Adaptive Teaching: Learning to Teach

by

Aazim Lakhani
Bachelor of Computer Engineering, Mumbai University, Mumbai, 2009

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Supervisory Committee

Dr. Nishant Mehta, Supervisor
(Department of Computer Science)

Dr. George Tzanetakis, Departmental Member
(Department of Computer Science)

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(Department of Computer Science)

Dr. George Tzanetakis, Departmental Member
(Department of Computer Science)

ABSTRACT

Traditional approaches to teaching were not designed to address individual students needs. We propose a new way of teaching one that personalizes the learning path for each student. We frame this use case as a contextual multi-armed bandit (CMAB) problem a sequential decision-making setting in which the agent must pull an arm based on context to maximize rewards. We customize a contextual bandit algorithm for adaptive teaching to present the best way to teach a topic based on contextual information about the student and the topic the student is trying to learn. To streamline learning, we add an additional feature which would allow our algorithm to skip topics that a student is unlikely to learn. We evaluate our algorithm over a synthesized unbiased heterogeneous dataset to show that our baseline learning algorithm can maximize rewards to achieve results similar to an omniscient policy.

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Almighty One for giving me the courage, belief and strength to pursue my ideas.

Mom for her blessings and prayers.

DEDICATION

I dedicate this project to my family, to whom I owe both the joy and pain of growing up.

Chapter 1

Introduction

The quest for a fully personalized learning experience began with the development of intelligent tutoring systems (ITSs) [6, 14, 29, 31]. However, to date, ITSs are primarily rules-based which requires domain experts to consider every possible learning scenario that students can encounter and then manually specify the corresponding learning actions in each case. This approach is not scalable since it is both labor-intensive and domain-specific [16].

Machine learning-based personalized learning systems have shown great promise in reaching beyond ITS to scale to large numbers of courses and students. These systems automatically create personalized learning actions for each individual student to maximize their learning. Examples of actions include reading a textbook section, watching a lecture video, interacting with a simulation or lab, solving a practice question, etc. Instead of domain-specific rules machine learning algorithms are used to select actions automatically by analyzing the data students generate as they interact with learning resources [16].

The goal of this project is to design a learning algorithm, which could adapt based on students feedback to help them learn effectively.

1.1 Use Case

There is no universal best way to explain a topic. The best way is subjective to every student. Unless we explore different ways to teach a topic, we cannot find a policy which would help map different students to explanations conducive for them. Once, we have such a policy we can use it to teach students effectively. **This is**

the exploration-exploitation dilemma in which we have to find a trade-off between two competing goals: maximizing students satisfaction in the long run, while exploring uncertainties in students preferences [3]. For example, an adaptive teaching system should present different explanations knowing students preferences on learning. However, unless we try different ways of teaching it is not possible to say with certainty whether or not an explanation would help a student learn effectively. We use the term adaptive teaching to avoid confusing it with adaptive learning used in machine learning literature. In the education domain, these terms are used interchangeably.

We represent this use case as a contextual bandits problem. We use contextual information about the student such as their preferences to learn through *visual, text, demo-based, practical, activity-based, step-by-step, lecture, audio-based explanations, as well as, self-evaluation and pre-assessment of students*. We also use contextual information about the contents used to teach a topic, by rating them in terms of *ease of understanding, simplicity, intuitiveness, depth in teaching, conciseness, thoroughness, ratings, abstractness, hands-on, experimental*. A **content item or arms are different actions or ways a topic can be taught**. The reward would be the students feedback to confirm their understanding of the topic they are trying to learn. The feedback can be through quizzes, interactions with a content item, tasks to name a few. **By pulling an arm, we obtain a reward drawn from some unknown distribution determined by the selected content item and the context. Our goal is to maximize the total cumulative reward**.

Let us make this more concrete by mapping this use case to teaching a class. In any school, a course comprises of multiple topics. However now instead of a single way to teach everyone, there would be multiple ways to teach. These different ways to teach are called content items. Students give their feedback on the presented content. Behind the scenes, our learning algorithm takes information about the student *(also referred to as student context)*, topic, content items*(also referred to as content context)* to find the best way to teach a student. This project extends the most cited contextual bandit learning algorithm, LinUCB (Linear Upper Confidence Bound) [18] to enhance it for our use case.

1.2 Motivation

The primary and perennial problem in education is the overwhelming challenge of teachers being responsible for accomplishing learning mastery among a demographically diverse set of students [23]. In traditional classrooms learning has largely remained a one-size-fits-all experience in which the teacher selects a single learning action for all students in their class regardless of their diversity in needs, prior knowledge, skills, learning styles, and backgrounds. It is not feasible for teachers to ensure their explanations can cater to all students. Hence there is a need for a system which could personalize teaching for students to help them learn effectively as well as increase course engagement and progression.

Such systems would be adaptive, recognize different levels of prior knowledge among students, as well as course progression based on students skill and feedback from learning. This could reduce teachers load to remediation to teaching and facilitating. These would adapt to individual students learning patterns instead of students having to adjust to the way of teaching. They would provide timely and comprehensive data-driven feedback to recognize potential challenges that students might come across as the course progresses.

1.3 Contribution

We suggest a novel baseline algorithm for our proposed adaptive teaching methodology which learns from students and contents for each topic to create a personalized learning path for each student. It adapts dynamically based on students feedback and learning preferences.

We also provide a skip feature which is meant to keep students engaged to increase student retention as well as provide feedback to teachers by recognizing the challenges faced by a student early in the course. Our online learning algorithm gives close to optimal results over a synthesized unbiased heterogeneous dataset.

1.4 Organization

Chapter 1 provided a brief overview of our use case along with the need for an adaptive teaching system and how this project contributes to realizing it. Chapter 2 introduces the technical concepts used to represent our use case along with the algorithm we customize for adaptive teaching. Chapter 3 describes prior work related to our use case using different approaches and how our work compares to them. Chapter 4 explains the algorithm created for adaptive teaching along with the skip feature. Chapter 5 describes the experimental setup which comprises the dataset synthesized to evaluate our algorithm. It also explains the evaluation strategy followed to examine our results. Chapter 6 presents the results of our experiments and compares its performance with respect to the best possible policy. Chapter 7 concludes this project by summarizing the contributions and outlines possible avenues for future work.

Chapter 2

Preliminaries

This chapter briefly explains the key concepts used in this project.

2.1 Multi-armed bandits

Multi-armed bandit is a problem setting where an agent needs to make a sequence of decisions in time $1, 2, ..., T$. At each time t the agent is given a set of K arms to choose and has to decide which arm to pull. After pulling an arm, it receives a reward for that arm, and the rewards of other arms are unknown. However, arm pulled does not change the state of the world. This problem setting can be stochastic or adversarial. In a stochastic setting the reward of an arm is sampled from some unknown distribution, and in an adversarial setting the reward of an arm is chosen by an adversary and is not necessarily sampled from any distribution [32]. In this project, we assume the problem setting as stochastic.

Personalized recommender systems recommend items (e.g., movies, news articles) to the users based on their predicted individual interests on these items. The users response helps the system improve their prediction [2]. However, the response to any particular item can only be available after these items are recommended. If an item is never shown to the users, the recommender systems cannot collect the response on these items. Such problems can be naturally modeled as a contextual bandit problem [30].

2.2 Contextual Bandits

In the theory of sequential decision-making, contextual bandit problems [28] occupy a middle ground between multi-armed bandit problems [7] and full-blown reinforcement learning (usually modeled using Markov decision processes along with discounted or average reward optimality criteria) [27]. Unlike bandit algorithms which cannot use any side-information or context, contextual bandit algorithms can learn to map the context into appropriate actions. However contextual bandits do not consider the impact of actions on the evolution of future contexts. Nevertheless, in many practical domains where the impact of the learners action on future contexts is limited contextual bandit algorithms have shown great promise. Examples include web advertising [1] and news article selection on web portals [18, 13].

