ICE-2231
(Data Structures and Algorithms)

Lecture on

Chapter-3: Stacks, Queues, Recursion

By

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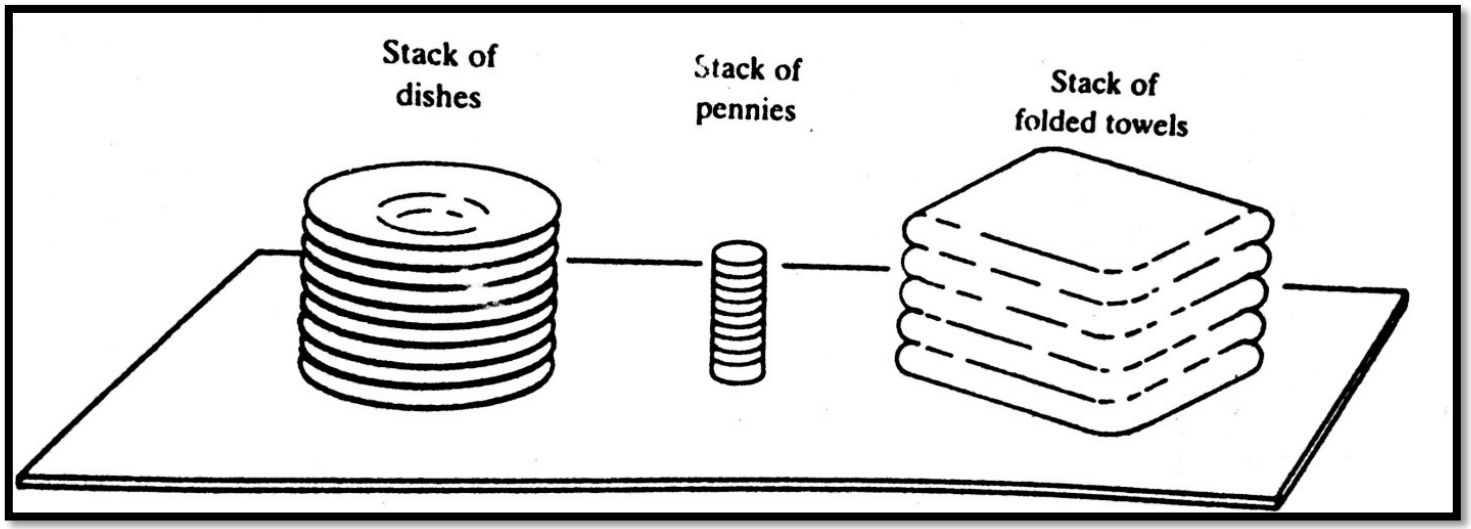
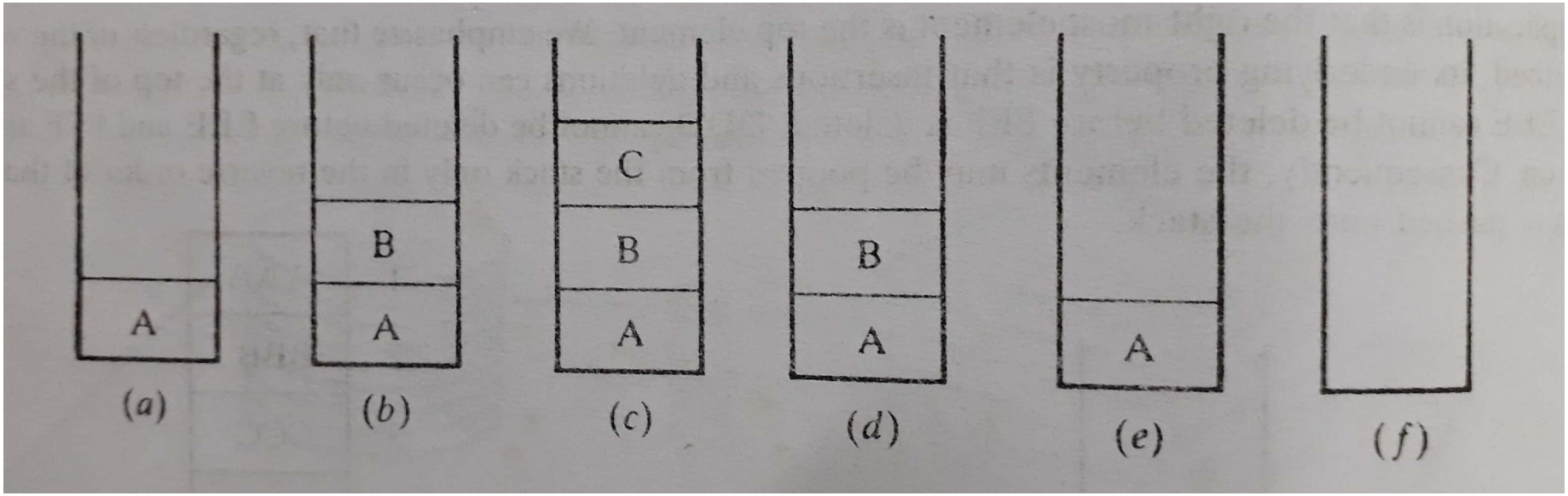
STACKS: Introduction



- A stack is a linear structure in which items may be added or removed only at one ends
- Stacks are also called Last-in-First-out (LIFO) list.
- Other names:
  - Piles
  - Push-down lists.
- Although the stack may be a very restricted type of data structure, it has many important applications in computer science.

STACKS: Dealing Postponed Decisions

Stacks are frequently used to indicate the order of the processing of data when certain steps of the processing must be postponed until other conditions are fulfilled.



STACKS: Special Terminology

Special terminology is used for two basic operations associated with stacks:

(a) “Push”-used to insert an element into a stack

(b) “Pop”-used to delete an element from a stack.

STACKS: Push Example

Suppose the following 6 elements are pushed, in order, onto an empty stack:

AAA, BBB, CCC, DDD, EEE, FFF

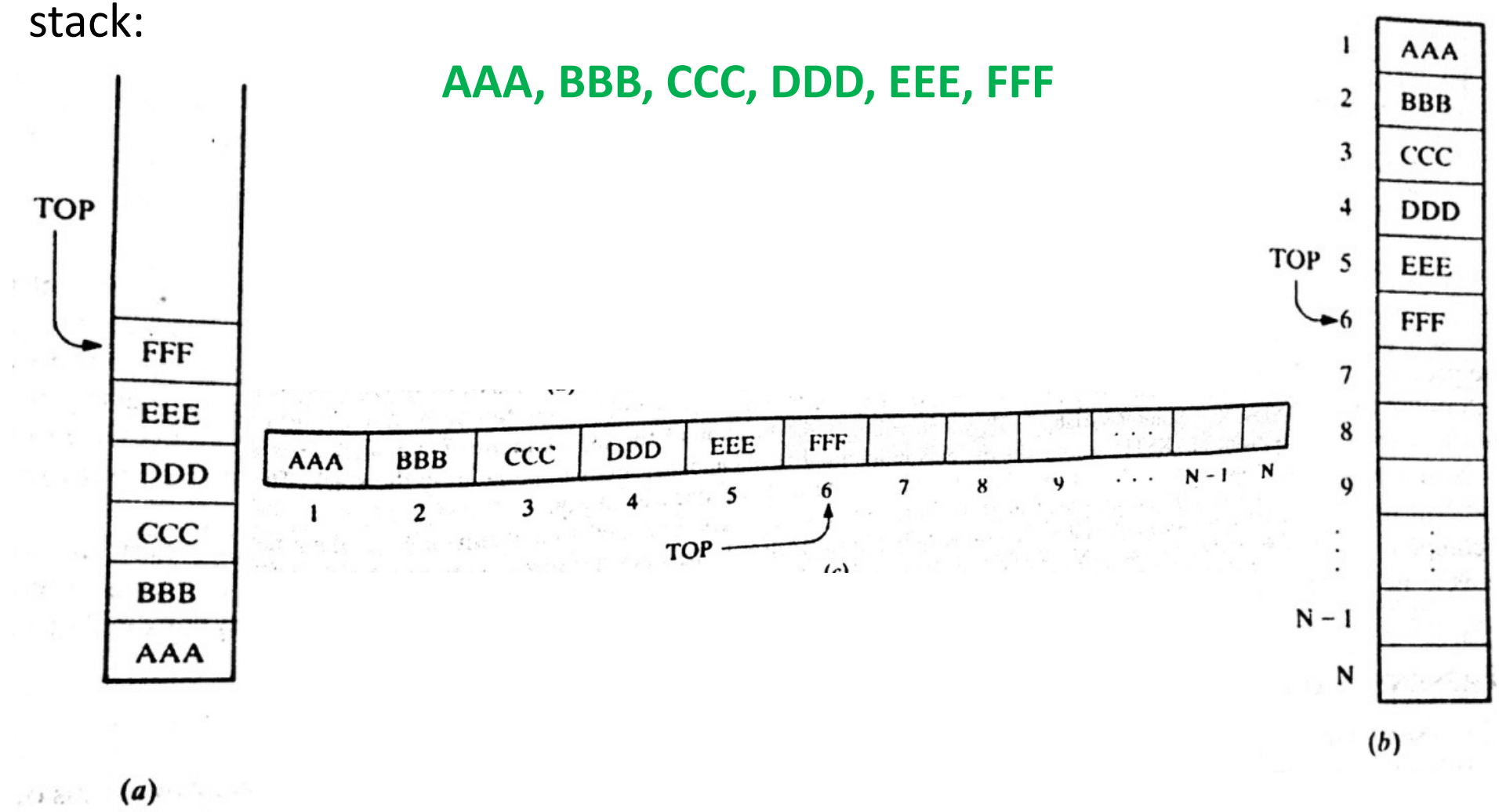


Diagram of Stack

STACKS: Array Representation

- Stack may be represented in the computer in various ways,
  - ✓ usually by means of a one-way list, or
  - ✓ A linear array
- Usually Stacks will be maintained by……
  - A linear array, **STACK**,**(**main lists**)**
  - A pointer variable, **TOP**, **(**contains the locations of the top element of the stack**)**
  - A variable **MAXSTK**, (gives the maximum number of elements that can be held by the stack)

The condition TOP=0 or **TOP=NULL** of a **STACK** indicates…..?

EMPTY

STACKS: Array Representation Example

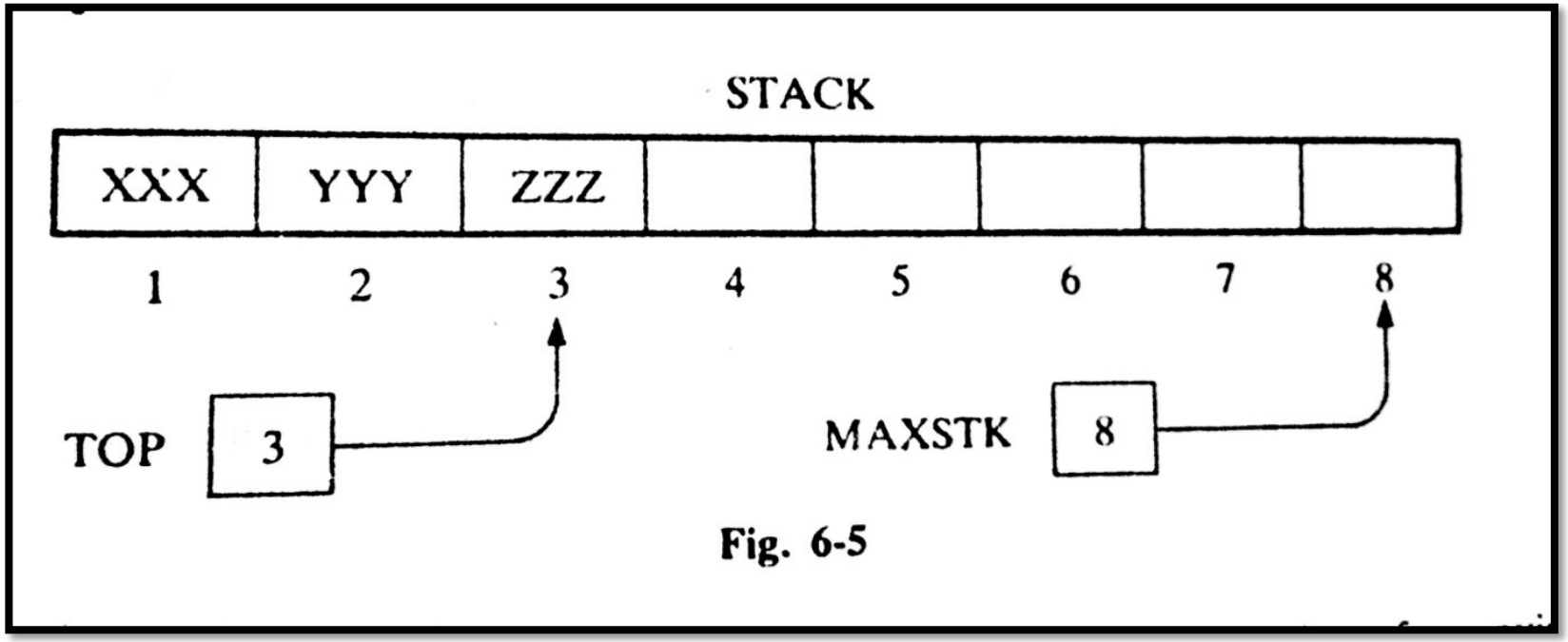


Fig. 6-5

Present Status: TOP=3;

MAXSTK=8;

There is room for 5 more items in the stack

Future Implementation:

Pushing an item onto a stack

Popping an item from a stack

STACKS: Pushing Algorithm

PUSH (STACK, TOP, MAXSTK, ITEM)

Step 1. [Stack already filled?]

