Physics Pre-U Revision Guide

Westminster School

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v0.6.0

Contents

About this revision guide

This is a revision guide for Pre-U Physics 2016-18. The specification is available at `http://www.cie.org.uk/images/163265-2016-2018-syllabus.pdf`.

This revision guide has been written by the Physics Department at Westminster School. This is a narrow revision guide, if you are looking for a textbook try Openstax College Physics or *Advanced Physics* by Adams and Allday.

The revision guide is written in LaTeX, is open source and hosted on Github (`https://github.com/mrpsharp/physics-PreU`). Most of the work was done using SageMathCloud, an excellent online development environment for LaTeX, Sage, Jupyter notebooks and more.

If you find a mistake, or would like to suggest an enhancement, please do submit bug reports and enhancement requests. Simply click the link and go to 'issues' in order to report your thoughts. If you would like to contribute to the revision guide, and this would be great, please ensure you read 'CONTRIBUTING.md' and then you can submit a pull request.

Structure of Assessment

Components	Weighting
Paper 1 Multiple Choice **1 hour 30 minutes** Candidates answer 40 multiple-choice questions based on Parts A and B of the syllabus content. 40 marks	20%
Paper 2 Written Paper **2 hours** Section 1: Candidates answer structured questions based on Part A of the syllabus content. Section 2: Candidates answer structured questions related to pre-released material. 100 marks	30%
Paper 3 Written Paper **3 hours** Section 1: Candidates answer structured questions requiring short answers or calculations and some longer answers. The questions are focused on Part B of the syllabus content, but may also draw on Part A. Section 2: Candidates answer three questions from a choice of six. Three questions will have a strong mathematical focus and three questions will focus on philosophical issues and/or physics concepts. Learning outcomes marked with an asterisk (*) will only be assessed in this section. 140 marks	35%
Practical Investigation **20 hours**	15%

Part A

1 Mechanics

Content

- scalars and vectors
- moment of a force
- kinematics
- Newton's laws of motion
- conservation of linear momentum
- density
- pressure

Candidates should be able to:

Scalars and Vectors

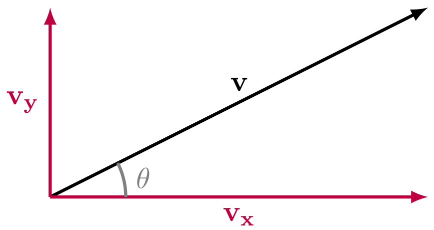
(a) distinguish between scalar and vector quantities and give examples of each
A scalar quantity[1] is one which has only a magnitude whereas a vector has *both* magnitude and direction. We often use positive and negative values to indicate direction (e.g. $v = -2\ ms^{-1}$) but this does not mean that all negative values are vectors!

Note that there are different ways of multiplying vectors and scalars. Two vectors can be multiplied to give a scalar *or* a vector. For example, word done is the (scalar) product of force and displacement, both vectors.

[1] strictly we are modelling a physical quantity as a mathematical object

(b) resolve a vector into two components at right angles to each other by drawing and by calculation

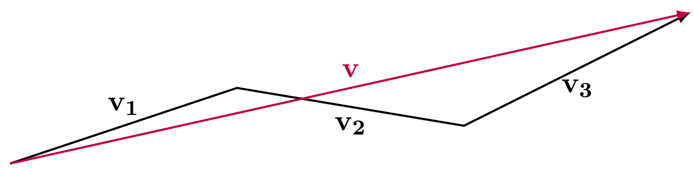
Vectors can be split into two components using trigonometry. The diagram below shows a velocity vector being split into horizontal and vertical components v_x and v_y.



$$\mathbf{v} = \mathbf{v_x} + \mathbf{v_y}$$
$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

(c) combine any number of coplanar vectors at any angle to each other by drawing

Vectors can be added by placing them end to end. The resultant vector is the one joining the start of the first vector to the end of the final vector. Its magnitude and direction can be calculated by trigonometry or scale drawing.



$$\mathbf{v} = \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}$$

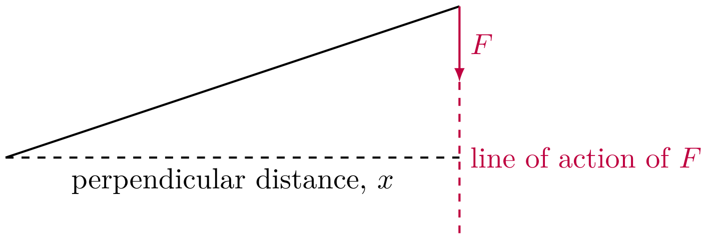
Static Equillibrium

(d) calculate the moment of a force and use the conditions for equilibrium to solve problems (restricted to coplanar forces)

The moment of a force is calculated by multiplying its magnitude by the perpendicular distance of the force's line of action to the pivot point. This is mathematically equivalent to multiplying the distance from the pivot by the component of the force perpendicular to that distance.

The conditions for equilibrium are:

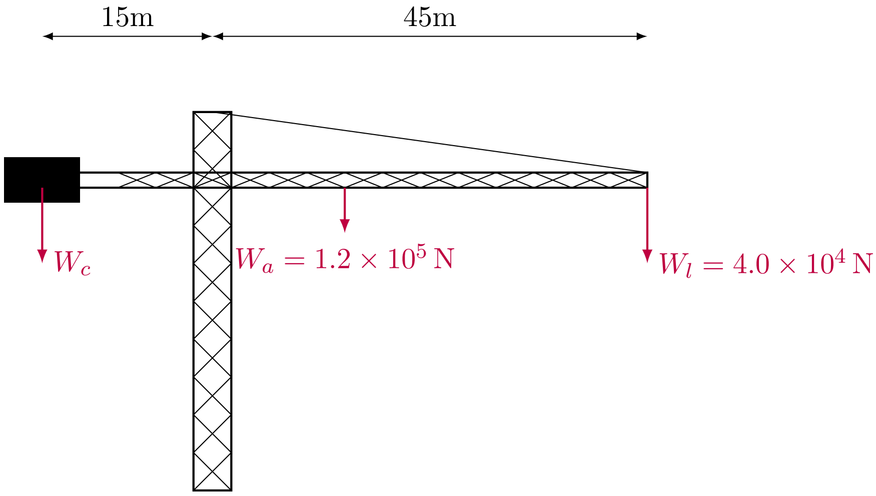
1. The sum of all the forces acting on the object must be zero.



$$\text{moment} = Fx$$

2. The sum of all the moments on an object must be zero.

Example Question
A Tower Crane lifts a load into position. The load has a weight of 4.0×10^4 N and the arm of the crane has a weight of 1.2×10^5 N. Calculate the required weight of the counterweight and the force the tower must support. Assume the centre of mass of the arm is at its centre.



Answer
We begin by taking moments around the tower of the crane. The weight of the arm, W_a, acts 15 m from the tower so solving for moments gives:

$$15W_c = 15W_a + 25W_l$$
$$W_c = 4.2 \times 10^5 \text{ N}$$

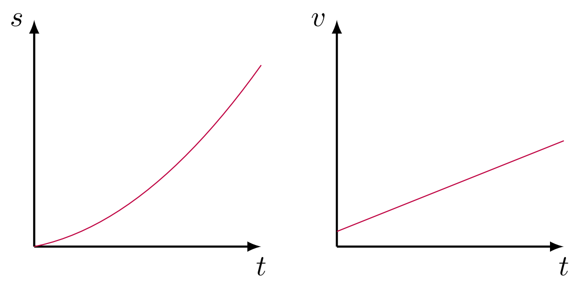
The sum of the downward forces must equal the reaction force of the tower so:

$$R = 4.0 \times 10^5 \text{ N}$$

Kinematics

(e) construct displacement-time and velocity-time graphs for uniformly accelerated motion

For uniform acceleration, a graph of velocity against time will be linear, with the formula $v = u + at$, and a graph of displacement against time will be parabolic, with the formula $s = ut + \frac{1}{2}at^2$.



(f) identify and use the physical quantities derived from the gradients of displacement–time and areas and gradients of velocity–time graphs, including cases of non-uniform acceleration

The quantities are given in the table below:

	gradient	area
displacement-time	velocity	–
velocity-time	acceleration	displacement

If the graph is non-linear then the gradient of a tangent must be taken. Note that areas below the axis in a velocity-time graph represent *negative* displacement.

(g) recall and use:

$$v = \frac{\Delta x}{\Delta t}$$
$$a = \frac{\Delta v}{\Delta t}$$

(h) recognise and use the kinematic equations for motion in one dimension with constant acceleration:

$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \left(\frac{u+v}{2}\right)t$$

(i) recognise and make use of the independence of vertical and horizontal motion of a projectile moving freely under gravity

When an object moves in a uniform gravitational field it motion can be modeled by considering the horizontal and vertical components of motion separately. The horizontal component has a constant velocity and the vertical has a constant acceleration.

Example Question
A ball is thrown with a velocity of $5\,\mathrm{m\,s^{-1}}$ from a height of $1.2\,\mathrm{m}$. If its initial angle to the horizontal is $50°$ calculate the distance it travels before it hits the ground.
Answer
The first step is to split the velocity into horizontal and vertical components:

$$v_x = 5\cos 50$$
$$v_y = 5\sin 50$$

The time for the ball to reach the ground can now be calculated using the vertical motion and the equation $s = ut + \frac{1}{2}at^2$, setting $s = -1.2\,\mathrm{m}$. This gives $t = 1.02\,\mathrm{s}$.
Finally, the horizontal distance is calculated using the simple constant velocity formula to give $x = 3.28\,\mathrm{m}$.

Forces

(j) recognise that internal forces on a collection of objects sum to zero vectorially

This is as a result of Newton's Third Law.

(k) recall and interpret statements of Newton's laws of motion

1. An object will remain at rest, or continue at a constant velocity, unless a resultant force acts upon it.
2. $F = ma$, where F is the vector sum of the forces acting on the body. Or, alternatively $F = \frac{dp}{dt}$ (see below).
3. For every force of object A acting on object B there exists a force of the same type, of equal magnitude and opposite direction of object B acting on object A.

It is important to be able to distinguish the 'equal and opposite' forces which may act on a single object in equilibrium from a Newton's Third Law pair of forces.

(l) recall and use $F = ma$ in situations where mass is constant

Remember that F is the *resultant* force acting on the body.

(m) understand the effect of kinetic friction and static friction

(n) use $F_k = \mu_k N$ and $F_s = \mu_s N$, where N is the normal contact force and μ_k and μ_s are the coefficients of kinetic friction and static friction, respectively

Friction occurs between two objects when they are pushed together by a normal force. A useful model is that the maximum size of the frictional force is proportional to the normal force. There is usually a difference between the constant of proportionality when the two surfaces are stationary compared to each other (static friction) compared to when they are sliding past each other (kinetic friction). It is usually the case that $\mu_k < \mu_s$.

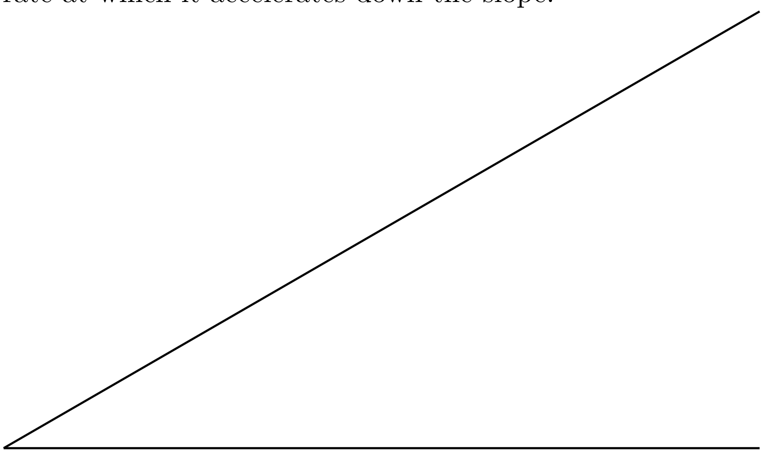
An interesting result is that blocks of different masses should take the same distance to slide to a halt:

$$s = \frac{\frac{1}{2}mv^2}{F} = \frac{\frac{1}{2}mv^2}{\mu_k N} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g}$$

(o) recall and use the independent effects of perpendicular components of a force

As with velocities, forces can be split into two perpendicular components and their effects considered independently.

Example Question
A block of mass 4 kg is on a frictionless slope of 30°. Calculate the rate at which it accelerates down the slope.



Answer
The weight should be split into components along the slope and perpendicular to the slope (shown in green). Only the component along the slope contributes to the acceleration.

$$a = \frac{F}{m} = \frac{mg\cos 30}{m} = 8.5\,\mathrm{ms}^{-2}$$

This question could be extended to include friction by calculating the normal force, the frictional force and hence a new acceleration. If $\mu_k = 0.4$ then the answers are 19.62 N, 7.85 N and 6.5 ms^{-2} respectively (Try it!)

(p) recall and use $p = mv$ and apply the principle of conservation of linear momentum to problems in one dimension

Momentum is a conserved quantity (along with energy and charge). It can be calculated using the formula $p = mv$ where p is the momentum. In any closed system the total momentum of the particules must remain constant. This can be used to predict the outcomes of collisions in certain cases.

(q) distinguish between elastic and inelastic collisions

An elastic collision is one in which *kinetic energy* is conserved. An inelastic collision is one in which it is not. In general, a collision in which two objects adhere will not conserve kinetic energy as the final velocity will be given by:

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

and therefore the final kinetic energy will be given by:

$$\text{KE} = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}\frac{(m_1u_1 + m_2u_2)^2}{m_1 + m_2}$$

which cannot be equal to the original kinetic energy.

(r) relate resultant force to rate of change of momentum in situations where mass is constant and recall and use $F = \frac{\Delta P}{\Delta t}$

Newton's second law is more properly given by:

$$F = \frac{dp}{dt}$$

This simplifies to the GCSE formulation for constant mass:

$$F = \frac{dp}{dt} = m\frac{dv}{dt} = ma$$

A simplified version is:

$$F = \frac{\Delta P}{\Delta t}$$

This will give the correct result for a constant force or otherwise give the average force.

(s) recall and use the relationship impulse = change in momentum

Multiplying both sides of the equation above by time gives:

$$F\Delta t = \Delta P$$

The quantity on the left hand side is the impulse.

(t) recall and use the fact that the area under a force-time graph is equal to the impulse

Using calculus to solve differential version of Newton's Second Law above gives:

$$\Delta P = \int_{t_0}^{t_1} F dt$$

The right-hand side of this equation represents the area under a force-time graph.

(u) apply the principle of conservation of linear momentum to problems in two dimensions

When objects are free to move in two dimensions then momentum must be conserved along two axis.

Example Question

Two objects are able to slide frictionlessly over a horizontal surface. The first object, $m_1 = 3\,\mathrm{kg}$ is propelled with an initial speed $u_1 = 5\,\mathrm{m\,s^{-1}}$ towards a second mass, $m_2 = 1.5\,\mathrm{kg}$, which is initially at rest. After the collision both objects move at 30° on either side of the line of the original motion. What are the final speeds of the two objects? Is the collision elastic?

Answer

Conservation of momentum along the x-axis gives

$$m_1u_1 = m_1v_1\cos\theta + m_2v_2\cos\theta$$

Conservation of momentum along the y-axis gives

$$m_1v_1\sin\theta = m_2v_2\sin\theta$$

These equations can be combined to give

$$v_1 = \frac{u_1}{2\cos\theta} = 2.887\,\mathrm{m\,s^{-1}}$$

and

$$v_2 = \frac{m_1}{m_2}v_1 = 5.773\,\mathrm{m\,s^{-1}}$$

The initial KE of the system is

$$K_i = \frac{1}{2}m_1u_1^2 = 37.5\,\mathrm{J}$$

and the final KE of the system is

$$K_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2u_2^2 = 37.5\,\mathrm{J}$$

since $K_i = K_f$, the collision is elastic

(v) recall and use density = mass / volume

(w) recall and use pressure = normal force / area

(x) recall and use $p = \rho g h$ for pressure due to a liquid. These are GCSE equations and should present no problems.

2 Gravitational Fields

(a) recall and use the fact that the gravitational field strength g is equal to the force per unit mass and hence that weight $W = mg$

A **field** is a region where a particle experiences a force. If this is applied to gravitation, then we can say that a **gravitational** field is a region where a **mass** experiences a force.

You can only tell if a field exists when it exerts a force on something. It is a way of envisaging (seeing in your mind's eye) the size and the direction of the force that would be exerted on a particle when placed in that field.

A gravitational field is produced by anything with mass.

Therefore, a gravitational field is a way of envisaging what would happen to a mass if it were placed in the field due to another mass.

The field is usually represented by lines which show both the **direction** and **strength** of the field.

The **strength** of a gravitational field (the field strength) at any point is the force felt **per unit mass** at that point. This is a **definition.**

It can be written as a word equation:

Gravitational field strength at a point (N/kg)= Force felt by mass (measured in Newtons)/Size of mass (measured in kilograms)

Or in symbols:

$$g = \frac{F}{m}$$

The force, F, felt by any object on the surface of the Earth due to the gravitational field strength of the Earth is known as its **weight.** It is given the symbol **W**.

This means that we can re-write equation above for the field strength at the surface of the Earth by putting W instead of F.

$$g = \frac{W}{m}$$

This then rearranges to an equation that you have all seen before:

$$W = mg$$

Thus the weight of an object on the surface of the Earth is its mass multiplied by the gravitational field strength g.

(b) recall that the weight of a body appears to act from its centre of gravity

The centre of gravity of an object is the point where the weight acts or appears to act.

Thus, when you draw a free-body force diagram for any object in a gravitational field, you draw **one** arrow from the centre of gravity of the object to represent the force due to the field. On the Earth this is, of course, the weight and the arrow points vertically downwards.

(c) sketch the field lines for a uniform gravitational field (such as near the surface of the Earth)

A uniform field is a field where the field strength is the same at all points in the field.

This means that for a gravitational field the force felt per unit mass (see definition) is the same at all points.

The surface of the Earth is a very good approximation to a uniform field.

Therefore if you draw a diagram of the Earth's gravitational field at the Earth's surface over a small area, it will look like Figure 2.1

As you can see, the field lines are **parallel** and **evenly-spaced.** This is always the case for a uniform field.

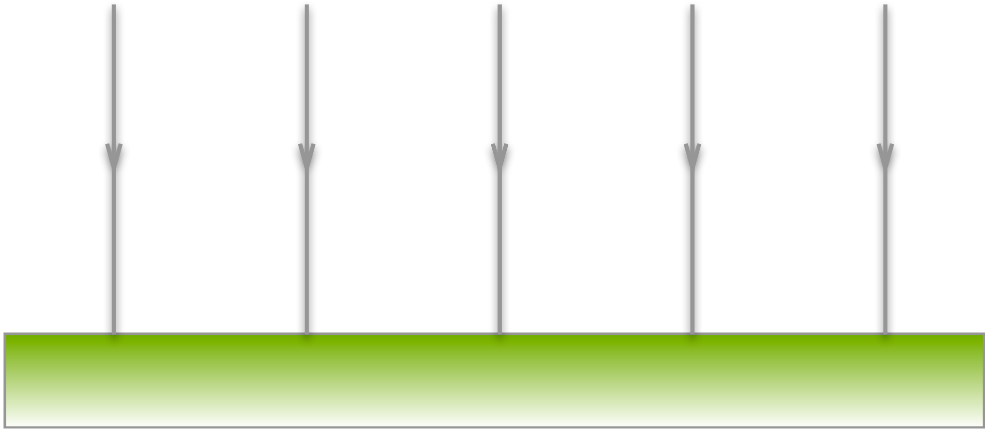


Figure 2.1: Uniform Field

(d) explain the distinction between gravitational field strength and force and explain the concept that a field has independent properties.

There is a very important distinction to make between **gravitational field strength** and **force** at this point: The field strength at any point is the same for all bodies in the field and is the force felt per kilogram, but the force is different and depends on the size of the mass there.

This is best illustrated with an example: If a mass of 60kg is in the Earth's gravitational field at the surface of the Earth, then we can calculate the force acting on it, its weight, using equation (3):

$$W = mg = 60\,\mathrm{kg} \times 9.8\,\mathrm{N\,kg^{-1}} = 590\,\mathrm{N}$$

So the force felt by the 60kg mass is 590N but the field strength for the mass **and for any other mass** is $9.8\mathrm{Nkg}^{-1}$. So the field strength is fixed by your position in the field and the size of the mass that is exerting the field, and nothing else. The force depends on the mass in the field as well.

3 Deformation of Solids

Content

- elastic and plastic behaviour
- stress and strain

Candidates should be able to:

(a) distinguish between elastic and plastic deformation of a material

Elastic deformation is defined as deformation where the sample returns to its original length when the load is removed. Plastic deformation involved a permanent change in length of the sample.

(b) recall the terms brittle, ductile, hard, malleable, stiff, strong and tough, explain their meaning and give examples of materials exhibiting such behaviour

Brittle Brittleness is an indicator of how soon after the yield point a material fractures. Failure will be through the propagation of cracks. A brittle material cannot absorb much energy before breaking. For example, glass and ceramics can be strong but brittle.

Ductile Ductility is a measure of plastic behaviour under tension. It gives an indication of how easily a material can be drawn into wires i.e. can withstand large strains without breaking. Copper is highly ductile.

Hard Hardness is a measure of a materials ability to resist impact or scratching. Diamond is an exceptionally hard material.

Malleable Malleability is a measure of plastic behaviour under compression. It gives an indication of how easily a material can be worked. Metals are ductile and hence relatively easy to form into shapes for use in manufacture.

Stiff Stiffness relates the resistance to change of shape a material possesses. A measure of stiffness is the Young Modulus. Glass fibres and steel are both stiff materials.

Strong A strong material is able to withstand a large stress without failing.

Tough Toughness is a measure of the ability of a material to resist failure through crack propagation. It is the opposite of brittleness. A tough material is able to absorb a lot of energy without breaking. Plastics/polymers are often tough.

(c) explain the meaning of, use and calculate tensile/compressive stress, tensile/compressive strain, spring constant, strength, breaking stress, stiffness and Young modulus

Firstly, compressive forces and deformations are those which reduce the length of the sample whereas tensile forces act to increase its length.

Stress Stress is defined as the force per unit of cross-sectional area applied to a material.

$$\sigma = \frac{F}{A}$$

Stress is measured in pascals (Pa).

Strain Strain is the fractional extension of a material.

$$\epsilon = \frac{x}{l}$$

where l is the original length.

Spring constant The spring constant k is the force per unit of extension of a material during its proportional phase of deformation. It is defined by Hooke's Law:

$$F = kx$$

The spring constant is often used as a measure of stiffness of an object.

Strength Strength is often measured as the maximum stress a material can withstand before permanent deformation. This is known as the yield stress.

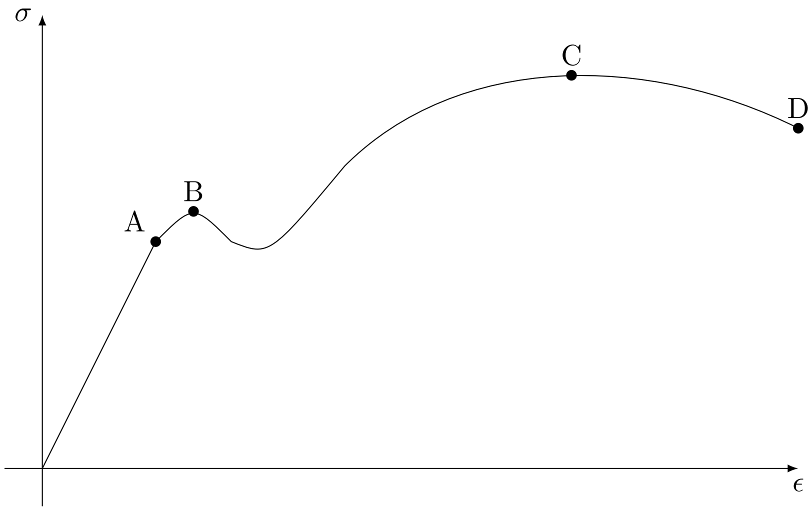
Breaking stress This is the stress at which the material fails.

Young modulus This is a quantitative measure of the stiffness of a material, defined as stress per unit of strain in the proportional region the material's behaviour.

$$E = \frac{\sigma}{\epsilon}$$

(d) draw force-extension, force-compression and tensile/compressive stress-strain graphs, and explain the meaning of the limit of proportionality, elastic limit, yield point, breaking force and breaking stress

The gradient of a force-extension graph gives the spring constant.



A	The limit of proportionality
B	The yield point
C	The ultimate stress (maximium stress)
D	The breaking point

Figure 3.1: Stress-strain curve for a ductile material

Figure 3.1 shows an example stress-strain curve. Note that the limit of proportionality is often a good approximation of the elastic limit of a metal. The "breaking stress" usually refers to the ultimate stress, i.e. the maximum stress the material can withstand, rather than the stress at the breaking point.

(e) state Hooke's law and identify situations in which it is obeyed

$$F = kx$$

Hooke's law is obeyed by an ideal spring and by a sample of metal up to the limit of proportionality.

(f) account for the stress-strain graphs of metals and polymers in terms of the microstructure of the material.

Metals

Metals consist of positive ions in a sea of delocalised electrons. During the elastic phase of deformation the spaces between the ions get larger and smaller. The metallic bonds resist this change from their equilibrium length and act like small springs acting to return the spacing to its original length.

An initial expectation of the plastic phase of deformation in a metallic lattice may be that the planes of ions slip past one another; however an analysis of the forces required for such movement gives an answer hundreds of times higher than the measured yield stress. Instead, the plastic deformation of metals must be explained in terms of *dislocations*. A dislocation occurs when there is a gap in the metallic lattice. Dislocations occur naturally in materials and enable plastic deformation to occur through the breaking of individual bonds in succession, rather than all at once.

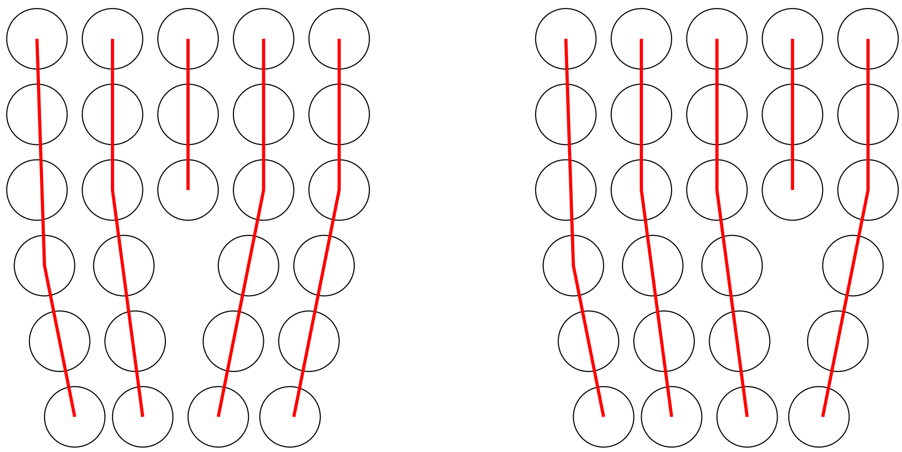


Figure 3.2: The movement of a dislocation

Figure 3.2 shows a dislocation moving within a metal which would allow the metal to deform by moving one atom at a time. As the movement of dislocations is the dominant mode of plastic deformation, changes to the ability of dislocations to move through the metal have significant effects on it properties. For example:

Work Hardening As a metal is deformed, the dislocations move through the structure. Slowly the dislocations reach grain-boundaries or other dislocations and are no longer able to move. The metal therefore becomes less ductile and more brittle. This may be a desired property in order to harden a metal, or the additional brittleness may be undesirable.

Alloying The addition of alloying atoms to the lattice can 'pin' a dislocation in place (as shown in figure 3.3. The metal is therefore no longer able to deform by the movement of dislocations so the metal has a greater yield stress and is less ductile. Examples include adding carbon to iron to produce steel or adding zinc to copper to produce brass.

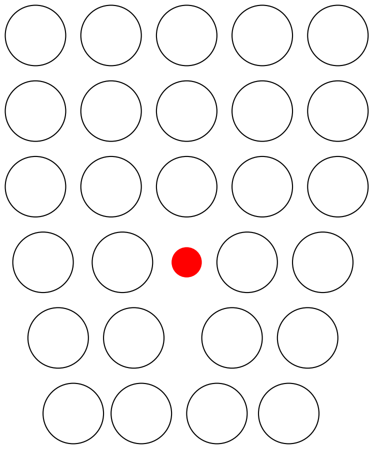


Figure 3.3: Alloying atom pinning a dislocation

The effect on a stress-strain graph can be seen in figure 3.4 below.

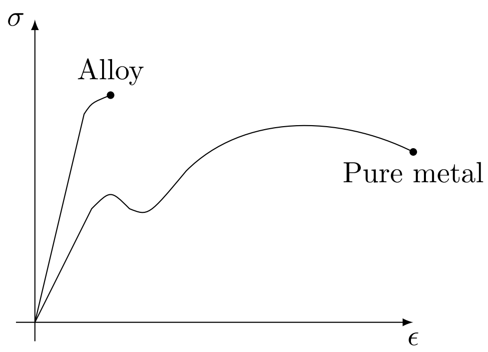


Figure 3.4: Stress-strain curve for a ductile material

Polymers

Polymers consist of long chain molecules weakly held together by intermolecular forces. Initially the molecules are likely to be tangled-up together. As force is applied it is initially difficult to move the polymer chains from this state (**A**). As the chains begin to unravel they straighten out by bond rotation, requiring relatively little force for a large increase in strain (**B**). As the polymer chains become straight it becomes much more difficult to extend the material any further without damaging the material (**C**).

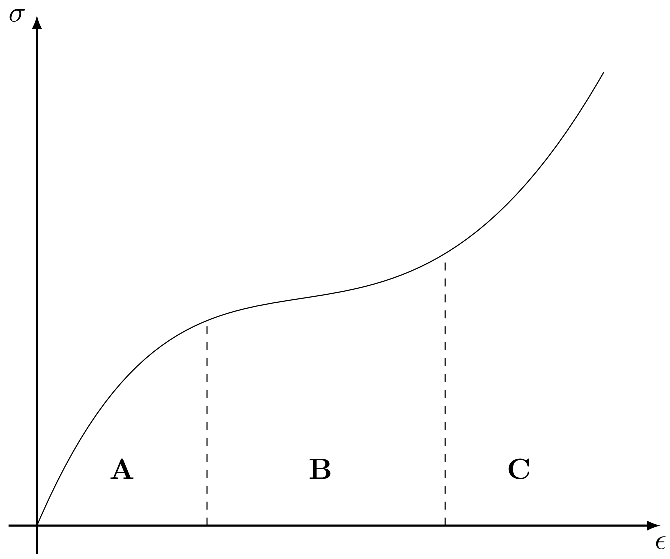


Figure 3.5: Stress-strain curve for a polymer

4 Energy Concepts

Content

- work
- power
- potential and kinetic energy
- energy conversion and conservation
- specific latent heat
- specific heat capacity

Candidates should be able to

(a) understand and use the concept of work in terms of the product of a force and a displacement in the direction of that force, including situations where the force is not along the line of motion

Work, in a scientific sense, is done whenever a force, F acts on an object which moves through a displacement s. When the force and the displacement are acting along the same line then work is simply calculated using $W = Fs$. However, when the force does not act in the same direction as the displacement, the work done is calculated by multiplying the component of the force in the direction of the displacement by the displacement as shown in figure 4.1.