Formally a contextual bandit problem is a repeated interaction which proceeds over T rounds. At each round $t = 1, 2, ...T$ the environment reveals contexts $x_t \in X$ which is used by the learner to pick an action $a_t \in A$ which gives a reward r_t revealed by the environment.The goal of the learner is to choose actions which would maximize cumulative reward $\sum_{t=1}^{T} r_t$.

We will now translate this problem setting for our adaptive teaching use case in which an algorithm A which proceeds in discrete rounds $t = 1, 2, 3, ...$ In round t:

1. The algorithm observes the student context x_s and a set A_t of content items together with their feature vectors x_c for $a \in A_t$. X_t encapsulates x_s and the context x_c of all content items available in round t.

2. Based on observed rewards in previous rounds, A chooses an arm $a_t \in A_t$. The arm a_t is estimated to have the highest expected reward. In a stochastic setting, the expected reward is given as the inner product of an unknown arm-dependent parameter $\theta_{a,t}$ and the context $x_{t,a}$, that is, $E[r_{t,a}|\mathbf{x_{t,a}}] = x_{t,a}^T \theta_{\mathbf{t,a}}$.

3. The student reveals the received reward r_t for arm a_t whose expectation depends on both the context X_t and the arm a_t.

4. The algorithm then improves its content item selection strategy with the new observation (x_t, a_t, r_t). It is important to emphasize here that no feedback namely, reward r_t is observed for unchosen arms $a \neq a_t$ [18].

2.3 Upper Confidence Bound (UCB)

A perpetual challenge in bandit algorithms is to find the right balance between exploration and exploitation (Section 1.1). Upper Confidence Bound (UCB) comprises of a family of algorithms which try to find the best trade-off between exploration and exploitation. It is based on the principle of *being optimistic* by choosing actions which have the highest potential for reward. The intuitive reason that this works is that when acting optimistically, one of two things happens. Either the optimism was justified, in which case the learner is acting optimally, or the optimism was not justified. In the latter case, the algorithm takes some action that they believed might give a reward when in fact it does not. If this happens sufficiently often, then the algorithm will learn the true reward of this action and not choose it in the future [17]. UCB algorithms estimate the expected reward for each arm by adding estimated sample mean of an arm with its upper confidence bound.

We will refer to Figure 2.1 as an example to understand UCB. Let us assume we have three arms a_1, a_2, a_3. The reward distribution for each arm after several rounds is a Gaussian distribution Q with mean μ and standard deviation σ. The y-axis is the probability of obtaining a certain reward for these arms.

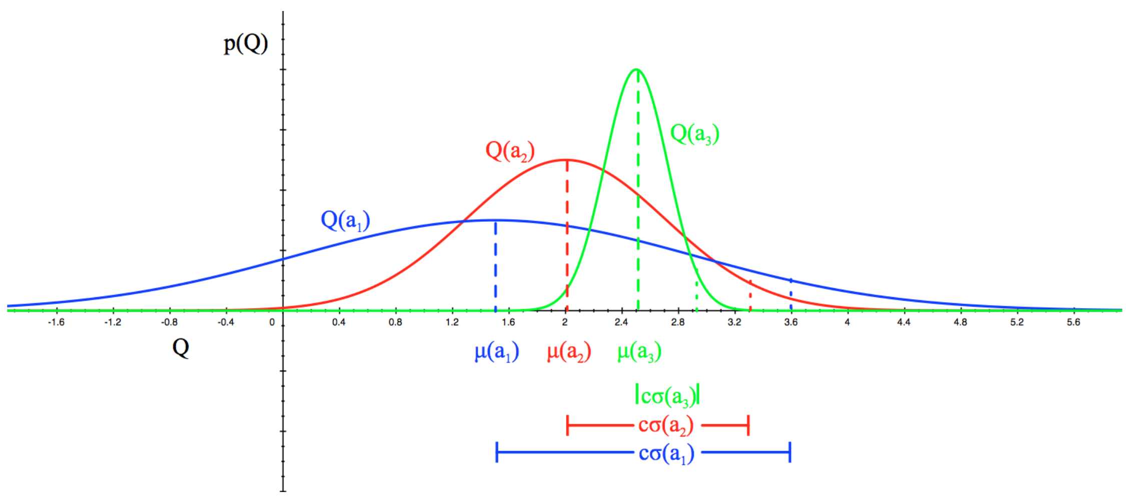


Figure 2.1: An example: UCB
[26]

The upper confidence bound for each arm is given by $c\sigma(a_i)$. The distribution shows that the sum of the expected mean and upper confidence bound is highest for a_1. Hence the UCB algorithm will select a_1. The reward received will reduce uncertainty around a_1. So for the next round, the algorithm once again finds the arm with the highest sum for the expected mean and upper confidence bound. This is repeated for T rounds [9].

2.4 Linear Upper Confidence Bound (LinUCB)

LinUCB is a way to apply UCB to a more general contextual bandits setting where the UCB of each arm is computed efficiently by assuming the reward is linear, given as $E[r_{t,a}|\mathbf{x_{t,a}}] = x_{t,a}^T\theta_{\mathbf{t,a}}$. The estimated expected mean is parameterized over the context x_a for each arm a. At round t this is given as $\widehat{\theta}_a^T x_{t,a}$. The upper confidence bound around each arm a at round t is given as $\sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$. Here, A_a is the co-variance over the context data $x_{t,a}$ for each arm a at round t.

LinUCB introduces a hyper-parameter α, which allows us to control exploration over arms. This is achieved by scaling the upper confidence bound by α. A higher value of α encourages exploration. As a result, the algorithm would need more rounds to explore before it begins exploiting. We can now compute **the expected estimated reward for an arm a at round t as** $p_{t,a} = \widehat{\theta}_a^T x_{t,a} + \alpha\sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$ [18].

Chapter 3

Related Work

Our use case could also be formulated using the partially observed Markov decision process (POMDP) framework. POMDPs model the student's latent knowledge states and their transitions to learn an action selection policy that maximizes reward received in the possibly distant future (long-term learning outcome). Previous work applying POMDPs to personalized learning has achieved some degree of success. However, learning a personalized learning schedule using a POMDP is greatly complicated by the curse of dimensionality. The solution quickly becomes intractable as the dimensions of the state and action spaces grow. Consequently, POMDPs have made only a limited impact in large-scale personalized learning applications involving large numbers of students and learning actions [16].

A more scalable approach to personalized learning is to learn a policy, which maps contexts to actions using the multi-armed bandits (MAB) framework, which is more suitable for optimizing students success. The simplicity of the MAB framework makes it more practical than the POMDP framework in real-world educational applications [16].

The work in [19] applies a MAB algorithm to educational games to find trade-off between exploring learning resources to accurately estimate arm means, while also trying to maximize users test performance. Their approach is context-free and does not consider diversity among individual users. The work in [21] collects high-dimensional student - computer interaction features as they play an educational game and uses them to search for a good teaching policy [16].

The work in [16] is focused on adaptive testing to assess a students performance. They use contextual MAB to find questions to assess a student. The question depends on a student's response to earlier questions. At each round, they have all questions to assess a student. Contrary to that we only have a restricted set of content items available at each round. Our use case is focused on adaptive teaching to enable students to learn.

The works in [10, 15] both use a form of expert knowledge to learn a teaching policy. The approach of [10], in particular, uses expert knowledge to narrow down the set of possible actions a student can take. Our approach, in contrast, requires no expert knowledge and is fully data-driven [16].

The work in [25] found that various student response models, including knowledge tracing (KT) [11], Item Response Theory (IRT) models [20, 24, 5], additive factor models (AFM) [8], and performance factor models (PFM) [12] can have similar predictive performance yet lead to very different teaching policies. [16]. While these results are interesting, we emphasize that the focus of the current work is to develop policy learning algorithms rather than comparing student models.

Chapter 4

Algorithm

This chapter presents the algorithm created for the adaptive teaching system. We first present the basic version of the algorithm (Section 4.1). We then explain the skip feature (Section 4.2), which could streamline learning.