If **TOP=MAXSTK,** then print: **OVERFLOW**, and Return.

Step 2. Set **TOP**=**TOP**+1. [Increase TOP by 1.]

Step 3. Set **STACK**[**TOP**]:=**ITEM.** [Insert ITEM in new TOP position]

Step 4. Return

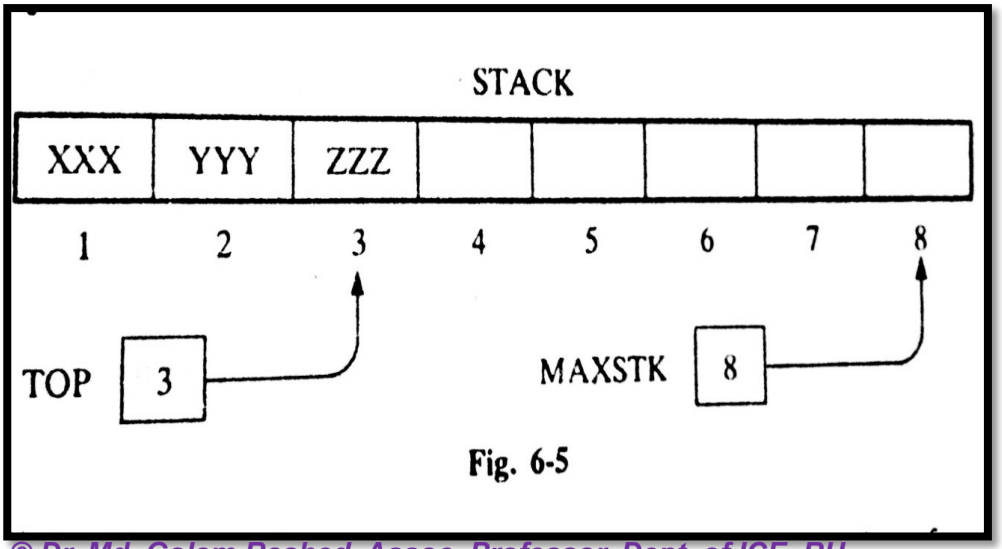


Fig. 6-5

1. Since TOP=3, go to Step 2

2. TOP=3+1=4

3. STACK [TOP]=STACK[4]= **www**

4. Return

STACKS: Popping Algorithm

POP (STACK, TOP, ITEM)

Step 1. [Stack has an item to be removed?]

If **TOP=0,** then print: **Underflow**, and Return.

Step 2. Set **ITEM**= **STACK**[**TOP**] [Assign TOP element to ITEM.]

Step 3. Set **TOP**:= **TOP-1.** [Decreases TOP by 1]

Step 4. Return

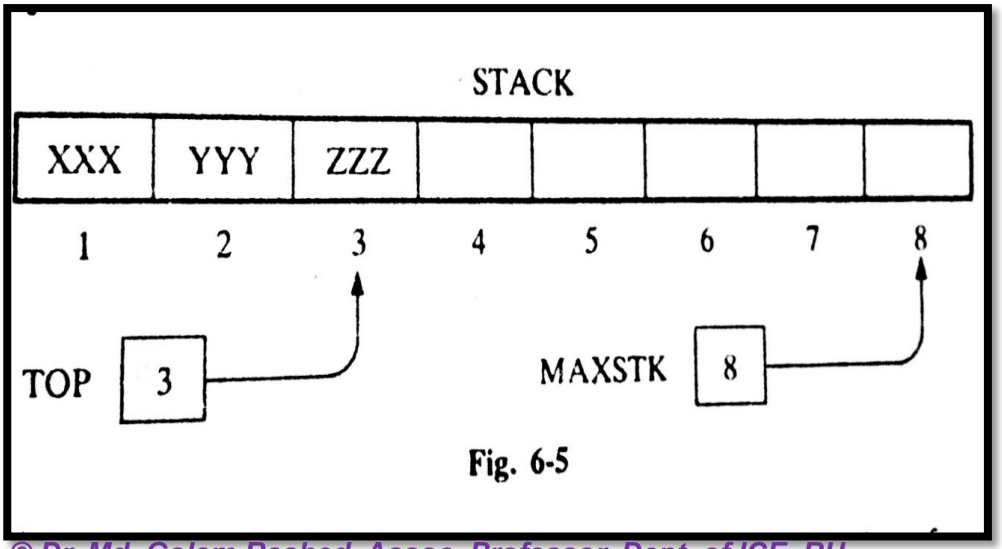


Fig. 6-5

1. Since TOP=3, go to Step 2

2. ITEM=ZZZ.

3. TOP=3-1=2.

4. Return.

STACKS:

PUSH

1. Since TOP=3, go to Step 2

2. TOP=3+1=4

3. STACK [TOP]=STACK[4]= **WWW**

4. Return

POP

1. Since TOP=3, go to Step 2

2. ITEM=ZZZ.

3. TOP=3-1=2.

4. Return.

✓ **TOP** is changed before the insertion in PUSH, whereas

✓ **TOP** is changed after the deletion in POP

STACKS: Arithmetic Expressions;

✓ Let Q be an arithmetic expression involving constants and operation.

Q: 2 ↑3+5*2 ↑2-12/6, we have to find the value of Q

✓ Q may have different levels of precedence in its binary operations.

✓ We assume the following three levels of precedence for the usual **FIVE** binary operations.

- Highest: Exponential (↑)
- Next Highest: Multiplication (*) and division(/)
- Lowest: Addition (+) and Subtraction(-)

✓ Thus, we obtain after exponentiations (8+5*4-12/6)

✓ After, multiplication and division (8+20-2)

✓ After. Addition and subtraction , 26

STACKS: Polish Notation

For most common arithmetic operations:

A+B, C-D, E*F, G/H (*infix notation*)

Polish notation refers to the notation in which the operator symbol is placed before its two operands.

For example: +AB, -CD, *EF, /GH

Instant Exercise: (***Infix expression*** to ***polish notation***)

(A+B)*C = ?

= *[A+B]C=?

= *+ABC

A+(B*C) = ?

= A+[*BC]=?

= +A*BC

STACKS: Polish Notation

$$(A+B)/(C-D) = ?$$
$$= [+AB]/[-CD] = ?$$
$$= /+AB-CD$$

- ✓ The fundamental property of polish notation is that the order in which the operations are to be performed is completely determined by
  - ✓ The positions of the operators, and
  - ✓ Operands in the expression.
- ✓ **One never needs parentheses when writing expressions in polish notation.**

STACKS: Reverse Polish Notation

Reverse Polish Notation refers to the analogous notation in which the operator symbol is placed after its two operands:

AB+, CD-, EF*, GH/

- ✓ This notation is frequently called ***postfix notation***
- ✓ One never needs parentheses to determine the order of the operations in any arithmetic expression in ***postfix notation***

STACKS: Evaluation of Postfix Expression

P: 5, 6, 2, +, *, 12, 4, /, -

Symbol Scanned	STACK
(1) 5	5
(2) 6	5, 6
(3) 2	5, 6, 2
(4) +	5, 8
(5) *	40
(6) 12	40, 12
(7) 4	40, 12, 4
(8) /	40, 3
(9) -	37
(10))	

STACKS:

Algorithm to find the VALUE of an arithmetic expression P written in postfix notation.

1. Add a right parenthesis “)” at the end of P.
2. Scan P from left to right and repeat Steps 3 and 4 for each of P until the “)” is encountered.
3. If an operand is encountered, put it on STACK
4. If an operator ⌘ is encountered, then:
   a) Remove the two top elements of the STACK, where A is the top element and B is the next-to-top element
   b) Evaluate B ⌘ A
   c) Place the result of (b) back on STACK.

   [End of If structure]

   [End of Step 2 loop]
5. Set VALUE equal to the TOP element on STACK.

Exit

STACKS: Transforming Q into P expression

Q: A + (B * C- (D / E ↑ F) * G) * H = (?) P expression

Symbol Scanned	Stack	Expression P
(1) A	(	A
(2) +	(+	A
(3) (	(+ (	A
(4) B	(+ (	A B
(5) *	(+ (*	A B
(6) C	(+ (*	A B C
(7) -	(+ (-	A B C *
(8) (	(+ (- (	A B C *
(9) D	(+ (- (	A B C * D
(10) /	(+ (- (/	A B C * D
(11) E	(+ (- (/	A B C * D E
(12) ↑	(+ (- (/ ↑	A B C * D E
(13) F	(+ (- (/ ↑	A B C * D E F
(14))	(+ (-	A B C * D E F ↑ /
(15) *	(+ (- *	A B C * D E F ↑ /

STACKS: Transforming Q into P expression

Q: A + (B * C- (D / E ↑ F) * G) * H = (?) P expression

Symbol Scanned	Stack	Expression P
(7) -	(+ (-	A B C *
(8) (	(+ (- (	A B C *
(9) D	(+ (- (	A B C * D
(10) /	(+ (- (/	A B C * D
(11) E	(+ (- (/	A B C * D E
(12) ↑	(+ (- (/ ↑	A B C * D E
(13) F	(+ (- (/ ↑	A B C * D E F
(14))	(+ (-	A B C * D E F ↑ /
(15) *	(+ (- *	A B C * D E F ↑ /
(16) G	(+ (- *	A B C * D E F ↑ / G
(17))	(+	A B C * D E F ↑ / G * -
(18) *	(+ *	A B C * D E F ↑ / G * -
(19) H	(+ *	A B C * D E F ↑ / G * - H
(20))		A B C * D E F ↑ / G * - H * +

Instant TEST

12 ,7 ,3 ,- ,/ ,2 ,1 ,5 ,+ ,* ,+

Which Expression?

Postfix or Prefix

✓ **Postfix**

Equivalent infix expression ?

P = 12, [7-3], / , 2 ,1, 5, +, *, +

= [12/(7-3)], 2, 1, 5, +, *, +

= [12/(7-3)], 2, [1+5], *, +

= [12/(7-3)], [2* (1+5)], +

= 12/(7-3) + 2* (1+5)

Result: 15

Instant TEST

Infix	Prefix	Postfix
A+B * C+D	++A*BCD	ABC*+D+
(A+B) * (C+D)	*+AB+CD	AB+CD+*
A*B + C*D	+*AB*CD	AB*CD*+
A+B+C+D	++AB+CD	AB+CD++

Instant TEST

Infix Expression	Prefix Expression	Postfix Expression
1-4^3+7*(9^1/5)-2	-1+^43-*7/^9152	143^-791^5/*+2-
A+B-(C*D^E)*X+Y	+A-B+**C^DEXY	AB+CDE^*X*-Y+

QUICKSORT: An Application of STACKS

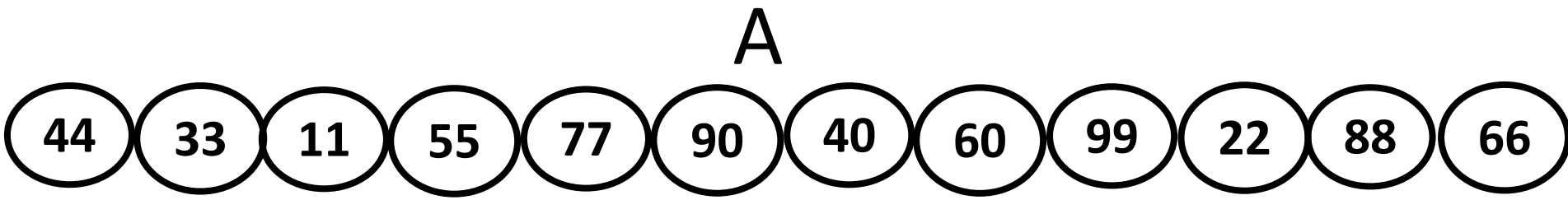
- What is Sorting?
  - ✓ The operation of rearranging the elements of a list so that they are in some logical order.
    - Numerically ordered (When list contain numerical data)
    - Alphabetically ordered (When list contains character data)

- Quicksort is an algorithm of the **DIVIDE-AND-CONQUER** type.
  - ➢ The problem of sorting a set is reduced to the problem of sorting two smaller sets.
- We illustrate this "Reduced Step" by means of a specific example
- Suppose A is the following list of 12 numbers.

