Collinear factorization in deep inelastic lepton-nucleus collisions

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(Dated: 17 Jan 2010)

We present a formalism for inelastic lepton-nucleus collisions at high energies that includes quasi-elestic and deep-inelastic nucleon scattering. The nuclear target dynamics is described by microscopic many-body theory with full account of relativistic kinematics. .... Initial results are presented and compared with available data on lepton-nucleus cross sections, with special attention to the large Bjorken x_B events. Predictions for future experiments are also given.
...Abstract in progress... TO BE CHANGED....

PACS numbers: 12.38.Bx, 13.60.Hb, 21.30.-x, 24.85.+p
Keywords: two- and three-nucleon correlations, scaling at large x_B, collinear factorization

I. INTRODUCTION.

II. THE NUCLEAR HADRONIC TENSOR

Within the Born approximation, the cross section of lepton-nucleus scattering is determined by the nuclear hadronic tensor $W_A^{\mu\nu}$ normalized to the number A of nucleons,

$$\begin{aligned} &AW_A^{\mu\nu}(P_A,q) \\ &\quad = \frac{1}{8\pi}\overline{\sum}_X \langle A_{\rm in}|J^{\dagger\mu}(0)|X_{\rm out}\rangle\langle X_{\rm out}|J^{\nu}(0)|A_{\rm in}\rangle \\ &\quad \times (2\pi)^4\delta^{(4)}(P_A+q-p_X)\,, \end{aligned} \tag{1}$$

with nuclear momentum $P_A = Ap_A$, virtual photon momentum q and corresponding current J, and total final state momentum p_X. The symbol $\overline{\sum}_X$ denotes the sum over final states X and integration over their phase space; the integration measure will be made explicit when needed to avoid confusion. The expectation values are taken over the target nucleus in-state $|A_{\rm in}\rangle$ and the final state particles out-state $|X_{\rm out}\rangle$. In the following, we will suppress the "in" labels unless needed. We picture the nucleus as a system of A bound nucleons whose momentum distributions ("Fermi motion") are given by nuclear many-body wave functions. Non-nucleonic degrees of freedom, such as a pion cloud, can be included in the formalism in a second step. Next, we assume the Hilbert space of the nucleus to be the direct product of A single-nucleon Hilbert spaces,

$$\mathcal{H}^{\rm in} \approx \mathcal{H}_1^{\rm in}\otimes\mathcal{H}_2^{\rm in}\otimes\cdots\otimes\mathcal{H}_A^{\rm in} \tag{2}$$

with an orthonormal, antisymmetric and complete basis

$$\big\{|k_1,k_2,\ldots,k_A\rangle = \mathcal{A}|k_1\rangle|k_2\rangle\cdots|k_A\rangle\big\}\ , \tag{3}$$

where $\mathcal{A}$ antysimmetrizes identical nucleons, $|k_i\rangle$ are free particle states of 4-momentum k_i normalized such that

$$\langle k_i|k_j\rangle = (2\pi)^4\delta^{(4)}(k_i-k_j)\ , \tag{4}$$

and complete in their own subspace,

$$\int (d^4k_i)|k_i\rangle\langle k_i| = \mathbb{I} \quad \forall i, \tag{5}$$

where we use the notation $(d^4k) = d^4k/(2\pi)^4$, and $\mathbb{I}$ is the unit operator. For simplicity, we are omitting the nucleon quantum numbers in our notation. The nuclear wave function ψ_A can be defined as

$$\begin{aligned} &\langle A|k_1,k_2,\ldots,k_A\rangle \\ &\qquad = \psi_A(k_1,k_2,\ldots,k_A)\delta^{(4)}\Big(P_A-\sum_{i=1}^A k_i\Big)\ , \end{aligned} \tag{6}$$

where we imposed 4-momentum conservation. At large Bjorken $x_B = Q^2/2p_A{\cdot}q$ and large Q^2 with $Q^2=-q^2$, the virtual photon is very localized in space-time, so that the virtual photon interacts with only one nucleon at a time, see Fig. 1. In this impulse approximation, the current operator J is the sum of 1-body current operators each acting on a different single-nucleon subspace,

$$J^\mu = \sum_{i=1}^A j_i^\mu \tag{7}$$

where $[j_i^\mu, j_l^\nu] = 0$ if $i \neq l$. As a consequence, we can decompose the final state Hilbert space as follows:

$$\mathcal{H}^{\rm out} \approx \mathcal{H}_1^{\rm out}\otimes\mathcal{H}_2^{\rm out}\otimes\cdots\otimes\mathcal{H}_A^{\rm out}\otimes\mathcal{R} \tag{8}$$

with j_i acting only on $\mathcal{H}_i^{\rm out}$. Note that we don't assume $\{j_i\}$ to be a complete set of operators, hence the presence of a subspace $\mathcal{R}$ in the above orthogonal decomposition. The orthonormal, antysimmetrized and complete basis of $\mathcal{H}^{\rm out}$ can be written as

$$\begin{aligned} &\big\{|h_{1,\rm out},h_{2,\rm out},\ldots,h_{A,\rm out},r_{\rm out}\rangle \\ &\qquad = \mathcal{A}|h_{1,\rm out}\rangle|h_{2,\rm out}\rangle\cdots|h_{A,\rm out}\rangle|r_{\rm out}\rangle\big\}\ , \end{aligned} \tag{9}$$

where for brevity we denoted by $|h_i\rangle$ a generic state of $\mathcal{H}_i^{\rm out}$, which can represent more than one final state hadron. States are normalized to 1 analogously to Eq. (4) and complete in each subspace:

$$\overline{\sum}_{h_i}|h_{i,\mathrm{out}}\rangle\langle h_{i,\mathrm{out}}| = \mathbb{I} \quad \forall i; \qquad \overline{\sum}_{r}|r_{\mathrm{out}}\rangle\langle r_{\mathrm{out}}| = \mathbb{I} \tag{10}$$

with $\mathbb{I}$ the identity operator. Assuming $|h_{out}\rangle = |h_{1,out}\rangle$ to be well separated in phase space from the other fragments, consistent with the large-Q^2 and impulse approximation, we neglect the antysimmetrization of $|h_{out}\rangle$ with $|h_{2,out}\rangle, \ldots, |h_{A,out}\rangle, |r_{out}\rangle$. With these approximations, the nuclear hadronic tensor (1) becomes

$$\begin{aligned} W_A^{\mu\nu}(P_A, q) = \frac{1}{8\pi}\overline{\sum}_{h,R}&\langle A|j^{\dagger\mu}(0)|h_{\mathrm{out}}\rangle|R_{\mathrm{out}}\rangle \\ &\times \langle R_{\mathrm{out}}|\langle h_{\mathrm{out}}|j^{\nu}(0)|A\rangle \\ &\times (2\pi)^4\delta^{(4)}(P_A + q - p_X) \ , \end{aligned} \tag{11}$$

where we wrote $j = j_1$ for ease of notation, and $|R_{\mathrm{out}}\rangle = |h_{2,\mathrm{out}}, \ldots, h_{A.\mathrm{out}}, r_{\mathrm{out}}\rangle$. Let's introduce the total 4-momentum p_h of the hadronic state connected to the 1-body current j, and the total 4-momentum p_R of the remnant R, such that $p_X = p_h + p_R$. Then, we can write

$$\overline{\sum}_{h,R} = \sum_{h,R}\int (d^4p_h)(d^4p_R)d\Pi_h d\Pi_R \ , \tag{12}$$

where $d\Pi_h$ and $d\Pi_R$ are the integration measures over the intrinsic degrees of freedom of h and R, and include continuous as well as discrete degrees of freedom. Next, we use twice the completeness identity in (5):

$$\begin{aligned} W_A^{\mu\nu}(P_A, q) = \overline{\sum}_h \sum_R \int &(d^4p_R)d\Pi_R\Big[\prod_{i=1}^{A}(d^4k_i)(d^4k'_i)\Big] \\ &\times \frac{1}{8\pi}\langle k_1|j^{\dagger\mu}(0)|h_{\mathrm{out}}\rangle\langle h_{\mathrm{out}}|j^{\dagger\mu}(0)|k'_1\rangle \\ &\times \langle A|k_1, k_2, \ldots, k_A\rangle\langle k'_1, k'_2, \ldots, k'_A|A\rangle \\ &\times \langle k_2, \ldots, k_A|R_{\mathrm{out}}\rangle\langle R_{\mathrm{out}}|k'_2, \ldots, k'_A\rangle \\ &\times (2\pi)^4\delta^{(4)}(P_A + q - p_h - p_R) \ . \end{aligned} \tag{13}$$

Using the exponential notation for the δ-function in (13), initial state momentum conservation, $P_A = \sum_{i=1}^{A} k_i$, and translation invariance of the $|k_i\rangle$ states, we obtain

$$\begin{aligned} (2\pi)^4&\delta^{(4)}(P_A + q - p_h - p_R)\langle k_2, \ldots, k_A|R_{\mathrm{out}}\rangle \\ &= (2\pi)^4\delta^{(4)}(k_1 + q - p_h)\langle k_2, \ldots, k_A|R_{\mathrm{out}}\rangle \ . \end{aligned} \tag{14}$$

