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MEC 208 Fluids Engineering

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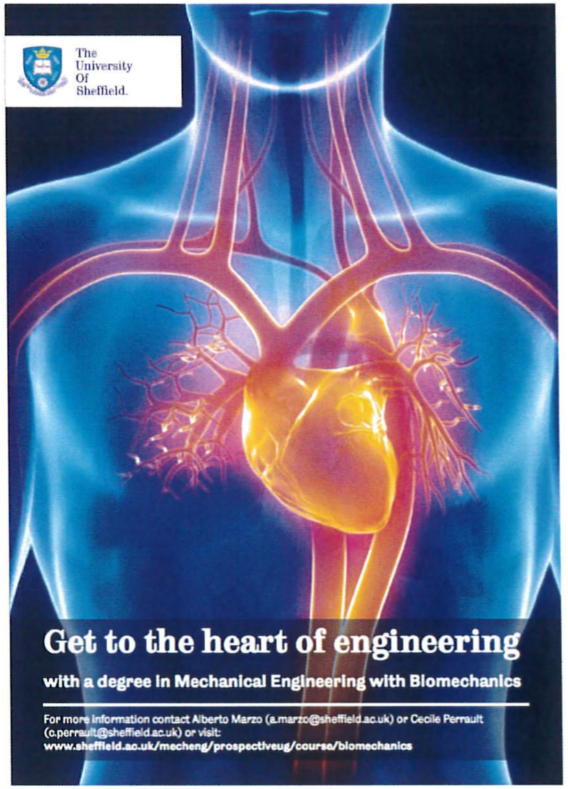
Before we start

- Mechanical Engineering with Biomechanics:
  - Time to register !



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Before we start

- Finishing topic 5 today and REVISION

- Week 12: your questions !

Responses 9

Polling Closed

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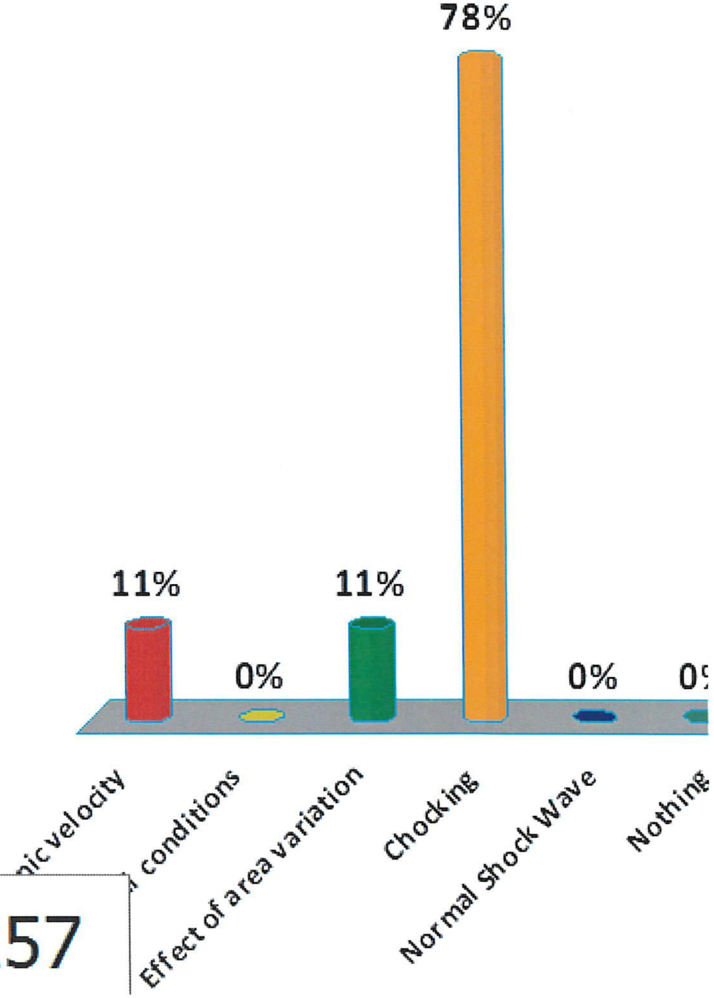
SESSION #: 744257