A

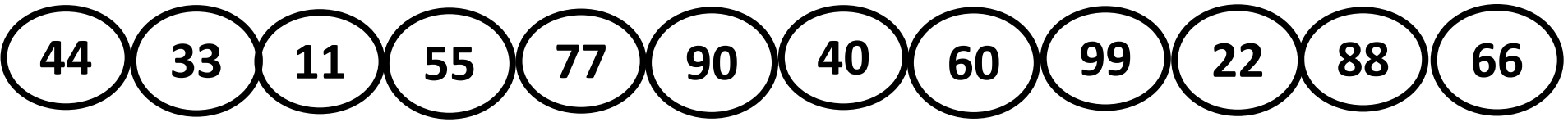
44	33	11	55	77	90	40	60	99	22	88	66

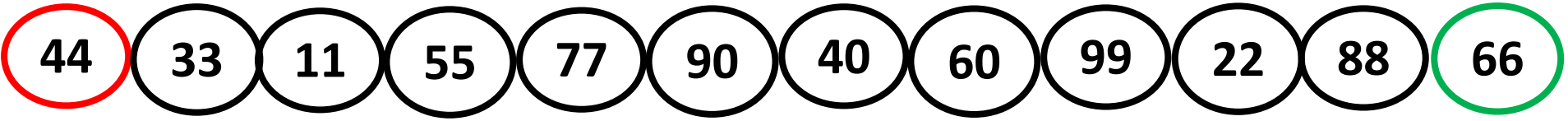
Reduction Step of the Quicksort Algorithm



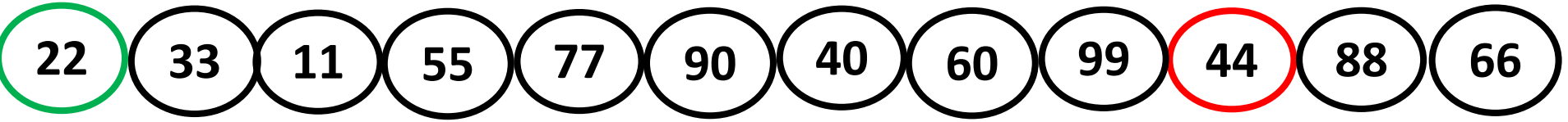
- The reduction step of the quicksort algorithm finds the final position of one of the numbers.
- In this illustration, we use the first number 44.

STACKS: Quicksort/ Reduction Step

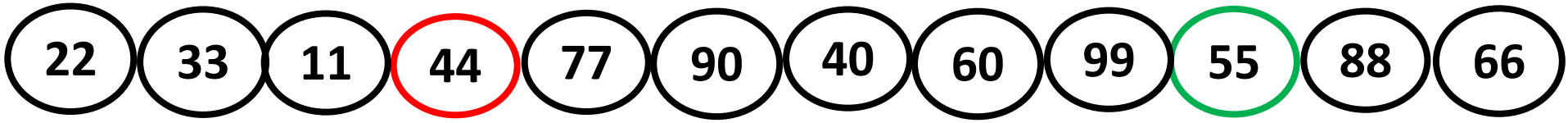




- ✓ Beginning with the last number, 66, scan the list from right to left till less than 44.
- ✓ Interchange 44 and 22

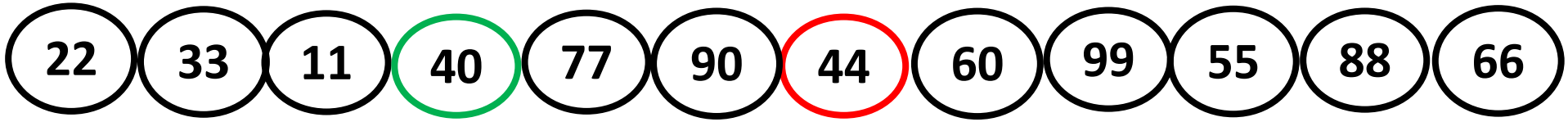


- ✓ Beginning with, 22 , scan left to right till greater than 44
- ✓ Interchange 44 and 55

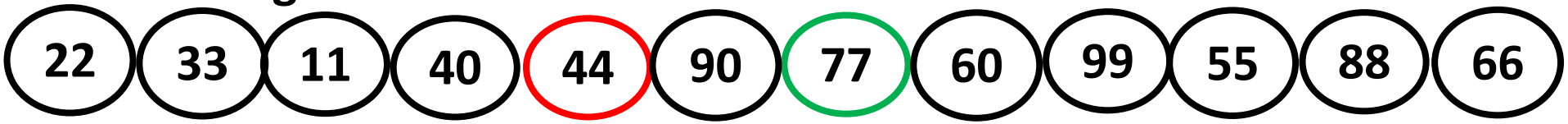


- ✓ Beginning with, 55, scan the list from right to left till less than 44.
- ✓ Interchange 44 and 40

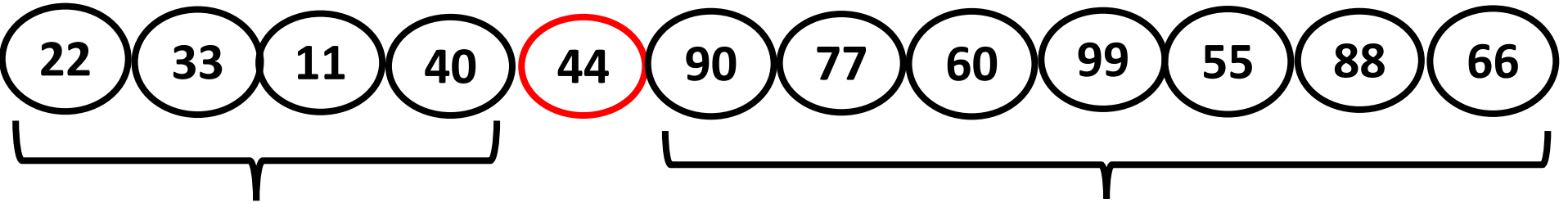
STACKS: Quicksort/ Reduction Step



- ✓ Beginning with, 40, scan left to right till greater than 44
- ✓ Interchange 44 and 77



- ✓ Beginning with, 77, scan the list from right to left till less than 44.
- ✓ Do not meet such a number before meeting 44.



- ✓ Thus, 44 is correctly placed in its final position.
- ✓ The task of sorting the original list A has now been reduced to the task of sorting each of the above sublists.

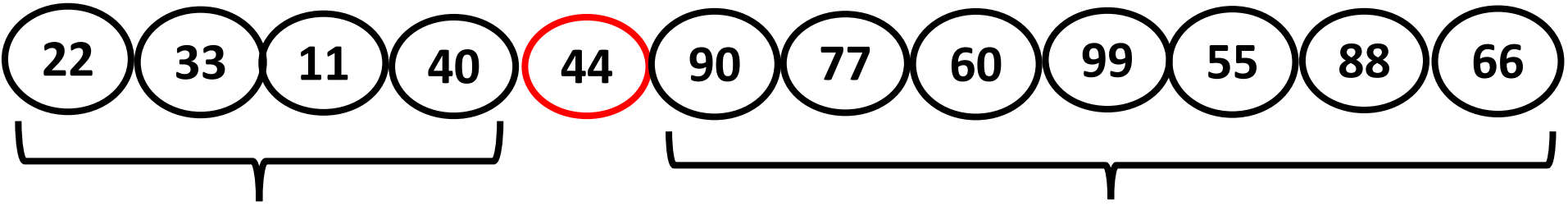
STACKS: Quicksort/ Reduction Step

- The reduction step is repeated with each sublist containing 2 or more elements.
- Since we can process one sublist at a time, we must be able to keep track of some sublist for future processing.
- **This is accomplished by using two STACKS**

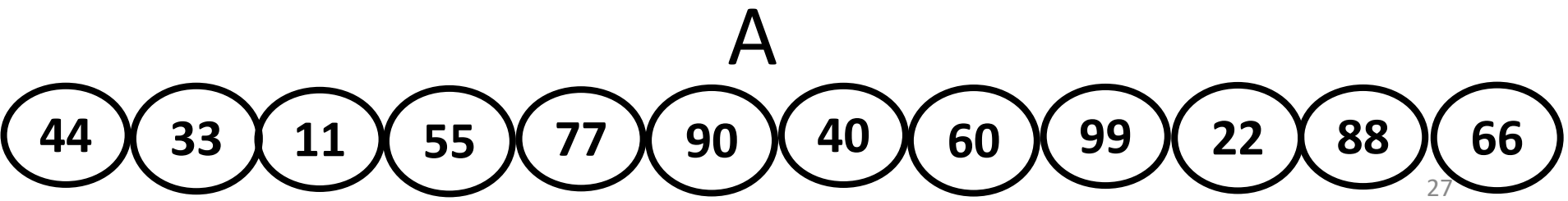
LOWER and UPPER

to temporarily hold such sub-lists.

- The addresses of the first and last elements of each sublist, called its *"**BOUNDARY VALUES**"* , are pushed onto the STACKs **LOWER** and **UPPER**, respectively.
- The reduction steps is applied to a sublist only after its ***BOUNDARY VALUES*** are removed from the STACKs.



STACKS: Illustration of the way the STACKS LOWER and UPPER are used



N=12 elements,

Thus,

Boundary values are ()?

1 and 12.

Now,

1 and 12 should be Stacked

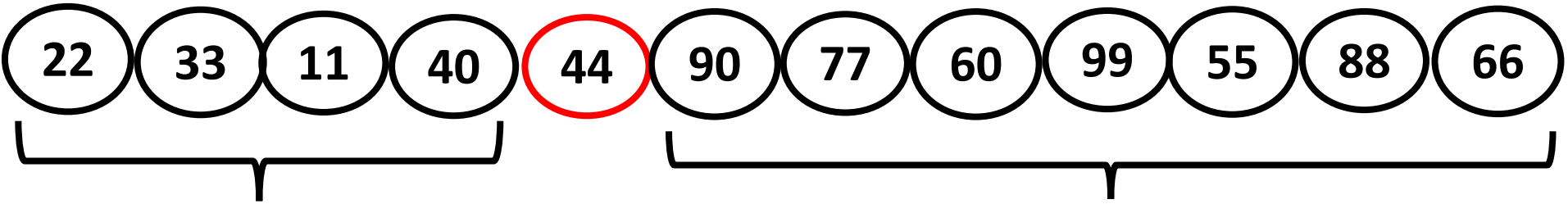
LOWER:1 and UPPER:12

- **In order to apply the REDUCTION STEP, the algorithm first removes the top values 1 and 12.**
- **After removing the top values 1 and 12 from the STACK, leaving**

  LOWER:(Empty) and UPPER: (Empty)
- **Then Applies the REDUCTION STEP to the corresponding list A[1], A[2],…,A[12].**

STACKS: Illustration of the way the STACKS LOWER and UPPER are used

- After executing **REDUCTION STEP** to the list A[1] to A[12]
  - Finally places the first element 44, in A[5].



First sublist **Second sublist**

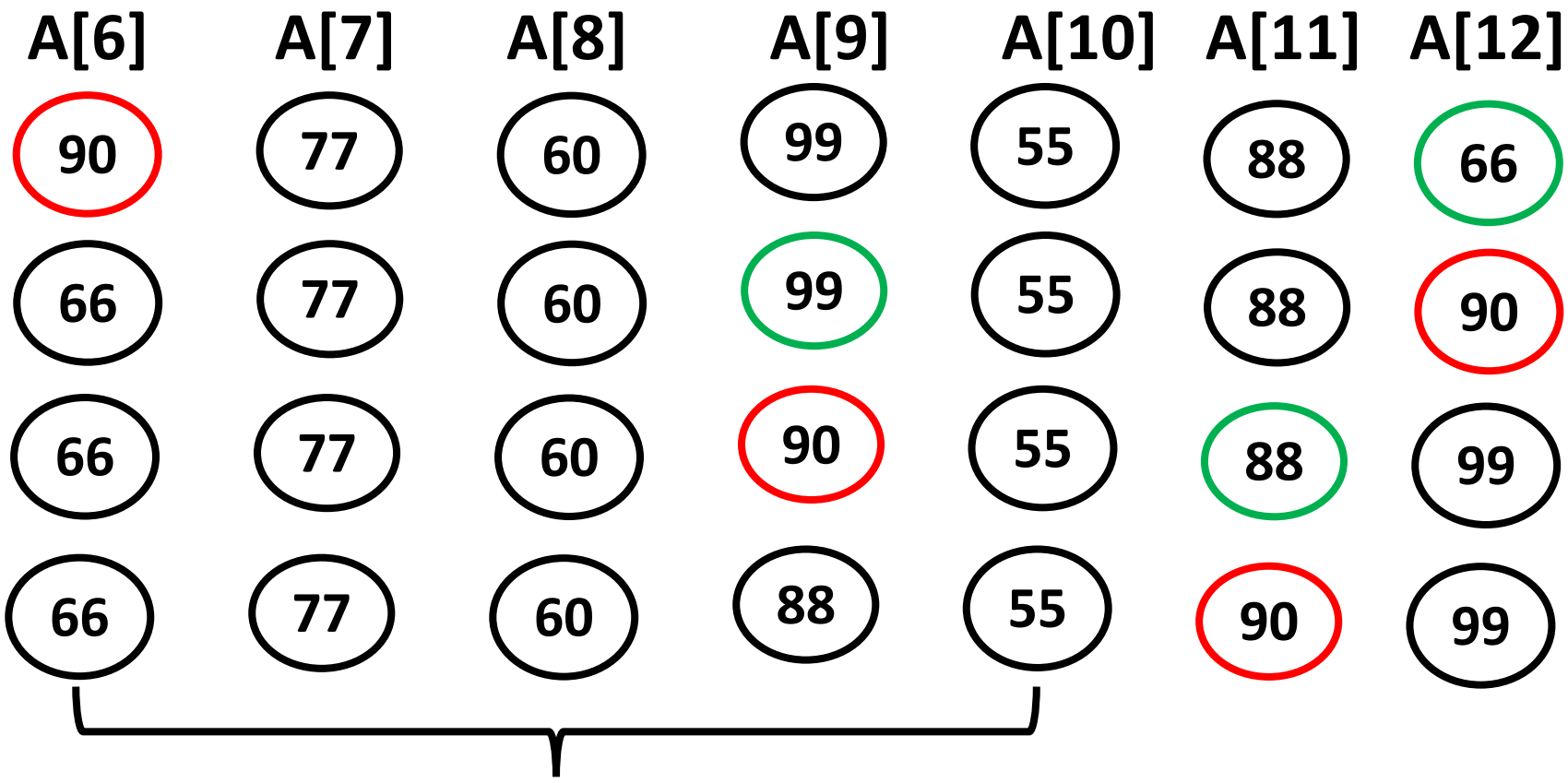
- Accordingly, the algorithm pushes the boundary values
  - 1 and 4 of the first sublist, and
  - 6 and 12 of the second sublist on to the STACK to yield

