(1) Follow the JFLAP tutorial at http://www.jflap.org/tutorial/fa/createfa/fa.html. Then use JFLAP to draw and simulate some of the DFAs/NFAs discussed in the lecture.

Solution

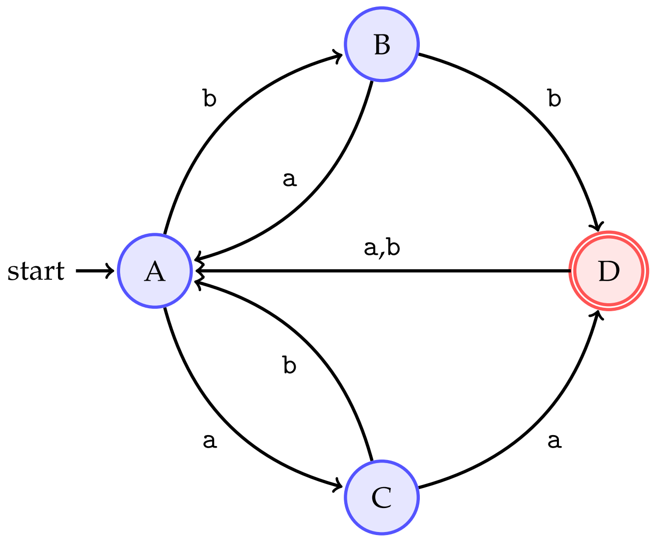
The aim of this exercise is to get used to JFLAP and learn from experimenting with it.

Play with it, develop your intuition and understanding of the concepts, experience the mistakes and errors, the failures and successes!

In particular, if something does not work then ask yourself: what is the source of the problem? How can I fix it?

If it works then what are the transferable skills/knowledge I can use elsewhere?

(2) Consider the following DFA:



Practice *simulating* the behaviour of the above DFA using the following strings

`abba` `babb` `aaa` `bbabba`

For each string, list the sequence of states visited by the DFA (e.g. $q_0, q_2, q_0, q_1, q_3, \ldots$).

Solution

One convenient way of listing the visited states is to use tables as follows:

	A
a	C
b	A
b	B
a	A
	reject

	A
b	B
a	A
b	B
b	D
	accept

	A
a	C
a	D
a	A
	reject

	A
b	B
b	D
a	A
b	B
b	D
a	A
	reject

Produce the formal definition of the above DFA. This should consist of: the alphabet Σ, the set of states Q, the transition function δ, in table form, the start state, and the set of final states F.

Solution

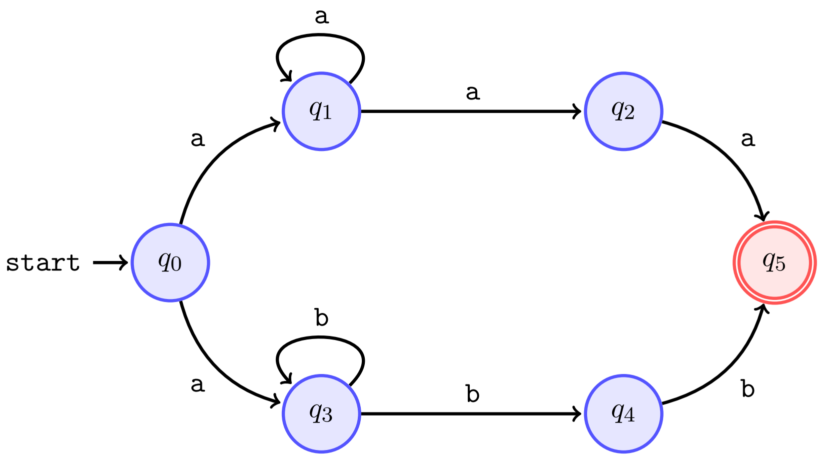
$$\Sigma = \{\mathtt{a}, \mathtt{b}\}$$
$$Q = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$$

δ :

Q	a	b
A	C	B
B	A	D
C	D	A
D	A	A

$$q_{\text{start}} = \mathrm{A}$$
$$F = \{\mathrm{D}\}$$

(3) Consider the following NFA:



Practice *simulating* the behaviour of the NFA using the following strings.