In this case the work done is given by

$$W = Fs\cos\theta \tag{4.1}$$

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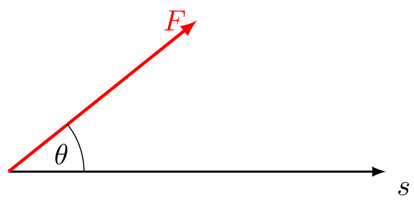


Figure 4.1: Non-aligned force doing work

Note that this means that the following cases work is *not* done:

- any stationary object (no displacement);
- an object in circular motion (the force is acting at right angles to the displacement).

Whenever work is done energy is transferred to or from the object. The type of energy this is transferred to or from varies depending on the circumstances.

(b) calculate the work done in situations where the force is a function of displacement using the area under a force-displacement graph

Equation 4.1 applies whenever a constant force acts over a displacement; however, if the force varies then a different approach is needed. Force a constant force acting in the same direction as the displacement, it can be seen that the area under a force-displacement graph is equal to Fs, i.e. the work done. This is generally true and work can be written in an integral form as:

$$W = \int F \, \mathrm{d}s \tag{4.2}$$

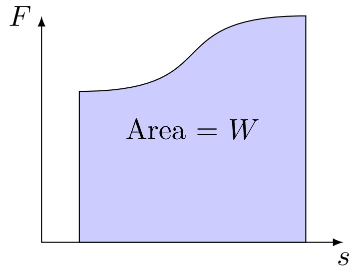


Figure 4.2: Work as area under a graph

(c) understand that a heat engine is a device that is supplied with thermal energy and converts some of this energy into useful work

A heat engine is a device which uses heat to do work. This is shown schematically in figure 4.3. The energy for the work done comes from the difference between Q_1 and Q_2. Examples of heat engines include internal combustion engines, jet engines and steam turbines.

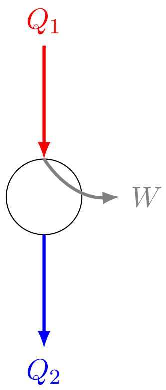


Figure 4.3: A heat engine

(d) calculate power from the rate at which work is done or energy is transferred

Power is defined as the rate at which energy is transferred and is measured in watts (W).

$$P = \frac{W}{t} \tag{4.3}$$

(e) recall and use $P = Fv$

For a constant force, this equation can be shown from equation 4.3 and 4.1:

$$P = \frac{W}{T} = F\,\frac{s}{t} = Fv$$

(f) recall and use $\Delta E = mg\Delta h$ for the gravitational potential energy transferred near the Earth's surface

This is familiar from GCSE.

(g) recall and use $g\Delta h$ as change in gravitational potential

Gravitational potential is defined as the energy per unit mass. Hence, the change in gravitational potential is given by

$$\frac{mg\Delta h}{m} = g\Delta h$$

(h) recall and use $E = \frac{1}{2}Fx$ for the elastic strain energy in a deformed material sample obeying Hooke's law

(i) use the area under a force-extension graph to determine elastic strain energy

This relies on equating the work done straining an object with the elastic strain energy stored in the object. Once this is done, the statement follows from equation 4.2 and figure 4.2.

The area of such a graph when the material obeys Hooke's Law is $\frac{1}{2}Fx$.

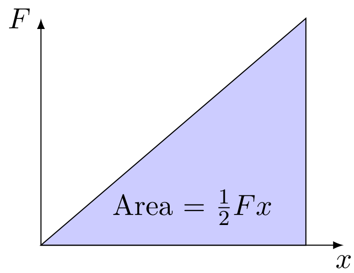


Figure 4.4: Work as area under a graph

(j) derive, recall and use $E = \frac{1}{2}kx^2$

This can be arrived at from Hooke's Law ($F = kx$) and the definition of work in equation 4.2, noticing that the extension of the spring is equal to the displacement of the object.

$$W = \int F \, \mathrm{d}s = \int_0^x kx \, \mathrm{d}x = \frac{1}{2}kx^2$$

This integration could equally be done by substituting $F = kx$ into the expression for elastic strain energy derived above from the graph.

(k) derive, recall and use $E = \frac{1}{2}mv^2$ for the kinetic energy of a body

Consider the work done accelerating an object from rest to a velocity v. Using the equations for uniform accleration with $u = 0$ we can see that

$$W = Fs = mas = m\,\frac{v}{t}\,\frac{v}{2}\,t = \frac{1}{2}mv^2$$

Since this work has gone into the kinetic energy of the object this formula gives us this kinetic energy.

(l) apply the principle of conservation of energy to solve problems

The principle of conservation of energy states that energy cannot be created or destroyed, only transferred between different forms.

(m) recall and use

$$\%\ \text{efficiency} = \frac{\text{useful energy (or power) out}}{\text{total energy (or power) in}} \times 100$$

This is familiar from GCSE.

(n) recognise and use $\Delta E = mc\Delta\theta$, where c is the specific heat capacity

The specific heat capacity is defined as the energy required to heat 1 kg of a substance by 1 °C.

Example Question
A kettle with a power rating of 2 kW heats 500 g of water from 15 °C to boiling. If the kettle is 80% efficient, calculate the time take for the water to boil.
The specific heat capacity of water is $4200\,\mathrm{J\,kg^{-1}\,{}^{\circ}C^{-1}}$
Answer
Total heat energy required by the water:

$$\Delta E = 0.5\,\mathrm{kg} \times 4200\,\mathrm{J\,kg^{-1}\,{}^{\circ}C^{-1}} \times 85\,{}^{\circ}\mathrm{C} = 178.5\,\mathrm{kJ}$$

Useful power provided by the kettle:

$$P = 0.8 \times 2\,\mathrm{kW} = 1.6\,\mathrm{kW}$$

Therefore the time taken is:

$$t = \frac{\Delta E}{P} = \frac{178.5\,\mathrm{kJ}}{1.6\,\mathrm{kW}} = 112\,\mathrm{s}$$

(o) recognise and use $\Delta E = mL$, where L is the specific latent heat of fusion or of vaporisation

When a substance changes state it releases or absorbs energy. This energy is known as the latent head. The specific latent heat is the energy absorbed or released when 1 kg of the substance changes state.

5 Electricity

(a) discuss electrical phenomena in terms of electric charge

(b) describe electric current as the rate of flow of charge and recall and use $I = \Delta Q/\Delta t$

Electric current is the flow of charge. The *current* is defined as the rate of flow of charge. Conventional current flows from positive to negative. This current can consist of positive charges flowing from positive to negative or, more usually, negative charges flowing from negative to positive. The total current depends on the charge carrier density, the cross-sectional area, the charge on the carrier and the drift velocity of the carriers.

(c) understand potential difference in terms of energy transfer and recall and use $VQ = W$

When a charge moves through an electric field it gains of loses potential energy. The energy change per unit charge is defined as the potential difference.

(d) recall and use the fact that resistance is defined by $R = V/I$ and use this to calculate resistance variation for a variety of voltage-current characteristics

As well as measuring resistance directly it can be found from a graph of V against I. *Note: the resistance is defined as V/I at all points and is not equal to the gradient of a V/I graph except in the case that V is proportional to I*

(e) define and use the concepts of emf and internal resistance and distinguish between emf and terminal potential difference

(f) derive, recall and use $E = I(R + r)$ and $E = V + Ir$

A real cell or battery can be represented by a cell circuit symbol in series with a resistor. This resistance represents the *internal resistance* of the cell and the fixed, theoretical potential difference across the cell symbol is the emf of the cell (the electromotive force provided). When a voltmeter is connected across the cell the terminal potential difference is measured.

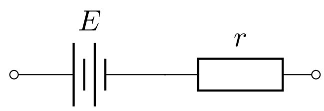


Figure 5.1: Terminal potential difference

In order to relate the terminal potential difference to the internal characteristics of the cell we must subtract the potential difference across the internal resistance from the emf. In symbols this gives:

$$V = E - Ir$$

which can be re-arranged to give the formula above.

If our real cell is connected into a circuit with a load resistance R, the circuit in figure 5.2 is produced.

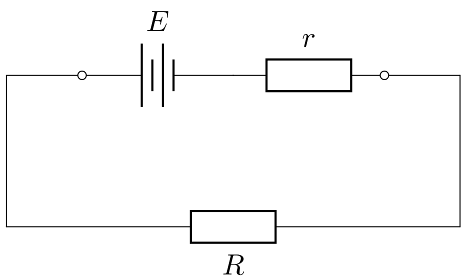


Figure 5.2: A loaded real cell

Now, the terminal potential difference must be equal to the potential difference across the load resistor, R.

$$IR = E = Ir$$

Which can be re-arranged to give the second equation in the specification.

(g) recall and use $P = VI$ and $W = VIt$, and derive and use $P = I^2R$

Power is defined as the energy transferred per unit of time. In the case of electrical power this is the product of current (charge per unit time) and potential difference (energy per unit charge). Given a constant voltage and current, the energy transferred (work done) is given by $P = Wt = IVt$

(h) recall and use $R = \rho l/A$

This formula allows the calculation of a regular sample of material. In this formula ρ is the resistivity, l is the length of the sample and A its cross-sectional area. Typical resistivities for conductors are of the order of $10^{-8}\,\Omega\,\mathrm{m}$ and above $10^{9}\,\Omega\,\mathrm{m}$ for insulators. Semi-conductors lie between these values.

(i) recall the formula for the combined resistance of two or more resistors in series and use it to solve problems $R_T = R_1 + R_2 + \ldots$

(j) recall the formula for the combined resistance of two or more resistors in parallel and use it to solve problems $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$

This are fairly simple to derive from Kirchoff's Laws (see below).

(k) recall Kirchhoff's first and second laws and apply them to circuits containing no more than two supply components and no more than two linked loops

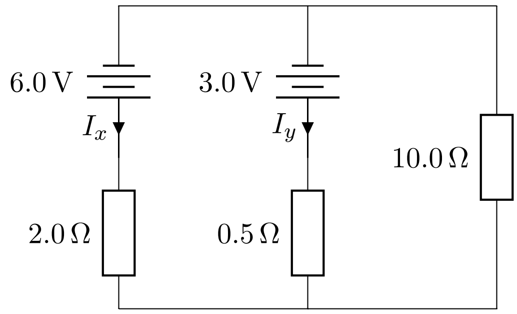
Kirchoff's First Law The current that flows into any junction is equal to the current which flows out.

Kirchoff's Second Law The sum of the emfs around any closed loop of a circuit must equal the sum of the potential differences across any components. It is important to note that direction matters and if the loop crosses emfs or components against the flow of current they must be subtracted.

These two laws, and the definition of resistance, are the most useful tools in circuit analysis. The important skill is to work methodically through the circuit applying the laws, rather than attempting to solve the circuit all in one go.

Example Question

Calculate the current through the 3 V cell.



Answer

We can use Kirchoff's Second Law to derive two expressions linking I_x and I_y. The first is formed by creating a loop consisting of the two branches containing cells.

$$6 - 3 = 2I_x - 0.5I_y$$

Note that I am using a clockwise loop so the signs of the two components in the 'Y' branch have negative signs.

The second expression is now arrived at by using the outermost loop of the circuit (and using Kirchoff's First Law to get the current through the 10 Ω resistor):

$$6 = 10(I_x + I_y) + 2I_x$$

These two equations can now be used to solve for I_y, giving

$$I_y = -0.923\,\mathrm{A}$$

Note the sign of I_y is negative, this means that current is flowing in the opposite direction to the arrow shown. This means that the cell is charging.

(l) appreciate that Kirchhoff's first and second laws are a consequence of the conservation of charge and energy, respectively

The charge flowing into or out of a junction in a given time, t, is given by $Q = It$. Given that a junction can neither store nor create charge, Kirchoff's First Law follows directly.

Each charge carrier can only take one loop around the circuit. Once it returns to its original position its energy must be equal to the amount it had when it left. The charge carrier gains energy passing through cells and loses it passing through components. Since the sum of these energies must be zero and $W = qV$, the sum of emfs must equal the sum of potential differences across components.

(m) use the idea of the potential divider to calculate potential differences and resistances

When two resistors are in series with a battery we say that the circuit is a *potential divider.*

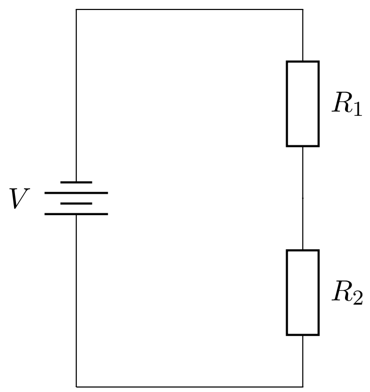


Figure 5.3: A potential divider

Since there are no junctions in the series circuit we can know that the current is the same in all parts of the circuit and that the total resistance is $R_1 + R_2$. The p.d. across resistor 1 is therefore given by:

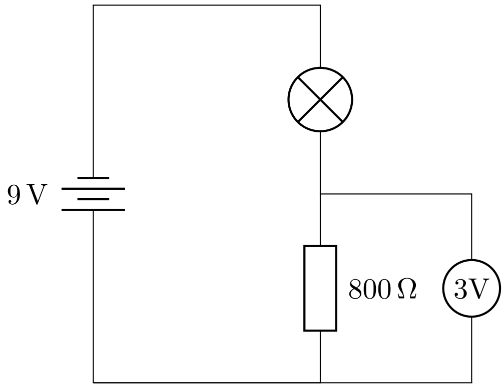
$$V_1 = IR_1 = \frac{V}{R_1 + R_2} R_1 = \frac{R_1}{R_1 + R_2} V$$

In other words, the ratio of p.d.s in the circuit is equal to the ratios of the resistances.

This can also be extended to the ratios between the components as they share the same current.

$$I_1 = I_2 \implies \frac{V_1}{R_1} = \frac{V_2}{R_2} \implies \frac{R_1}{R_2} = \frac{V_1}{V_2}$$

Example Question
Calculate the resistance of the bulb in the circuit below.



Answer
The potential difference across the bulb must be 6 V by Kirchoff's Second Law. Therefore:

$$\frac{3\,\text{V}}{9\,\text{V}} = \frac{800\,\Omega}{R}$$

$$\implies R = 800\,\Omega \times \frac{9\,\text{V}}{3\,\text{V}} = 2400\,\Omega$$

6 Waves

Content

- progressive waves
- longitudinal and transverse waves
- electromagnetic spectrum
- polarisation
- refraction

Candidates should be able to:

(a) understand and use the terms displacement, amplitude, intensity, frequency, period, speed and wavelength

All waves consist of oscillations. The oscillations could be of particles, for example in a sound wave, or of an electromagnetic field, as in a light wave.

The following terms are used to describe properties of waves:

- **displacement**: This is a measurement of the distance and direction away from the equilibrium position.
- **amplitude**: The maximum displacement of the oscillation, represented by A.
- **intensity**: The power of the wave per unit area, represented by I. The unit of intensity is Wm^{-2}.
- **frequency**: The number of oscillations per second, represented by f.

- **period**: The time taken for one oscillation, represented by T.
- **speed**: The speed of a wave represented by v. This will depend on the medium through which the wave is travelling.
- **wavelength**: The distance over which a wave's shape repeats, represented by λ.

(b) recall and apply $f = \frac{1}{T}$ to a variety of situations not limited to waves

This equation follows from the definition of the frequency and time period of a wave. Remember to use Hertz as the unit for frequency and seconds as the unit for period.

(c) recall and use the wave equation $v = f\lambda$

$$speed = \frac{distancetravelled}{timetaken}$$

For a wave, the distance travelled in one time period, T, is the wavelength, λ. Therefore we can write

$$v = \frac{\lambda}{T}$$

Then, using the equation $f = \frac{1}{T}$, we can write:

$$v = f\lambda$$

This is known as the wave equation and can be applied to all waves. The frequency of the wave generally depends on the source of the wave or how it is produced and the speed depends on the medium through which the wave is travelling.

(d) recall that a sound wave is a longitudinal wave which can be described in terms of the displacement of molecules or changes in pressure

When a sound wave travels through a material, the collisions of molecules are parallel to the direction of travel. Energy is transferred through these collisions and the speed of the sound wave will depend on factors such as the density of the material and the temperature.

When a sound wave is viewed on an oscilloscope, it looks as though the oscillations are perpendicular to the direction of travel, as in a transverse wave. The y-axis can represent either the displacement of molecules (still in the parallel direction) from their equilibrium position, or the difference in pressure.

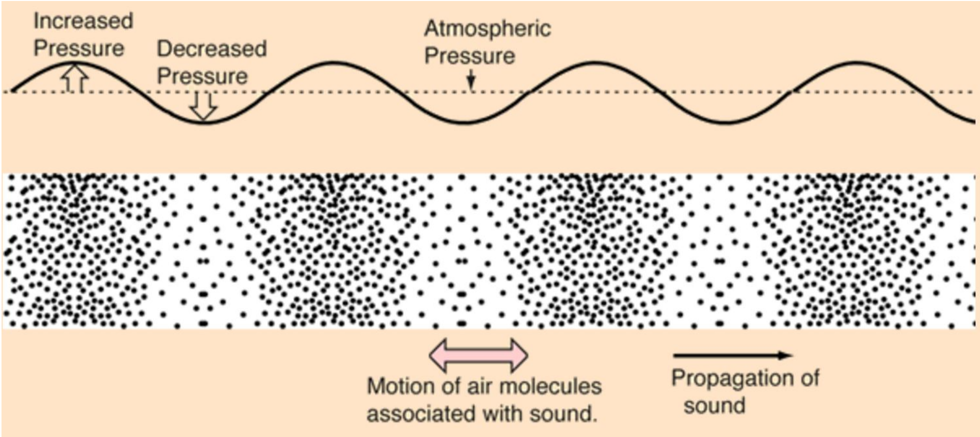


Figure 6.1: Sound wave in air (credit: hyperphysics)

(e) recall that light waves are transverse electromagnetic waves, and that all electromagnetic waves travel at the same speed in a vacuum

(f) recall the major divisions of the electromagnetic spectrum in order of wavelength, and the range of wavelengths of the visible spectrum

Electromagnetic waves are transverse waves where the oscillations are perpendicular to the direction of travel. In all electromagnetic waves there are actually two waves oscillating perpendicular to each other and to the direction of travel. One is an oscillating magnetic field; the other an oscillation electric field.

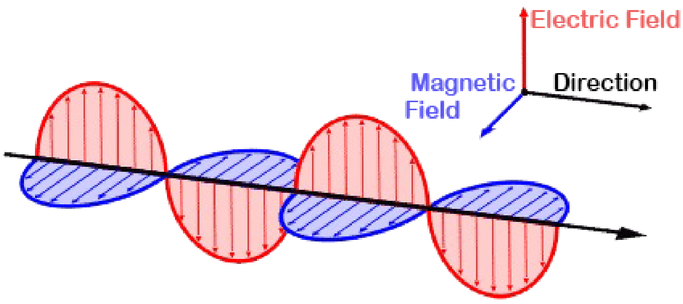


Figure 6.2: oscillations in an electromagnetic wave

The electromagnetic spectrum is the name for the arrangement and classification of electromagnetic waves in order of their wavelengths or frequencies.

The electromagnetic spectrum is shown below in order of increasing wavelength.

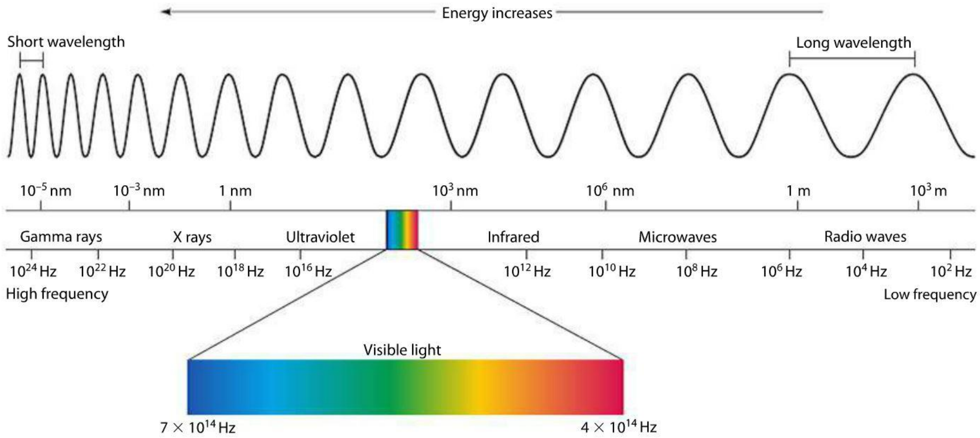


Figure 6.3: The electromagnetic spectrum (Credit:miniphysics.com)

You can see that the visible light spectrum makes up a small part of the electromagnetic spectrum, with wavelengths between 400 - 700 nm.

(g) recall that the intensity of a wave is directly proportional to the square of its amplitude

If the amplitude of a wave varies sinusoidally, the intensity will vary as sine squared. Therefore the following expression can be used:

$$I \propto A^2$$

(h) use graphs to represent transverse and longitudinal waves, including standing waves

Note: Standing waves will be covered in Chapter 7 on Superposition

There are two types of graphs used to represent transverse and longitudinal waves, shown in Figure 6.4. You need to be careful as they look similar.

The first graph plots the motion of one part of the wave with time, for example the motion of one water molecule as a water wave goes by. The x-axis on this graph can give you the time period of the wave.

The second graph is a snapshot of a section of the wave at one particular instant in time. On this graph the wavelength can be measured from the x-axis.

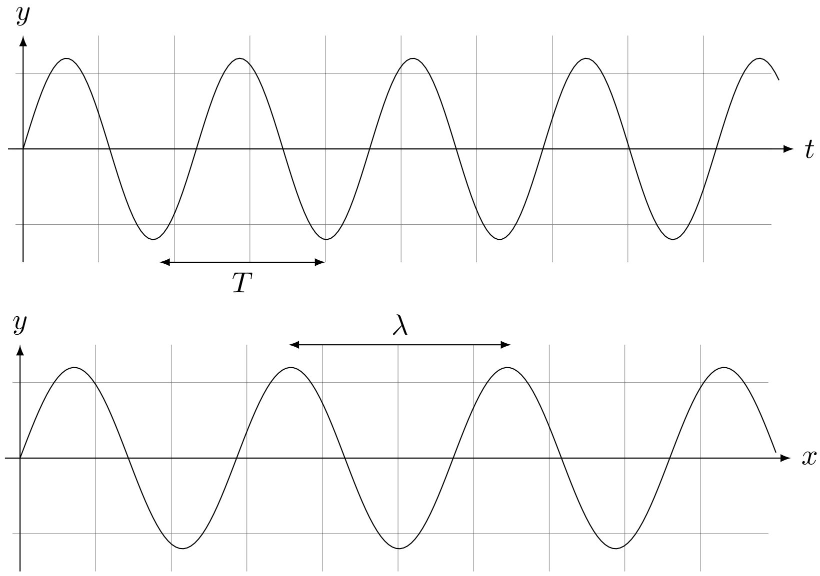


Figure 6.4: Two graphs of a wave

(i) explain what is meant by a plane-polarised wave

(j) recall Malus' Law ($I \propto \cos^2\theta$) and use it to calculate the amplitude and intensity of transmission through a polarising filter

A plane-polarised wave is one where there is only **one** allowed direction of oscillation. This is only applicable to transverse waves where there are multiple allowed modes of oscillation which are all perpendicular to the direction of travel. A longitudinal wave cannot be polarised as there is already only one direction of oscillation - the direction parallel to that of travel. All electromagnetic waves can be polarised.

Consider visible light as an example of a polarised wave. There are 4 ways in which light can be polarised.

- **Transmission**: A polarising filter can be used to polarise light. A filter is made up of chains of molecules that will absorb one direction of oscillation of the light wave, therefore only letting through the perpendicular direction. Note that this 'one' direction is a simplificiation as it encompasses oscillations in both the electric and magnetic fields. The *axis of* *transmission* of a filter is the direction of oscillation that the filter will let through.

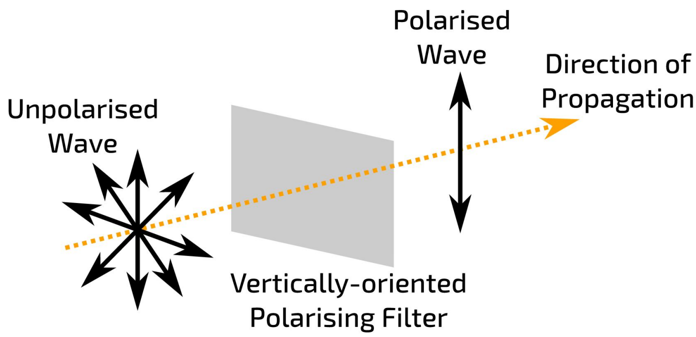


Figure 6.5: diagram showing the operation of a polarising filter (Credit: isaac-physics)

As unpolarised light passes through a polaroid filter, its intensity will drop of 50% of what it originally was. If light that is already polarised is incident on a filter with a perpendicular axis of transmission, none will pass through. If light that is already polarised is incident on a filter with a parallel axis of transmission, then all of the light will pass through. For cases other than parallel or perpendicular, Malus' Law can be used.

Malus' Law can be used to work out how the intensity of polarised light changes as it passes through a polaroid filter. The angle θ is the angle *between* the direction of polarisation of the incident light and the axis of transmission of the polaroid. If you start with unpolarised light, θ is the angle between the two polaroids.

Malus' Law states that the intensity of the transmitted light is proportional to the square of $\cos\theta$.

$$I \propto \cos^2\theta$$

If the incident intensity is I_0, then we can write Malu's Law as:

$$I = I_0 \cos^2\theta$$

Note that if you are dealing with *amplitude* instead of intensity then you must take the square root to give $\cos\theta$.

- **Reflection**: Light can be partially polarised on reflection from certain non metallic surfaces, such as water. The reflected light will be polarised parallel to the surface. This is why polaroid sunglasses are useful as they can cut out the glare from water or roads.

- **Refraction**: Light can be partially polarised, often in two perpendicular directions, when passing through some materials, such as calcite. Specific details will always be given to you in a question.
- **Scattering**: Light from the Sun scatters of molecules in our atmosphere and is partially polarised depending on the direction that you are looking at the sky. Again, specific details will always be provided in a question.

(k) recognise and use the expression for refractive index

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

When a wave crosses a boundary which involves a change in speed, refraction occurs. This concept should be familiar from GCSE.

For light, the refractive index of a medium is the ratio of the speed of light in a vacuum, c, to the speed of light in the medium, v.

$$n = \frac{c}{v}$$

Therefore the refractive index of a material is always greater than one.

If a wave now crosses a boundary between material 1 and material 2, with the angle of incidence being θ_1 and the angle of refraction being θ_2, the following relationship (Snell's Law) applies:

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

As the refractive index of a material is inversely proportional to the speed of light in that material, we know that

$$\frac{n_2}{n_1} = \frac{v_1}{v_2}$$

Snell's Law now becomes

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

This is the most general form of Snell's Law. For the specific case where material 1 is air we can take $n_1 = 1$ as the speed of light in air is so close to the speed of light in a vacuum. Now, replacing n_2 with n, the equation is:

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

This is the equation given in the specification. Be careful as it only applies to the case where material 1 is air and this might not always be the case.

(l) derive and recall $\sin c = \frac{1}{n}$ *and use it to solve problems*

If we take Snell's Law for the case where light is travelling from a material of higher refractive index into a material with lower refractive index, $n_1 > n_2$, we know that the light will bend away from the normal with the angle of refraction, θ_2 being larger than the angle of incidence, θ_1. If the angle of incidence is increased until the angle of refraction is 90°, then the angle of incidence is now called the *critical angle*, as above this angle, *total internal reflection* will occur.

Now we can put this into Snell's Law. θ_1 is now c, the critical angle and θ_2 is now 90°.

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

This now becomes:

$$\frac{n_2}{n_1} = \frac{\sin c}{\sin 90^\circ}$$

As $\sin 90^\circ = 1$, the most general equation to find the critical angle is:

$$\frac{n_2}{n_1} = \sin c$$

or

$$\frac{n_1}{n_2} = \frac{1}{\sin c}$$

In the specification, the equation is given for the specific case where material 2 is air, therefore n_2 can be taken to be 1. This gives the equation:

$$\sin c = \frac{1}{n}$$

(m) recall that optical fibres use total internal reflection to transmit signals

(n) recall that, in general, waves are partially transmitted and partially reflected at an interface between media.

Should be familiar from GCSE.

7 Superposition

Content

- phase difference
- diffraction
- interference
- standing waves

Candidates should be able to:

(a) explain and use the concepts of coherence, path difference, superposition and phase

(b) understand the origin of phase difference and path difference, and calculate phase differences from path differences

(c) understand how the phase of a wave varies with time and position

These terms are all used when considering more than one wave.

The phase of a wave is related to how far through an oscillation a wave is. This is expressed in radians or degrees, where one complete oscillation corresponds to 360° or 2π radians.

The phase difference between two waves is more useful than the phase of one wave. This refers to the fraction of an oscillation by which one wave 'leads' or 'lags' behind another. If the phase difference is $2n\pi$, where n is an integer, then two waves are said to be *in phase* and if the phase difference is $(2n-1)\pi$ then the waves are completely *out of phase*.

Two waves are said to be coherent if they have a constant phase difference. Most often, this is a phase difference of zero, which means that the waves are in phase, but this does not always have to be the case. For interference patterns to occur, coherence is often a necessary condition.

If two waves, from two different sources, meet at a point, the path difference is the difference in distance travelled between the two waves. To calculate the path difference the smaller distance should be taken away from the larger distance. Path difference is normally expressed as a multiple of wavelength, as this then allows the phase difference to be calculated easily.

When two or more waves meet at a point, superposition will occur. This means that the displacements of the individual waves add up to give a resultant displacement. If the two waves are in phase, then constructive interference will occur and if they are out of phase then destructive interference will occur.

Path difference	**Phase difference**	**Superposition**
$n\lambda$	$2n\pi$	constructive interference
$(n+\frac{1}{2})\lambda$	$(2n-1)\pi$	destructive interference

(d) determine the resultant amplitude when two waves superpose, making use of phasor diagrams

When two waves superpose, the resultant displacement at any point is the vector sum of the individual displacements.

