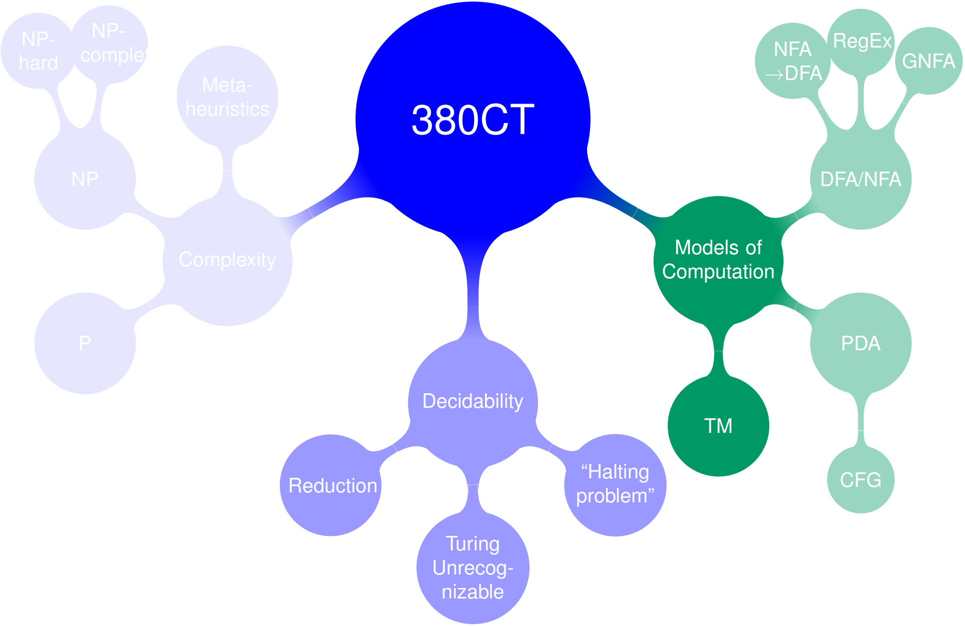
Turing Machines (TMs)

Dr Kamal Bentahar

School of Engineering, Environment and Computing
Coventry University

7/11/2016

History
Turing Machines
Examples
Generalizations


380CT
Complexity
NP
NP-hard
NP-complete
Meta-heuristics
P
Decidability
Reduction
Turing Unrecog-nizable
"Halting problem"
Models of Computation
DFA/NFA
NFA →DFA
RegEx
GNFA
PDA
CFG
TM

History: nature of computing

Questions about this first arose in the context of pure Mathematics:

- Gottlob Frege (1848–1925)
- David Hilbert (1862–1943)
- George Cantor (1845–1918)
- Kurt Gödel (1906–1978)
- 1936:
  - Gödel and Stephen Kleene (1909-1994): **Partial Recursive Functions**
  - Gödel, Kleene and Jacques Herbrand (1908–1931)
  - Alonzo Church (1903–1995): **Lambda Calculus**
  - Alan Turing (1912–1954): **Turing Machine**
- 1943: Emil Post (1897–1954): **Post Systems**
- 1954: A.A. Markov: Theory of Algorithms – **Grammars**
- 1963: Shepherdson and Sturgis: **Universal Register Machines**

The Church-Turing Thesis

- It turns out that the “Turing Machine model” and *all* the other models of general purpose computation that have been proposed are equivalent!

The Church-Turing Thesis

- It turns out that the "Turing Machine model" and *all* the other models of general purpose computation that have been proposed are equivalent!
- They all share the essential feature of **unrestricted access to unlimited memory.**

  As opposed to the DFA/NFA/PDA models for example.

The Church-Turing Thesis

- It turns out that the “Turing Machine model” and *all* the other models of general purpose computation that have been proposed are equivalent!
- They all share the essential feature of **unrestricted access to unlimited memory.**

  As opposed to the DFA/NFA/PDA models for example.
- They all satisfy reasonable requirements such as the ability to perform only a finite amount of work in a single step.

The Church-Turing Thesis

- It turns out that the "Turing Machine model" and *all* the other models of general purpose computation that have been proposed are equivalent!
- They all share the essential feature of **unrestricted access to unlimited memory.**

  As opposed to the DFA/NFA/PDA models for example.
- They all satisfy reasonable requirements such as the ability to perform only a finite amount of work in a single step.
- They all can **simulate** each other!