`abbaa` `babb` `aaaba` `abbbbbbbaab`

For each string, list the sets of states visited by the NFA (e.g. $\{q_0\}, \{q_1, q_2\}, \{q_2, q_3\}, \ldots$).

Solution

	$\{q_0\}$
a	$\{q_1, q_3\}$
b	$\{q_3, q_4\}$
b	$\{q_3, q_4, q_5\}$
a	$\emptyset$
	reject

	$\{q_0\}$
b	$\emptyset$
a	$\emptyset$
b	$\emptyset$
b	$\emptyset$
	reject

	$\{q_0\}$
a	$\{q_1, q_3\}$
a	$\{q_1, q_2\}$
a	$\{q_1, q_2, q_5\}$
b	$\emptyset$
a	$\emptyset$
	reject

	$\{q_0\}$
a	$\{q_1, q_3\}$
b	$\{q_3, q_4\}$
b	$\{q_3, q_4, q_5\}$
b	$\{q_3, q_4, q_5\}$
b	$\{q_3, q_4, q_5\}$
b	$\{q_3, q_4, q_5\}$
b	$\{q_3, q_4, q_5\}$
b	$\{q_3, q_4, q_5\}$
a	$\emptyset$
a	$\emptyset$
b	$\emptyset$
	reject

Produce the formal definition $(\Sigma, Q, \delta, q_{\text{start}}, F)$ of the above NFA.

Solution

$$\Sigma = \{\mathtt{a}, \mathtt{b}\}$$
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

δ :

Q	a	b
q_0	$\{q_1, q_3\}$	$\emptyset$
q_1	$\emptyset$	$\{q_1, q_2\}$
q_2	$\{q_5\}$	$\emptyset$
q_3	$\emptyset$	$\{q_3, q_4\}$
q_4	$\emptyset$	$\{q_5\}$
q_5	$\emptyset$	$\emptyset$

$$q_{\text{start}} = q_0$$
$$F = \{q_5\}$$

(4) The formal description $(Q, \Sigma, \delta, q_{\text{start}}, F)$ of a DFA is given by

$$(\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{u}, \mathtt{d}\}, \delta, q_1, \{q_3\}),$$

where δ is given by the following table

	u	d
$\rightarrow q_1$	q_1	q_2
q_2	q_1	q_3
$*q_3$	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

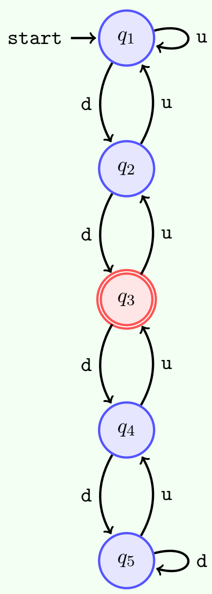
Give the *state diagram* of this machine.

Solution

Hint:

It looks like a ladder or a lift going between floors.

(u: go up, d: go down.)



(5) Use JFLAP to design simple DFAs which recognize the following languages over $\Sigma = \{a, b\}$

a) The language of strings which begin with a.

b) The language of strings which end with b.

c) The language of strings which either begin **or** end with b.

d) The language of strings which begin with a **and** end with b.

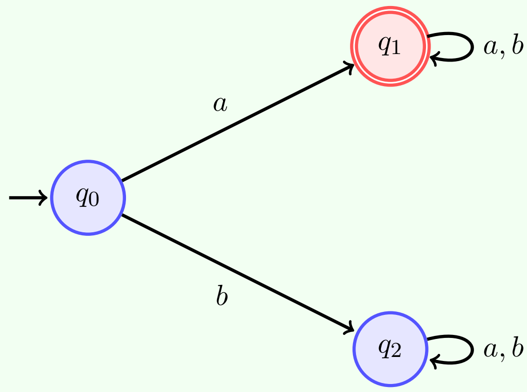
e) The language of strings which contain the substring ba.

f) The language of strings with all the a's on the left and b's on the right

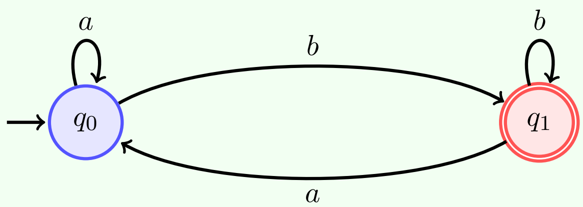
g) The language strings consisting of alternating a's and b's.

