

Adaptive Teaching: Learning to Teach

by

Aazim Lakhani

Bachelor of Computer Engineering, Mumbai University, Mumbai, 2009

A Project Submitted in Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

in the Department of Computer Science

© Aazim Lakhani 2018
University of Victoria

All rights reserved. This dissertation may not be reproduced in whole or in part, by
photocopying or other means, without the permission of the author.

Adaptive Teaching: Learning to Teach

by

Aazim Lakhani

Bachelor of Computer Engineering, Mumbai University, Mumbai, 2009

Supervisory Committee

Dr. Nishant Mehta, Supervisor
(Department of Computer Science)

Dr. George Tzanetakis, Departmental Member
(Department of Computer Science)

Supervisory Committee

Dr. Nishant Mehta, Supervisor
(Department of Computer Science)

Dr. George Tzanetakis, Departmental Member
(Department of Computer Science)

ABSTRACT

Traditional approaches to teaching were not designed to address individual students needs. We propose a new way of teaching, one that personalizes the learning path for each student. We frame this use case as a contextual multi-armed bandit (CMAB) problem, a sequential decision-making setting in which the agent must pull an arm based on context to maximize rewards. We customize a contextual bandit algorithm for adaptive teaching to present the best way to teach a topic based on contextual information about the student and the topic the student is trying to learn. To streamline learning, we add an additional feature which would allow our algorithm to skip topics that a student is unlikely to learn. We evaluate our algorithm over a synthesized unbiased heterogeneous dataset to show that our baseline learning algorithm can maximize rewards to achieve results similar to an omniscient policy.

Contents

Supervisory Committee	ii
Abstract	iii
Table of Contents	iv
List of Tables	vi
List of Figures	vii
Acknowledgements	viii
Dedication	ix
1 Introduction	1
1.1 Use Case	1
1.2 Motivation	3
1.3 Contribution	3
1.4 Organization	3
2 Preliminaries	5
2.1 Multi-armed bandits	5
2.2 Contextual Bandits	5
2.3 Upper Confidence Bound (UCB)	6
2.4 Linear Upper Confidence Bound (LinUCB)	7
3 Related Work	9
4 Algorithm	11
4.1 Basic Version	14
4.2 Skipping	14

5	Experiments	17
5.1	Dataset	17
5.1.1	Courses	17
5.1.2	Context	18
5.2	Environment	21
5.3	Evaluation Strategy	21
5.4	Omniscient Policy	21
5.5	Learning Algorithm	22
5.6	Skip Topics	22
6	Results and Evaluation	24
6.1	Confidence Bound α	24
6.2	Confidence Threshold	25
6.2.1	Without confidence threshold	26
6.2.2	With optimal confidence threshold	26
6.3	Learning Algorithm	27
6.3.1	Skipping Disabled	27
6.3.2	Skipping Enabled	28
7	Conclusions	31
	Bibliography	32

List of Tables

Table 4.1 Important Algorithmic Notations	12
Table 5.1 Student Context Description	19
Table 5.2 Content Context Description	20
Table 6.1 Confusion Matrix without Confidence Threshold	26
Table 6.2 Confusion Matrix with Optimal Confidence Threshold	27
Table 6.3 Confusion Matrix with Confidence Threshold of 30	27

List of Figures

Figure 2.1 An Example: UCB [26]	7
Figure 5.1 Student Context Template	20
Figure 5.2 Content Context Template	20
Figure 6.1 Cumulative Rewards For Values of α	25
Figure 6.2 Compare Cumulative Rewards Without Skipping.	28
Figure 6.3 Cumulative Rewards With optimal hyper-parameters	29

ACKNOWLEDGEMENTS

I would like to thank:

Dr. Nishant Mehta for his dedication, commitment and insights, without which I would not have been able to overcome the gaps I could not see.

Almighty One for giving me the courage, belief and strength to pursue pursue my ideas

Mom for her blessings and prayers.

DEDICATION

I dedicate this project to my family, to whom I owe both the joy and pain of growing up.

Chapter 1

Introduction

The quest for a fully personalized learning experience began with the development of intelligent tutoring systems (ITSs) [6, 14, 29, 31]. However, to date, ITSs are primarily rules-based, which requires domain experts to consider every possible learning scenario that students can encounter and then manually specify the corresponding learning actions in each case. This approach is not scalable since it is both labor-intensive and domain-specific [16].

Machine learning-based personalized learning systems have shown great promise in reaching beyond ITS to scale to large numbers of courses and students. These systems automatically create personalized learning actions for each individual student to maximize their learning. Examples of actions include reading a textbook section, watching a lecture video, interacting with a simulation or lab, solving a practice question, etc. Instead of domain-specific rules, machine learning algorithms are used to select actions automatically by analyzing the data students generate as they interact with learning resources[16].

The goal of this project is to design a learning algorithm, which could adapt based on students feedback to help them learn effectively.

1.1 Use Case

There is no universal best way to explain a topic. The best way is subjective to every student. Unless we explore different ways to teach a topic, we cannot find a policy, which would help map different students to explanations conducive for them. Once, we have such a policy we can use it to teach students effectively. **This is**

the exploration-exploitation dilemma in which we have to find a trade-off between two competing goals: maximizing students satisfaction in the long run, while exploring uncertainties in students preferences [3]. For example, an adaptive teaching system should present different explanations knowing students preferences on learning. However, unless we try different ways of teaching, it is not possible to say with certainty whether or not an explanation would help a student learn effectively. We use the term adaptive teaching to avoid confusing it with adaptive learning, used in machine learning literature. In the education domain, these terms are used interchangeably.

We represent this use case as a contextual bandits problem. We use contextual information about the student, such as their preferences to learn through *visual, text, demo-based, practical, activity-based, step-by-step, lecture, audio-based explanations, as well as, self-evaluation and pre-assessment of students*. We also use contextual information about the contents used to teach a topic, by rating them in terms of *ease of understanding, simplicity, intuitiveness, depth in teaching, conciseness, thoroughness, ratings, abstractness, hands-on, experimental*. **A content item or arms are different actions or ways a topic can be taught.** The reward would be the students feedback to confirm their understanding of the topic they are trying to learn. The feedback can be through quizzes, interactions with a content item, tasks to name a few. **By pulling an arm, we obtain a reward, drawn from some unknown distribution determined by the selected content item and the context. Our goal is to maximize the total cumulative reward.**

Let us make this more concrete by mapping this use case to teaching a class. In any school, a course comprises of multiple topics. However, now instead of a single way to teach everyone, there would be multiple ways to teach. These different ways to teach are called content items. Students give their feedback on the presented content. Behind the scenes, our learning algorithm takes information about the student (*also referred to as student context*), topic, content items (*also referred to as content context*) to find the best way to teach a student. This project extends the most cited contextual bandit learning algorithm, LinUCB (Linear Upper Confidence Bound) [18] to enhance it for our use case.

1.2 Motivation

The primary and perennial problem in education is the overwhelming challenge of teachers being responsible for accomplishing learning mastery among a demographically diverse set of students [23]. In traditional classrooms, learning has largely remained a one-size-fits-all experience in which the teacher selects a single learning action for all students in their class, regardless of their diversity in needs, prior knowledge, skills, learning styles, and backgrounds. It is not feasible for teachers to ensure their explanations can cater to all students. Hence, there is a need for a system which could personalize teaching for students to help them learn effectively, as well as, increase course engagement and progression.