Least understood part of the lecture

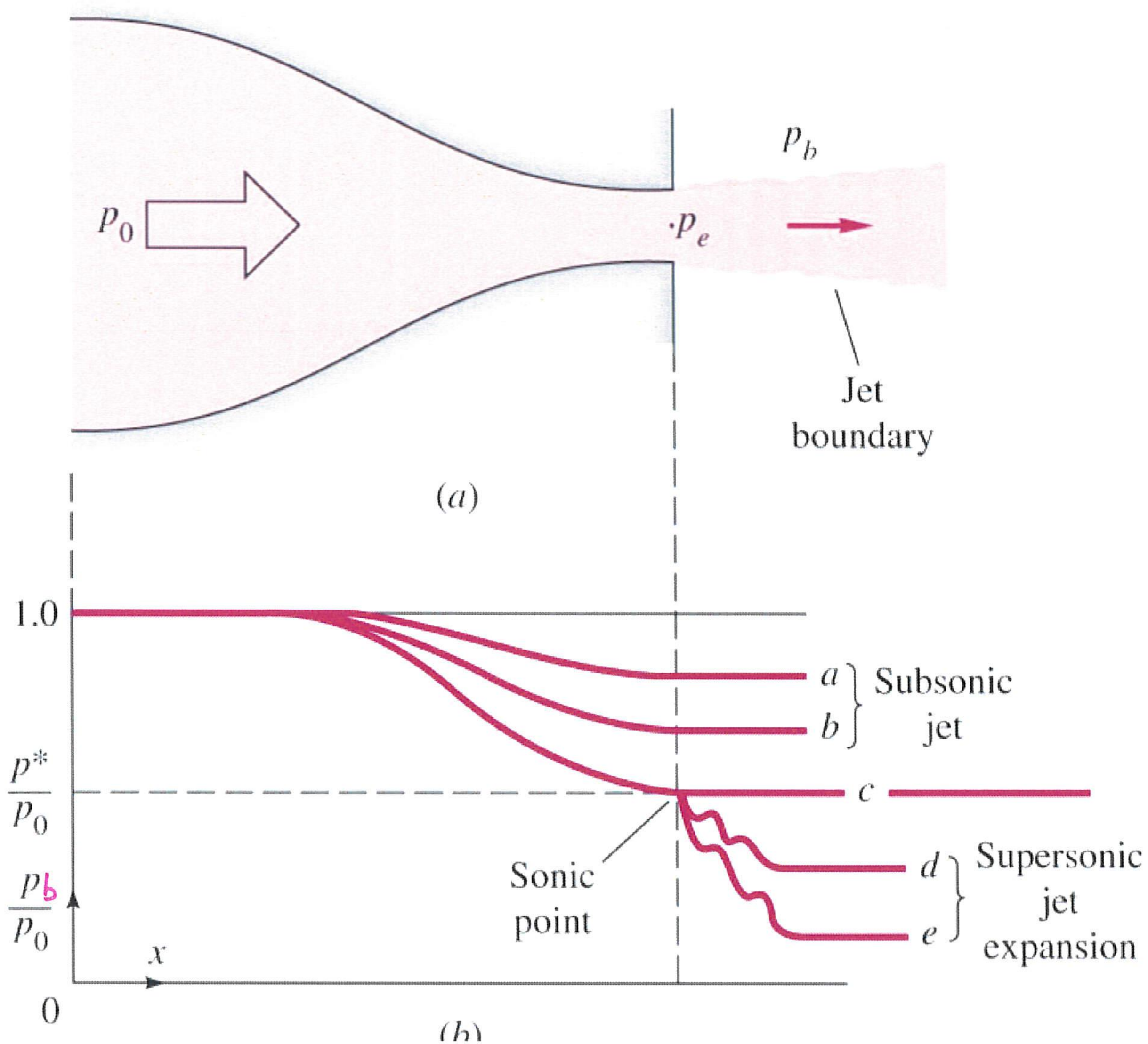
A. Sonic velocity
B. Critical conditions
C. Effect of area variation
D. Chocking
E. Normal Shock Wave
F. Nothing



Session ID 744257

5.10. Choking

Consider isentropic flow through a convergent nozzle. If the back pressure p_b is reduced, what happens?