Therefore,

$$\begin{aligned} W_A^{\mu\nu}(P_A, q) = \overline{\sum}_{h,R}\int &\Big[\prod_{i=1}^{A}(d^4k_i)(d^4k'_i)\Big] \\ &\times \frac{1}{8\pi}\langle k_1|j^{\dagger\mu}(0)|h_{\mathrm{out}}\rangle\langle h_{\mathrm{out}}|j^{\dagger\mu}(0)|k'_1\rangle \\ &\times (2\pi)^4\delta^{(4)}(k_1 + q - p_h) \\ &\times \langle A|k_1, k_2, \ldots, k_A\rangle\langle k'_1, k'_2, \ldots, k'_A|A\rangle \\ &\times \langle k_2, \ldots, k_A|R_{\mathrm{out}}\rangle\langle R_{\mathrm{out}}|k'_2, \ldots, k'_A\rangle \ . \end{aligned} \tag{15}$$

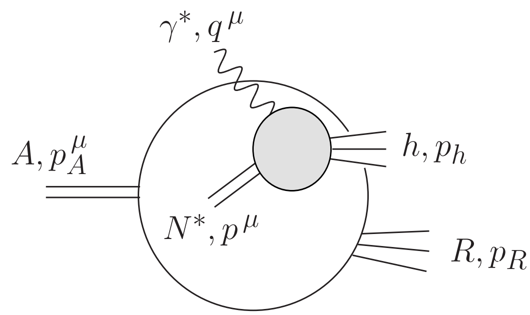

FIG. 1: Impulse approximation for the electron-nucleus scattering, with definition of the 4-momenta. The final state is decomposed into the hadron system h produced in the interaction of the virtual photon γ^* with the off-shell nucleon N^*, and a remnant R.

From this equation, we can derive the hadronic tensor for inclusive lepton-nucleus scattering, $\ell + A \to \ell' + X$, or for more exclusive processes like quasi-elastic nucleon knock-out or semi-inclusive deep inelastic scattering with an identified excited $(A-1)^*$ nucleus in the final state, $\ell + A \to \ell' + h + (A-1)^*$. Inclusive and semi-inclusive processes will be dealt with in turn.

A. Inclusive lepton-nucleus scattering

In inclusive lepton-nucleus scattering, we place no restrictions over the final states, so that the sum over R in (15) runs over all possible R's. The completeness relation

$$\overline{\sum}_R|R_{\mathrm{out}}\rangle\langle R_{\mathrm{out}}| = \mathbb{I} \tag{16}$$

and $\langle k_i|k'_i\rangle = (2\pi)^4\delta^{(4)}(k_i - k'_i)$ in Eq. (15) constrain $k_i = k'_i$ with $i \geq 2$, so that we obtain

$$\begin{aligned} W_A^{\mu\nu}(P_A, q) = \int &(d^4k_1)(d^4k'_1)\Big[\prod_{i=2}^{A}(d^4k_i)\Big] \\ &\times \langle A|k_1, k_2, \ldots, k_A\rangle\langle k'_1, k_2, \ldots, k_A|A\rangle \\ &\times \frac{1}{8\pi}\overline{\sum}_h\langle k_1|j^{\dagger\mu}(0)|h_{\mathrm{out}}\rangle\langle h_{\mathrm{out}}|j^{\dagger\mu}(0)|k'_1\rangle \\ &\times (2\pi)^4\delta^{(4)}(k_1 + q - p_h) \ . \end{aligned} \tag{17}$$

In the third and fourth lines, we can recognize an off-diagonal nucleon hadronic tensor. By combining the δ-functions which enforce momentum conservation in the nuclear wave function, Eq. (6), we can set $k_1 = k'_1 \equiv p$ and obtain

$$W_A^{\mu\nu}(P_A, q) = \int (d^4p)\rho_A(p)W_N^{\mu\nu}(p, q) \tag{18}$$

where

$$\rho_A(p) = \int \Big[\prod_{i=2}^{A} (d^4 k_i')\Big] |\psi_A(p, k_2', \ldots, k_A')|^2 \\ \times (2\pi)^4 \delta^{(4)}(p + \sum_{i=2}^{A} k_i' - P_A) \tag{19}$$

is the single nucleon 4-momentum distribution, and

$$W_N^{\mu\nu}(p,q) = \frac{1}{8\pi} \overline{\sum\nolimits_h} \langle p | j^{\dagger\mu}(0) | h_{\rm out}\rangle \langle h_{\rm out} | j^{\dagger\mu}(0) | p\rangle \\ \times (2\pi)^4 \delta^{(4)}(p + q - p_h) \tag{20}$$

is the nucleon hadronic tensor. We note that Eq. (18) could have been obtained directly from the definition of W_A in Eq. (1), using the exponential notation for the δ-function in (13) as done to obtain Eq. (14). However, it would have needed an antisymmetrization approximation on the initial state nucleons, which is not justified.

The longer derivation presented here makes it explicit that the obtained decomposition of the nuclear hadronic tensor is possible because we are considering fully inclusive lepton-nucleus scattering, therefore we do not impose any restriction on R and we are able to use the completeness relation (16). In semi-inclusive measurements, in which part of the final state is identified, the nuclear hadronic tensor can still be written as a convolution with the nucleon hadronic tensor, but with a more complicated weight function than the nucleon momentum distribution ρ_A. An example is discussed in the next subsection.

The 4-momentum nucleon distribution ρ_A is the generalization of the non-relativistic spectral function

$$S_A(\vec{p}, p^0) = \langle A | a^\dagger(\vec{p}) \delta(p^0 + \hat{H} - P_A^0) a(\vec{p}) | A\rangle \\ = \sum_f \Big| \langle A | (A-1)^f, \vec{P}_A - \vec{p}; N, \vec{p}\rangle \Big|^2 \\ \times \delta(p^0 + P_{A-1}^0|^f - P_A^0) \ , \tag{21}$$

which describes the joint momentum $\vec{p}$ and off-shell energy p^0 distribution of a single nucleon $|N\rangle$ in the A nucleus. The sum runs over all possible excited states f of an $A-1$ nucleus of momentum $\vec{P}_A - \vec{p}$. This definition slightly differs from the more conventional ones [11, 33], as we will discuss later. In (21), we denoted with $a(\vec{p})$ and $a^\dagger(\vec{p})$ the non relativistic destruction and creation operators of a nucleon of 3-momentum $\vec{p}$, with quantum numbers understood for ease of notation, and with $\hat{H}$ the many-body nuclear Hamiltonian. To develop the relationship between ρ_A and S_A, we perform a non relativistic reduction of Eq. (19) by substituting

$$|\psi_A(p, k_2, \ldots, k_A)|^2 \longrightarrow \mathcal{N} (2\pi)^A |\psi_A^{\rm nr}(\vec{p}, \vec{k}_2, \ldots, \vec{k}_A)|^2 \tag{22}$$

with $\mathcal{N}$ a normalization factor to be discussed shortly, and ψ^{nr} the non-relativistic wavefunction. Then,

$$\rho_A(\vec{p}, p^0) \longrightarrow (2\pi)^4 \mathcal{N} \int \Big[\prod_{i=2}^{A} d^3 k_i dk_i^0\Big] |\psi_A^{\rm nr}(\vec{p}, \vec{k}_2, \ldots, \vec{k}_A)|^2 \\ \times \delta^{(3)}(\vec{p} + \sum_{i=2}^{A} \vec{k}_i - \vec{P}_A) \\ \times \delta(p^0 + \sum_{i=2}^{A} k_i^0 - P_A^0) \ . \tag{23}$$

If we identify

$$\int \Big[\prod_{i=2}^{A} d^3 k_i'\Big] |\psi_A^{\rm nr}(\vec{p}, \vec{k}_2', \ldots, \vec{k}_A')|^2 \delta^{(3)}(\vec{p} + \sum_{i=2}^{A} \vec{k}_i - \vec{P}_A) \\ = \langle A | a^\dagger(\vec{p}) a(\vec{p}) | A\rangle \ , \tag{24}$$

we obtain

$$\rho_A(\vec{p}, p^0) \longrightarrow (2\pi^4) \mathcal{N} \int \Big[\prod_{i=2}^{A} dk_i^0\Big] \\ \times \langle A | a^\dagger(\vec{p}) \delta(p^0 + \hat{H} - P_A^0) a(\vec{p}) | A\rangle \ . \tag{25}$$

The covariant normalization of ρ_A is dictated by baryon number conservation [37]

$$\int (d^4 p) \frac{x_B}{x_N} \rho_A(\vec{p}, p^0) = 1 \ . \tag{26}$$

where $x_N = -q^2/(2p \cdot q)$. Note that this normalization differs by a factor $(2\pi)^4$ from the normalization of Ref. [37] due to the different normalization of nucleon states. The mismatch with the non-relativistic normalization

$$\int d^3 p \, dp^0 S_A(\vec{p}, p^0) = 1 \tag{27}$$

can be absorbed introducing a constant [35] or a momentum-dependent normalization factor[37]:

$$\mathcal{N} = \mathcal{N}' \Big(\int \prod_{i=2}^{A} dk_i^0 \Big)^{-1} \tag{28}$$

with, respectively,

$$\mathcal{N}' = \begin{cases} C \\ x_N / x_B \end{cases} \tag{29}$$

and C to be determined by Eq. (26). Finally,

$$\rho_A(\vec{p}, p^0) \longrightarrow (2\pi)^4 \mathcal{N}' S_A(\vec{p}, p^0) \ , \tag{30}$$

with S_A defined in Eq. (21). Note that with such normalization the nucleons do not carry all of the nucleus momentum. This violation of the momentum sum rule indicates the necessity of considering non-nucleonic degrees of freedom in the nuclear wave function [37]. The 2 possible choices for $\mathcal{N}'$ can be considered 2 extreme cases, the constant normalization overestimating the correction at small $|\vec{p}|$, and underestimating it at large $|\vec{p}|$, compared to the momentum-dependent normalization. This ambiguity should be regarded as an inherent theoretical uncertainty [38] in the non relativistic reduction. It would be absent in a fully relativistic treatment of the nuclear many-body problem, which at present is lacking. A step towards it is light-front quantization [8, 42]: it allows at the same time to satisfy baryon number conservation and the momentum sum rule, but is at present limited to ${}^2H$ targets or to the mean field approximation, which is insufficient for a fully quantitative understanding of $e + A$ collisions.