Solution

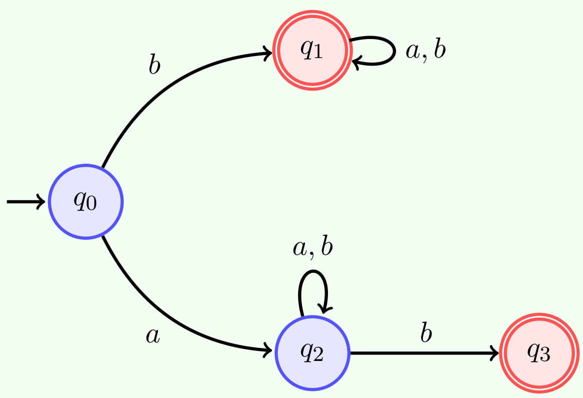
a)



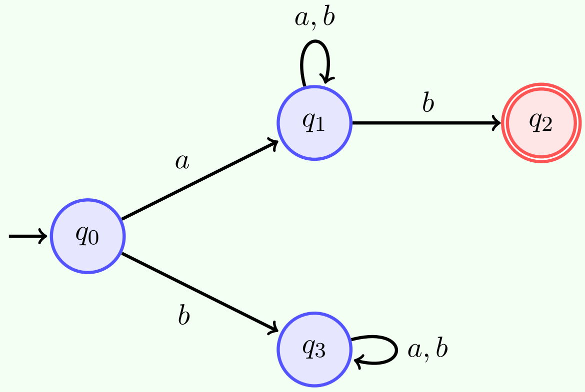
b)



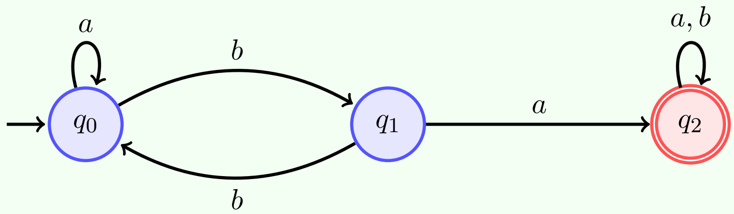
c)



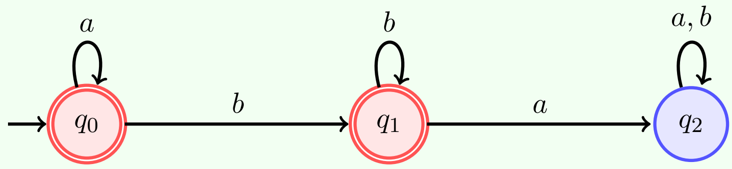
d)



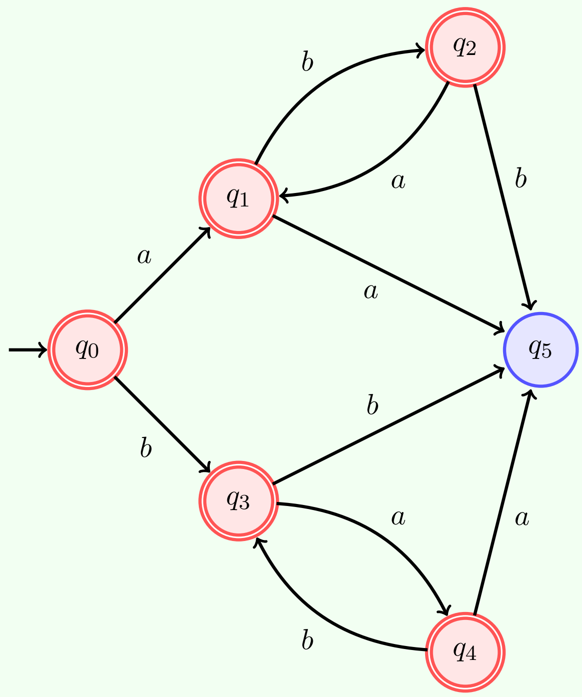
e)



f)



g) Assume that ε (the empty string) also satisfies the required property.



q_5 here only plays the role of a "trap" state.