Such systems would be adaptive, recognize different levels of prior knowledge among students, as well as course progression based on students skill and feedback from learning. This could reduce teachers load to remediation to teaching and facilitating. These would adapt to individual students learning patterns instead of students having to adjust to the way of teaching. They would provide timely and comprehensive data-driven feedback, to recognize potential challenges that students might come across as the course progresses.

1.3 Contribution

We suggest a novel baseline algorithm, for our proposed adaptive teaching methodology, which learns from students and contents for each topic to create a personalized learning path for each student. It adapts dynamically, based on students feedback and learning preferences.

We also provide a skip feature which is meant to keep students engaged, increase student retention, as well as provide feedback to teachers by recognizing the challenges faced by a student, early in the course.

Our online learning algorithm gives close to optimal results, over a synthesized unbiased heterogeneous dataset.

1.4 Organization

Chapter 1 provided a brief overview of our use case, along with the need for an adaptive teaching system and how this project contributes to realizing it. **Chap-**

ter 2 introduces the technical concepts used to represent our use case, along with the algorithm we customize for adaptive teaching. **Chapter 3** describes prior work related to our use case, using different approaches and how our work compares to them. **Chapter 4** explains the algorithm created for adaptive teaching, along with the skip feature. **Chapter 5** describes the experimental setup, which comprises the dataset synthesized to evaluate our algorithm. It also explains the evaluation strategy followed to examine our results. **Chapter 6** presents the results of our experiments and compares its performance with respect to the best possible policy. **Chapter 7** concludes this project by summarizing the contributions and outlines possible avenues for future work.

Chapter 2

Preliminaries

In this chapter, we explain key concepts that should be understood for this project.

2.1 Multi-armed bandits

Multi-armed bandits is a problem setting, where an agent needs to make a sequence of decisions in time $1, 2, \dots, T$. At each time t the agent is given a set of K arms to choose and has to decide which arm to pull. After pulling an arm, it receives a reward for that arm, and the rewards of other arms are unknown. The reward of each arm are assumed to be drawn i.i.d from an unknown arm dependent distribution. This problem setting is stochastic in nature[32].

Personalized recommender systems recommend items (e.g., movies, news articles) to the users based on their predicted individual interests on these items. The users response helps the system improve their prediction[2]. However, the response to any particular item can only be available after these items are recommended. If an item is never shown to the users, the recommender systems cannot collect the response on these items. Such problems can be naturally modeled as a contextual bandit problem [30].

2.2 Contextual Bandits

In the theory of sequential decision-making, contextual bandit problems [28] occupy a middle ground between multi-armed bandit problems [7] and full-blown reinforcement learning (usually modeled using Markov decision processes along with discounted or

average reward optimality criteria) [27]. Unlike bandit algorithms, which cannot use any side-information or context, contextual bandit algorithms can learn to map the context into appropriate actions. However, contextual bandits do not consider the impact of actions on the evolution of future contexts. Nevertheless, in many practical domains where the impact of the learners action on future contexts is limited, contextual bandit algorithms have shown great promise. Examples include web advertising [1] and news article selection on web portals [18] [13].

Formally, a contextual bandit problem is a repeated interaction which proceeds over T rounds. At each round $t = 1, 2, \dots, T$, the environment reveals contexts $x_t \in X$, which is used by the learner to pick an action $a_t \in A$, which gives a reward r_t revealed by the environment. The goal of the learner is to choose actions which would maximize cumulative reward $\sum_{t=1}^T r_t$

We will now translate this problem setting for our adaptive teaching use case in which an algorithm A which proceeds in discrete rounds $t = 1, 2, 3, \dots$. In round t :

1. The algorithm observes the student context x_s and a set A_t of content items together with their feature vectors x_c for $a \in A_t$. X_t summarizes context for both the student x_s and content item x_c .
2. Based on observed rewards in previous rounds, A chooses an arm $a_t \in A_t$. The arm a_t is estimated to have the highest expected reward. The student reveals the received reward r_t for arm a_t whose expectation depends on both the context X_t and the arm a_t .
3. The algorithm then improves its content item selection strategy with the new observation, (x_t, a_t, r_t) . It is important to emphasize here that no feedback namely, reward r_t is observed for unchosen arms $a \neq a_t$ [18].

2.3 Upper Confidence Bound (UCB)

A perpetual challenge in bandit algorithms is to find the right balance between exploration and exploitation (Section 1.1). Upper Confidence Bound (UCB) comprises of a family of algorithms which try to find the best trade-off between exploration and exploitation. It is based on the principle of *being optimistic* by choosing actions which have the highest potential for reward. The intuitive reason that this works is that when acting optimistically, one of two things happens. Either the optimism was

justified, in which case the learner is acting optimally, or the optimism was not justified. In the latter case, the algorithm takes some action that they believed might give a large reward when in fact it does not. If this happens sufficiently often, then the learner will learn what is the true reward of this action and not choose it in the future [17]. UCB algorithms estimate the expected reward for each arm by adding estimated sample mean of an arm and its upper confidence bound around it.

We will refer to Figure 2.1 as an example to understand UCB. Let us assume we have three arms a_1, a_2, a_3 . The reward distribution for each arm after several rounds is a Gaussian distribution Q with mean μ and standard deviation σ . The y-axis is the probability of obtaining the reward for these arms.

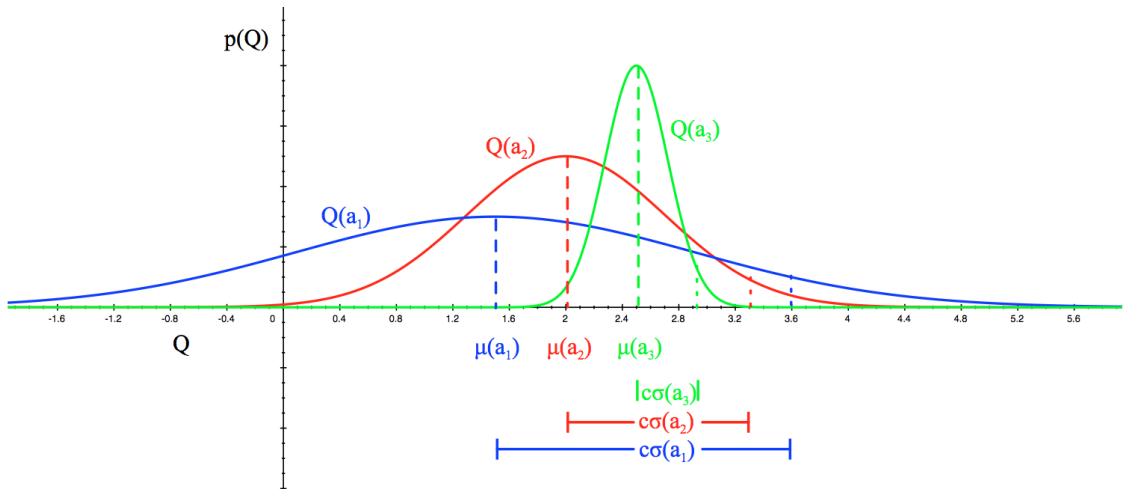


Figure 2.1: An Example: UCB [26]

The upper confidence bound for each arm is given by $c\sigma(a_i)$. The distribution shows that the sum of the expected mean and confidence bound is highest for a_1 . Hence, the UCB algorithm, will select a_1 . The reward received will reduce uncertainty around a_1 . So for the next round, the algorithm once again finds the arm with the highest sum for the expected mean and confidence bound. This is repeated for T rounds[9].