An important special case of non-relativistic reduction is for the $A = 2$ deuterium target, D. We will use the subscript s for the spectator nucleon, i.e., the nucleon which does not interact with the virtual photon. Eq. (25) becomes

$$\begin{aligned}\rho_D(\vec{p}, p^0) &\longrightarrow (2\pi)^4 \rho_D^{nr}(\vec{p}) \\ &\quad \times \delta(p^0 + \sqrt{m_s^2 + (\vec{P}_D - \vec{p})^2} - P_D^0) \ .\end{aligned} \tag{31}$$

The delta function relates the spectator momentum to the struck nucleon invariants y and p^2. In the Deuteron rest frame one finds

$$p^2 = M_D^2 + m_s^2 - 2M_D\sqrt{\vec{p}_s^{\,2} + m_s^2} \tag{32}$$

$$\tilde{y} = M_D - \sqrt{m_s^2 + \vec{p}_s^{\,2}} - |\vec{p}_s|\gamma\cos\theta_s \ , \tag{33}$$

where θ_s is the angle between the spectator and the virtual photon. In particular, the larger $|p_s|$ the more off-shell the struck nucleon. Note also that denoting by E_D the Deuteron binding energy $\sqrt{p^2}\to m_N - E_D$ and $\tilde{y}\to 1 - E_D/M$ at small spectator momentum, $\vec{p}_s\to 0$.

Finally, let us comment on the relationship of our non-relativistic spectral function (21) with others commonly used. In [33] the spectral function is defined as a function of the nucleon (positive) binding energy

$$E = m_N - p^0 \ , \tag{34}$$

So that in the rest frame of the nucleus Eq. (21) reads

$$\begin{aligned}S_A^{\rm nr}(\vec{p}, p^0) &= \sum_f \left|\langle A|(A-1)^f, -\vec{p}; N, \vec{p}\rangle\right|^2 \\ &\quad \times \delta(m_N - E + P_{A-1}^0|^f - P_A^0) \ .\end{aligned} \tag{35}$$

In Ref. [11], which uses non relativistic kinematics, the spectral function is defined in terms of the nucleon removal energy

$$E_{\rm rem} = m_N - p^0 - \frac{\vec{p}^{\,2}}{2M_{A-1}^f} \tag{36}$$

and of the negative of the nucleus binding energy

$$E_A = M_A - Am_N . \tag{37}$$

Then, Eq. (21) reads

$$\begin{aligned}S_A^{\rm nr}(\vec{p}, p^0) &= \sum_f \left|\langle A|(A-1)^f, -\vec{p}; N, \vec{p}\rangle\right|^2 \\ &\quad \times \delta(E_{\rm rem} - E_{A-1}^f + E_A) \ .\end{aligned} \tag{38}$$

B. Semi-inclusive lepton-nucleus scattering

As an example of semi-inclusive lepton-nucleus scattering, we discuss processes of the kind

$$\ell + A \to \ell' + h + (A-1)^* \tag{39}$$

in which an excited $(A-1)$ nucleus with momentum $\vec{p}_R$ is identified in the final state. The remaining particles are denoted by h, and may consists of a single nucleon (quasi-elastic nucleon knock-out) or other hadrons (inelastic scattering). Assuming as before that the final state excited nucleus is made of $A - 1$ nucleons,

$$\begin{aligned}\overline{\sum}_R |R_{\rm out}\rangle\langle R_{\rm out}| &= \sum_f \int \Big[\prod_{j=2}^{A}(d^4l_j)\Big](d^4p_R) \\ &\quad \times |\psi_{A-1}^f(l_2,\ldots,l_A)|^2 \\ &\quad \times |l_{2,\rm out},\ldots,l_{A,\rm out}\rangle\langle l_{2,\rm out},\ldots,l_{A,\rm out}| \\ &\quad \times (2\pi)^4\delta^{(4)}(p_R - l_2 - \ldots - l_A) \ ,\end{aligned} \tag{40}$$

where $|l_{2,\rm out},\ldots,l_{A,\rm out}\rangle$ is an antisymmetric orthonormal and, by assumption, complete basis for the final states. It is normalized analogously to Eqs. (4)-(5), and the superscript f labels the quantum numbers of the excited states of an $(A-1)^*$ nuclueus, as well as its off-shell states and any unbound state of $A-1$ nucleons. We also introduced the definition of the total recoil momentum $p_R = l_2 + \ldots + l_A$. We supplement the impulse approximation by assuming that the out-states not directly coupled to the electromagnetic current act as spectators at least on the short time scale of a large Q^2 interaction, i.e.,

$$|l_{2,\rm out},\ldots,l_{A,\rm out}\rangle = |l_2,\ldots,l_A\rangle \ . \tag{41}$$

Final state interactions, which happen on a longer time scale, may be considered separately **[AA] Give some Ref.**. Then, we can write

$$\begin{aligned}&\overline{\sum}_R \langle k_2,\ldots,k_A|R\rangle\langle R|k_2',\ldots,k_A'\rangle \\ &\quad = \sum_f \int (d^4p_R)|\psi_{A-1}^f(k_2,\ldots,k_A)|^2 \\ &\quad \times (2\pi)^4\delta^{(4)}(p_R - \sum_{j=2}^{A}k_j)\prod_{j=2}^{A}(2\pi)^4\delta^{(4)}(k_j - k_j')\end{aligned} \tag{42}$$

Using it in Eq. (15) and integrating over k'_j with $j = 2,\ldots,A$ leads to

$$\begin{aligned} W_A^{\mu\nu}(P_A,q) = &\int (d^4k_1)(d^4k'_1)(d^4p_R)\Big[\prod_{i=2}^{A}(d^4k_i)\Big] \\ &\times \frac{1}{8\pi}\overline{\sum}_h \langle k_1|j^{\dagger\mu}(0)|h_{\rm out}\rangle\langle h_{\rm out}|j^{\dagger\mu}(0)|k'_1\rangle \\ &\times (2\pi)^4\delta^{(4)}(k_1+q-p_h) \\ &\times \sum_f |\psi^f_{A-1}(k_2,\ldots,k_A)|^2 \\ &\times \langle A|k_1,k_2,\ldots,k_A\rangle\langle k'_1,k_2,\ldots,k_A|A\rangle \\ &\times (2\pi)^4\delta^{(4)}(p_R-k_2-\ldots-k_A) \ . \end{aligned} \tag{43}$$

In the second and third lines we can recognize the off-diagonal nucleon hadronic tensor. As for the inclusive case, $k_1 = k'_1 = p$ is obtained by combining the δ-functions which enforce momentum conservation in the nuclear wave function, Eq. (6). We finally obtain

$$W_A^{\mu\nu}(P_A,q) = \sum_f \int (d^4p)(d^4p_R)\mathcal{S}^f_A(p,p_R)W_N^{\mu\nu}(p,q) \ , \tag{44}$$

where $p \equiv k_1$ is the struck nucleon momentum and we defined the semi-inclusive spectral function $\mathcal{S}^f_A$ as

$$\begin{aligned} \mathcal{S}^f_A(p,p_R) = &\int \Big[\prod_{i=2}^{A}(d^4k_i)\Big] \\ &\times |\psi_A(p,k_2,\ldots,k_A)|^2(2\pi)^4\delta^{(4)}(P_A-p-p_R) \\ &\times |\psi^f_{A-1}(k_2,\ldots,k_A)|^2(2\pi)^4\delta^{(4)}(p_R-\sum_{j=2}^{A}k_j) \ . \end{aligned} \tag{45}$$

Hence, the differential hadaronic tensor for production of an on-shell $(A-1)^*$ nucleus reads

$$\begin{aligned} &\frac{dW_A^{\mu\nu}(P_A,q)}{d^3p_R} \\ &\quad = 2p_R^0\int (d^4p)\sum_f \mathcal{S}^f_A(p,p_R)W_N^{\mu\nu}(p,q)\Big|_{p_R^0=\sqrt{(M^f_{A-1})^2+\vec{p}_R^{\,2}}} \ , \end{aligned} \tag{46}$$

where we restricted f to run over all bound states of the $A-1$ nucleus, with masses $\sqrt{p_R^2} = M^f_{A-1}$.

