# Welcome ☺

Agenda : Log Basics
Quiz
Time Complexity
Big O
TLE
Constraints.

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## Logarithm

$\log_a b = c$ ⟹ to what value we need to raise 'a' so that its value becomes 'b'

⇅

$a^c = b$

$\log_2 64 = 2^? = 64 = 2^6 = 64$

$\log_3 27 = 3^? = 27 = 3^3 = 27$

$\log_2 2^{10} = 10$

$$\boxed{\begin{array}{c} 2^K = N \\ \log_2 N = K \end{array}}$$

$$\boxed{\log_a a^m = m}$$

Q How many times we need to divide 9 by 2 till it reaches 1

⟹ $9/2 = 4$

$4/2 = 2$

$2/2 = 1$

$2^K = 9$

$K = \log_2 9$

Q How many times we need to divide 27 by 2 till it reaches 1

$27/2 = 13$

$13/2 = 6$ Ans = 4

$6/2 = 3$

$3/2 = 1$

## No. of Iterations.

```
N>0
i = N
while ( i >1)
{
    i = i/2
}
```

$\Rightarrow$ $i \rightarrow i/2 \rightarrow i/2^2 \rightarrow i/2^3 \dots\dots$

Ans = $\log_2 N$ $O(\log N)$

Quiz

```
for( i=1 ; i<N ; i= i*2)
{
    . . . .
}
```

| $i_b$ | iterations | $i_a$ |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 2 | $2*2 = 4$ |
| 4 | 3 | $4*2 = 8$ |
| ⋮ | ⋮ | ⋮ |
| $2^{K-1}$ | $K$ | $2^K$ |

$$2^K = N \Rightarrow K = \log_2 N$$

## Quiz 5

```
for( i=0 ; i<N ; i=i*2)
{
    . . . .
}
```

# iterations =

| $i_b$ | iterations | $i_a$ |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 2 | 0 |
| 0 | 3 | 0 |
| . | ⋮ | |
| 0 | K | 0 |

## Quiz 6

```
for( i= 1 ;  i≤10;  i++)
{
    for( j=1 ; j<=N ; j++)
    {
        . . . .
    }
}
```

| i | j | # iteraⁿ |
|---|---|---|
| 1 | 1→N | N |
| 2 | 1→N | N |
| 3 | 1→N | N |
| | ⋮ | |
| 10 | 1→N | N |
| | | 10N |

# iteraⁿs = 10N

$O(N)$

## Quiz 7

```
for( i= 1 ;  i≤N ;  i++)
{
    for( j=1 ; j<=N ; j++)
    {
        . . . .
    }
}
```

# iteraⁿ = N*N

$= N^2$

$O(N^2)$

Quiz 8

```
for( i = 1 ;  i ≤ N ;  i++)
{
    for( j = 1 ; j <= N ; j = j*2)
    {
        . . . .
    }
}
```

| i | j | # iteraⁿ |
|---|---|---|
| 1 | 1 → logN | log N |
| 2 | 1 — logN | log N |
| 3 | 1 — log N | log N |
| ⋮ | | |
| N | 1 — log N | logN |
| | | N logN |

# iterations = $N \log_2 N$        O( N logN)

Quiz 9

```
for( i = 1 ;  i ≤ 4 ;  i++)
{
    for( j = 1 ; j <= i ; j++)
    {
        . . . .
    }
}
```

| i | j | # iteraⁿ |
|---|---|---|
| 1 | [1, 1] | 1 |
| 2 | [1, 2] | 2 |
| 3 | [1, 3] | 3 |
| 4 | [1, 4] | 4 |
| | | 10 |

# iteraⁿ = 10  ⇒ O(1)

Quiz 10

```
for( i = 1 ;  i ≤ N ;  i++)
{
    for( j = 1 ; j <= i ; j++)
    {
        . . . .
    }
}
```

| i | j | # iteraⁿ |
|---|---|---|
| 1 | [1, 1] | 1 |
| 2 | [1, 2] | 2 |
| 3 | [1, 3] | 3 |
| 4 | [1, 4] | 4 |
| ⋮ | | ⋮ |
| N | [ 1, N ] | N |
| | | N(N+1)/2 |