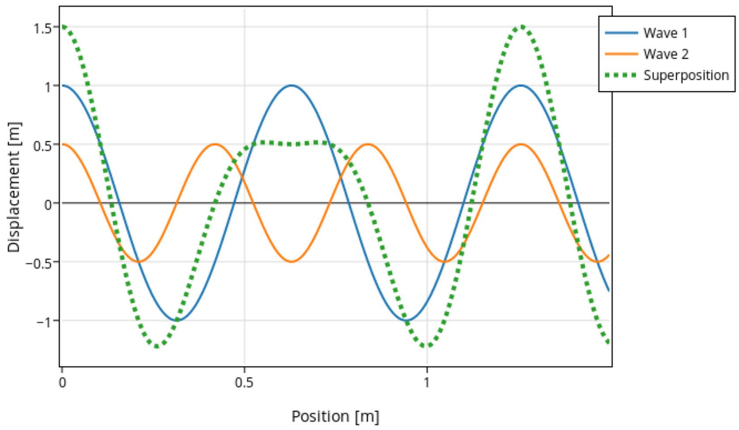


Figure 7.1: Graph showing superposition of two waves

Phasors are rotating arrows that can be used to describe waves. A phasor arrow rotates anticlockwise and one full oscillation of the wave corresponds to one complete oscillation of the phasor arrow. The length of the phasor arrow corresponds to the amplitude of the wave.

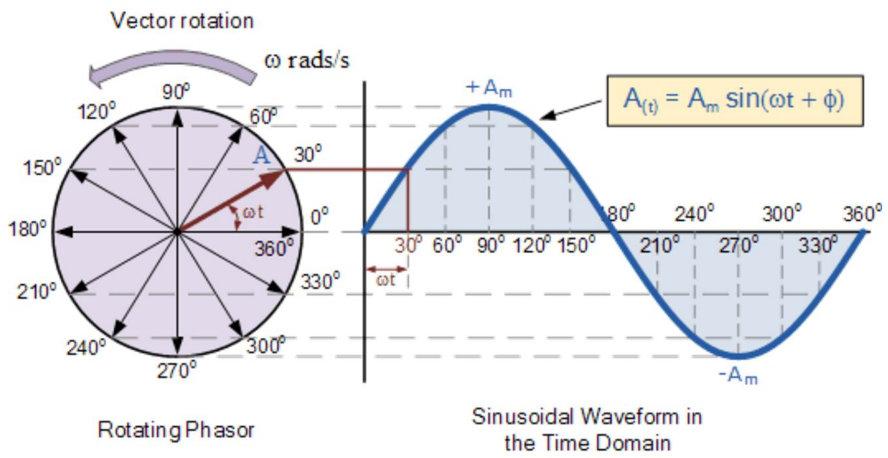


Figure 7.2: The circle on the left shows the rotating phasors that correspond to this sine wave

If we have two waves that superpose, their individual phasor arrows at a particular point can be added up as vectors as shown in the diagram below.

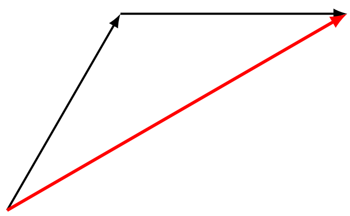


Figure 7.3: The sum of two phasors placed end to end

(e) explain what is meant by a standing wave, how such a wave can be formed, and identify nodes and antinodes

A standing wave arises from a combination of reflection and interference. Consider the set up below.

The vibration generator leads to a progressive wave travelling to the right. This waves reflects off the fixed end and so there are now two waves on the string, travelling in opposite directions.

These two waves superpose and, in some cases (conditions discussed below), a standing wave can form on the string. If these conditions are met, there will be

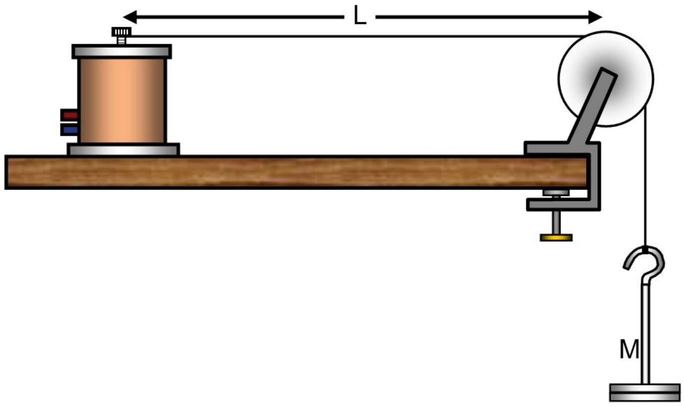


Figure 7.4: Vibration generator connected to a horizontal string under tension

points where the two waves always meet in phase and interfere constructively. These are called *antinodes*. The points where the two waves always meet out of phase and interfere destructively are called *nodes*.

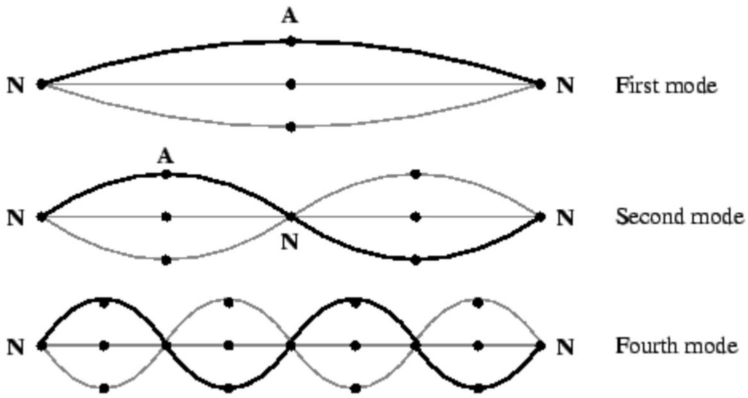


Figure 7.5: modes of oscillation on a string

Unlike a progressive wave, all points on a stationary wave do not have the same amplitude. The amplitude is at a minimum (often zero) at a node and at a maximum at an antinode.

If the length of the string is L, and the speed of waves on the string is v, we can work out the frequencies of the various modes of oscillation. The lowest frequency (first mode) shown in the diagram is called the fundamental frequency. You can see that half of a wavelength fits on the string. Therefore we can write:

$$\frac{\lambda}{2} = L$$

$$\lambda = 2L$$

Putting this into the wave equation gives an expression for the fundamental frequency:

$$v = f\lambda = f.2L$$
$$f = \frac{v}{2L}$$

The other modes of oscillation can be worked out in similar ways, by looking at the relationship between L and λ and substituting into the wave equation. For a given string under a certain tension the speed is constant. The frequency is the frequency of the vibration generator which can be changed to give the different modes of oscillation.

The boundary conditions for this string were that both ends had to be nodes. In other cases where standing waves occur, for example sound waves, the boundary conditions could be different. Closed ends of tubes are always nodes and open ends are always antinodes.

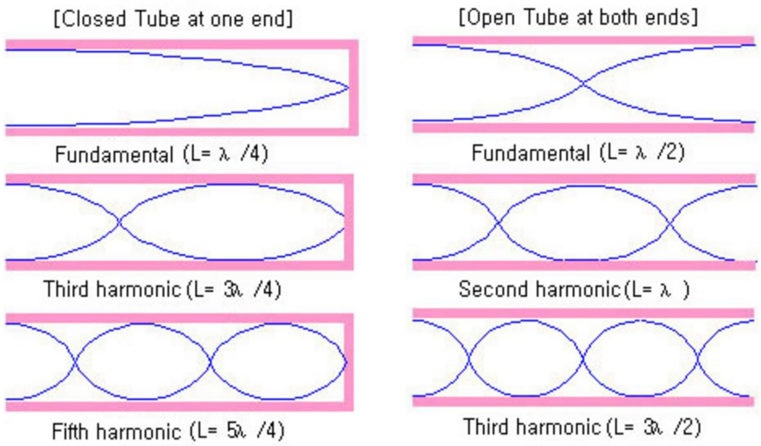


Figure 7.6: Standing waves in a tube credit:Yonsei Phylab

(f) understand that a complex wave may be regarded as a superposition of sinusoidal waves of appropriate amplitudes, frequencies and phases

If more than two waves superpose, the resultant wave can get very complicated! This means that *any* waveform can always be broken down into sinusoidal waves. With combinations of sinusoidal waves of various frequencies, amplitudes and phase differences, any waveform can be made.

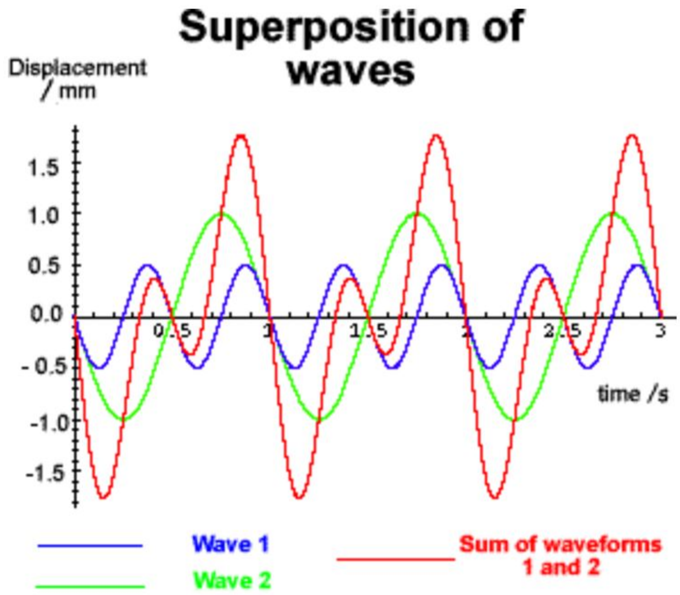


Figure 7.7: E

xample showing how 2 waves combine to form a more complex wave (credit:cyberphysics.co.uk)

(g) recall that waves can be diffracted and that substantial diffraction occurs when the size of the gap or obstacle is comparable to the wavelength

(h) recall qualitatively the diffraction patterns for a slit, a circular hole and a straight edge

When waves pass through an opening, or around a barrier, diffraction occurs and the waves can change in direction and spread out. Diffraction is most significant when the size of the gap or obstacle is comparable to the wavelength. For example, sound waves will diffract through open doors as they have wavelengths of similar orders of magnitudes to the size of the door. However, light will not diffract as much through a door as the wavelength of light is many orders of magnitude smaller than the size of the door.

When waves diffract, diffraction patterns will be formed due to interference of the waves. Specific cases will be discussed below. Here are some examples of diffraction patterns:

A slit

When a wave passes through a slit, and the wave is observed a certain distance away from the slit, a diffraction pattern consisting of points of constructive interference and destructive interference will be formed. Visible patterns will occur when the size of the slit is comparable to the wavelength. For example, with visible light and a very narrow slit, the following pattern will be observed.

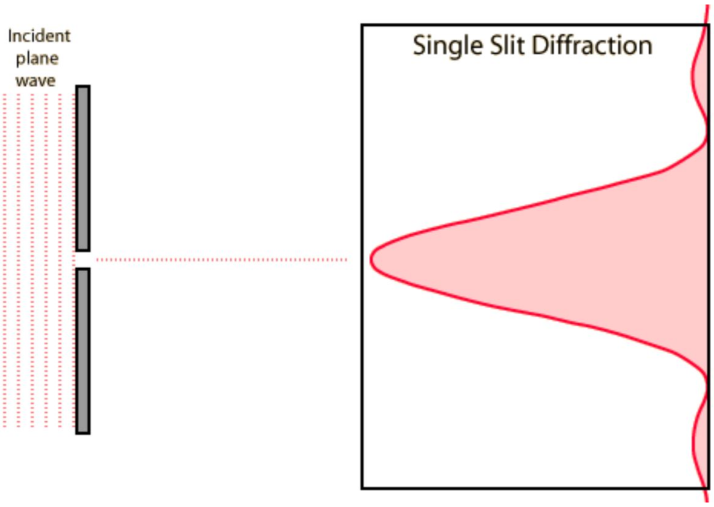


Figure 7.8: Diffraction pattern for a slit (credit:hyperphysics)

There is a central maximum which is twice the width of the maxima on either side and the pattern is symmetrical.

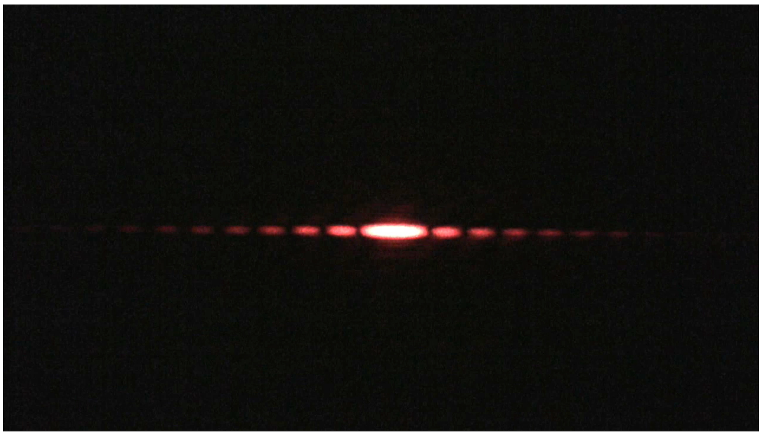


Figure 7.9: Photograph of single slit diffraction pattern

A circular hole

The diffraction pattern for a circular hole is similar to a single slit, except that instead of fringes, the pattern consists of rings.

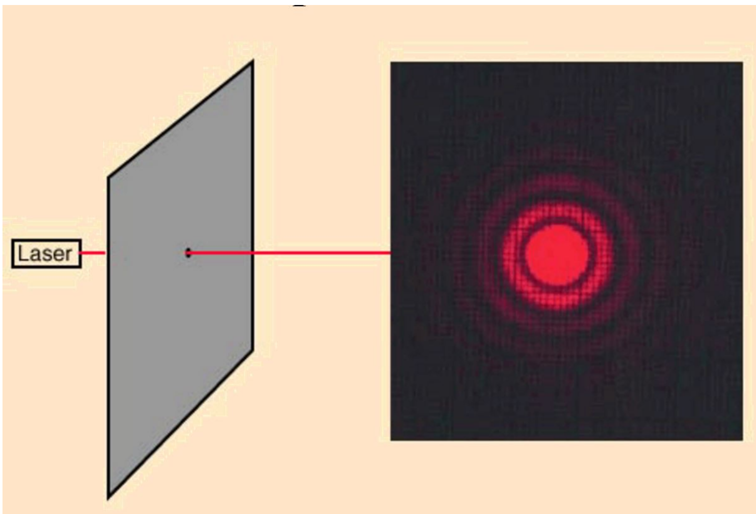


Figure 7.10: Circular hole diffraction pattern credit:hyperphysics

Although the examples given here are for visible light, remember that *all* waves can diffract.

A straight edge

A similar pattern of fringes is seen, due to constructive and destructive interference. Here the example given is for radio waves.

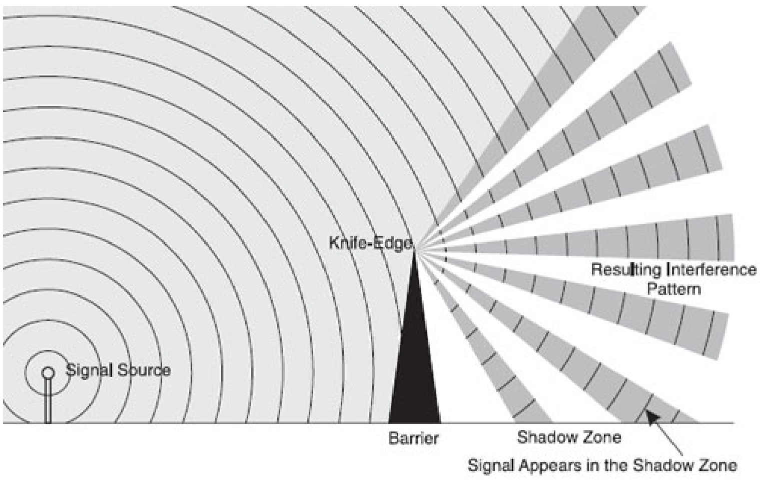


Figure 7.11: Diffraction at a straight edge credit:University of Alberta

(i) recognise and use the equation $n\lambda = bsin\theta$ to locate the positions of destructive superposition for single slit diffraction, where b is the width of the slit

When an electromagnetic wave, such as light, travels through a single slit, it will diffract. Therefore light from the slit reaches many points on a screen placed at some distance from the slit.

Consider a point on the screen. Light will reach this point from all points within the slit. The distances travelled from various points within the slit to the point on the screen will be different, and so there will be a path and phase difference. Therefore, as we move along the screen, there will be points of constructive interference and points of destructive interference.

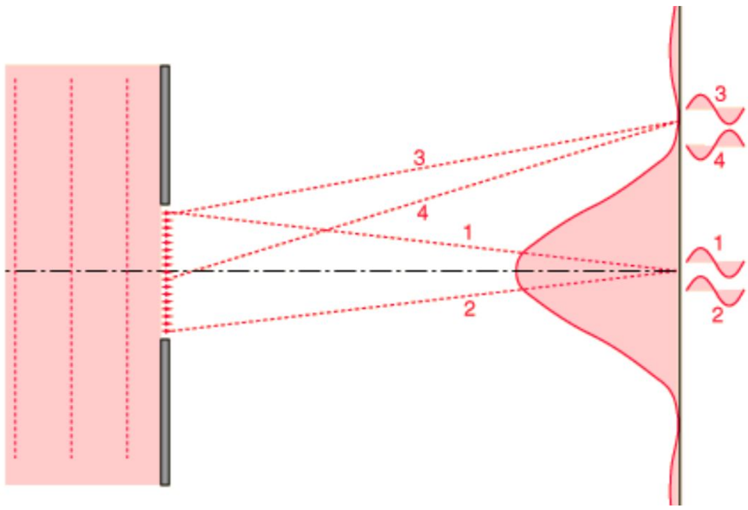


Figure 7.12: Single slit diffraction credit:hyperphysics

It can be shown that for a single slit of width b, the points of **destructive interference** will be at an angle θ given by the equation $n\lambda = bsin\theta$ Here, n, refers to the order of the minimum being considered.

(j) recognise and use the Rayleigh criterion $\theta \approx \frac{\lambda}{b}$ for resolving power of a single aperture, where b is the width of the aperture

As light travels through an instrument, such as the eye, or a telescope, it passes through a gap or aperture and will diffract. The diffraction pattern will depend on the shape of the aperture and the width.

If light from two different objects passes through the aperture, there will be two diffraction patterns that overlap. The Rayleigh criterion tells us the minimum angle between the two objects at which it is still possible to see them as separate. This is when the first diffraction minimum of one pattern coincides with the central maximum of another.

Working in radians, so that the approximation $\sin\theta \approx \theta$, then this minimum angle can be approximated by:

$$\theta \approx \frac{\lambda}{b}$$

(k) describe the superposition pattern for a diffraction grating and for a double slit and use the equation $d \sin\theta = n\lambda$ to calculate the angles of the principal maxima

(l) use the equation $\lambda = \frac{ax}{D}$ for double-slit interference using light

When light, or any other coherent waves, pass through a double slit, a diffraction pattern consisting of evenly spaced fringes is seen. This is due to the waves from each slit interfering with each other.

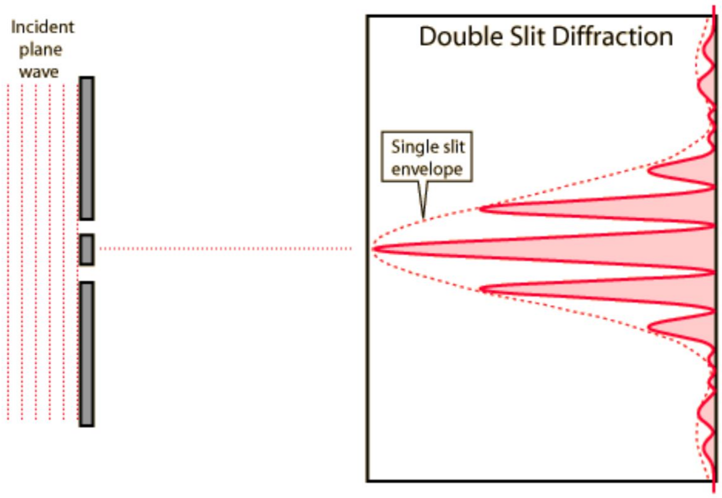


Figure 7.13: Double slit diffraction (credit:hyperphysics)

You can see that there is also a single slit envelope, which is much wider than the double slit pattern.

If the number of slits is increased, the pattern becomes more defined, and the fringes get narrower. Therefore for a diffraction grating, the pattern consists of sharp, evenly spaced peaks or bright spots.

If we consider two adjacent slits, you can see that for constructive interference to occur the following must be true:

$$d \sin\theta = n\lambda \tag{7.1}$$

d is the distance between adjacent slits and n is the order of the maxima that is being considered.

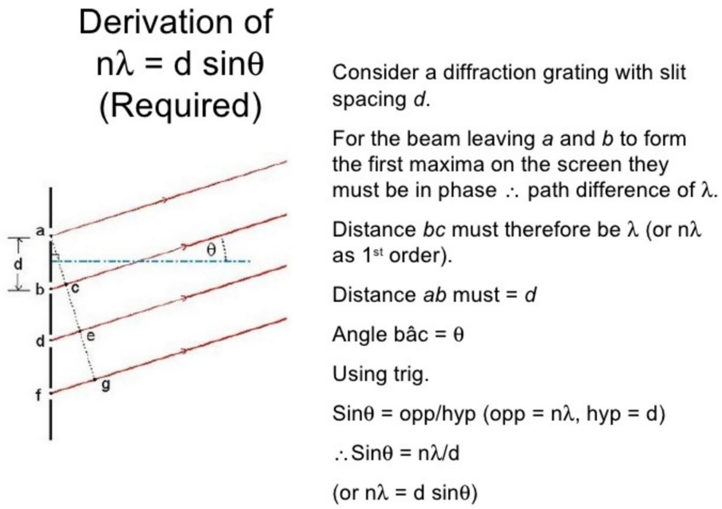


Figure 7.14: Derivation of grating formula

Equation 7.1 can be re-arranged to give

$$\sin\theta = \frac{n\lambda}{d} \tag{7.2}$$

This formula can be used for any waves, where there is diffraction from more than one slit.

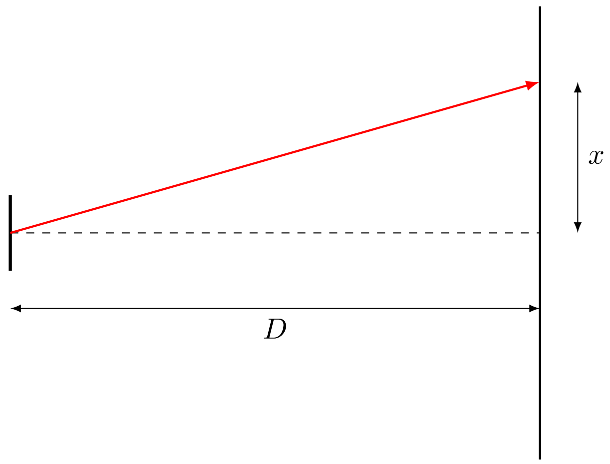


Figure 7.15: The double slit pattern

Now consider a double slit pattern, where the screen is now a distance D away from the slits and the slit separation is a instead of d. Bright fringes will be equally spaced on the screen and we can call the fringe separation x.

For the first maxima, $n = 1$, the equation from before now becomes:

$$\sin\theta = \frac{\lambda}{a}$$

but from the diagram you can see that:

$$\tan\theta = \frac{x}{D}$$

If θ is small, then the small angle approximation can be used. This is true if D is much larger than a.

$$\sin\theta \approx \tan\theta$$
$$\frac{\lambda}{a} = \frac{x}{D}$$
$$\lambda = \frac{ax}{D}$$

8 Atomic and Nuclear Processes

Content

- the nucleus
- nuclear processes
- probability and radioactive decay
- fission and fusion

Candidates should be able to:

(a) understand the importance of the α-particle scattering experiment in determining the nuclear model

In the famous scattering experiment an alpha source was placed in a lead container to produce a beam of alpha particles which were directed at a piece of gold leaf. Given the high energies of these alpha particles it was expected that they would simply crash through the gold leaf like a bullet through tissue paper. Instead, it was found that a few alpha particles were scattered through large angles and a small number came straight back.

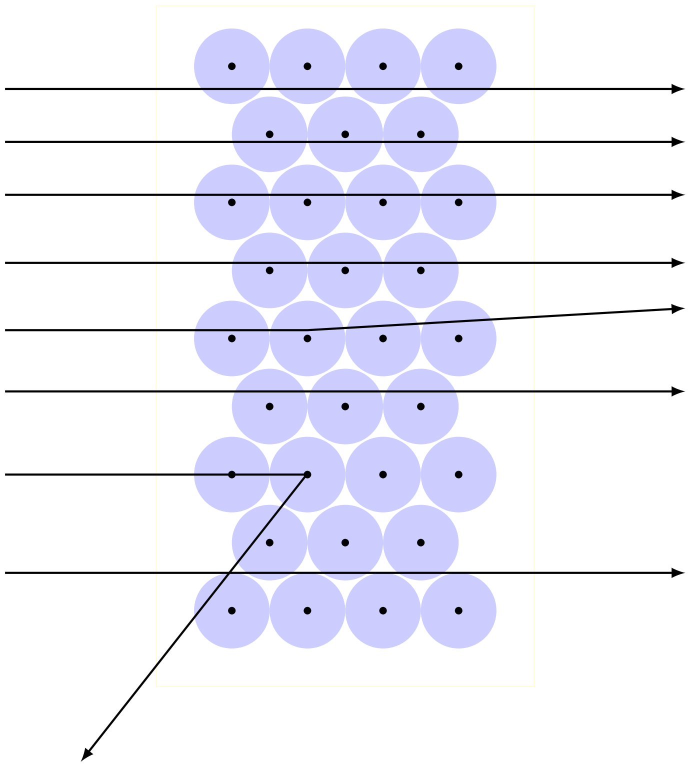


Figure 8.1: Scattering of alpha particles by gold atoms

This occasional scattering implied that the gold nuclei are very small compared to the size of the gold atom and contain almost all of the atom's mass.

(b) describe atomic structure using the nuclear model

The nuclear model has a very small (10^{-15} m) nucleus which is positively charged and made up of protons and neutrons. Around this nucleus move the much lighter, negatively charged electrons.

(c) show an awareness of the existence and main sources of background radiation

The radiation referred to in this part of the specification is *ionising* radiation. This is always present and is composed of both natural and artificial sources. The most common sources are shown in figure 8.2.

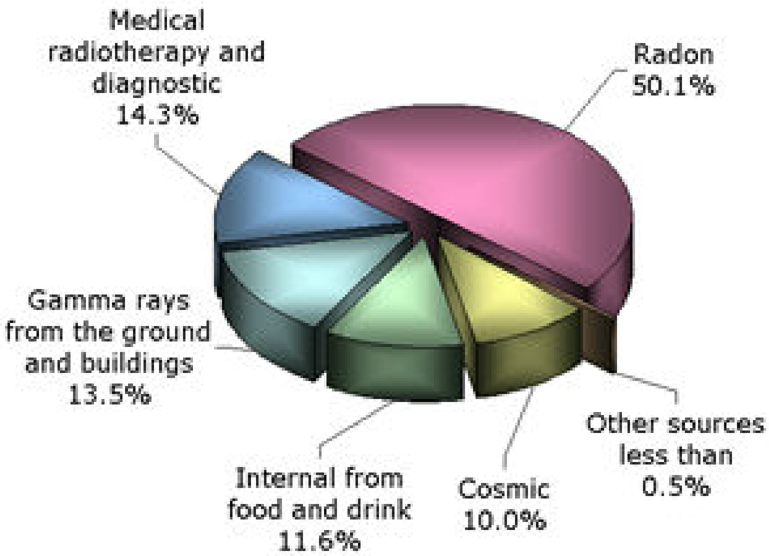


Figure 8.2: Background Radiation Sources in the UK (http://www.npl.co.uk/educate-explore/factsheets/ionising-radiation/)

(d) recognise nuclear radiations (α, $\beta-$, γ) from their penetrating power and ionising ability, and recall the nature of these radiations

Radiation	Penetrating Power	Ionising Ability	Nature
α	low	high	helium-4 nucleus
β	medium	medium	electron
γ	high	low	electromagnetic wave

Table 8.1: Three types of radiation

Table 8.1 shows the properties and nature of three types of radiation. Note that the penetrating power and ionising ability are inversely proportional as if a type of radiation is highly ionising it will interact with matter and therefore be stopped easily.

(e) write and interpret balanced nuclear transformation equations using standard notation

In nuclear equations the individual species are written as ${}^{A}_{Z}X$ where A is the *nucleon* number, Z is the *proton* number and X is the chemical symbol for the element. The three types of radiation are written as follows:

$$\alpha \quad {}^{4}_{2}He \qquad\qquad \beta \quad {}^{0}_{-1}e \qquad\qquad \gamma \quad {}^{0}_{0}\gamma$$

The electron is given a 'proton number' to enable the balancing of nuclear equations. In beta decay a neutron is transformed into a proton and an electron, therefore the nucleon number of the daughter does not change but the proton number increases by one. The beta particle's proton number of -1 balances this change.

In a balanced equation the sum of the proton numbers on each side must be equal, as must the sums of the nucleon numbers.

Example Question
Write a balanced nuclear equation for the decay of carbon-14 by beta minus decay.
Answer

$$\begin{aligned} {}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\,e \\ A : 14 = 14 + 0 \\ Z : 6 = 7 - 1 \end{aligned}$$

(f) understand and use the terms nucleon number (mass number), proton number (atomic number), nuclide and isotope

Nucleon number The total number of protons and neutrons in the nucleus.

Proton number The number of protons in the nucleus.

Nuclide A specific number of protons and neutrons which makes a unique nucleus. Usually defined by a combination of proton number / element name and mass number. E.g. carbon-14, ${}^{14}_{6}C$.

Isotope Isotopes are nuclei of the same element with different mass numbers, i.e. the same number of protons but different numbers of neutrons.

(g) appreciate the spontaneous and random nature of nuclear decay

Radionuclei decay spontaneously and randomly because of the fundamentally quantum nature of the process. This means that while one can make predictions about a collection of nuclei using half-life it is impossible to predict when any particular nucleus will decay.

(h) define and use the concept of activity as the number of decays occurring per unit time

Activity is defined in this way, but measured by counting the number of ionisation events occurring in a detector per unit time.

(i) understand qualitatively how a constant decay probability leads to the shape of a radioactive decay curve

A constant probability of decay means that the total rate of decay of a sample is proportional to the number of nuclei remaining. This means that the number of nuclei remaining decreases with decreasing rate. The activity is proportional to the number of nuclei remaining so it also decreases with decreasing rate.

(j) determine the number of nuclei remaining or the activity of a source after a time which is an integer number of half-lives

One half-life is the time taken for half of a sample to decay. Therefore after three half-lives the amount remaining is $\left(\frac{1}{2}\right)^3 N_0$ where N_0 is the original amount.

