ that are not in M ($v \notin M$) as visited
4. Start at visited vertices and do a vertices search (DFS) from L to R along edges from $V \setminus M$ and R to L along edges from M . Each such visited vertex is marked as visited
5. All unvisited vertices in L and all visited in R are part of the minimal vertex cover

9.11 Connected Component

- For undirected graphs
- Connected component is a subgraph of a graph where any two vertices in the subgraph are connected by a path, but no path exists to vertices to ones outside the subgraph

9.12 Strongly Connected Component

- For directed graphs
- Partitions a graph into subgraphs where each subgraph is strongly connected.
- **Strongly Connected:** A graph is strongly connected iff every vertex of the graph is reachable from every other vertex



From: <https://commons.wikimedia.org/wiki/File:Scc.png>

9.13 Biconnected Component / Articulation Points

See BGL documentation, it's pretty good. Otherwise this might help:

- **Biconnected Graph:** A connected graph is biconnected iff removing any single vertex (and all edges from/to this vertex) can't disconnect the graph. Such a graph doesn't contain *articulation points*!
- **Biconnected Components:** A subset of vertices where removing one of these vertices doesn't result in a disconnected graph inside the subgraph. Vertices can belong to multiple biconnected components!
- **Articulation Points:** These are the vertices that belong to more than one biconnected component. Such vertices are called *articulation points*. Removing such a articulation point would result in an increase of connected components (i.e. increase the number of graphs). No such articulation points means the graph is biconnected!

9.14 BGL: DFS / BFS

```
#include <iostream>

#include <boost/graph/adjacency_list.hpp>
#include <boost/graph/breadth_first_search.hpp>
#include <boost/graph/depth_first_search.hpp>

using namespace std;
using namespace boost;

typedef adjacency_list<vecS, vecS, undirectedS, no_property, no_property> Graph;
typedef graph_traits<Graph>::edge_descriptor Edge;
typedef graph_traits<Graph>::vertex_descriptor Vertex;

class BFSVisitorClass: public default_bfs_visitor {
public:
    BFSVisitorClass() {
        cout << "here do some initialisation before passing to breadth_first_search()" << endl;
    }

    void initialize_vertex(Vertex v, const Graph& graph) {
        cout << "initialize_vertex() for vertex: " << v << endl;
    }

    void discover_vertex(Vertex v, const Graph& graph) {
        cout << "discover_vertex() for vertex: " << v << endl;
    }

    // this is called right before all edges are taken and tree_edge() calls done
    void examine_vertex(Vertex v, const Graph& graph) {
        cout << "examine_vertex() for vertex: " << v << endl;
    }

    void examine_edge(Edge e, const Graph& graph) {
        cout << "examine_edge() for edge: " << e << endl;
    }

    // this one is interesting if you want to know the visited/followed edges
    // this edge forms the search tree
    void tree_edge(Edge e, const Graph& graph) {
        cout << "tree_edge() for edge: " << e << endl;
    }

    void non_tree_edge(Edge e, const Graph& graph) {
        cout << "non_tree_edge() for edge: " << e << endl;
    }

    void gray_target(Edge e, const Graph& graph) {
        cout << "gray_target() for edge: " << e << endl;
    }

    void black_target(Edge e, const Graph& graph) {
        cout << "black_target() for edge: " << e << endl;
    }

    void finish_vertex(Vertex v, const Graph& graph) {
        cout << "finish_vertex() for vertex: " << v << endl;
    }
};

class DFSVisitorClass: public default_dfs_visitor {
public:
    DFSVisitorClass() {
        cout << "here do some initialisation before passing to depth_first_search()" << endl;
    }

    void initialize_vertex(Vertex v, const Graph& graph) {
        cout << "initialize_vertex() for vertex: " << v << endl;
    }

    void start_vertex(Vertex v, const Graph& graph) {
        cout << "start_vertex() for vertex: " << v << endl;
    }

    void discover_vertex(Vertex v, const Graph& graph) {
        cout << "discover_vertex() for vertex: " << v << endl;
    }

    void examine_edge(Edge e, const Graph& graph) {
        cout << "examine_edge() for edge: " << e << endl;
    }

    void tree_edge(Edge e, const Graph& graph) {
        cout << "tree_edge() for edge: " << e << endl;
    }

    void back_edge(Edge e, const Graph& graph) {
        cout << "back_edge() for edge: " << e << endl;
    }

    void forward_or_cross_edge(Edge e, const Graph& graph) {
        cout << "forward_or_cross_edge() for edge: " << e << endl;
    }
}
```

```

void finish_vertex(Vertex v, const Graph& graph) {
    cout << "finish_vertex() for vertex: " << v << endl;
}
};

int main() {
    /*
        0
       / \
      1   2
     / \  |
    3  4  5
       |
       6
    */

    Graph graph(8);
    add_edge(0, 1, graph);
    add_edge(1, 3, graph);
    add_edge(1, 4, graph);
    add_edge(2, 5, graph);
    add_edge(4, 6, graph);

    cout << "Depth First Search:" << endl;
    depth_first_search(graph, visitor(DFSVisitorClass()));
    cout << endl << endl;

    cout << "Breath First Search" << endl;
    breadth_first_search(graph, 0, visitor(BFSVisitorClass()));
    //      ^- source to start from

    return 0;
}