(6) Use JFLAP to produce NFAs to recognize the following languages over $\Sigma = \{0, 1\}$

a) The language of strings which begin and end with `01`.

b) The language of strings which do not end with `01`.

c) The language of strings which begin and end with different symbols.

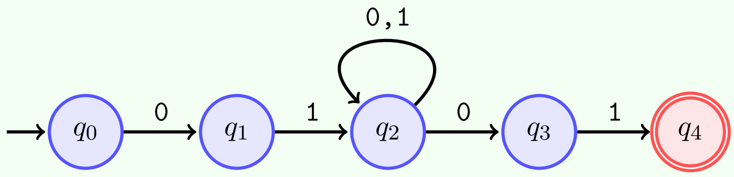
d) The language of strings of odd length.

e) The language of strings which contain an even number of `0`'s.

f) The language of binary numbers which are divisible by 4.

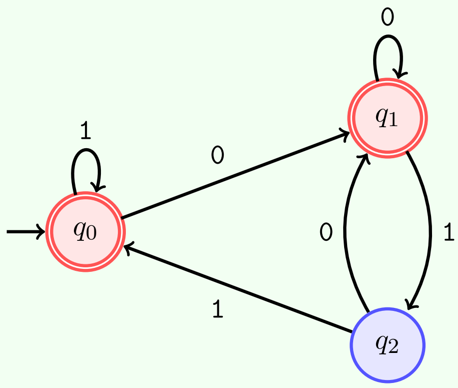
Solution

a)

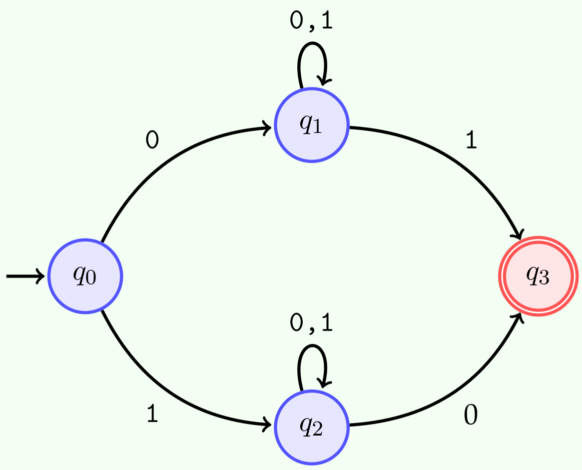


b) Recall that DFAs are a special case of NFAs. For this problem, it may be easier to first create a DFA that accepts *strings that* ***end*** *with* `01`, then we flip the accepting states into non-accepting ones, and vice versa. This will produce a DFA that accepts *strings that* ***do not end*** *with* `01`, as required.

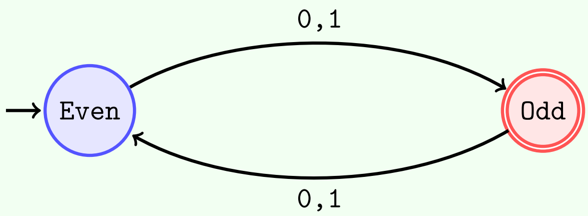
This is a useful technique, but note that **it only works for DFAs – it does not work for NFAs.**



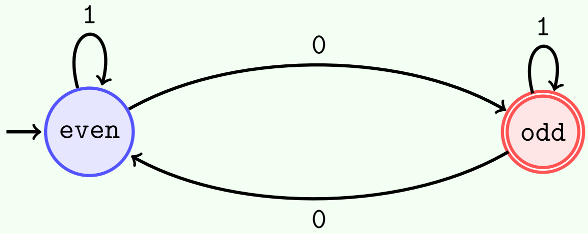
c)



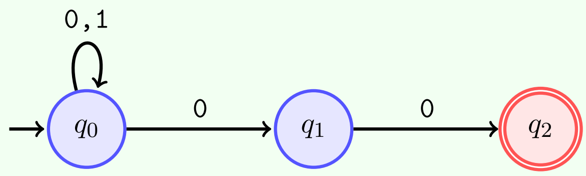
d)



e)



f) A number is divisible by 4 if its binary representation ends with 00.