The algorithm used is an extension of upper confidence bound (UCB)-based algorithms [4] (Section 2.3). These algorithms maintain estimates of the expected reward of each arm together with confidence bound around it. It then pulls the arm with the highest expected reward which is equal to the sample mean plus the confidence bound. Based on the observed reward it optimizes the arm parameters iteratively after each pull. In this project we are using the most cited contextual bandit algorithm, namely LinUCB (Section 2.4).

Before we dive in it is important to note, that to better understand the algorithm I have divided the explanation into two halves. *The first half explains the overall flow without skipping whereas the second explains in-depth the function calls made in the first half along with skipping.* I would be using bandit terminology to explain. **Arm** refers to a content item. **Payoff** refers to the estimated reward computed by the algorithm. A **round** comprises of computing the expected payoff for each content item; then presenting a content item with maximum expected payoff and getting student feedback for the content item.

estimated, or actual?

Before we proceed to the algorithm below are the notations you will come across.

Symbol	Meaning
α	Parameter to scale Confidence bound.
C	Confidence threshold to skip.
$\mathbf{x_s}$	Student context vector.
x_c	Content items context.
$\mathbf{x_t}$	Context vector at round t.
X_t / X_t^i	Context at round t. It combines $\mathbf{x_s}$ and all available x_c for topic i.
X_t^{i+1}	Context at round t. It combines $\mathbf{x_s}$ and all available x_c^{i+1} for topic $i+1$.
x_c^{i+1}	Content items contexts for topic $i+1$.
a	An arm a for topic i.
a'	An arm a' for topic $i+1$.
A_t	Arms available at round t.
$A_{t'}^{i+1}$	Arms available for topic $i+1$ at round t'.
a_t^{i+1}	Arm a for topic $i+1$ at round t.
t	Current round t.
t'	Possible next round t'.
T	Total number of rounds.
i	Topic being taught.
$i+1$	Next Topic in the sequence.
$p_{t,a}$	Expected payoff from arm a at round t.
$p_{t,a}^i$	Expected payoff from arm a at round t for topic i.
$p_{t',a'}^{i+1}$	Expected payoff from arm a' at round t' for next topic $i+1$.
X	Input features for skip classifier.
Y	Label to train the skip classifier.

I thought you do not limit this; instead, you let it be variable according to how many rounds it takes the algorithm to finish, right?

Table 4.1: Algorithmic notations

Note

- We are always on the current topic i, unless we explicitly specify next topic $i+1$.
- All vectors are **bold** faced lower cased. what about x_c ?
- All sets are upper cased.

Algorithm 1 Teach with LinUCB

Hyper Parameters : $\alpha \in \mathbb{R}_+$
C : Confidence threshold to skip
Inputs : Student context $\mathbf{x_s}$ and content context x_c of available arms $a \in A_t$ for topic i at round t
Prepare context $X_t = \begin{pmatrix} \mathbf{x_s} \\ x_c \end{pmatrix}$
skip-enabled $\leftarrow$ False
while $A_t \neq \emptyset$ **do**
    a_t^i , $p_{t,a}^i \leftarrow$ EXPECTED-PAYOFF(X_t,A_t)
    skip-decision , $p_{t',a'}^{i+1} \leftarrow$ SKIPTOPIC($\mathbf{x_s}$, $p_{t,a}^i$, i)
    if skip-decision , skip-enabled is True **then**   if skip-decision what?
        Move to next topic $i \leftarrow i + 1$
        break
    else
        Pull arm a_t and observe reward r_t
        $A_{a_t} \leftarrow A_{a_t} + \mathbf{x_{t,a_t}} \mathbf{x_{t,a_t}^T}$
        $\boldsymbol{b_{a_t}} \leftarrow \boldsymbol{b_{a_t}} + r_t \mathbf{x_{t,a_t}}$
        label $\leftarrow$ SETLABEL(r_t)
        TRAIN($\mathbf{x_s}$, $p_{t,a}^i$, $p_{t',a'}^{i+1}$,label)
        $t \leftarrow t + 1$
    if $r_t \neq 1$ **then**
        Remove $a_t \in A_t$
        skip-enabled $\leftarrow$ True
    else
        Move to next topic : $i \leftarrow i + 1$
        break

4.1 Basic Version

The basic version is without skipping. It explains the main flow of the algorithm. The next section explains the functions used along with skipping.

The algorithm requires two hyper-parameters to be configured. The first one is α which scales the confidence bound (Section 2.4). The second hyper-parameter is the confidence threshold C which decides confidence threshold that must be exceeded to skip a topic. Skipping is a feature to help reduce students who are unlikely to learn from content items available for a topic. It is meant to streamline learning. This could also be used by teachers to recognize topics that should be addressed in class.

We now explain how LinUCB (Section 2.4) helps the algorithm decide an arm to pull. Before we recommend a content item to a student, we need to prepare context X_t for the round t. It is prepared by combining the student context $\mathbf{x_s}$ with content items context x_c for the topic i being taught. With the context X_t and arms A_t, we use LinUCB to compute the expected payoff from each arm and return the arm a_t^i with the maximum expected payoff $p_{t,a}^i$ which must be pulled for topic i at round t.

Assuming the classifier does not recommend skipping, a student is presented with the content item a_t for topic i. After being taught the student sends a reward r_t to complete the round t. Now the round t is complete we update the arm parameters A_{a_t} , $\mathbf{b_{a_t}}$ of the arm pulled. We then use this reward r_t to train the skip classifier to make better predictions in upcoming rounds. The features for the classifier comprise of students contextual information $\mathbf{x_s}$, expected payoff $p_{t,a}^i$ from the current topic i and the expected payoff $p_{t',a'}^{i+1}$ for the topic $i+1$.

If no reward r_t was sent by the student $\mathbf{x_s}$, then it implies the student was unable to understand topic i. In which case the algorithm removes the presented arm a_t and remains on the same topic i. However if a reward r_t was sent, then the student is moved to the next topic $i+1$. That completes the first half. The second half explains the functions briefly described above.

4.2 With Skipping

On line 6 (Section 4.1) of the algorithm we get the expected payoff $p_{t,a}^i$ estimated on pulling the arm a_t^i for the current topic i. Now to decide whether or not it should pull the arm or move to the next topic it calls the skip topic function.

The *SKIPTOPIC* function takes the student context $\mathbf{x_s}$, the expected payoff $p_{t,a}^i$

function SKIPTOPIC(x_s, $p^i_{t,a}$, i)
    Get next topic $i+1$ from topic i
    Get arms $A^{i+1}_{t'}$ and content context x^{i+1}_c for topic $i+1$
    Prepare context vector $X^{i+1}_{t'} = \begin{pmatrix} x_s \\ x^{i+1}_c \end{pmatrix}$
    $a^{i+1}_{t'}$, $p^{i+1}_{t',a'} \leftarrow$ EXPECTED-PAYOFF($X^{i+1}_{t'}$,$A^{i+1}_{t'}$)
    skip-decision $\leftarrow$ PREDICT(x_s, $p^i_{t,a}$, $p^{i+1}_{t',a'}$) to decide on skip
    return skip-decision , $p^{i+1}_{t',a'}$
function EXPECTED-PAYOFF(X_t,A_t)

weird name for this function, considering this function returns the arm with the highest UCB and the UCB for that arm, as opposed to the arm with the highest sample mean

    for $a \in A_t$ **do**
        Get $x_{t,a} \in X_t$
        if a is new **then**
            $A_a \leftarrow I_d$ (d-dimensional identify matrix)
            $\boldsymbol{b_a} \leftarrow 0_{d\times 1}$ (d-dimensional zero vector)
        $\widehat{\theta}_a \leftarrow A_a^{-1} b_a$
        $p_{t,a} \leftarrow \widehat{\theta}_a^T x_{t,a} + \alpha\sqrt{x^T_{t,a} A_a^{-1} x_{t,a}}$

