Assignment #3 - CPSC 4310

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Non Programming Part

1. Consider the following data sets comprising of 3 boolean input and 1 boolean output

Example	A_1	A_2	A_3	$output$
x_1	1	0	0	0
x_2	1	0	1	0
x_3	0	1	0	0
x_4	1	1	1	1
x_5	1	1	0	1

- Classify the following example using a native Bayes classifier with Laplace 1 smoothing with the addition of $x_6 = (0, 1, 1, ?)$ to the table

Prior Probability:

$output$:

	0	1	$total$
$output$	3	2	5

$P_{LAP,1}(output = 0) = \frac{3+1}{5+1(2)} = 4/7$
$P_{LAP,1}(output = 1) = \frac{2+1}{5+1(2)} = 3/7$
$P_{LAP,1}(output) = \langle \frac{4}{7}, \frac{3}{7} \rangle$

A_i:

	0	1	$total$
A_1	1	4	5
A_2	2	3	5
A_3	3	2	5

	0	1	$total$
$total$	6	9	15

$P_{LAP,1}(A_1 = 0) = \frac{1+1}{5+1(2)} = 2/7$
$P_{LAP,1}(A_1 = 1) = \frac{4+1}{5+1(2)} = 5/7$
$P_{LAP,1}(A_1) = \langle \frac{2}{7}, \frac{5}{7} \rangle$

$P_{LAP,1}(A_2 = 0) = \frac{2+1}{5+1(2)} = 3/7$
$P_{LAP,1}(A_2 = 1) = \frac{3+1}{5+1(2)} = 4/7$
$P_{LAP,1}(A_2) = \langle \frac{3}{7}, \frac{4}{7} \rangle$

$P_{LAP,1}(A_3 = 0) = \frac{3+1}{5+1(2)} = 4/7$
$P_{LAP,1}(A_3 = 1) = \frac{2+1}{5+1(2)} = 3/7$
$P_{LAP,1}(A_3) = \langle \frac{4}{7}, \frac{3}{7} \rangle$

Conditional Probability:

$P_{LAP,1}(x_i | A_i = 0) = 1/2$
$P_{LAP,1}(x_i | A_i = 1) = 1/2$
$P_{LAP,1}(x_i | A_i) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

$P_{LAP,1}(A_1 = 0 | output = 0) = \frac{1+1}{3+1(2)} = 2/5$
$P_{LAP,1}(A_1 = 1 | output = 0) = \frac{2+1}{3+1(2)} = 3/5$
$P_{LAP,1}(A_1 | output = 0) = \langle \frac{2}{5}, \frac{3}{5} \rangle$

$P_{LAP,1}(A_1 = 0 | output = 1) = \frac{0+1}{2+1(2)} = 1/4$
$P_{LAP,1}(A_1 = 1 | output = 1) = \frac{2+1}{2+1(2)} = 3/4$
$P_{LAP,1}(A_1 | output = 1) = \langle \frac{1}{4}, \frac{3}{4} \rangle$

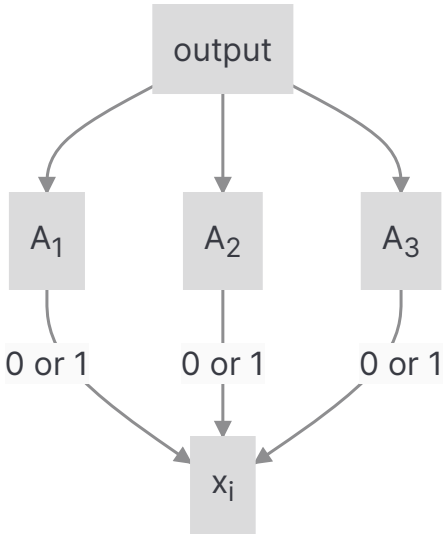
$P_{LAP,1}(A_2 = 0 | output = 0) = \frac{2+1}{3+1(2)} = 3/5$
$P_{LAP,1}(A_2 = 1 | output = 0) = \frac{1+1}{3+1(2)} = 2/5$
$P_{LAP,1}(A_2 | output = 0) = \langle \frac{3}{5}, \frac{2}{5} \rangle$

$P_{LAP,1}(A_2 = 0 | output = 1) = \frac{0+1}{2+1(2)} = 1/4$
$P_{LAP,1}(A_2 = 1 | output = 1) = \frac{2+1}{2+1(2)} = 3/4$
$P_{LAP,1}(A_2 | output = 1) = \langle \frac{1}{4}, \frac{3}{4} \rangle$