(7) If a is a *symbol* from an alphabet Σ then $\mathtt{a}^n$ denotes the string which consists of n successive copies of a.

Similarly, if x is a *string* of symbols then x^n denotes the string which consists of n successive copies of x. For example, $\mathtt{a}^2 = \mathtt{aa}$ and $(\mathtt{ab})^2 = \mathtt{abab}$.

Let $\Sigma = \{0, 1\}$. Write $0^4, 1^4, (10)^3, 10^3$ explicitly as strings in the usual form.

Solution

$$\begin{aligned} 0^4 &= 0000 \\ 1^4 &= 1111 \\ (10)^3 &= 101010 \\ 10^3 &= 1000 \end{aligned}$$

(8) If Σ is an alphabet then Σ^n denotes the set of all strings over Σ which have length exactly n symbols.

a) Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Find Σ^2.

b) Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Find Σ^3.

Solution

a) For $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

$$\Sigma^2 = \{\mathtt{aa}, \mathtt{ab}, \mathtt{ac}, \mathtt{ba}, \mathtt{bb}, \mathtt{bc}\}$$

b) For $\Sigma = \{\mathtt{a}, \mathtt{b}\}$:

$$\Sigma^3 = \{\mathtt{aaa}, \mathtt{aab}, \mathtt{aba}, \mathtt{abb}, \mathtt{baa}, \mathtt{bab}, \mathtt{bba}, \mathtt{bbb}\}$$

(9) If Σ is an alphabet then the set of all finite-length strings over it is denoted by Σ^*.

Let $\Sigma_1 = \{\mathtt{a}\}$ and $\Sigma_2 = \{\mathtt{a}, \mathtt{b}\}$. List the strings of length 0, 1, 2, 3, and 4 over these two alphabets. Write these in the form $\Sigma_1^* = \{\dots\}$ and $\Sigma_2^* = \{\dots\}$.

Note that

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \cup \cdots.$$

Solution

For $\Sigma_1 = \{\mathtt{a}\}$:

Length	Strings
0	ε
1	a
2	aa
3	aaa
4	aaaa

So,

$$\Sigma_1^* = \{\varepsilon, \mathtt{a}, \mathtt{aa}, \mathtt{aaa}, \mathtt{aaaa}, \dots\}$$

For $\Sigma_2 = \{\mathtt{a}, \mathtt{b}\}$:

Length	Strings							
0	ε							
1	a	b						
2	aa	ab	ba	bb				
3	aaa	aab	aba	abb	baa	bab	bba	bbb
4	aaaa	aaab	aaba	aabb	abaa	abab	abba	abbb
	baaa	baab	baba	babb	bbaa	bbab	bbba	bbbb

So,

$$\begin{aligned}\Sigma_2^* = \{&\varepsilon, \mathtt{a}, \mathtt{b}, \mathtt{aa}, \mathtt{ab}, \mathtt{ba}, \mathtt{bb}, \mathtt{aaa}, \mathtt{aab}, \mathtt{aba}, \mathtt{abb}, \mathtt{baa}, \mathtt{bab}, \mathtt{bba}, \mathtt{bbb},\\ &\mathtt{aaaa}, \mathtt{aaab}, \mathtt{aaba}, \mathtt{aabb}, \mathtt{abaa}, \mathtt{abab}, \mathtt{abba}, \mathtt{abbb},\\ &\mathtt{baaa}, \mathtt{baab}, \mathtt{baba}, \mathtt{babb}, \mathtt{bbaa}, \mathtt{bbab}, \mathtt{bbba}, \mathtt{bbbb}, \dots\}\end{aligned}$$