2.4 Linear Upper Confidence Bound (LinUCB)

LinUCB is a way to apply UCB to a more general contextual bandits setting, where the UCB of each arm is computed efficiently by assuming the reward is linear. The estimated expected mean is parameterized over the context x_a for each arm a . At

round t this is given as $\hat{\theta}_a^T x_{t,a}$. The upper confidence bound around each arm a at round t is given as $\sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$. Here, A_a is the co-variance over the context data $x_{t,a}$ for each arm a at round t .

LinUCB introduces a hyper-parameter α , which allows us to control exploration over arms. This is achieved by scaling the upper confidence bound by α . A higher value of α encourages exploration. As a result, the algorithm would need more rounds to explore before it begins exploiting. We can now compute **the expected estimated reward for an arm a at round t as** $p_{t,a} = \hat{\theta}_a^T x_{t,a} + \alpha \sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$ [18].

Chapter 3

Related Work

Our use case could also be formulated using the partially observed Markov decision process (POMDP) framework. POMDPs model the student’s latent knowledge states and their transitions to learn an action selection policy that maximizes reward received in the possibly distant future (long-term learning outcome). Previous work applying POMDPs to personalized learning has achieved some degree of success. However, learning a personalized learning schedule using a POMDP is greatly complicated by the curse of dimensionality. The solution quickly becomes intractable as the dimensions of the state and action spaces grow. Consequently, POMDPs have made only a limited impact in large-scale personalized learning applications involving large numbers of students and learning actions [16].

A more scalable approach to personalized learning is to learn a policy, which maps contexts to actions using the multi-armed bandits (MAB) framework, which is more suitable for optimizing students success. The simplicity of the MAB framework makes it more practical than the POMDP framework in real-world educational applications[16].

The work in [19] applies a MAB algorithm to educational games in order to trade off scientific discovery (learning about the effect of each learning resource) and student learning. Their approach is context-free and thus not ideally suited for applications with significant variation among the knowledge states of individual students. Indeed, it can be seen as a special case of our work when there is no context information available. The work in [21] collects high-dimensional student - computer interaction features as they play an educational game and uses them to search for a good teaching policy [16].

The works in [10, 15] both use a form of expert knowledge to learn a teaching

policy. The approach of [10], in particular, uses expert knowledge to narrow down the set of possible actions a student can take. Our approach, in contrast, requires no expert knowledge and is fully data-driven [16].

The work in [25] found that various student response models, including knowledge tracing (KT) [11], Item Response Theory (IRT) models [20, 24, 5], additive factor models (AFM) [8], and performance factor models (PFM) [12] can have similar predictive performance yet lead to very different teaching policies. [16]. While these results are interesting, we emphasize that the focus of the current work is to develop policy learning algorithms rather than comparing student models.

Chapter 4

Algorithm

This chapter presents the algorithm created for the adaptive teaching system. We first present the basic version of the algorithm (Section 4.1). We then explain the skip feature (Section 4.2), which could streamline learning.

The algorithm used is an extension of upper confidence bound (UCB)-based algorithms [4] (Section 2.3). These algorithms maintain estimates of the expected reward of each arm together with confidence bound around it. It then pulls the arm with the highest expected reward, which is equal to the sample mean plus the confidence bound. Based on the observed reward, it optimizes the arm parameters iteratively after each pull. In this project, we are using the most cited contextual bandit algorithm, namely LinUCB (Section 2.4).

Before we dive in, it is important to note, that to better understand the algorithm, I have divided the explanation into two halves. *The first half, explains the overall flow without skipping, whereas the second explains in-depth the function calls made in the first half along with skipping.* I would be using bandit terminology to explain. **Arm** refers to a content item. **Payoff** refers to the estimated reward computed by the algorithm. A **round** comprises of computing the expected payoff for each content item; then selecting a content item with maximum expected payoff and getting student feedback for the content item.

Before we proceed to the algorithm, below are the notations, you will come across.

Symbol	Meaning
α	Parameter to scale Confidence bound.
C	Confidence threshold to skip.
\mathbf{x}_s	Student context vector.
x_c	Content items context.
\mathbf{x}_t	Context vector at round t .
X_t / X_t^i	Context at round t . It combines \mathbf{x}_s and all available x_c for topic i .
X_t^{i+1}	Context at round t . It combines \mathbf{x}_s and all available x_c^{i+1} for topic $i + 1$.
x_c^{i+1}	Content items contexts for topic $i + 1$.
a	An arm a for topic i .
a'	An arm a' for topic $i + 1$.
A_t	Arms available at round t .
$A_{t'}^{i+1}$	Arms available for topic $i + 1$ at round t' .
a_t^{i+1}	Arm a for topic $i + 1$ at round t .
t	Current round t .
t'	Possible next round t' .
T	Total number of rounds.
i	Topic being taught.
$i + 1$	Next Topic in the sequence.
$p_{t,a}$	Expected payoff from arm a at round t .
$p_{t,a}^i$	Expected payoff from arm a at round t for topic i .
$p_{t',a'}^{i+1}$	Expected payoff from arm a' at round t' for next topic $i + 1$.
X	Input features for skip classifier.
Y	Label to train the skip classifier.

Table 4.1: Important Algorithmic Notations

Note

- We are always on the current topic i , unless we explicitly specify next topic $i + 1$.
- All vectors are **bold** faced lower cased.
- All sets are upper cased.

Algorithm 1 Teach with LinUCB

```

1: Hyper Parameters :  $\alpha \in \mathbb{R}_+$ 
2:  $C$  : Confidence threshold to skip
3: Inputs : Student context  $\mathbf{x}_s$  and content context  $x_c$  of available arms  $a \in A_t$  for
   topic  $i$  at round  $t$ 
4: Prepare context  $X_t = \begin{pmatrix} \mathbf{x}_s \\ x_c \end{pmatrix}$ 
5: skip-enabled  $\leftarrow$  False
6: while  $A_t \neq \emptyset$  do
7:    $a_t^i, p_{t,a}^i \leftarrow \text{EXPECTED-PAYOFF}(X_t, A_t)$ 
8:   skip-decision,  $p_{t',a'}^{i+1} \leftarrow \text{SKIPTOPIC}(\mathbf{x}_s, p_{t,a}^i, i)$ 
9:   if skip-decision, skip-enabled is True then
10:    Move to next topic  $i \leftarrow i + 1$ 
11:    break
12:   else
13:    Pull arm  $a_t$  and observe reward  $r_t$ 
14:     $A_{a_t} \leftarrow A_{a_t} + \mathbf{x}_{t,a_t} \mathbf{x}_{t,a_t}^T$ 
15:     $\mathbf{b}_{a_t} \leftarrow \mathbf{b}_{a_t} + r_t \mathbf{x}_{t,a_t}$ 
16:    label  $\leftarrow \text{SETLABEL}(r_t)$ 
17:     $\text{TRAIN}(\mathbf{x}_s, p_{t,a}^i, p_{t',a'}^{i+1}, \text{label})$ 
18:     $t \leftarrow t + 1$ 
19:   if  $r_t \neq 1$  then
20:    Remove  $a_t \in A_t$ 
21:    skip-enabled  $\leftarrow$  True
22:   else
23:    Move to next topic :  $i \leftarrow i + 1$ 
24:    break

```

4.1 Basic Version

The basic version is without skipping. It explains the main flow of the algorithm. The next section explains the functions used along with skipping.

