**Assignment 2**

1. **Simplify the following symbollic statements as much as you can, leaving your answer in the standard symbolic form. (In case you are not familiar with the notation, I'll answer the first one for you.)**
   1. **(pi > 0) ^ (pi < 10)**

0 < pi < 10

* 1. **(p >= 7) ^ (p < 12)**

7 <= p < 12

* 1. **(x > 5) ^ (x < 7)**

5 < x < 7

If x were an integer, we could simplify this further to x = 6.

* 1. **(x < 4) ^ (x < 6)**

x < 4

* 1. **(y < 4) ^ (y\*y < 9)**

y < 3 -- Wrong: y \* y < 9 (missed negative 3)

If y were an integer, y <= 2 would also be true.

* 1. **(x >= 0) ^ (x <= 0)**

x = 0

1. **Express each of your simplified statements from question 1 in natural English.**
   1. pi is a number greater than 0 and less than 10.

pi is a number between 0 and 10 is a simpler statement but also ambiguous about the inclusiveness of the limits.

* 1. p is a number greater than or equal to 7 and less than 12.
  2. x is a number greater than 5 and less than 7.
  3. x is a number less than 4.
  4. y is a number less than 3.
  5. x is equal to 0.

1. **What strategy would you adopt to show that the conjunction s1 ^ s2 ^ ... ^ sn is true?**

I would prove that each of the conjuncts, s1, s2, ... , sn are true.

1. **What strategy would you adopt to show that the conjunction s1 ^ s2 ^ ... ^ sn is false?**

I would prove that at-least one of the conjuncts, s1, s2, ... , sn is false.

1. **Simplify the following symbolic statements as much as you can, leaving your answer in a standard symbolic form (assuming you are familiar with the notation):**
   1. **(pi > 3) V (pi > 10)**

pi > 3

* 1. **(x < 0) V (x > 0)**

x /= 0

* 1. **(x = 0) V (x > 0)**

x >= 0

* 1. **(x > 0) V (x >= 0)**

x >= 0

* 1. **(x > 3) V (x2 > 9)**

x2 > 9

1. **Express each of your simplified statements from question 5 in natural English.**
   1. pi is a number greater than 3.
   2. x is a number less than 0 or greater than 0.
   3. x is a number greater than or equal to 0.
   4. x is a number greater than or equal to 0.
   5. x squared is a number greater than 9.
2. **What strategy would you adopt to show that the disjunction s1 V s2 V ... V sn is true**?

I would prove that at-least one of the disjuncts, s1, s2, ... , sn is true.

1. **What strategy would you adopt to show that the disjunction s1 V s2 V ... V sn is false?**

I would prove that all the disjuncts, s**1**, s2, ... , sn are false.

1. **Simplify the following symbolic statements as much as you can, leaving your answer in a standard symbolic form (assuming you are familiar with the notation):**
   1. **~(pi > 3.2)**

pi <= 3.2

* 1. **~(x < 0)**

x >= 0

* 1. **~(x\*x > 0)**

x = 0 (if x is a real number) x\*x <= 0 (if x is imaginary)

* 1. **~(x = 1)**

x /= 1

* 1. **~~phi**

phi

1. **Express each of your simplified statements from question 9 in natural English.**
   1. pi is a number less than or equal to 3.2.
   2. x is a number greater than or equal to 0.
   3. x is equal to 0 if x is a real number (or) x is less than or equal to 0 if x can be complex.
   4. x is a number not equal to 1.
   5. The value phi.
2. **Let D be the statement "The dollar is strong", Y the statement "The Yuan is strong" and T the statement "New US-China trade agreement signed". Express the main content of each of the following (fictitious) newspaper headlines in logical notation. (Note that logical notation captures truth, but not the many nuances and inferences of natural language.) How would you justify and defend your answers?**
   1. **Dollar and Yuan both strong**

D ^ Y

* 1. **Yuan weak despite new trade agreement, but Dollar remains strong.**

T ^ ~Y ^ D

* 1. **Dollar and Yuan can't both be strong at the same time.**

~(D ^ Y)

* 1. **New trade agreement does not prevent fall in Dollar and Yuan.**

T ^ ~(D V Y)

* 1. **US-China trade agreement fails but both currencies remain strong.**

~T ^ D ^ Y

Since the trade agreement failed, the statement T is negated. Since both the currencies are strong, both D and Y assert themselves.

**Two to think about and discuss with other students**

1. **In US law, a trial verdict of "Not guilty" is given when the prosecution fails to prove guilt. This, of course, does not mean the defendant is, as a matter of actual fact, innocent. Is this state of affairs captured accurately when we use "not" in the mathematical sense? (i.e., Do "Not guilty" and "~ guilty" mean the same?) What if we change the question to ask if "Not proven" and "~ proven" mean the same?**

US court system is designed to provide the benefit of doubt to the accused to make sure no innocents are dealt with penalty. The verdict "Not guilty" nicely summarizes this position. "Not guilty" only means that the justice system in its investigative and reasoning pursuit has failed to prove beyond reasonable doubt that a person is guilty.

As the question above states, it does not allude to the innocence of the accused at all.

When we consider the above mathematically, it is impossible to reason about the expression without establishing the context and the rules.

If we assume that the following rules are true:

guilty = ~ innocent (and) innocent = ~ guilty

Then innocent and guilty are logical opposites. In this context, the mathematical statement "~ guilty" is not at all the same as the court verdict "Not guilty".

If we take the context away from the above argument, then "Not guilty" is the same as "~ guilty" as there is no inference that can be derived from "~ guilty" other than the "guilty is not true". In this case, the court verdict and the logical expression are equivalent.

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Whereas, Guilt => Innocence forms a spectrum of possibilities, Proven vs Not Proven has no such ambiguity.

We can easily conclude that "Not proven" is the same as "~ proven".

1. **The truth table for ~~si is clearly the same as that for si itself, so the two expressions make identical truth assertions. This is not necessarily true for negation in everyday life. For example, you might find yourself saying "I was not displeased with the movie." In terms of formal negation, this has the form ~(~ PLEASED), but your statement clearly does not mean that you were pleased with the movie. Indeed, it means something considerably less positive. How would you capture this kind of use of language in the formal framework we have been looking at?**

The conflict here is again comparing a binary system with that of a spectrum. Human emotions are shades of gray whereas Boolean logic is black and white. To capture this would mean we need to capture the various states of the spectrum (state machine in system terms).

Pleased = ~Not Pleased

Displeased = ~Not Displeased

So we will have two different variables for these. si and phi perhaps.

This can also extend to other degrees of being "pleased":

Very pleased = ~Not very pleased

Pretty pleased = ~Not that pleased (perhaps).