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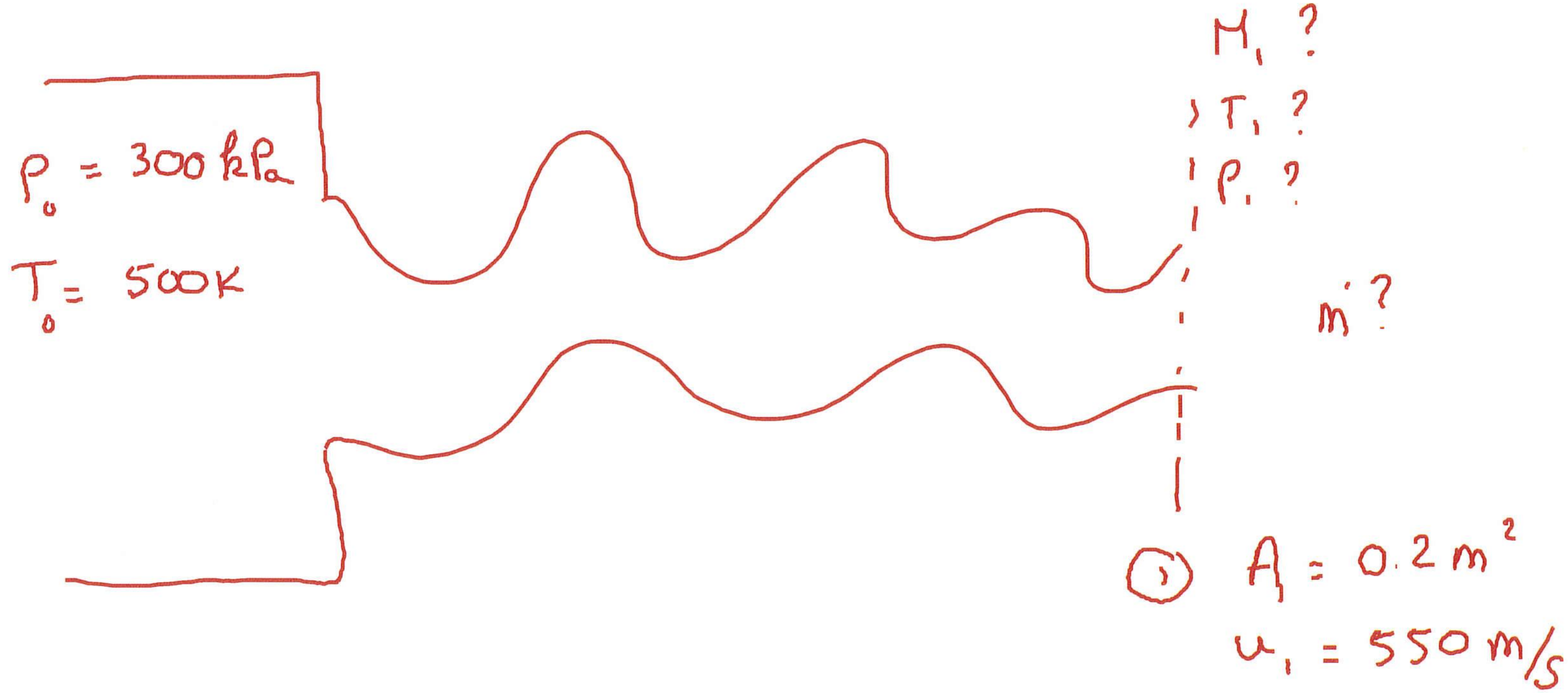
Example 5.6. Supersonic Expansion

Air is flows isentropically from a reservoir, where p=300 kPa and T=500K to a section 1 in a duct where A_1=0.2 m^2 and u_1=550 m/s. Determine (a) M_1, (b)T_1, (c) p_1, (d) $\dot{m}$. Is the flow choked?