$P_{LAP,1}(A_3 = 0 | output = 0) = \frac{2+1}{3+1(2)} = 3/5$
$P_{LAP,1}(A_3 = 1 | output = 0) = \frac{1+1}{3+1(2)} = 2/5$
$P_{LAP,1}(A_3 | output = 0) = \langle \frac{3}{5}, \frac{2}{5} \rangle$

$P_{LAP,1}(A_3 = 0|output = 1) = \frac{1+1}{2+1(2)} = 2/4 = 1/2$
$P_{LAP,1}(A_3 = 1|output = 1) = \frac{1+1}{2+1(2)} = 2/4 = 1/2$
$P_{LAP,1}(A_3|output = 1) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

Calculations:



$P_{LAP,1}(output = 0|x_1)$

$\propto P_{LAP,1}(output = 0)$
$P_{LAP,1}(A_1 = 1|output = 0)P_{LAP,1}(x_1|A_1 = 1)$
$P_{LAP,1}(A_2 = 0|output = 0)P_{LAP,1}(x_1|A_2 = 0)$
$P_{LAP,1}(A_3 = 0|output = 0)P_{LAP,1}(x_1|A_3 = 0)$

$\propto \frac{4}{7} * (\frac{3}{5} * \frac{1}{2}) * (\frac{3}{5} * \frac{1}{2}) * (\frac{3}{5} * \frac{1}{2}) = \frac{4}{7} * (\frac{3}{5} * \frac{1}{2})^3 = \frac{4}{7} * (\frac{3}{10})^3 = \frac{4}{7} * \frac{27}{1000} = \frac{108}{7000} = \frac{27}{1750}$

$P_{LAP,1}(output = 0|x_1) \propto \frac{27}{1750} \approx 0.015$

$P_{LAP,1}(output = 0|x_2)$

$\propto P_{LAP,1}(output = 0)$
$P_{LAP,1}(A_1 = 1|output = 0)P_{LAP,1}(x_2|A_1 = 1)$
$P_{LAP,1}(A_2 = 0|output = 0)P_{LAP,1}(x_2|A_2 = 0)$
$P_{LAP,1}(A_3 = 1|output = 0)P_{LAP,1}(x_2|A_3 = 1)$

$\propto \frac{4}{7} * (\frac{3}{5} * \frac{1}{2}) * (\frac{3}{5} * \frac{1}{2}) * (\frac{2}{5} * \frac{1}{2}) = \frac{4}{7} * (\frac{3}{10})^2 * \frac{1}{5} = \frac{4}{7} * \frac{9}{100} * \frac{1}{5} = \frac{4}{7} * \frac{9}{500} = \frac{36}{3{,}500} = \frac{9}{875}$

$P_{LAP,1}(output = 0|x_2) \propto \frac{9}{875} \approx 0.010$

$P_{LAP,1}(output = 0|x_3)$

$\propto P_{LAP,1}(output = 0)$
$P_{LAP,1}(A_1 = 0|output = 0)P_{LAP,1}(x_3|A_1 = 0)$

$P_{LAP,1}(A_2 = 1|output = 0)P_{LAP,1}(x_3|A_2 = 1)$
$P_{LAP,1}(A_3 = 0|output = 0)P_{LAP,1}(x_3|A_3 = 0)$

$\propto \frac{4}{7} * (\frac{2}{5} * \frac{1}{2}) * (\frac{2}{5} * \frac{1}{2}) * (\frac{3}{5} * \frac{1}{2}) = \frac{4}{7} * (\frac{1}{5})^2 * \frac{3}{10} = \frac{4}{7} * \frac{1}{25} * \frac{3}{10} = \frac{4}{7} * \frac{3}{250} = \frac{12}{1750} = \frac{6}{875}$

$P_{LAP,1}(output = 0|x_3) \propto \frac{6}{875} \approx 0.007$

$P_{LAP,1}(output = 1|x_4)$

$\propto P_{LAP,1}(output = 1)$
$P_{LAP,1}(A_1 = 1|output = 1)P_{LAP,1}(x_4|A_1 = 1)$
$P_{LAP,1}(A_2 = 1|output = 1)P_{LAP,1}(x_4|A_2 = 1)$
$P_{LAP,1}(A_3 = 1|output = 1)P_{LAP,1}(x_4|A_3 = 1)$