mismatch

    Choose arm $a_t = \arg\max_{a\in A_t} p_{t,a}$ with ties broken arbitrarily
    return $a, argmaxp_{t,a},$
function PREDICT(x_s, $p^i_{t,a}$, $p^{i+1}_{t',a'}$)
    X $\leftarrow$ x_s ,i+1 , $p^i_{t,a}$, $p^{i+1}_{t',a'}$
    Y , confidence-score $\leftarrow$ Prediction from classifier
    if confidence-score <C **then**
        decision $\leftarrow$ 0
    return decision , confidence-score
function TRAIN(x_s, $p^i_{t,a}$, $p^{i+1}_{t',a'}$, label)
    $X \leftarrow x_s, p^i_{t,a}, p^{i+1}_{t',a'}$, topic ,
    $Y \leftarrow$ label
    Train online SGD classifier
function SETLABEL(r_t)
    if r_t is 0 **then**
        label $\leftarrow$ 1
    else
        label $\leftarrow$ 0
    return label

for pulling arm a at round t for topic i and the current topic i. It uses the topic i to get a reference to the next topic $i+1$. Through the topic $i+1$ it gets content items $A_{t'}^{i+1}$ and context data x_c^{i+1} associated those content items. After combining the contexts to prepare $X_{t'}^{i+1}$ it gets the maximum expected payoff $p_{t',a'}^{i+1}$ and the arm $a_{t'}^{i+1}$ to pull by passing the context vector $X_{t'}^{i+1}$ and arms available for next topic $A_{t'}^{i+1}$. The expected payoff function returns an arm with the maximum estimated payoff. Skip topic function then calls the skip classifier to predict a skip-decision for the student context $\mathbf{x_s}$, along with the expected payoff from the current and the next topic to make a prediction.

The *EXPECTED-PAY0FF* function takes the context X_t, along with the arms A_t available at round t. After an arm a_t is initialized with parameters A_a, b_a they are used to calculate the expected mean $\widehat{\theta}_a^T x_{t,a}$ and confidence bound $\sqrt{x_{t,a}^T A_a^{-1} x_{t,a}}$ for the arm. The confidence bound is scaled by α. The expected mean and the scaled confidence bound are added to give the expected payoff $p_{t,a}$ for arm a at round t. It then finds an arm a with maximum expected payoff $p_{t,a}$ and returns the expected payoff along with the arm a to be pulled.

The *PREDICT* function is used to predict whether the student should be moved to the next topic $i+1$ or should remain on the same topic i. It combines student context vector $\mathbf{x_s}$, the expected payoff $p_{t,a}^i$ for the current topic i and the expected payoff $p_{t',a'}^{i+1}$ for the next topic $i+1$ to prepare a feature vector X. Its then asks a prediction from the binary supervised online Support Vector classifier with hinge loss to make a prediction Y and a confidence-score for its prediction. If the confidence-score is less than the confidence threshold, then set the *decision* variable is set to 0 which implies no skipping. This is because a confidence score lower than the threshold implies that the classifier is not sufficiently confident about its prediction.

The *TRAIN* function is used to train the skip classifier to make better predictions. Similar to the predict function it combines student context vector $\mathbf{x_s}$, the expected payoff $p_{t,a}^i$ for the current topic i and the expected payoff $p_{t',a'}^{i+1}$ for the next topic $i+1$ to prepare a feature vector X. It sets the *label* to the output Y. Together they train the skip classifier.

The *SETLABEL* function is used to set the *label* to train the skip classifier. If the reward r_t for round t is set to 0 then the *label* is set to 1. This implies that since staying on the same topic did not give any reward, it would have been better to skip. If the reward r_t for round t is set to 1, then the *label* is set to 0. This implies that staying on the same topic was a good decision. The set *label* is then returned.

Chapter 5

Experiments

This chapter explains the dataset (Section 5.1) used to evaluate the learning algorithm. It then describes the environmental setup (Section 5.2) used for these experiments. The next section explains how we evaluate our algorithm (Section 5.3) in absence of pre-existing benchmarks using an omniscient policy (Section 5.4). This is followed by sections which explain how the learning algorithm (Section 5.5) and the skip feature (Section 5.6) work in these experiments.

5.1 Dataset

Machine learning algorithms are data-driven. Due to the novelty of our approach to the best of our knowledge, there is no similar dataset available. Hence **we synthesized datasets to represent data generated by students taking courses in an adaptive teaching environment.**

An honest attempt is made to synthesize an unbiased dataset representative of the heterogeneous students and content items. Biased datasets tend focus on targeted student groups (for instance, having many students who give positive feedback). This could result in higher rewards. Contrary to this, our dataset is representative of diverse student and content data and is not skewed towards a particular student group or content type.

The contextual data is created from a uniform distribution U(0,1] sampled randomly to simulate the diverse nature of student preferences and content features.

5.1.1 Courses

We use the following courses for our experiments.

1. *Course 1 :* A course which comprises of 10 topics. Its taken by 50 students. There are a total of 119 content items for 10 topics. So on an average, there are 12 content items per topic. We use this course to find optimal hyper-parameters (α and C) for our learning algorithm.

2. *Course 2 :* A course which has 25 topics. Its taken by 100 students. There are 329 content items for 25 topics. So on an average, there are 13 content items per topic. We use this course for evaluation.

5.1.2 Context

We will assume there was a survey conducted among students who were asked how should teaching streamline learning? Students gave their preferences on a scale of 1 to 10 with 1 being least preferred and 10 being most preferred. These preferences were normalized.

Research has shown that students prefer to learn a certain way. Though there is no unanimous consensus, there is a fair bit of research and understanding on the needs of a student. The features we consider are by no means exhaustive but a representative subset of the main features. The tables 5.1 and 5.2 describe the student and content context used for these experiments.

Apart from the above contextual data, there is a course which is taught. For our experiments, we consider a typical course which comprises of topics to be taught. These topics are labeled as *T_1, T_2 ... T_25*. For e.g: T_1 refers to the first topic of the course. Each topic has between 5 to 20 different content items. Each content is labeled in the format *C_topic-id_content-number*. For e.g: C_1_2 refers to the second content item for topic T_1.

Not true. you haven't yet described the context information for content items

We now have the required contextual information. Topics in the course are taught in a sequence outlined by the teacher. This allows them to control the course sequence. Let us take an example to understand the data.

what is a vlog? I've heard thi phrase maybe once in my life. Perhaps add a footnote explaining

Student Context	**Description**
Visual (S_V)	How much preference is given to visual explanations (video, short-film, movie-clip, vlogs)?
Text (S_T)	How much preference is given to written explanations (books, articles, blogs, research papers)?
Demo-based (S_D)	How much preference is given to live experiments to help understand a concept?
Practical (S_P)	How much preference is given to an explanation, followed by a demo of the topic, and enabling students to perform it?
Step-by-step (S_S)	How much preference is given to a guide to practice, try and understand a topic in a systematic way?
Activity / Task-based (S_AT)	How much preference is given to content items which are interactive and require students to participate?
Lecture (S_L)	How much preference is given to being passive and listen to an expert explain the topic?
Audio (S_A)	How much preference is given to audio explanations (podcasts, music)?
Self-evaluation (S_SE)	Students self-evaluate their readiness, motivation, excitement for the course.
Pre-assessment (S_PA)	Teachers conduct a pre-assessment of the pre-requisites required for the course.

Table 5.1: Student context

Below (Figure 5.1) is a student context data point which shows a student preference. It tells us that this student prefers visual (S_V), text(S_T), demo-based(S_D) methods of learning, but does not prefer practical (S_P), activity-based(S_AT), and did not fare well in the pre-assessment(S_PA). The student does not mind step-by-step(S_S), lectures(S_L) , audios(S_A) methods of learning and believes he/she is ready for the course (S_SE).

for the final version can you make a good version of this table? it looks like you took a low-quality image and enlarged it, so it looks bad

S_V	S_T	S_D	S_P	S_S	S_AT	S_L	S_A	S_SE	S_PA
0.87	0.82	0.88	0.36	0.6	0.06	0.66	0.56	0.66	0.07

Figure 5.1: Student context template

A lot of these are vague and similar to each other. For instance, isn't C_A just the opposite of C_P? And, C_E is heavily subjective, depending on the student, so it isn't a good content context feature. Also, C_ID implies C_P. I don't know much about this area, but frankly these features don't seem to be very good.