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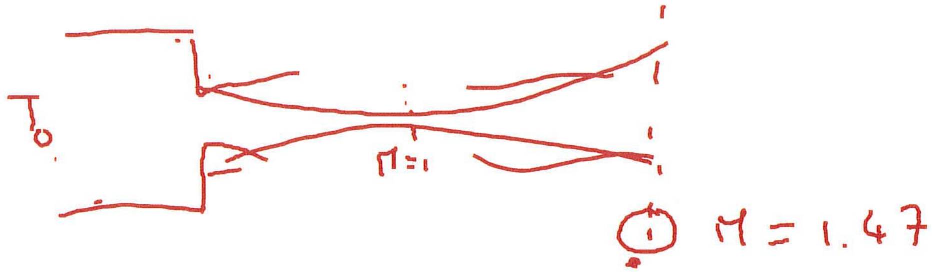

The air in the reservoir is at stagnation conditions (velocity is zero). So p_0=300 kPa and T_0=500K. For an adiabatic process $T_0 = T_{01}$

So applying

$$\frac{T_{01}}{T_1} = 1 + \frac{u_1^2}{2c_pT_1} \qquad \frac{500}{T_1} = 1 + \frac{550^2}{2 \times 993 \times T_1} \quad \text{gives } T_1 = 368\ \text{K}$$

347,68 K

Now we can determine a_1;

373.764 m/s

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 368} = \cancel{374\ \text{m/s}}$$

So $M_1 = \dfrac{u_1}{a_1} = \dfrac{550}{374} = 1.47$

Therefore at 1 the flow is supersonic. Since in the reservoir the flow was at stagnation, there must be a throat somewhere between the reservoir and 1. The flow at that throat must then be sonic so the nozzle is choked.

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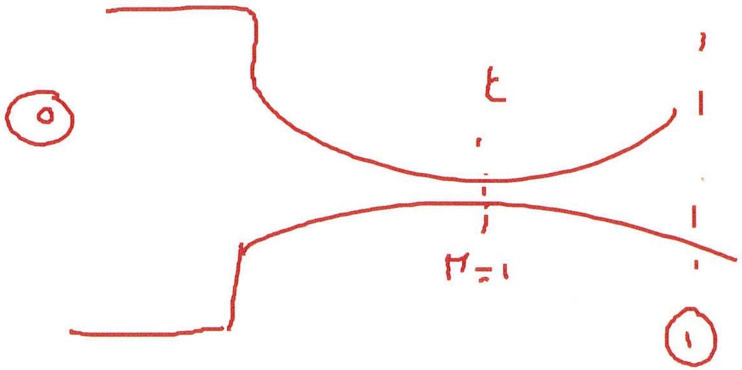

Now for the isentropic process 0 to 1

$$\frac{P_0}{P} = \left[1+\left(\frac{\gamma-1}{2}\right)M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{360\times 10^3}{P_1} = \left[1+\left(\frac{1.4-1}{2}\right)1.47^2\right]^{\frac{1.4}{0.4}}$$

$$P_1 = \cancel{86\ kPa}$$
$$85.33\ kPa$$

We use the perfect gas equation to find the density;

$$\rho_1 = \frac{P_1}{RT_1} = \frac{86\times 10^3}{287\times 368} = 0.861\ kg/m^3$$

and hence the mass flow rate;

$$\dot{m} = \rho_1 u_1 A_1 = 0.861\times 550\times 0.2 = \cancel{94.7\ kg/s}$$
$$93.87\ kg/s$$