The algorithm requires two hyper-parameters to be configured. The first one is α which scales the confidence bound (Section 2.4). The second hyper-parameter is the confidence threshold C which decides confidence threshold that must be exceeded to skip a topic. Skipping is a feature to help reduce students who are unlikely to learn from content items available for a topic. It is meant to streamline learning. This could also be used by teachers to recognize topics that should be addressed in class.

We now explain how LinUCB (Section 2.4) helps the algorithm decide an arm to pull. Before we recommend a content item to a student, we need to prepare context X_t for the round t . It is prepared by combining the student context \mathbf{x}_s with content items context x_c for the topic i being taught. With the context X_t and arms A_t , we use LinUCB to compute the expected payoff from each arm and return the arm a_t^i with the maximum expected payoff $p_{t,a}^i$ which must be pulled for topic i at round t .

Assuming the classifier does not recommend skipping, a student is presented with the content item a_t for topic i . After being taught the student sends a reward r_t to complete the round t . Now the round t is complete We update the parameters A_{a_t} , \mathbf{b}_{a_t} of the arm pulled. We then use this reward r_t to train the skip classifier, to make better predictions in upcoming rounds. The features for the classifier comprise of students contextual information \mathbf{x}_s , expected payoff $p_{t,a}^i$ from the current topic i and the expected payoff $p_{t',a'}^{i+1}$ for the topic $i + 1$.

If no reward r_t was sent by the student \mathbf{x}_s , then it implies the student was unable to understand topic i . In which case, the algorithm removes the presented arm a_t and remains on the same topic i . However, if a reward r_t was sent, then the student is moved to the next topic $i + 1$. That completes the first half. The second half explains the functions briefly described above.

4.2 Skipping

On line 6 (Section 4.1) of the algorithm, we get the expected payoff $p_{t,a}^i$ estimated on pulling the arm a_t^i for the current topic i . Now, to decide whether or not it should pull the arm or move to the next topic, it calls the skip topic function.

The *SKIPTOPIC* function, takes the student context \mathbf{x}_s , the expected payoff $p_{t,a}^i$

```

25: function SKIPTOPIC( $x_s, p_{t,a}^i, i$ )
26:   Get next topic  $i + 1$  from topic  $i$ 
27:   Get arms  $A_{t'}^{i+1}$  and content context  $x_c^{i+1}$  for topic  $i + 1$ 
28:   Prepare context vector  $X_{t'}^{i+1} = \begin{pmatrix} x_s \\ x_c^{i+1} \end{pmatrix}$ 
29:    $a_{t'}^{i+1}, p_{t',a'}^{i+1} \leftarrow \text{EXPECTED-PAYOFF}(X_{t'}^{i+1}, A_{t'}^{i+1})$ 
30:   skip-decision  $\leftarrow \text{PREDICT}(x_s, p_{t,a}^i, p_{t',a'}^{i+1})$  to decide on skip
31:   return skip-decision,  $p_{t',a'}^{i+1}$ 
32: function EXPECTED-PAYOFF( $X_t, A_t$ )
33:   for  $a \in A_t$  do
34:     Get  $x_{t,a} \in X_t$ 
35:     if  $a$  is new then
36:        $A_a \leftarrow I_d$  (d-dimensional identity matrix)
37:        $b_a \leftarrow 0_{d \times 1}$  (d-dimensional zero vector)
38:        $\hat{\theta}_a \leftarrow A_a^{-1} b_a$ 
39:        $p_{t,a} \leftarrow \hat{\theta}_a^T x_{t,a} + \alpha \sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$ 
40:   Choose arm  $a_t = \arg \max_{a \in A_t} p_{t,a}$  with ties broken arbitrarily
41:   return  $a, \text{argmax} p_{t,a}$ ,
42: function PREDICT( $x_s, p_{t,a}^i, p_{t',a'}^{i+1}$ )
43:    $X \leftarrow x_s, i+1, p_{t,a}^i, p_{t',a'}^{i+1}$ 
44:    $Y$ , confidence-score  $\leftarrow$  Prediction from classifier
45:   if confidence-score  $< C$  then
46:     decision  $\leftarrow 0$ 
47:   return decision, confidence-score
48: function TRAIN( $x_s, p_{t,a}^i, p_{t',a'}^{i+1}$ , label)
49:    $X \leftarrow x_s, p_{t,a}^i, p_{t',a'}^{i+1}$ , topic,
50:    $Y \leftarrow$  label
51:   Train online SGD classifier
52: function SETLABEL( $r_t$ )
53:   if  $r_t$  is 0 then
54:     label  $\leftarrow 1$ 
55:   else
56:     label  $\leftarrow 0$ 
57:   return label

```

for pulling arm a at round t for topic i and the current topic i . It uses the topic i to get a reference to the next topic $i + 1$. Through the topic $i + 1$ it gets content items $A_{i'}^{i+1}$ and context data x_c^{i+1} associated those content items. After combining the contexts to prepare $X_{i'}^{i+1}$, it gets the maximum expected payoff $p_{i',a'}^{i+1}$ and the arm $a_{i'}^{i+1}$ to pull by passing the context vector $X_{i'}^{i+1}$ and arms available for next topic $A_{i'}^{i+1}$. The expected payoff function returns an arm with the maximum estimated payoff. Skip topic function then calls the skip classifier to predict a skip-decision for the student context \mathbf{x}_s , along with the expected payoff from the current and the next topic to make a prediction.

The *EXPECTED-PAYOFF* function takes the context X_t , along with the arms A_t available at round t . After an arm a_t is initialized with parameters A_a, b_a , they are used to calculate the expected mean $\hat{\theta}_a^T x_{t,a}$ and confidence bound $\sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$ for the arm. The confidence bound is scaled by α . The expected mean and the scaled confidence bound are added to give the expected payoff $p_{t,a}$ for arm a at round t . It then finds an arm a with maximum expected payoff $p_{t,a}$ and returns the expected payoff along with the arm a to be pulled.

The *PREDICT* function is used to predict whether the student should be moved to the next topic $i + 1$ or should remain on the same topic i . It combines student context vector \mathbf{x}_s , the expected payoff $p_{t,a}^i$ for the current topic i and the expected payoff $p_{i',a'}^{i+1}$ for the next topic $i + 1$ to prepare a feature vector X . It then asks a prediction from the binary supervised online Support Vector classifier, with hinge loss to make a prediction Y and a confidence-score for its prediction. If the confidence-score is less than the confidence threshold, then set the *decision* variable is set to 0, which implies no skipping. This is because a confidence score lower than the threshold implies that the classifier is not sufficiently confident about its prediction.

The *TRAIN* function is used to train the skip classifier to make better predictions. Similar to the predict function, it combines student context vector \mathbf{x}_s , the expected payoff $p_{t,a}^i$ for the current topic i and the expected payoff $p_{i',a'}^{i+1}$ for the next topic $i + 1$ to prepare a feature vector X . It sets the *label* to the output Y . Together they train the skip classifier.

The *SETLABEL* function is used to set the *label* to train the skip classifier. If the reward r_t for round t is set to 0, then the *label* is set to 1, as it implies that since staying on the same topic did not give any reward, it would have been better to skip. If the reward r_t for round t is set to 1, then the *label* is set to 0, as it implies that staying on the same topic was a good decision. The set *label* is then returned.

