**Week 4 - Assignment 6**

**1. Show that ¬[∃𝑥𝐴(𝑥)] is equivalent to ∀𝑥[¬𝐴(𝑥)].**

In order to prove that the statements:

¬[∃𝑥𝐴(𝑥)] (p) and ∀𝑥[¬𝐴(𝑥)] (q)

are equivalent, we need to prove that each statement implies the other (𝑝 ⇔ 𝑞).

Lets take the case 𝑝 ⇒ 𝑞 first.

Suppose p is true.

The statement ¬[∃𝑥𝐴(𝑥)] can be expressed as "It is not the case that there exists an x where A(x) is true". It follows that, "A(x) is not true for any value of x". Symbolically, ∀[¬𝐴(𝑥)].

Since we started with statement p and ended with statement q, we can conclude that 𝑝 ⇒ 𝑞.

Now let’s take the case 𝑞 ⇒ 𝑝.

Suppose q is true.

The statement ∀[¬𝐴(𝑥)] can be expressed as "For every value of x, A(x) is false". It follows that, "There is no value of x where we can say A(x) is true". It also means that the statement "There is at

least one value of x where A(x) is true" is false.

Symbolically, ¬[∃𝑥𝐴(𝑥)].

Since we started with statement q and ended with statement p, we can conclude that 𝑞 ⇒ 𝑝.

Having proved that 𝑞 ⇒ 𝑝 and 𝑝 ⇒ 𝑞, we can conclude that 𝑝 ⇔ 𝑞.

**2. Prove that the following statement is false:**

**There is an even prime number bigger than 2**

The statement can be expressed symbolically as:

(∃𝑥 > 2)[[(∀𝑝 > 1)(∀𝑞 > 1)(𝑝𝑞 ̸= 𝑥)] ∧ [(∃𝑟)(2𝑟 = 𝑥)]]

Assume that the above statement is true and there exists a value x that is an even prime number.

In order for x to exist the following have to be true:

a) There are no numbers p, q greater than 1 whose product will produce x. (Since x is prime)

b) There is a number r greater than 1 when multiplied by 2 will produce x. (Since x is even, it has to be evenly divisible by 2).

The above two statements contradict each other. Hence our original assumption cannot be true. This follows that: "There is an even prime number bigger than 2" is false.

**3. Translate the following sentences into symbolic form using quantifiers. In each case the assumed domain is given in parentheses.**

**a. All students like pizza. (All people)**

∀𝑠 ∈ 𝑃[𝑠𝑡𝑢𝑑𝑒𝑛𝑡(𝑠) ⇒ 𝑙𝑖𝑘𝑒𝑠(𝑠, 𝑝𝑖𝑧𝑧𝑎)]

**b. One of my friends does not have a car. (All people)**

∃𝑓 ∈ 𝑃[𝑀𝑦𝐹 𝑟𝑖𝑒𝑛𝑑(𝑓) ∧ ¬𝐻𝑎𝑠𝐶𝑎𝑟(𝑓)]

**c. Some elephants do not like muffins. (All animals)**

∃𝑒 ∈ 𝐴[𝐸𝑙𝑒𝑝ℎ𝑎𝑛𝑡(𝑒) ∧ ¬𝑙𝑖𝑘𝑒𝑠(𝑒, 𝑚𝑢𝑓𝑓 𝑖𝑛𝑠)]

**d. Every triangle is isoceles. (All geometric figures)**

∀𝑖 ∈ 𝐺[𝑇 𝑟𝑖𝑎𝑛𝑔𝑙𝑒(𝑖) ⇒ 𝑖𝑠𝑜𝑐𝑒𝑙𝑒𝑠(𝑖)]

**e. Some of the students in the class are not here today. (All people)**

∃𝑠 ∈ 𝑃[𝑆𝑡𝑢𝑑𝑒𝑛𝑡(𝑠, 𝑐𝑙𝑎𝑠𝑠) ∧ ¬ℎ𝑒𝑟𝑒(𝑠, 𝑡𝑜𝑑𝑎𝑦)]

**f. Everyone loves somebody. (All people).**

∀𝑒 ∈ 𝑃∃𝑠 ∈ 𝑃[𝑒♡𝑠]

**g. Nobody loves everybody. (All people).**

¬∃𝑠 ∈ 𝑃∀𝑒 ∈ 𝑃[𝑠♡𝑒]