The Church-Turing Thesis

- It turns out that the “Turing Machine model” and *all* the other models of general purpose computation that have been proposed are equivalent!
- They all share the essential feature of **unrestricted access to unlimited memory.**

  As opposed to the DFA/NFA/PDA models for example.
- They all satisfy reasonable requirements such as the ability to perform only a finite amount of work in a single step.
- They all can **simulate** each other!

The Church-Turing Thesis

- It turns out that the "Turing Machine model" and *all* the other models of general purpose computation that have been proposed are equivalent!
- They all share the essential feature of **unrestricted access to unlimited memory.**

  As opposed to the DFA/NFA/PDA models for example.
- They all satisfy reasonable requirements such as the ability to perform only a finite amount of work in a single step.
- They all can **simulate** each other!

Philosophical Corollary: Church-Turing Thesis

Every *effective computation* can be carried out by a TM.

i.e. *algorithmically computable* $\iff$ computable by a TM.

The Church-Turing Thesis

- It turns out that the “Turing Machine model” and *all* the other models of general purpose computation that have been proposed are equivalent!
- They all share the essential feature of **unrestricted access to unlimited memory.**

  As opposed to the DFA/NFA/PDA models for example.
- They all satisfy reasonable requirements such as the ability to perform only a finite amount of work in a single step.
- They all can **simulate** each other!

Philosophical Corollary: Church-Turing Thesis

Every *effective computation* can be carried out by a TM.

i.e. *algorithmically computable* $\iff$ computable by a TM.

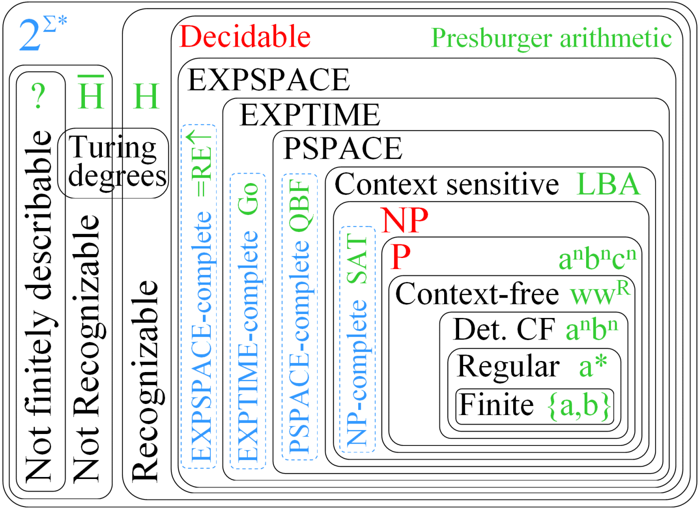
See http://plato.stanford.edu/entries/church-turing/ and http://en.wikipedia.org/wiki/Church-Turing_thesis for discussion.

Algorithms

In a sense, the Church-Turing thesis implies that the underlying class of "algorithms" described by all these models of computation is the same, and corresponds to the natural intuitive concept of *algorithms*.

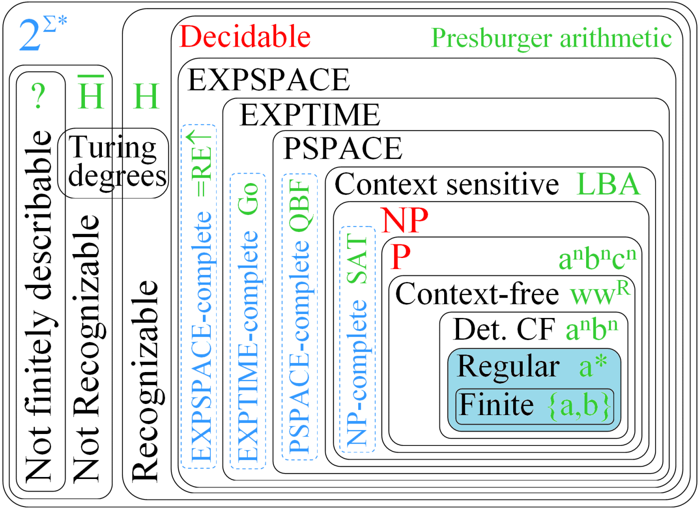
Intuitive concept of algorithms $=$ Turing machine algorithms