Chapter 5

Experiments

This chapter explains the dataset (Section 5.1) used to evaluate the learning algorithm. Next, it describes the environmental setup (Section 5.2) used for these experiments. The next section explains how we evaluate our algorithm (Section 5.3) in absence of pre-existing benchmarks using an omniscient policy (Section 5.4). This is followed by sections which explain how the learning algorithm (Section 5.5) and the skip feature (Section 5.6) work in these experiments.

5.1 Dataset

Machine learning algorithms are data-driven. Due to the novelty of our approach to the best of our knowledge there is no similar dataset available. Hence **we synthesized datasets to represent data generated by students taking courses in an adaptive teaching system.**

An honest attempt is made to synthesize an unbiased dataset representative of the heterogeneous students and content items. The contextual data is created from a uniform distribution $U(0,1]$ sampled randomly to simulate the diverse nature of student preferences and content features.

5.1.1 Courses

We use the following courses for our experiments.

1. *Course 1* : A course which comprises of 10 topics. Its taken by 50 students. There are a total of 119 content items for 10 topics. So on an average, there are

12 content items per topic. We use this course to find optimal hyper-parameters (α and C) for our learning algorithm.

2. *Course 2* : A course which has 25 topics. Its taken by 100 students. There are 329 content items for 25 topics. So on an average, there are 13 content items per topic. We use this course for evaluation.

5.1.2 Context

We will assume, there was a survey conducted among students, who were asked how should teaching be, to streamline learning? Students gave their preferences on a scale of 1 to 10, with 1 being least preferred and 10 being most preferred. These preferences were normalized.

Research has shown that students prefer to learn a certain way. Though there is no unanimous consensus, there is a fair bit of research and understanding on the needs of a student. The features we consider are by no means exhaustive, but a representative subset of the main features. The tables 5.1 and 5.2 describe the student and content context used for these experiments.

Apart from the above contextual data, there is a course which is taught. For our experiments, we consider a typical course which comprises of topics to be taught. These topics are labeled as $T_1, T_2 \dots T_{25}$. For e.g: T_1 refers to the first topic of the course. Each topic has between 5 to 20 different content items. Each content is labeled in the format $C_{topic-id_content-number}$. For e.g: C_{1_2} refers to the second content item for topic T_1 .

We now have the required contextual information. Topics in the course are taught in a sequence outlined by the teacher. This allows them to control the course sequence.

Student Context	Description
Visual (S_V)	How much preference is given to visual explanations (video, short-film, movie-clip, vlogs)?
Text (S_T)	How much preference is given to written explanations (books, articles, blogs, research papers)?
Demo-based (S_D)	How much preference is given to live experiments to help understand a concept?
Practical (S_P)	How much preference is given to an explanation, followed by a demo of the topic, and enabling students to perform it?
Step-by-step (S_S)	How much preference is given to a guide to practice, try and understand a topic in a systematic way?
Activity / Task-based (S_AT)	How much preference is given to content items which are interactive and require students to participate?
Lecture (S_L)	How much preference is given to being passive and listen to an expert explain the topic?
Audio (S_A)	How much preference is given to audio explanations (podcasts, music)?
Self-evaluation (S_SE)	Students self-evaluate their readiness, motivation, excitement for the course.
Pre-assessment (S_PA)	Teachers conduct a pre-assessment of the pre-requisites required for the course.

Table 5.1: Student Context Description

Let us take an example to understand the data.

Below (Figure 5.1) is a student context data point which shows a student preference. It tells us that this student prefers visual (S_V), text(S_T), demo-based(S_D) methods of learning, but does not prefer practical (S_P), activity-based(S_{AT}), and did not fare well in the pre-assessment(S_{PA}). The student does not mind step-by-step(S_S), lectures(S_L), audios(S_A) methods of learning and believes he/she is ready for the course (S_{SE}).

Content Context	Description
<i>Ease of understanding (C_E)</i>	<i>How relatively easy is it to understand the content?</i>
Simple / Intuition (C_I)	Does it provide a surface level or deep understanding of the topic?
Surface / In-depth (C_ID)	How much preference is given to live experiments to help understand a concept?
Brief / Concise (C_C)	Is it short, to the point or descriptive, verbose and elaborative, keeping in mind that learners have different levels of concentration and capacity to remember?
Thorough (C_T)	How well does the content item cover the topic?
Preference / Well reviewed / Well rated (C_R)	How well rated is the explanation?
Theoretical / Abstract (C_A)	How theoretical or abstract is the content item?
Practical / Hands on (C_P)	Is it something that can be tried or experienced?
Experimental / Task-based (C_ETB)	Does it require a task to be completed to fully understand it, like collaboration with other students or some research/findings?

Table 5.2: Content Context Description

S_V	S_T	S_D	S_P	S_S	S_AT	S_L	S_A	S_SE	S_PA
0.87	0.82	0.88	0.36	0.6	0.06	0.66	0.56	0.66	0.07

Figure 5.1: Student Context Template

Below is a content context data point prepared for the course. This content item is thorough(C_T), practical(C_P), and experimentally sound(C_{ETB}), but not in-depth(C_{ID}),concise(C_C), and abstract(C_A). Its moderate in terms of understanding(C_E), intuitiveness(C_I) and has positive reviews(C_R).

	C_E	C_I	C_ID	C_C	C_T	C_R	C_A	C_P	C_ETB
C_1_1	0.45	0.72	0.31	0.05	0.91	0.75	0.06	0.88	0.97

Figure 5.2: Content Context Template

5.2 Environment

We run a simulation of a course being taken by students with the omniscient policy and the learning algorithm, deciding the content item to be presented for each student. It is an environment where several students are taking the course at the same time. Both the omniscient policy and the learning algorithm work in online mode. The learning algorithm optimizes its parameters in each round to give better predictions.

5.3 Evaluation Strategy

Since there are no readily available benchmarks to compare our algorithm, we assume there exist an omniscient policy. This policy has optimal parameters to recommend the best arm to pull.

We run the same course with an omniscient policy and the learning algorithm, to evaluate our learning algorithm relative to the omniscient policy. The evaluation is conducted with and without skipping. Due to the stochastic nature of students feedback, both the omniscient policy and the learning algorithm will run for a different number of rounds. However, the total cumulative reward available is the same for both of them. Hence, we evaluate them based on cumulative reward accumulated over all rounds.

We simulate the student feedback as a Bernoulli distribution. Here, the probability of success is the maximum expected reward computed by the omniscient policy. Based on the reward received by the learning algorithm the arm parameters are updated to make better decisions in the upcoming rounds. This experiment aims to find, how well does our algorithm optimize the arm parameters to match the omniscient policy.

5.4 Omniscient Policy

This policy knows all the probability distributions. It knows every step of the way the best decision based on its knowledge of the true distributions. It does not have to learn anything. It has optimal parameters θ^* for each arm. Hence it is expected to maximize the cumulative reward.

This policy calculates the expected payoff for each arm a available for a topic. It then selects the arm which has maximum expected payoff. The expected payoff for

an arm a with optimal parameters $\theta_{t,a}^*$ and with context vector $x_{t,a}$ at round t is given by $E[r_{t,a}|\mathbf{x}_{t,\mathbf{a}}] = x_{t,a}^T \theta_{t,\mathbf{a}}^*$

5.5 Learning Algorithm

The learning algorithm can adapt to several students at the same time to present a content item personalized for each student. For every topic a student is trying to learn, it gets the expected payoff for all available content items. It checks whether it should skip to next topic or remain on the current topic. Skipping is activated only if the student gave no reward for a content item presented for the topic.