LOWER= **1, 6** and UPPER= **4, 12**

- In order to apply the **REDUCTION STEP** again, the algorithm removes the TOP values 6 and 12 from the STACKs, leaving

LOWER= **1,** and UPPER= **4**

STACKS: Illustration of the way the STACKS LOWER and UPPER are used



- The second sublist has only one element, Accordingly
- The algorithm pushes only the boundary values 6 and 10 of the first sublist on the STACKs to yield
  - LOWER= **1, 6** and UPPER= **4, 10**

QUICKSORT

- The quick sort is regarded as the best sorting algorithm.
- This is because of its significant advantage in terms of efficiency because it is able to deal well with a huge list of items.
- Because it sorts in place, no additional storage is required as well.
- The slight disadvantage of quick sort is that its worst-case performance is similar to average performances of the bubble, insertion or selections sorts.
- In general, the quick sort produces the most effective and widely used method of sorting a list of any item size

STACKS:

- Observe that the reduction step in the k^{th} level finds the location of 2^{k-1} elements.

Recursion: Procedure

- ✓ Recursion is an important concept in Computer Science.
- ✓ Suppose, P is a *procedure* containing either
  - ➢ a Call statement to itself or
  - ➢ a Call statement to a second procedure that may eventually result in a Call statement back to the original *procedure*, P
- ✓ Then P is called a *Recursive Procedure*.

Recursion: *Procedure*

- ✓ A *Recursive Procedure* must have the following two properties:
  - There must be certain criteria, called *base criteria*, for which the procedure does not call itself
  - Each time the *procedure* does call itself (directly or indirectly), it must be closer to the *base criteria.*
- ✓ A *Recursive Procedure* with these two properties is said to be *well-defined.*

Recursion: Function

- ✓ A function is said to be *Recursively defined* if the function definition refers to itself.
- ✓ A *Recursive Function* must have the following two properties:
  - ➢ There must be certain arguments, called **BASE VALUE**, for which the function does not refer to itself.
  - ➢ Each time the function does refer to itself, the argument of the function must be closer to a **BASE VALUE**.
- ✓ A *Recursive Function* with two properties is also said to be *well-defined*.

Recursion: Factorial Function

- ✓ In some problems, it may be natural to define the problem in terms of the problem itself.
- ✓ Recursion is useful for problems that can be represented by a **SIMPLER VERSION** of the same problem.
- ✓ Example: the factorial function

```
6! = 6 * 5 * 4 * 3 * 2 * 1
```

We could write:

```
6! = 6 * 5!
```

Recursion: Factorial Function

- ✓ In general, we can express the factorial function as follows:

```
n! = n * (n-1)!
```

Is this correct? **Well… almost.**

- ✓ The factorial function is **ONLY DEFINED** for *positive* integers. So we should be a bit more precise:

```
i)  n! = 1                  (if n is equal to 1)
ii) n! = n * (n-1)!         (if n is larger than 1)
```

- ✓ Observe that, this definition of `n!` is recursive, since it refers to itself when it uses `(n-1)!` , However,
- ✓ **i)** the value of `n!` is explicitly given when `n=0` (**BASE VALUE**)
- ✓ **ii)** the value of `n!` for arbitrary `n` is defined in terms of a smaller value of n which is closer to the **BASE VALUE**

Recursion: Factorial Function

EXAMPLE: Let's calculate `3!` Using the recursive definition.

(1) `3! = 3 . 2!`

(2) `2! = 2 . 1!`

(3) `1! = 1 . 0!`

(4) `0! = 1 (BASE VALUE)`

(5) `1! = 1 . 1 = 1`

(6) `2! = 2 . 1 = 2`

(7) `3! = 3 . 2 = 6`

- ✓ **Observe that we back track in the reverse order of the original postponded evaluations.**
- ✓ **Recall that this type of postponed processing tends itself to the use of STACKS.**

Recursion: Function

Assume the number typed is 3, that is, numb=3.

```
fac(3) :
3 <= 1 ?                    No.
fac(3) = 3 * fac(2)
   fac(2) :
      2 <= 1 ?              No.
      fac(2) = 2 * fac(1)
         fac(1) :
            1 <= 1 ?        Yes.
            return 1
      fac(2) = 2 * 1 = 2
      return fac(2)
fac(3) = 3 * 2 = 6
return fac(3)
```

```
i)   n! = 1              (if n is equal to 1)
ii)  n! = n * (n-1)!     (if n is larger than 1)
```

```
int fac(int numb){
  if(numb<=1)
      return 1;
   else
      return numb * fac(numb-1);
}
```

fac(3) has the value 6

Recursion: Function

For certain problems (such as the factorial function), a recursive solution often leads to short and elegant code. Compare the recursive solution with the iterative solution:

Recursive solution

```
int fac(int numb){
   if(numb<=1)
      return 1;
   else
      return numb*fac(numb-1);
}
```

Iterative solution

```
int fac(int numb){
int product=1;
    while(numb>1){
        product *= numb;
            numb--;
                }
     return product;
                }
```

Recursion: Cost

- ✓ You have to pay a price for recursion:
  - ➢ Calling a function **CONSUMES MORE TIME AND MEMORY** than adjusting a loop counter.
  - ➢ High performance applications (graphic action games, simulations of nuclear explosions) hardly ever use recursion.
- ✓ In **LESS DEMANDING APPLICATIONS** recursion is an attractive alternative for iteration.

Recursion: Precautions

You must always make sure that the recursion *bottoms out*:

- A recursive function must contain at least one non-recursive branch.
- The recursive calls must eventually lead to a non-recursive branch.

```
int fac(int numb){
  if(numb<=1)
      return 1;
   else
      return numb * fac(numb-1);
}
```

Recursion: Function

How many pairs of rabbits can be produced from a single pair in a year's time?

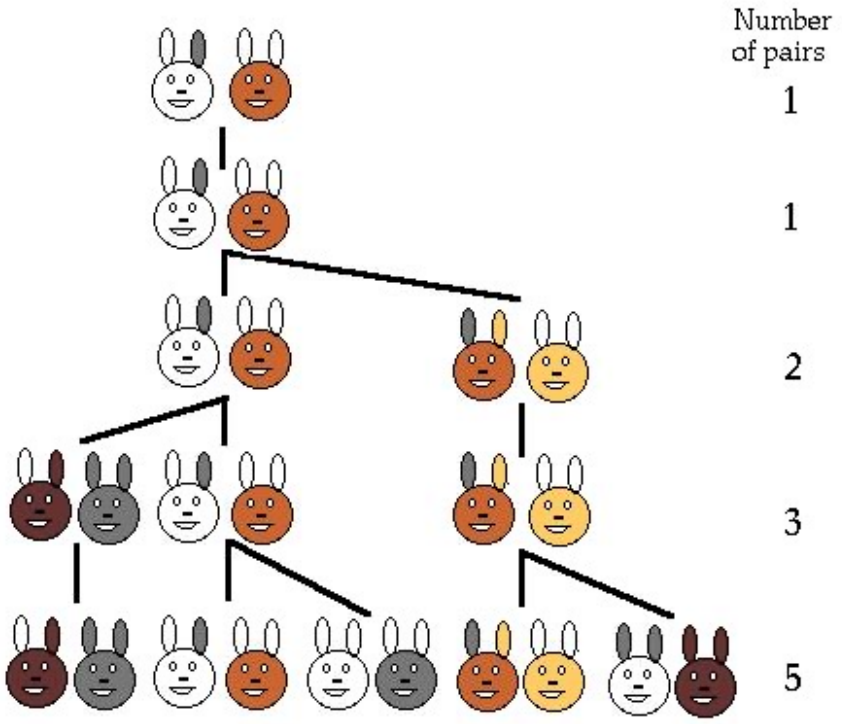
Assumptions:

- Each pair of rabbits produces a new pair of offspring every month;
- each new pair becomes fertile at the age of one month;
- none of the rabbits dies in that year.

Example:

- After 1 month there will be 2 pairs of rabbits;
- after 2 months, there will be 3 pairs;
- after 3 months, there will be 5 pairs (since the following month the original pair and the pair born during the first month will both produce a new pair and there will be 5 in all).

Population Growth in Nature



- ✓ Leonardo Pisano (Leonardo Fibonacci, son of Bonaccio) proposed the (Fibonacci) sequence in 1202 in *The Book of the Abacus*.
- ✓ Fibonacci numbers are believed to model nature to a certain extent, such as Kepler's observation of leaves and flowers in 1611.

Direct Computation Method

Fibonacci numbers:

```
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...
```

where each number is the sum of the preceding two.

Recursive definition:

```
F(0) = 0;
F(1) = 1;
F(number) = F(number-1)+ F(number-2);
```

Recursion: Function

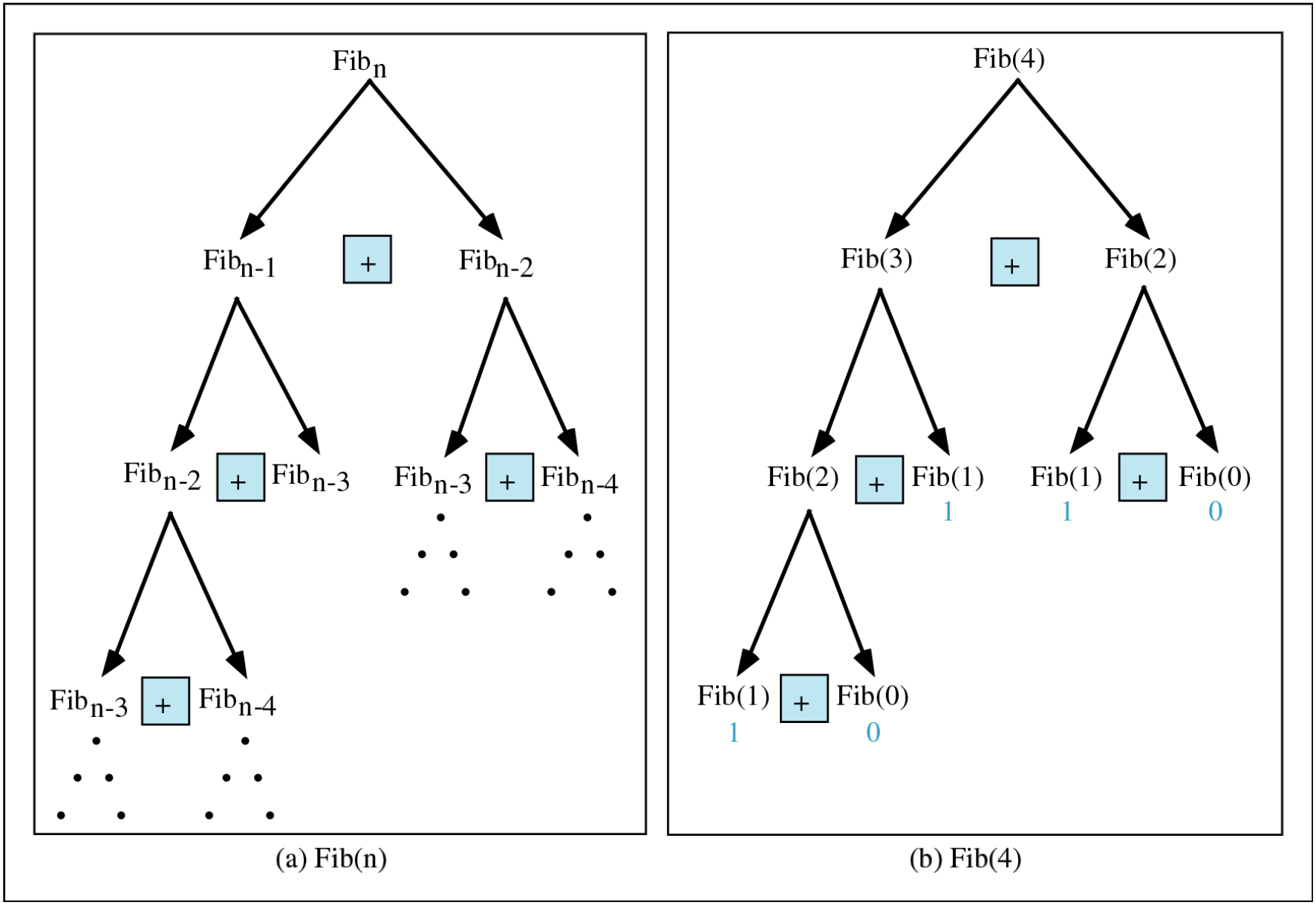
```
// Calculate Fibonacci numbers using recursive function.
// A very inefficient way, but illustrates recursion well
int fib(int number)
{
        if (number == 0) return 0;
        if (number == 1) return 1;
        return (fib(number-1) + fib(number-2));
}

int main(){   // driver function
        int inp_number;
        printf("Please enter an integer: ");
        scanf("%d"&inp_number);
        cout << "The Fibonacci number for "<< inp_number
                << " is "<< fib(inp_number)<<endl;
  return 0;
}
```