# iteraⁿ = $\frac{N(N+1)}{2}$

$\frac{N^2 + \cancel{N}}{\cancel{2}}$ ⇒ O(N²)

Quiz 11

```
for( i = 1 ; i ≤ N ; i++)
{
    for( j = 1 ; j <= (2^i); j++)
    {
        . . . .
    }
}
```

| i | j | # itera^n |
|---|---|---|
| 1 | $[1, 2^1]$ | 2 |
| 2 | $[1, 2^2]$ | 4 |
| 3 | $[1, 2^3]$ | 8 |
| ⋮ | | |
| N | $[1, 2^N]$ | $2^N$ |

$2 + 4 + 8 + \dots\dots 2^N$

$\boxed{10:45}$

$$\#\ \text{itera}^n\text{'s} = 2 + 4 + 8 + 16 + \dots\dots 2^N$$

$$\text{Sum of G.P} = \frac{a(r^N - 1)}{r - 1}$$

$$= \frac{2(2^N - 1)}{2 - 1} = 2(2^N - 1) \Rightarrow 2^{N+1} \cancel{-2}$$

$$\Rightarrow 2 * 2^N$$

$O(2^N)$

Comparing itera^n's of 2 algo.

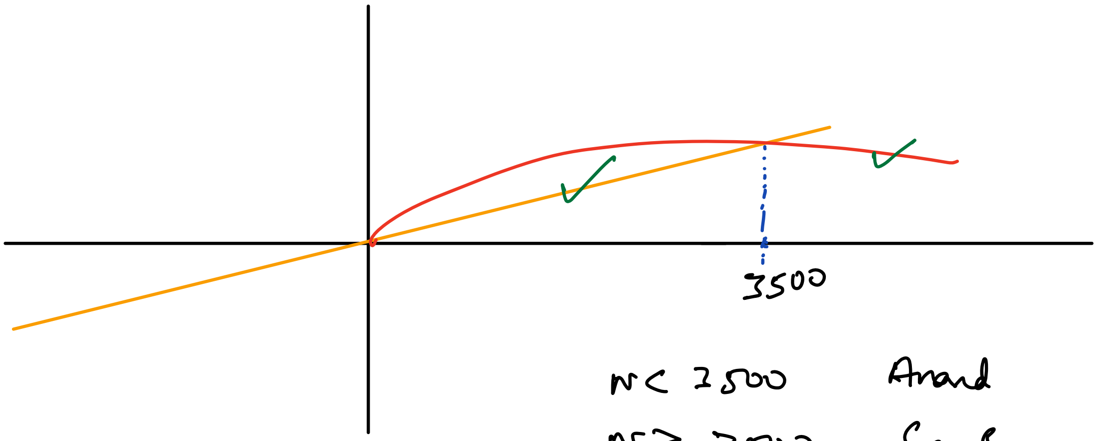
Sanjay

$100 \log_2 N$ (winner)

large inputs.

Anand.

$N/10$



| | |
|---|---|
| N < 3500 | Anand |
| N > 3500 | Sanjay |

obs → in real life, inputs can be very large.

# Asymptotic Analysis of Algorithm. $\rightarrow$ Big(O)

⇒ analyzing performance of algo. for larger i/Ps

⇒ Big(O)

Sanjay (winner) Anand.

$100 \log_2 N \Rightarrow$ $N/10$

To calculate Big(O)

1) calculate iterations
2) Take higher order terms (neglect small order terms)
3) Ignore constant coefficients.