The Extended Chomsky Hierarchy

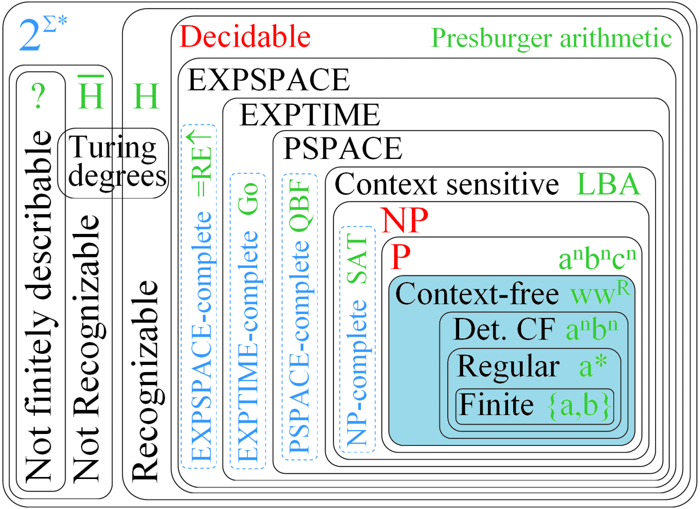


Chomsky Hierarchy

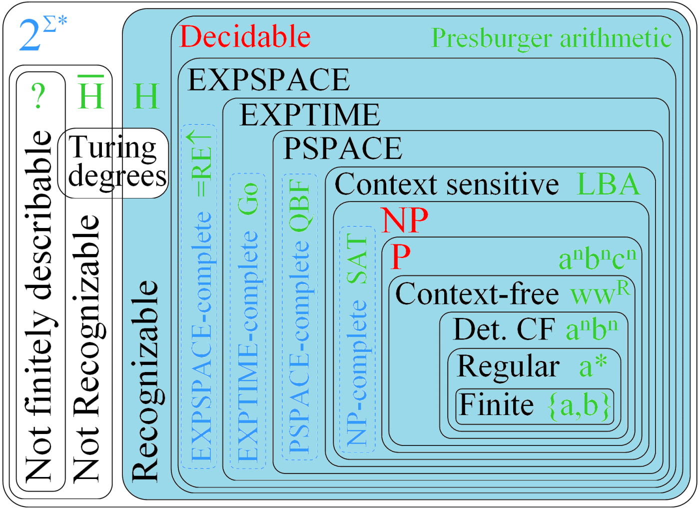
The Extended Chomsky Hierarchy



The Extended Chomsky Hierarchy



The Extended Chomsky Hierarchy



Chomsky Hierarchy

Grammar	**Languages**	**Automaton**	**Production rules**
Type-0	Recursively Enumerable	Turing Machine (TM)	$\alpha \rightarrow \beta$ (no restrictions)
Type-1	Context Sensitive	Linear-bounded TM	$\alpha A \beta \rightarrow \alpha \gamma \beta$
Type-2	Context Free	PDA	$A \rightarrow \gamma$
Type-3	Regular	NFA/DFA	$A \rightarrow aB \mid a$

$a, b, \ldots$ *Terminals – constitute the strings of the language*
$A, B, \ldots$ *Non-terminals – should be replaced*
$\alpha, \beta, \ldots$ *Combinations of the above*

TMs vs NFAs/PDAs

- TM are similar to NFA/PDA, but has access to unlimited memory.

TMs vs NFAs/PDAs

- TM are similar to NFA/PDA, but has access to unlimited memory.
- No known model of computation is more powerful than a TM.

TMs vs NFAs/PDAs

- TM are similar to NFA/PDA, but has access to unlimited memory.
- No known model of computation is more powerful than a TM.

TMs vs NFAs/PDAs

- TM are similar to NFA/PDA, but has access to unlimited memory.
- No known model of computation is more powerful than a TM.

The main differences are:

1. TM may store the entire input string and refer to it **as often as needed**.

TMs vs NFAs/PDAs

- TM are similar to NFA/PDA, but has access to unlimited memory.
- No known model of computation is more powerful than a TM.