$\propto \frac{3}{7} * (\frac{3}{4} * \frac{1}{2}) * (\frac{3}{4} * \frac{1}{2}) * (\frac{1}{2} * \frac{1}{2}) = \frac{3}{7} * (\frac{3}{8})^2 * \frac{1}{4} = \frac{3}{7} * \frac{9}{16} * \frac{1}{25} = \frac{3}{7} * \frac{9}{400} = \frac{27}{2800}$

$P_{LAP,1}(output = 1|x_4) \propto \frac{27}{2800} \approx 0.010$

$P_{LAP,1}(output = 1|x_5)$

$\propto P_{LAP,1}(output = 1)$
$P_{LAP,1}(A_1 = 1|output = 1)P_{LAP,1}(x_5|A_1 = 1)$
$P_{LAP,1}(A_2 = 1|output = 1)P_{LAP,1}(x_5|A_2 = 1)$
$P_{LAP,1}(A_3 = 0|output = 1)P_{LAP,1}(x_5|A_3 = 0)$

$\propto \frac{3}{7} * (\frac{3}{4} * \frac{1}{2}) * (\frac{3}{4} * \frac{1}{2}) * (\frac{1}{2} * \frac{1}{2}) = \frac{3}{7} * (\frac{3}{8})^2 * \frac{1}{4} = \frac{3}{7} * \frac{9}{16} * \frac{1}{25} = \frac{3}{7} * \frac{9}{400} = \frac{27}{2800}$

$P_{LAP,1}(output = 1|x_5) \propto \frac{27}{2800} \approx 0.010$

With $x_6 = (0, 1, 1, ?)$:

Example	A_1	A_2	A_3	$output$
x_1	1	0	0	0
x_2	1	0	1	0
x_3	0	1	0	0
x_4	1	1	1	1
x_5	1	1	0	1
x_6	0	1	1	?

Since we don't know what's the $output$ for x_6, we can calculate with each cases.

Prior Probability:

$output$:

	$true$	$false$	$total$
$output = 0$	4	2	6

$P_{LAP,1}(output = 0) = \frac{4+1}{6+1(2)} = 5/8$
$P_{LAP,1}(output \neq 0) = \frac{2+1}{6+1(2)} = 3/8$
$P_{LAP,1}(output = 0) = \langle \frac{5}{8}, \frac{3}{8} \rangle$

	$true$	$false$	$total$
$output = 1$	3	3	6

$P_{LAP,1}(output = 1) = \frac{3+1}{6+1(2)} = 4/8 = 1/2$
$P_{LAP,1}(output \neq 1) = \frac{3+1}{6+1(2)} = 4/8 = 1/2$
$P_{LAP,1}(output = 1) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

A_i:

	0	1	$total$
A_1	2	4	6
A_2	2	4	6
A_3	3	3	6
$total$	7	11	18

$P_{LAP,1}(A_1 = 0) = \frac{2+1}{6+1(2)} = 3/8$
$P_{LAP,1}(A_1 = 1) = \frac{4+1}{6+1(2)} = 5/8$
$P_{LAP,1}(A_1) = \langle \frac{3}{8}, \frac{5}{8} \rangle$

$P_{LAP,1}(A_2 = 0) = \frac{2+1}{6+1(2)} = 3/8$
$P_{LAP,1}(A_2 = 1) = \frac{4+1}{6+1(2)} = 5/8$
$P_{LAP,1}(A_2) = \langle \frac{3}{8}, \frac{5}{8} \rangle$

$P_{LAP,1}(A_3 = 0) = \frac{3+1}{6+1(2)} = 4/8 = 1/2$
$P_{LAP,1}(A_3 = 1) = \frac{3+1}{6+1(2)} = 4/8 = 1/2$
$P_{LAP,1}(A_3) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

Conditional Probability:

$P_{LAP,1}(x_i | A_i = 0) = 1/2$
$P_{LAP,1}(x_i | A_i = 1) = 1/2$

$P_{LAP,1}(x_i|A_i) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

$P_{LAP,1}(A_1 = 0|output = 0) = \frac{2+1}{4+1(2)} = 3/6 = 1/2$
$P_{LAP,1}(A_1 = 1|output = 0) = \frac{2+1}{4+1(2)} = 3/6 = 1/2$
$P_{LAP,1}(A_1|output = 0) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

