Dzień dobry

12^{15} - MATEMATYKA

$f(x), g(x)$

$$\int f(x)g'(x)\,dx = \underbrace{f(x)}\cdot\underbrace{g(x)} - \int g(x)f'(x)\,dx$$

$$\int \underbrace{e^x}_{f(x)} \underbrace{\overset{A}{\sin x}\,dx}_{g'(x)} =$$

$f(x) = e^x \qquad g'(x) = \sin x$

$f'(x) = e^x \qquad g(x) = \int \sin x\,dx =$

$= -\cos x + C$

$$= e^x\cdot(-\cos x) - \int (-\cos x)e^x\,dx =$$

$$= e^x(-\cos x) + \int \underbrace{e^x}_{f(x)} \underbrace{\cos x \, dx}_{g'(x)} =$$

$$= e^x(-\cos x) + \left\{ \begin{array}{ll} f(x) = \underline{e^x} & g'(x) = \cos x \\ f'(x) = e^x & g(x) = \int \cos x \, dx = \underline{\sin x + C} \end{array} \right\}$$

$$= e^x(-\cos x) + \left[e^x \sin x - \int \sin x \, e^x \, dx \right] =$$

$$= \underbrace{-e^x \cos x}_{B} + \underbrace{e^x \sin x}_{C} - \underbrace{\int e^x \sin x \, dx}_{A}$$

$$A = B + C - A$$

$$2A = B + C \quad / : 2$$

$$A = \frac{B + C}{2}$$

$$\overset{A}{\int e^x \sin x\,dx} = \overset{B}{-e^x \cos x} + \overset{C}{e^x \sin x} - \overset{A}{\int e^x \sin x\,dx}$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x \quad / : 2$$

$$\int e^x \sin x\,dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

$$\overset{A}{\int \sin^2 x\,dx} = \int \underbrace{\sin x}_{f(x)}\, \underbrace{\sin x\,dx}_{g'(x)} =$$

$$\left\{\begin{array}{ll} f(x) = \sin x & g'(x) = \sin x \\ f'(x) = \cos x & g(x) = \int \sin x\,dx = -\cos x + C \end{array}\right\}$$

$$= \sin x(-\cos x) - \int(-\cos x \cdot \cos x)\,dx =$$

$$= -\sin x \cos x + \int \cos^2 x\,dx =$$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$= -\sin x \cos x + \int (1 - \sin^2 x)\,dx =$$

$$= -\sin x \cos x + \int 1\,dx - \int \sin^2 x\,dx =$$

$$= -\overset{B}{\sin x \cos x} + \overset{C}{x} - \overset{A}{\int \sin^2 x\,dx}$$

$$2\int \sin^2 x \, dx = -\sin x \cos x + x \quad \Big/ : 2$$

$$\int \sin^2 x \, dx = \frac{-\sin x \cos x}{2} + C$$

$$\int \underbrace{\cos^2 x}\cdot \overbrace{\underbrace{\sin x}\, dx}^{-dt}$$

$$\cancel{\int f(x) g(x)\,dx = \int f(x)\,dx \cdot \int g(x}$$

$$\cos^2 x = (\cos x)^2$$

$$= \left\{ \begin{array}{l} \cos x = t \\ -\sin x\,dx = 1\,dt \quad \big| \cdot (-1) \\ \sin x\,dx = -dt \end{array} \right\}$$

$$= \int t^2 (-1)\,dt = -\int t^2\,dt = -\frac{t^3}{3} + C =$$

$$= \underbrace{-\frac{(\cos x)^3}{3} + C}$$

$$\int x\cos(x^2)\,dx = \left\{\begin{array}{l} x^2 = t \\ 2x\,dx = 1\,dt \quad /:2 \\ x\,dx = \frac{1}{2}dt \end{array}\right\}$$

$$= \frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t + C = \frac{1}{2}\sin x^2 + C$$

$$\int \underbrace{x}_{f(x)}\underbrace{\cos(x^2)\,dx}_{g'(x)} = \left\{\begin{array}{ll} f(x) = x & g'(x) = \cos x^2 \\ f'(x) = 1 & g(x) = \int \cos(x^2)\,dx = ? \end{array}\right.$$

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$$\int \ln x \, dx = \int \underbrace{1}_{g'(x)} \cdot \underbrace{\ln x}_{f(x)} \, dx = \left| \begin{array}{ll} f(x) = \ln x & g'(x) = 1\,dx \\ f'(x) = \frac{1}{x}\,dx & g(x) = \int 1\,dx = \\ & = x + C \end{array} \right.$$

$$= x \cdot \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx =$$

$$= x \ln x - \int 1 \, dx = \underline{x \ln x - x} + C$$

$$\int \underbrace{x^3}_{f(x)} \underbrace{\sin x}_{g'(x)} dx = \left\{ \begin{array}{ll} f(x) = x^3 & g'(x) = \sin x \\ f'(x) = 3x^2 & g(x) = \int \sin x \, dx = \\ & = -\cos x + C \end{array} \right\}$$

$$= x^3(-\cos x) - \int 3x^2 \cdot (-\cos x)\, dx =$$

$$= -x^3 \cos x + 3 \int \underbrace{x^2}_{f(x)} \underbrace{\cos x \, dx}_{g'(x)} = -x^3 \cos x + 3 \left\{ \begin{array}{l} f(x) = x^2 \\ f'(x) = 2x \\ g'(x) = \cos x \\ g(x) = \int \cos x \, dx = \\ = \sin x + C \end{array} \right.$$

$$= -x^3 \cos x + 3 \left[x^2 \sin x - \int 2x \sin x \, dx \right] =$$

$$= -x^3 \cos x + 3x^2 \sin x - 6 \int \underbrace{x}_{f(x)} \underbrace{\sin x \, dx}_{g'(x)} =$$

$$= -x^3\cos x + 3x^2\sin x - 6 \quad \left[\begin{array}{ll} f(x) = x & g'(x) = \sin x \\ f'(x) = 1 & g(x) = \int \sin x\,dx = \\ & = -\cos x + C \end{array}\right.$$

$$= -x^3\cos x + 3x^2\sin x - 6\left[x\cdot(-\cos x) - \int(-\cos x)\,dx\right] =$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\int \cos x\,dx =$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$$

$$\int \frac{\ln x}{x}\,dx = \left\{ \begin{array}{c} \ln x = t \\ \frac{1}{x}\,dx = 1\,dt \end{array} \right\} =$$

$$= \int \ln x \cdot \frac{1}{x}\,dx = \int t\,dt = \frac{t^2}{2} + C$$

$$= \frac{(\ln x)^2}{2} + C$$

$$\int \sin x \, e^{-\cos x} \, dx = \left\{ \begin{matrix} -\cos x = t \\ \sin x \, dx = 1 \, dt \end{matrix} \right\}$$

$$= \int e^t \, dt = e^t + C = e^{-\cos x} + C$$

$$\int \frac{dx}{x \ln x} = \int \frac{1}{x} \frac{1}{\ln x} dx = \left\{ \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = 1 dt \end{array} \right.$$

$$= \int \frac{1}{t} dt = \ln|t| + C = \underline{\ln|\ln x| + C}$$