**h. If a man comes, all the women will leave. (All people)**

∃𝑚 ∈ 𝑃[(𝑀 𝑎𝑛(𝑚)∧𝑐𝑜𝑚𝑒𝑠(𝑚)) ⇒ ∀𝑤 ∈ 𝑃[𝑊 𝑜𝑚𝑎𝑛(𝑤) ⇒ 𝑙𝑒𝑎𝑣𝑒𝑠(𝑤)]]

**i. All people are tall or short. (All people)**

∀𝑥 ∈ 𝑃[𝑡𝑎𝑙𝑙(𝑥) ∨ 𝑠ℎ𝑜𝑟𝑡(𝑥)]

**j. All people are tall or all people are short. (All people)**

[(∀𝑥 ∈ 𝑃)𝑡𝑎𝑙𝑙(𝑥)] ∨ [(∀𝑥 ∈ 𝑃)𝑠ℎ𝑜𝑟𝑡(𝑥)]

**k. Not all precious stones are beautiful. (All stones)**

¬(∀𝑠 ∈ 𝑆)[𝑝𝑟𝑒𝑐𝑖𝑜𝑢𝑠(𝑠) ⇒ 𝑏𝑒𝑎𝑢𝑡𝑖𝑓𝑢𝑙(𝑠)]

(𝑜𝑟)

∃𝑠 ∈ 𝑆[𝑝𝑟𝑒𝑐𝑖𝑜𝑢𝑠(𝑠) ∧ ¬𝑏𝑒𝑎𝑢𝑡𝑖𝑓𝑢𝑙(𝑠)]

**l. Nobody loves me. (All people)**

∀𝑒 ∈ 𝑃[¬(𝑒♡𝑚𝑒)]

**m. At least one American snake is poisonous. (All snakes)**

∃𝑠 ∈ 𝑆[𝐴𝑚𝑒𝑟𝑖𝑐𝑎𝑛(𝑠) ∧ 𝑝𝑜𝑖𝑠𝑜𝑛𝑜𝑢𝑠(𝑠)]

**n. At least one American snake is poisonous. (All animals)**

∃𝑠 ∈ 𝐴[𝑆𝑛𝑎𝑘𝑒(𝑠) ∧ 𝐴𝑚𝑒𝑟𝑖𝑐𝑎𝑛(𝑠) ∧ 𝑝𝑜𝑖𝑠𝑜𝑛𝑜𝑢𝑠(𝑠)]

**4. Negate each of the symbolic statements you wrote in the last question, putting your answers in positive form. Then express each negation in natural idiomatic English.**

**a. All students like pizza. (All people)**

∀𝑠 ∈ 𝑃[𝑠𝑡𝑢𝑑𝑒𝑛𝑡(𝑠) ⇒ 𝑙𝑖𝑘𝑒𝑠(𝑠, 𝑝𝑖𝑧𝑧𝑎)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑠 ∈ 𝑃[𝑠𝑡𝑢𝑑𝑒𝑛𝑡(𝑠) ∧ ¬𝑙𝑖𝑘𝑒𝑠(𝑠, 𝑝𝑖𝑧𝑧𝑎)]

Not all students like pizza (or) Some students don’t like pizza.

**b. One of my friends does not have a car. (All people)**

∃𝑓 ∈ 𝑃[𝑀𝑦𝐹 𝑟𝑖𝑒𝑛𝑑(𝑓) ∧ ¬𝐻𝑎𝑠𝐶𝑎𝑟(𝑓)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∀𝑓 ∈ 𝑃[𝑀𝑦𝐹 𝑟𝑖𝑒𝑛𝑑(𝑓) ⇒ 𝐻𝑎𝑠𝐶𝑎𝑟(𝑓)]

All my friends have a car.

**c. Some elephants do not like muffins. (All animals)**

∃𝑒 ∈ 𝐴[𝐸𝑙𝑒𝑝ℎ𝑎𝑛𝑡(𝑒) ∧ ¬𝑙𝑖𝑘𝑒𝑠(𝑒, 𝑚𝑢𝑓𝑓 𝑖𝑛𝑠)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∀𝑒 ∈ 𝐴[𝐸𝑙𝑒𝑝ℎ𝑎𝑛𝑡(𝑒) ⇒ 𝑙𝑖𝑘𝑒𝑠(𝑒, 𝑚𝑢𝑓𝑓 𝑖𝑛𝑠)]

All elephants like muffins.