Content Context	**Description**
Ease of understanding (C_E)	How relatively easy is it to understand the content?
Simple / Intuition (C_I)	Does it provide a surface level or deep understanding of the topic?
Surface / In-depth (C_ID)	How much preference is given to live experiments to help understand a concept?
Brief / Concise (C_C)	Is it short, to the point or descriptive, verbose and elaborative, keeping in mind that learners have different levels of concentration and capacity to remember?
Thorough (C_T)	How well does the content item cover the topic?
Preference / Well reviewed / Well rated (C_R)	How well rated is the explanation?
Theoretical / Abstract (C_A)	How theoretical or abstract is the content item?
Practical / Hands on (C_P)	Is it something that can be tried or experienced?
Experimental / Task-based (C_ETB)	Does it require a task to be completed to fully understand it, like collaboration with other students or some research/findings?

Table 5.2: Content context

Below is a content context data point prepared for the course. This content item is thorough(C_T), practical(C_P), and experimentally sound(C_ETB), but not in-depth(C_ID),concise(C_C), and abstract(C_A). Its moderate in terms of understanding(C_E), intuitiveness(C_I) and has positive reviews(C_R).

same comment as in Figure 5.1 – make table of the same form / content so the figure doesn't look bad

	C_E	C_I	C_ID	C_C	C_T	C_R	C_A	C_P	C_ETB
C_1_1	0.45	0.72	0.31	0.05	0.91	0.75	0.06	0.88	0.97

Figure 5.2: Content context template

5.2 Environment

We run a simulation of a course being taken by students with the omniscient policy and the learning algorithm deciding the content item to be presented for each student.

It is an environment where several students are taking the course at the same time. Both the omniscient policy and the learning algorithm work in online mode. The learning algorithm ~~optimizes~~ updates its parameters in each round to give better predictions.

5.3 Evaluation Strategy

Since there are no readily available benchmarks to compare our algorithm, we assume there exist an omniscient policy. This policy has optimal parameters to recommend the best arm to pull.

We run the same course with an omniscient policy and the learning algorithm to evaluate our learning algorithm relative to the omniscient policy. The evaluation is conducted with and without skipping. Due to the stochastic nature of students feedback both the omniscient policy and the learning algorithm will run for a different number of rounds. However, the total cumulative reward available is the same for both of them. Hence we evaluate them based on cumulative reward accumulated over all rounds.

as per the comment I made earlier, this is backwards. The probability of success is defined first. Then it just so happens that the omniscient policy obtains maximum expected reward equal to this probability

We simulate the student feedback as a Bernoulli distribution. Here, the probability of success is the maximum expected reward computed by the omniscient policy. This is passed to the Bernoulli distribution as the probability of reward for the presented content item. Based on the reward received by the learning algorithm the arm parameters are updated to make better decisions in the upcoming rounds. This experiment aims to find how well does our algorithm optimize the arm parameters to match the omniscient policy.

5.4 Omniscient Policy

This policy knows all the probability distributions. It knows every step of the way the best decision ~~based~~ because it knows the true distributions. It does not have to learn anything. It has optimal parameters θ^* for each arm. Hence it is expected to maximize the cumulative reward.

You should move this to the third paragraph of Section 5.3, where you explain what the mean success probability of an arm is, given the context

This policy calculates the expected payoff for each arm a available for a topic. It then selects the arm which has maximum expected payoff. The expected payoff for an arm a with optimal parameters $\theta^*_{t,a}$ and with context vector $x_{t,a}$ at round t is given by $E[r_{t,a}|\mathbf{x_{t,a}}] = x^T_{t,a}\theta^*_{\mathbf{t,a}}$.

5.5 Learning Algorithm

The learning algorithm can adapt to several students at the same time to present a content item personalized for each student. For every topic, a student is trying to learn it gets the expected payoff for all available content items. It checks whether it should skip to next topic or remain on the current topic. Skipping is activated only if the student gave no reward for a content item presented for the topic.

When a student is on a topic, the algorithm presents a content item that could maximize rewards. After working through the content item, the student shares feedback on the content item. If a reward is sent, then this implies that the student understood the concept and can be taken to the next topic. If no reward was sent, then the student may be presented with the next best content item for the same topic or could be moved to the next topic in the course sequence.

Once the student has shared feedback on the content item, the data is sent to train the skip classifier to make a better prediction in forthcoming rounds.

5.6 Skip Topics

The learning algorithm checks with skip topics to decide whether or not the content item should be presented for the current topic. Skip topic predicts this by using the student context along with the estimated payoff for the current topic and the estimated payoff for the next topic in the course sequence.

It makes this decision using an online supervised learning stochastic gradient descent classifier with student context along with the estimated payoff of the current and next topic to make a decision. The label for the classifier depends on the reward received for the topic. If a reward was sent then the label is set to 0 or else it is set to 1. Thus the classifier makes use of the feedback sent by all students to recognize common topics and content items that students find difficult so it could make a confident decision.

The aim to create skip topics feature is to streamline learning for students. If a student has been taught a topic once and was not satisfied with it, then there is the option to skip to the next topic or explain the same topic with a different content item.

The skip classifier is a linear Support Vector Machine estimator with hinge loss. The estimator is a regularized linear model with stochastic gradient descent (SGD) learning. The gradient of the loss is estimated, each sample at a time and the model is updated along the way with a decreasing learning rate. The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using squared Euclidean norm ~~L2~~ [22].

Chapter 6

Results and Evaluation

This chapter presents results using the experimental set-up given in the previous chapter (Chapter 5). We evaluate the learning algorithm with respect to the omniscient policy. Before evaluation we need to first find optimal values for hyper-parameters α (Section 6.1) and confidence threshold C (Section 6.2). We then proceed to use these optimal values to evaluate the learning algorithm with and without the skip feature. (Section 6.3). Up until now, we did not penalize content items which gave no rewards. However, in section 6.4 we penalize such content items.

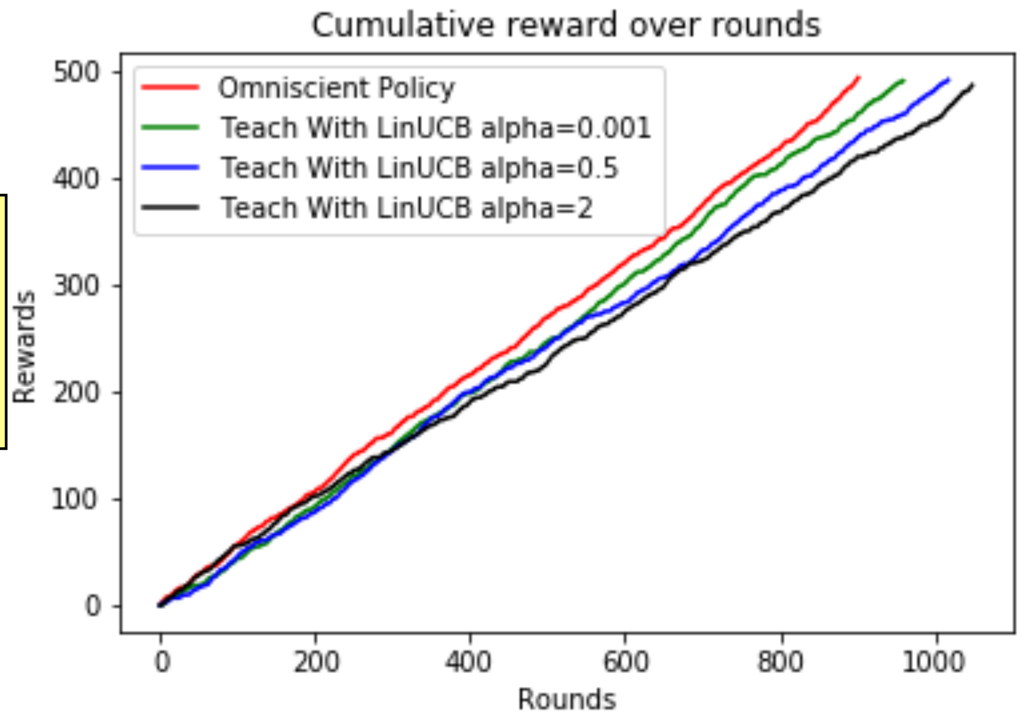
6.1 Confidence Bound α

Finding an optimal value for α is important to learn faster as it scales the confidence bound of each content item. An optimal value would find the right balance between exploration and exploitation. A higher value of α would imply the learning algorithm takes more rounds exploring which can lead to sub-optimal results.