The main differences are:

1. TM may store the entire input string and refer to it **as often as needed**.
2. Special states for **accepting** and **rejecting** which take **immediate effect**. (No need to reach the end of the input string.)
   The TM has the potential to go on with its computation for ever, without reaching either an accept or reject state $\rightarrow$ the **“Halting Problem”**.

Turing Machine

- A Turing machine has an **infinite tape** (memory).

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.
- Initially: tape contains only the *input string*; *blank everywhere else*.

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.
- Initially: tape contains only the *input string*; *blank everywhere else*.
- If the machine needs to store information, it can write it on the tape.

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.
- Initially: tape contains only the *input string*; *blank everywhere else*.
- If the machine needs to store information, it can write it on the tape.
- It has designated **accept** and **reject** states.

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.
- Initially: tape contains only the *input string*; *blank everywhere else*.
- If the machine needs to store information, it can write it on the tape.
- It has designated **accept** and **reject** states.
- The machine can only terminate on reaching one or the other; otherwise, it will just keep going. . . !

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.
- Initially: tape contains only the *input string*; *blank everywhere else*.
- If the machine needs to store information, it can write it on the tape.
- It has designated **accept** and **reject** states.
- The machine can only terminate on reaching one or the other; otherwise, it will just keep going. . . !
- The TM's **transition function** is defined according to the *state* of the machine and the *symbol currently being read* by the tape head.

Turing Machine

- A Turing machine has an **infinite tape** (memory).
- It has a **tape head**, which may **read** and **write** symbols and **move** around the tape.
- Initially: tape contains only the *input string*; *blank everywhere else*.
- If the machine needs to store information, it can write it on the tape.
- It has designated **accept** and **reject** states.
- The machine can only terminate on reaching one or the other; otherwise, it will just keep going. . . !
- The TM's **transition function** is defined according to the *state* of the machine and the *symbol currently being read* by the tape head.
- Given a **state and symbol** pair, the machine may **change state, write a symbol** onto the tape and **move left or right** by one space.

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.
- As the TM computes, changes occur in the current state, the current tape contents and the current head location.

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.
- As the TM computes, changes occur in the current state, the current tape contents and the current head location.
- A set of these three items is called a **configuration**

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.
- As the TM computes, changes occur in the current state, the current tape contents and the current head location.
- A set of these three items is called a **configuration**

Configuration may be represented in the form uqv, where

e.g. If the tape contents are 10010, the machine is in state q_6, and the head is over the second zero we write: $10q_6010$

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.
- As the TM computes, changes occur in the current state, the current tape contents and the current head location.
- A set of these three items is called a **configuration**

Configuration may be represented in the form uqv, where

- u is the string of all symbols to the left of the head

e.g. If the tape contents are 10010, the machine is in state q_6, and the head is over the second zero we write: $10q_6010$

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.
- As the TM computes, changes occur in the current state, the current tape contents and the current head location.
- A set of these three items is called a **configuration**

Configuration may be represented in the form uqv, where

- u is the string of all symbols to the left of the head
- v is the string consisting of the symbol at the current head location, followed by all symbols to the right of it

e.g. If the tape contents are 10010, the machine is in state q_6, and the head is over the second zero we write: $10q_6010$

Turing Machine Computation

- The input is placed on the tape. The rest of the tape is blank.
- The head starts on the leftmost square of the input string.
- The computation proceeds according to the rules of δ.
- Computation continues until it enters either an accept or reject state.
- As the TM computes, changes occur in the current state, the current tape contents and the current head location.
- A set of these three items is called a **configuration**

Configuration may be represented in the form uqv, where

- u is the string of all symbols to the left of the head
- v is the string consisting of the symbol at the current head location, followed by all symbols to the right of it
- q is the current state

e.g. If the tape contents are 10010, the machine is in state q_6, and the head is over the second zero we write: $10q_6010$

Turing Machines (TMs) – languages

Decidable languages

A language is (Turing) **decidable** if some TM *decides* it.

i.e. given a string w:

- if w is **in** the language: the TM will **accept** it.
- it w is **not in** the language: the TM will **reject** it.

