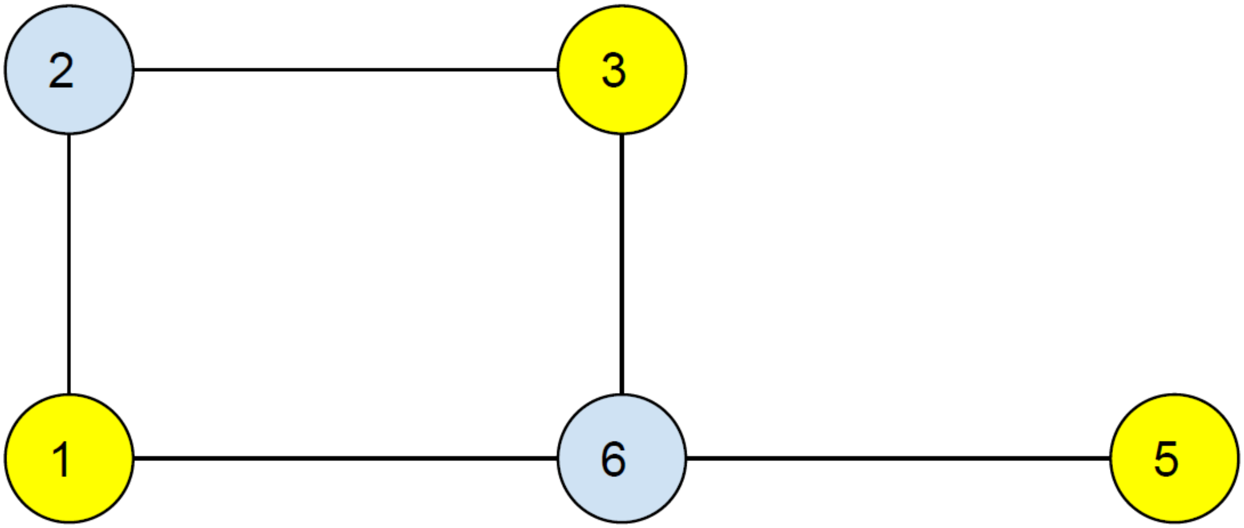
Write a flow ILP for the Steiner tree problem for the graph (see attached) with yellow terminals 1, 3, 5.



Solution:

$S = \{1, 3, 5\}$

X_e : whether edge e is chosen or not chosen.

Minimize: $\sum_{e \in E} c_e X_e$,

$\sum_{e \in (S, S')} X_e >= 1$, for all S separating some (s_i, t_i) pair.

$X_e >= 0$.

Flow: $f_{ij}^{13} >= 0 \ \forall \ (i, j) \in E, \ \forall \ K \in V$

$\sum f_{ik}^{13} = \sum f_{kj}^{13}$

Source:

1: $\sum f_{i1}^{13}$ (incoming) = $\sum f_{1j}^{13} - 1$(outgoing)

Destination:

3: $\sum f_{i3}^{13}$ (incoming) = $\sum f_{3j}^{13} + 1$(outgoing)

Therefore,

$X_{1,2} + X_{1,6} >= 1$ (Separating vertex 1)

$X_{2,3} + X_{3,6} >= 1$ (separating vertex 3)

Also, $x_{56} >= 1$ (Separating vertex 5)

$X_{1,2} >= f_{1,2}^{1,3}$
$X_{1,2} >= f_{2,1}^{1,3}$
$X_{1,2} >= f_{1,2}^{1,5}$
$X_{1,2} >= f_{2,1}^{1,5}$

Let's Consider the edge 2,3

$X_{2,3} >= f_{2,3}^{1,3}$
$X_{2,3} >= f_{3,2}^{1,3}$
$X_{2,3} >= f_{2,3}^{1,5}$
$X_{2,3} >= f_{3,2}^{1,5}$

Let's Consider the edge 1,6

$X_{1,6} >= f_{1,6}^{1,3}$
$X_{1,6} >= f_{6,1}^{1,3}$
$X_{1,6} >= f_{1,6}^{1,5}$
$X_{1,6} >= f_{6,1}^{1,5}$

Let's Consider the edge 3,6

$X_{3,6} >= f_{3,6}^{1,3}$
$X_{3,6} >= f_{6,3}^{1,3}$
$X_{3,6} >= f_{3,6}^{1,5}$
$X_{3,6} >= f_{6,3}^{1,5}$

Let's Consider the edge 5,6

$X_{5,6} >= f_{6,5}^{1,5}$
$X_{5,6} >= f_{5,6}^{1,5}$