(k) understand the terms thermonuclear fusion, induced fission and chain reaction

Thermonuclear fusion This is the combining of two nuclei to form a new, heavier nucleus. It is the type of nuclear reaction which occurs in stars where hydrogen nuclei fuse to form helium. The electrostatic repulsion between the nuclei is overcome by the thermal energy of the particles involved, meaning that thermonuclear fusion can only occur at extremely high temperatures.

Induced fission This is the splitting of a large nucleus into two smaller, daughter nuclei. This does not occur spontaneously but must be induced by the collision of a neutron with the nucleus.

Chain reaction When a neutron induces fission in a large nucleus it is usually the case that as well as the two daughter nuclei, several further neutrons are released. These neutrons can then go on to cause further induced fission events in other heavy nuclei. This is known as a chain reaction.

9 Quantum Ideas

Content

- the photoelectric effect
- the photon
- wave-particle duality

Candidates should be able to:

(a) recall that, for monochromatic light, the number of photoelectrons emitted per second is proportional to the light intensity and that emission occurs instantaneously

(b) recall that the kinetic energy of photoelectrons varies from zero to a maximum, and that the maximum kinetic energy depends on the frequency of the light, but not on its intensity

(c) recall that photoelectrons are not ejected when the light has a frequency lower than a certain threshold frequency which varies from metal to metal

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(d) understand how the wave description of light fails to account for the observed features of the photoelectric effect and that the photon description is needed

Each of the above features *(a)* - *(c)* is discussed in turn below.

(a) In the wave description of light the energy an electron in the metal receives energy from the light arrives continuously. An electron therefore gradually absorbs enough energy to escape from the surface of the

metal, something which is not seen in practice. Additionally, increasing the intensity of a wave corresponds to increasing the amplitude of the oscillation. This would lead us to expect that increasing the intensity of light would give photoelectrons of a higher energy, rather than simply more of them.

The photon description of light accounts of both of these effects. When light arrives on the surface of the metal a single photon interacts with a single electron. Assuming the photon gives the electron enough energy to escape, the electron will leave the metal instantaneously. In the photon model, the intensity of the light is due to the number of photons arriving per second. Therefore if the intensity doubles, the number of photons doubles and the number of photoelectrons doubles.

(b) When the photoelectrons leave the metal some of the photon energy is used to break away from the metal and the remainder goes into their kinetic energy. Different photoelectrons have different amounts of energy, however there is a maximum kinetic energy the photoelectrons are found to have and this depends *only* on the frequency of the incoming radiation.

The wave model of light would allow different electrons to absorb different amounts of energy and therefore this relationship would not be seen.

(c) The energy from a photon of light is split between the energy required to escape the metal and the kinetic energy of the photoelectron. If the photon does not have enough energy to enable to the electron to escape the metal then no emission of photoelectrons is seen.

The wave model would still allow emission as a single electron could absorb energy from the wave over a longer period of time. However, since there are so many electrons in the surface of the metal it is vanishingly unlikely that a single photoelectron will interact with two photons.

(e) recall that the absorption of a photon of energy can result in the emission of a photoelectron

As described above.

(f) recognise and use $E = hf$

The energy of a photon of light is related to the frequency of that radiation using the equation $E = hf$ where h is Planck's Constant which has a value of $6.626 \times 10^{-34}\,\mathrm{J\,s}$.

(g) understand and use the terms threshold frequency and work function and recall and use

$$hf = \phi + \frac{1}{2}mv_{max} \tag{9.1}$$

Threshold Frequency, f_0. This is the minimum frequency required for the emission of photoelectrons to occur. This varies from metal to metal.

Work Function, ϕ. This is the energy required to remove an electron from the surface of the metal.

Equation 9.1 expresses the sharing of the energy of the photon (hf) between the work function (ϕ) and the kinetic energy of the photoelectron.

(h) understand the use of stopping potential to find the maximum kinetic energy of photoelectrons and convert energies between joules and electron-volts

We can measure the energy of the photoelectrons by placing a photocell in a circuit and using a potential difference to stop the flow of electrons.

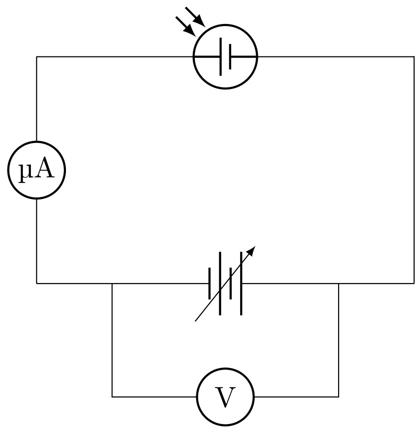


Figure 9.1: Measuring stopping potential

Figure 9.1 shows a simple set-up to measure the stopping potential for photoelectrons. Monochromatic light is shone on the photocell. Electrons leave the surface of the metal and travel around the circuit creating a small current. The variable power supply is gradually increased until no current flows through the circuit. At this point none of the electrons leaving the surface of the metal in the photocell have enough energy to cross the potential difference and create a current in the circuit. At this point the maximum energy of the photoelectrons can be equated to the energy required to cross a potential difference V:

$$\frac{1}{2}mv_{\max}^2 = eV \tag{9.2}$$

Since we are measuring the energies of electrons using potential differences, it makes sense to define a unit of energy in terms of these potential differences. Hence, 1 electronvolt is defined as the energy transferred by an electron moving through a potential difference of 1 volt.

$$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J} \tag{9.3}$$

(i) plot a graph of stopping potential against frequency to determine the Planck constant, work function and threshold frequency

By repeating the experiment shown in Figure 9.1 for different frequencies of light and measuring the stopping potential for each frequency we get the graph shown in Figure 9.2.

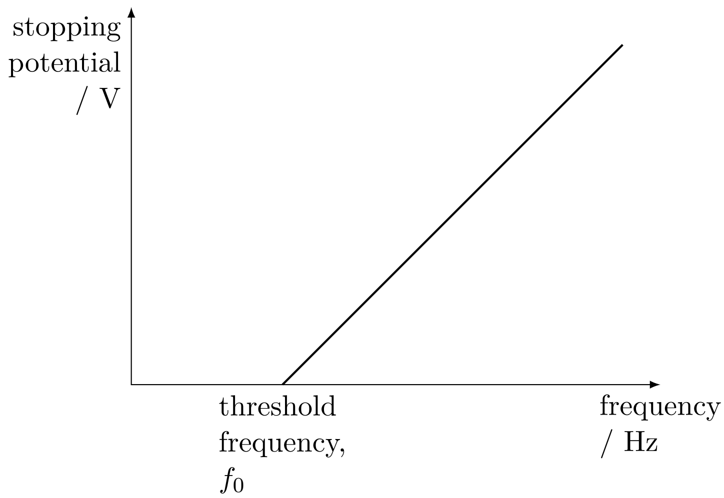


Figure 9.2: Stopping potential against threshold frequency

Equation 9.1 and 9.2 can be combined with and re-arranged to give

$$V = \frac{h}{e}f - \frac{\phi}{e} \tag{9.4}$$

Equation 9.4 describes the linear relation seen on the graph in Figure 9.2. The graph has a gradient of $\frac{h}{e}$ which enables the determination of the Planck Constant and the intercept with the x-axis gives the threshold frequency. The work function is simply the energy of a photon with the threshold frequency, i.e.

$$\phi = hf_0 \tag{9.5}$$

(j) understand the need for a wave model to explain electron diffraction

When electrons are fired at two closely spaced slits the result is entirely unlike what one would expect from a particle model. A particle model would predict that each electron would either go through one slit or the other, creating two regions at which electrons are found as shown in Figure 9.3.

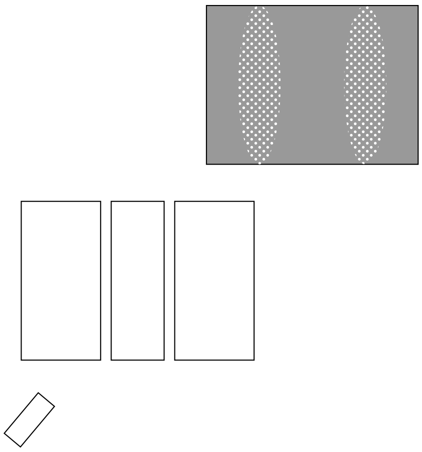


Figure 9.3: Expected pattern for electron diffraction

However, this behaviour is not seen but rather electrons form an interference pattern similar to that seen for light, as shown in Figure 9.4.

The mystery here is how can single particles form an interference pattern? If electrons are fired through two slits one at a time then they initially appear to arrive randomly. As more and more arrive they fill in the interference pattern. We explain this by saying that the probability of their arrival is determined by a form of wave-mechanics and as particles arrive they fill in the probability pattern as shown in figure 9.5.

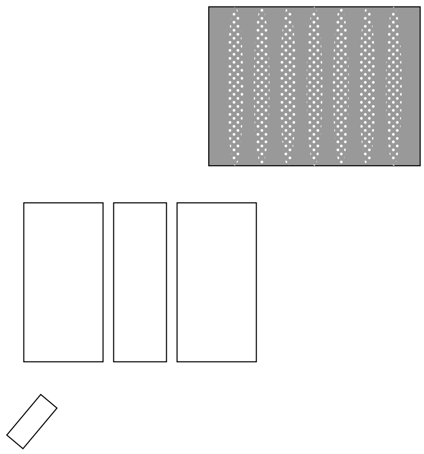


Figure 9.4: Observed pattern for electron diffraction

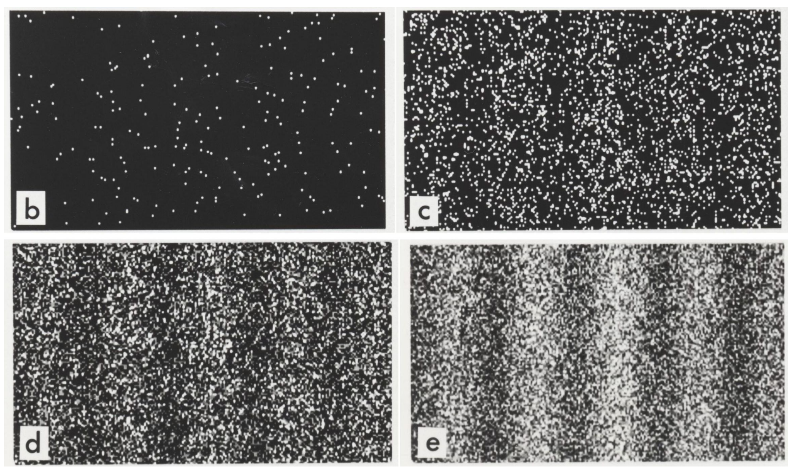


Figure 9.5: Build up of the interference pattern of electrons. *Credit: Belsazar, Wikimedia Commons*

(k) recognise and use

$$\lambda = \frac{h}{p} \tag{9.6}$$

for the de Broglie wavelength.

In order to explain the wave-like nature of electrons we need to be able to assign them a wavelength. de Broglie's thesis is that *all* particles have a wavelength which is defined in equation 9.6. This brings wave-particle duality from photons and electrons to all particles. The natural question here is why isn't interference ordinarily observed? The answer is that the de Broglie wavelength for macroscopic objects is extremely small and therefore these objects behave as particles, travelling in straight lines and not undergoing interference.

Example Question
Electrons will be diffracted by crystal lattices if they have a wavelength of around 0.1 nm. Calculate the speed and energy of such electrons.
Answer
The speed of these electrons can therefore be calculated as follows:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda} = 7.27 \times 10^{6}\,\mathrm{m\,s^{-1}}$$

$$E = \frac{1}{2}mv^2 = 151\,\mathrm{eV}$$

Part B

10 Rotational Mechanics

This chapter contains revision on the topics of:

- kinematics of uniform circular motion
- centripetal acceleration
- moments of inertia
- kinematics of rotational motion

Candidates should be able to:

(a) Define and use the radian

An angle in radians is defined by the length of the arc of circle it subtends divided by the radius of the circle. Numerically 2π radians is equivalent to 360 degrees.

(b) Understand the concept of angular velocity, and recall and use the equations $v = r\omega$ and $T = \frac{2\pi}{\omega}$

These equations are valid for an object travelling in a circle in a uniform manner.

Angular velocity is defined as the rate of change of angle, $\omega = \frac{d\theta}{dt}$ ie how many radians per second the rotating object passes through. Hence the time for one complete rotation will be $T = \frac{2\pi}{\omega}$

From the definition of the radian, in a small time interval δt we can say that the displacement of the rotating object is $\delta s \approx r\delta\theta$ (see figure 10.1), and hence in the limit as we make the time interval smaller that means $v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$

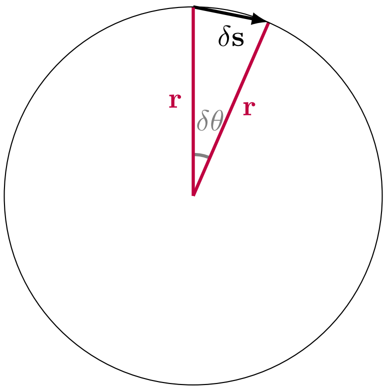


Figure 10.1:

Example Question
Calculate the linear velocity of the Earth relative to the sun, given the Earth-Sun distance is 1.5 x 10^{11}m
Answer
First calculate the angular velocity of the Earth. It performs a complete orbit (ie 2π radians) in 1 year, so $\omega = \frac{2\pi}{365\times24\times3600}$
Then $v = r\omega = 30000\ ms^{-1}$

(c) Derive, recall and use the equations for centripetal acceleration $a = \frac{v^2}{r}$

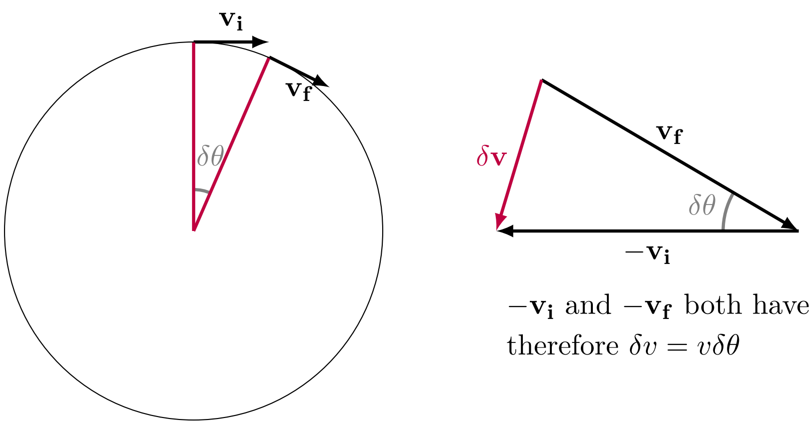
and $a = r\omega^2$

Since acceleration is defined as change in velocity, we can see from the following diagram (figure 10.2) that the velocity change when a uniformly rotating object moves through a small angle $\delta\theta$ can be written as $\delta v = v\delta\theta$ since the magnitudes of the initial and final velocity are both equal to v.

The acceleration is therefore $a \approx \frac{v\delta\theta}{\delta t}$ and as we let $\delta t \to 0$ we get $a = v\frac{d\theta}{dt} = v\omega$. since $v = r\omega$ we also get $a = \frac{v^2}{r} = r\omega^2$

(d) Recall that $F = ma$ applied to circular motion gives resultant $F = \frac{mv^2}{r}$

Since $a = \frac{v^2}{r}$ and $F = ma$ we can combine these to give $F = \frac{mv^2}{r}$. This can be very useful for example when combined with Newton's law of Gravity to explain planetary orbits etc.



$-\mathbf{v_i}$ and $-\mathbf{v_f}$ both have magnitude v, therefore $\delta v = v\delta\theta$

Figure 10.2:

Example Question
Show that, for a circular orbit, the time period squared is proportional to the radius cubed.
Answer
Start by equating $F = mr\omega^2$ with $F = \frac{GMm}{r^2}$ (we can ignore the minus sign)

$mr\omega^2 = \frac{GMm}{r^2}$

Then rearrange and cancel m to find

$r^3 = \frac{GM}{\omega^2}$

then using $\omega = \frac{2\pi}{T}$ we get

$r^3 = \frac{GM}{4\pi^2}T^2$

(e) Describe qualitatively the motion of a rigid solid object under the influence of a single force in terms of linear acceleration and rotational acceleration.

When a rigid solid object has a single force applied to it, it may just result in linear acceleration, if the force acts through the centre of mass of the object. However it is more likely that the force will not act through this point, and therefore also cause some rotational acceleration, causing the object's angular velocity to change.

The remaining sections in this chapter deal with how we describe this rotational acceleration, but it should be noted that they are asterisked sections so will only form part of section 2 of paper 3.

*(f) *Recall and use $I = \Sigma mr^2$ to calculate the moment of inertia of a body consisting of three or fewer point particles fixed together*

The moment of inertia of a body is the rotational analogue of mass, and basically describes how resistant an object is to angular acceleration when a torque is applied (in the same way that mass describes how resistant an object is to accelerating linearly when a force is applied...)

A point mass m at a distance r from an axis of rotation will have Moment of Inertia $I = mr^2$

More generally the moment of inertia is the sum of all the mr^2 values for all the point masses that make up an object. For a small number of particles this can just be found by adding the values of individual moments of inertia.

*(g) *Use integration to calculate the moment of inertia of a ring, a disk and a rod*

For more complex bodies, the summing of moments of inertia needs to be done by setting up an integration. This is acheived by summing moments of inertia for a series of small sections of the object, strips or rings of thickness δx say, and using the fact that as $\delta x \to 0$ the sum $\Sigma \delta x \equiv \int dx$

As an example, consider a disk of radius r , thickness t and density ρ. The moment of inertia about its central axis could be found by splitting it up into rings of radius x and thickness δx. See figure 10.3.

Each ring has moment of inertia equal to its mass multiplied by x^2, as all the particles in it have (approximately) the same distance from the axis.[1]

Hence $I_{ring} = (2\pi x t \rho \delta x)x^2 = 2\pi \rho x^3 \delta x$

To find the moment of inertia of the disk, we sum all the rings and let their size δx tend to 0.

[1] This can be proved by another, much simpler, sum. If you imagine the ring as the sum of many small points of mass δm each at the same radius r, the moment of inertia becomes $\Sigma r^2 \delta m$ which is the same as $r^2 \Sigma \delta m$ and hence $I = mr^2$

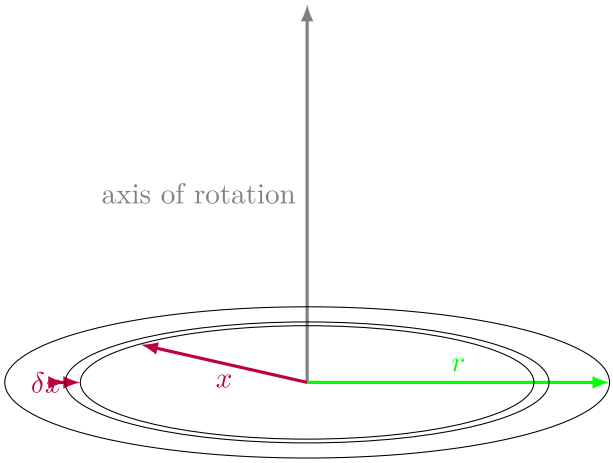


Figure 10.3:

$I_{disk} = \Sigma 2\pi\rho x^3 \delta x$

$I_{disk} = \int_0^r 2\pi\rho t x^3 \delta x$

giving

$I_{disk} = \frac{2\pi\rho t r^4}{4}$

which is equal to

$I_{disk} = \frac{mr^2}{2}$

Other objects would be derived in a similar way. You should get $I = \frac{mr^2}{3}$ for a rod rotating about one end, and $I = \frac{mr^2}{12}$ for a rod rotating about its centre of mass.

*(h) *Deduce equations for rotational motion by analogy with Newton's laws for linear motion, including $E = \frac{1}{2}I\omega^2$, $L = I\omega$ and $\Gamma = I\frac{d\omega}{dt}$*

Here is a table outlining the analogies between linear and rotational moton:

Linear Motion	**Rotational Motion**
Mass m	Moment of Inertia I
linear velocity v	Angular velocity ω
Force F	Torque Γ
Linear Momentum p	Angular Momentum L
$p = mv$	$L = I\omega$
$F = ma = m\frac{dv}{dt}$	$\Gamma = I\frac{d\omega}{dt}$
$KE_{linear} = \frac{1}{2}mv^2$	$KE_{rotational} = \frac{1}{2}I\omega^2$

*(i) *Apply the laws of rotational motion to perform kinematic calculations regarding a rotating object when the moment of inertia is given.*

Example Question
The moment of inertia of a large flywheel in a factory is 60 kgm^2. Calculate how long it would take the flywheel to obtain an angular velocity of 6.0 rad s^{-1} when a torque of 24 Nm was applied.
Answer
First use $\Gamma = I\frac{d\omega}{dt}$ to find the angular acceleration:
$\frac{d\omega}{dt} = \frac{\Gamma}{I} = \frac{24}{60} = 0.4$ rad s^{-2}
Then the time taken will be
$T = \frac{6}{0.4} = 15$ seconds

11 Oscillations

Simple Harmonic Motion

(a) Recall the condition for simple harmonic motion and hence identify situations in which simple harmonic motion will occur

Any oscillation where the acceleration is proportional to the displacement from an equilibrium position and in the opposite direction to the displacement is described as Simple Harmonic Motion (SHM).

These two conditions can be expressed in equation form as:

$$a \propto -x$$

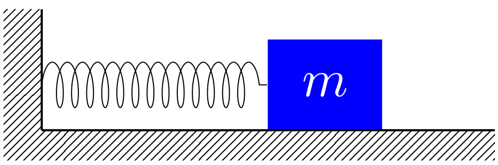
Notice the minus sign to signify that the acceleration is in the opposite direction to the displacement.

We will mainly be looking at idealised springs and pendulums to understand the maths behind it but the real reason for studying SHM is that it is an excellent approximation for many of the oscillations that we come across in the natural world.

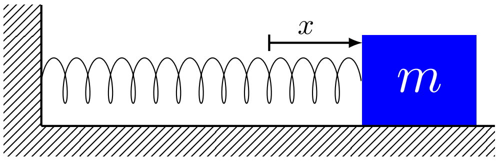
So while the topic is introduced with some rather prosaic examples this provides the building blocks to understanding earthquakes or how atoms vibrate in lattices as well as any musical instrument you can think of.

So without further ado let's look at our first and perhaps simplest example.

A mass on a spring on a smooth horizontal surface.



What happens if we move the mass by a distance x and then let it go?



Before going into any mathematical detail we can think about what will happen to the mass.

- We know that there is now a force from the stretched string which is pulling the mass back to its original position.
- This will cause the mass to accelerate to the left.
- Once the mass reaches its starting point it will overshoot and start compressing the spring.
- This creates a resultant force which will again restore the mass to its original position.
- This will cause the mass to accelerate to the right.
- The mass will overshoot again and the cycle will continue.

Note that the direction of the acceleration is always opposite to the displacement.

The mathematical treatment for this starts very simply with a basic knowledge of Hooke's law.

The force on a stretched or compressed spring is given by

$$F = -kx$$

where k is the spring constant.

Newton's second law tells us that

$$F = ma$$

Putting these together we get

$$a = \frac{F}{m} = -\frac{k}{m}x$$

Because k and m are constant this satisfies the condition for SHM,

$$a \propto -x$$

Example Question
A 2kg mass attached to a horizontal spring of spring constant $0.3Nm^{-1}$ is stretched by 10cm and then released.
Find the maximum acceleration of the mass

Answer
Using the formula $a = -\frac{k}{m}x$ The maximum acceleration will occur when x is a maximum so

$$acceleration_{max} = -\frac{0.3}{2} \times 0.1$$

$$acceleration_{max} = -0.015ms^{-2}$$

*(b) * show that the condition for simple harmonic motion leads to a differential equation of the form*

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

and that

$$x = Acos\omega t$$

is a solution to this equation

Acceleration is the rate of change of velocity

$$a = \frac{dv}{dt}$$

and velocity is the rate of change of displacement

$$v = \frac{dx}{dt}$$

Putting these two together gives the condition for simple harmonic motion as.

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where ω^2 is a (strange choice of) constant.

This is a second order differential equation and to solve it we need to find a function which when differentiated twice gives us the negative of the original function.

By inspection we can see that functions with $\sin \omega t$ and $\cos \omega t$ are both possible solutions.

e.g.

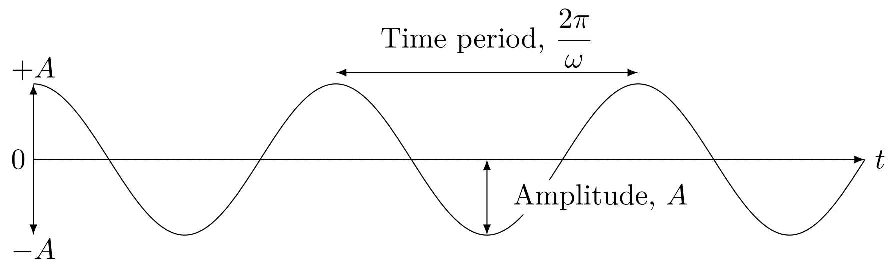
$$x = Acos\omega t$$
$$\frac{dx}{dt} = -A\omega sin\omega t$$
$$\frac{d^2x}{dt^2} = -A\omega^2 cos\omega t = -\omega^2 x$$

So $x = Acos\omega t$ is a solution (using sin is equivalent but with a phase offset.)

Looking at this function we can see that it will give us a cosine wave with an Amplitude of A and a time period of $\frac{2\pi}{\omega}$.

This gives us a frequency on $\frac{1}{T} = \frac{\omega}{2\pi}$

So ω is the angular frequency (which explains our strange choice of constant).



We now have a general expression for the displacement x of the object after a time t.

The value of ω will be given by the physical properties of the system.

e.g. For the mass on a spring we looked at earlier

$$
\begin{aligned}
F &= -kx && \text{Force on mass by Hooke's Law} \\
a &= -\frac{k}{m}x && \text{gives acceleration} \\
\frac{d^2x}{dt^2} &= -\omega^2 x && \text{comparing with condition for SHM} \\
\omega^2 &= \frac{k}{m} && \text{gives value for constant } \omega \\
\omega &= \sqrt{\frac{k}{m}}
\end{aligned}
$$

*(c) * use differential calculus to derive the expressions*

$$v = -A\omega sin\omega t$$

and

$$a = -A\omega^2 cos\omega t$$

for simple harmonic motion.

This is achieved very simply by differentiating our expression for displacement with respect to time as velocity $=\frac{dx}{dt}$ and acceleration $= \frac{dv}{dt}$

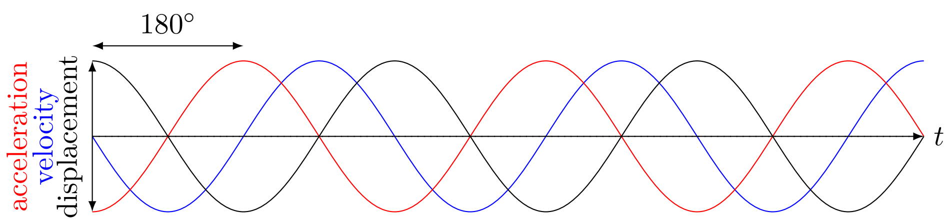
Remember that the differential of cos is -sin.

so

$$
\begin{aligned}
x &= Acos\omega t \\
v = \frac{dx}{dt} &= -A\omega sin\omega t \\
a = \frac{dv}{dt} &= -A\omega^2 cos\omega t
\end{aligned}
$$

(d) understand the phase differences between displacement, velocity and acceleration in simple harmonic motion

Plotting these equations onto a graph gives the following.



Note the phase shift between displacement, velocity and acceleration.

Velocity is $\frac{\pi}{2}$ or 90°out of phase with displacement.

Acceleration is π or 180° out of phase with displacement.

So when acceleration or displacement has a maximum magnitude, the velocity is zero

*(e) *recognise and use the expressions $x = Acos\omega t, v = -A\omega sin\omega t, a = -A\omega^2 cos\omega t$ and $F = -m\omega^2 x$ to solve problems*

We can use these equations to solve problems by identifying the variables and substituting.

The last equation is a combination of $a = -\omega^2 x$ and $F = ma$

Example Question
A mass attached to a spring is set into motion on a smooth horizontal surface. The amplitude of oscillation is 15mm and it takes 5 seconds to perform 20 oscillations.
Calculate the time period and frequency.
Hence calculate the velocity and acceleration after 9.3 seconds.

Answer
The first part doesn't require any knowledge of SHM. If there are 20 oscillations in 5 seconds this means $\frac{20}{5}$ oscillations in one second.
So $f = 4Hz$
and $T = \frac{1}{f} = 0.25s$
Now that we have the frequency we can calculate the angular frequency
$\omega = 2\pi f = 8\pi$
We are given the amplitude A as 15mm and the time is 9.3 seconds so we can now use our SHM formulae.
So for the velocity

$$v = -A\omega sin\omega t$$

$$v = -15 \times 8\pi sin(8 \times \pi \times 9.3)$$

$$v = -360mms^{-1} = .36ms^{-1}$$

and for acceleration

$$a = \frac{dv}{dt} = -A\omega^2 cos\omega t$$

$$a = 15(8 \times \pi)^2 cos(8 \times \pi \times 9.3)$$

$$a = 2900mms^{-2} = 2.9ms^{-2}$$

(f) recall and use $T = \frac{2\pi}{\omega}$ as applied to a simple harmonic oscillator

Angular frequency w is how many radians per second the oscillator goes through with one complete cycle being 2π radians.

So $\frac{1}{\omega}$ is how many seconds is takes to complete one radian.

and the time for one complete cycle is $T = \frac{2\pi}{\omega}$

*(g) *show that the total energy of an undamped simple harmonic system is given by $E = \frac{1}{2}mA^2\omega^2$ and recognise that this is a constant*

For an undamped system no energy is lost to the surroundings so the total energy is conserved.

The total energy will take the form of Potential Energy and Kinetic Energy.

Considering our mass on a spring at maximum displacement the spring will be at maximum extension and the mass will be stationary. So the total energy will be in the form of P.E.

Similarly when the displacement is zero all the energy will be K.E.

At any point in between the total energy will be a mixture of K.E. and P.E.

As the total energy is the same at any point we can pick what's easiest to derive an equation for it.