Recursive definition:

```
F(0) = 0;
F(1) = 1;
F(number)=F(number-1)+ F(number-2);
```

Recursion: Fibbonacci

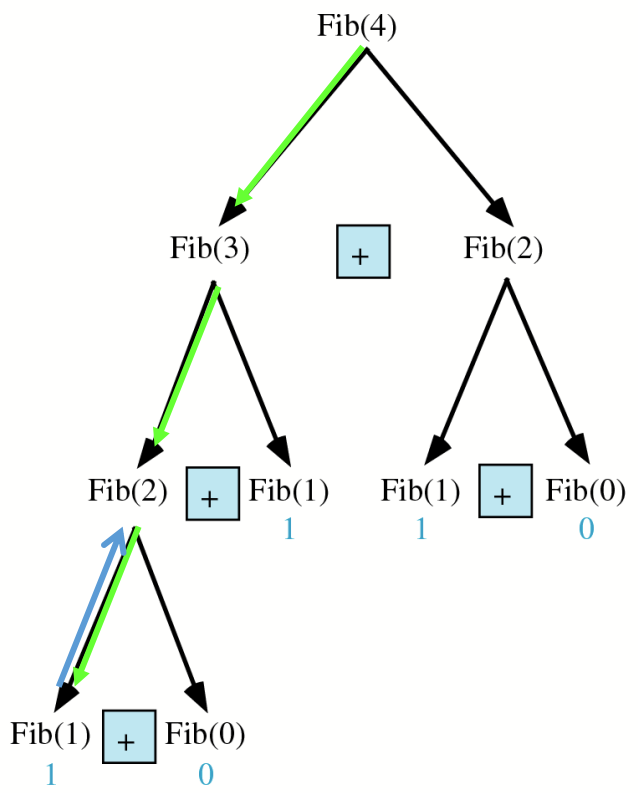


Recursion: Trace a Fibonacci Number

Assume the input number is 4, that is, num=4:

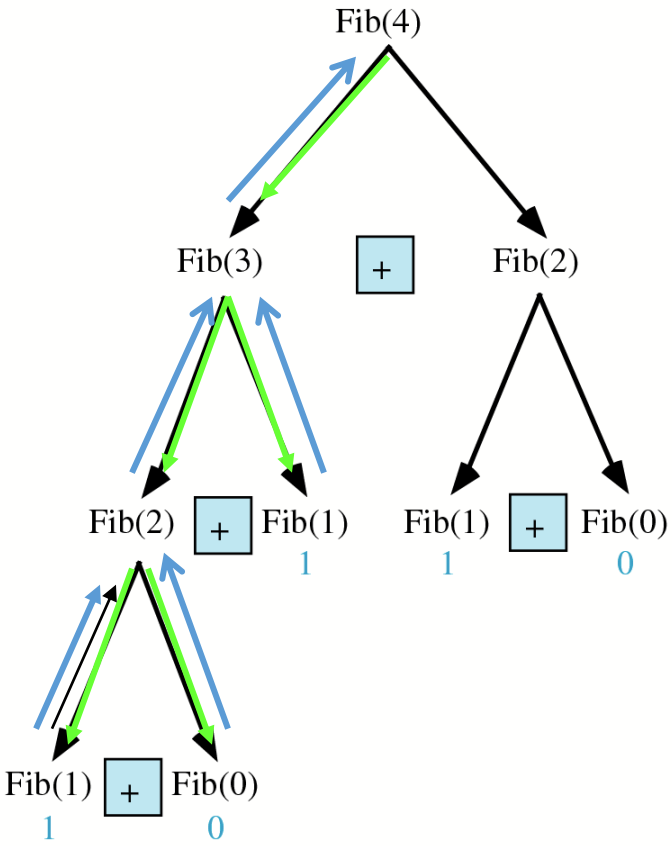
```
fib(4):
    4 == 0 ? No;    4 == 1?  No.
    fib(4) = fib(3) + fib(2)
    fib(3):
        3 == 0 ? No; 3 == 1? No.
        fib(3) = fib(2) + fib(1)
        fib(2):
            2 == 0? No; 2==1? No.
            fib(2) = fib(1)+fib(0)
            fib(1):
                1== 0 ? No; 1 == 1? Yes.
                 fib(1) = 1;
                        return fib(1);
```

```
int fib(int num)
{
    if (num == 0) return 0;
    if (num == 1) return 1;
    return
      (fib(num-1)+fib(num-2));
}
```



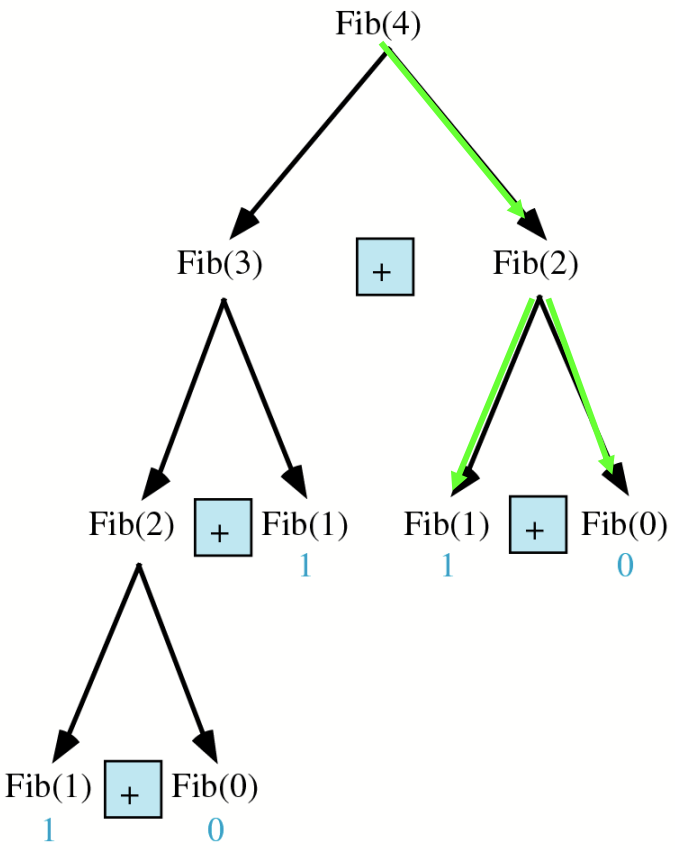
Recursion: Trace a Fibonacci Number

```
        fib(0):
              0 == 0 ?  Yes.
              fib(0) = 0;
              return fib(0);
    fib(2) = 1 + 0 = 1;
    return fib(2);
fib(3) = 1 + fib(1)
    fib(1):
     1 == 0 ? No; 1 == 1? Yes
     fib(1) = 1;
      return fib(1);
fib(3) = 1 + 1 = 2;
return fib(3)
```



Recursion:

```
        fib(2):
    2 == 0 ? No; 2 == 1?    No.
    fib(2) = fib(1) + fib(0)
    fib(1):
       1== 0 ? No; 1 == 1?  Yes.
        fib(1) = 1;
        return fib(1);
    fib(0):
        0 == 0 ?    Yes.
        fib(0) = 0;
        return fib(0);
    fib(2) = 1 + 0 = 1;
    return fib(2);
fib(4) = fib(3) + fib(2)
       = 2 + 1 = 3;
return fib(4);
```



Recursion: Divide-and-Conquer Algorithms

- ✓ Consider a problem **P** associated with a set S.
- ✓ Suppose A is an algorithm which partitions S into smaller sets such that the solution of the problem **P** for S is reduced to the solution of P for one or more of the smaller sets.
- ✓ Then A is called a divide-and-conquer Algorithm.
- ✓ Examples:

The Quicksort Algorithm

-use reduction step to find the location of a single element and to reduce the problem of sorting the entire set to the problem of sorting smaller sets

The Binary Search Algorithm

-divides the given sorted set into two halves so that the problem of searching for an item in the entire set is reduced to the problem of searching for the item in one of the two halves.

Recursion: Divide-and-Conquer Algorithms

- ✓ A divide-and-conquer algorithm A may be viewed as a recursive procedure. But Why?
  - ➢ The divide-and-conquer algorithm A may be viewed as calling itself when it is applied to the smaller sets.
  - ➢ The base criteria for these algorithms are usually the one-element sets.
  - ➢ For example, with a sorting algorithm, a one-element set is automatically sorted, and
  - ➢ with a searching algorithm, one-element set requires only a single comparison.

TOWER of HANOI Problem

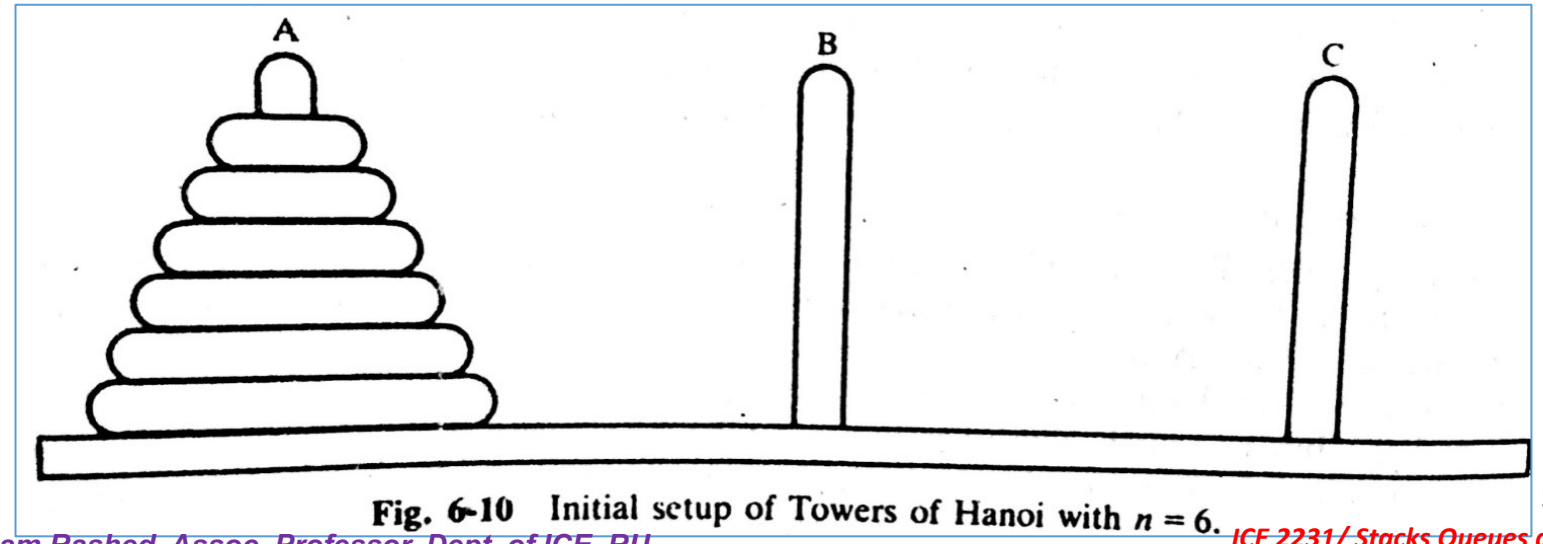


Fig. 6-10 Initial setup of Towers of Hanoi with $n = 6$.

