**Mathematical Thinking: Week 2 - Lecture 1 - Assignment 4 - Optional**

1. Express the combinator

P unless S

in terms of the standard logical combinators.

P is true unless S is true.

The word unless suggests equivalence.

P <=> ~S

(S => ~P) ^ (~S => P)

(~S V ~P) ^ (S V P)

((~S V ~P) ^ S) V ((~S V P) ^ P)

((~S ^ S) V (~P ^ S)) V ((~S ^ P) V (P ^ P))

(~P ^ S) V (~S ^ P)

Answer:

(~P ^ S) V (~S ^ P)

1. Let # denote the 'exclusive or' that corresponds to the English expression "either one or the other but not both". Construct the truth table for this connective.

| **P** | **S** | **P # S** |
| --- | --- | --- |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

1. Express P # S in terms of the basic combinators ^, V, ~.

(~P ^ S) V (~S ^ P)

1. Give, if possible, an example (one example in each case) of a true conditional sentence for which:
   1. the converse is true

If n is a multiple of 10, n\*n is a multiple of 100.

* 1. the converse is false

If you are human, you have opposable thumbs.

* 1. the contrapositive is true

The crops will die, if it doesn't rain.

* 1. the contrapositive is false

Since the contra postive is equivalent to the original statement it is not possible to make true conditional statement where the contrapositive is fale.

1. Mod-2 arithmetic has just the two numbers 0 and 1 and follows the usual rules of arithmetic together with the additional rule 1 + 1 = 0.

(It is the arithmetic that takes place in a single bit location in a digital computer.) Complete the following table:

| **M** | **N** | **M x N** | **M + N** |
| --- | --- | --- | --- |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 |

1. In the table you obtained in the above exercise, interpret 1 as T and 0 as F and view M,N as denoting statements:
   1. Which of the logical combinators ^,V corresponds to x?

^ (Since its true only both operands are true)

* 1. Which logical combinator corresponds to +?

It is XOR. In terms of ^,V and ~:

(~M ^ N) V (M ^ ~N)

* 1. Does ~ correspond to - (minus)?

No.

1. Repeat the above exercise, but interpret 0 as T and 1 as F. What conclusions can you draw?

| **M** | **N** | **M x N** | **M + N** |
| --- | --- | --- | --- |
| F | F | F | T |
| F | T | T | F |
| T | F | T | F |
| T | T | T | T |

* 1. Which of the logical combinators ^,V corresponds to x?

V (Since its true when one of the operands is true)

* 1. Which logical combinator corresponds to +?

It is ~(D # Y). In terms of ^,V and ~:

~((~M ^ N) V (M ^ ~N))

~(~M ^ N) ^ ~(M ^ ~N)

(M V ~N) ^ (~M V N)

((M V ~N) ^ ~M) V ((M V ~N) ^ N)

((M ^ ~M) V (~N ^ ~M)) V ((M ^ N) V (~N ^ N))

(~N ^ ~M) V (M ^ N)

(M ^ N) V (~N ^ ~M)

M <=> N

* 1. Does ~ correspond to - (minus)?

No.

1. The following puzzle was introduced by the psychologist Peter Wason in 1966, and is one of the most famous subject tests in the psychology of reasoning. Most people get it wrong. (So you have been warned!)

Four cards are placed on the table in front of you. You are told (truthfully) that each has a letter printed on one side and a digit on the other, but of course you can only see one face each. What you see is:

B E 4 7

You are now told that the cards you are looking at were chosen to follow the rule "If there is a vowel on one side, then there is an odd number on the other side." What is the least number of cards you have to turn over to verify this rule, and which cards do you in fact have to turn over?

* 1. Two cards
  2. E -> to make sure there is an odd number. 4 -> to make sure there is NOT a vowel.

1. Let m, n denote any two natural numbers. Prove that mn is odd iff m and n are odd.
2. 1) M = m is odd {let}
3. 2) N = n is odd {let}
4. 3) T = m \* n is odd {let}
5. 4) ~M => ~T {If m is even then m \* n is even}
6. 5) ~N => ~T {If n is even then m \* n is even}
7. 6) (~M V ~N) => ~T {From 4 and 5}
8. 7) ~(M ^ N) => ~T {From 6}
9. 8) T => (M ^ N) {From 7, applying contrapostive rule}
10. 9) M ^ N => T {If M is odd, T cannot be even if N is also odd}
11. 10) T <=> (M ^ N) {From 8 and 9}
12. With reference to the previous question, is it true that mn is even iff m and n are even? Prove your answer.

False.

1) M = m is even {let}

2) N = n is even {let}

3) T = m \* n is even {let}

4) T <=> M ^ N {Assumption}

5) T => M ^ N {From 4, since A <=> B means A => B ^ B => A}

6) M => T {If m is even, then T has to be even}

7) T </=> M ^ N {Since the statements 5 and 6 are in contradiction,

our assumption in step 4) must be False}

The contradiction stems from the fact that M is sufficient for T. Since M is sufficient for T, T => M ^ N becomes False for all False values of N.

1. You are in charge of a party where there are young people. Some are drinking alcohol, others soft drinks, Some are old enough to drink alcohol legally, others are under age. You are responsible for ensuring that the drinking laws are not broken, so you have asked each person to put his or her photo ID on the table. At one table are four young people. One person has a beer, another has a Coke, but their IDs happen to be face down so you cannot see their ages. You can, however, see the IDs of the other two people. One is under the drinking age, the other is above it. Unfortunately, you are not sure if they are drinking Seven-up or vodka and tonic. Which IDs and/or drinks you need to check to make sure that no one is breaking the law?

A = Beer ^ Unknown B = Coke ^ Unknown C = Unknown ^ Legal D = Unknown ^ Underage

Check to make sure the person drinking Beer is legal (check ID) and check to make sure the underage person is having a soft drink.

1. Compare the logical structure of the previous questions with Wason's problem (Optional problem 8 above.) Comment on your answers to those questions. In particular, identify any logical rules you have used in solving each problem, say which one was easier, and why you felt it was easier.

**Logical Rules**:

Both the problems involve deciphering conditionals to their combinatorial form.

*Wasons rule*:

A = Side one is vowel B = Side two is odd

A => B (If Side one is vowel, side two has to be odd)

(or)

~A v B

So in the Problem 8 above, the only cases that had to be checked were:

* 1. A vowel facing us A is true - so we need to check to make sure B is true
  2. An even number is facing us B is not true, so we need to check to make sure A is not true

*Youth Party*:

L = Legal Age A = Alcoholic drink

A => L

(or)

~A v L

So in Problem 11, the only cases that had to be checked were:

* 1. If the person was drinking alcohol we need to check their id A is true, so we need to check to make sure L is true as well.
  2. If the person was underage we need to check their drink L is not true, so we need to check to make sure A is not true

**Difficulty**:

Despite the fact that they are essentially same problems, the youth party one was harder as the original rule had to be deduced from the details.