So at zero displacement.

$$\begin{aligned} T.E. &= K.E. \\ &= \frac{1}{2}mv_{max}^2 \\ &= \frac{1}{2}mA^2\omega^2 \end{aligned}$$

(h) recognise and use $E = \frac{1}{2}mA^2\omega^2$ to solve problems

Example Question
A mass of 8kg is attached to a spring with a spring constant of $5Nm^{-1}$ on a smooth horizontal surface. It is pulled back a distance of 20 cm and then released.
Calculate the total energy of the system, the time period of oscillation and the velocity of the mass when the displacement is 10 cm.
Answer
One way to approach this question is by using our equation for the energy stored in a spring.

$$P.E = \frac{1}{2}kx^2$$

$$P.E = \frac{1}{2} \times 5 \times .2^2 = 0.1J$$

We also know that all the energy at this point is in the form of potential as the mass is not moving so this is the total energy of the system and will not change.
For the time period we can first calculate the angular frequency using our energy equation.

$$\begin{aligned} E = \frac{1}{2}mA^2\omega^2 &= 0.1J \\ \therefore \omega &= \sqrt{\frac{2E}{mA^2}} \\ &= \sqrt{\frac{2 \times 0.1}{5 \times 0.2^2}} \\ &= 1rads^{-1} \\ \omega &= \frac{2\pi}{T} \\ \therefore T &= 2\pi = 6.2seconds \end{aligned}$$

When the mass has a displacement of 10cm, the total energy will be the kinetic and potential energy.

$$\begin{aligned} T.E. = P.E + K.E. &= 0.1J \\ \frac{1}{2} \times 5 \times .1^2 + K.E. &= 0.1J \\ 0.025 + K.E. &= 0.1J \\ \implies K.E. &= 0.075J \\ \frac{1}{2}mv^2 &= 0.075J \\ v &= 0.17ms^{-1} \end{aligned}$$

(i) distinguish between free, damped and forced oscillations

A free oscillator is one which is set in motion and left to oscillate without any external forces.

e.g. a pendulum.

A forced oscillation is one where an external periodic force is applied.

e.g. pushing a child on a swing.

Damping is where frictional forces remove energy from the system and the oscillations die down.

This can be increased intentionally for example adding thick oil to car suspension to prevent you bouncing around every time the car hits a bump.

If a system is lightly damped it will gradually come to rest after a number of cycles.

An important case is critical damping where the system comes to rest without overshooting and in the shortest possible time.

(j) recall how the amplitude of a forced oscillation changes at and around the natural frequency of a system and describe, qualitatively, how damping affects resonance.

Every system has a natural frequency of oscillation which will depend on its physical properties. In the case of a mass on a spring it will depend on the mass and spring constant but more complex examples e.g. washing machines or the millennium bridge will also have natural frequencies.

If we apply a driving force to an oscillator, the effect it has will depend on the amplitude of the driving force and its frequency compared to the natural frequency of the system.

For a simple example try holding a spring with a mass on the end of it and move your hand up and down rhythmically.

If you move your hand very slowly the spring won't stretch or contract and the mass will simply follow the movement of your hand. So the amplitude will be the same as the amplitude of the driving frequency.

As you start to increase the frequency of your hand movement the spring will start to stretch and contract and the mass will move more than your hand. So the amplitude of oscillation increases.

Once the frequency of your hand is close to the natural frequency you are adding energy at just the right point in the cycle to increase the amplitude with each cycle and the amplitude gets very large. We call this resonance.

If we know move our hand with a very high frequency, the spring stretches and compresses but the mass doesn't much have time to accelerate hence the amplitude starts to get very small.

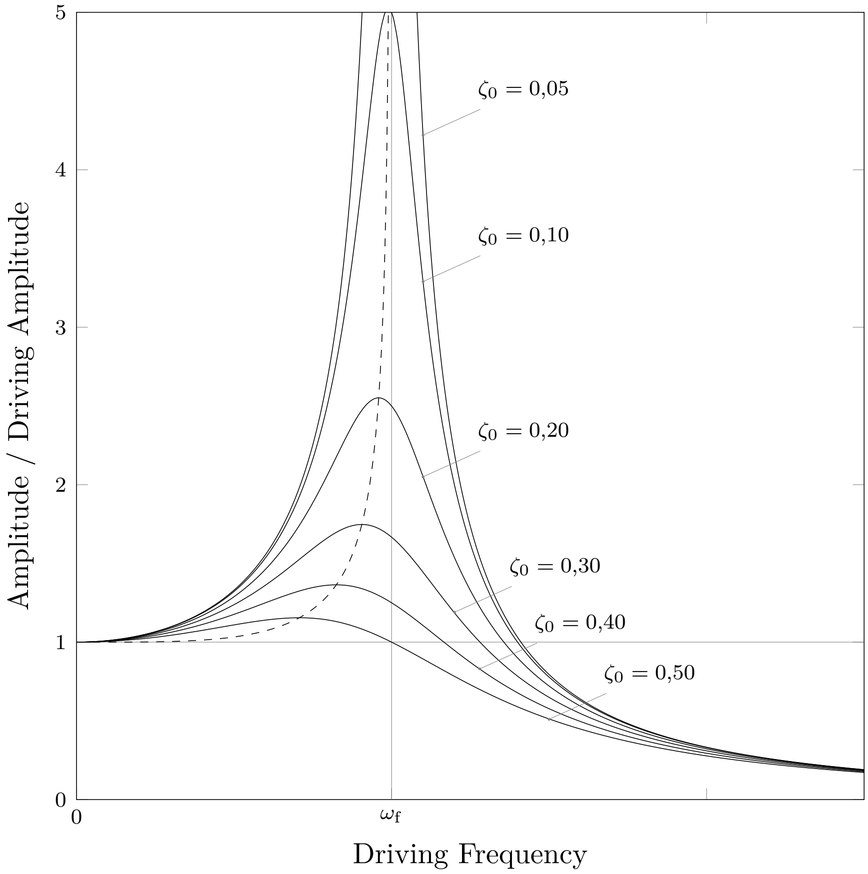
- Low frequency driving force, amplitude of oscillator = amplitude of driver.
- Driving frequency around natural frequency, resonance - very high amplitude oscillations.
- High frequency driving force, low amplitude oscillations.

In the real world we need to consider damping which will affect all systems to some degree. Damping will come from frictional forces and will remove energy from the system every cycle. If the energy added by the driving force is more than is removed by damping then the amplitude will increase. As the amplitude increases, the energy removed each cycle will also increase until a maximum amplitude is reached.

From this it is clear that the more damping there is, the lower the amplitude.

Another feature is that as we have more damping, the frequency at which resonance occurs is less. You can think of this as damping slowing down the system and so reducing the natural frequency (although strictly speaking the natural frequency refers to an undamped system).

This is all expressed in the following graph where I have used the damping ratio ζ as a measure of how much damping there is. Its not on the syllabus, you will only need to know qualitatively how damping affects resonance e.g. light damping and heavy damping but I though it would be good to introduce you to another Greek letter.



- At very low driving frequencies the amplitude is the same as the driving amplitude.
- With low damping we get a sharp resonance peak.
- More damping creates a broader resonance peak.
- Increasing damping reduces the amplitude at all frequencies.
- The resonance peak shifts lower with increased damping.
- More damping creates a broader resonance peak.
- As the frequency gets high the amplitude gets low regardless of damping.

12 Electric Fields

Content

- concept of an electric field
- uniform electric fields
- capacitance
- electric potential
- electric field of a point charge

Candidates should be able to:

(a) explain what is meant by an electric field and recall and use $E = \frac{F}{q}$ for electric field strength

An electric field is the region of space in which electrical forces are exerted on charged bodies. The definition of electric field strength is force per unit charge, and this is written in symbols as:

$$E = \frac{F}{q}$$

Electric field therefore has units of $\mathrm{N\,C^{-1}}$.

(b) recall that applying a potential difference to two parallel plates stores charge on the plates and produces a uniform electric field in the central region between them

If an electric potential is created between two parallel plates a distance d apart then an electric field exists between them. Except at the edges, the field is *uniform*. This is shown in figure 12.1 by the fact that the field lines are parallel.

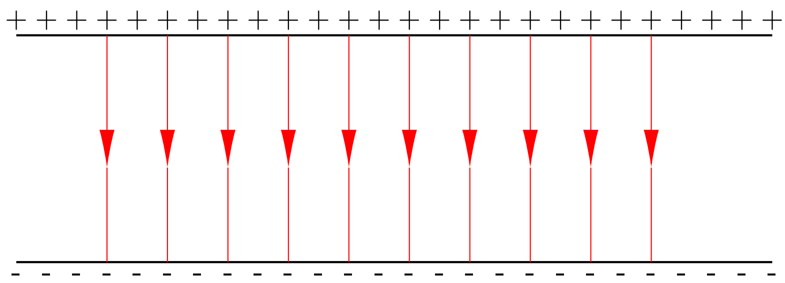


Figure 12.1: Uniform field between parallel plates

Note that field lines show the direction in which a force would act on a *postive* charge so they are from positive to negative.

(c) derive and use the equations $Fd = QV$ and $E = \frac{V}{d}$ for a charge moving through a potential difference in a uniform electric field

From the definition potential difference we can say that the work done moving through a potential difference is

$$W = QV$$

and that is equal to the force multiplied by the distance moved,

$$Fd$$

If the movement is through a uniform field we can equate these two to give

$$Fd = QV$$

The definition of electric field strength gives $F = QE$ and therefore substitution and re-arrangement give

$$E = \frac{V}{d}$$

(d) recall that the charge stored on parallel plates is proportional to the potential difference between them

This is an application of Gauss' law which relates the electric field strength around an object to the charge contained within a surface. It is enough to recall this fact at Pre-U.

(e) recall and use $C = \frac{Q}{V}$ for capacitance

This is the definition of capacitance and should be learnt. It can also be calculated from the gradient of a graph of Q against V.

(f) recognise and use $W = \frac{1}{2}QV$ for the energy stored by a capacitor, derive the equation from the area under a graph of charge stored against potential difference, and derive and use related equations such as $W = \frac{1}{2}CV^2$

If a capacitor is partially charged with a charge Q then to increase its potential difference by δV will require a small amount of work δW such that

$$\delta W = Q\delta V$$

If a graph is plotted of Q against V then this can be seen as the area of a small section. Thus, the energy required to charge a capacitor from uncharged to a p.d. of V is given by the area of a graph of Q against V from zero to V, hence

$$W = \frac{1}{2}QV$$

We can substitute for each quantity in turn to give the following variations of the equation:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

This can, of course, also be done with integration

$$W = \int_0^V QdV = \int_0^V CVdV = \frac{1}{2}CV^2$$

(g) analyse graphs of the variation with time of potential difference, charge and current for a capacitor discharging through a resistor

When a capacitor discharges through a resistor the potential difference on the capacitor drives current around the circuit. Since it is this current which removes charge from the capacitor, it is the case that the rate of change of charge on the capacitor is proportional to the charge on the capacitor.

$$\frac{dQ}{dt} = -I = -\frac{V}{R} = -\frac{Q}{RC}$$

This is a first order differential equation and has a solution

$$Q = Q_0 e^{-\frac{t}{RC}}$$

The substitutions $Q = CV$ and then $I = \frac{V}{R}$ can be used to get similar equations for each of the above. The graph of this change is exponential decay. The key features are the initial value (e.g. V_0) and the fact that all three quantities tend to zero.

(h) define and use the time constant of a discharging capacitor

The time constant, τ, is defines as follows:

$$\tau = RC$$

This means that in one time constant the current, voltage and charge on a capacitor have declined to $\frac{1}{e}$ of their initial values.

A common rule-of-thumb from electronics is that a capacitor takes five time constants to discharge. If we plug this into the equation

$$V = V_0 e^{-\frac{t}{RC}} = V_0 e^{-5} = 0.067\ V_0$$

So the voltage across the capacitor has declined to less than 1% of its initial value.

(i) analyse the discharge of a capacitor using equations of the form $x = x_0 e^{\frac{-t}{RC}}$

Much of this has been covered above. One important point to note is that the analysis of capacitor decay is usually carried out by plotting a graph of $\ln V$ against time. This changes the equation to give

$$\ln V = -\frac{t}{RC} + \ln V_0$$

Therefore the gradient of the graph is $-\frac{1}{RC}$ and the intercept $\ln V_0$.

(j) understand that the direction and electric field strength of an electric field may be represented by field lines (lines of force), and recall the patterns of field lines that represent uniform and radial electric fields

Field lines are a way of visualising the field. The direction of the field lines is from north to south and show the direction in which a positively charged particle will move. The density of field lines represent the strength of the field. For example in figure 12.2 the region **A** contains a uniform strong field which is stronger than the field at **B** as the field lines are more closely spaced.

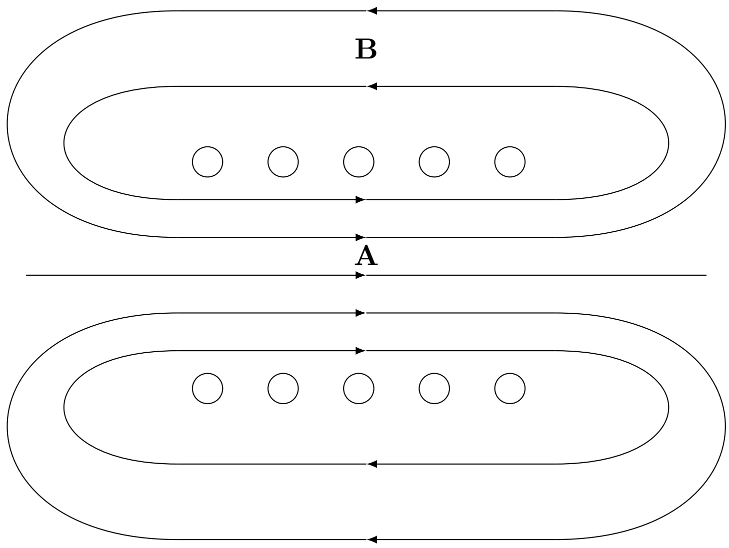


Figure 12.2: Field around a solenoid

(k) understand electric potential and equipotentials

Electric potential is defined as the energy per unit charge due to an electric field. Space without an electric field is defined as having zero potential. An equipotential is a line or surface on which the potential is a constant value. Therefore no work is done against the electric force moving along an equipotential. On diagrams equipotentials always cross field lines at right angles.

(l) understand the relationship between electric field and potential gradient, and recall and use $E = -\frac{dV}{dX}$

The strength of the electric field at any point is equal to the negative of the potential gradient. This can be seen most easily in a uniform field. If a unit charge is moved through a distance Δx within a uniform field E then the work down equals $Fx = -Ex$ (for a unit charge). The negative sign comes from the fact that if the force is doing work on the charge it must be moving in the opposite direction to the force on that charge. Thus, the change in energy per unit charge is given by $\Delta V = -E\Delta x$ or $E = -\frac{\Delta V}{Deltax}$. This can be generalised for a non-unform field as

$$E = -\frac{dV}{dx}$$

(m) recognise and use

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

for point charges

The equation above is known as Coulomb's law and enables the calculation of the force between two point charges separated by a distance r.

(n) derive and use $E = \frac{Q}{4\pi\epsilon_o r^2}$ for the electric field due to a point charge

This is simply from the definition of electric field strength:

$$E = \frac{F}{Q} = \frac{1}{Q_2}\frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_o r^2}$$

*(o) *use integration to derive $W = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$ from $F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$ for point charges*

Since free space is defined as having zero potential, the electrostatic potential energy, W, of a particle is equal to the work done by the field moving the particle from a distance r to infinity. Since work done is equal to $\int F dx$ it follows:

$$W = \int_r^\infty \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} dr = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

You can define this alternatively by thinking about the work done bringing a charged particle from infinity to a distance r. If you do this it is important to remember that F acts in the opposite direction to r so the limits of integration are reversed and the formula has a minus sign, thus reaching the same answer.

*(p) *recognise and use $W = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$ for the electrostatic potential energy for point charges.*

This is simply applying the equation above. This will allow the calculation of changes in potential energy and possibly transfer to other forms of energy (e.g. kinetic).

13 Gravitation

(a) state Kepler's laws of planetary motion Before you learn Kepler's laws (which you MUST learn) you should spend some time looking through the chapter on rotational mechanics and making absolutely sure that you know how to apply Newton's laws of motion to an orbiting body. This is vital or you won't get the maths in this chapter.

The specification dictates that you need to be able to state Kepler's Laws. They are as follows:

1. Planets move in elliptical orbits with the Sun at one focus. *(motion in ellipses is not part of the specification, so don't worry about the mathematics of this – we approximate to a circle for pre-U)*

2. The Sun-planet line sweeps out equal areas in equal times.

3. The orbital period squared of a planet is proportional to its mean distance from the Sun cubed.

First Law

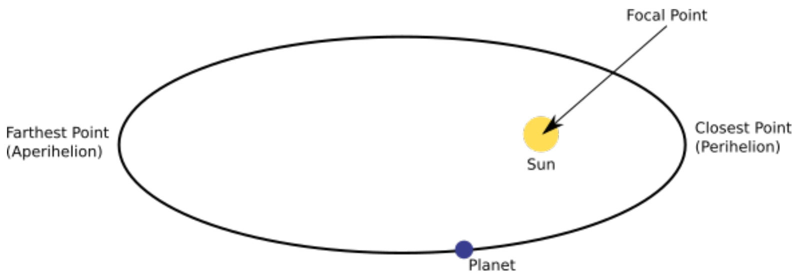


Figure 13.1: Kepler's First Law

Of course, it looks nothing like this – this is MASSIVELY exaggerated for the sake of seeing what is going on. The Earth's orbit around the Sun is very nearly circular, which is why it took so long for astronomers to realize that it wasn't.

Second Law

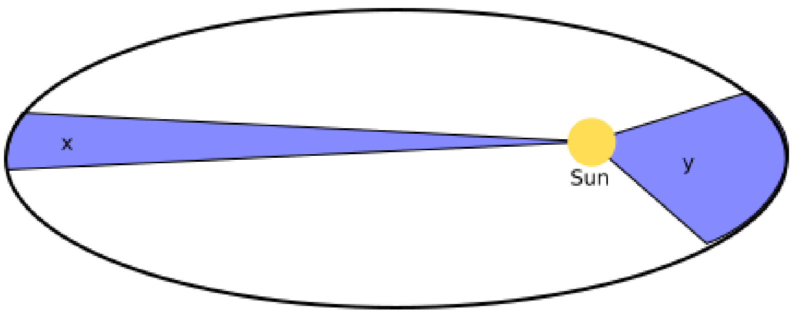


Figure 13.2: Kepler's Second Law

Figure 13.2 shows the areas swept out in the same amount of time at different parts of the orbit. As you can see from the diagram, areas x and y are the same. The reason for this, put simply, is that the objects move more quickly when they are closer to the Sun and more slowly when they are further away.

Third Law

This will be described in more detail below.

(b) recognise and use $F = -\frac{Gm_1m_2}{r^2}$

"The gravitational force between two objects is proportional to the product of their masses and inversely proportional to the square of the distance between their centres."

That is a lot of words and it is much more easily explained with an equation:

$$F = -\frac{Gm_1m_2}{r^2}$$

Where: F is the force, measured in newtons (N)

G is the universal gravitational constant, which has a value of $6.67\text{x}10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$

m_1 and m_2 are the two masses measured in kilograms (kg)

r is the distance between their centres, measured in metres (m).

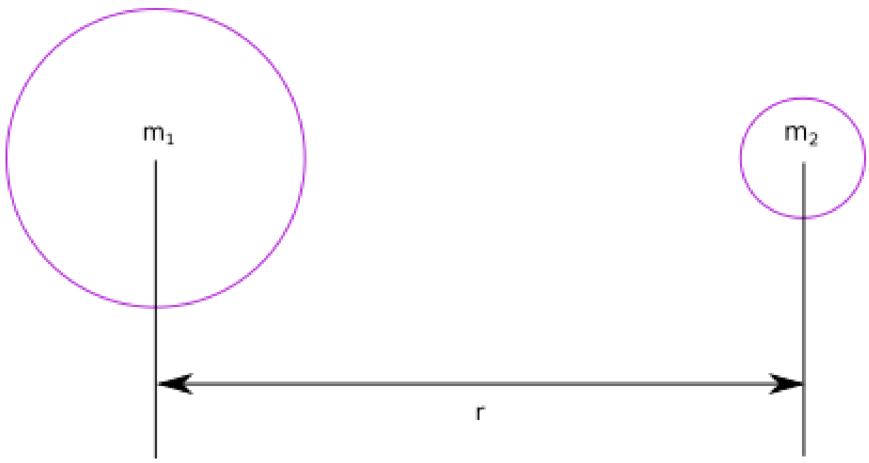


Figure 13.3: Two masses

Important things to note:

1. The force is negative. This is because it is always attractive and attractive forces are always negative, by definition.
2. The force exerted by mass 1 on mass 2 is the same magnitude but opposite in direction to the force exerted by mass 2 on mass 1. This is a direct consequence of Newton's third law. In most cases that we study

the **effect** of the force on the smaller mass is much greater than it is on the larger mass and we ignore the effects of the force on the larger mass.

(c) use Newton's law of gravity and centripetal force to derive $r^3 \propto T^2$for a circular orbit

Now that we know the size of the force acting on a body moving in circular motion due to the gravitational force acting on it, we can prove Kepler's third law:

For a body orbiting another body, the centripetal force is provided by the gravitational force.

$$F = -\frac{Gm_1m_2}{r^2} \quad (1)$$

As the body is moving in circular motion, the gravitational force causes the body to accelerate towards the centre of the circle, as in the diagram:

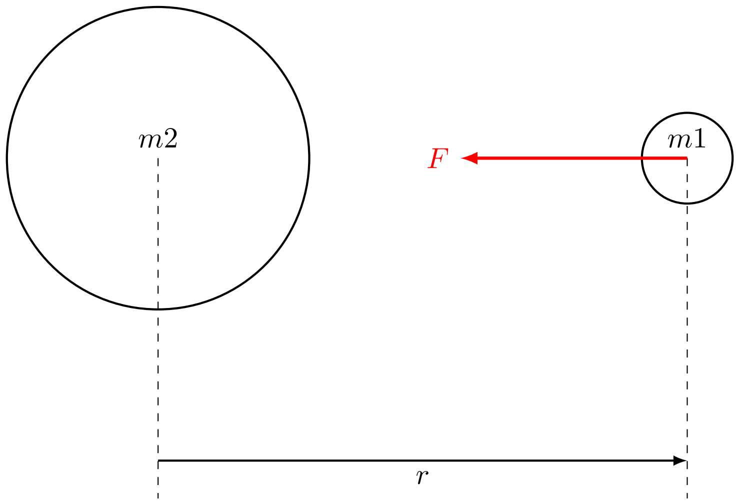


Figure 13.4: Two masses and a force

Thus we apply F=ma, with F being the gravitational force, as on the diagram, and the acceleration being equal to $r\omega^2$.

$$F = ma$$

So

$$G\frac{m_1 m_2}{r^2} = m_1 r\omega^2$$

m_1 is the object that is orbiting, so it is the mass of m_1 that is feeling the acceleration and thus m_1 goes into the right hand side of the equation.

Thus we can cancel m_1 and collect the terms in r to give:

$$Gm_2 = r^3\omega^2 \quad (2)$$

But we know from our revision of circular motion that the period of the orbit is related to the angular velocity by:

$$\omega = \frac{2\pi}{T} \quad (3)$$

Therefore we substitute equation (3) into equation (2) to give:

$$Gm_2 = \frac{4\pi^2 r^3}{T^2}$$

Now re-arrange to make r the subject of the formula:

$$r^3 = \frac{Gm_2}{4\pi^2}T^2 \quad (4)$$

G, m_2 and π are all constants, so we can finally write:

$$r^3 \propto T^2$$

You need to be able to do this for your examination, so make sure that you learn this proof.

(d) understand energy transfer by analysis of the area under a gravitational force-distance graph

If you apply a force through a distance, you do work on it. This is a definition.

At GCSE, you learnt that it was given by the equation:

Work done = Force x distance

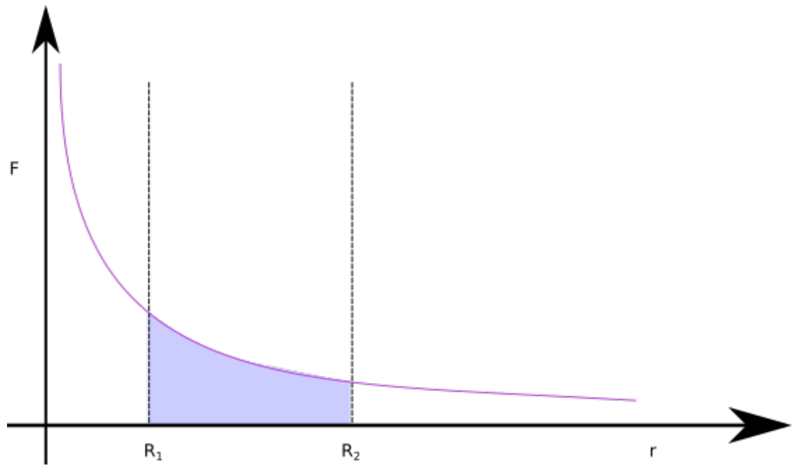


Figure 13.5: Graviational PE as area of a graph

Later you learned that in fact it was the area under the Force-distance graph.

(Of course, it's a little bit more difficult than that. It is actually given by:

$$Work\ done = \int_r^{\infty} F dx \qquad (8)$$

You use this integral in other parts of the specification, but not this part.)

Therefore if we want to know the gravitational potential energy gained or lost by an object in a gravitational field, we look at the area under the force-distance graph.

The zero of gravitational potential energy is taken as being at infinity, which makes sense. If the object isn't in a field, then it isn't experiencing any force, so it doesn't have any GPE.

This does mean, however, that all gravitational potential energies are negative as they lose GPE as they fall towards a mass.

For example, if you want to know the GPE gained/lost by an object as it moves from point R_1 to point R_2 in the field you look at the area under the force-distance graph (see figure 13.5)

(Generally, we tend to look at the gravitational potential rather than the GPE, and use the field strength-distance graph, but the specification asks for GPE and a force-distance graph, so that is what we are looking at!)

(e) derive and use $g = -\frac{Gm}{r^2}$ for the magnitude of the gravitational field strength due to a point mass

If you look back to chapter 2 on gravitational fields, you will know the definition of the gravitational field strength. It is given by the equation:

$$g = \frac{F}{m} \quad (5)$$

But we now know how the force is provided by a spherical mass from Newton's Law of Gravitation. It is given by equation (1) as:

$$F = \frac{Gm_1m_2}{r^2} \quad (6)$$

So we can substitute equation (6) into equation (5) to give us:

$$g = \frac{\mathrm{Gm}}{r^2} \quad (7)$$

This is the gravitational field strength due to a spherical mass m at a distance r from its centre and you need to be able to derive this equation.

A graph of the field strength due to a spherical body against distance looks like this:

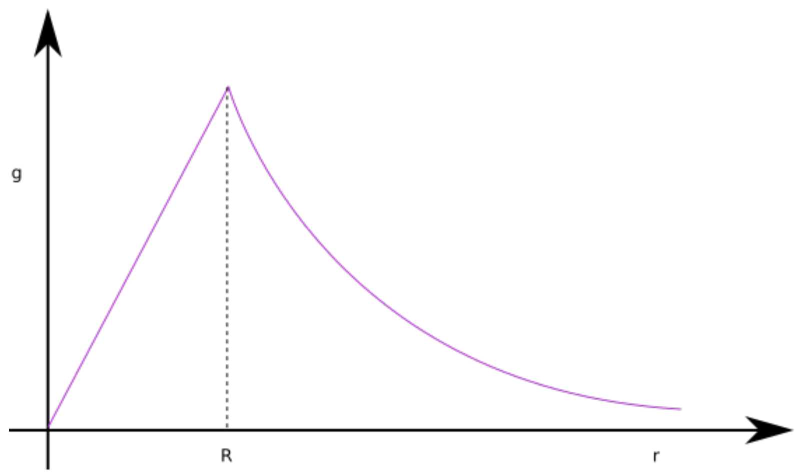


Figure 13.6: Field strength due to a sphere

From the centre of the object up to its radius (0 to R) the variation with field strength inside the body varies linearly **if the density is constant.** R is the radius of the body, so therefore the distance from the centre. After R it drops off as $1/r^2$.

(f) recall similarities and differences between electric and gravitational fields

	Gravitational Fields	Electric Fields
Similarities	Both obey an inverse square law for the field at a distance r from a point mass/charge.	
	Both have a potential that drops off as 1/r from a point mass/charge.	
	Field strength is defined as the force per unit mass/charge	
Differences	Only radial. All uniform gravitational fields are approximations.	Can have both radial and uniform fields
	Only ever attractive	Can be both attractive and repulsive
	Very weak	Very strong

Figure 13.7: Similarities and Differences

(g) recognise and use the equation for gravitational potential energy for point masses $E = -\frac{Gm_1m_2}{r}$

There is an equation for the GPE which you need to be able to use. It is found from integrating the expression for the force as mentioned earlier, and it is given by:

$$E = -\frac{Gm_1m_2}{r} \qquad (9)$$

(h) calculate escape velocity using the ideas of gravitational potential energy (or area under a force-distance graph) and energy transfer

We can now use these methods of working out the GPE gained by an object in a gravitational field to calculate a quantity called the **escape velocity.**

NB. A lot of people get escape velocity wrong. It is the velocity needed to be given to an object **on the surface of the planet** in order for it to escape the gravitational field of the planet and have zero KE at that time. Once it has been given this velocity (by, for example, a cannon) **no more energy is put into the system**. From that moment on it is a projectile and is constantly losing KE as it gains GPE.

Of course, this means that when it finally escapes the gravitational field it has no energy at all, which also means that it has zero overall energy to start with as well!

Therefore we use the law of conservation of energy to work out what the escape velocity must be:

Total energy before = Total energy after = 0

Therefore Initial KE + Initial GPE = 0

$$\frac{1}{2}mv_e^2 - \frac{\text{GMm}}{R} = 0 \qquad (10)$$

Where M is the mass of the planet in kg

R is the radius of the planet in m

m is the mass of the projectile in kg

v_e is the escape velocity in ms^{-1}

You can cancel the mass of the projectile and re-arrange for v_e from equation (10) to give:

$$v_e = \sqrt{\frac{2GM}{R}} \qquad (11)$$

This is the escape velocity and you can work it out for the Earth. You should get about 12 kms^{-1}.

There is a graphical way of looking at this as well. The GPE gained by the body as it moves from the surface of the planet, radius R, is given by the area under the force-distance graph from the surface of the planet to infinity (i.e. as in Figure 13.5, but with R_2 at infinity). If it is launched from the surface of the planet then that also equals its initial KE.

(i) calculate the distance from the centre of the Earth and the height above its surface required for a geostationary orbit.

A geostationary orbit is one which stays above the same position on the Earth's equator at all times.

This means, therefore, that it has a period of 24 hours.

It is therefore possible to calculate, using equation (4), r for a geostationary orbit.

N.B. A very common mistake is to say that r is the height of the orbit. This isn't the case – it is the radius of the orbit, so it is the distance of the satellite from the **centre** of the Earth, not the surface of the Earth.

You should make sure that you can do this. Have a go at working out r, given the following data:

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

$M_E = 5.98 \times 10^{24}$ kg

and remembering that T = 24 hours (but don't forget to convert to seconds!)

You should have got an answer of 4.23×10^7 m.

If I now tell you that the radius of the Earth is 6.36×10^6 m, you can also write down the height of the satellite above the surface of the Earth, and this comes to 3.6×10^6 m.

So a geostationary satellite orbits at a height that is about 6 times greater than the radius of the Earth.