The semi-inclusive spectral function (45) is interpreted as the joint probability distribution to find inside the nucleus A an $(A-1)$ nucleon system in a quantum state f with total momentum p_R and a nucleon with recoil momentum $p = P_A - p_R$. It generalizes the single-nucleon distribution in the sense that

$$\sum_f \int (d^4p_R)\mathcal{S}^f_A(p,p_R) = \rho_A(p) \ , \tag{47}$$

where now no restrictions are imposed on f. Indeed, we can use Eqs. (45) and (40) to write $\mathcal{I} \equiv \int (d^4p_R)\mathcal{S}_A(p,p_R)$ as

$$\begin{aligned} \mathcal{I} = &\int \Big[\prod_{i=2}^{A}(d^4k_i)(d^4k'_i)\Big] \\ &\times (2\pi)^4\delta^{(4)}(P_A-p-\sum_{j=2}^{A}k_j)(2\pi)^4\delta^{(4)}(p_R-\sum_{j=2}^{A}k'_j) \\ &\times |\psi_A(p,k_2,\ldots,k_A)|^2 \\ &\times \overline{\sum}_R \langle k_2,\ldots,k_A|R\rangle\langle R|k'_2,\ldots,k'_A\rangle \ . \end{aligned} \tag{48}$$

Since now f is unrestricted, we can use $\overline{\sum}_{R\in(A-1)^*}|R\rangle\langle R| = \mathbb{I}$ to obtain

$$\begin{aligned} \mathcal{I} = &\int \Big[\prod_{i=2}^{A}(d^4k_i)\Big]|\psi_A(p,k_2,\ldots,k_A)|^2 \\ &\times \delta^{(4)}(p+\sum_{i=2}^{A}k_i-P_A) \ . \end{aligned} \tag{49}$$

By definition (19) we obtain $\mathcal{I} = \rho_A(p)$, as we wanted to prove. In the same way, one can easily verify that the right hand side of Eq. (44) coincides with the right hand side of Eq. (18).

As a special case, we consider the $D(e,e'N)X$ process on a deuterium target, where N is assumed to be a spectator nucleon. The last condition can be realized experimentally by detecting the nucleon at backward angles compared to the virtual photon, where final state interactions between the spectator and the struck nucleon debris are minimized [?]. We label as usual with p the 4-momentum of the nucleon interacting with γ^*, and with $p_s \equiv p_R$ the 4-momentum of the detected spectator nucleon. The deuterium semi-inclusive spectral function (45) reads

$$\begin{aligned} \mathcal{S}^f_D(p,p_s) = &|\psi_D(p,p_s)|^2|\psi_s(p_s)|^2 \\ &\times (2\pi)^4\delta^{(4)}(P_D-p-p_s) \ . \end{aligned} \tag{50}$$

Since the probability of finding a nucleon of momentum k in an $A=1$ nucleus of momentum k is 1, we can set $|\psi_s(k)|^2 = 1$; moreover, by definition $\rho_D(p_s) = |\psi_D(P_D - p_s, p_s)|^2$. Hence,

$$\mathcal{S}^f_D(p,p_s) = \rho_D(p_s)(2\pi)^4\delta^{(4)}(P_D-p-p_S) \ , \tag{51}$$

and

$$\begin{aligned} &\frac{dW_D^{\mu\nu}(P_D,q)}{d^3p_s} \\ &\quad = 2p_s^0\rho_D(p_s)W_N^{\mu\nu}(P_D-p_s,q)\Big|_{p_s^0=\sqrt{m_s^2+\vec{p}_s^{\,2}}} \ , \end{aligned} \tag{52}$$

Performing the non relativistic reduction (22) in Eq. (50), and setting $\mathcal{N} = \mathcal{N}'/(2p_s^0)$, to satisfy the baryon sum rule and absorb the relativistic normalization of the $|p\rangle$ states, we analogously obtain

$$\frac{dW_D^{\mu\nu}(P_D,q)}{d^3p_s} \to \mathcal{N}' \rho_D^{nr}(\vec{p}_s) W_N^{\mu\nu}(P_D - p_s, q)\Big|_{p_s^0=\sqrt{m_s^2+\vec{p}_s^2}} . \quad (53)$$

The same derivation holds for $A-1$ nuclei with no excited states. For example, on can consider scattering on a ^4He with detected ^3H or ^3He and Eqs. (51)-(53) hold with the obvious substitution $m_s \to m_{^3H}$ and $m_s \to m_{^3He}$.

III. STRUCTURE FUNCTIONS FOR INCLUSIVE SCATTERING

We continue the study of inclusive lepton-nucleus scattering, define the nucleus and nucleon structure functions and derive their relationship. Using the nucleon, nucleus, and virtual photon momentum, we can form 6 invariants

$$x_B = \frac{-q^2}{2p_A \cdot q} \qquad Q^2 = -q^2 \qquad m_N^2 = p_A^2$$
$$x_N = \frac{-q^2}{2p \cdot q} \qquad p_A \cdot p \qquad p^2$$

where $p_A = (m_N/M_A)P_A \approx P_A/A$ is the rescaled nucleus mometum, x_B is the rescaled nucleus Bjorken invariant (with $x_B \leq M_A/m_N$), x_N is the Bjorken invariant at the nucleon level, and Q^2 is the photon virtuality. The "external" invariants in the first line are experimentally measurable. The "internal" invariants in the second line depend on the nucleon 4-momentum which is not directly measurable, but whose distribution is assumed to be known theoretically. We did not assign a special symbol to the internal invariant $p_A \cdot p$ or the nucleon virtuality p^2.

For ease of notation, we can rewrite the inclusive nuclear hadronic tensor (18) as

$$W_A^{\mu\nu}(p_A, q) = \int d\mu_A \, W_{N_A}^{\mu\nu}(p,q) \ , \quad (54)$$

with $W_N^{\mu\nu}$ the hadronic tensor of a bound nucleon, see Eq. (20), and relativistic Fermi smearing measure

$$d\mu_A = (d^4p)\rho_A(p) \ . \quad (55)$$

Note that the struck nucleon energy, p^0, or equivalently its virtuality p^2, is an integration variable. Gauge invariance at the nucleon level and Eq. (54) imply

$$0 = q_\mu W_A^{\mu\nu} = q_\mu W_N^{\mu\nu} = 0 \ , \quad (56)$$

so we can define the nuclear and nucleon structure functions as follows:

$$\begin{aligned} W_A^{\mu\nu}(p_a, q) &= -\tilde{g}^{\mu\nu} F_{1A}(x_B, Q^2) \\ &\quad + \frac{\tilde{p}_A^\mu \tilde{p}_A^\nu}{p_A \cdot q} F_{2A}(x_B, Q^2) \qquad (57) \\ W_{N/A}^{\mu\nu}(p, q) &= -\tilde{g}^{\mu\nu} F_{1,N/A}(x_N, Q^2, p^2) \\ &\quad + \frac{\tilde{p}^\mu \tilde{p}^\nu}{p \cdot q} F_{2,N/A}(x_N, Q^2, p^2) \qquad (58) \end{aligned}$$

where $\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$, and $\tilde{a}^\mu = a^\mu - (a \cdot q/q^2) q^\mu$ for any 4-vector a. The nucleon hadronic tensor can in principle depend on the virtuality p^2 and the nucleus A explicitly listed as argument of W_i, see the discussion below. Actually, Eq. (56) for W_N is an approximation due to the fact that in Section II we effectively treated the struck nucleons as a non-identical scalar particle in deriving the inclusive hadronic tensor (18). Nevertheless, for deep inelastic scattering in collinear factorization, Eq. (56) turns out to be exact, as we will discuss in Sect. III D. A formalism to take into account the Dirac nature of the nucleons is desribed in [18]. In general, there are 3 kinds of corrections to the nuclear structure functions F_{iA}. The first is of purely kinematic nature, related to the target mass corrections for an off-shell nucleon with $p^2 < m_N^2$, and will be explicitly factored out in the nuclear structure functions, see Eq. (69) below. The second is the dependence of the nucleon wave function on its virtuality, and leads to an explicit dependence of $W_N^{\mu\nu}$ on p^2, and of F_{iA} on A because of the A-dependence of the probability to have off-shell fluctuations encoded in $\rho_A(p)$. The third are dynamical modifications of the nucleon hadronic tensor due to the embedding of the nucleon in the nuclear matter, which can further modify the off-shell nucleon wave-function $|p\rangle$. They are included in the formal dependence of $W_N^{\mu\nu}$ on A. A further source of dynamical nuclear effects on lepton-nucleus scattering, is lepton scattering on non-nucleonic degrees of freedom, which can be included in the formalism presented in this paper, but we will not further investigate it at this stage. We refer to [18] for an extended discussion of off-shell effects, and to [9, 22, 23] for a review of nuclear modifications of structure functions. **[AA] Perhaps give explicit reference for effects 1,2,3 above?**

A. Choice of reference frame

We choose a frame such that

$$\begin{aligned} p_A^\mu &= p_A^+ \overline{n}^\mu + \frac{m_N^2}{2p_A^+} n^\mu \\ q^\mu &= -\xi p_A^+ \overline{n}^\mu + \frac{Q^2}{2\xi p_A^+} n^\mu \\ p^\mu &= y p_A^+ \overline{n}^\mu + \frac{p^2 + p_T^2}{2y p_A^+} n^\mu + \vec{p}_\perp^{\,\mu} \ . \end{aligned} \quad (59)$$

