Investigating 3SAT
(Guide presentation for 380CT Coursework 2)

Kamal Bentahar

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Notation

Let $x_1, x_2, \ldots, x_n$ be Boolean variables, and let ϕ be a Boolean formula written in 3-cnf form

$$\phi = c_1 \wedge c_2 \wedge \cdots \wedge c_\ell,$$

where each $c_m = x_i \vee x_j \vee x_k$ for some $i, j, k = 1, 2, \ldots, n$.

Definition of the problem

Decisional 3SAT

Given a boolean expression in 3-CNF form, decide if it is satisfiable.

Computational 3SAT

Given a boolean expression in 3-CNF form, find a satisfying assignment if satisfiable.

Optimization 3SAT

Given a boolean expression in 3-CNF form, find an assignment that minimizes the number of non-satisfying clauses.
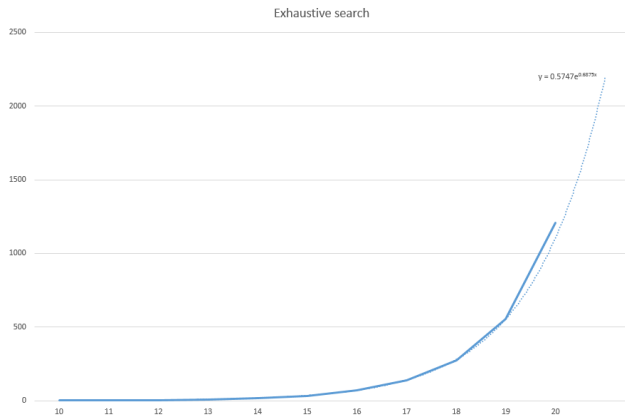
Exhaustive search – theory

1: **for** all possible variable assignments of $x_1, x_2, \ldots, x_n$ **do**
2:     **if** $\phi(x_1, x_2, \ldots, x_n)$ evaluates to True **then**
3:         **return** True
4:     **end if**
5: **end for**
6: **return** False

There are 2^n possible assignments, and each evaluation of ϕ costs $O(\ell)$. So this algorithm costs

$$O(\ell\, 2^n).$$

Exhaustive search – empirical results

Average time in seconds for randomly generated instances with $n = \ell$ for $n = 10, \ldots, 20$.
Dotted line: fitted exponential curve.