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$$M = \frac{u}{a} = 1 \Rightarrow u^* = a^*$$

using throat area and tables, use $\dot{m} = \rho_c u_c A_c = \rho^* u^* A^*$

$$\rho^* = \frac{p^*}{RT^*}$$

at $M = 1.0$

$$\frac{P_0}{P^*} = 1.89 \qquad \frac{T_0}{T^*} = 1.20$$

$$p^* = \frac{300 \times 10^3}{1.89} = 158.730 \times 10^3 \text{ Pa}$$

$$T^* = \frac{500}{1.20} = 416.67 \text{ K} \Rightarrow a^* = \sqrt{1.4 \times 287 \times T^*} = 409.166 \text{ m/s}$$

$$\rho^* = \frac{p^*}{RT^*} = \frac{158.730 \times 10^3}{287 \times 416.67} = 1.32 \text{ kg/m}^3$$

from $M = 1.47$, $\frac{A^*}{A} = 0.865$ $\qquad A^* = 0.173 \text{ m}^2$

$$\dot{m} = \rho^* a^* A^* = 1.32 \times 409.166 \times 0.173 = 93.44 \text{ kg/s}$$

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5.11. Normal Shock Waves

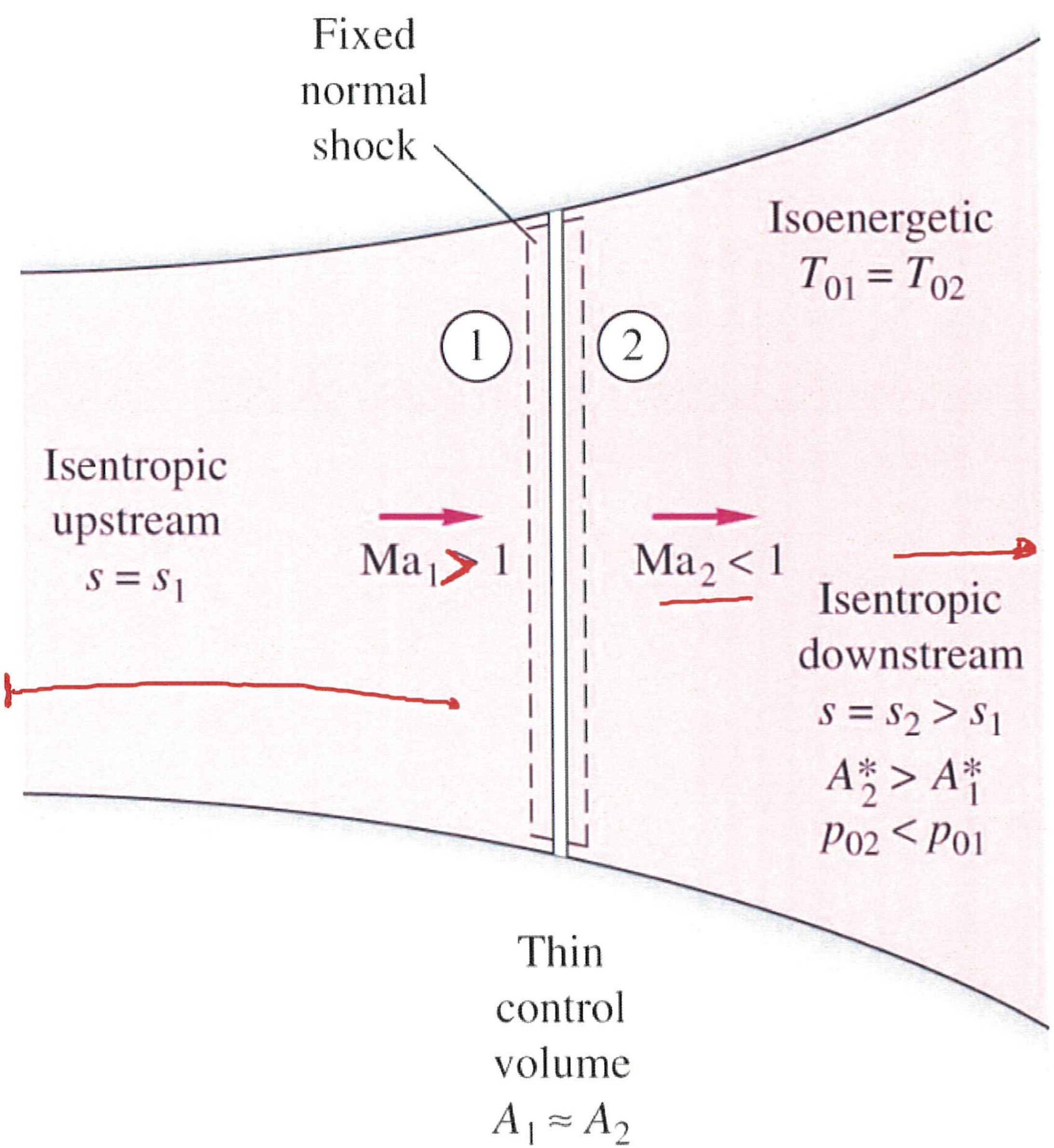
A shock wave is a rapid discontinuity of flow properties. The velocity suddenly drops over a few micrometers. Upstream $M>1$ of the shock whilst downstream $M<1$. Because it occurs over a very short distance there is no time for heat transfer to occur – so the shock wave is adiabatic. This means that (from the SFEE):