The light-cone vectors n^μ and $\overline{n}^\mu$ satisfy

$$n^2 = \overline{n}^2 = 0 \qquad n \cdot \overline{n} = 1 \ , \tag{60}$$

and define the light-cone plus and minus directions. A 4-vector plus- and minus-components are defined by

$$a^+ = a \cdot n \qquad a^- = a \cdot \overline{n}. \tag{61}$$

If we choose $\overline{n} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$ and $n = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$, we obtain $a^\pm = (a_0 \pm a_3)/\sqrt{2}$. The nucleon fractional light-cone momentum with respect to the nucleus is defined as

$$y = p^+/p_A^+ \ , \tag{62}$$

and

$$\xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m_N^2/Q^2}} \tag{63}$$

is the nuclear-level Nachtmann variable [15]. It is also convenient to define the nucleon transverse mass $m_\perp^2 = p^2 + p_\perp^2$. The nucleus plus-momentum, p_A^+, can be interpreted as a parameter for boosts along the z-axis, connecting the target rest frame to the nucleus infinite-momentum frame. In this class of frames, the internal invariant x_N reads

$$x_N = \frac{\xi/y}{1 - (\xi/y)^2 m_\perp^2/Q^2} \ . \tag{64}$$

The Fermi smearing measure can also be usefully expressed as

$$d\mu_A = \frac{dp^2}{2\pi} \frac{dy d^2 p_\perp}{(2\pi)^3 2y} \rho_A(y, p^2, \vec{p}_\perp) \ . \tag{65}$$

B. Structure functions and cross section

Contracting Eq. (57) with $g_{\mu\nu}$ and $\hat{p}_{A\mu}\hat{p}_{A\nu}$, where $\hat{p}_{A\mu} = p_A^+ \overline{n}_\mu$, and we find

$$\begin{aligned} F_{2A}(x_B, Q^2) &= \frac{x_B}{\gamma^2} \Big[-g_{\mu\nu} + \frac{12\xi^2}{Q^2} \hat{p}_A^\mu \hat{p}_A^\nu \Big] W_A^{\mu\nu}(x_B, Q^2) \\ F_{1A}(x_B, Q^2) &= \frac{1}{2} \Big[-g_{\mu\nu} + \frac{4\xi^2}{Q^2} \hat{p}_A^\mu \hat{p}_A^\nu \Big] W_A^{\mu\nu}(x_B, Q^2) \end{aligned} \tag{66}$$

where

$$\gamma^2 = 1 + 4x_B^2 \frac{m_N^2}{Q^2} \ . \tag{67}$$

Upon substitution of Eq. (54) in Eq. (66), it is straightforward to see that

$$\begin{aligned} F_{2A}(x_B, Q^2) &= \int d\mu_A \frac{3\gamma_{NT}^2 - \gamma_N^2}{2\gamma^2} \frac{x_B}{x_N} F_{2,N/A}(x_N, Q^2, p^2) \\ F_{1A}(x_B, Q^2) &= \int d\mu_A \Big\{ F_{1,N/A}(x_N, Q^2, p^2) \\ &\quad + \frac{\gamma_{NT}^2 - \gamma_N^2}{4x_N} F_{2,N/A}(x_N, Q^2, p^2) \Big\} \end{aligned} \tag{68}$$

where

$$\begin{aligned} \gamma_N^2 &= 1 + 4x_N^2 \frac{p^2}{Q^2} \\ \gamma_{NT}^2 &= 1 + 4x_N^2 \frac{p^2 + p_T^2}{Q^2}. \end{aligned} \tag{69}$$

The formulae for the transverse and longitudinal structure functions can be found in Appendix A. The per-nucleon cross section for lepton-nucleus scattering in terms of nuclear strucure functions reads

$$\begin{aligned} \frac{d\sigma_A}{dQ^2 dx_B} = \frac{4\pi\alpha^2}{Q^4} \Big\{ & y_e^2 F_{1A}(x_B, Q^2) \\ & + \Big(1 - y_e - \frac{m_N^2}{Q^2} x_B^2 y_e^2\Big) \frac{F_{2A}(x_B, Q^2)}{x_B} \Big\} \end{aligned} \tag{70}$$

where $y_e = P_A \cdot q / P_A \cdot p_l$ with p_l the lepton initial 4-momentum. In the nucleus rest frame, $y_e = Q^2/(2E_{lab} m_N x_B)$, where E_{lab} is the lepton beam energy.

The limits on the d^4p integration in Eq. (69) are

$$p^2 \leq m_N^2 \tag{71}$$

for bound nucleons, and

$$\frac{x_B}{\frac{M_A}{m_N} + x_B \frac{m_N^2 - p^2}{Q^2}} \leq x_N \leq \frac{1}{1 + \frac{m_N^2 - p^2}{Q^2}} \tag{72}$$

imposed by energy-momentum conservation. This also implies $x_B \leq M_A/m_N$, as it should be independently of the impulse approximation. To prove Eq. (72), let's consider the kinematics of Fig. 1. By assumption, in the impulse approximation, the baryon number β is distributed in the final state such that $\beta(h) = 1$ and $\beta(R) = A - 1$. Then the final state momenta must satify $p_h^2 \geq m_N^2$ and $p_Y^2 \geq m_N^2$. Using $p_h^2 = (p+q)^2 = (1/x_N - 1)Q^2 + p^2$ we obtain

$$x_N \leq \frac{1}{1 + \frac{m_N^2 - p^2}{Q^2}} \ . \tag{73}$$

Let's consider the invariant energy squared of the process, $s = (P_A + q)^2 = (p_R + p_h)^2$. The remnant R must contain at least $A-1$ nucleons by baryon number conservation. Let us label their momenta $p_1, \ldots, p_{A-1}$ and the remaining particles momenta collectively by $\{k_i\}$, so that

$$p_R = p_1 + \ldots + p_{A-1} + \{k_i\} \ . \tag{74}$$

Likewise, h must contain at least one hadron, so that with analogous conventions for the momenta,

$$p_h = p'_1 + \{k'_i\} \ . \tag{75}$$

For any 2 on-shell particles of 4-momentum a, b and masses m_a, m_b one has $a \cdot b \geq m_a m_b$. The $A-1$ baryon

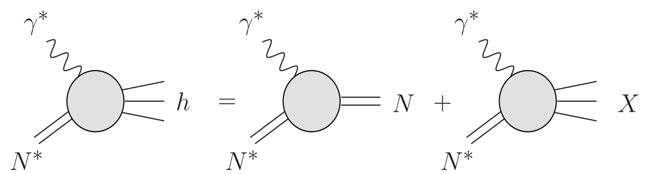

FIG. 2: Decomposition of the scattering of a virtual photon on an off-shell nucleon into its quasi-elastic and inelastic components.

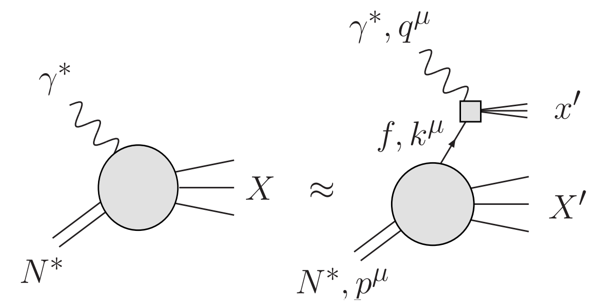

FIG. 3: Inelastic $\gamma^* + N^*$ scattering in perturbative QCD at arbitrary order in α_s: the virtual photon γ^* scatters on a parton (quark or gluon) of flavor f belonging to the off-shell nucleon N^*. The definition of the particles momenta are given beside their symbol.

system may be a $(A-1)$ nucleus bound state, or any combinations of nucleons and lighter nuclei with total baryon number $A-1$. Hence,

$$p_R^2 \geq M_{A-1}^2 \ ; \qquad p_R \cdot p_h \geq M_{A-1} m_N \ . \tag{76}$$

This implies that

$$(P_A+q)^2 = (p_R+p_h)^2 \geq p_h^2 + M_{A-1}^2 + 2M_{A-1}m_N \ . \tag{77}$$

Using $(P_A+q)^2 = [M_A/(m_N x_B) - 1]Q^2 + M_A^2$, and neglecting the small binding energy compared to the total nucleus mass so that $M_A \approx M_{A-1} + m_N$, we obtain

$$x_N \geq \frac{x_B}{\frac{M_A}{m_N} + x_B \frac{m_N^2 - p^2}{Q^2}} \ . \tag{78}$$

We note that in the Bjorken limit ($Q^2, \nu \to \infty$ at fixed x_B) $\gamma_{B,N,NT} \to 1$ and $x_B/x_N \to y$, so that

$$\begin{aligned} F_{2A}(x_B,Q^2) &\longrightarrow \int d\mu_A \, y \, F_{2,N}\Big(\frac{x_B}{y}, Q^2, p^2; A\Big) \\ F_{1A}(x_B,Q^2) &\longrightarrow \int d\mu_A \, F_1\Big(\frac{x_B}{y}, Q^2, p^2; A\Big) \ . \end{aligned} \tag{79}$$

where the integration over d^4p is restricted by

$$x_B \leq y \leq A \tag{80}$$

because of Eq. (72). Thus, we recover the relativistic convolution formula [37], where the factor y in front of $F_{2,N}$, absent in front of F_1, is usually called "flux factor". In the non-relativistic limit ($\vec{p}^{\,2}/p^2 \to 0$), the flux factor disappears, $y \to m/m_N \approx 1$. Finally, note that in the free nucleon limit, defined by $S_A(p) \to (2\pi)^4 \delta^{(4)}(p - p_A)$ and $A \to 1$, the nuclear structure functions tend to the free nucleon ones: $F_{iA}(x_B) \to F_i(x_B)$.