Turing Machines (TMs) – languages

Decidable languages

A language is (Turing) **decidable** if some TM *decides* it.

i.e. given a string w:

- if w is **in** the language: the TM will **accept** it.
- it w is **not in** the language: the TM will **reject** it.
  Such TMs are called **deciders.**

Turing Machines (TMs) – languages

Decidable languages

A language is (Turing) **decidable** if some TM *decides* it.

i.e. given a string w:

- if w is **in** the language: the TM will **accept** it.
- it w is **not in** the language: the TM will **reject** it.
  Such TMs are called **deciders.**

Turing recognizable languages

A language is **Turing recognizable** if some TM *recognizes* it.

i.e. given a string w:

- if w is **in** the language: the TM will **accept** it.
- it w is **not in** the language then the TM may **reject** it or **never halt**.

Description of algorithms and TMs

Specification can be at one of 3 levels of detail:

1. Formal description (Transition diagrams, etc.).

Description of algorithms and TMs

Specification can be at one of 3 levels of detail:

1. Formal description (Transition diagrams, etc.).
2. Implementation description.
   (Describe how TM manages tape and moves head).

Description of algorithms and TMs

Specification can be at one of 3 levels of detail:

1. Formal description (Transition diagrams, etc.).
2. Implementation description.
   (Describe how TM manages tape and moves head).
3. High-level description (Pseudocode or higher).

Description of algorithms and TMs

Specification can be at one of 3 levels of detail:

1. Formal description (Transition diagrams, etc.).
2. Implementation description.
   (Describe how TM manages tape and moves head).
3. High-level description (Pseudocode or higher).

Description of algorithms and TMs

Specification can be at one of 3 levels of detail:

1. Formal description (Transition diagrams, etc.).
2. Implementation description.
   (Describe how TM manages tape and moves head).
3. High-level description (Pseudocode or higher).

Also, we usually specify how to **encode** objects (if not “standard”), and the exact **input** and **output**.

Example (TM to recognize $\{w\#w \mid w = \{0,1\}^*\}$)

- Scan the input to check it contains only a single # symbol.
  If not, reject.

Task: Trace the following inputs

`01#01` `01#011` `011#01` `01##01`

Example (TM to recognize $\{w\#w \mid w = \{0,1\}^*\}$)

- Scan the input to check it contains only a single # symbol.
  If not, reject.
- Zig-zag across the tape to corresponding symbols on either side of the # symbol, crossing off each one.
  If they are not the same, reject.

Task: Trace the following inputs

$\texttt{01}\#\texttt{01}$ $\texttt{01}\#\texttt{011}$ $\texttt{011}\#\texttt{01}$ $\texttt{01}\#\#\texttt{01}$

Example (TM to recognize $\{w\#w \mid w = \{0,1\}^*\}$)

- Scan the input to check it contains only a single # symbol.
  If not, reject.
- Zig-zag across the tape to corresponding symbols on either side of the # symbol, crossing off each one.
  If they are not the same, reject.
- When all symbols to the left of the # are crossed off, check for remaining symbols to the right. If there are reject, otherwise accept.

Task: Trace the following inputs

$$\texttt{01\#01} \quad \texttt{01\#011} \quad \texttt{011\#01} \quad \texttt{01\#\#01}$$

Example (TM to recognize $\{\mathtt{0}^{2^n} \mid n \geq 0\}$)

This language consists of all strings of 0's whose length is a power of 2.

1. Sweep left to right across the tape, crossing off every other 0.

Task: Trace the following inputs

$$\mathtt{0}, \mathtt{0}^2, \mathtt{0}^3, \mathtt{0}^4, \mathtt{0}^7$$

Example (TM to recognize $\{\mathtt{0}^{2^n} \mid n \geq 0\}$)

This language consists of all strings of 0's whose length is a power of 2.

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0 then accept.

Task: Trace the following inputs

$$\mathtt{0}, \mathtt{0}^2, \mathtt{0}^3, \mathtt{0}^4, \mathtt{0}^7$$

Example (TM to recognize $\{\mathtt{0}^{2^n} \mid n \geq 0\}$)

This language consists of all strings of `0`’s whose length is a power of 2.

1. Sweep left to right across the tape, crossing off every other `0`.
2. If in stage 1 the tape contained a single `0` then accept.
3. If in stage 1 the tape contained more than a single `0` and the number of `0`’s was odd then reject.