14 Electromagnetism

Candidates should be able to:

(a) understand and use the terms magnetic flux density, flux and flux linkage

Electromagnetic induction depends crucially on the concept of *flux*. This is the product of the magnetic field strength B and the area of a loop perpendicular to the field, A.

$$\Phi = BA$$

The unit of flux is the weber, Wb. Given its relation to flux, the field strength B can also be referred to as the magnetic flux density. It also follows that the tesla is equivalent to one weber per metre-squared.

When a coil of wire encloses an area of flux we can calculate the *flux-linkage* which is the product of the flux through the coil and the number of turns on the coil, $N\Phi$.

Magnetic flux density can be defined as the force per unit of current in a wire of unit length in a magnetic field.

(b) understand that magnetic fields are created by electric current

An electric current in a wire creates a magnetic field. The field strength depends on the size of the current and the distance from the wire. In the case of a solenoid or electromagnet the field strength is also proportional to the number of turns on the coil.

(c) recognise and use $F = BIl\sin\theta$

This is the equation for the force on a current-carrying wire of length l with a current of I at an angle θ to a magnetic field of strength B.

(d) recognise and use $F = BQv \sin\theta$

This is the equation for a particle of charge Q moving at a velocity v at an angle θ to a magnetic field of strength B. Note that this is sometimes called the Lorentz force.

(e) use Fleming's left-hand rule to solve problems

Fleming's left-hand rule allows the calculation of the direction of the force on either a charged particle or a current-carrying wire. The geometry is shown below.

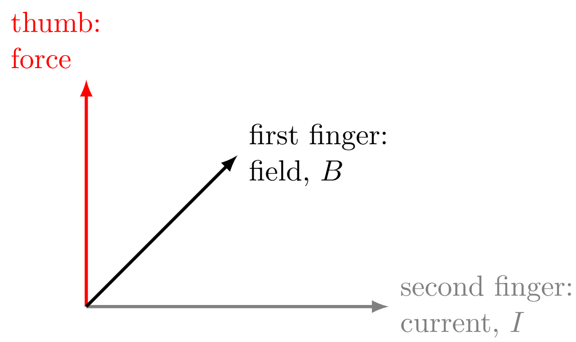


Figure 14.1: Fleming's left-hand rule

It is important to note that if you are using the left-hand rule to predict the direction of force on a moving negatively charged particle then the *current* is flowing in the opposite direction to the velocity of the particle.

Note that vector form of the equations explains the left-hand rule and sine terms as follows:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

$$\mathbf{F} = \mathbf{I}L \times \mathbf{B}$$

(f) explain qualitatively the factors affecting the emf induced across a coil when there is relative motion between the coil and a permanent magnet or when there is a change of current in a primary coil linked with it

Since the emf depends on the rate of change of flux linkage the size of the emf is proportional to the number of turns on a coil. The rate of change of flux is determined by strength of the magnet and how quickly it is moving through the coil.

(g) recognise and use $E = -\frac{d(N\Phi)}{dt}$ and explain how it is an expression of Faraday's and Lenz's laws

Since $N\Phi$ is the flux linkage then $\frac{d(N\Phi)}{dt}$ is the rate of change of flux linkage. The negative sign indicates that the induced emf acts to oppose the change in flux linkage which created it (which is Lenz's Law).

(h) derive, recall and use $r = \frac{mv}{BQ}$ for the radius of curvature of a deflected charged particle

When a charged particle moves through a uniform magnetic field at a constant speed it experiences a constant force at right-angles to its velocity. This results in it moving in circular motion with the Lorentz force providing the centripetal force.

$$BQv = \frac{mv^2}{r}$$

$$\implies r = \frac{mv}{BQ}$$

This allows the calculation of the mass to charge ratio of the particle providing its velocity can be determined (e.g. from an accelerating potential).

(i) explain the Hall effect, and derive and use $V = Bvd$

When an electron moves through a magnetic field it will experience a force at right-angles to its motion. If a current flows through a thin film of material then a negative charge will accumulate on one edge of the film. Figure 14.2 shows this force acting and charge building up.

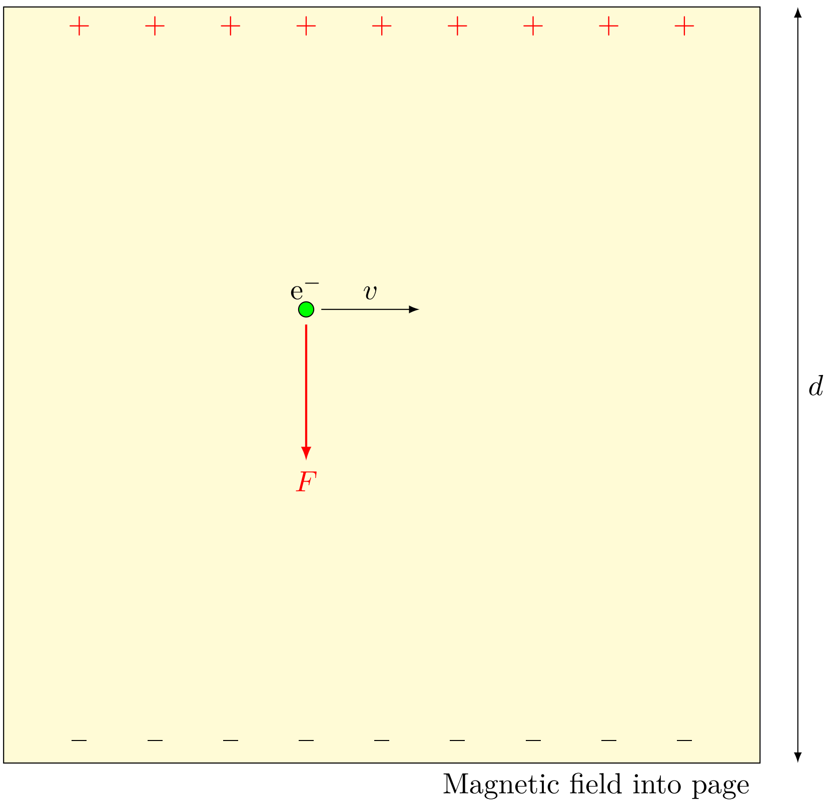


Figure 14.2: The Hall Effect

Over time the charges build up on either side of the wager until equilibrium is reached. At this point the the force due to the electric field on the charge equals the force due to the Lorentz force on the charge.

$$q\frac{V}{d} = Bqv$$

$$V_{\text{hall}} = Bvd$$

This relationship is the basis of a Hall Probe which is used to measure magnetic fields. In these circumstances the velocity v is the drift velocity of the electrons.

15 Special Relativity

This chapter contains revision on the topics of:

- Einstein's special principle of relativity
- Time dilation
- Length contraction

Candidates should be able to:

(a) recall that Maxwell's equations describe the electromagnetic field and predict the existence of electromagnetic waves that travel at the speed of light (Maxwell's equations are not required)

Maxwell related electricity and magnetism in a mathematically elegant way, and in doing so predicted that the speed of electromagnetic waves was:

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \text{ x } 10^8 \text{ ms}^{-1}$

*(b) *recall that analogies with mechanical wave motion led most physicists to assume that electromagnetic waves must be vibrations in an electromagnetic medium (the aether) filling absolute space*

Just what was it that was oscillating? Most physicists thought there must be a medium...

*(c) *recall that experiments to measure variations in the speed of light caused by the Earth's motion through the ether gave null results*

...However experiments such as the Michelson-Morley experiment[1] showed that there was no difference in the measurement of the speed of light over

[1] http://scienceworld.wolfram.com/physics/Michelson-MorleyExperiment.html

the course of a year - if the Earth was travelling through an aether you would expect the discrepancy to be measurable. But crucially no differences were found.

*(d) *understand that Einstein's theory of special relativity dispensed with the aether and postulated that the speed of light is a universal constant*

Einstein put forward the idea that there was no medium, and the speed of light was fixed.

*(e) *state the postulates of Einstein's special principle of relativity*

His two postulates were

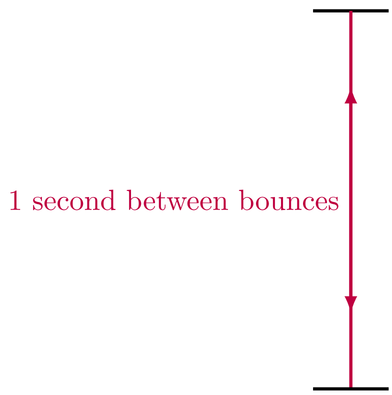
- The laws of physics are the same in all inertial rest frames
- The speed of light in free space is a fixed value

From these two postulates, it is possible to deduce the idea that time is not fixed for all observers, and that distances measured by two observers moving relative to each other may be different! A whole new avenue of physics was opened.

*(f) *explain how Einstein's postulates lead to the idea of time dilation and length contraction that undermines the idea of absolute time and space*

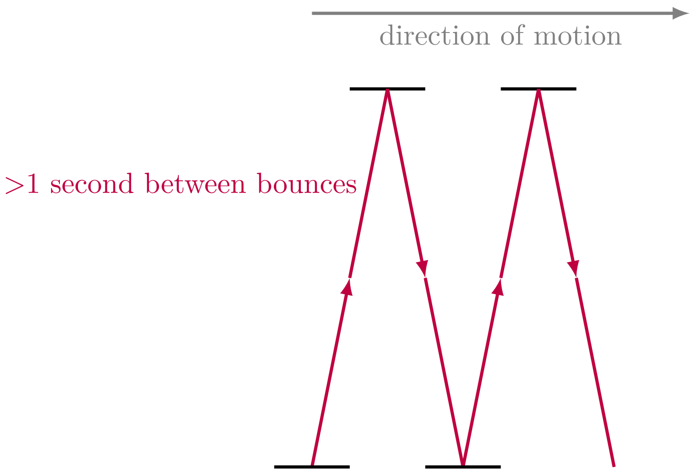
Light clock

Imagine a beam of light bouncing up and down between two mirrors, spaced a distance of 3 x 10^8 m apart



Now imagine that you are moving right-to-left relative to the 'light clock' (or that the light clock is moving left-to-right relative to you - it doesn't matter...)

If the speed of light is constant for all observers, as Einstein's postulates require it will take longer between bounces, because it has to travel further due to the extra horizontal component:



*(g) *recognise and use* $t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$ *and* $l = l\sqrt{1-\frac{v^2}{c^2}}$

Example Question
Cosmic radiation constantly bombards the Earth from outer space. The majority of these cosmic rays are protons which, when they hit the Earth's upper atmosphere, create sub-atomic particles called pions, which then quickly decay into muons. Suppose a cosmic ray proton hits a molecule of nitrogen in the upper atmosphere at a distance of 50 km above the Earth's surface. If the pion produced has a velocity of 0.9999c and has an average lifetime (in its own time) of t = 2.60×10^{-8} s. How far would the pion travel before decaying taking into the effects of time dilation?

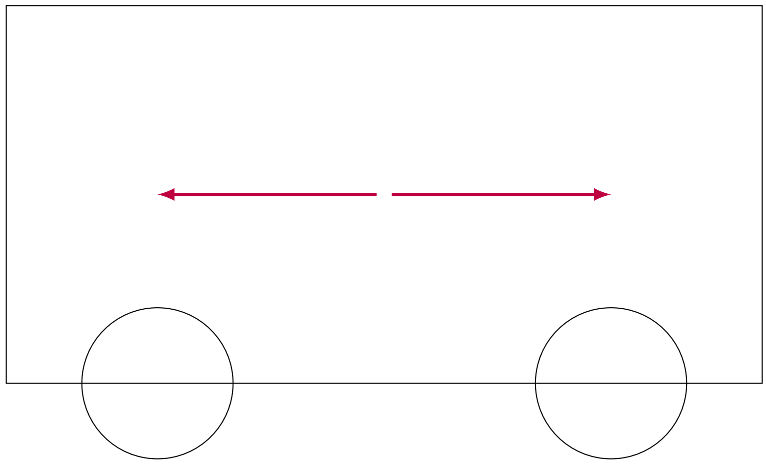
Answer
First we must calculate the time it takes to decay according to a stationary observer. We do this by calculating the Lorentz factor γ and multiplying this by the lifetime in its own frame of reference:
$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 70.7$

so time taken to decay = 70.7 x 2.60 x 10^{-8} = 1.83 x 10^{-6} seconds
Then distance travelled = speed x time = 550 m

*(h) *understand that two events which are simultaneous in one frame of reference may not be simultaneous in another; explain this in terms of the fundamental postulates of relativity and distinguish this from the phenomenon of time dilation*

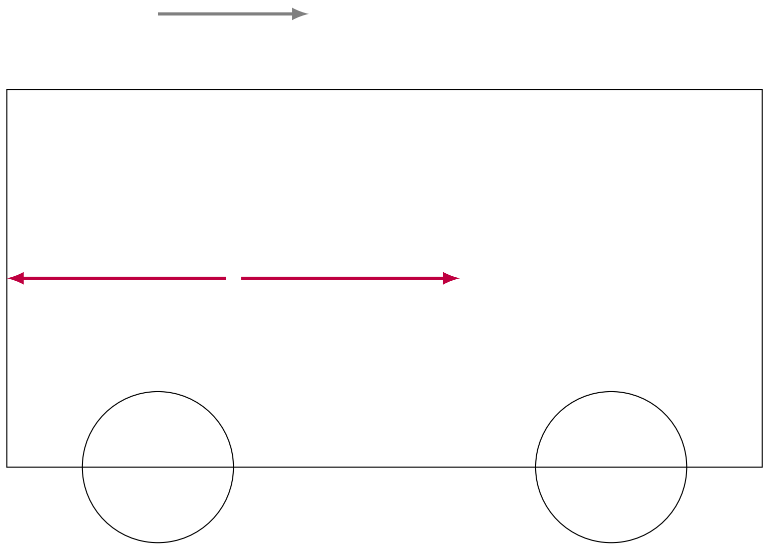
Imagine a situation in which a light is turned on in the middle of a train carriage, and light rays travel out in both forward and backwards directions.

From the reference frame of a person riding along with the train, the light

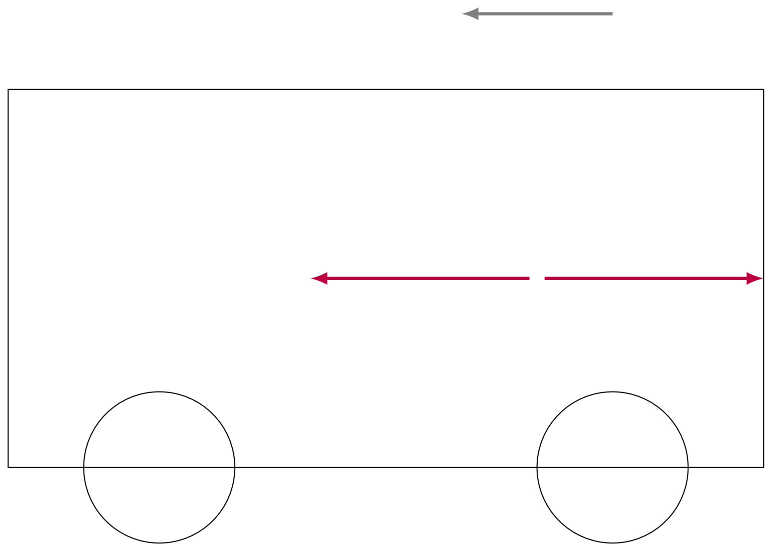


reaches each end of the carriage at the same time. Those events appear to be simultaneous.

In the reference frame of a stationary observer, the train is moving forward



(to the right in this diagram), but the light rays travel at c, so the light ray travelling backwards will reach the rear end of the carriage first.



In the reference frame of somebody travelling faster than the train, the train appears to be going backwards, and the forward travelling light ray will reach the front end first.
The idea of two events being simultaneous is therefore impossible - it depends on your reference frame.

This effect is a completely different effect to time dilation. The effect we are dealing with here is called **loss of simultaneity**.

16 Molecular Kinetic Theory

Content

- absolute scale of temperature
- equation of state
- kinetic theory of gases
- kinetic energy of a molecule
- first law of thermodynamics
- entropy
- second law of thermodynamics

Candidates should be able to:

(a) explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature

In the seventeenth century Robert Boyle began making quantitative measurements on gases at different pressures. By observing the results of varying the volumes of gases on their pressures he postulated what has become known as Boyles' Law.

The product of the pressure and volume of a sample of gas at constant termperature remains constant.

Developing a quantitative scale for temperature was a difficult process[1] and it was not until the end of the eighteenth century that a relationship between

[1] see Hasok Chang's *Inventing Temperature* for more details

pressure and temperature was discovered. Pressure was found to be linearly related to temperature as measured in the celsius scale. It was found that all samples of gases had the same intercept with the x-axis and this lead to the idea of *absolute zero*, the temperature at which the pressure of the gas would be zero.

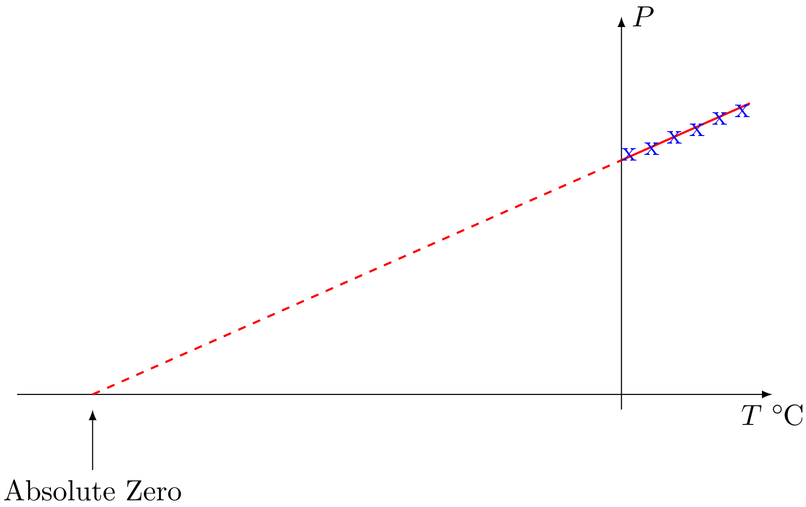


Figure 16.1: Pressure against Temperature of a Gas

The temperature of Absolute Zero was defined as zero kelvin, $0\,\mathrm{K}$ with this being equal to $-273\,°\mathrm{C}$. This scale is known as the absolute temperature scale.

If the absolute temperature scale is used, then two more empirical laws can be stated. The first is Charles' Law:

> For a fixed volume of gas, the pressure is proportional to the absolute temperature.

The second is Gay-Lussac's law:

> For gas at a fixed pressure, the volume of a sample of gas is proportional to its absolute temperature

(b) use the units kelvin and degrees Celsius and convert from one to the other

Conversion between kevin and degrees celsius is simply a matter of adding or subtracting 273 as the two scale share the same size of unit:

$$0\,\mathrm{K} = -273\,°\mathrm{C}$$

(c) recognise and use the Avogadro number $N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$

The Avogadro number is the number of particles (atoms or molecules) which are present in one mole of the substance. It can also be thought of as the constant of proportionality between the mass of the particle and the mass of one mole of the substance.

For example, using carbon-12:

$$N = \frac{0.012\,\mathrm{kg}}{1.99 \times 10^{-26}\,\mathrm{kg}} = 6.0302 \times 10^{23}$$

The number of moles of a substance can be found by:

1. dividing the number of particles by the Avogadro number;
2. dividing the mass of the sample by the molar mass.

(d) recall and use $pV = nRT$ *as the equation of state for an ideal gas*

This equation includes the following quantities:

Symbol	Quantity	Standard Unit
p	pressure	Pa
V	volume	$\mathrm{m^3}$
n	no. of moles of the gas	N/A
R	the molar gas constant	$\mathrm{J\,mol^{-1}\,K^{-1}}$
T	the absolute temperature	K

R, the molar gas constant, has a value of $8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$ and is equal to $N_A k$.

(e) describe Brownian motion and explain it in terms of the particle model of matter

When a small, visible particle such as a pollen grain or smoke particle is observed under a microscope it is seen to move around in an erratic, random manner. This is explained by the fact that it is being constantly bombarded by air molecules whose effects do not quite cancel out. Since the pollen grain or smoke particle is much more massive than the air molecules it follows that these must be moving very rapidly. In 1905 Albert Einsein published a detailed statistical treatment of Brownian motion using the theory of atoms and thus Brownian motion provides very strong evidence for the atomic hypothesis.

(f) understand that the kinetic theory model is based on the assumptions that the particles occupy no volume, that all collisions are elastic, and that there are no forces between particles until they collide

Notes on these assumptions:

1. $1\,\mathrm{m}^3$ of air contains approximately

$$N = \frac{PV}{KT} = \frac{101\,\mathrm{kPa}\,1\,\mathrm{m}^3}{1.38 \times 10^{-23}\,\mathrm{J\,K}^{-1} 298\,\mathrm{K}} = 2.5 \times 10^{25}$$

particles. If these particles are modeled as spheres with a diameter of 300 pm then the particles take up a fraction of the total volume of around 10^{-27}.

2. Elastic collisions mean that the kinetic energy of the particles is not lost.

3. The particles travel in straight lines between collisions. Forces between the particles would cause them to clump together. The lack of forces between particles also means that all the energy is in the form of kinetic energy of the particles.

(g) understand that a model will begin to break down when the assumptions on which it is based are no longer valid, and explain why this applies to kinetic theory at very high pressures or very high or very low temperatures

High pressures At high pressures particles will be forces together. This means that the first assumption about negligible volume may break down as the gaps between particles decreases. In addition, the gas may get close to the point of condensation and particles may begin to attract each other.

Very low temperatures Similarly to high pressures, at low temperatures the molecules may be very close to one another and may begin to condense, implying forces between particles.

Very high temperatures At very high temperatures atoms become a plasma, i.e. separate into positive ions and free electrons. Under such circumstances forces exist between the particles.

(h) derive $PV = \frac{1}{3}Nm\langle c^2\rangle$ from first principles to illustrate how the microscopic particle model can account for macroscopic observations

This theory assumes that pressure is caused by the averaging the many elastic collisions of a number of particles with the walls of the container.

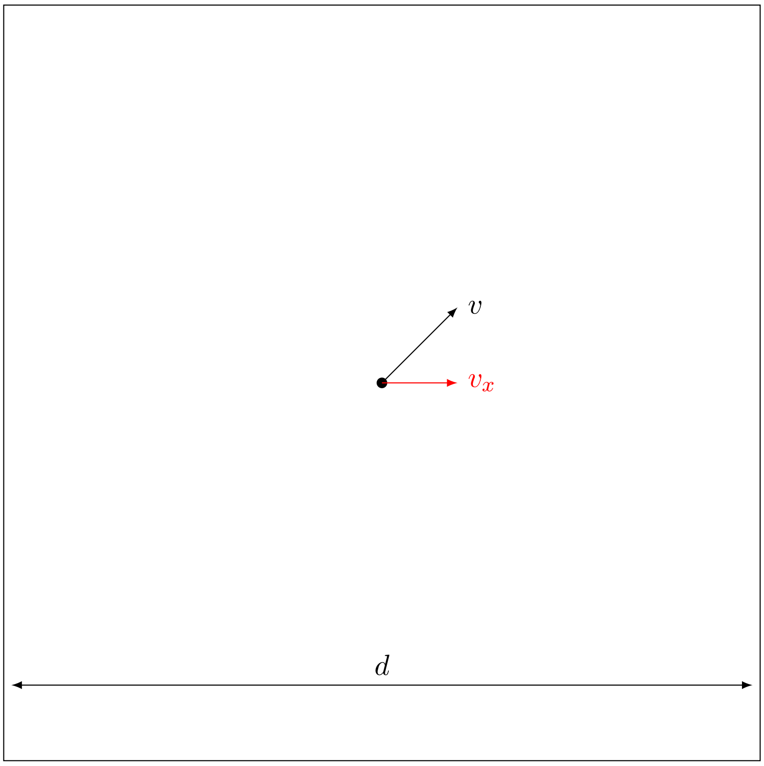


Figure 16.2: A particle in a box

When the particle, i, collides with the right-hand wall of the box it rebounds with the same y-velocity and a negative x-velocity. The change in momentum of the particle is therefore $\Delta p = 2mv_x$, where m is the mass of the particle. This is equal to the impulse delivered on the wall during the particle collision.

$$\text{Impulse} = 2mv_x$$

The particle will now travel to the left-hand side of the box and back. The time taken to do this is equal to $2d/v_x$. We can therefore think of the impulse due to a single collision being averaged over this period of time. If this is the case then the force due to an individual particle is given as:

$$f_i = \frac{2mv_x}{2d/v_x} = \frac{m{v_x}^2}{d}$$

If we take a system of N particles and replace the properties of our particle with the average properties of the particles in the system we get:

$$F = \sum_i f_i = \frac{Nm\langle {v_x}^2 \rangle}{d}$$

Note that this includes the expression $\langle {v_x}^2 \rangle$ - the mean of the squared x-velocity. The order here is important and the mean velocity of the particles is zero. The square of a component of velocity is related to the velocity by pythagoras:

$$c^2 = {v_x}^2 + {v_y}^2 + {v_z}^2$$

Now we are using many particles we can make the assumption that a particle is equally likely to be travelling in any direction, therefore

$$\langle {v_x}^2 \rangle = \langle {v_y}^2 \rangle = \langle {v_y}^2 \rangle$$

giving

$$\langle {v_x}^2 \rangle = \frac{1}{3}\langle c^2 \rangle$$

Substitution into the equation for F gives

$$F = \frac{\frac{1}{3}Nm\langle c^2 \rangle}{d}$$

and

$$P = \frac{F}{A} = \frac{\frac{1}{3}Nm\langle c^2 \rangle}{Ad}$$

We now note that Ad is equal to the volume of the box and re-arrange to give

$$PV = \frac{1}{3}Nm\langle c^2 \rangle$$

(i) recognise and use $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$

The formula for PV derived above can be equated with the formula from the empirical gas laws ($PV = nRT$) to give:

$$\frac{1}{3}Nm\langle c^2 \rangle = nRT$$

Since $n = N/N_A$ and $R = N_A k$ the right-hand side becomes NkT. This is usually re-arranged to give

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

as now the left-hand side represents the average kinetic energy of a molecule in the gas. Since this is an ideal gas and has no potential energies this is also the total energy per molecule. Hence the link between the macroscopic quantity of temperature and the microscopic energy per molecule is arrived at.

(j) understand and calculate the root mean square speed for particles in a gas

The root mean square (RMS) is usually calculated by re-arranging the equation above and square-rooting:

$$\sqrt{\langle c^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

(k) understand the concept of internal energy as the sum of potential and kinetic energies of the molecules

In an ideal gas the internal energy is equal to the sum of kinetic energyies of the molecules, i.e.

$$U = \frac{1}{2}Nm\langle c^2 \rangle = \frac{3}{2}NkT$$

(l) recall and use the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and the work done on the system

This can be written as:

$$\Delta U = \Delta Q + \Delta W$$

Note that in some textbooks the first law is written in terms of the work done by *the system, giving* ΔW *a different sign. If you think in terms of conservation of energy in the particular example you will be alright.*

The Carnot Cycle shown in Firgure 16.3 is commonly used to test the understanding for the first law. The cycle begins and ends at point **A**. This means that the internal energy of the system at the start and end of the cycle is the same (as $PV = nRT$ and T is proportional to the internal energy). The changes at each stage of the cycle are as follows:

1. **Isothermal Expansion** In stage 1 the gas expands at contant temperature. Since it is expanding, the gas is doing work on the surroundings and since it is at constant temperature the internal energy of the gas remains constant. Therefore the second law implies that the gas must be absorbing heat. *Note that isothermal changes are often indicated by describing the change as occuring slowly.*

2. **Adiabatic Expansion** In stage 2 the gas expands without transferring heat to/from the surroundings. Since it is doing work the gas's internal energy drops, as does its temperature.

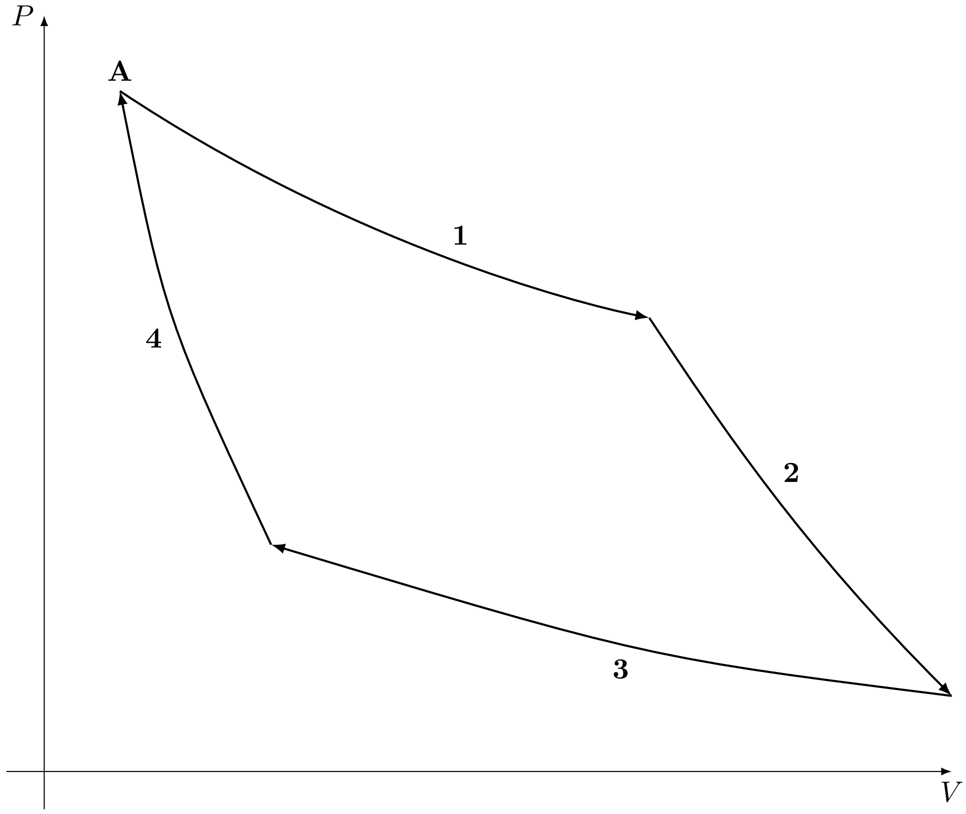


Figure 16.3: The Carnot Cycle

3. **Isothermal Compression** In stage 3 the gas is compressed at constant temperature. Since work is done on the gas and the internal energy remains constant it must be the case that heat is lost to the surroundings.

4. **Adiabatic Compression** Finally, the gas is compressed without heat transfer to/from the surroundings. Work is done on the gas and therefore the internal energy, and termperature, increase.

Example Question

Complete the table for the Carnot Cycle shown in figure 16.3.

stage	thermal energy supplied **to** the gas / J	work done **on** the gas / J	**increase** in internal energy of the gas / J
1	(a)	-936	(b)
2	0	(c)	(d)
3	(e)	+702	0
4	(f)	+844	+844

Answer

In stage 1 there is no change in internal energy of the gas, so (b) equals zero. In order to satisfy the first law of thermodynamics the thermal energy supplied must equal the work done **by** the gas so (a) equals 936 J.

