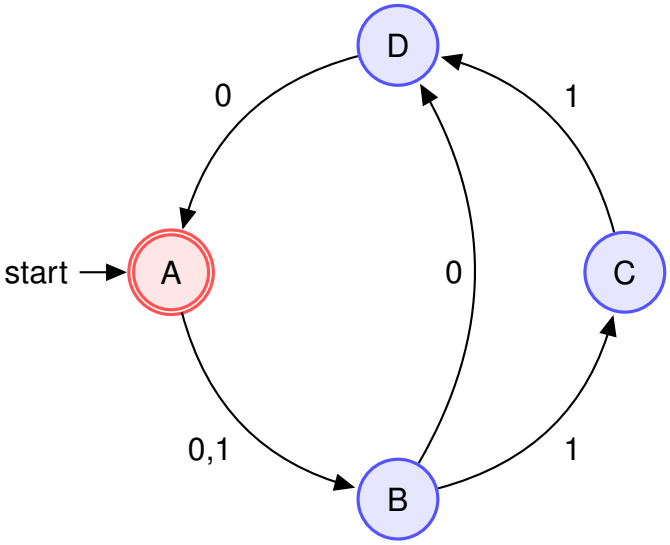
Write your solutions electronically then submit as a PDF file through the Turnitin link on Moodle before **11:55pm on Friday 18 November 2016.**

1 Consider the finite automaton $\mathcal{A}$ given by the following transition diagram:



(2 marks) a) Is $\mathcal{A}$ a Deterministic Finite Automaton (DFA) or a Nondeterministic Finite Automaton (NFA)? Justify your answer.

(5 marks) b) Produce the formal specification of $\mathcal{A}$.

(8 marks) c) List all the strings of length ≤ 4 over the alphabet $\Sigma = \{0, 1\}$, and for each one of them state if it will be accepted or rejected by $\mathcal{A}$.

(10 marks) d) Use the GNFA algorithm to give a regular expression for the language recognized by $\mathcal{A}$.

Make sure to show all the steps – do not only give the final expression.

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2 Prove that the following languages are not regular.

(10 marks) (a) $A = \{\mathtt{a}^n\,\mathtt{b}^{2n}\mathtt{c}^{3n} \mid n \geq 0\}$

(10 marks) (b) $B = \{1^{n^2} \mid n \geq 0\} = \{\varepsilon, 1, 1111, 1^9, 1^{16}, \ldots\}$

(5 marks) (c) $C = \{1^a \mid a \text{ is a prime number}\} = \{11, 111, 11111, 1^7, 1^{11}, 1^{13}, \ldots\}$

3 Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and

- $L_{\mathtt{a}}$ be the language of all strings that start with $\mathtt{a}$ and have odd length.
- $L_{\mathtt{b}}$ be the language of all strings that end with $\mathtt{b}$.

i.e.

$$\begin{aligned} L_{\mathtt{a}} &= \{\mathtt{a}, \mathtt{aaa}, \mathtt{aab}, \mathtt{aba}, \mathtt{abb}, \mathtt{a}^5, \ldots\} \\ L_{\mathtt{b}} &= \{\mathtt{b}, \mathtt{ab}, \mathtt{bb}, \mathtt{aab}, \mathtt{abb}, \mathtt{bab}, \mathtt{bbb}, \ldots\} \end{aligned}$$

(6 marks) (a) Produce regular expressions for $L_{\mathtt{a}}, L_{\mathtt{b}}$, and $L_{\mathtt{a}} \cup L_{\mathtt{b}}$.

(9 marks) (b) Produce ε-NFAs for the languages $L_{\mathtt{a}}, L_{\mathtt{b}}$, and then for $L_{\mathtt{a}} \cup L_{\mathtt{b}}$ using the construction for the union of regular languages.

(c) Design a PDA to recognize the following language

$$L = \{w \mid w = \mathtt{a}^n\mathtt{b}^{2n} \text{ or } w = \mathtt{a}^{3n}\mathtt{b}^{2n} \text{ for } n = 0, 1, 2, \ldots\}.$$

Do this in two steps:

(5 marks) - Explain the idea used.

(5 marks) - Give a state diagram for the PDA.

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4 Consider the following three languages over $\Sigma = \{0, 1\}$:

$$\begin{aligned} \mathcal{L} &= \{w \mid w = 0^{a+b}1^a0^b\} \\ \mathcal{M} &= \{w \mid w = 0^a1^{a+b}0^b\} \\ \mathcal{R} &= \{w \mid w = 0^a1^b0^{a+b}\} \end{aligned}$$

where $a = 0, 1, \ldots$ and $b = 0, 1, \ldots$.

a) Design three Push-Down Automata (PDA) to recognize $\mathcal{L}, \mathcal{M}$ and $\mathcal{R}$.

Do this in two steps:

(7 marks) - Explain the idea(s) used.

- Give the PDA state diagrams.
  You may use JFLAP to draw and export the diagrams, in a suitable format, to

(9 marks) be included in your portfolio.

(9 marks) b) Give Context-Free Grammars (CFGs) for $\mathcal{L}, \mathcal{M}$ and $\mathcal{R}$.