Task: Trace the following inputs

$$\mathtt{0}, \mathtt{0}^2, \mathtt{0}^3, \mathtt{0}^4, \mathtt{0}^7$$

Example (TM to recognize $\{0^{2^n} \mid n \geq 0\}$)

This language consists of all strings of 0's whose length is a power of 2.

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0 then accept.
3. If in stage 1 the tape contained more than a single 0 and the number of 0's was odd then reject.
4. Return to the left hand end of the tape.

Task: Trace the following inputs

$$0, 0^2, 0^3, 0^4, 0^7$$

Example (TM to recognize $\{\mathtt{0}^{2^n} \mid n \geq 0\}$)

This language consists of all strings of 0's whose length is a power of 2.

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0 then accept.
3. If in stage 1 the tape contained more than a single 0 and the number of 0's was odd then reject.
4. Return to the left hand end of the tape.
5. Go to stage 1.

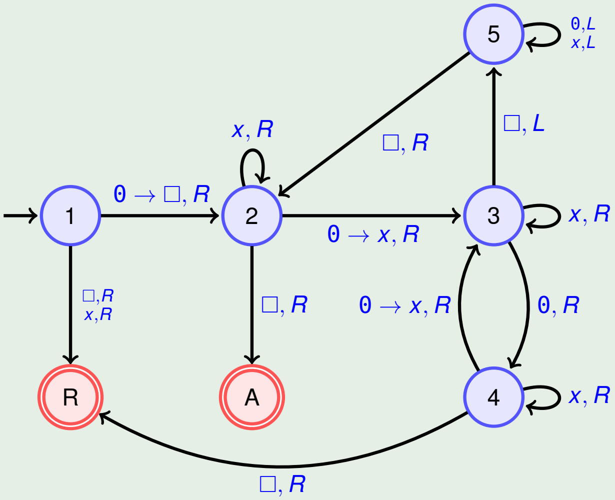
Task: Trace the following inputs

$$\mathtt{0}, \mathtt{0}^2, \mathtt{0}^3, \mathtt{0}^4, \mathtt{0}^7$$

Example (TM to recognize $A = \{0^{2^n} \mid n \geq 0\}$)

Formal description:

- $Q = \{1, 2, 3, 4, 5, A, R\}$
- $\Sigma = \{0\}$
- $\Gamma = \{0, x, \square\}$
- The start, accept and reject states are 1, A and R, respectively.
- δ is given by the state diagram:



Notation:
$a \to b, R$: on reading a on the tape: replace it with b,then move to the right.
a, R: shorthand for $a \to a, R$

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states

History
Turing Machines
Examples
Generalizations

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states
- Σ is the input alphabet, not containing the special *blank symbol*: $\square$

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states
- Σ is the input alphabet, not containing the special *blank symbol*: $\square$
- Γ is the tape alphabet, where $\square \in \Gamma$ and $\Sigma \subset \Gamma$

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states
- Σ is the input alphabet, not containing the special *blank symbol*: $\square$
- Γ is the tape alphabet, where $\square \in \Gamma$ and $\Sigma \subset \Gamma$
- $\delta \colon Q \times \Gamma \to Q \times \Gamma \times \{R, L\}$ is the transition function

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states
- Σ is the input alphabet, not containing the special *blank symbol*: $\square$
- Γ is the tape alphabet, where $\square \in \Gamma$ and $\Sigma \subset \Gamma$
- $\delta \colon Q \times \Gamma \to Q \times \Gamma \times \{R, L\}$ is the transition function
- q_{start} is the start state

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states
- Σ is the input alphabet, not containing the special *blank symbol*: $\square$
- Γ is the tape alphabet, where $\square \in \Gamma$ and $\Sigma \subset \Gamma$
- $\delta \colon Q \times \Gamma \to Q \times \Gamma \times \{R, L\}$ is the transition function
- q_{start} is the start state
- q_{accept} is the accept state

Formal Definition of a TM

A Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_{\text{start}}, q_{\text{accept}}, q_{\text{reject}})$ where

- Q is the finite set of states
- Σ is the input alphabet, not containing the special *blank symbol*: $\square$
- Γ is the tape alphabet, where $\square \in \Gamma$ and $\Sigma \subset \Gamma$
- $\delta \colon Q \times \Gamma \to Q \times \Gamma \times \{R, L\}$ is the transition function
- q_{start} is the start state
- q_{accept} is the accept state
- q_{reject} is the reject state, where $q_{\text{accept}} \neq q_{\text{reject}}$

Multi-tape TMs

- A **multi-tape TM**, is a TM with *more than one tape*.