When a student is on a topic, the algorithm presents a content item that could maximize rewards. After working through the content item, the student shares feedback on the content item. If a reward is sent, then this implies that the student understood the concept and can be taken to the next topic. If no reward was sent, then the student may be presented with the next best content item for the same topic or could be moved to the next topic in the course sequence.

Once the student has shared feedback on the content item, the data is sent to train the skip classifier, to make a better prediction in forthcoming rounds.

5.6 Skip Topics

The learning algorithm checks with skip topics to decide whether or not the content item should be presented for the current topic. Skip topic predicts this by using the student context along with the estimated payoff for the current topic and the estimated payoff for the next topic in the course sequence.

It makes this decision using an online supervised learning stochastic gradient descent classifier with student context along with the estimated payoff of the current and next topic to make a decision. The label for the classifier depends on the reward received for the topic. If a reward was sent then the label is set to 0 or else it is set to 1. Thus the classifier makes use of the feedback sent by all students to recognize common topics and content items that students find difficult so it could make a confident decision.

The aim to create skip topics feature is to streamline learning for students. If a student has been taught a topic once and was not satisfied with it, then there is the

option to skip to the next topic or explain the same topic with a different content item.

The skip classifier is a linear Support Vector Machine estimator with hinge loss. The estimator is a regularized linear model with stochastic gradient descent (SGD) learning. The gradient of the loss is estimated, each sample at a time and the model is updated along the way with a decreasing learning rate. The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using squared Euclidean norm L2 [22].

Chapter 6

Results and Evaluation

This chapter presents results using the experimental set-up given in the previous chapter (Chapter 5). We evaluate the learning algorithm with respect to the omniscient policy. Before evaluation we need to first find optimal values for hyper-parameters α (Section 6.1) and confidence threshold C (Section 6.2). We then proceed to use these optimal values to evaluate the learning algorithm with and without the skip feature. (Section 6.3)

6.1 Confidence Bound α

Finding an optimal value for α is important to learn faster as it scales the confidence bound of each content item. An optimal value would find the right balance between exploration and exploitation. A higher value of α would imply the learning algorithm takes more rounds exploring which can lead to sub-optimal results.

This parameter is configured for the learning algorithm and not the omniscient policy. We empirically evaluated an optimal value for α using course 1 (Section: 5.1.1). The graph (Figure 6.1) shows the cumulative reward for different values of α .

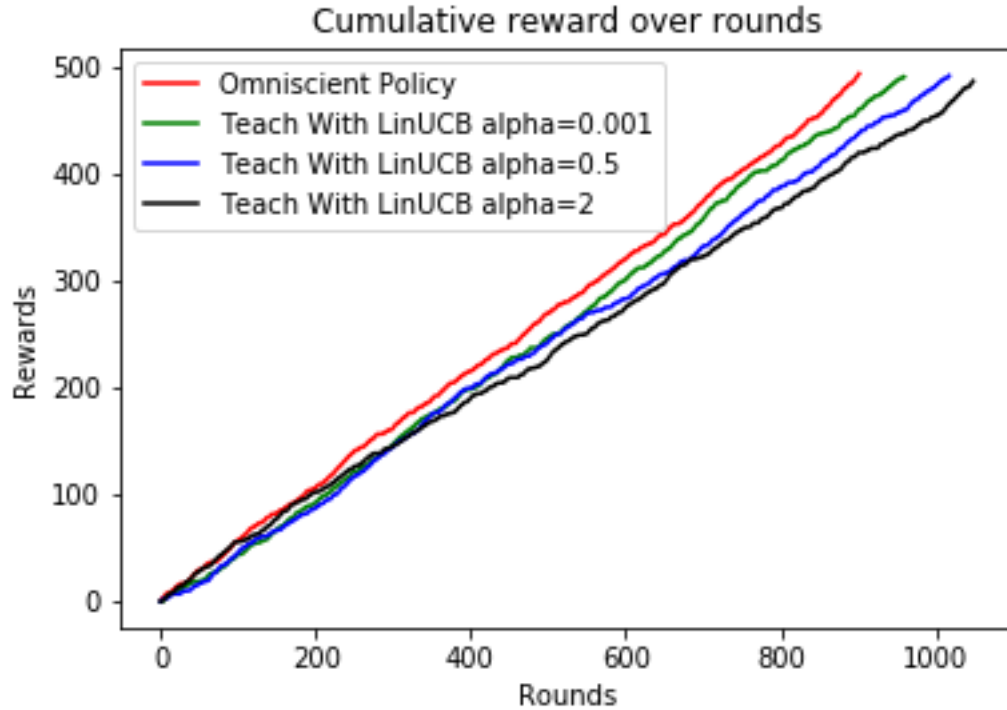


Figure 6.1: Cumulative Rewards For Values of α .

The graph compares the omniscient policy with the learning algorithm for different values of α . It shows that the **cumulative reward needs fewer rounds to maximize rewards when $\alpha = 0.001$** for the learning algorithm. On running repeated experiments, we found that a value of α between 0 to 0.5 could give optimal results. We would be using $\alpha = 0.001$ to evaluate the learning algorithm.

6.2 Confidence Threshold

This is a threshold on the confidence score the skip classifier should exceed for its prediction to be accepted. Skipping is enabled for a topic only after a student gives no reward to a content item. The threshold helps:

- To keep students engaged by skipping topics they are unable to understand.
- Give teachers control on their preference to skipping.
- Allow the learning algorithm to skip content items that are less likely to give rewards.

We do not want the confidence threshold to be too high as students might have to go through each content item nor do we want it to be too low such that students are taken to the next topic on the first occurrence of not understanding a topic. Hence finding an optimal value for the confidence threshold is important to have a good learning experience.

We evaluate the performance for different values of the confidence threshold over course 1 (Section: 5.1.1). Below are the results.

6.2.1 Without confidence threshold

We evaluated the skip classifier with no confidence threshold. Below is a confusion matrix of the results.

		Predictions		Total
		Stay (0)	Skip (1)	
Reward	0	181	40	221
	1	260	50	310
Total		441	90	1062

Table 6.1: Confusion Matrix without Confidence Threshold

The classifier is evaluated on how well it helps the learning algorithm maximize cumulative reward. It shows that in 310 rounds its decision helped increase rewards whereas, in 221 rounds its decision gave no rewards. This shows us that about **58.38%** times its decision helped increase the cumulative reward.

6.2.2 With optimal confidence threshold

We evaluate the skip classifier with confidence threshold. We will only consider data points where the skip classifiers decision was overruled as its confidence score was below the threshold. This would be when the skip classifier had predicted skipping to the next topic, but since the confidence score was below the threshold, the prediction was ignored. This gives us the true measure of the effectiveness of the confidence threshold.

We evaluated the classifier for different values of confidence threshold. For different threshold values performance ranged consistently between 55 - 58 %. We found the skip classifier performed most optimally when the confidence threshold is 10. The below table 6.2 shows the results.

