1 Напоминание: дифференциал

$$df(x) := f'(x)dx$$

$$d(f(g(x))) = f'(g(x))dg(x)$$

$$(f(g(x)))'dx = f'(g(x))g'(x)dx$$

$$d(\sin^3 x) = 3\sin^2 x \cos x dx$$

$$d(\sin(x^3)) = \cos(x^3)3x^2dx = \cos x^3 dx^3$$

$$d(\ln(x^2+3x+4)) = \frac{d(x^2+3x+4)}{x^2+3x+4} = \frac{(2x+3)dx}{x^2+3x+4}$$

$$d(\sin(5x+2)) = \cos(5x+2) \cdot 5 \cdot dx$$

$$\cos x dx = d\sin x \text{ по определению } dx$$

$$3\sin^2 x\cos x dx = 3\sin^2 x d\sin x = d(\sin^3 x)$$

$$\text{Не у всего можно взять интеграл: } \nexists f(x) : d(f(x)) = e^{x^2}dx$$

2 Первообразная — антипроизводная

$$F' = f$$

F' — первообразная f, f — производная F

Первообразных много *(с точностью до прибавления константы)*

$f(x) := \frac{1}{x}$

$F(x) = \ln|x| + C$

Но еще есть другие первообразные:

$$F(x) = \begin{cases} \ln|x| + C_1, & x > 0 \\ \ln|x| + C_2, & x < 0 \end{cases}$$

В произвольной точке так поделить нельзя, т.к. разрыв и не будет производной

3 Неопределенный интеграл

$$\int f(x)dx := \{F : F' = f\}$$

$$\int f(x)dx := F'(x) + C$$

$$\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \left(\frac{1+x^2}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)} \right) = -x^{-1} + \operatorname{arctg} x + C$$

$$\int \frac{(1+x)^2}{x(1+x^2)}dx = \int \left(\frac{1}{x} + \frac{2}{1+x^2} \right) dx = \ln|x| + 2\operatorname{arctg} x + C$$

$$\int \frac{dx}{\cos 2x + \sin^2 x} = \int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{(\arcsin x + \arccos x)dx}{\frac{\pi}{2}} = \frac{\pi}{2}x + C$$

$$\int \operatorname{tg}^3 x d\operatorname{tg} x = [t = \operatorname{tg} x] = \int t^3 dt = \frac{t^4}{4} + C = \frac{\operatorname{tg}^4 x}{4} + C$$

$$\int \frac{d(1+x^2)}{\sqrt{1+x^2}} = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{1+x^2}$$

$$\int \sqrt[5]{(3+8x)^6}dx = \frac{1}{8}\int (3+8x)^{\frac{6}{5}} d(3+8x) = \frac{1}{8}\frac{5}{11}(3+8x)^{\frac{11}{5}} + C$$

$$\int \frac{\operatorname{arctg}^2 x}{1+x^2}dx \int \operatorname{arctg}^2 x d\operatorname{arctg} x = \int t^2 dt = \frac{t^3}{3} = \frac{\operatorname{arctg}^3 x}{3} + C$$

$$\int f(g(x)) \cdot g'(x)dx = [g(x) = t, dt = g'(x)dx] = \int f(t)dt$$

$$\int \frac{dx}{1+\sqrt[3]{x+1}} = [\sqrt[3]{x+1} = t, x = t^3 - 1, dx = 3t^2 dt] = \int \frac{3t^2 dt}{1+t} = \int \frac{3t^2 - 3 + 3}{1+t} dt = 3 \int (t - 1 + \frac{1}{t+1}) dt$$

$$= 3(\frac{t^2}{2} - t + \ln(t+1))$$

$$\int \frac{\ln \operatorname{tg} x}{\sin x \cos x} dx = \left[\operatorname{tg} x = t, dt = \frac{dx}{\cos^2 x} \right] = \int \frac{lnt \cos^2 x dt}{\sin x \cos x} = \int \frac{\ln t}{t} dt = \int \ln t dt = [y = \ln t] = \frac{y^2}{2} + C =$$

$$= \frac{(\ln \operatorname{tg} x)^2}{2}$$

Проверка:

$$\left(\frac{(\ln \operatorname{tg} x)^2}{2} \right) = \frac{1}{2} \cdot 2 \cdot \ln \operatorname{tg} x \cdot \frac{1}{\operatorname{tg} x} \frac{1}{\cos^2 x} = \ln \operatorname{tg} x \frac{1}{\sin x \cos x}$$

4 Интегрирование по частям

$$(uv)' = uv' + v'u$$

$$d(uv) = udv + vdu$$

$$udv = d(uv) - vdu$$

$$\int udv = \int d(uv) - \int vdu$$

$$\int udv = uv - \int vdu$$

u должна не “портиться” при дифф., а dv не “портится” при интегрировании.

$$\int e^{3x}(x+1)dx = [u = x+1, dv = e^{3x}dx, du = dx, v = \frac{e^{3x}}{3}] = \int udx = uv - \int vdu =$$

$$= (x+1)\frac{e^{3x}}{3} - \frac{1}{3} \int e^{3x} dx = (x+1)\frac{e^{3x}}{3} - \frac{1}{9} e^{3x} dx + C$$

$$\int \sin x(x^2+x)dx = [u = x^2+x, dv = \sin x dx, du = (2x+1)dx, v = -\cos x] =$$

$$= uv - \int v du = -(x^2+x)\cos x + \int \cos x(2x+1)dx = [u = 2x+1, dv = \cos x dx, du = 2dx, v = \sin x] =$$

$$= -(x^2+x)\cos x + ((2x+1)\sin x - \int 2dx \sin x) = -(x^2+x)\cos x + (2x+1)\sin x + 2\cos x$$

$$\int \ln x dx = [u = \ln x, dv = dx, du = \frac{1}{x}dx, v = x] = x\ln x - \int dx = x\ln x - x + c$$

Проверка:

$$(x\ln x - x + c)' = \ln x + 1 - 1 = \ln x$$

$$\int \ln(x^2+1)dx = [u = \ln(x^2+1), dv = dx, v = x, du = \frac{1}{x^2+1}2xdx] =$$

$$= \ln(x^2+1)x - \int x\frac{1}{x^2+1}2xdx = x\ln(x^2+1) - 2\int(1-\frac{1}{x^2+1})dx = x\ln(x^2+1) - 2(x - \operatorname{arctg} x) + C$$

$$\int e^x \sin 2x dx = [u = e^x, dv = \sin 2x dx, du = e^x, v = \frac{-1}{2}\cos 2x] =$$

$$= e^x\frac{-1}{2}\cos 2x + \int \frac{1}{2}e^x\cos 2x dx = -e^x\frac{1}{2}\cos 2x + \frac{1}{4}e^x\sin 2x - \frac{1}{4}\int e^x \sin 2x dx$$

$$\frac{5}{4}\int e^x \sin 2x dx = e^x\left(-\frac{1}{2}\cos 2x + \frac{1}{4}\sin 2x\right)$$

$$\int e^x \sin 2x dx = \frac{2}{5}e^x\left(-\cos 2x + \frac{1}{2}\sin 2x\right) +$$