Multi-tape TMs

- A **multi-tape TM**, is a TM with *more than one tape*.
- More transitions need to be defined, but it **simplifies computations**

Multi-tape TMs

- A **multi-tape TM**, is a TM with *more than one tape*.
- More transitions need to be defined, but it **simplifies computations**

Example ($\{w\#w \mid w = \{0, 1\}^*\}$)

Multi-tape TMs

- A **multi-tape TM**, is a TM with *more than one tape*.
- More transitions need to be defined, but it **simplifies computations**

Example ($\{w\#w \mid w = \{0,1\}^*\}$)

One-tape: zig-zag around $\#$ crossing off matching symbols.
Requires two loops.

Multi-tape TMs

- A **multi-tape TM**, is a TM with *more than one tape*.
- More transitions need to be defined, but it **simplifies computations**

Example ($\{w\#w \mid w = \{0,1\}^*\}$)

One-tape: zig-zag around $\#$ crossing off matching symbols. Requires two loops.

Multi-tape: write the second half in the second tape, then use a single loop to check it matches the first half.

Multi-tape TMs

- A **multi-tape TM**, is a TM with *more than one tape*.
- More transitions need to be defined, but it **simplifies computations**

Example ($\{w\#w \mid w = \{0,1\}^*\}$)

One-tape: zig-zag around $\#$ crossing off matching symbols. Requires two loops.

Multi-tape: write the second half in the second tape, then use a single loop to check it matches the first half.

Equivalence

Every multi-tape TM has an equivalent single-tape TM.

Nondeterministic Turing Machines (NTMs)

- A configuration can have **zero or more** subsequent configurations.
  $\rightarrow$ The machine may be in many configurations at the same time.
  Imagine the TM self-replicating as it goes along.

  Interestingly, the closest real thing we have to an NTM is **DNA computation**, as the processed units are artificially manufactured chromosomes (capable of self-replication). This still is not really nondeterministic as there is a finite limit to the number of DNA strands which may exist during computation.

Nondeterministic Turing Machines (NTMs)

- A configuration can have **zero or more** subsequent configurations. $\rightarrow$ The machine may be in many configurations at the same time. Imagine the TM self-replicating as it goes along.

  Interestingly, the closest real thing we have to an NTM is **DNA computation**, as the processed units are artificially manufactured chromosomes (capable of self-replication). This still is not really nondeterministic as there is a finite limit to the number of DNA strands which may exist during computation.

- Subtlety: If a non-deterministic TM is a decider then **all** branches need to reject for it to **reject** a string.

Nondeterministic Turing Machines (NTMs)

- A configuration can have **zero or more** subsequent configurations.
  $\rightarrow$ The machine may be in many configurations at the same time. Imagine the TM self-replicating as it goes along.

  Interestingly, the closest real thing we have to an NTM is **DNA computation**, as the processed units are artificially manufactured chromosomes (capable of self-replication). This still is not really nondeterministic as there is a finite limit to the number of DNA strands which may exist during computation.
- Subtlety: If a non-deterministic TM is a decider then **all** branches need to reject for it to **reject** a string.
- Deterministic and nondeterministic TMs recognize the same languages!

Nondeterministic Turing Machines (NTMs)

- A configuration can have **zero or more** subsequent configurations.
  $\rightarrow$ The machine may be in many configurations at the same time.
  Imagine the TM self-replicating as it goes along.

  Interestingly, the closest real thing we have to an NTM is **DNA computation**, as the processed units are artificially manufactured chromosomes (capable of self-replication). This still is not really nondeterministic as there is a finite limit to the number of DNA strands which may exist during computation.
- Subtlety: If a non-deterministic TM is a decider then **all** branches need to reject for it to **reject** a string.
- Deterministic and nondeterministic TMs recognize the same languages!

Equivalence

Every NTM has an equivalent deterministic TM.

Next week...

Limits of computation...

Even a TM cannot solve certain problems!

Such problems are beyond the theoretical limits of computation (unsolvable)