		Predictions		Total
		Stay (0)	Skip (1)	
Reward	0	51	36	87
	1	70	55	125
Total		121	91	424

Table 6.2: Confusion Matrix with Optimal Confidence Threshold

The above table shows us that in 125 rounds its decision helped increase rewards whereas in 87 rounds its decision gave no rewards. This results in about **58.96 %** times it made the correct decision.

As the value of the confidence threshold was increased the number of skips decreased. Table 6.3 shows the results for confidence threshold of 30.

		Predictions		Total
		Stay (0)	Skip (1)	
Reward	0	128	7	135
	1	178	9	187
Total		306	16	644

Table 6.3: Confusion Matrix with Confidence Threshold of 30

The above table shows us that in 187 rounds its decision helped increase rewards whereas in 135 rounds its decision gave no rewards. This results in about **58.07 %** times it made the correct decision.

Our results may not justify the need for confidence threshold in terms of rewards for the learning algorithm. However, we believe that besides the reasons stated in section 5.6 it is also needed for our online learning algorithm, which suffers from *cold start* and its learning path is erratic before it stabilizes.

6.3 Learning Algorithm

We now evaluate the learning algorithm with and without the skip feature.

6.3.1 Skipping Disabled

With skipping disabled the only way a student can move to the next topic is by understanding it or until all content items have failed to explain the student. This

could increase the number of rounds required by a student to complete a course.

The figure 6.2 shows the cumulative reward of the learning algorithm with respect to the omniscient policy. The reward for the omniscient policy increases linearly, whereas that of the learning algorithm is sub-optimal. This is expected as it does not have optimal arm parameters pre-configured and learns them in each round.

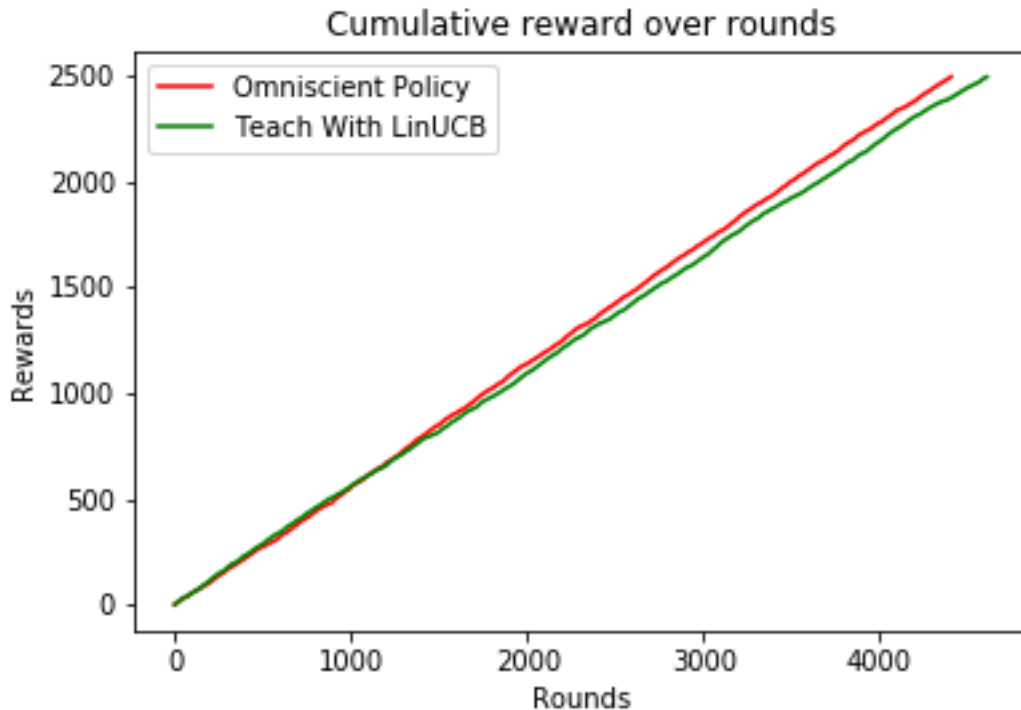


Figure 6.2: Compare Cumulative Rewards Without Skipping.

The omniscient policy required 4404 rounds to get a cumulative reward of 2494. This implies it needs 1.77 rounds for a reward (of 1). The learning algorithm required 4608 rounds to get a reward of 2492. This implies it needs 1.85 rounds for a reward (of 1). The cumulative reward graph shows that our learning algorithm is close to the optimal policy.

6.3.2 Skipping Enabled

If a topic is not understood by a student then skipping is enabled. This does not directly imply the student would be taken to the next topic. For it to happen, the skip classifier should be confident beyond the confidence threshold to predict that it would be better to take the student to the next topic.

Skipping tells the learning algorithm to skip sub-optimal content items and instead move to content items that have a higher estimated reward. This ensures that we do not present content items which are unlikely to help the student understand the topic. The figure 6.3 shows results of the learning algorithm with optimal confidence threshold $C = 10$ and $\alpha = 0.001$.

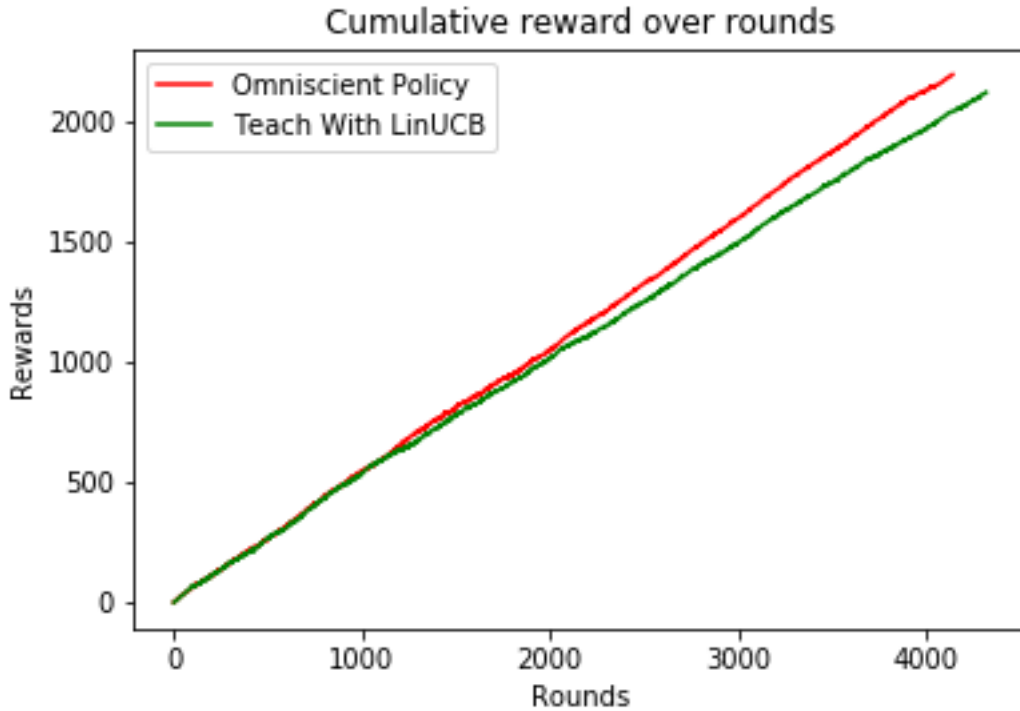


Figure 6.3: Cumulative Rewards With optimal hyper-parameters

The graph shows the performance of the learning algorithm with respect to the omniscient policy. The number of rounds and the cumulative reward reduce with skipping enabled. The cumulative reward reduces as there would be topics that the student did not understand and the skip classifier predicted with high confidence that it would be better to move to the next topic.