As the total change in internal energy throughout the cycle must be zero, we can now calculate (d) as being -844 J.

(c) can be calculated using the first law as -844 J.

A similar argument to that used in stage 1 can be used in stage 3 to give (e) as -702 J.

In stage 4 no energy is transferred by heating therefore (f) is equal to zero.

The final table is therefore:

stage	thermal energy supplied **to** the gas / J	work done **on** the gas / J	**increase** in internal energy of the gas / J
1	**+936**	-936	**0**
2	0	**-844**	**-844**
3	**-702**	+702	0
4	**0**	+844	+844

(m) recognise and use $W = p\Delta V$ for the work done on or by a gas

This follows from the definition of work done. If the cross-sectional area is a contant A, then derivation is as follows:

$$W = F\Delta s = PA \times \Delta s = P\Delta V$$

(n) understand qualitatively how the random distribution of energies leads to the Boltzmann factor $e^{-\frac{E}{kT}}$ as a measure of the chance of a high energy

When a large number of particles share a fixed amount of energy between them and are able to transfer energy through random collisions, it turns out that the most likely distribution of energies follows a specific distribution - the Boltzmann distribution. One of the features of this distribution is that the fraction of particles with an energy greater than a certain amount of energy, E, is given by the Boltzmann factor, i.e.

$$\frac{N_E}{N_{total}} = e^{-\frac{E}{kT}}$$

(o) apply the Boltzmann factor to activation processes including rate of reaction, current in a semiconductor and creep in a polymer

From the point above, it follows that the rate of any process which depends on an activation energy is likely to be proportional to the Boltzmann factor.

Process	**Depends on**	**Energy E represents**
rate of reaction	the number of particles with an energy above the activation energy	the activation energy of the process
current in a semiconductor	the number of free charge carriers	the energy required for a charge carrier to move the the conduction band, i.e. the band gap
creep in a polymer	the ability of the polymer chains to break the weak intermolecular forces between them	the energy required to break the intermolecular bonds between polymer chains

*(p) *describe entropy qualitatively in terms of the dispersal of energy or particles and realise that entropy is related to the number of ways in which a particular macroscopic state can be realised*

A common way of describing the concept of entropy is to focus on the idea of disorder. When discussing entropy scientifically it is important to be precise about what this means. A more precise way of characterising entropy is to think about the number of ways in which a macroscopic state can be realised. For example, imagine seven particles in a quantised system with four units of energy to share between them. Each state can be denoted using the notation $S = \{N_0, N_1, N_2, \ldots\}$ where N_0 is the number of particles with this much energy.

For example, one particle could have all four units of energy and the rest have zero. This would be denoted $\{6, 0, 0, 0, 1\}$. There are seven different ways in which this macrostate could come about. The table below shows the different possible macrostates and the number of ways these could be attained.

State	**No. of microstates**
$\{6,0,0,0,1\}$	7
$\{5,1,0,1\}$	$7 \times 6 = 42$
$\{4,2,1\}$	$7 \times 6 \times 5/2 = 105$
$\{3,4\}$	*TODO*

Table 16.1: Macrostates and microstates

As you can see, there are far more ways to achieve the macrostate $\{4, 2, 1\}$ than the others. This makes it the state with the highest entropy.

*(q) *recall that the second law of thermodynamics states that the entropy of an isolated system cannot decrease and appreciate that this is related to probability*

If the energy available to a system, or a distribution of particles, is dispersed through random process then over time the most probable configuration will dominate. As the highest entropy macrostate is also the most probable one it is highly likely that the entropy will increase with time.

As an example, consider 100 coins which are all heads-up. There is only one way to arrange for this particular macrostate to occur. There are 1.27×10^{30} possible microstates available and therefore if the coins are tossed it is overwhelmingly likely that the disorder in the system is going to increase. If this idea is multiplied up to the billions of particles in even the smallest sample of matter it is seen that all systems will move towards their highest entropy state.

The only way to return to the original state of the coins would be to intervene and sort them individually one-by-one, thus breaking the isolation of the system.

*(r) *understand that the second law provides a thermodynamic arrow of time that distinguishes the future (higher entropy) from the past (lower entropy)*

The idea here is that nature tends towards higher entropy states. For example, consider the two distributions of gas particles in figure 16.4. In this case it is overwhelmingly likely that **B** occured after **A**. Thus, the second law of theormodynamics provides an arrow of time despite the fact that individual particles may be governed by Newton's laws of motion which are fully reversible and do not give any information about the direction of time.

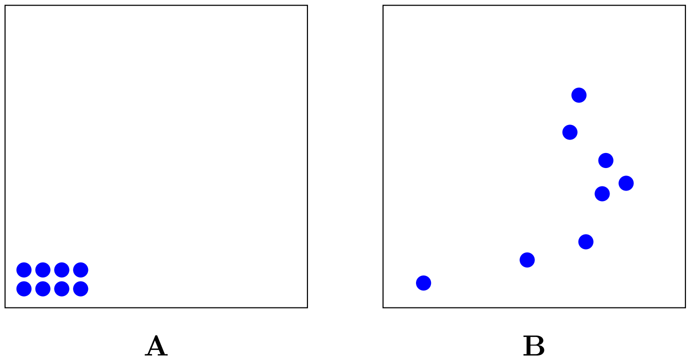


Figure 16.4: Two distributions of particles

*(s) *understand that systems in which entropy decreases (e.g. humans) are not isolated and that when their interactions with the environment are taken into account their net effect is to increase the entropy of the Universe*

Living organisms including humans take in relatively simple molecules and create highly complex, ordered structures from them. This appears to violate the second law of thermodynamics and entropy appears to be decreasing. However, an individual *part* of a system can have a net increase in entropy if it is not isolated. In such a scenario the overall entropy of the universe (the ultimate isolated system) must still decrease. In the case of life on earth, the whole biosphere can be seen as a complex heat engine extracting work from the heat energy input from the sun whilst radiating heat into the coldness of space.

*(t) *understand that the second law implies that the Universe started in a state of low entropy and that some physicists think that this implies it was in a state of extremely low probability.*

If the universe is seen as the ultimate isolated system then it follows that the second law of thermodynamics should apply. In this case the universe must be moving from a low entropy state to a high entropy. In isolated systems, low entropy states are also highly improbably and this leads to the idea that our universe's initial state is a highly improbable one. However, this requires us to accept that we can extend the idea of probability to the initial state of the universe and that we are not committing a 'category error' by considering the universe as a thermodynamic system in this way.

17 Nuclear Physics

Content

- equations of radioactive decay
- mass excess and nuclear binding energy
- antimatter
- the standard model

Candidates should be able to:

(a) show that the random nature of radioactive decay leads to the differential equation

$$\frac{dN}{dt} = -\lambda N \tag{17.1}$$

and that

$$N = N_0 e^{-\lambda t} \tag{17.2}$$

is a solution to this equation.

Radioactive decay is characterised by the fact that the number of nuclei which disintegrate per unit time is directly proportional to the number of unchanged nuclei remaining. Since a disintegrating nucleus reduces the number remaining, there is a negative sign in the proportionality. This relationship can be expressed mathematically as equation 17.1. Where N is the number of nuclei and λ is called the decay constant, with units s^{-1}.

Equation 17.1 is a differential equation with respect to time and therefore the solution to it is a function of time. We can show that equation 17.2 is a solution to this equation by differentiating it.

$$
\begin{aligned}
N &= N_0 e^{-\lambda t} \\
\frac{dN}{dt} &= (-\lambda)\, N_0 e^{-\lambda t} \\
&= -\lambda N
\end{aligned}
$$

(b) recall that activity

$$A = -\frac{dN}{dt} \tag{17.3}$$

and show that $A = \lambda N$ and $A = A_0 e^{-\lambda t}$

Every time a radioactive nucleus disintegrates it emits a particle of ionising radiation. Thus the activity is simply the negative of the rate of change of the number of unchanged nuclei remaining. Simple substitutions allow the derivation of the following equations.

$$
\begin{aligned}
A &= -\frac{dN}{dt} \\
&= -(-\lambda N) \\
&= \lambda N
\end{aligned}
\qquad\qquad
\begin{aligned}
N &= N_0 e^{-\lambda t} \\
-\lambda N &= -\lambda N_0 e^{-\lambda t} \\
A &= A_0 e^{-\lambda t}
\end{aligned}
$$

Note that we do not measure the true activity as that would mean detecting all of the radiation given off by the sample. However, we assume that the measured activity is proportional to the true activity and therefore all our measurements behave in the same way.

(c) show that the half-life

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

The half-life is defined as the time take for half of the nuclei to decay. Therefore I can substitute $N = \frac{N_0}{2}$ into equation 17.1 to give:

$$
\begin{aligned}
\frac{1}{2} &= e^{-\lambda t_{\frac{1}{2}}} \\
\ln \frac{1}{2} &= -\lambda t_{\frac{1}{2}} \\
t_{\frac{1}{2}} &= \frac{\ln 2}{\lambda}
\end{aligned}
$$

(d) use the equations in (a), (b) and (c) to solve problems

(e) recognise and use the equation

$$I = I_0 e^{-\mu x} \tag{17.4}$$

as applied to attenuation losses

When a wave or ionising radiation travels through a medium its amplitude will reduce due to *attenuation*. This is due to scattering and/or absorption by the medium. Note that is this different to a reduction in intensity due to the radiation spreading out. It is assumed that if a given fraction of the intensity is absorbed in a unit length then eqaution 17.1 can be used, replacing N with intensity, t with distance, x and the constant of proportionality with μ. This gives

$$\frac{dI}{dx} = -\mu x \tag{17.5}$$

Using a similar logic to that for equation 17.2 we can show that equation 17.4 is a solution to this equation.

(f) recall that radiation emitted from a point source and travelling through a non-absorbing material obeys an inverse square law and use this to solve problems

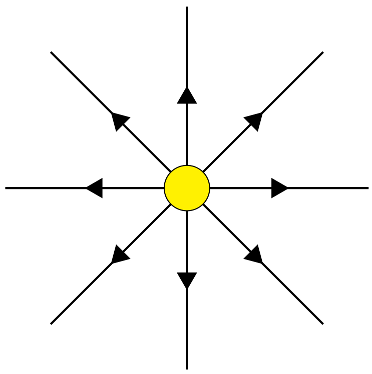


Figure 17.1: Radial radiation

Radiation emitted from a point source spreads out symmetrically in all directions as shown in figure 17.1. The intensity is defined as the power per unit area of radiation. A distance r from the centre of the source the radiation is spread over the surface of a sphere and is therefore calculated using

$$I = \frac{P}{4\pi r^2} \tag{17.6}$$

Since $I \propto r^{-2}$ this is known as an inverse-square law.

(g) estimate the size of a nucleus from the distance of closest approach of a charged particle

When particles approach a nucleus head-on they begin with kinetic energy E_K. All of this kinetic energy is converted to electrical potential energy, EPE. If the energy of the alpha particles is known then a distance of minimum separation can be calculated.

Example Question
Calculate the minimum distance of separation of a alpha particle of kinetic energy 4.0 MeV travelling directly towards a gold nucleus ($Z = 79$).
Answer
Equating the initial kinetic energy to the electrostatic potential gives

$$E_k = \frac{Q_1 Q_2}{4\pi\epsilon_0 d}$$

therefore

$$\begin{aligned} d &= \frac{Q_1 Q_2}{4\pi\epsilon_0 E_k} \\ &= \frac{2 \times 79 \times (1.6 \times 10^{-19})^2}{4\pi\epsilon_0 \times 4.0 \times 10^6 \times 1.6 \times 10^{-19}} \\ &= 5.7 \times 10^{-14}\,\mathrm{m} \end{aligned}$$

It turns out that if high energy alpha particles are fired at the nucleus then Rutherford's deflection formulae break down and the distribution of alpha particles no longer fits the expectation. Under these circumstances it can be assumed the alpha particle is interacting with the nucleus and therefore has been able to approach to within the nuclear radius.

(h) understand the concept of nuclear binding energy, and recognise and use the equation $\Delta E = c^2 \Delta m$ (binding energy will be taken to be positive)

It turns out that the mass of a nucleus is always smaller than the total mass of the constituent protons and neutrons. This difference is called the *mass deficit* and can be converted to an energy using $\Delta E = c^2 \Delta m$. The reduction in mass corresponds to the energy released by combining the nucleons.

Example Question
Calculate the binding energy of a helium-4 nucleus of mass 4.0015 u
Answer
The helium-4 nucleus is composed of two protons and two neutrons. Their total mass is

$$2 \times 1.007\,28\,\mathrm{u} + 2 \times 1.008\,67\,\mathrm{u} = 4.0319\,\mathrm{u}$$

The mass deficit, Δm, is given by

$$\Delta m = 4.0319\,\mathrm{u} - 4.002\,15\,\mathrm{u} = 0.0304\,\mathrm{u}$$

Therefore the binding energy is calculated as

$$0.0304 \times 1.66 \times 10^{-27} \times c^2 = 4.54 \times 10^{-12}\,\mathrm{J} = 28.4\,\mathrm{MeV}$$

(i) recall, understand and explain the curve of binding energy per nucleon against nucleon number

A useful measure of the stability of the nucleus is given by its *binding energy per nucleon*. This is the binding energy divided by the nucleon number. A plot of the binding energy per nucleon against nucleon number is show in figure 17.2. The nucleus with the largest binding energy is iron-56 and this therefore is the most stable nucleus. Nuclei with nucleon numbers below iron-56 will release energy when they undergo nuclear fusion and those above iron-56 will release energy when they undergo fission.

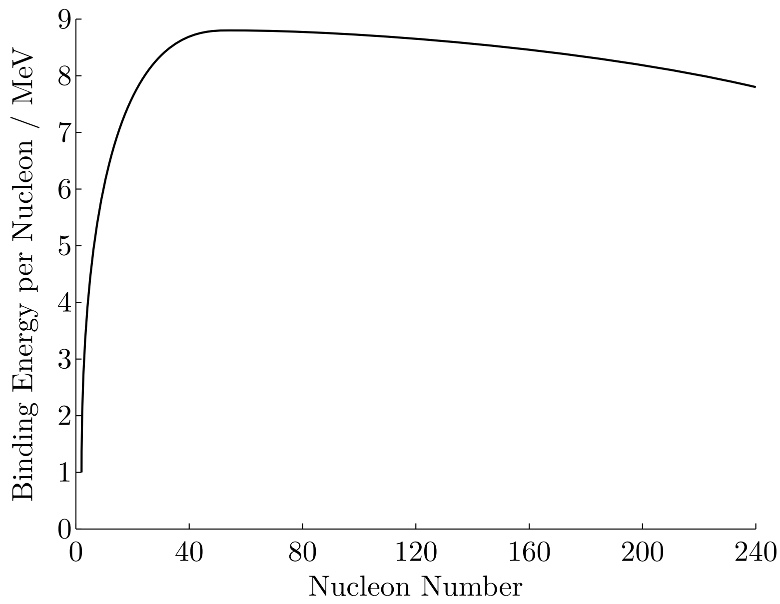


Figure 17.2: Variation of binding energy per nucleon with nucleon number

(j) recall that antiparticles have the same mass but opposite charge and spin to their corresponding particles

All normal particles have an antiparticle partner with the same mass, but some properties which are opposite including electrical charge.

(k) relate the equation $\Delta E = c^2 \Delta m$ to the creation or annihilation of particle-antiparticle pairs

A particle-antiparticle pair can be created whenever there is enough energy present to do so. For example, if a photon of light is near an atomic nucleus it can spontaneously convert into an electron-positron pair.

Example Question
Calculate the minimum energy a photon must have in order to create an electron-positron pair.
Answer

$$E = c^2\Delta m = c^2 \times 2 \times 9.11 \times 10^{-31} = 1.02\,\mathrm{MeV}$$

(l) recall the quark model of the proton (uud) and the neutron (udd)

The theory of quarks was developed to explain the large number of particles discovered in the early particle colliders. Particles made of quarks are called *hadrons*. Normal matter is made up of two types of quark, the up quark (charge $+\frac{2}{3}e$) and the down quark (charge $-\frac{1}{3}e$). From the charges it is possible to see that the proton must be uud and the neutron udd.

(m) understand how the conservation laws for energy, momentum and charge in beta-minus decay were used to predict the existence and properties of the antineutrino

The existence of the antineutrino was first predicted from the energy spectrum of beta decay. Beta particles are produced when a neutron in the nucleus is converted into a proton and a high energy electron. These electrons leave the nucleus at high speed and are detected as beta particles. In such a scenario (a two body process) the electrons should have a fixed amount of energy due to the conservation of momentum. However, the electron was found to have a range of energies. The explanation provided was that whenever an electron was emitted with little energy a third particle has carried away a lot of energy (and vice-versa). The particle had to be neutral (to conserve charge) and of very small mass and was named the **neutrino**.

The full beta decay equation now becomes

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}$$

Further evidence for the existence of this third particle comes from bubble chamber tracks left by beta decay which show the nucleus and electron both recoiling away from a particle which does not leave a trace in the bubble chamber - the neutrino. A photo of this decay can be seen at the science photo library here: `http://www.sciencephoto.com/media/1210/view`

(n) balance nuclear transformation equations for alpha, beta-minus and beta-plus emissions

When these nuclear decays occur total nucleon number remains the same and charge is conserved. In order to make life easier we give beta-minus particles a proton number of -1 and a nucleon number of zero. Beta-plus particles (positrons) are given a proton number of $+1$. Alpha particles are helium-4 nucleii ($^4_2$He).

As an example, if we know that magnesium-23 decays by beta-plus decay we can write

$$^{23}_{12}\text{Mg} \rightarrow ^{A}_{Z}\text{X} + ^{0}_{1}e^{+}$$

We can deduce the identity of X by conserving nucleon number ($A + 0 = 23$) and proton number ($Z + 1 = 12$). Therefore:

$$^{23}_{12}\text{Mg} \rightarrow ^{23}_{11}\text{Na} + ^{0}_{1}e^{+}$$

(o) recall that the standard model classifies matter into three families: quarks (including up and down),leptons (including electrons and neutrinos) and force carriers (including photons and gluons)

An important feature to note is that quarks and gluons are never observed on their own. There are further 'generations' of quarks and leptons but these only exist at higher energies.

(p) recall that matter is classified as baryons and leptons and that baryon numbers and lepton numbers are conserved in nuclear transformations.

Baryons are made up of three quarks and have a baryon number of +1. Similarly, leptons have a lepton number of +1. Anti-baryons and anti-leptons have respective numbers of -1.

For example, conservation of baryon number prohibits the following:

$$\begin{aligned} p + n &\rightarrow p + e^{+} + e^{-} \\ B = 1 + 1 &\neq 1 + 0 + 0 \end{aligned}$$

Conservation of lepton number also shows why the anti-electron neutrino is required in beta decay:

$$\begin{aligned} n &\rightarrow p + e^{-} + \overline{\nu}_e \\ L = 0 &= 0 + 1 + (-1) \\ B = 1 &= 1 + 0 + 0 \end{aligned}$$

18 The Quantum Atom

Content

- linespectra
- energy levels in the hydrogen atom

Candidates should be able to:

(a) explain atomic line spectra in terms of photon emission and transitions between discrete energy levels

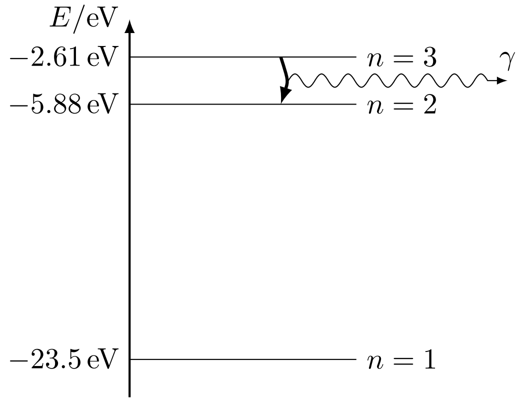
When a gas of atoms is given energy (e.g. by an electric field) that energy is emitted at electromagnetic waves at a few, specific frequencies. These frequencies are the same for every atom of a specific element; however they differ between elements. At typical line spectrum is shown in Figure

The explanation for this behaviour is the quantisation of energy levels within the atom. Electrons are only able have occupy certain (discrete) energies and when they move between these energies they emit (or absorb) photons of electromagnetic radiation. These photons have different frequencies depending on the amount of energy they carry away.

(b) apply $E = hf$ to radiation emitted in a transition between energy levels

When an electron in an atom falls from one energy level to a lower one the excess energy is emitted as a photon. The energy of this photon is equal to the energy lost.

Example Question
An electron in an atom falls from the $n = 3$ state to the $n = 2$ state as shown below. Calculate the wavelength of the photon emitted.



Answer
The difference in energy between the two levels is given by

$$E = (-2.61\,\text{eV}) - (-5.88\,\text{eV}) = 3.27\,\text{eV}$$

The wavelength can be calculated using

$$E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{hc}{5.24 \times 10^{-19}\,\text{J}} = 379\,\text{nm}$$

(c) show an understanding of the hydrogen line spectrum, photons and energy levels as represented by the Lyman, Balmer and Paschen series

The example above shows a single transition between energy levels. In reality when an atom is excited it will emit photons corresponding to many transitions at once. The example of the Hydrogen atom is shown in Figure 18.1. These transitions can be grouped into series based on which energy level the electron ends up in. Here there are three series shown which correspond to the Lyman (falling to $n = 1$), Balmer (falling to $n = 2$) and Paschen (falling to $n = 3$) Series. For simplicity only six energy levels are shown but in reality each series has potentially infinitely many possible starting states, although most of them will have very similar energies (as the starting energy approaches zero).

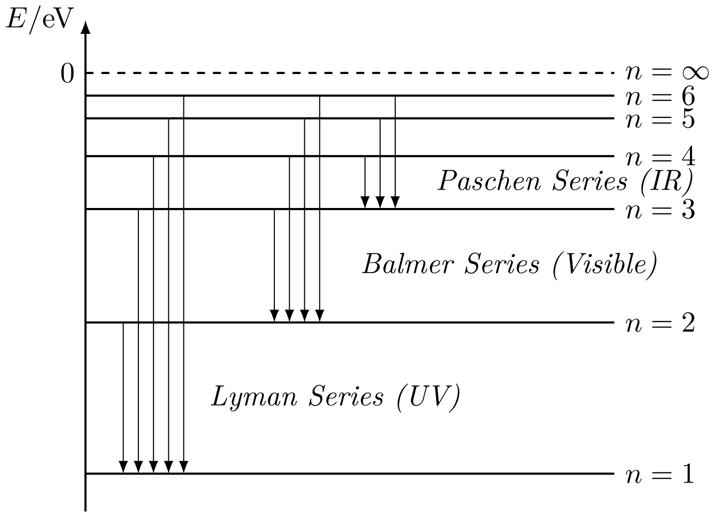


Figure 18.1: Transitions corresponding to the Hydrogen spectrum

(d) recognise and use the energy levels of the hydrogen atom as described by the empirical equation

$$E_n = \frac{-13.6\,\mathrm{eV}}{n^2} \tag{18.1}$$

By studying the line spectra of hydrogen equation 18.1 can be determined from experiment. The equation can be used to determine the wavelengths of the emission lines of the Hydrogen spectrum:

$$E_\gamma = -13.6\,\mathrm{eV}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

*(e) *explain energy levels using the model of standing waves in a rectangular one-dimensional potential well*

The orbital model of electrons in an atom allows electrons to have any energy we require. However, if one considers the electron to be acting as a standing wave then the idea of discrete energy levels comes naturally.

The simplest model of an electron as a standing wave is to consider the standing wave as a linear wave bound at each end. With an electron we define a ‘potential well’, i.e. a region of space in which the electron has zero potential energy and the rest of space the electron would have infinite potential energy. The electron is therefore bound within this space.

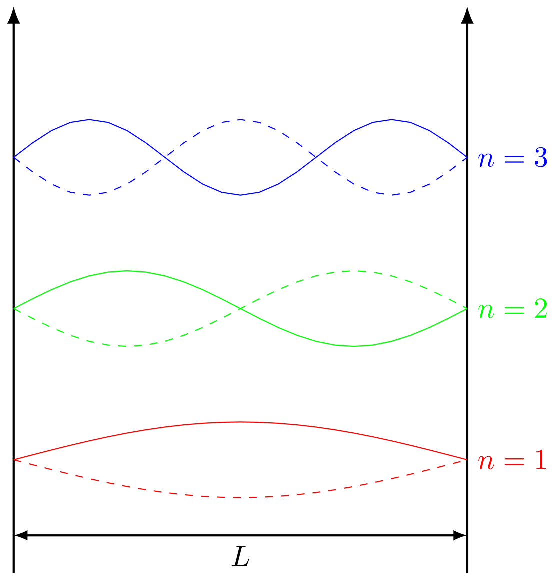


Figure 18.2: Three standing waves in a potential well

For each of the standing waves described in Figure 18.2 the wavelength can be calculated using

$$\lambda_n = \frac{2L}{n} \tag{18.2}$$

If the wavelength is related to the de Broglie wavelength of the electron then each of these standing waves can be given an momentum and hence energy.

$$E_n = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2 n^2}{8mL^2} \tag{18.3}$$

Equation 18.3 clearly can not match the empirical equation (18.1), but it does show a dependence on n and discrete energy levels.

*(f) *derive the hydrogen atom energy level equation $E_n = \frac{-13.6\,\mathrm{eV}}{n^2}$ algebraically using the model of electron standing waves, the de Broglie relation and the quantisation of angular momentum.*

In order to adapt the above model to fit an atom, the idea of the potential well was adapted to say that instead of fitting inside a potential well, a whole number of wavelengths should fit around the circumference of the atom. This gives a new criterion:

$$2\pi r = n\lambda \tag{18.4}$$

The de Broglie wavelength equation can be substituted into equation 18.4 to give

$$mvr = \frac{nh}{2\pi} \tag{18.5}$$

The quantity on the left is the *angular momentum*. It turns out that our quantisation rule based on wavelength is equivalent to stating that the angular momentum is quantised. The value $\frac{h}{2\pi}$ is so common in quantum theory that it has its own symbol, $\hbar$.

In fact, equation 18.5 was Bohr's starting point for his model of the atom.

Now we have a rule for the quantisation we can apply it to the classical model of the hydrogen atom. The electron in the classical model has electrostatic potential energy due to its attraction to the nucleus and kinetic energy due to its orbit around the nucleus. In order to calculate the kinetic energy we calculate the v^2 by equating the centripetal force to the electrostatic attraction.

$$\begin{aligned}\frac{mv^2}{r} &= \frac{e^2}{4\pi\epsilon_0 r^2} \\ v^2 &= \frac{e^2}{4m\pi\epsilon_0 r} \qquad (18.6)\end{aligned}$$

Note that we use this express for v^2 *twice* in our derivation.

Energy can now be calculated:

$$\begin{aligned}E &= \mathrm{KE} + \mathrm{PE} \\ &= \frac{1}{2}mv^2 + -\frac{e^2}{4\pi\epsilon_0 r} \qquad (18.7) \\ &= \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} \\ &= -\frac{e^2}{8\pi\epsilon_0 r} \qquad (18.8)\end{aligned}$$

Note that in equation 18.7 the PE is negative due to the opposite signs of the electron and the nucleus and that the total energy (18.8) is negative due to the bound state of the electron.

We can now introduce our quantisation criteria (18.5) by calculating the allowed values of r. This makes use of the v^2 term from equation 18.6. The difficult point is to remember that equation 18.5 should be rearranged to give r *squared*.

$$r^2 = \frac{n^2h^2}{4\pi^2m^2v^2} \tag{18.9}$$

$$= \frac{n^2h^2}{4\pi^2m^2}\frac{4m\pi\epsilon_0 r}{e^2} \tag{18.10}$$

$$r = \frac{n^2h^2\epsilon_0}{\pi m e^2} \tag{18.11}$$

Finally, equation (18.11) is substituted into the equation for energy (18.8)

$$\begin{aligned} E &= -\frac{e^2}{8\pi\epsilon_0 r} \\ &= -\frac{e^2}{8\pi\epsilon_0}\frac{\pi m e^2}{n^2h^2\epsilon_0} \\ &= -\frac{me^4}{8\epsilon_0^2 n^2 h^2} \\ &= \frac{E_1}{n^2} \end{aligned}$$

$$\text{where } E_1 = -\frac{me^4}{8\epsilon_0^2h^2} = -2.17\times10^{-18}\,\text{J} = -13.6\,\text{eV}$$

which matches the empirical formula (18.1)!

Note that many calculators give a value of zero if you type this equation in as one calculation. This is because $me^4 = 5.97\times10^{-106}$ *which casio calculators cannot cope with. A way around this is to calculate directly in electron-volts by dividing through by* e *thus requring only* me^3 *to be calculated.*

This powerful piece of reasoning also gives a value for the radius of the hydrogen atom which matches that measured by experiment.

This reasoning can be extended to nuclei with different charges; however it only works with a single electron as further inter-electron interactions are not taken into account.

19 Interpreting Quantum Theory

Background Information

(This is not part of the Pre U syllabus but it will help with your understanding.)

Wave Equation

We have already seen how oscillations can be expressed as second order differential equations in Simple Harmonic Motion.

To describe waves we need to go a step further and consider how it changes with time and distance.

If we imagine ocean waves, if we stand on the end of a jetty, i.e. in a *fixed position*, we can see the water moving up and down as time goes by.

Equally we could take a photograph, i.e. a *fixed time*, and see how the wave changes with distance.

So for a general wave equation we need a second order differential equation for the oscillations but with respect to time, with position kept constant, and with respect to position with time kept constant. To do this we use partial differentiation which is written with a curly ∂ and means keep the other variables constant.

Here is the equation which describes wave motion where u is a function of time and position and c is the speed of the wave.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

The derivation of this is slightly beyond the level of the Pre U although it was first done in the 18th Century.

Schrödinger's Wave Equation

In the 1920's, Austrian physicist Erwin Schrödinger used de Broglie's idea of particles having wavelengths to try and formulate a wave equation which could explain some of the quantum phenomena which had been observed.

He came up with the following equation originally to describe the energy levels in Hydrogen but it has been incredibly successful (matching experimental data to a high degree of accuracy) in explaining a wide number of quantum mechanical phenomena.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$$

Note the partial differentials and Planck's constant as well as the complex numbers.

You will not need to know this but you need to know that it exists. Also note the wave function Ψ (Psi). It is a function of space and time but when Schrödinger first came up with the equation, he didn't know what it represented.

*(a) *interpret the double-slit experiment using the Copenhagen interpretation (and collapse of the wavefunction), Feynman's sum-over-histories and Everett's many-worlds theory*

The double-slit experiment involved shining a beam of light, electrons or other particles through two small slits and observing the interference pattern cast on a screen. This can be explained using the wave theory, path difference and superposition. (see earlier chapter.)

If the beam of light is reduced in intensity until only one photon passes through the slits at a time, the individual photon will hit the screen at a unique place although it is impossible to predict exactly where. If we continue to send individual photons through we end up with the same diffraction pattern as before.