**d. Every triangle is isoceles. (All geometric figures)**

∀𝑖 ∈ 𝐺[𝑇 𝑟𝑖𝑎𝑛𝑔𝑙𝑒(𝑖) ⇒ 𝑖𝑠𝑜𝑐𝑒𝑙𝑒𝑠(𝑖)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑖 ∈ 𝐺[𝑇 𝑟𝑖𝑎𝑛𝑔𝑙𝑒(𝑖) ∧ ¬𝑖𝑠𝑜𝑐𝑒𝑙𝑒𝑠(𝑖)]

Not all triangles are isosceles.

**e. Some of the students in the class are not here today. (All people)**

∃𝑠 ∈ 𝑃[𝑆𝑡𝑢𝑑𝑒𝑛𝑡(𝑠, 𝑐𝑙𝑎𝑠𝑠) ∧ ¬ℎ𝑒𝑟𝑒(𝑠, 𝑡𝑜𝑑𝑎𝑦)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∀𝑠 ∈ 𝑃[𝑆𝑡𝑢𝑑𝑒𝑛𝑡(𝑠, 𝑐𝑙𝑎𝑠𝑠) ⇒ ℎ𝑒𝑟𝑒(𝑠, 𝑡𝑜𝑑𝑎𝑦)]

All students in the class are here today.

**f. Everyone loves somebody. (All people).**

∀𝑒 ∈ 𝑃∃𝑠 ∈ 𝑃[𝑒♡𝑠]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑒 ∈ 𝑃∀𝑠 ∈ 𝑃¬[𝑒♡𝑠]

Not everyone loves somebody.

**g. Nobody loves everybody. (All people).**

¬∃𝑠 ∈ 𝑃∀𝑒 ∈ 𝑃[𝑠♡𝑒]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑠 ∈ 𝑃∀𝑒 ∈ 𝑃[𝑠♡𝑒]

Somebody loves everybody.

**h. If a man comes, all the women will leave. (All people)**

∃𝑚 ∈ 𝑃[(𝑀 𝑎𝑛(𝑚)∧𝑐𝑜𝑚𝑒𝑠(𝑚)) ⇒ ∀𝑤 ∈ 𝑃[𝑊 𝑜𝑚𝑎𝑛(𝑤) ⇒ 𝑙𝑒𝑎𝑣𝑒𝑠(𝑤)]]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑚 ∈ 𝑃[(𝑀 𝑎𝑛(𝑚)∧𝑐𝑜𝑚𝑒𝑠(𝑚)) ; ∀𝑤 ∈ 𝑃[𝑊 𝑜𝑚𝑎𝑛(𝑤) ⇒ 𝑙𝑒𝑎𝑣𝑒𝑠(𝑤)]]

(𝑜𝑟)

∃𝑚 ∈ 𝑃[[(𝑀 𝑎𝑛(𝑚)∧𝑐𝑜𝑚𝑒𝑠(𝑚))]∧[∃𝑤 ∈ 𝑃[𝑊 𝑜𝑚𝑎𝑛(𝑤)∧¬𝑙𝑒𝑎𝑣𝑒𝑠(𝑤)]]]

Not all women will leave if a man comes.

(or)

Some women won’t leave if a man comes.

**i. All people are tall or short. (All people)**

∀𝑥 ∈ 𝑃[𝑡𝑎𝑙𝑙(𝑥) ∨ 𝑠ℎ𝑜𝑟𝑡(𝑥)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑥 ∈ 𝑃[¬𝑡𝑎𝑙𝑙(𝑥) ∧ ¬𝑠ℎ𝑜𝑟𝑡(𝑥)]

Not all people are tall or short.

(or)

There are some people that are neither tall nor short.

**j. All people are tall or all people are short. (All people)**

[(∀𝑥 ∈ 𝑃)𝑡𝑎𝑙𝑙(𝑥)] ∨ [(∀𝑥 ∈ 𝑃)𝑠ℎ𝑜𝑟𝑡(𝑥)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

[(∃𝑥 ∈ 𝑃)¬𝑡𝑎𝑙𝑙(𝑥)] ∧ [(∃𝑥 ∈ 𝑃)¬𝑠ℎ𝑜𝑟𝑡(𝑥)]

There are some people that are not tall and there are some

people that are not short.