$$h_{01} = h_{02}$$

And so:

$$T_{01} = T_{02} \cdot$$

However it is not reversible (there are friction losses). So it is not isentropic (i.e. equations 5.13 and 5.14 don't apply). So how do the properties change across a shock wave? Consider a CV around a stationary shock wave.



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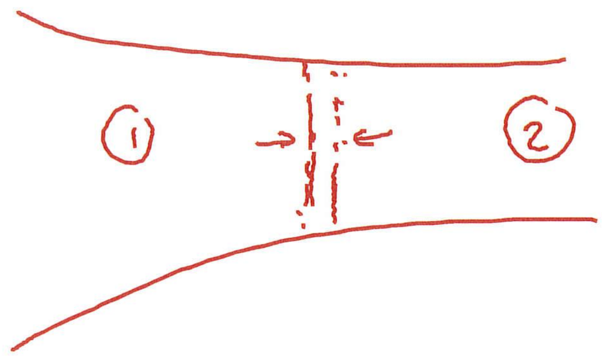

Applying the MCE:

$$\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \qquad (5.32)$$

Applying the FME:

Forces on the CV

$$P_1 A_1 - P_2 A_2$$

$$\sum \dot{M}_{in} = \dot{m} u_1$$

$$\sum \dot{M}_{\cancel{in}\ out} = \dot{m} u_2$$

$$P_1 A_1 - P_2 A_2 = \dot{m}(u_2 - u_1) \quad ; \; A_1 = A_2 \qquad (5.33)$$

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Combining (5.32) and (5.33) gives:

$$u_2^2 - u_1^2 = (p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right)$$

Since it is adiabatic:

$$h_1 - h_2 = \frac{1}{2}\left(u_2^2 - u_1^2\right)$$

[see p 5.2 of notes]

Gives

$$h_2 - h_1 = \frac{(p_2 - p_1)}{2}\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right) \tag{5.34}$$

This is called the Rankin-Hugoniot relation. Note that it is independent of the equation of state for the gas. If we assume the gas is perfect then:

$$h = c_p T = \frac{\gamma p}{(\gamma - 1)\rho}$$

Using this to replace h in equation (5.34) gives:

$$\frac{\rho_2}{\rho_1} = \frac{1 + \beta\left(\dfrac{p_2}{p_1}\right)}{\beta + \left(\dfrac{p_2}{p_1}\right)} \quad \text{where } \beta = \left(\frac{\gamma + 1}{\gamma - 1}\right) \tag{5.35}$$

Compare this with an isentropic change in pressure (e.g. a weak pressure wave):

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \tag{5.36}$$

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Entropy change across the shock wave (a measure of irreversibility)

Remember:

$$Tds = dh - \frac{dp}{\rho}$$

$$\int ds = \int \frac{dh}{T} - R \int \frac{dp}{p} \qquad \rho T = \frac{p}{R}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = c_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}$$

$$s_2 - s_1 = c_v \ln \left[\frac{p_2}{p_1} \left(\frac{\rho_1}{\rho_2} \right)^{\gamma} \right] \qquad (5.37)$$

The table below (for the case where γ=1.4) uses the non-isentropic relationship for the pressure change across the shock wave compared with an isentropic change.