The problem here is that the diffraction pattern needs a wave interpretation whereas we are sending individual particles through. A particle can't split up between two slits and interfere with itself? So when the photon is released it is a particle, as it passes through the slits it is a wave and when it hits the screen it becomes a particle again.

What causes it to change from one thing to another?

Where is the photon just before it hits the screen?

What is it a wave of?

There hasn't yet been a satisfactory explanation to these questions but here are a few attempts.

- Copenhagen Interpretation

  The Danish physicist Niels Bohr set up a conference in Copenhagen with some of the best scientific minds of the time and came up with the first explanation.

  Schrodinger had already developed his famous wave equation which had great success at calculating quantum interactions although he didn't establish what the wave function represented.

  It was Max Born who later suggested that the wave function squared (similar to intensity) represented the probability of finding a particle at that point.

  Just before the photon hits the screen it could be anywhere where the wave function isn't zero and yet once it hits the screen and a measurement is made, it is in one precise point on the screen. We call this change from having a probability of being in a number of places to being in one definite location the *wave function collapsing*.

  This is the Copenhagen interpretation, until a measurement is made, the particle is simultaneously in all the possible different states and it is only the act of making a measurement which forces it into one unique outcome.

- Everett's Many World's Theory

  A weakness with the Copenhagen interpretation is the question of what is needed to make the wave function collapse? We have slightly skirted around the issue by saying that it is when a measurement is made. But what constitutes a measurement? Do we need a human observer? Particles will undergo many interactions, do they really exist in all possible states until an observation is made?

  Everett proposed a solution to this by saying that whenever multiple opportunities occur, for example where our photon hits the screen, the universe splits into multiple versions of itself. Each universe is identical apart from this one difference so one universe will have the photon hitting the centre of the screen whilst in another it will hit the first maximum. When you make a reading the result will depend on which branch of reality you are in. There will be multiple universes with different versions of you in them but you will never meet up. The only reality you will know is what happens in the branch you are in.

This is great fun to think about, there will be a parallel universe where you have already taken your pre U Physics a year early, scored full marks and are now sipping cocktails on a beach. It also does away with all the arbitrary reasons for the wave function collapsing. The problem with it is the number of universes which are continually being generated. Every time a subatomic particle interacts with another one, the universe splits. Thinking about how many interactions happen every second leads to a mind blowing number of universes and trying to visualise how they can exist alongside each other, perhaps using extra dimensions, is near impossible.

- Feynman's sum-over-histories

  The last interpretation was proposed by Feynman as a mathematical way of dealing with quantum phenomena. In the case of the double slit experiment we can only know that a photon leaves the laser and where it arrives on the screen after it has been detected. We have no way of knowing which slit it passed through or what route it took to get from the laser to the screen. What Feynman did was to assume that the photon takes *every* possible path. Not just straight lines, not just direct routes. A photon could go to the ends of the universe and back as it makes its way to the screen. If we add all the possible paths together some of them will cancel out if they arrive out of phase, or reinforce if they are in phase. What end up with is a probability of the photon being at any given point on the screen. This probability corresponds exactly with the observed diffraction pattern.

*(b) *describe and explain Schrödinger and appreciate the use of a thought experiment to illustrate and argue about fundamental principles*

Unhappy with the Copenhagen interpretation for the quantum world, Schrödinger set up a now famous thought experiment. The new quantum physics was very counter intuitive in terms of observed physical phenomena and yet gave an incredibly accurate model of the sub atomic world. But the world we see around us is made up of particles so, by extension, the same physics should hold true for both. What Schrödinger did was to take a purely random quantum phenomena and use it to control a macroscopic event.

Schrödinger's Cat Paradox A cat is put inside a box with a vial of poison and a radioactive material. If there is a radioactive decay, it will break the vial and release the poison and kill the cat. There is a 50/50 chance of the substance decaying in a given time so there is a 50/50 chance of the cat being dead or alive. According to the Copenhagen interpretation, until a measurement is made, both states exist. So before the box is opened, the particle has and hasn't decayed and consequently the cat is simultaneously dead and alive. Schrödinger's use of a dead cat was inspired but he put it forward to show how ridiculous the whole situation was. Later, after it gathered an inordinate amount of attention he regretted ever coming up with it.

In the many world's interpretation, the universe splits into two. One has a box with a decayed particle and a dead cat whilst in the other the cat is alive. When we open the box we see the outcome according to whichever branch of the universe we are in.

*(c) *recognise and use $\Delta p \Delta x \geq \frac{h}{2\pi}$ as a form of the Heisenberg uncertainty principle and interpret it*

Imagine two waves, one is a sine wave and the other is a short pulse.

The sine wave has a definite frequency and wavelength but stretches out indefinitely. The pulse is made up of many sine waves of different frequencies (see Fourier transforms for more on this) but has a fixed size.

So with waves we can see that there is a trade-off between a clearly defined position and wavelength.

We also know that the deBroglie wavelength of a particle is related to the momentum.

$$\lambda = \frac{h}{p}$$

Mathematically the combination of position and wavelength which gives the lowest combined uncertainty is with a Gaussian (normal) wave and from this we can place a lower bound on the uncertainty in momentum and position.

Heisenberg's Uncertainty Principle

$$\Delta p \Delta x \geq \frac{h}{2\pi}$$

so the more accurately we know the momentum of a particle, the less we can know about its position.

note: this is not the effect of taking the measurement which introduces the uncertainty but is an intrinsic limitation due to the wave nature of particles.

*(d) *recognise that the Heisenberg uncertainty principle places limits on our ability to know the state of a system and hence to predict its future*

If we roll a dice we consider it a random process but theoretically if we could measure the exact velocity, spin, gravitational force, air resistance, frictional forces of the table etc. we would know what side the dice would land on.

In the case of quantum particles we can't know the initial conditions precisely because of the limits imposed by Heisenberg's principle. The more accurately we know one thing, the less we know about the other and so we cannot accurately predict what will happen in the future.

*(e) *recall that Newtonian physics is deterministic, but quantum theory is indeterministic*

With Newtonian physics, the physics we observe in the everyday world, the initial conditions will set into motion a chain of events which determine what happens in the future. If you watch cricket on television you may have seen a computer making leg before wicket decisions by continuing the trajectory of the ball and seeing if it would have hit the wicket had the batsman's leg not been in the way. By knowing the flight of the ball the subsequent path could be determined.

In quantum physics we cannot know all the initial conditions and so the future cannot be determined.

We say that Newtonian physics is deterministic whilst quantum physics is indeterministic.

*(f) *understand why Einstein thought that quantum theory undermined the nature of reality by being:*

(i) indeterministic (initial conditions do not uniquely determine the future)

(ii) non-local (for example, wave-function collapse)

(iii) incomplete (unable to predict precise values for properties of particles).

Einstein was not happy with quantum theory and couldn't accept that nature could be governed by probability. His famous quote "God does not play dice with the universe" sums up his frustration.

Einstein wanted the universe to be deterministic but we have already seen that this is not what quantum theory predicts.

Spukhaft Fernwirkung

Spooky action at a distance. Einstein was concerned that quantum physics seemed to allow faster than light travel. Going back to the double slit experiment, the instant before the particle hits the screen it could be in a whole range of different places and yet the moment that the measurement is made, the wave function collapses and it is in one unique place. How did it get there so quickly?

He also proposed another famous thought experiment with Podoslki and Rosen known as the EPR paradox.

If we start with two particles fired with equal velocities from a common origin their subsequent motion can be described using a single, two-particle wave function. After some time a measurement of the position of one of the particles will instantaneously fix the other particle even though it could be a great distance away and was not being measured directly.

The final thing which upset Einstein was not being able to know everything about the state of a particle i.e. incomplete knowledge. We cannot know the position and momentum simultaneously.

20 Astronomy and cosmology

Content

- standard candles
- stellar radii
- Hubble's law
- the Big Bang theory
- the age of the Universe

(a) understand the terms luminosity and luminous flux

(b) recall and use the inverse square law for flux

$$F = \frac{L}{4\pi d^2} \tag{20.1}$$

Stars are described as LUMINOUS because they emit electromagnetic waves.

The LUMINOSITY of a luminous ("hot") object is defined as the amount of electromagnetic wave energy emitted by the object per second i.e. it is the emission Power of the object and is thus measured in Watts (W). It is a measure of the absolute "brightness" of the object.

FLUX (F) is Power per unit area (an Intensity). For a spherical emitter of radius R (and thus surface area of $4\pi R^2$), this means that $F = \frac{L}{4\pi d^2}$.

(c) understand the need to use standard candles to help determine distances to galaxies

Standard Candles are types of stars or galaxies for which the Luminosity (the absolute brightness) can be determined directly from observations. By measuring the observed brightness and comparing it to the absolute brightness of the object, it is possible to determine how far away that object is.

The best-known and most widely used Standard Candles are Cepheid Variables and Type 1a Supernovae.

Cepheid Variables (named after the star Delta Cephei) pulsate and vary in brightness with a frequency that is related to the star's Luminosity. The periods of some Cepheids have been measured as a few days, others a few months. Once the absolute brightness has been found from the observed period of the pulsation, it can be compared with the brightness the star appears to have as observed from the Earth and hence the distance can be determined.

Type 1a supernovae occur when one of the stars (a white dwarf) in a binary system gains mass, becomes unstable and catastrophically explodes emitting vast quantities of light and other electromagnetic energy in the process. The maximum absolute brightness achieved is related to the rate at which the emission fades (the so-called Light Curve). Thus, again, once the absolute brightness is known, a comparison with how bright the object appears to be from the Earth will yield its distance. Because they are so bright, these objects can be easily seen in distant galaxies, so that distances well beyond the limits of the Milky Way can be established.

(d) recognise and use Wien's displacement law

$$\lambda_{max} \propto \frac{1}{T} \tag{20.2}$$

to estimate the peak surface temperature of a star either graphically or algebraically

Black bodies of a particular temperature T will emit radiation over a range of wavelengths (a radiation distribution known as a Planck Curve):

The distribution peaks at a wavelength λ_{max} which is determined by the reciprocal of the absolute temperature T of the object. This is known as Wien's Displacement Law. The constant of proportionality has a value of 2.90 x 10^{-3} m K:

$$\lambda_{max} = \frac{2.90 \times 10^{-3}}{T}$$

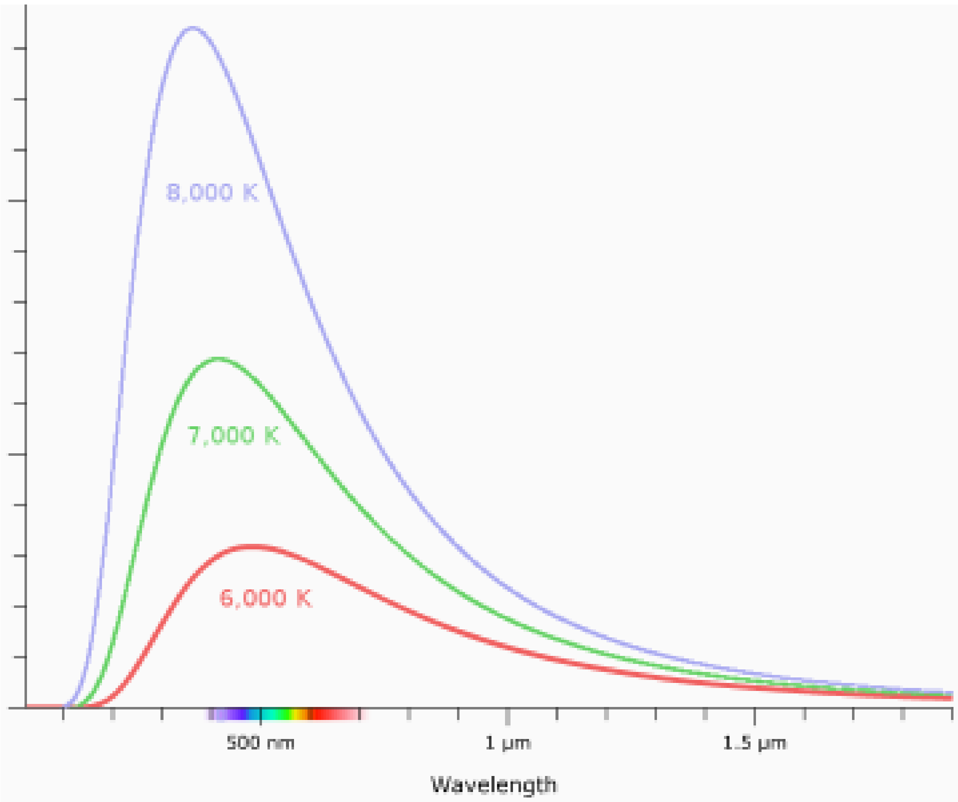


Figure 20.1: Planck curves

The greater the temperature, the larger the area under the curve, suggesting that more radiation is being emitted altogether, consistent with the Stefan-Boltzmann relation. Refer to Figure 20.1 and consider the T = 8000 K and the T = 6000 K graphs. The area beneath the T = 8000 K graph is $(8000/6000)^4$ times that beneath the T = 6000 K graph, i.e. 3.16 times greater.

The Sun's photosphere is at a temperature of a little under 6000 K; notice that the Planck Curve for this temperature peaks at a wavelength lying in the visible part of the electromagnetic spectrum.

Wien's Displacement Law enables astronomers to determine the temperature of a star from observations of the light (and other radiation) emitted by that star.

(e) recognise and use Stefan's law for a spherical body

$$L = 4\pi\sigma r^2 T^4 \tag{20.3}$$

The Stefan-Boltzmann Law (sometimes simply called Stefan's Law) states that the Flux from a hot object is proportional to the fourth power of the absolute temperature, T. Strictly speaking, this applies only to idealised radiation emitters (referred to as "black bodies"); the constant of proportionality is the Stefan-Boltzmann constant, σ, which has a value of 5.67 x 10^{-8} W m^{-2} K^{-4}:

$$F = \sigma T^4$$

Therefore, for an emitted with surface area A, the Luminosity will be given by

$$L = \sigma A T^4$$

and in the case of a spherical emitter (such as a star) of radius R, this becomes

$$L = 4\pi R^2 \sigma T^4$$

(f) use Wien's displacement law and Stefan's law to estimate the radius of a star

Having determined the temperature, T, one can then use the Stefan-Boltzmann relation to determine the Luminosity, L. This is a measure of the actual amount of radiation being emitted by the star. By measuring the actual amount of radiation received per second per unit area (i.e. the flux, F), one can then calculate how big the star must be (the surface area from which the radiation is being emitted) in order to produce that amount of Flux.

Example Question

Worked example: Proxima Centauri, the nearest star to the Sun

Data:

Parallax angle = 0.8 seconds of arc

Peak wavelength, λ_{max} = 967 nm

Flux measured at Earth = 3.56 x 10^{11} W m^{-2}

1. Calculate the distance to Proxima Centauri.
2. Calculate its surface temperature.
3. Calculate the Flux at the star's surface
4. Hence calculate the radius of the star.

Answer

1. D (in parsecs) = 1 / angle of parallax in seconds of arc = 1.25 pc = *3.85 x 10^{16} m*
2. From Wien's Displacement Law, T = 2.90 x 10^{-3} m K / 967 x 10^{-9} m = 3000 K
3. From Stefan's Law, F = σ T^4 = 5.67 x 10^{-8} x 8.10 x 10^{13} = *4.6 x 10^6 W m^{-2}*
4. Flux measured at a distance of 3.85 x 10^{16} m is 3.56 x 10^{-11} W m^{-2}.

   So, $F_{\text{at Earth}}$ / $F_{\text{star's surface}}$ =. (radius of star)^2 / (distance to star)^2

- Radius of star = *1.07 x 10^8 m*

(g) understand that the successful application of Newtonian mechanics and gravitation to the Solar System and beyond indicated that the laws of physics apply universally and not just on Earth

Newton's law of Universal Gravitation states that two masses, M and m, whose centres are separated by a distance r, will mutually attract with a gravitational force given by

$$F = \frac{GMm}{r^2}$$

where G, the Universal Gravitational Constant, = 6.67 x 10^{-11} N m^2 kg^{-2}.

One of the great triumphs of the law was to demonstrate consistency with Kepler's Laws of Planetary Motion, formulated empirically some 70 years earlier. Kepler's Laws state that the planets move about the Sun in elliptical orbits and whilst Newton's Law can be applied to such orbits, for simplicity we consider a planet moving in a circular orbit. If the radius of the orbit is r, then from rotational mechanics, the planet will experience a constant centripetal force of mv^2/r. This origin of this force is the gravitational attraction given by Newton's equation and by equating the two formulae it is possible to show that

$$T^2 = \frac{4\pi^2 r^3}{GM} \tag{20.4}$$

i.e. that $T^2 \propto r^3$ as stated by Kepler's 3rd Law. Newton's Law, when combined with his Laws of Motion, was applied to other objects observed to be in gravitational orbits with great success. The movements of planetary satellites, binary star systems, galactic spiral arms and even clusters of galaxies themselves have all been shown to be consistent with the relationships. Famously, the relationships demonstrated orbital irregularities in the motion of the planet Uranus, high led to the discovery of the planet Neptune beyond it. Similar anomalous behaviour in the rotations of spiral galaxies observed by Vera Rubin has led to the speculation of the existence of Dark Matter.

Example Question
From the orbital data for the Earth, calculate the mass of the Sun (assuming a circular orbit).
Answer
Radius of Earth's orbit = 150 x 10^6 km
Orbital period of earth = 1 year = 3.2 x 10^7 s

- From Kepler's 3rd Law (above), *M = 1.96 x 10^{30} kg*

(h) recognise and use

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c} \tag{20.5}$$

for a source of electromagnetic radiation moving relative to an observer

When a source of waves moves away from an observer (in a stationary medium), the observer will receive waves of a longer wavelength (and thus lower frequency) than those emitted by the source. If the source moves towards the observer then the observed wavelength is shorter (frequency is higher). This is called Doppler Shift. For example, with sound waves this can be perceived as a change in the pitch of the emitted sound.

The greater the speed of the source, v, the greater the change in the wavelength $\Delta\lambda$ (or frequency, Δf, whichever is being measured) of the waves received by the observer in which c is the velocity of the emitted waves (a relationship strictly only true for if c is much greater than v).

The dark lines observed in the visible light spectra of stars and galaxies are caused by the absorption of specific frequencies (colours) by elements present in those objects, enabling astronomers to determine their composition. In the 1920s, Edwin Hubble discovered that the absorption lines for distant galaxies were shifted towards the red end of the colour (emission) spectrum. This Red Shift indicated that the galaxies were moving away (receding) from the Earth. But there were two particular features of his discovery that made a special impact:

a) Galaxies were receding in all directions.

b) The more distant the galaxy, the higher its recessional velocity.

(The distances to the galaxies were established using Standard Candles, especially Cepheid Variables.)

(i) state Hubble's law and explain why galactic redshift leads to the idea that the Universe is expanding and to the Big Bang theory

These observations have led to the conclusion that the universe began with a Big Bang and what Hubble observed was the expansion of space itself. A helpful picture is that of the infinite scaffolding by M C Escher (Figure 20.2):

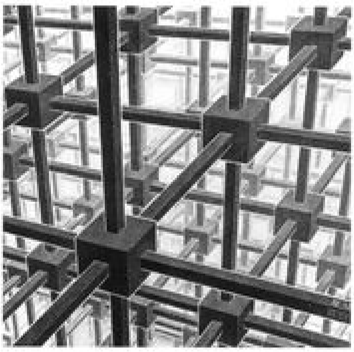


Figure 20.2: M C Escher drawing

No matter which junction you view from, if each scaffold pole is expanding, junctions in all directions will appear to recede. Note that, in effect, the junctions themselves do not move: the space in between them does. Also, more distant junctions (with more expanding poles between them and the observer) will appear to recede with greater speeds.

In such a model, the speed of "recession" (expansion) is proportional to the distance of the observed galaxy and observations are by and large consistent with this:

$V = H_o d$

in which the constant of proportionality, $H_{o,}$ is known as the Hubble Constant, the value of the gradient of the graph of v plotted against d (Figure 20.3):

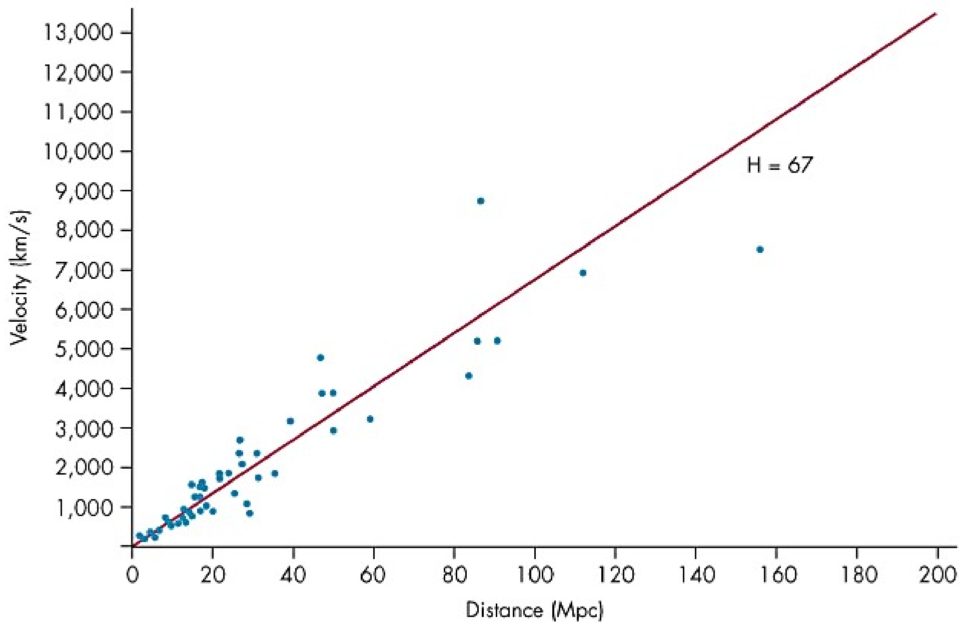


Figure 20.3: Hubble's Law

Recent measurements (see graph) suggest a value of H_o of a little over 67 km s^{-1} Mpc^{-1}

Given that, in the observable universe, the greatest distance from which radiation could be received is given by the speed of light x the age of the universe, it follows that the age of the universe will be given by $1/H_o$. For the value of H_o quoted, this equates to an age of

When measuring the red shifts of distant galaxies, the calculated velocities thus more properly indicate the rate at which the intervening space is stretching. The value of $\Delta\lambda/\lambda$ (or $\Delta f/f$) is called the Cosmological Red Shift, z, indicating that the space has expanded by a factor of 1 + z in order to produce the observed Doppler Shift.

The quasar 3C273 was the first object of its kind to be identified. So called because they were star-like but much more luminous ("quasi-stellar objects") they are now known to consist of supermassive black holes that draw in a huge disc of orbiting gas, causing large emissions of radiation across a wide range of wavelengths. Their spectra exhibit large red shifts.

Example Question

1. One emission line in the spectrum of 3C273 appears at a wavelength of 475.0 nm; in the laboratory the same line is measured at 410.2 nm. Calculate the recessional velocity of the quasar.

2. Using the value of H_0 quoted above, hence calculate the distance of 3C273.

Answer

1. $\Delta\lambda$ = 475.0 – 410.2 nm = 64.8 nm.

From Doppler equation, $\Delta\lambda/\lambda$ = v/c, this gives v = 4.74 x 10^7 ms^{-1} or *4.74 x 10^4 km s^{-1}*.

1. Assume a value of H_0 of 67 km s^{-1} Mpc^{-1}.

- d = 4.74 x 10^4 km s^{-1} / 67 km s^{-1} Mpc^{-1} = *707 Mpc.*

(j) explain how microwave background radiation provides empirical support for the Big Bang theory

(k) understand that the theory of the expanding Universe involves the expansion of space-time and does not imply a pre-existing empty space into which this expansion takes place or a time prior to the Big Bang

(l) recall and use the equation

$$v \approx H_0 d \qquad (20.6)$$

for objects at cosmological distances

(m) derive an estimate for the age of the Universe by recalling and using the Hubble time

$$t = \frac{1}{H_0} \tag{20.7}$$

In the 1940s, George Gamow suggested that the observed ratio of hydrogen to helium in the universe could be explained by assuming the universe was much hotter and denser a long time ago, consistent with the idea of a Big Bang origin stemming from Hubble's work. His theory predicted the existence of a "leftover" radiation which was eventually discovered by chance in 1965. This is today known as the Cosmic Microwave Background Radiation and it has the biggest cosmological red shift. It was produced when the early universe had cooled down to about 3000K, low enough for electrons to combine with protons to produce atoms, a process resulting in the emission of photons of wavelengths around 1 mm. Because of cosmological expansion, the wavelength is now a thousand times bigger (millimetres) and the temperature a thousand times smaller, about 3 K.

Refinements to the Big Bang model have included a period of Inflation very early on which gave rise to the eventual clumping of matter, which accounts for the later existence of stars and galaxies (and planets and humans). The Cosmic Background Explorer satellite revealed such clumping (measured as tiny temperature fluctuations in the background radiation) and further evidence is still being sought to confirm Inflation as a part of the model.

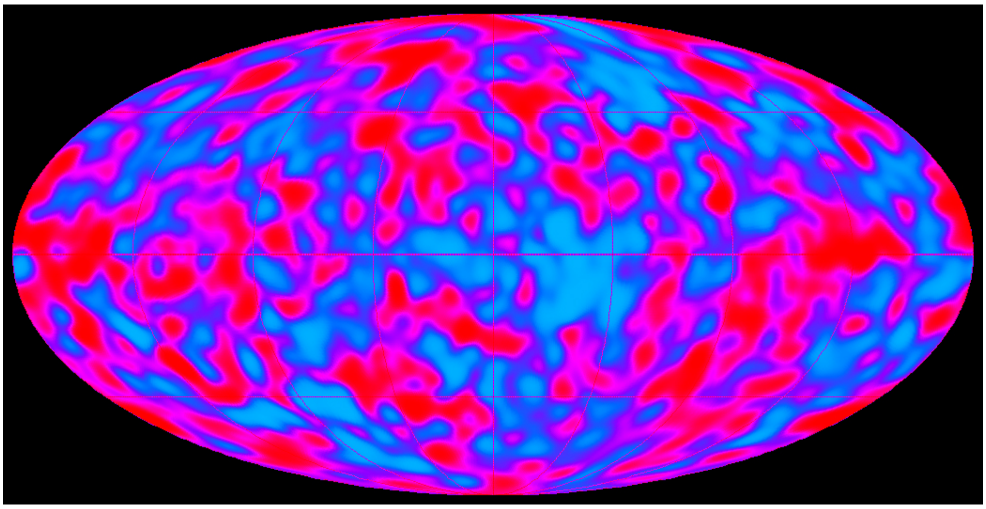


Figure 20.4: COBE satellite image

Appendices

A Equations

Equations to be learnt

Equations you need to remember for pre-U

velocity	$v = \frac{\Delta x}{\Delta t}$
acceleration	$a = \frac{\Delta v}{\Delta t}$
resultant force	$F = ma$
momentum	$p = mv$
resultant force	$F = \frac{\Delta p}{\Delta t}$
impulse	Impulse = change in momentum
density	Density = mass/volume
pressure	Pressure = force/area
pressure in a liquid	$P = \rho g h$
weight	$W = mg$
power	$P = Fv$
GPE	$\Delta E = mg\Delta h$
change in gravitational potential	$= g\Delta h$
energy in a spring	$E = \frac{1}{2}Fx$

efficiency	$\%\ \text{efficiency} = \frac{\text{useful energy or power}}{\text{total energy or power in}} \times 100$
current	$I = \frac{\Delta Q}{\Delta t}$
potential difference	$V = \frac{W}{Q}$
resistance	$R = \frac{V}{I}$
electrical power	$P = VI$
electrical work done	$W = VIt$
resistance	$R = \frac{\rho l}{A}$
resistors in series	$R_T = R_1 + R_2 + \ldots$
resistors in parallel	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
frequency	$f = 1/T$
wave speed	$v = f\lambda$
Malus' law	$\text{Intensity} \propto \cos^2\theta$
photoelectric equation	$hf = \phi + \frac{1}{2}mv_{\text{max}}^2$
angular velocity	$v = r\omega$
period	$T = 2\pi/\omega$
circular motion	$F = mv^2/r$
electric field strength	$E = F/Q$
capacitance	$C = Q/V$
field strength/potential	$E = -\frac{\mathrm{d}V}{\mathrm{d}X}$
the ideal gas law	$pV = nRT$
Boltzmann factor	$e^{-\frac{E}{kT}}$
activity	$A = -\frac{\mathrm{d}N}{\mathrm{d}t}$

luminous flux	$F = \frac{L}{4\pi d^2}$
Hubble's law	$v \approx H_0 d$
Hubble time	$t = 1/H_0$

Equations you need to remember for paper 3, part B, but not for any other part of the examination.

moment of inertia	$I = \Sigma m r^2$
shm	$\frac{d^2x}{dt^2} = -\omega^2 x$
	$x = A\cos\omega t$

Derivations

Equations you need to derive and remember for pre-U

energy in a spring	$E = \frac{1}{2}kx^2$
kinetic energy	$E = \frac{1}{2}mv^2$
emf	$E = I(R + r)$
emf	$E = V + Ir$
electrical power	$P = I^2R$
critical angle	$\sin c = 1/n$
centripetal acceleration	$a = v^2/r$
centripetal acceleration	$a = r\omega^2$
uniform electric field	$Fd = QV$
uniform electric field	$E = V/d$
energy in a capacitor	$W = \frac{1}{2}CV^2$
energy in a capacitor	$W = \frac{1}{2}\frac{Q^2}{C}$
electric field due to a point charge	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$
Kepler's third law	$r^3 \propto T^2$
gravitational field strength	$g = \frac{\mathrm{Gm}}{r^2}$
radius of curvature of particle in B-field	$r = \frac{\mathrm{mv}}{\mathrm{BQ}}$
Hall effect	$V = Bvd$
kinetic theory of gases	$pV = \frac{1}{3}Nm\langle c^2\rangle$
activity of a source	$A = \lambda N$

activity at time t	$A = A_0 e^{-\lambda t}$
half-life	$t_{\frac{1}{2}} = \frac{ln2}{\lambda}$

Equations you need to derive for paper 3, part B, but not for any other part of the examination.

moment of inertia of a ring	$I = mr^2$
moment of inertia of a disc	$I = mr^2/2$
moment of inertia of a rod about one end	$I = ml^2/3$
moment of inertia of a rod about its centre	$I = ml^2/12$
velocity in shm	$v = -A\omega \sin \omega t$
acceleration in shm	$a = -A\omega^2 \cos \omega t$
total energy in shm	$E = \frac{1}{2}mA^2\omega^2$
electric potential (from electric force)	$W = \frac{Q_1 Q_2}{4\pi\varepsilon_o r}$
hydrogen atom energy levels	$E_n = \frac{-13.6eV}{n^2}$