TOWER of HANOI Problem

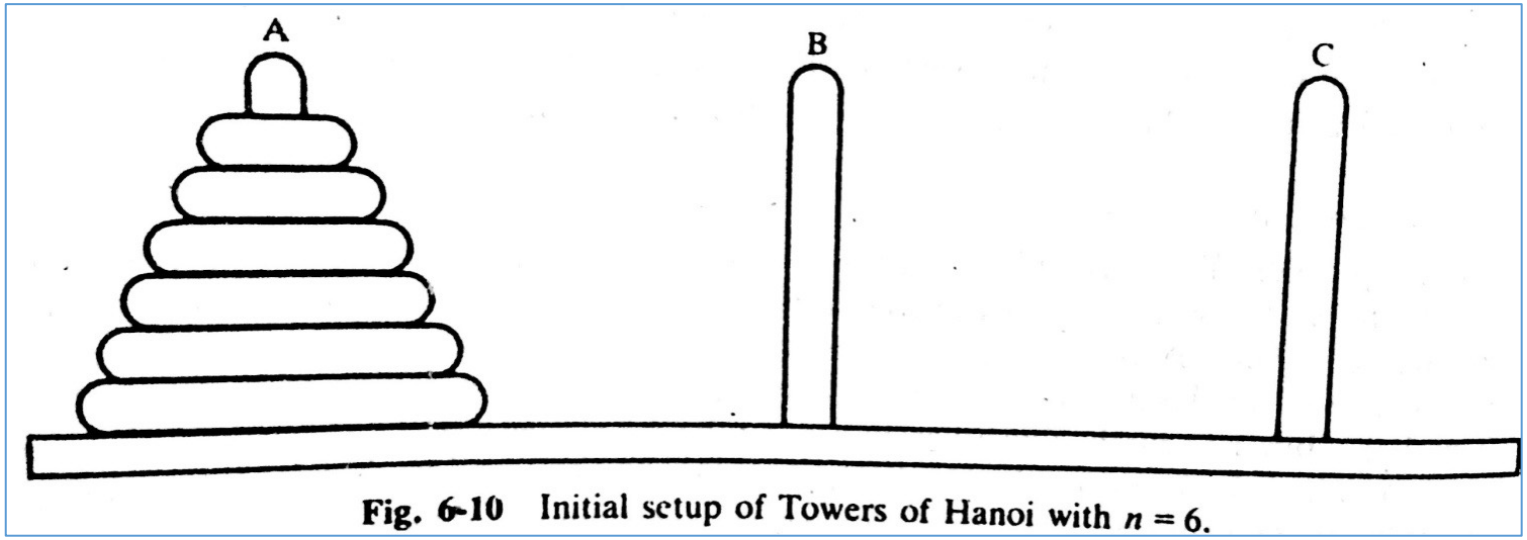
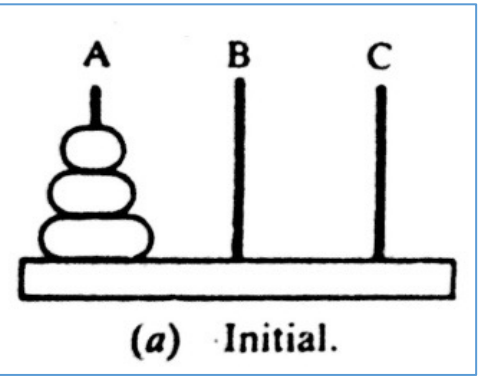


Fig. 6-10 Initial setup of Towers of Hanoi with $n = 6$.

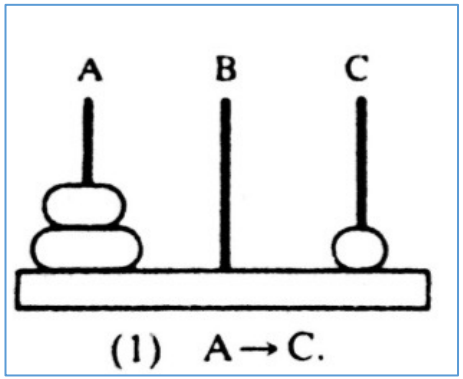
- ✓ Only one disc could be moved at a time
- ✓ A larger disc must never be stacked above a smaller one
- ✓ One and only one extra needle could be used for intermediate storage of discs

TOWER of HANOI Problem Solution

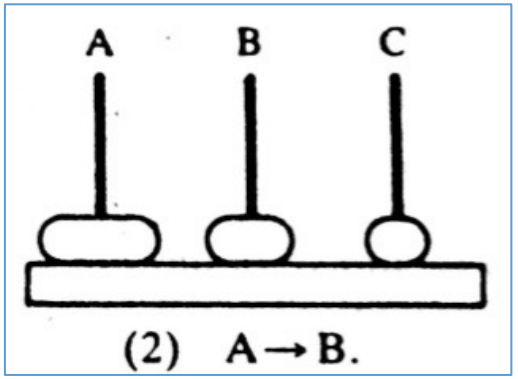
The solution to the Tower of Hanoi problem for n=3 appears



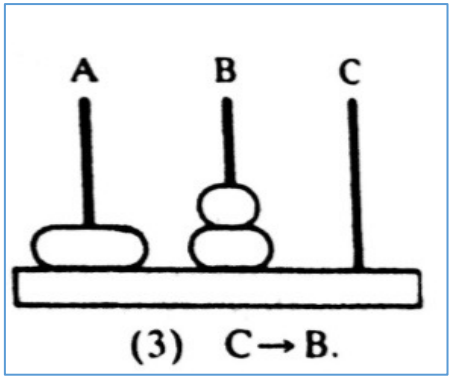
(a) Initial.



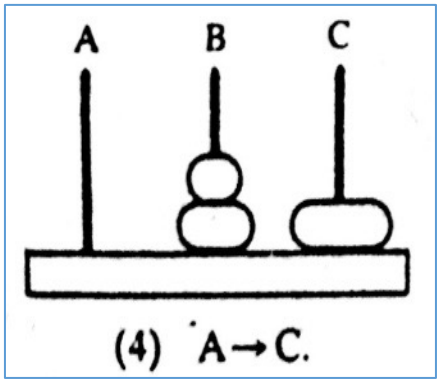
(1) A→C.



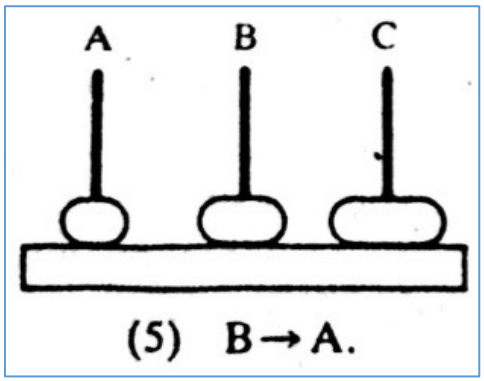
(2) A→B.



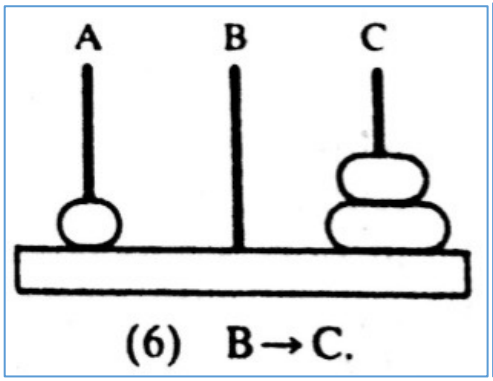
(3) C→B.



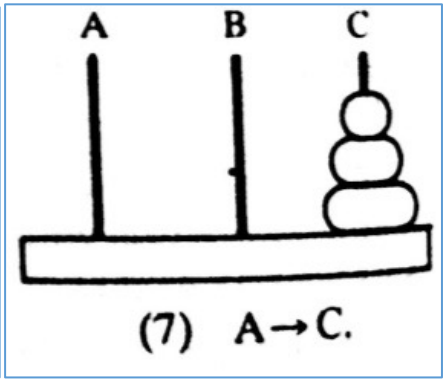
(4) A→C.



(5) B→A.



(6) B→C.



(7) A→C.

N=3: A→C, A→B, C→B, A→C, B→A, B→C, A→C,

TOWER of HANOI Problem Solution

The **SEPARATE SOLUTION** to the Tower of Hanoi problem

for **n=1**

Solution: A→C

n=2

Solution: A→B, A→C, B→C

n=3

Solution: A→C, A→B, C→B, A→C, B→A, B→C, A→C

- ✓ **General Solution to the Tower of Hanoi for any # of Disk is PREFERRED.**
- ✓ **Which can be done in RECURSIVE way.**

Solution of TOWER of HANOI in RECURSIVE WAY

- ✓ The solution to the Tower of Hanoi problem for n>1 disks may be reduced to the following sub-problems:
  - ➢ Move the top n-1 disks from peg A to peg B.
  - ➢ Move the top disk from A to peg C: **A→C**
  - ➢ Move the top n-1 disks from peg B to peg C

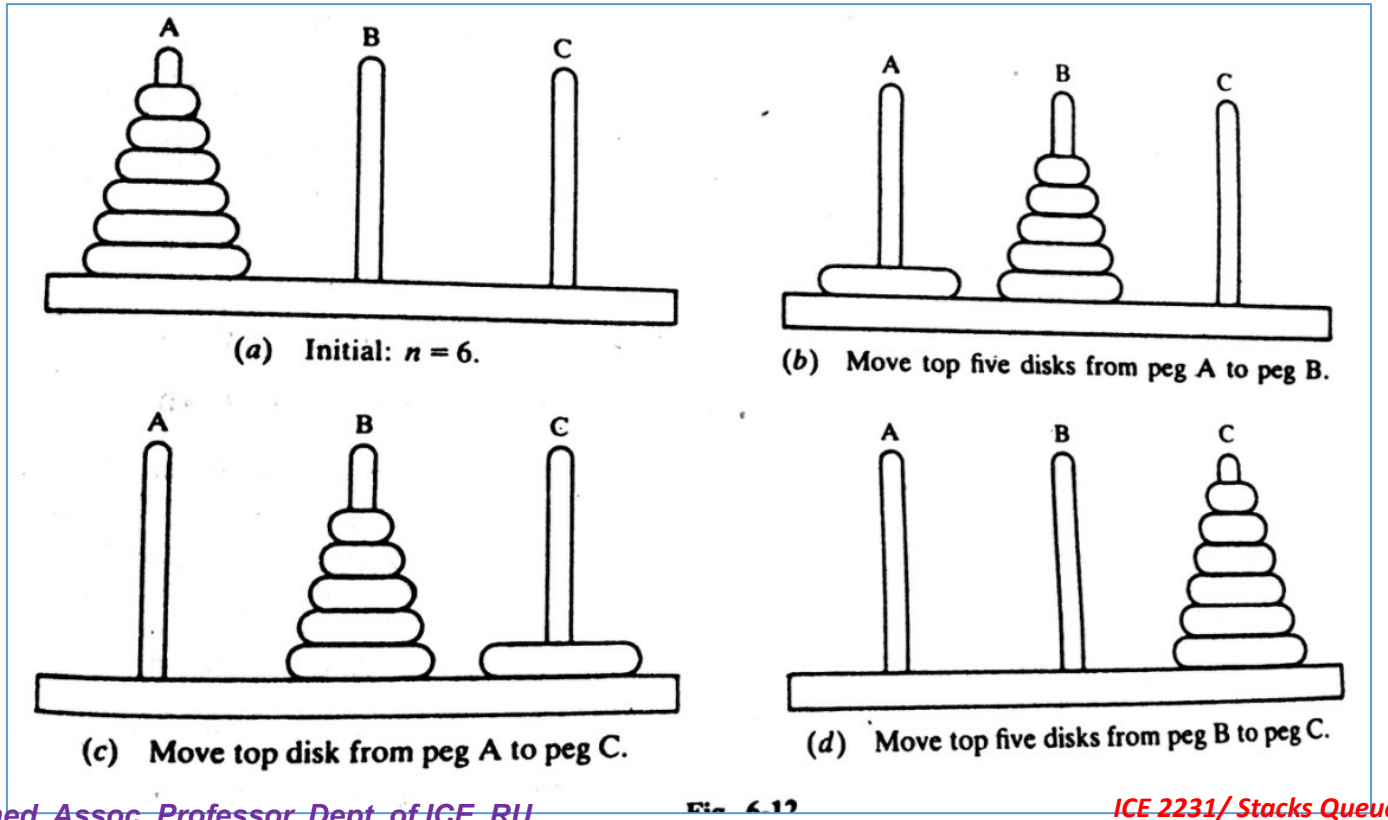


Fig. 6.12

Solution of TOWER of HANOI in RECURSIVE WAY

The general notation of the solution for any # of disk:

TOWER(N, BEG, AUX, END)