	$\dfrac{\rho_2}{\rho_1}$		
$\dfrac{p_2}{p_1}$	Irreversible Equation (~~5.28~~) 5.35	Isentropic Equation (~~5.29~~) 5.36	$\dfrac{s_2 - s_1}{c_v}$ 5.37 Equation (~~5.30~~)
0.5	0.6154	0.6095	-0.0134
.9	0.9275	0.9275	-0.00005
1.0	1.01	1.0	0
1.1	1.00704	1.00705	0.00004
1.5	1.3333	1.3359	0.0027
2.0	1.6250	1.6407	0.0134

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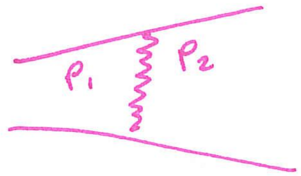

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0.5	0.6154	0.6095	-0.0134
.9	0.9275	0.9275	-0.00005
1.0	1.01	1.0	0
1.1	1.00704	1.00705	0.00004
1.5	1.3333	1.3359	0.0027
2.0	1.6250	1.6407	0.0134

- A decrease in enthalpy i.e. $(s_2-s_1)<0$ is not possible – violates the 2nd law of thermodynamics. This means that the pressure must always increase across a shock wave.
- For the shock wave the pressure ratio is greater than its isentropic value
- For small pressure ratios the difference is negligible and the shock may be considered as reversible and isentropic.

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5.12. Property Changes across a Shock Wave & Shock Tables

(a) Temperature Ratio

We know $T_{01} = T_{02}$ since across the shock wave it is adiabatic.
Then:

$$T_{01} = T_{02}$$

$$\frac{T_2}{T_1} = \frac{T_2}{T_{02}} \quad \frac{T_{02}}{T_{01}} \quad \frac{T_{01}}{T_1}$$

But remember the definition of static and stagnation temperature $\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right) M^2$

$$\frac{T_2}{T_1} = \frac{1+\left(\frac{\gamma-1}{2}\right)M_1^2}{1+\left(\frac{\gamma-1}{2}\right)M_2^2} \tag{5.38}$$

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(b) Density Ratio

From MCE $\rho_1 u_1 = \rho_2 u_2$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{M_1 a_1}{M_2 a_2}$$

$$a = \sqrt{\gamma R T} \qquad \frac{a_1}{a_2} = \sqrt{\frac{T_1}{T_2}}$$

Substituting equation (5.38):

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{1+\left(\frac{\gamma-1}{2}\right)M_1^2}{1+\left(\frac{\gamma-1}{2}\right)M_2^2}\right]^{0.5} \qquad (5.39)$$

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(c) Static Pressure Ratio

5.34

From the FME equation (~~5.26~~):

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

Replacing ρ in this equation $\qquad \rho = \dfrac{p}{RT}$

And u=Ma

Gives: $\qquad p_1 + \dfrac{p_1}{RT_1} u_1^2 = p_2 + \dfrac{p_2}{RT_2} u_2^2$

$$u_1 = M_1 \sqrt{\gamma R T_1}$$
$$u_2 = M_2 \sqrt{\gamma R T_2}$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{5.40}$$

Notice how this is always greater than one. The static pressure increases across a shock wave.