**k. Not all precious stones are beautiful. (All stones)**

¬(∀𝑠 ∈ 𝑆)[𝑝𝑟𝑒𝑐𝑖𝑜𝑢𝑠(𝑠) ⇒ 𝑏𝑒𝑎𝑢𝑡𝑖𝑓𝑢𝑙(𝑠)]

(𝑜𝑟)

∃𝑠 ∈ 𝑆[𝑝𝑟𝑒𝑐𝑖𝑜𝑢𝑠(𝑠) ∧ ¬𝑏𝑒𝑎𝑢𝑡𝑖𝑓𝑢𝑙(𝑠)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∀𝑠 ∈ 𝑆[𝑝𝑟𝑒𝑐𝑖𝑜𝑢𝑠(𝑠) ⇒ 𝑏𝑒𝑎𝑢𝑡𝑖𝑓𝑢𝑙(𝑠)]

All precious stones are beautiful.

**l. Nobody loves me. (All people)**

∀𝑒 ∈ 𝑃[¬(𝑒♡𝑚𝑒)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∃𝑒 ∈ 𝑃[𝑒♡𝑚𝑒]

Somebody loves me.

**m. At least one American snake is poisonous. (All snakes)**

∃𝑠 ∈ 𝑆[𝐴𝑚𝑒𝑟𝑖𝑐𝑎𝑛(𝑠) ∧ 𝑝𝑜𝑖𝑠𝑜𝑛𝑜𝑢𝑠(𝑠)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∀𝑠 ∈ 𝑆[𝐴𝑚𝑒𝑟𝑖𝑐𝑎𝑛(𝑠) ⇒ ¬𝑝𝑜𝑖𝑠𝑜𝑛𝑜𝑢𝑠(𝑠)]

American snakes aren’t poisonous.

**n. At least one American snake is poisonous. (All animals)**

∃𝑠 ∈ 𝐴[𝑆𝑛𝑎𝑘𝑒(𝑠) ∧ 𝐴𝑚𝑒𝑟𝑖𝑐𝑎𝑛(𝑠) ∧ 𝑝𝑜𝑖𝑠𝑜𝑛𝑜𝑢𝑠(𝑠)]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

∀𝑠 ∈ 𝐴[(𝑆𝑛𝑎𝑘𝑒(𝑠) ∧ 𝐴𝑚𝑒𝑟𝑖𝑐𝑎𝑛(𝑠) ⇒ ¬𝑝𝑜𝑖𝑠𝑜𝑛𝑜𝑢𝑠(𝑠)]

American snakes aren’t poisonous.

**5. Which of the following are true? The domain for each is given in parentheses.**

**a. ∃(2𝑥 + 3 = 5𝑥 + 1) (Natural numbers)**

False.

2𝑥 + 3 = 5𝑥 + 1

5𝑥 − 2𝑥 = 3 − 1

3𝑥 = 2

𝑥 = 2/3

**b. ∃(𝑥2 = 2) (Rational numbers)**

False. Square root of 2 is irrational.

**c. ∀𝑥∃(𝑦 = 𝑥2) (Real numbers)**

True. You can raise any real number to the power of 2 and get a real number.

**d. ∀𝑥∃(𝑦 = 𝑥2) (Natural numbers)**

True. You can raise any natural number to the power of 2 and get a natural number.

**e. ∀𝑥∃𝑦∀(𝑥𝑦 = 𝑥𝑧) (Real numbers)**

False. There is no value of y, for which xy will be equal to xz for any value of z or x.

**f. ∀𝑥∃𝑦∀(𝑥𝑦 = 𝑥𝑧) (Prime numbers)**

False. There is no value of y, for which xy will be equal to xz for any value of z or x.

**g. ∀[𝑥 < 0 => ∃𝑦(𝑦2 = 𝑥)] (Real numbers)**

False. 𝑦2 cannot be negative when y is a Real number.

**h. ∀[𝑥 < 0 => ∃𝑦(𝑦2 = 𝑥)] (Positive real numbers)**

True. In the set of positive real numbers, 𝑥 < 0 will always

be false. A false antecedent makes the implication True.

**6. Negate each of the statements in the last question, putting your answers in postive form.**

**a. ∃(2𝑥 + 3 = 5𝑥 + 1) (Natural numbers)**

∀(2𝑥 + 3 ̸= 5𝑥 + 1)

True

**b. ∃(𝑥2 = 2) (Rational numbers)**

∀(𝑥2 != 2)

True.