The omniscient policy required 4141 rounds to get a cumulative reward of 2200. This implies it needs 1.88 rounds for a reward (of 1). The learning algorithm required 4320 rounds to get a reward of 2126. This implies it needs 2.03 rounds for a reward (of 1).

Comparing the cumulative reward graph with and without skipping shows us that our learning algorithm performs better without skipping than with skipping.

We believe this happens because the skip classifier is learning online without any prior training.

Chapter 7

Conclusions

This project presents a student-centric approach to teaching. An approach, which could make classrooms more interactive by providing a personalized learning experience for students. We synthesized an unbiased dataset for our adaptive learning system, to represent heterogeneous student and content data to evaluate our learning algorithm. Since there were no benchmarks available, we created an omniscient policy which has optimal parameters pre-configured. We use these parameters to optimize the learning algorithm.

We then present a feature which would be useful when there are several different content items for a topic, to avoid students from getting frustrated with being stuck on a topic. This not only helps students but also helps teachers recognize topics students struggle with. We evaluated the learning algorithm to set a baseline for this new teaching methodology.

Our future work would involve creating an actual course that follows the teaching methods outlined in this project. This would give real student data, to evaluate the learning algorithms. We would also like to customize other algorithms to evaluate their performance against our baseline algorithm.

Bibliography

- [1] Naoki Abe and Atsuyoshi Nakamura. Learning to optimally schedule internet banner advertisements. In *ICML*, volume 99, pages 12–21, 1999.
- [2] Gediminas Adomavicius and Alexander Tuzhilin. Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions. *IEEE Transactions on Knowledge & Data Engineering*, (6):734–749, 2005.
- [3] Deepak Agarwal, Bee-Chung Chen, Pradheep Elango, Nitin Motgi, Seung-Taek Park, Raghu Ramakrishnan, Scott Roy, and Joe Zachariah. Online models for content optimization. In *Advances in Neural Information Processing Systems*, pages 17–24, 2009.
- [4] Peter Auer, Nicolo Cesa-Bianchi, and Paul Fischer. Finite-time analysis of the multiarmed bandit problem. *Machine learning*, 47(2-3):235–256, 2002.
- [5] Yoav Bergner, Stefan Droschler, Gerd Kortemeyer, Saif Rayyan, Daniel Seaton, and David E Pritchard. Model-based collaborative filtering analysis of student response data: Machine-learning item response theory. *International Educational Data Mining Society*, 2012.
- [6] Peter Brusilovsky and Christoph Peylo. Adaptive and intelligent web-based educational systems. *International Journal of Artificial Intelligence in Education (IJAIED)*, 13:159–172, 2003.
- [7] Sébastien Bubeck, Nicolo Cesa-Bianchi, et al. Regret analysis of stochastic and nonstochastic multi-armed bandit problems. *Foundations and Trends® in Machine Learning*, 5(1):1–122, 2012.
- [8] Hao Cen, Kenneth Koedinger, and Brian Junker. Learning factors analysis—a general method for cognitive model evaluation and improvement. In *International Conference on Intelligent Tutoring Systems*, pages 164–175. Springer, 2006.

- [9] Ankit Choudhary. Reinforcement learning guide: Solving the multi-armed bandit problem from scratch in python, September 24, 2018.
- [10] Benjamin Clement, Didier Roy, Pierre-Yves Oudeyer, and Manuel Lopes. Multi-armed bandits for intelligent tutoring systems. *arXiv preprint arXiv:1310.3174*, 2013.
- [11] Albert T Corbett and John R Anderson. Knowledge tracing: Modeling the acquisition of procedural knowledge. *User modeling and user-adapted interaction*, 4(4):253–278, 1994.
- [12] Yue Gong, Joseph E Beck, and Neil T Heffernan. Comparing knowledge tracing and performance factor analysis by using multiple model fitting procedures. In *International conference on intelligent tutoring systems*, pages 35–44. Springer, 2010.
- [13] Kristjan Greenewald, Ambuj Tewari, Susan Murphy, and Predag Klasnja. Action centered contextual bandits. In *Advances in neural information processing systems*, pages 5977–5985, 2017.
- [14] Kenneth R Koedinger, John R Anderson, William H Hadley, and Mary A Mark. Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education (IJAIED)*, 8:30–43, 1997.
- [15] Kenneth R Koedinger, Emma Brunskill, Ryan Sjd Baker, Elizabeth A McLaughlin, and John Stamper. New potentials for data-driven intelligent tutoring system development and optimization. *AI Magazine*, 34(3):27–41, 2013.
- [16] Andrew S Lan and Richard G Baraniuk. A contextual bandits framework for personalized learning action selection. In *EDM*, pages 424–429, 2016.
- [17] Tor Lattimore. The upper confidence bound algorithm, September 18, 2016.
- [18] Lihong Li, Wei Chu, John Langford, and Robert E Schapire. A contextual-bandit approach to personalized news article recommendation. In *Proceedings of the 19th international conference on World wide web*, pages 661–670. ACM, 2010.

- [19] Yun-En Liu, Travis Mandel, Emma Brunskill, and Zoran Popovic. Trading off scientific knowledge and user learning with multi-armed bandits. In *EDM*, pages 161–168, 2014.
- [20] Frederic M Lord. *Applications of item response theory to practical testing problems*. Routledge, 2012.
- [21] Travis Mandel, Yun-En Liu, Sergey Levine, Emma Brunskill, and Zoran Popovic. Offline policy evaluation across representations with applications to educational games. In *Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems*, pages 1077–1084. International Foundation for Autonomous Agents and Multiagent Systems, 2014.
- [22] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay. Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, 12:2825–2830, 2011.
- [23] Lou Pugliese. Adaptive learning systems: Surviving the storm, October 17, 2016.
- [24] Mark D Reckase. Multidimensional item response theory models. In *Multidimensional Item Response Theory*, pages 79–112. Springer, 2009.
- [25] Joseph Rollinson and Emma Brunskill. From predictive models to instructional policies. *International Educational Data Mining Society*, 2015.
- [26] David Silver. Exploration and exploitation.
- [27] Richard S Sutton and Andrew G Barto. *Introduction to reinforcement learning*, volume 135. MIT press Cambridge, 1998.
- [28] Ambuj Tewari and Susan A Murphy. From ads to interventions: Contextual bandits in mobile health. In *Mobile Health*, pages 495–517. Springer, 2017.
- [29] Kurt Vanlehn, Collin Lynch, Kay Schulze, Joel A Shapiro, Robert Shelby, Linwood Taylor, Don Treacy, Anders Weinstein, and Mary Wintersgill. The andes physics tutoring system: Lessons learned. *International Journal of Artificial Intelligence in Education*, 15(3):147–204, 2005.

- [30] Joannes Vermorel and Mehryar Mohri. Multi-armed bandit algorithms and empirical evaluation. In *European conference on machine learning*, pages 437–448. Springer, 2005.
- [31] Beverly Park Woolf. *Building intelligent interactive tutors: Student-centered strategies for revolutionizing e-learning*. Morgan Kaufmann, 2010.
- [32] Li Zhou. A survey on contextual multi-armed bandits. *arXiv preprint arXiv:1508.03326*, 2015.