C. Deep inelastic scattering and collinear factorization

The nucleon hadronic tensor $W_N^{\mu\nu}$ (20) describes in general the scattering of a virtual photon γ^* on an off-shell nucleon N^*. We will discuss separately the case when the out-state h is an on-shell nucleon, $h = \{N\}$ (quasi-elastic $\gamma^* N^*$ scattering) and when $h = X \neq \{N\}$ (inelastic $\gamma^* N^*$ scattering), see Fig. 2. Correspondingly, we can split the nucleon hadronic tensor into a quasi-elastic and an inelastic part:

$$\begin{aligned} & W_{N/A}^{\mu\nu}(x_N, Q^2, p^2) \\ & \quad = W_{N/A,\mathrm{q.e.}}^{\mu\nu}(x_N, Q^2, p^2) + W_{N/A,\mathrm{inel}}^{\mu\nu}(x_N, Q^2, p^2) \ . \end{aligned} \tag{81}$$

Of course, this decomposition carries over to the structure functions and defines the analogous decomposition at the nuclear level via Eq. (54). The inelastic tensor can be further decomposed into a resonance production term ($h = \Delta, \ldots$) and a deep inelastic term. However, the deep inelastic structure functions computed in terms of photon-parton scattering are known to describe, on average, the resonance region. This phenomenon is known as quark-hadron duality [24]. Furthermore Fermi motion in nuclear targets smears out the resonance structures present in $F_{2,N}$, with the exception of the Δ peak on a Deuterium target. Hence, for the present time, we will content ourselves with the decomposition (94). In this paper we concentrate on DIS structure functions, and will leave the inclusion of quasi-elastic scatterings to a future effort.

We compute the Deep Inelastic Scattering (DIS) hadronic tensor in the QCD collinear factorization framework, see Fig. 3 [14, 43]. In the impulse approximation, the lepton-nucleon interaction proceeds through the scattering of the virtual photon with a parton (quark or gluon) belonging to the nucleon. We will limit our discussion to light quarks and gluons, all assumed to be massless; the generalization to massive quarks is discussed in [43]. To evaluate the hadronic tensor or the structure functions in terms of the QCD collinear factorization approach, we need to expand the parton momentum around the positive light-cone direction defined by the nucleon momentum [14]. To this purpose, we define the positive and negative light cone vectors $\overline{n}_*^\mu$ and n_*^μ through the following decomposition of the nucleon and virtual photon momenta:

$$p^\mu = p_*^+ \overline{n}_*^\mu + \frac{p^2}{2p_*^+} n_*^\mu$$
$$q^\mu = -\xi_N p_*^+ \overline{n}_*^\mu + \frac{Q^2}{2\xi_N p_*^+} n_*^\mu \tag{82}$$

where $\overline{n}_*^2 = n_*^2 = 0$, $n_* \cdot \overline{n}_* = 1$,

$$p_*^+ = p \cdot n_*$$
$$p_*^- = p \cdot \bar{n}_* \ , \tag{83}$$

and

$$\xi_N = \frac{2x_N}{1 + \sqrt{1 + 4x_N^2 p^2/Q^2}} \tag{84}$$

is the nucleon Nachtmann variable. Note that $\xi_N = \xi_N(x_N, Q^2, p^2)$ depends also on the nucleon virtuality P^2. When the proton moves fast in the positive z-axis direction (and the photon in the opposite direction) its p^+ can be considered the large light-cone component. Following QCD collinear factorization, we expand the parton momentum k in Fig. 3 around its positive light-cone component,

$$k^\mu = x p_*^+ \overline{n}_*^\mu + O(k - x p_*^+ \overline{n}_*) \tag{85}$$

where we defined the parton light-cone fractional momentum as

$$x = \frac{k_*^+}{p_*^+} \ . \tag{86}$$

According to the QCD factorization theorem [14], the nucleon hadronic tensor can be factorized as follows:

$$\begin{aligned} W_{N/A}^{\mu\nu}&(x_N, Q^2, p^2) \\ &= \sum_f \int \frac{dx}{x}\, \mathcal{H}_f^{\mu\nu}(\hat{x}, Q^2)\, \varphi_{f/N/A}(x, Q^2, p^2) \\ &+ O(1/Q^2) \end{aligned} \tag{87}$$

where the parton-level Bjorken invariant, $\hat{x}$, is defined as

$$\hat{x} \equiv \frac{Q^2}{2k \cdot q} = \frac{\xi_N}{x}, \tag{88}$$

and the second identity holds only for massless quarks. Energy-momentum conservation limits the integration to

$$\frac{x_N}{1 + x_N \frac{p^2 - m_N^2}{Q^2}} \leq \hat{x} \leq 1 \ , \tag{89}$$

as can be proved in analogy with the derivation of Eq. (72), so that

$$\xi_N \leq x \leq \frac{\xi_N}{x_N} \left(1 + \frac{p^2 - m_N^2}{Q^2} \right) \ . \tag{90}$$

This also implies $x_N \leq [1 + (p^2 - m_N^2)/Q^2] \leq 1$. Note that for on-shell nucleons, $p^2 = m_N^2$, one recovers the results of Ref. [43].

In Eq. (100), $\varphi_{f/N}$ is the leading twist parton distribution function (PDF) of a bound off-shell nucleon for a parton of flavor $f = g, q, \bar{q}$, and $\mathcal{H}_f^{\mu\nu}$ is the partonic tensor for an on-shell parton of momentum $k^\mu = x p^+ \overline{n}^\mu$ with all perturbative collinear divergences along the parton momentum k absorbed into the PDFs. With no surprise, Eq. (100) has the same factorized form as that of a free nucleon [16, 17] because the factorization of short-distance partonic dynamics is insensitive to the details of long-distance hadron physics, which is encoded in the PDF ϕ_f. Consider as an example the quark PDF at leading order (LO) in the strong coupling constant α_s:

$$\varphi_q(x, Q^2, p^2) = \int \frac{dz^-}{2\pi} e^{-ixp_*^+ z^-} \langle p | \overline{\psi}(z^- n_*)\, \frac{\gamma^+}{2}\, \psi(0) | p \rangle \tag{91}$$

Other than the state $|p\rangle$, this definition is the same for both bound and free nucleon [17] since the n_*^μ and $\overline{n}_*^\mu$ are defined in terms of the nucleon momentum p. The off-shellnes of $|p\rangle$ is reflected in an additional dependence of ϕ_f on the nucleon off-shell mass p^2, such that

$$\varphi_{f/N/A}(x, Q^2, p^2) = \varphi_{f/N/A}(x, Q^2) + O((m_N^2 - p^2)/m_N^2) \ . \tag{92}$$

We can identify $\varphi_{f/N_A}(x, Q^2)$ with the (bound) nucleon PDF because of our choice of expanding k around $(k \cdot n_*)\overline{n}_*^\mu$, instead of other equally large light-cone components like $(k \cdot n)\bar{n}^\mu$. The alternative choice $\overline{n}_* = \overline{n}$, $n_* = n$ is useful for defining nuclear PDF as in [27–29], and the connection with the underlying nucleon parton distributions will be discussed in Section **??**.

Since the parton in the collinear expansion is on shell, gauge invariance implies

$$q_\mu \mathcal{H}_f^{\mu\nu} = 0 \ . \tag{93}$$

Therefore we can decompose the partonic tensor as

$$\mathcal{H}_f^{\mu\nu}(\hat{x}, Q^2) = -\tilde{g}^{\mu\nu}\, h_{1/f}(\hat{x}, Q^2) + \frac{\tilde{k}^\mu \tilde{k}^\nu}{k \cdot q}\, h_{2/f}(\hat{x}, Q^2)\,, \tag{94}$$

where the scalar functions $h_{i/f}$ can be computed order-by-order in powers of α_s from the factorized expression (100) with the nucleon state $|p\rangle$ replaced by an on-shell parton state of flavor f, regardless of the nucleon state. It should be noted that gauge invariance (106) at the parton level and Eq. (100) imply $q_\mu W_{N|DIS}^{\mu\nu}(p, q) = 0$ even for off-shell nucleons. The same is not guaranteed for the quasi-elastic part of the nucleon tensor [18].