This parameter is configured for the learning algorithm and not the omniscient policy. We empirically evaluated an optimal value for α using course 1 (Section: 5.1.1). The graph (Figure 6.1) shows the cumulative reward for different values of α.

I've been thinking about the best type of figure to show here. I do not think this kind of figure is that helpful. Here is an idea. Switch the axes, so that reward is the x-axis and round # is the y-axis. Now, for each amount of cumulative reward, you can plot the ratio of rounds/reward. If near the end Teach with LinUCB is performing optimally, then near the end we should see its rounds/reward converging toward the rounds/reward of Omniscient. I really think this is the right kind of plot, and much needed. I'd want to see that before "signing off". If it does look good, then we should have that plot for Figure 6.1, 6.2, and 6.3. Maybe it will take time, but finally there will be a figure that makes sense. In fact, what you can do is plot the *difference* between Omniscient and each other algorithm. So, finally, there will be some notion of regret!

For the final version, please make PDF versions of all figures; in Python you can save figures directly to PDF, making them vector graphs, so they don't look bad (your figures look bad when zooming)



Is each curve a plot from a single run? To get a more meaningful measure of reward/round, you should repeat the experiment several times per algorithm configuration

Figure 6.1: Cumulative rewards per α.

The graph compares the omniscient policy with the learning algorithm for different values of α. It shows that the **cumulative reward needed 960 rounds to maximize rewards when** $\alpha = \mathbf{0.001}$ for the learning algorithm compared to 1018 required by $\alpha = 0.5$ and 1049 required by $\alpha = 2$. On running repeated experiments, we found that a value of α between 0 to 0.5 give better results. We would be using $\alpha = 0.001$ to evaluate the learning algorithm.

The below table shows the content items explored for different values of α.

what happens for alpha = 0? The problem is probably too easy if alpha = 0 (no exploration) works well

Values of α	**Content Items Explored**
0.001	64
0.5	79
2.0	81

you need to say what you mean by "content items explored"

Table 6.1: Content items explored per α

.

6.2 Confidence Threshold (C)

This is a threshold on the confidence score the skip classifier should exceed for its prediction to be accepted. Skipping is enabled for a topic only after a student gives no reward to a content item. The threshold helps:

- To keep students engaged by skipping topics they are unable to understand.
- Give teachers control on their preference to skipping.
- Allow the learning algorithm to skip content items that are less likely to give rewards.

We do not want the confidence threshold to be too high as students might have to go through each content item nor do we want it to be too low such that students are taken to the next topic on the first occurrence of not understanding a topic. Hence finding an optimal value for the confidence threshold is important to have a good learning experience.

We evaluate the performance for different values of the confidence threshold over course 1 (Section: 5.1.1). Below are the results.

6.2.1 Without confidence threshold

We evaluated the skip classifier with no confidence threshold. Below is a confusion matrix of the results.

This table is hard to understand because we really care about the percentages. Please just show the four percentages, and if you want to show the raw numbers, do so in a different table. I do not think the table with raw numbers is interesting though

		Predictions		
		Stay (0)	Skip (1)	Total
Reward	0	70	51	121
	1	90	68	158
	Total	160	119	558

Table 6.2: Confusion matrix without confidence threshold

The classifier is evaluated on how well it helps the learning algorithm maximize cumulative reward. It shows that in 158 rounds its decision helped increase rewards whereas in 121 rounds its decision gave no rewards. This shows us that about **56.63%** times its decision helped increase the cumulative reward.

6.2.2 With confidence threshold

We evaluate the skip classifier with confidence threshold. We will only consider data points where the skip classifiers decision was overruled as its confidence score was below the threshold. This would be when the skip classifier had predicted skipping to the next topic, but since the confidence score was below the threshold, the prediction was ignored. This gives us the true measure of the effectiveness of the confidence threshold.

We evaluated the classifier for different values of confidence threshold. For different threshold values performance ranged consistently between 56 - 60 %. We found the skip classifier performed most optimally when the confidence threshold is 10. The below table 6.3 shows the results.

Again, percentages

		Predictions		
		Stay (0)	Skip (1)	Total
Reward	0	28	37	65
	1	28	60	98
	Total	56	97	316

Table 6.3: Confusion matrix with confidence threshold of 10

The above table shows us that in 98 rounds its decision helped increase rewards whereas in 65 rounds its decision gave no rewards. This results in about **60.12%** times it made the correct decision.

As the value of the confidence threshold was increased the number of skips decreased. Table 6.4 shows the results for confidence threshold of 30.

		Predictions		
		Stay (0)	Skip (1)	Total
Reward	0	81	9	90
	1	111	19	130
	Total	192	28	440

Table 6.4: Confusion matrix with confidence threshold of 30

The above table shows us that in 130 rounds its decision helped increase rewards whereas in 90 rounds its decision gave no rewards. This results in about **59.09 %** times it made the correct decision.

6.3 Learning Algorithm

We now evaluate the learning algorithm with and without the skip feature.

6.3.1 No Skipping

With skipping disabled the only way a student can move to the next topic is by understanding it or until all content items have failed to explain the student. This could increase the number of rounds required by a student to complete a course.

The figure 6.5 shows the cumulative reward of the learning algorithm with respect to the omniscient policy. The reward for the omniscient policy increases linearly, whereas that of the learning algorithm is similar to the optimal policy. This is expected as it does not have optimal arm parameters pre-configured and learns them in each round.

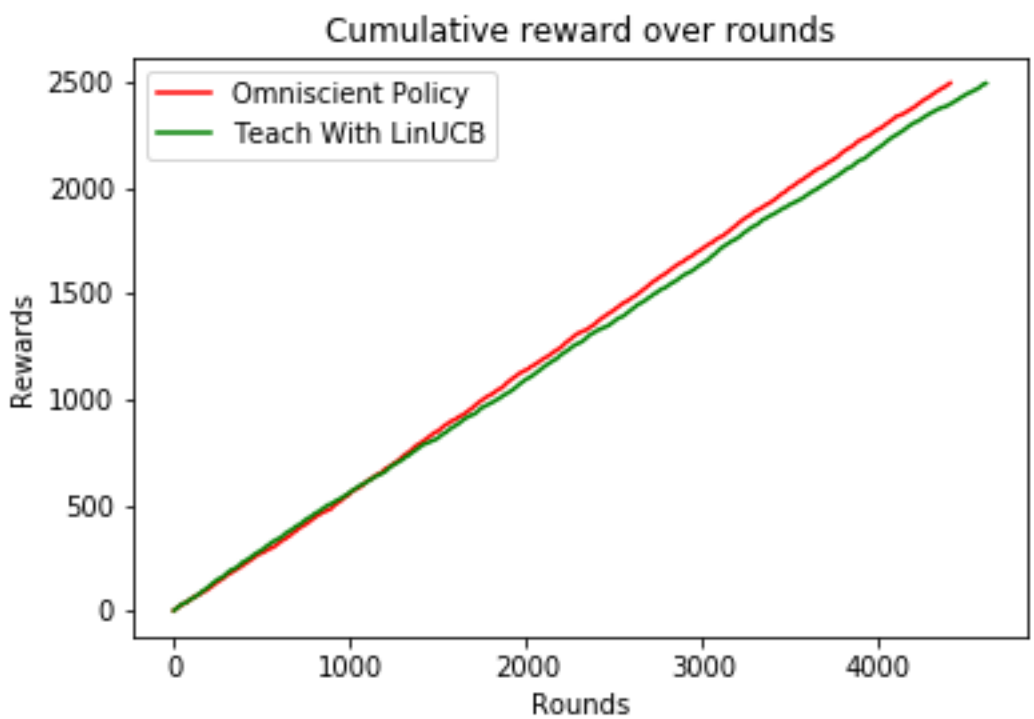


Figure 6.2: Cumulative rewards without skipping.