```

```

# OUTPUT
Depth First Search:
here do some initialisation before passing to depth_first_search()
initialize_vertex() for vertex: 0
initialize_vertex() for vertex: 1
initialize_vertex() for vertex: 2
initialize_vertex() for vertex: 3
initialize_vertex() for vertex: 4
initialize_vertex() for vertex: 5
initialize_vertex() for vertex: 6
initialize_vertex() for vertex: 7
start_vertex() for vertex: 0
discover_vertex() for vertex: 0
examine_edge() for edge: (0,1)
tree_edge() for edge: (0,1)
discover_vertex() for vertex: 1
examine_edge() for edge: (1,0)
back_edge() for edge: (1,0)
examine_edge() for edge: (1,3)
tree_edge() for edge: (1,3)
discover_vertex() for vertex: 3
examine_edge() for edge: (3,1)
back_edge() for edge: (3,1)
finish_vertex() for vertex: 3
examine_edge() for edge: (1,4)
tree_edge() for edge: (1,4)
discover_vertex() for vertex: 4
examine_edge() for edge: (4,1)
back_edge() for edge: (4,1)
examine_edge() for edge: (4,6)
tree_edge() for edge: (4,6)
discover_vertex() for vertex: 6
examine_edge() for edge: (6,4)
back_edge() for edge: (6,4)
finish_vertex() for vertex: 6
finish_vertex() for vertex: 4
finish_vertex() for vertex: 1
finish_vertex() for vertex: 0
start_vertex() for vertex: 2
discover_vertex() for vertex: 2
examine_edge() for edge: (2,5)
tree_edge() for edge: (2,5)
discover_vertex() for vertex: 5
examine_edge() for edge: (5,2)
back_edge() for edge: (5,2)
finish_vertex() for vertex: 5
finish_vertex() for vertex: 2
start_vertex() for vertex: 7
discover_vertex() for vertex: 7
finish_vertex() for vertex: 7

Breath First Search
here do some initialisation before passing to breadth_first_search()
initialize_vertex() for vertex: 0
initialize_vertex() for vertex: 1
initialize_vertex() for vertex: 2
initialize_vertex() for vertex: 3
initialize_vertex() for vertex: 4

```

```

initialize_vertex() for vertex: 5
initialize_vertex() for vertex: 6
initialize_vertex() for vertex: 7
discover_vertex() for vertex: 0
examine_vertex() for vertex: 0
examine_edge() for edge: (0,1)
tree_edge() for edge: (0,1)
discover_vertex() for vertex: 1
finish_vertex() for vertex: 0
examine_vertex() for vertex: 1
examine_edge() for edge: (1,0)
non_tree_edge() for edge: (1,0)
black_target() for edge: (1,0)
examine_edge() for edge: (1,3)
tree_edge() for edge: (1,3)
discover_vertex() for vertex: 3
examine_edge() for edge: (1,4)
tree_edge() for edge: (1,4)
discover_vertex() for vertex: 4
finish_vertex() for vertex: 1
examine_vertex() for vertex: 3
examine_edge() for edge: (3,1)
non_tree_edge() for edge: (3,1)
black_target() for edge: (3,1)
finish_vertex() for vertex: 3
examine_vertex() for vertex: 4
examine_edge() for edge: (4,1)
non_tree_edge() for edge: (4,1)
black_target() for edge: (4,1)
examine_edge() for edge: (4,6)
tree_edge() for edge: (4,6)
discover_vertex() for vertex: 6
finish_vertex() for vertex: 4
examine_vertex() for vertex: 6
examine_edge() for edge: (6,4)
non_tree_edge() for edge: (6,4)
black_target() for edge: (6,4)
finish_vertex() for vertex: 6