We can relate the nucleon and partonic structure functions in the following way. First, we invert Eq. (107):

$$\begin{aligned}\frac{h_{2/f}(\hat{x},Q^2)}{\hat{x}} &= \Big[-g_{\mu\nu}+\frac{12\hat{x}^2}{Q^2}k^\mu k^\nu\Big]\mathcal{H}^{\mu\nu}(\hat{x},Q^2)\\ 2h_{1/f}(\hat{x},Q^2) &= \Big[-g_{\mu\nu}+\frac{4\hat{x}^2}{Q^2}k^\mu k^\nu\Big]\mathcal{H}^{\mu\nu}(\hat{x},Q^2)\ .\end{aligned} \tag{95}$$

Then, we contract the nucleon hadronic tensor (58) with $g_{\mu\nu}$ and $\hat{p}^\mu\hat{p}^\nu$, where $\hat{p}^\mu=(p\cdot n_*)\overline{n}_*^\mu=k^\mu/x$ is the massless limit of the nucleon momentum p in Eq. (95). Finally, upon use of Eq. (108) we obtain

$$\begin{aligned}F_{2,N/A}(x_N,Q^2,p^2) &= \frac{x_N}{\gamma_N^2}\left(\frac{1}{\hat{x}}h_2\right)\otimes\varphi_{N/A}(\xi_N,Q^2,p^2)\\ F_{1,N/A}(x_N,Q^2,p^2) &= h_1\otimes\varphi_{N/A}(\xi_N,Q^2,p^2)\end{aligned} \tag{96}$$

where we defined the convolution symbol as follows,

$$\begin{aligned}&h\otimes\varphi_{N/A}(\xi_N,Q^2,p^2) \\ &\quad=\sum_f\int_{x_N}^1\frac{d\hat{x}}{\hat{x}}\,h_f(\hat{x},Q^2)\varphi_{f/N/A}(\xi_N/\hat{x},Q^2,p^2)\ .\end{aligned} \tag{97}$$

The dependence of $F_{2,N}$ on p^2 comes from 3 sources:

- the kinematic limits on the $d\hat{x}$ integration, Eq. (102);
- the purely kinematic target mass corrections, which for a bound nucleon are governed by p^2 instead of m_N^2, as found in the definiton of ξ_N and γ_N entering Eqs. (109);
- The dynamical p^2 and A dependence of $F_{iN/A}$ is due to the off-shellness of the nucleon state and its binding to surrounding nucleons, which modify the nucleon state $|p\rangle$ entering the definition of the PDF φ_f, see Eq. (104).

We finally note that because of the mixing of $F_{1,N}$ and $F_{2,N}$ in the formula for F_{1A} in Eq. (69), the Callan-Gross relation is broken for a nuclear target even at leading order in α_s, but is recovered in the Bjorken limit.

D. Convolution formulae at finite Q^2

The nuclear DIS structure functions can be in general expressed as a 2-dimensional convolution over the virtuality p^2 and the nucleon fractional momentum $\tilde{y}$,

$$\tilde{y}=\frac{x_B}{x_N}=\frac{p\cdot q}{p_A\cdot q}\ , \tag{98}$$

which reduces to the light-cone nucleon momentum fraction y in the Bjorken limit:

$$\tilde{y}=y\,\frac{1+\gamma}{2}\left(1-\frac{\xi^2}{y^2}\frac{p^2+p_T^2}{Q^2}\right)\ . \tag{99}$$

Taking F_{2A} as an example, we can write

$$\begin{aligned}F_{2A}(x_B,Q^2) = \int_{x_B}^{\infty}&d\tilde{y}\int_{p^2_{\rm min}}^{p^2_{\rm max}}dp^2\,C_A(\tilde{y},p^2;\gamma)\\ &\times F_{2,N/A}(x_N,Q^2,p^2)\ ,\end{aligned} \tag{100}$$

where

$$\begin{aligned}p^2_{\rm min} &= m_N^2+\frac{Q^2}{x_B}(x_B-\tilde{y})\\ p^2_{\rm max} &= \min\left[m_N^2, m_N^2+\frac{Q^2}{x_B}\Big(\frac{M_A}{m_N}-\tilde{y}\Big)\right]\ .\end{aligned} \tag{101}$$

Note that in Eq. (83), all the kinematic limits have been written explicitly. The function C_A is defined as

$$\begin{aligned}C_A(\tilde{y},p^2;\gamma) &= \frac{\tilde{y}}{\gamma^2}\int(d^4p)\rho_A(p)\delta(p^2-p_\mu p^\mu)\\ &\times\left[\frac{3\gamma_{NT}^2-\gamma_N^2}{2}\right]\delta(\tilde{y}-p\cdot q/p_A\cdot q)\ ,\end{aligned} \tag{102}$$

and depends parametrically only on γ because in the nucleus rest frame $\tilde{y}=(p^0+\gamma p^z)/m_A$. An analogous formula for F_{1A} can be easily derived. In general,

$$\begin{aligned}F_{2A} &= C_A\otimes F_{2N_A}\\ F_{2A} &= D_{1A}\otimes F_{1N_A}+D_{2A}\otimes F_{2N_A}\ ,\end{aligned} \tag{103}$$

where the convolution symbol denotes an integration over $\tilde{y}$ and p^2 as in Eq. (83), and the D_{iN_A} functions can be easily derived from Eq. (69).

Considering again F_{2A} as an example, a 1-dimensional convolution formula can be obtained by writing the bound nucleon structure functions in terms of the free nucleon structure functions $F_{2,N}$,

$$F_{2,N/A}(x_N,Q^2,p^2)=F_{2,N}(x_N,Q^2)\Delta_{2,N/A}(x_N,Q^2,p^2)\ , \tag{104}$$

so that

$$F_{2A}(x_B)=\int_{x_B}^{\infty}d\tilde{y}\mathcal{S}_A(\tilde{y},\gamma,x_B)F_{2,N}\Big(\frac{x_B}{\tilde{y}}\Big)\ , \tag{105}$$

where the "smearing function" $\mathcal{S}_A$ reads

$$\mathcal{S}_A(\tilde{y},\gamma,x_B)=\int_{p^2_{\rm min}}^{p^2_{\rm max}}dp^2\,C_A(\tilde{y},p^2;\gamma)\Delta_{2,N/A}(x_B/\tilde{y},Q^2,p^2)\ , \tag{106}$$

and $\Delta_{2,N/A}$ contains the off-shell modifications of the nucleon structure function. Expanding this in a power series one obtains

$$\Delta_{2,N/A}=1+\delta_{2,N/A}(x_N,Q^2)\frac{p^2-m_N^2}{m_N^2}+\cdots \tag{107}$$

with

$$\delta_{2,N/A}(x_N,Q^2)=\frac{1}{F_{2,N}}\left.\frac{dF_{2,N/A}}{dp^2}\right|_{p^2=m_N^2}\ . \tag{108}$$

Note that off-shell effects do not disappear in the Bjorken limit. Since δ contains the same structure function at the numerator and denominator, it is likely to be almost independent of the PDF parametrization, which makes Eq.(89) suitable for global PDF fits. It is also of interest to notice that the phenomenological analysis by Kulagin and Petti [30] indicates that δ is independent of A at least for large nuclei.

Eq.(89) can be further simplified when neglecting the dynamical off-shell effects (i.e., by setting $p^2 \approx m_N^2$ in Eq. (**??**)). This might be a reasonable approximation for light nuclei [26]. Furthermore, one needs to computing F_{2A} either (i) in the Bjorken limit or (ii) at small enough x_B values, in order to neglect the p^2 dependence in the integration limits over $\tilde{y}$. The small-x_B approximation is the most interesting for this work, where we are interested in finite-Q^2 corrections to the nuclear structure functions. In this case, the convolution of $F_{2,N}$ over the nuclear spectral function samples small values of the nucleon momentum so that $\langle|\vec{p}|\rangle \approx k_F$ (with k_f the Fermi momentum) and the nucleon is the least off-shell. At the same time, for small x_B, the C_A function is nearly zero at the borders of the integration region, so that little changes in the integration region will not affect significantly the result. This implies that one can use the Bjorken limit integration region ($-\infty < p^2 < m_N^2$, $x_B \leq \tilde{y} \leq A$). As a result **[AA] Clash of notation between $\mathcal{S}$ here and in Section I ??**

$$F_{2A}(x_B) \approx \int_{x_B}^{A} d\tilde{y}\mathcal{S}_A^*(\tilde{y},\gamma)F_{2,N/A}\Big(\frac{x_B}{\tilde{y}},Q^2,m_N^2\Big) \ , \tag{109}$$

where the smearing function $\mathcal{S}_A^*$ does not depend anymore on x_B:

$$\mathcal{S}_A^*(\tilde{y},\gamma) = \int_{-\infty}^{m_N^2} dp^2 C_A(\tilde{y},p^2,\gamma) \ . \tag{110}$$

However, at large values of $x_B \gtrsim 0.85$ the integration limit $\tilde{y} > x_B$ approaches the region where C_A has most strength (see, e.g., Ref [25]), and the sampled nucleon momenta and binding energy place the nucleon increasingly off-shell. Hence, the dependence on $\tilde{y}$ of the integration limits (84) cannot be ignored, and one needs the full Eq. (89) even when neglecting dynamical off-shell effects.