When **n=1** then

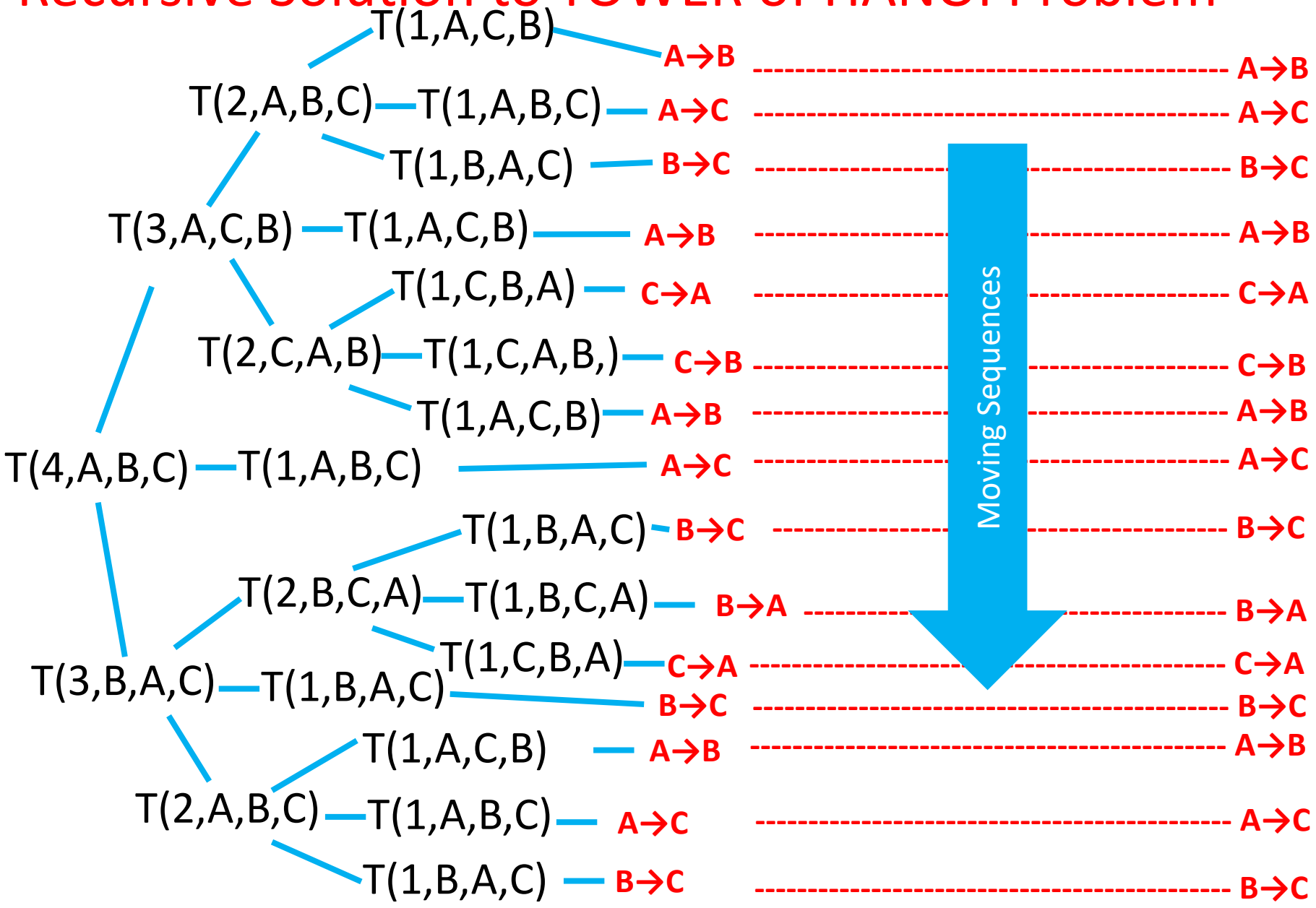
TOWER(1, BEG, AUX, END)

BEG → END, But

When **n> 1** then then solution may be reduced to the solution of the following three sub-problem:

(1) TOWER (N-1, BEG, END, AUX)
(2) TOWER (1, BEG, AUX, END)
(3) TOWER (N-1, AUX, BEG, END)

Recursive Solution to TOWER of HANOI Problem



Formal TOWER of HANOI Problem Solving Procedure

TOWER (N, BEG, AUX, END)

Step1. If N=1, then:

(a) Write: BEG→END.

(b) Return.

[End of If Structure]

Step2. [Move N-1 disks from peg BEG to peg AUX.]

Call TOWER (N-1, BEG, END, AUX).

Step3. Write: BEG → END.

Step4. [Move N-1 disks from peg AUX to peg END.]

Call TOWER (N-1), AUG, BEG, END).

Step5. Return

Implementation of Recursive Pro. By STACKS

Before implementing Recursive procedures…..

Imagine a SUBPROGRAM…

- ✓ It may contain both parameters, and local variables.
- ✓ The parameters are the variables which receive values from objects in the **CALLING PROGRAM**.
- ✓ It transmit values back to the **CALLING PROGRAM**.
- ✓ It must also keep track of the return address in the **CALLING PROGRAM**.
- ✓ The return address is essential, since control must be transferred back to its proper place in the **CALLING PROGRAM.**
- ✓ Once the subprogram finished executing and control is transferred back to the **CALLING PROGRAM.**
- ✓ The values of the local variables and the return address are no longer needed.

Implementation of Recursive Pro. By STACKS

Suppose SUBPROGRAM is a recursive program…then

- ✓ Each level of execution of the subprogram may contain different values for the parameters and local variables, and for the return address.
- ✓ Furthermore, if the recursive program does call itself, then these current values must be saved, since they will be used again when the program is reactivated.

Queues

- ✓ A queue is a linear list of elements in which…
  - ➢ deletions can take place only at one end, called the *front*, and
  - ➢ insertions can take place only at the other end, called the *rear.*
- ✓ Queues are also called first-in first-out (FIFO) list
- ✓ This contrasts with stacks, which are LIFO

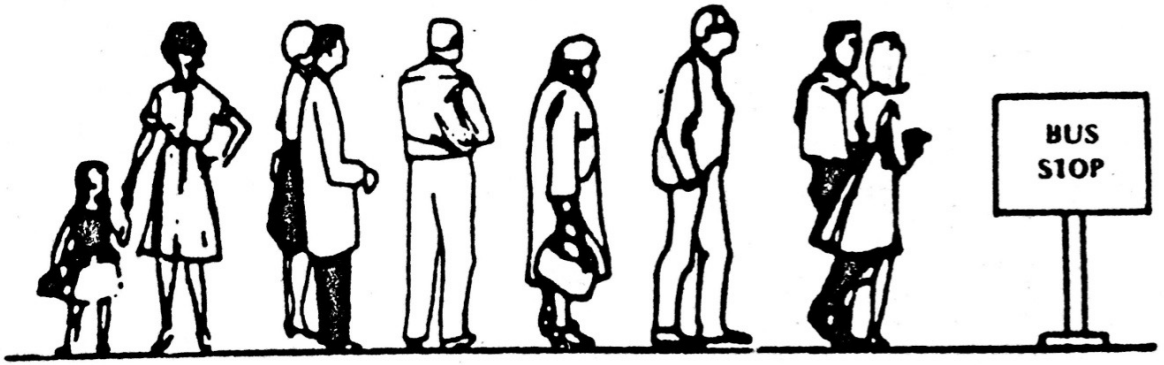
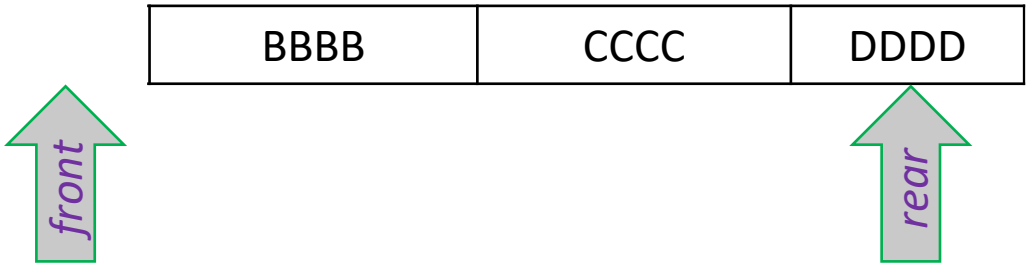


Fig. 6-2 Queue waiting for a bus.

- ✓ An important example of a queue in computer science occurs in a timesharing system in which programs with the same priority form a queue while waiting to be executed.

Queues: Example

Consider a Queue with 4 elements



✓**Suppose an element is deleted from the 4 element queue.**

Which is we can delete??

AAAA

Then which one is considered to be the front element?

BBBB

Suppose EEEE is added to the queue

Then which one is considered to be the rear element?

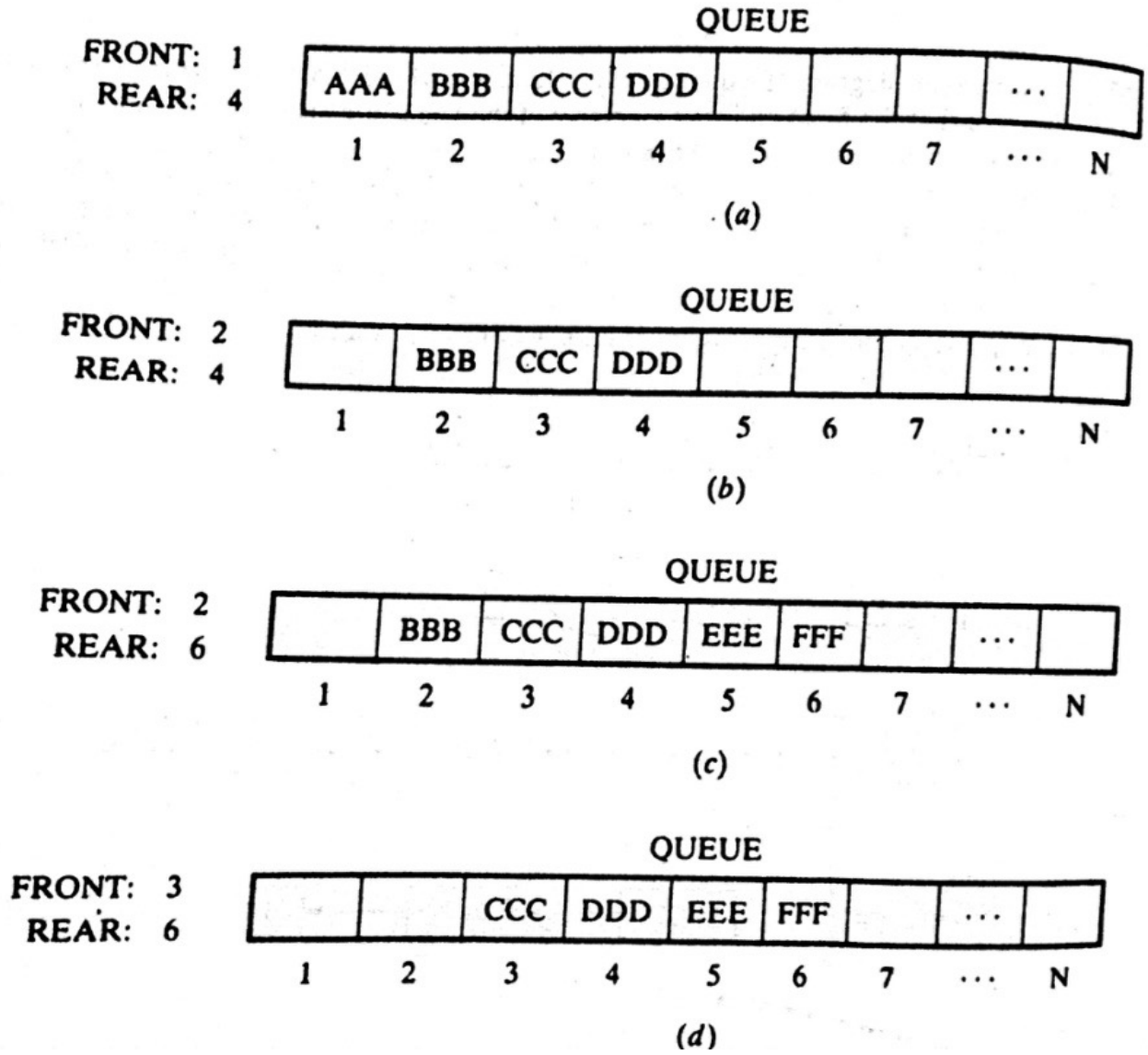
EEEE

Queues: Representation

- ✓ Queues may be represented in the computer in various ways.
- ✓ Usually by means of
  - ➢ One-way list
  - ➢ Linear arrays
- ✓ Each of the queues will be maintained by …
  - ➢ A Linear Array **QUEUE**
  - ➢ Two pointer variables:
    - ○ **FRONT** (containing the location of the front element of the Queue)
    - ○ **REAR** (containing the location of the rear element of the Queue)
- ✓ The condition **FRONT= NULL** will indicate that the Queue is……

EMPTY.