$\cancel{100} \log_2 N \Rightarrow \log_2 N$

Big O

$N > \log N$

$N/\cancel{10} \Rightarrow N$

eg: $\cancel{4}N^2 + \cancel{3N} + \cancel{1}$

$\Rightarrow N^2 \Rightarrow O(N^2)$

Quiz 12

$F(N) = \cancel{4N} + \cancel{3}N \log N + \cancel{1} \Rightarrow O(N \log N)$

Quiz 13

$F(N) = \cancel{4N\log N} + \cancel{3}N\,sqrt(N) + \cancel{10^6}$

$\Rightarrow O(N\sqrt{N})$

$\log N < sqrt(N) < N < N\log N < N\,sqrt(N) < N^2 < N^3 < 2^N < N! < N^N$

## Neglect lower order terms. ?

Aniket's algo → $N^2 + 10N$

| I/P | # itera^n's | % contribu^n of lower order terms. |
|---|---|---|
| $N = 10$ | 200 | 50% |
| $N = 100$ | $10^4 + 10^3$ | $\frac{10^3}{10^4 + 10^3} \simeq 9\%$ |
| $N = 10^4$ | $10^8 + 10^5$ | $\frac{10^5}{10^8 + 10^5} = 0.1\%$ |

**Obs:** as input size increases, contri^n of lower order terms decreases.

## Neglect constant coefficients ?

| Saketh | Aman | For larger i/p. |
|---|---|---|
| $10 \log N$ | $N$ | Saket |
| $100 \log N$ | $N$ | Saket |
| $10^3 \log N$ | $N$ | Saket |
| $10N$ | $N^2/10$ | Saket |
| $N \log N$ | $100N$ | Aman. |

## Drawbacks of Big O

1) $N \log N$    $10^5 N$

⇒ When coefficient is very large, it can affect itera^n's. But since we neglect coefficient while calculating Big O, it can sometime give incorrect comparison.

2)

Biswa — $2N^2 + 4N$ — $O(N^2)$

Ramiz — $3N^2$ — $O(N^2)$

Obs: Since Big(o) is same for both algo, we are not able to compare these 2 algo.

⇒ Sometimes, we cannot compare 2 algo. using Big(O)

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## Time Limit Exceeded (TLE)

Ayush → online amazon interview
↳ reads → idea → code → TLE

## Online Editors

1 GHz ⇒ $10^9$ instrucⁿs / sec

Obs At max our code can have $10^9$ instrucⁿ

## Pseudo code

```
bool func^n (int N)
{
    int c = 0 → 1
  → 1 i = 1 ; i ≤ N → 1 ; i++ → 1
    {
        if ( ______ ) → 1
            c++ → 1
    }
    return c → 1
}
```

6-7 $\text{instruc}^{n}\text{s}$

6N - 7N $\text{instruc}^{n}\text{s}$

## Apprach 1

Our code    1 $\text{itera}^{n}$ → 10 $\text{instruc}^{n}\text{s}$.

Atmam ⟹ $10^9$ $\text{instruc}^{n}\text{s}$

$10 \times 10^8$ ''

$10^8$ iterations

## Apprach 2

Our code    1 $\text{itera}^{n}$ → 100 $\text{instruc}^{n}\text{s}$.

Atmam ⟹ $10^9$ $\text{instruc}^{n}\text{s}$

$100 \times 10^7$ ''

$10^7$ iterations

*⇒ In general, your code can have $10^6, 10^7 \sim 10^8$ itera$^{n}$s.

## Constraints

$$1 \le N \le 10^5$$

Manikant ⇒ T.C ⇒ $O(N^2)$

$N = 10^5$ ⇒ $10^{10}$ iterations. ⇒ no need to code.

→ optimize further.

Shreeram ⇒

$$1 \le N \le 10^3$$

T.C ⇒ $O(N^2)$

$\#$ itera$^{n}$ = $10^6$ ✓

Rohith

$$1 \le N \le 10^4$$

T.C $O(N^2)$

$\#$ itera$^{n}$s = $10^8$ ⇐ grey area

→ need to check.

→ not sure if I will get TLE.