The omniscient policy required 4410 rounds to get a cumulative reward of 2490. This implies it needs 1.77 rounds for a reward (of 1). The learning algorithm required 4688 rounds to get a reward of 2491. This implies it needs 1.88 rounds for a reward (of 1). The cumulative reward graph shows that our learning algorithm is close to the optimal policy.

6.3.2 With Skipping

If a topic is not understood by a student then skipping is enabled. This does not directly imply the student would be taken to the next topic. For it to happen, the skip classifier should be confident beyond the confidence threshold to predict that it would be better to take the student to the next topic.

Skipping tells the learning algorithm to skip sub-optimal content items and instead move to content items that have a higher estimated reward. This ensures that we do not present content items which are unlikely to help a student understand the topic. The figure 6.6 shows results of the learning algorithm with optimal confidence threshold $C = 30$ and $\alpha = 0.001$.

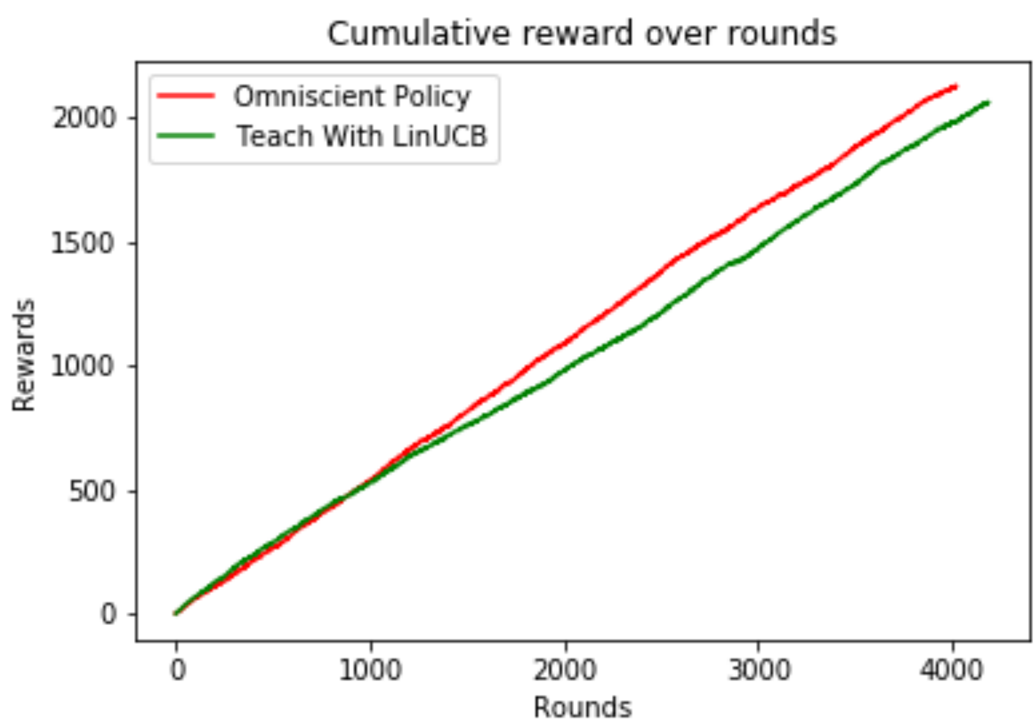


Figure 6.3: Cumulative rewards with skipping

The graph shows the performance of the learning algorithm with respect to the omniscient policy. The number of rounds and the cumulative rewards reduces with skipping enabled. The cumulative reward reduces for some topics that the student did not understand the skip classifier predicted with high confidence that it would be better to move to the next topic.

The omniscient policy required 4019 rounds to get a cumulative reward of 2128. This implies it needs 1.89 rounds for a reward (of 1). The learning algorithm required 4185 rounds to get a reward of 2062. This implies it needs 2.03 rounds for a reward (of 1).

Comparing the cumulative reward graph with and without skipping shows us that our learning algorithm performs better without skipping than with skipping. However, it needs more rounds which could affect student experience.

I don't understand the point of this section. What is the interpretation of a negative reward? Positive reward means the student learneed. So, negative reward means they forget? I would delete this entire Section 6.4 as it doesn't really make sense. On the other hand, maybe you could say 'reward' now means 'student satisfaction', and hence a failed quiz means negative feedback. But, it seems harshly unreasonable to me that the cost of failing a quiz is exactly equal to the negation of passing a quiz. Shouldn't the penalty for failing be lower than the reward for passing, as passing implies learning?

6.4 Penalizing Rewards

We now update our reward assignment strategy to penalize content items which gave no rewards. This presents a different perspective as earlier in Section 6.3 we did not penalize content items which gave no rewards. In this section rewards received by content items change from $\{0, 1\}$ to $\{-1, 1\}$.

When content items receive no reward, the learning algorithm reduces the estimated expected reward from it. This reduces the possibility of such content items being selected which results in more content items being explored as seen in table 6.5.

We follow a similar approach as before. We first find the optimal values for the hyper-parameters before we evaluate our learning algorithm.

6.4.1 Comparing α

α is a scaling factor to control exploration. A higher value of α reduces cumulative reward as more rounds are spent exploring to find the optimal content items. The graph below shows the cumulative reward for different values of α.

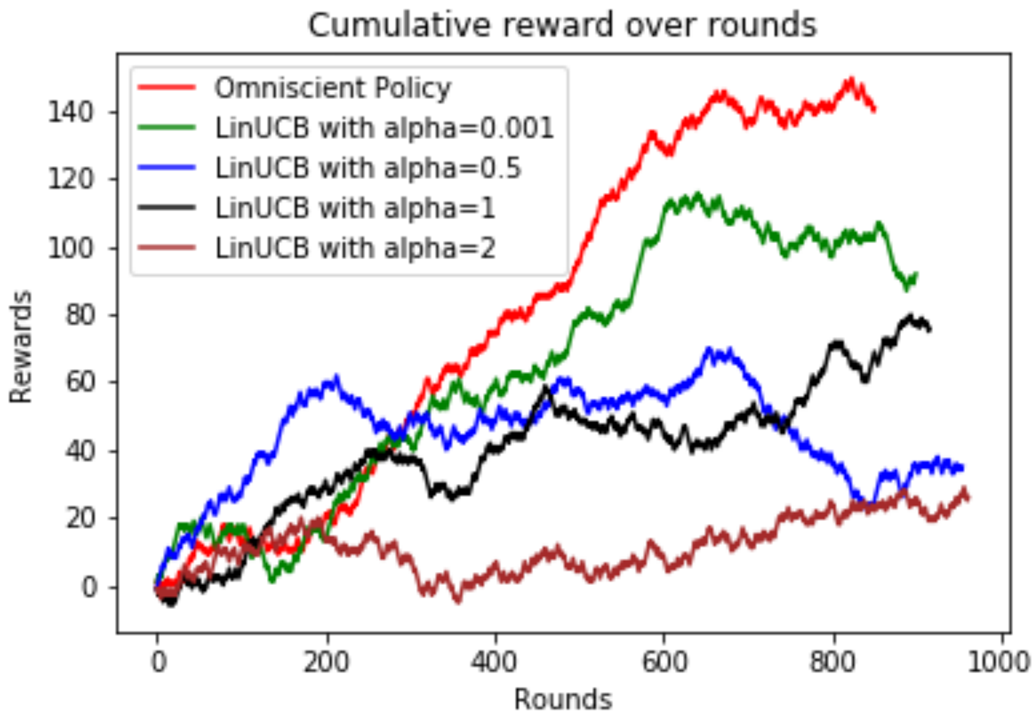


Figure 6.4: Cumulative penalized rewards per alpha

As before $\alpha = 0.001$ gives the best results compared to the omniscient policy. Below is the table which shows the number of content items explored for different values of α.