Queues: Representation



Queues: Representation

- ✓ After N insertion, the REAR element of the QUEUE will occupy QUEUE [N]
- ✓ This occurs even though the queue itself may not contain many elements

- ✓ Suppose, we want to insert an element **ITEM** into a queue at the time the queue does occupy the last part of the array, that is **REAR=N.**

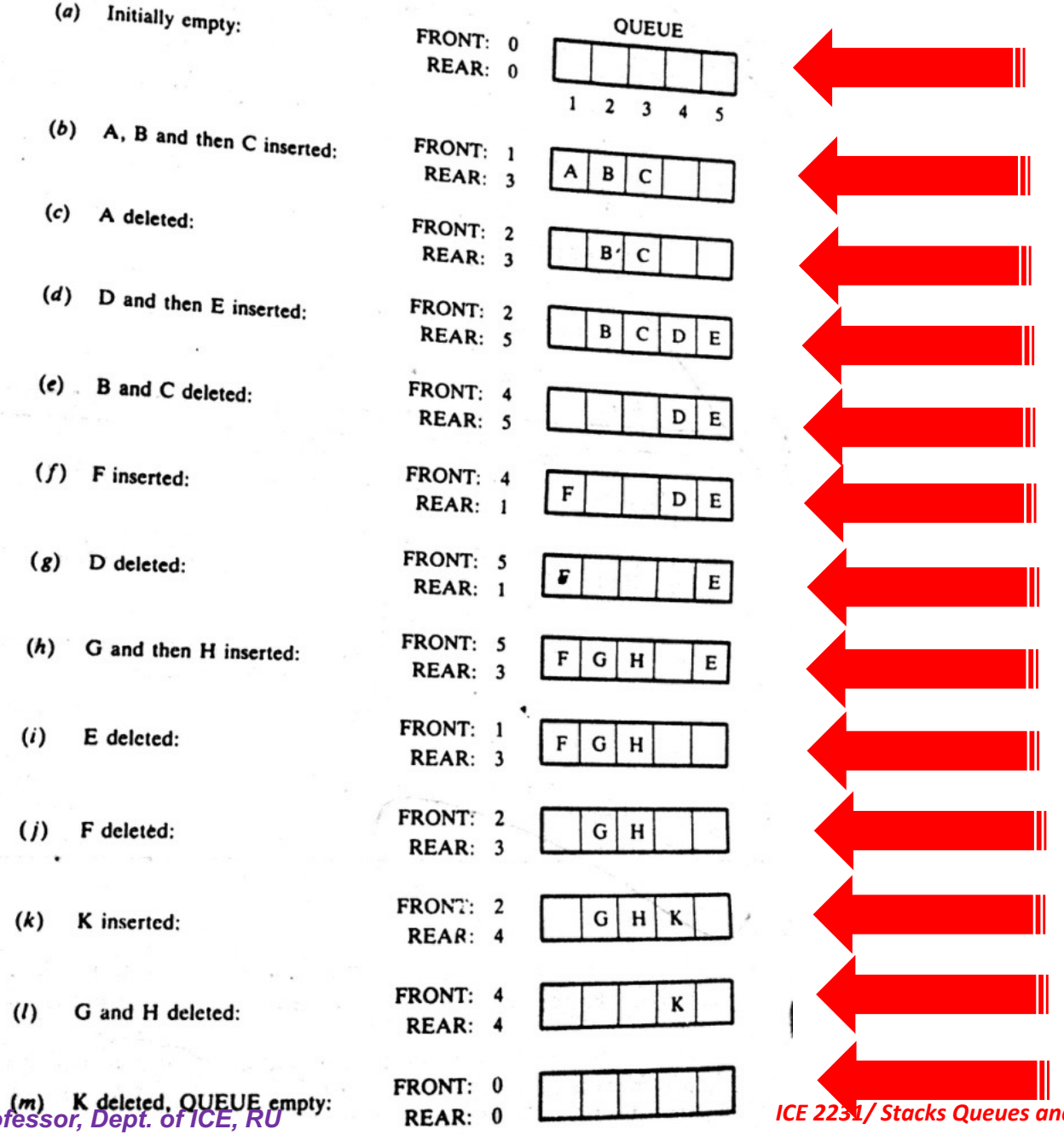
- ✓ One way to do this is to simply move the entire queue to the beginning of the array changing **FRONT** and **REAR** accordingly.
- ✓ Then insert item as above.

- ✓ This procedure is very expensive
- ✓ The procedure we adopt is to assume that the array **QUEUE is Circular. ie, QUEUE[1] comes after QUEUE[N]**

Queues: Representation

- ✓ Similarly, if FRONT=N and an element of Queue is deleted.
- ✓ We reset FRONT=1, instead of increasing FRONT to N+1.

Queues: Representation



Queues: Insertion Algorithm

QINSERT (QUEUE, N, FRONT, REAR, ITEM)

Step1. [QUEUE already filled?]

If FRONT=1 and REAR=N, or FRONT=REAR+1, then:

Write: OVERFLOW, and Return

Step2. [Find new value of REAR.]

If **FRONT:=NULL**, then: [Queue initially empty]

Set FRONT:=1 and REAR:=1

Else if **REAR=N**, then:

Set REAR:=1,

Else:

Set REAR:=REAR+1. [End of If structure]

Step3. **Set QUEUE[REAR]=ITEM** [This inserts new item]

Step4. Return

Queues: Deletion Algorithm

QDELETION (QUEUE, N, FRONT, REAR, ITEM)

Step1. [QUEUE already empty?]

If FRONT=NULL then:

Write: UNDERFLOW, and Return

Step2. **Set** ITEM:= QUEUE[FRONT],

Set3. [Find new value of FRONT.]

If **FRONT:=REAR**, then: [Queue has only one element to start]

Set FRONT:=NULL and REAR:=NULL

Else if **FRONT=N**, then:

Set FRONT:=1,

Else:

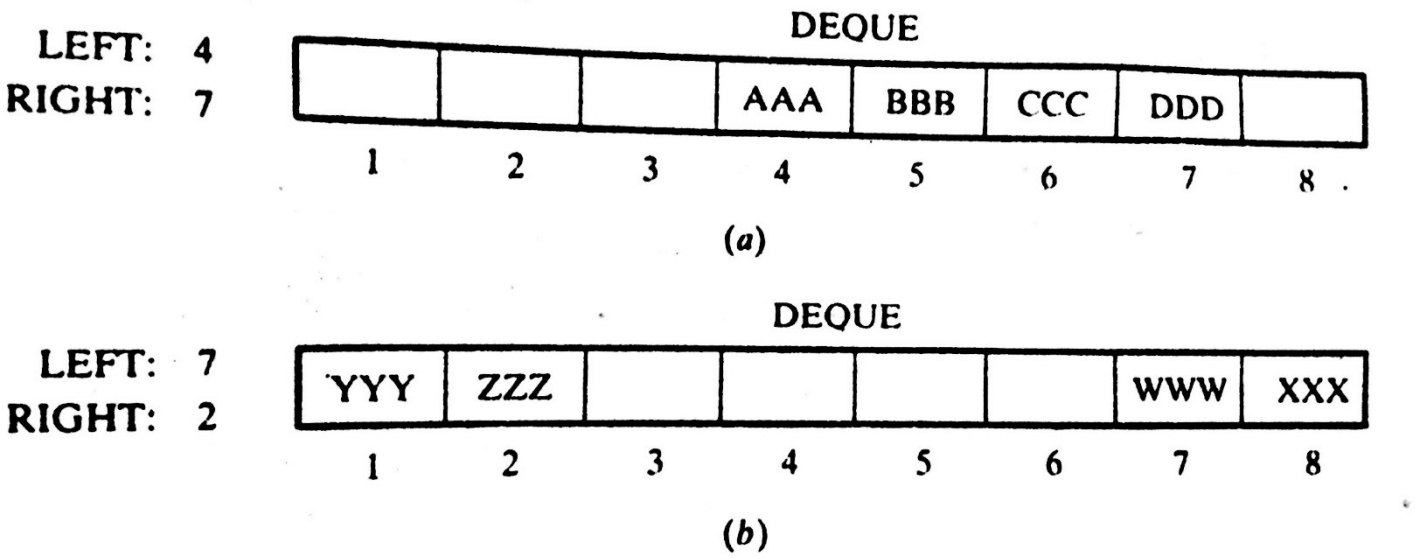
Set FRONT:=FRONT+1.

[End of If structure]

Step4. Return

Deques: Definition

- ✓ A deque is a linear linear list in which elements can be added or removed at either end but not in the middle.
- ✓ Deque can be maintained by……
  - ➢ a CIRCULAR array
  - ➢ Pointer LEFT-which points left end of the deque
  - ➢ Pointer RIGHT-which points right end of the deque.



Deques: Variation

- ✓ There are TWO variations of a Deque-
  - ➢ An Input-restricted deque, and
  - ➢ An Output-restricted deque
- ✓ This are intermediate between a Deque and Queue

- ✓ An Input-restricted deque is a deque which allow insertion at only one end of the list but allows deletions at both ends of the list
- ✓ An output-restricted deque is a deque which allows deletions at only one end of the list but allows insertions at both ends of the list.

Priority Queues: Definition

A priority queue is a collection of elements such that each elements has been assigned a priority and such that the order in which elements are deleted and processed comes from the following rules:

- An element of higher priority is processed before any element of lower priority
- Two elements with same priority are processed according to the order in which they were added to the queue.

Priority Queues: One-way List Representation

One way to maintain a priority queue in memory is by means of a one-way list, as follows

a) Each node in the list will contain three items of information:
   - An Information field INFO,
   - A priority number PRN, and
   - A link number LINK

b) A node X precedes a node Y in the list

(1) when X has higher priority then Y

(2) When both have the same priority but X was added to the list before Y

❖ Priority numbers will operate in the usual way: the Lower the priority number, the higher the priority.

Priority Queues: Schematic Diagram with 7 element

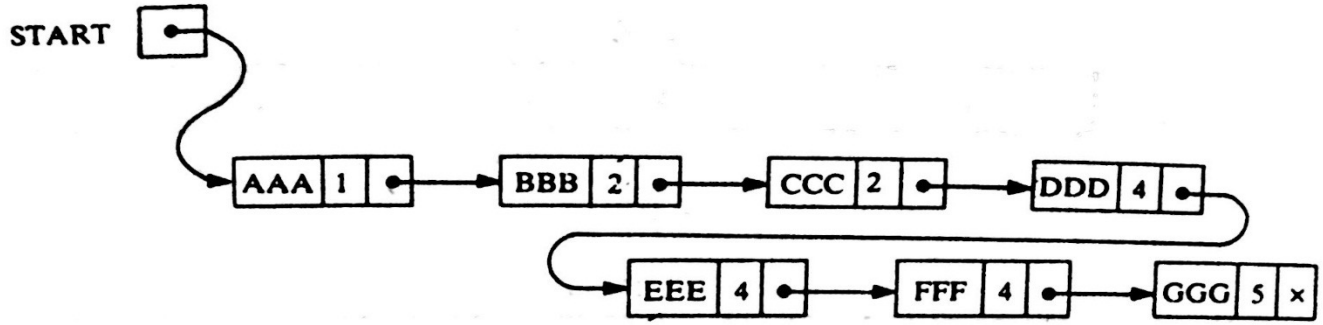


Fig. 6-19

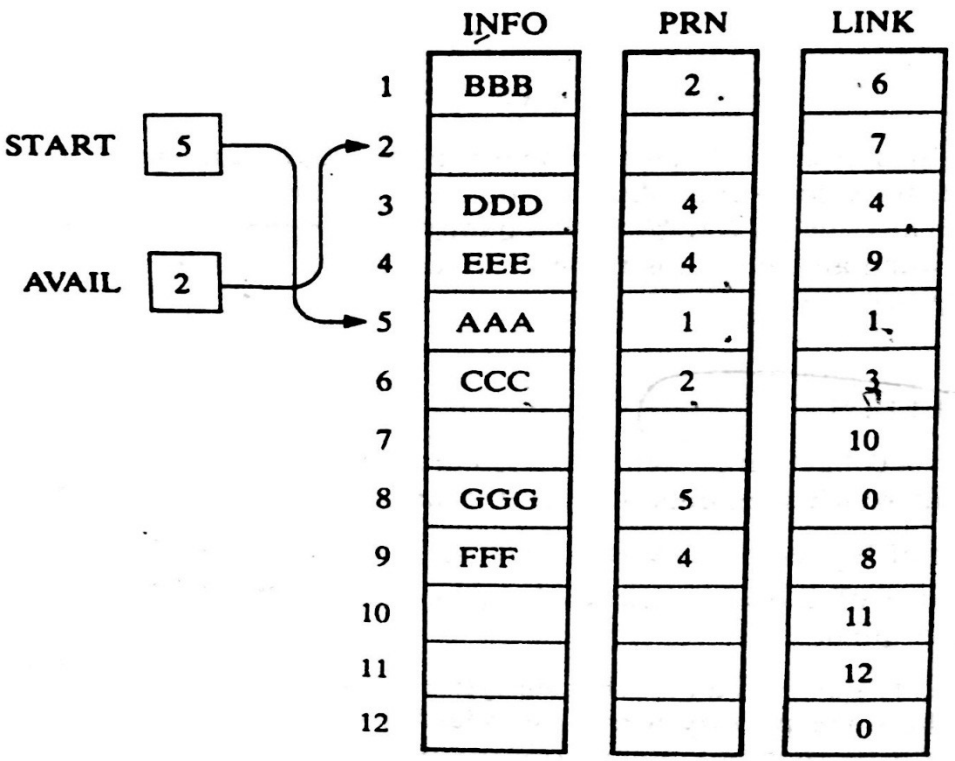


Fig. 6-20

Upcoming Presentation from Students:

- Implement STACK using one QUEUE
- Implements STACK using two QUEUES
- Implement QUEUE using one STACK
- Implement QUEUE using two STACK

LOOK at to the SOLVED Problems

6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 6.10
6.11, 6.12
6.14, 6.15, 6.16, 6.17
6.21,