E. Nuclear parton distributions

TO BE WRITTEN

IV. APPLICATIONS

This section is to be updated - ignore it for the moment

Lacking a full solution to the relativistic many-body problem, we approximate the covariant nucleon distribution by the non-relativistic spectral function $S_A^{\rm nr}$ as discussed in Section II A, Eq (30):

$$\rho_A(p) \approx (2\pi)^4 \mathcal{N}' S_A^{\rm nr}(\vec{p},p^0) \ , \tag{111}$$

but otherwise working with relativistic kinematics. We choose the nucleus rest frame, where computations of $S_A^{\rm nr}$ are performed. In this frame, $\tilde{y} = (p_0 + \gamma p^z)/m_A$, and $d\tilde{y}dp^2 = 2(p^0\gamma - p^z)/m_A dp^0 dp^z$, where we denoted $\vec{p} = (\vec{p}_T, p^z)$. Therefore,

$$C_A(\tilde{y},x_B,\gamma) = 2\pi\frac{\tilde{y}}{\gamma^2}\int_0^\infty dp_T^2\,\frac{p_0\gamma - p^z}{m_A} S_A^{\rm nr}(\vec{p},p^0) \tag{112}$$

where

$$\begin{aligned} p^0 &= \frac{\eta m_A^2\tilde{y}^2 + (p^2+p_T^2)}{\gamma\eta m_A^2\tilde{y}^2} \\ p^z &= \frac{m_A^2\tilde{y}^2 - (p^2+p_T^2)}{\gamma\eta m_A^2\tilde{y}^2} \end{aligned} \tag{113}$$

and

$$\eta = \sqrt{1 - \Big(1-\frac{1}{\gamma^2}\Big)\Big(1-\frac{p^2+p_T^2}{m_A^2\tilde{y}^2}\Big)} \ . \tag{114}$$

A. Deuterium F_2 structure function

Under the approximation that the spectator nucleon is on shell, the non-relativistic spectral function for ^{2}He is given by Eq. (31). In terms of the $\tilde{y}, p^2, p_T^2$ variables it reads

$$\begin{aligned} S_D^{\rm nr}(\vec{p},p^0) &= 2(M_D - p^0) n_D^{\rm nr}(\vec{p}) \\ &\quad \times \delta\big(p_T^2 - (M_D-p^0)^2 + m_s^2 + (p^z)^2\big) \ . \end{aligned} \tag{115}$$

This is the most convenient form to insert in Eq. (112). In order to concentrate on the effects of binding energy and Fermi motion, we will neglect off-shell modifications of the PDFs, and neglect A-dependent nuclear medium effects. Therefore, we can use free-nucleon parton distribution functions φ in Eq.(109), for which we choose the MRST2002 parametrization. The neutron PDFs are obtained from the proton PDFs by isospin symmetry.

It is interesting to compare our formalism with the nuclear smearing model employed by Kahn, Melnitchouk and Kulagin in Ref. [25]. They tacitly approximate $F_{2,N} = F_{2,N}|_{p^2=m_N^2}$, so their result should be compared with the 1-dimensional convolution (**??**), which for a deuterium target reads

$$\begin{aligned} C_A(\tilde{y},\gamma) &= \int d^3p\mathcal{K}(p)|\psi_D(\vec{p})|^2\frac{\tilde{y}}{\gamma^2}\frac{3\gamma_{NT}^2-\gamma_N^2}{2} \\ &\quad \times \delta\left(\tilde{y} - 1 - \frac{\varepsilon+\gamma_N p_z}{m_N}\right)|_{p_0=m_N+\varepsilon} \ , \end{aligned} \tag{116}$$

where $\varepsilon = \varepsilon_D - |\vec{p}|^2/(2m_N^2) + O(|\vec{p}|^4/m_N^4)$, and $\varepsilon_D = -2.2$ MeV is the deuterium binding energy. Using

$$\gamma_N^2 = 1 + \frac{\gamma^2-1}{\tilde{y}^2}\left(1+\frac{2\varepsilon}{m_N}-\frac{\vec{p}^2}{m_N^2}\right) + O\left(\frac{\vec{p}^4}{m_N^4}\right) + O\left(\frac{\varepsilon_D}{m_N}\right) \tag{117}$$

$$\gamma_{NT}^2 = 1 + \frac{\gamma^2-1}{\tilde{y}^2}\left(1+\frac{2\varepsilon}{m_N}-\frac{p_z^2}{m_N^2}\right) + O\left(\frac{\vec{p}^4}{m_N^4}\right) + O\left(\frac{\varepsilon_D}{m_N}\right) , \tag{118}$$

one easily obtains

$$C_A(\tilde{y},\gamma) = ... \tag{119}$$

$$\int d^3p|\psi_D(\vec{p})|^2 \frac{\tilde{y}-\varepsilon/m_N}{\gamma^2}\frac{3\gamma_{NT}-\gamma_N}{2} \times \delta\left(\tilde{y}-1-\frac{\varepsilon+\gamma_N p_z}{m_N}\right)|_{p_0=m_N+\varepsilon} + O\left(\frac{\vec{p}^4}{m_N^4}\right) + O\left(\frac{\varepsilon_D}{m_N}\right) . \tag{120}$$

which differs from the result of Ref. [25] only by a $\tilde{y} \longrightarrow \tilde{y} - \varepsilon/m_N$ substitution.

B. Neutron structure functions

V. DISCUSSION AND CONCLUSIONS

APPENDIX A: LONGITUDINAL AND TRANSVERSE STRUCTURE FUNCTIONS

TO BE CHANGED, to reflect latest projections of the hadronic tensor

Let's define the transverse and longitudinal photon polarization vectors $\epsilon_\pm^\mu$ and ϵ_L^μ such that

$$\begin{aligned}
&\epsilon_\pm^* \cdot \epsilon_\pm = -1 \\
&\epsilon_L^* \cdot \epsilon_\pm = 1 \\
&\epsilon_\pm \cdot q = \epsilon_L \cdot q = 0 .
\end{aligned} \tag{A1}$$

In the A-frame defined in Eq. (59) they read

$$\begin{aligned}
\epsilon_\pm^\mu &= \frac{1}{\sqrt{2}}(0,1,\pm i,0) \\
\epsilon_L^\mu &= -\frac{q^+}{Q}\overline{n}^\mu + \frac{q^-}{Q}n^\mu
\end{aligned} \tag{A2}$$

Then we can define the transverse and longitudinal projection tensors

$$\begin{aligned}
d^{\mu\nu} &= \sum_{\lambda=\pm}\epsilon_\lambda^{*\mu}\epsilon_\lambda^\nu = \overline{n}^\mu n^\nu + n^\mu \overline{n}^\nu - g^{\mu\nu} \\
l^{\mu\nu} &= \epsilon_L^{*\mu}\epsilon_L^\nu ,
\end{aligned} \tag{A3}$$

which satisfy $d_{\mu\nu}d^{\mu\nu} = l_{\mu\nu}l^{\mu\nu} = 1$ and $d_{\mu\nu}l^{\mu\nu} = 0$. The transverse and longitudinal structure functions are defined by

$$\begin{aligned}
W_A^{\mu\nu}(x_B,Q^2) &= d^{\mu\nu}F_{TA}(x_B,Q^2) + l^{\mu\nu}F_{LA}(x_B,Q^2) \\
W^{\mu\nu}(x_B,Q^2) &= d^{\mu\nu}F_T(x_B,Q^2) + l^{\mu\nu}F_L(x_B,Q^2) .
\end{aligned} \tag{A4}$$

Using these definitions in Eq. (57)-(58) it is straightforward to show that

$$\begin{aligned}
F_{TA}(x_B,Q^2) &= 2F_{1A}(x_B,Q^2) \\
F_{LA}(x_B,Q^2) &= -F_{1A}(x_B,Q^2) + \frac{1+\delta_A}{2x_B}F_{2A}(x_B,Q^2) ,
\end{aligned} \tag{A5}$$

and

$$\begin{aligned}
F_{T,N/A}(x_N,Q^2,p^2) &= 2F_{1,N/A}(x_N,Q^2,p^2) \\
F_{L\,N/A}(x_N,Q^2,p^2) &= -F_{1,N/A}(x_N,Q^2,p^2) \\
&\quad + \frac{1+\delta_N}{2x_N}F_{2,N/A}(x_N,Q^2,p^2) .
\end{aligned} \tag{A6}$$

From Eq. (69), we obtain the convolution formula for the nuclear structure functions in terms of nucleon structure functions:

$$\begin{aligned}
F_{TA}(x_B,Q^2) &= \int d\mu_A \Big\{\frac{5\Delta-1}{4}F_{T,N/A}(x_N,Q^2,p^2) \\
&\quad + \frac{\Delta-1}{2}F_{L\,N/A}(x_N,Q^2,p^2)\Big\} \\
F_{LA}(x_B,Q^2) &= \int d\mu_A \Big\{\frac{\Delta-1}{8}F_{T,N/A}(x_N,Q^2,p^2) \\
&\quad + \frac{7\Delta-3}{4}F_{L\,N/A}(x_N,Q^2,p^2)\Big\} .
\end{aligned} \tag{A7}$$

In the Bjorken limit,

$$\begin{aligned}
F_{TA}(x_B,Q^2) &= \int d\mu_A F_{T,N/A}(x_N,Q^2,p^2) \\
F_{LA}(x_B,Q^2) &= \int d\mu_A F_{L,N/A}(x_N,Q^2,p^2) ,
\end{aligned} \tag{A8}$$

without flux factor y. The computation of $F_{T,L}$ including target mass corrections is described in Ref. [43].

[AA] Other points to discuss:

- Diagonalization of (A7) to obtain decoupled nuclear smearing
- Extraction of suitable combination of $F_{T,L/A}$ such that $\alpha F_{TA} + \beta F_{LA} = \int dy F_{L\,N}(x_B/y)$.

Acknowledgments

We thank G. Miller, M. Strikman, V. Guzey, S. C. Pieper, G. Sterman, B. Wiringa for informative discussions. We acknowledge support from DOE grant DE-FG02-87ER40371. **[AA] any other grant/person?**

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