$P_{LAP,1}(A_1 = 0|output = 1) = \frac{1+1}{3+1(2)} = 2/5$
$P_{LAP,1}(A_1 = 1|output = 1) = \frac{2+1}{3+1(2)} = 3/5$
$P_{LAP,1}(A_1|output = 1) = \langle \frac{2}{5}, \frac{3}{5} \rangle$

$P_{LAP,1}(A_2 = 0|output = 0) = \frac{2+1}{4+1(2)} = 1/2$
$P_{LAP,1}(A_2 = 1|output = 0) = \frac{2+1}{4+1(2)} = 1/2$
$P_{LAP,1}(A_2|output = 0) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

$P_{LAP,1}(A_2 = 0|output = 1) = \frac{1+1}{3+1(2)} = 2/5$
$P_{LAP,1}(A_2 = 1|output = 1) = \frac{2+1}{3+1(2)} = 3/5$
$P_{LAP,1}(A_2|output = 1) = \langle \frac{2}{5}, \frac{3}{5} \rangle$

$P_{LAP,1}(A_3 = 0|output = 0) = \frac{2+1}{4+1(2)} = 1/2$
$P_{LAP,1}(A_3 = 1|output = 0) = \frac{2+1}{4+1(2)} = 1/2$
$P_{LAP,1}(A_3|output = 0) = \langle \frac{1}{2}, \frac{1}{2} \rangle$

$P_{LAP,1}(A_3 = 0|output = 1) = \frac{1+1}{3+1(2)} = 2/5$
$P_{LAP,1}(A_3 = 1|output = 1) = \frac{2+1}{3+1(2)} = 3/5$
$P_{LAP,1}(A_3|output = 1) = \langle \frac{2}{5}, \frac{3}{5} \rangle$

Calculations:

$P_{LAP,1}(output = 0|x_6)$

$\propto P_{LAP,1}(output = 0)$
$P_{LAP,1}(A_1 = 0|output = 0)P_{LAP,1}(x_6|A_1 = 0)$
$P_{LAP,1}(A_2 = 1|output = 0)P_{LAP,1}(x_6|A_2 = 1)$
$P_{LAP,1}(A_3 = 1|output = 0)P_{LAP,1}(x_6|A_3 = 1)$

$\propto \frac{5}{8} * (\frac{1}{2} * \frac{1}{2}) * (\frac{1}{2} * \frac{1}{2}) * (\frac{1}{2} * \frac{1}{2}) = \frac{5}{8} * (\frac{1}{4})^3 = \frac{5}{8} * \frac{1}{16} = \frac{5}{128}$

$P_{LAP,1}(output = 1|x_5) \propto \frac{5}{128} \approx 0.039$

$P_{LAP,1}(output = 1|x_6)$

$\propto P_{LAP,1}(output = 1)$
$P_{LAP,1}(A_1 = 0|output = 1)P_{LAP,1}(x_6|A_1 = 0)$

$P_{LAP,1}(A_2 = 1|output = 1)P_{LAP,1}(x_6|A_2 = 1)$
$P_{LAP,1}(A_3 = 1|output = 1)P_{LAP,1}(x_6|A_3 = 1)$

$\propto \frac{1}{2} * (\frac{2}{5} * \frac{1}{2}) * (\frac{2}{5} * \frac{1}{2}) * (\frac{2}{5} * \frac{1}{2}) = \frac{1}{2} * (\frac{1}{5})^3 = \frac{1}{2} * \frac{1}{125} = \frac{1}{250}$

$P_{LAP,1}(output = 1|x_6) \propto \frac{1}{250} \approx 0.004$

$P_{LAP,1}(output|x_6)$

$\propto \max(P_{LAP,1}(output = 0|x_6), P_{LAP,1}(output = 0|x_1)) = \max(0.039, 0.004) = 0.039$

$P_{LAP,1}(output|x_6) \propto P_{LAP,1}(output = 0|x_6) = \frac{5}{128} \approx 0.039$

Therefore, $x_6 = (0, 1, 1, 0)$

Example	A_1	A_2	A_3	$output$
x_1	1	0	0	0
x_2	1	0	1	0
x_3	0	1	0	0
x_4	1	1	1	1
x_5	1	1	0	1
x_6	0	1	1	0