```

9.15 Sorting Algorithms Implemented

```

// http://www.martinbroadhurst.com/cpp-sorting.html
// https://codereview.stackexchange.com/questions/49459/merge-sort-algorithm-implementation-using-c

/*
Merge Sort
*/

template<typename I>
void doMerge(I begin, I midPoint, I end)
{
    typename std::vector<typename std::iterator_traits<I>::value_type> TmpVec;

    TmpVec tmp(std::make_move_iterator(begin), std::make_move_iterator(end));

    TmpVec::iterator beginAlt = std::begin(tmp);
    TmpVec::iterator endAlt = std::end(tmp);
    TmpVec::iterator midAlt = std::next(beginAlt, std::distance(begin, midPoint));

    TmpVec::iterator l = beginAlt
    TmpVec::iterator r = midAlt;
    I i = begin;

    while(l < midAlt && r < endAlt)
    {
        *i = std::move((*l < *r) ? *l++ : *r++);
        ++i;
    }
    while(l < midAlt)
    {
        *i = std::move(*l++);
        ++i;
    }
    while(r < endAlt)
    {
        *i = std::move(*r++);
        ++i;
    }
}

template<typename I>
void mergeSort(I begin, I end)
{
    std::size_t length = std::distance(begin, end);
    if (length <= 1)
    {
        return;
    }

    std::size_t mid = length/2;
    I midPoint = std::next(begin, mid);

    mergeSort(begin, midPoint);
    mergeSort(midPoint, end);

    doMerge(begin, midPoint, end);
}

```



```

}

int main()
{
    std::vector<int> data {{ 5,12,45,2,67,8}};
    mergeSort(std::begin(data), std::end(data));

    std::copy(std::begin(data), std::end(data), std::ostream_iterator<int>(std::cout, ", "));
    std::cout << "\n";
}

/*
Bubble Sort

In a bubble sort, a series of passes is made over the data, and in each pass, every pair of adjacent elements is compared
. If they are in the wrong order, they are swapped. The sort completes when no changes have been made in a pass. In
each pass the maximum element is "bubbled" to the end, as it will be swapped with every succeeding element, and
consequently becoming a part of the next comparison. The effect of this is that a sorted sublist is built up at the
right of the input list. The end of the sublist to be sorted is shortened by one element after each pass to avoid
examining the sorted sublist.
*/

template <class RandomAccessIterator>
void bubble_sort(RandomAccessIterator first, RandomAccessIterator last)
{
    bool finished = false;
    RandomAccessIterator it, end = last - 1;
    while (!finished) {
        bool changed = false;
        for (it = first; it < end; ++it) {
            if (*it > *(it + 1)) {
                std::swap(*it, *(it + 1));
                changed = true;
            }
        }
        if (changed) {
            --end;
        }
        else {
            finished = true;
        }
    }
}

/*
Selection Sort

In a selection sort the data is searched for the minimum (or maximum) element, and this is swapped with the element in
its place at the end. As with a bubble sort, the end of the range to be sorted is adjusted after each pass as a
sorted sublist is built up at the end. The construction of the sorted range is identical to that in a bubble sort,
but it is performed much more efficiently because there are fewer swaps.
*/

template <class RandomAccessIterator>
void selection_sort(RandomAccessIterator first, RandomAccessIterator last)
{
    RandomAccessIterator begin = first;
    while (begin < last) {
        RandomAccessIterator it, min;
        bool started = false;
        for (it = begin; it < last; ++it) {
            if (!started || *it < *min) {
                min = it;
            }
            started = true;
        }
        std::swap(*min, *begin);
        ++begin;
    }
}

/*
Min-Max Sort

One can perform a selection sort and simultaneously search for the minimum and maximum element and move them to their
proper places in each pass. This has the effect of building up a sorted sublist at each end, with the unsorted part
becoming smaller from each ends, as in a cocktail sort. Again, the construction of the sorted ranges is performed
much more efficiently in this sort than a cocktail sort because there are fewer swaps.
*/

template <class RandomAccessIterator>
void minmax_sort(RandomAccessIterator first, RandomAccessIterator last)
{
    RandomAccessIterator begin = first;
    RandomAccessIterator end = last - 1;
    while (begin < end) {
        RandomAccessIterator it, min, max;
        bool started = false;
        for (it = begin; it <= end; ++it) {
            if (!started) {
                min = it;
                max = it;
                started = true;
            }
            else {
                if (*it < *min) {

```

```

        min = it;
    }
    else if (*it > *max) {
        max = it;
    }
}
}
if (max == begin && min == end) {
    // Swap min and max only
    std::swap(*max, *min);
}
else if (max == begin) {
    // Swap max and end first
    std::swap(*max, *end);
    std::swap(*min, *begin);
}
else {
    // Swap min and begin first
    std::swap(*min, *begin);
    std::swap(*max, *end);
}
++begin;
--end;
}
}
}

```

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