**c. ∀𝑥∃(𝑦 = 𝑥2) (Real numbers)**

∃𝑥∀(𝑦 != 𝑥2)

False.

**d. ∀𝑥∃(𝑦 = 𝑥2) (Natural numbers)**

∃𝑥∀(𝑦 != 𝑥2)

False.

**e. ∀𝑥∃𝑦∀(𝑥𝑦 = 𝑥𝑧) (Real numbers)**

∃𝑥∀𝑦∃𝑧(𝑥𝑦 != 𝑥𝑧)

False for x = 0.

**f. ∀𝑥∃𝑦∀(𝑥𝑦 = 𝑥𝑧) (Prime numbers)**

∃𝑥∀𝑦∃𝑧(𝑥𝑦 != 𝑥𝑧)

True for all values of != 𝑦

**g. ∀[𝑥 < 0 => ∃𝑦(𝑦2 = 𝑥)] (Real numbers)**

∃[(𝑥 < 0) ∧ ∀𝑦(𝑦2 != 𝑥)]

True for all values of x.

**h. ∀[𝑥 < 0 => ∃𝑦(𝑦2 = 𝑥)] (Positive real numbers)**

∃[(𝑥 < 0) ∧ ∀𝑦(𝑦2 != 𝑥)]

False.

**7. Negate the following statements and put each answer into positive form:**

**a. (∀𝑥 ∈ N)(∃𝑦 ∈ N)(𝑥 + 𝑦 = 1)**

(∃𝑥 ∈ N)(∀𝑦 ∈ N)(𝑥 + 𝑦 ̸= 1)

**b. (∀𝑥 > 0)(∃𝑦 < 0)(𝑥 + 𝑦 = 0) (where x, y are real number variables)**

(∃𝑥 > 0)(∀𝑦 < 0)(𝑥 + 𝑦 ̸= 0)

**c. ∃(∀𝜖 > 0)(−𝜖 < 𝑥 < 𝜖) (where x, 𝜖 are real number variables)**

∀(∃𝜖 > 0)((𝑥 ≤ −𝜖) ∨ (𝑥 ≥ 𝜖))

**d. (∀𝑥 ∈ N)(∀𝑦 ∈ N)(∃𝑧 ∈ N)(𝑥 + 𝑦 = 𝑧2)**

(∃𝑥 ∈ N)(∃𝑦 ∈ N)(∀𝑧 ∈ N)(𝑥 + != 𝑧2)

**8. Give a negation (in positive form) of the famous "Abraham Lincoln sentence" which we met in the previous assignment: "You may fool all the people some of the time, you can even fool some of the people all of the time, but you cannot fool all of the people all of the time."**

[(∃𝑡)(∀𝑝)(𝑓𝑜𝑜𝑙(𝑝, 𝑡))] ∧ [(∃𝑝)(∀𝑡)(𝑓𝑜𝑜𝑙(𝑝, 𝑡))] ∧ ¬[(∀𝑝)(∀𝑡)(𝑓𝑜𝑜𝑙(𝑝, 𝑡))]

𝑁𝑒𝑔𝑎𝑡𝑖𝑜𝑛 :

[(∀𝑡)(∃𝑝)(¬𝑓𝑜𝑜𝑙(𝑝, 𝑡))]∧[(∃𝑝)(∀𝑡)(¬𝑓𝑜𝑜𝑙(𝑝, 𝑡))]∧[(∀𝑝)(∀𝑡)(𝑓𝑜𝑜𝑙(𝑝, 𝑡))]

At any time, you can always find some people that cannot be fooled, there are some people that can never be fooled, but all of the people can be fooled all of the time.

**9. The standard definition of a real function f being continuous at a point x = a is**

**(∀𝜖 > 0)(∃𝛿 > 0)(∀𝑥)[|𝑥 − 𝑎| < 𝛿 ⇒ |𝑓(𝑥) − 𝑓(𝑎)| < 𝜖]**

**Write down a formal definition for f being discontinuous at a. Your definition should be in positive form.**

(∃𝜖 > 0)(∀𝛿 > 0)(∃𝑥)[|𝑥 − 𝑎| < 𝛿 ∧ |𝑓(𝑥) − 𝑓(𝑎)| ≥ 𝜖]