**Assignment 4**

1. **Build a truth table to prove the claim I made earlier that P <=> S is true if P and S are both true or both false, and P <=> S is false if exactly one of P, S is true and the other false. (To constitute a proof, your table should have columns that show how the entries P <=> S are derived one operator at a time.)**

| **P** | **S** | **P => S** | **S => P** | **P <=> S** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

1. **Build a truth table to show that (P => S) <=> (~P V S) is true for all truth values of P and S. A statement whose truth values are all T is called a logical validity, or sometimes a tautology.**

| **P** | **S** | **P => S** | **~P V S** | **(P => S) <=> (~P V S)** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

1. **Build a truth table to show that (P /=> S) <=> (P ^ ~S) is a tautology.**

| **P** | **S** | **P /=> S** | **P ^ ~S** | **(P => S) <=> (~P V S)** |
| --- | --- | --- | --- | --- |
| T | T | F | F | T |
| T | F | T | T | T |
| F | T | F | F | T |
| F | F | F | F | T |

1. **The ancient Greeks formulated a basic rule of reasoning for proving mathematical statements. Called modus ponens, it says that if you know P and you know P => S, then you can conclude S.**
   1. **Construct a truth table for the logical statement [P ^ (P => S)] => S**

| **P** | **S** | **P => S** | **[P ^ (P => S)]** | **[P ^ (P => S)] => S** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

* 1. **Explain how the truth table you obtain demonstrates that modus ponens is a valid rule of inference.**

Proof:

* + 1. [P ^ (P => S)] => S
    2. [P ^ (~P v S)] => S -----------------------{Using P => S <=> ~P v S}
    3. [(P ^ ~P) v (P ^ S)] => S ----------------{Using A ^ (B v C) <=> (A ^ B) v (A ^ C)}
    4. P ^ S => S ---------------------------------{Using A ^ ~A is always false}
    5. ~(P ^ S) v S -------------------------------{Using P => S <=> ~P v S}
    6. (~P v ~S) v S ------------------------------{Using ~(A ^ B) <=> ~A v ~B}
    7. ~P v (~S v S) ------------------------------{Using (A v B) v C <=> A v (B v C)}
    8. ~P v True ----------------------------------{Using A v ~A is always true}
    9. True

1. **[This question has a long set-up. The question itself is the very last sentence. TAKE YOUR TIME]. One way to prove that ~(P ^ S) and (~P) V (~S) are equivalent is to show that they have the same truth table.**

| **P** | **S** | **P ^ S** | **~(P ^ S)\*** | **~P** | **~S** | **~P v ~S\*** |
| --- | --- | --- | --- | --- | --- | --- |
| **T** | **T** | **T** | **F** | **F** | **F** | **F** |
| **T** | **F** | **F** | **T** | **F** | **T** | **T** |
| **F** | **T** | **F** | **T** | **T** | **F** | **T** |
| **F** | **F** | **F** | **T** | **T** | **T** | **T** |

**Since the two columns marked \* are identical, we know that the two expressions are equivalent.**

**Thus negation has the effect that it changes ‘v’ into ‘^’ and changes ‘^’ into ‘v’. An alternative way to prove this is to argue directly with the meaning of the first statement:**

* 1. **P ^ S means both P and S are true.**
  2. **Thus ~(P ^ S) means it is not the case that both P and S are true.**
  3. **If they are not both true, then at least one of P, S must be false.**
  4. **This is clearly the same as saying that at least one of ~P and ~S is true. (By the definition of negation).**
  5. **By the meaning of or, this can be expressed as (~P) v (~S).**

**Provide an analogous logical argument to show that ~(P v S) and (~P) ^ (~S) are equivalent.**

1. P v S means at least one of P and S is true.

2. ~(P v S) means that it is not the case that "at least one of P and S is true".

3. The only case where "at least one of P and S true" is false is when both P and

S are false.

4. This can be said that both ~P and ~S are true. (By the rule of negation.)

5. By the meaning of and, this can be expressed as (~P) ^ (~S)

1. **By a denial of a statement P we mean any statement equivalent to ~P. Give a useful (and hence natural sounding) denial of each of the following statements.**

**a)34,159 is a prime number.**

34, 159 is not a prime number.

**b) Roses are red and violets are blue.**

Either roses aren't red or violets aren't blue.

**c) If there are no hamburgers, I'll have a hot dog.**

I will not have hamburgers and hot dogs.

**d) Fred will go but he will not play**.

Given: G ^ ~P

The negation of which is, ~G v P, which can be expressed as:

Either Fred will not go or he will play.