Values of α	**Content Items Explored**
0.001	75
0.5	89
1.0	104
2.0	105

Table 6.5: Content items explored per α

6.4.2 Confidence Threshold

We ran a simulation over Course 1 to find the optimal value of confidence threshold C. An optimal value would be one for which the skip classifier has the best performance. We ran experiments for different values of confidence threshold. The table 6.6 shows the results of the predictions made by the skip classifier with no threshold set.

		Predictions		
		Stay (0)	Skip (1)	Total
Reward	0	74	39	113
	1	82	64	146
	Total	156	103	518

Table 6.6: Confusion matrix without confidence threshold

Without any threshold, our classifier would have an accuracy of 56.37 %. The table 6.7 shows the results for the classifier for confidence threshold of 20. This gives better results.

		Predictions		
		Stay (0)	Skip (1)	Total
Reward	0	41	20	61
	1	59	39	98
	Total	100	59	257

Table 6.7: Confusion matrix with confidence threshold of 20

With a threshold set to it, our classifier would have an accuracy of 61.64%.

6.4.3 Learning Algorithm

We now configure these optimal values and run a simulation of Course 2 for both the omniscient policy and the learning algorithm. The omniscient policy knows the best arm to pull in each round. On the other hand, our learning algorithm explores to find the best content item.

No Skipping

We now evaluate our learning algorithm without skipping.

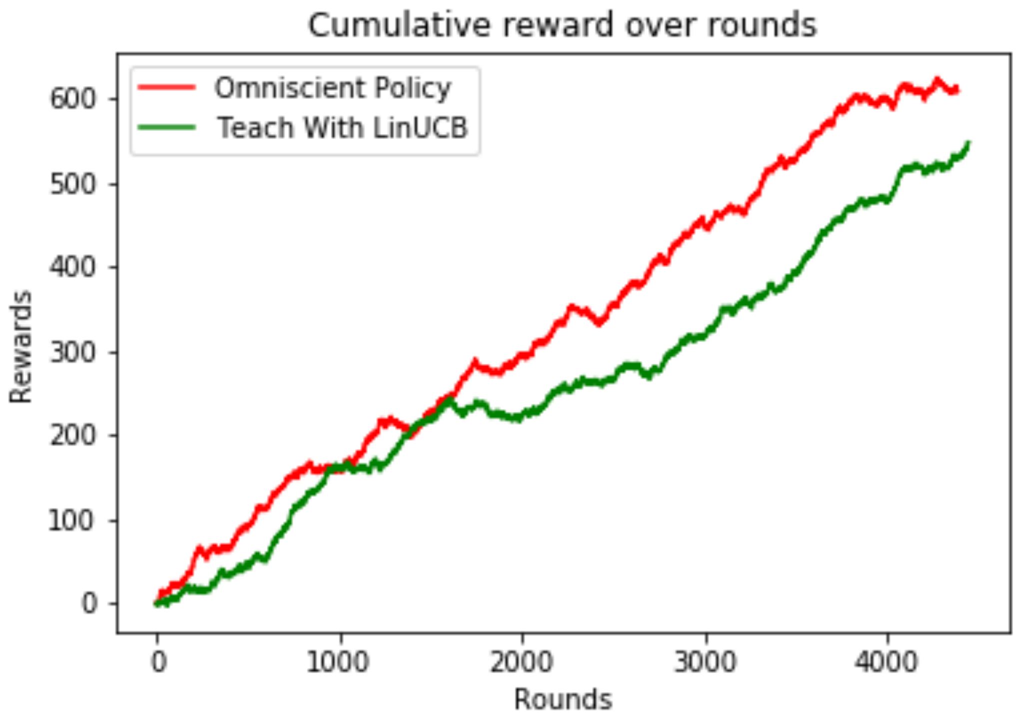


Figure 6.5: Cumulative penalized rewards without skipping

The omniscient policy took 4384 rounds compared to 4443 taken by the learning algorithm to complete the course for all students. The rewards accumulated by the omniscient policy is 608 compared to 547 by the learning algorithm. The learning algorithm performs sub-optimally, which is expected.

With Skipping

With skipping enabled the ability of the learning algorithm to optimize its content item selection strategy is restricted. Skipping restricts the learning algorithm from exploring different content items to find the optimal content item in each round. The graph below shows the performance of the learning algorithm with respect to the omniscient policy with skipping enabled.

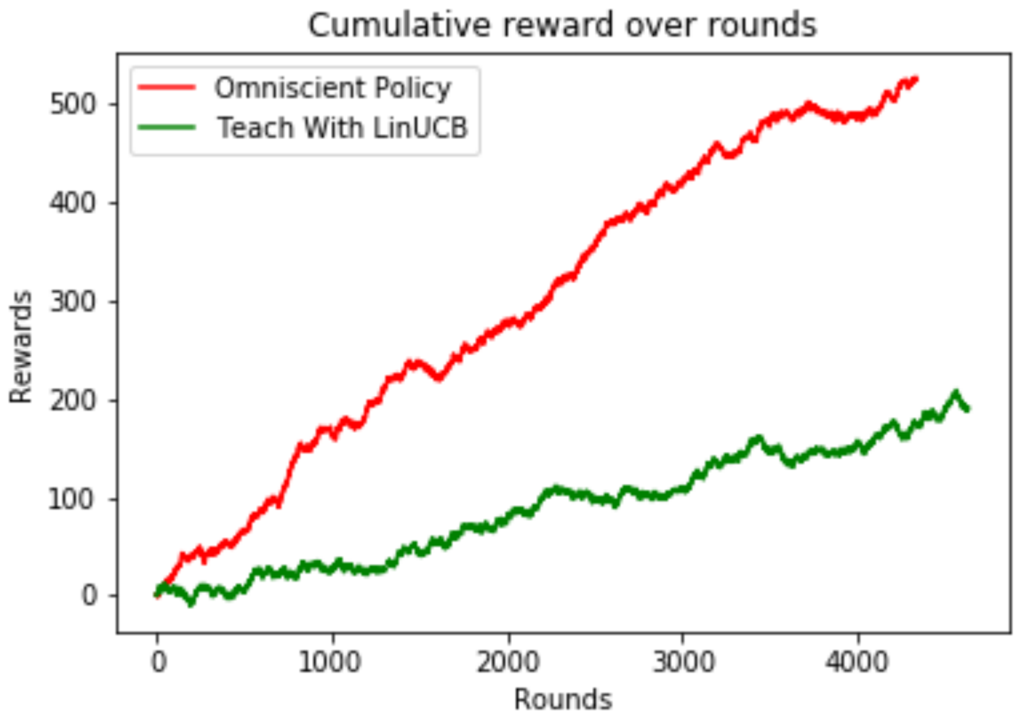


Figure 6.6: Cumulative penalized rewards with skipping

The omniscient policy took 4335 rounds compared to 4634 taken by the learning algorithm to complete the course for all students. The rewards accumulated by the omniscient policy is 527 compared to 191 received by the learning algorithm. The learning algorithm performs poorly, when exploration is restricted.

With this experiment, we find that skipping although useful should only be introduced once the learning algorithm has sufficiently explored all content items. If introduced too early then skipping prevents the learning algorithm from exploring. However if introduced too late then its value diminishes.

Chapter 7

Conclusions

This project presents a student-centric approach to teaching. An approach which could make classrooms more interactive by providing a personalized learning experience for students. We synthesized an unbiased dataset to represent heterogeneous student and content data to evaluate our learning algorithm. Since there were no benchmarks available, we created an omniscient policy which has optimal parameters pre-configured. The algorithm learns these parameters to find an optimal content item for each student.

We then present a feature which would be useful when there are several different content items for a topic to avoid students from getting frustrated by being unable to understand a topic. This not only helps students but also helps teachers recognize topics students are less likely to understand. We evaluated the learning algorithm to set a baseline for this new teaching methodology.

Our future work would involve creating an actual course that follows the teaching methods outlined in this project. This would give real-world student data to evaluate the algorithm. We would also like to design other algorithms to evaluate their performance against our baseline algorithm. An additional optimization would be to find an optimal strategy to introduce skipping such that it does not restrict exploration and still provides a good student experience.

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