**e) The number x is either negative or greater than 10**.

(x < 0) v (x > 10)

The negation of this is:

~(x < 0) ^ ~(x > 10) or (x >= 0) ^ (x <= 10)

The number x is greater than or equal to zero and less than or equal to 10.

**f) We will win the first game or the second.**

The negation is:

~(O V T) or ~O ^ ~T

1. **Show that P <=> S is equivalent to (~P) <=> (~S)**

p = P <=> S (let)

p = (P => S) ^ (S => P) ([A <=> B] <=> [(A => B) ^ (B => C)])

p = (~P V S) ^ (~S V P) ([A => B] <=> (~A V B))

q = (~P) <=> (~S) (let)

q = (~P => ~S) ^ (~S => ~P) ([A <=> B] <=> [(A => B) ^ (B => C)])

q = (P V ~S) ^ (S V ~P) ([A => B] <=> (~A V B))

q = (S V ~P) ^ (P V ~S) ((A ^ B) <=> (B ^ A))

q = (~P V S) ^ (~S V P) ((A V B) <=> (B V A))

q = p

1. **Construct the truth tables to illustrate the following:**

**P <=> S**

| **P** | **S** | **P => S** | **S => P** | **P <=> S** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

**P => (S V T)**

| **P** | **S** | **T** | **S V T** | **P => (S V T)** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | F | T | T |
| F | F | T | T | T |
| F | F | F | F | T |

1. **Use truth tables to prove that the following are equivalent: P => (S ^ T) --A and (P => S) ^ (P => T) --B**

| **P** | **S** | **T** | **S ^ T** | **P => (S ^ T) --A** | **P => S** | **P => T** | **(P => S) ^ (P => T) --B** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| T | T | T | T | T | T | T | T |
| T | T | F | F | F | T | F | F |
| T | F | T | F | F | F | T | F |
| T | F | F | F | F | F | F | F |
| F | T | T | T | T | T | T | T |
| F | T | F | F | T | T | T | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

1. **Verify the equivalence in the previous question by means of a logical argument. (So you must show that assuming P and deducing S ^ T is the same as both deducing S from P and T from P.**

Unanswered

1. **Use truth tables to prove the equivalence of P => S and (~S) => (~P). (~S) => (~P) is called the contrapositive of P => S. The logical equivalence of a conditional and its contrpositive means that one way to prove an implication is to verify the contrapostive. This is a common form of proof in mathematics that we'll encounter later.**

| **P** | **S** | **P => S** | **~S** | **~P** | **~S => ~P** |
| --- | --- | --- | --- | --- | --- |
| T | T | T | F | F | T |
| T | F | F | T | F | F |
| F | T | T | F | T | T |
| F | F | T | T | T | T |

1. **Write down the contrapositives of the following statements:**

* **If two rectangles are congruent, they have the same area.**

Antecedent (A): Two rectangles are congruent.

Consequent (C): They have the same area.

Contrapositive(Cnt):

If two rectangles do not have the same area, they are not congruent.

* **If a triangle with sides a, b, c (c largest) is right angled, then a\*\*2 + b\*\*2 = c\*\*2.**

A: If a triangle with sides a, b, c (c largest) is right angled

C: a\*\*2 + b\*\*2 = c\*\*2

*Cnt: If a\*\*2 + b\*\*2 not equals c\*\*2 (c largest) then the triangle with sides* a, b, c is not right angled.

* **If 2\*\*n - 1 is prime, then n is prime.**

A: 2\*\*n - 1 is prime

C: n is prime

Cnt: If n is not prime, then 2\*\*n -1 is not prime.

* **If the Yuan rises, the Dollar will fall.**

A: The Yuan rises

C: The Dollar will fall

Cnt: If the Dollar has not fallen, the Yuan has not risen.

1. **It is important not to confuse the contrapostive of a conditional P => S with its converse S => P. Use truth tables to show that the contrapostive and the converse of P => S are not equal.**

| **S** | **P** | **S => P** | **~S** | **~P** | **~S => ~P** |
| --- | --- | --- | --- | --- | --- |
| T | T | T | F | F | T |
| T | F | F | F | T | T |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

1. **Write down the converses of the four statements in question 12**.

* **If two rectangles are congruent, they have the same area.**

If two rectangles have the same area, they are congruent.

* **If a triangle with sides a, b, c (c largest) is right-angled, then a\*\*2+b\*\*2=c\*\*2.**

If a\*\*2 + b\*\*2 == c\*\*2, then the triangle with sides a, b, c is right angled.

* **If 2\*\*n - 1 is prime, then n is prime.**

If n is prime, then 2\*\*n - 1 is prime.

* **If the Yuan rises, the Dollar will fall.**

If the dollar falls, the Yuan will rise.