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(d) Stagnation Pressure Ratio

Let's write:

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2} \quad \frac{p_2}{p_1} \quad \frac{p_1}{p_{01}}$$

Remember the relationship between static and stagnation conditions

$$\frac{p_0}{p} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}$$

Gives:

$$\frac{p_{02}}{p_{01}} = \frac{p_2}{p_1}\left[\frac{1+\left(\frac{\gamma-1}{2}\right)M_2^2}{1+\left(\frac{\gamma-1}{2}\right)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{5.41}$$

(e) Mach Number Ratio

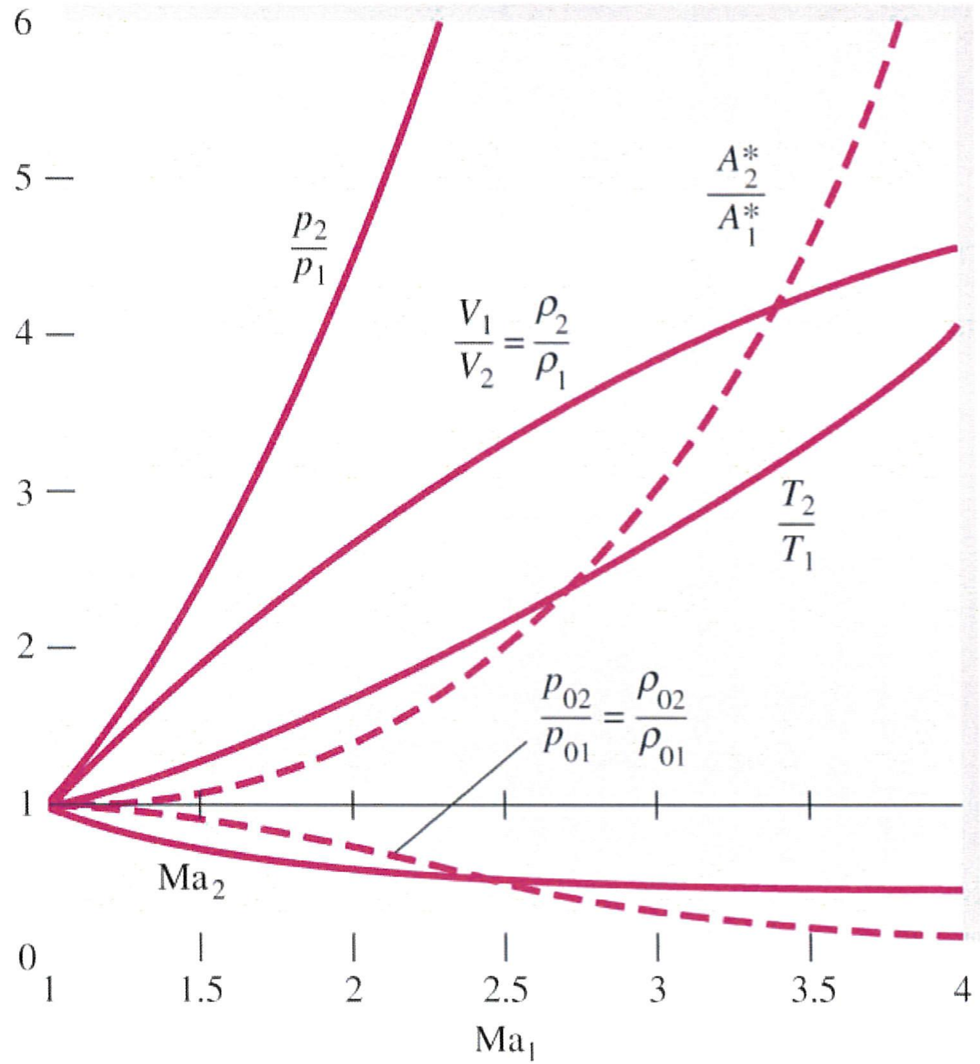
For a perfect gas $\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$

5.38 5.39 . 5.40

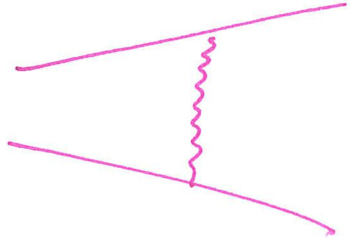
Combining (~~5.32~~), (5.~~3~~2), (5.~~33~~) gives:

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\left(\frac{2\gamma}{\gamma - 1}\right) M_1^2 - 1} \tag{5.42}$$

These equations show how the properties of the gas change before and after a shock wave. The following picture shows this graphically.



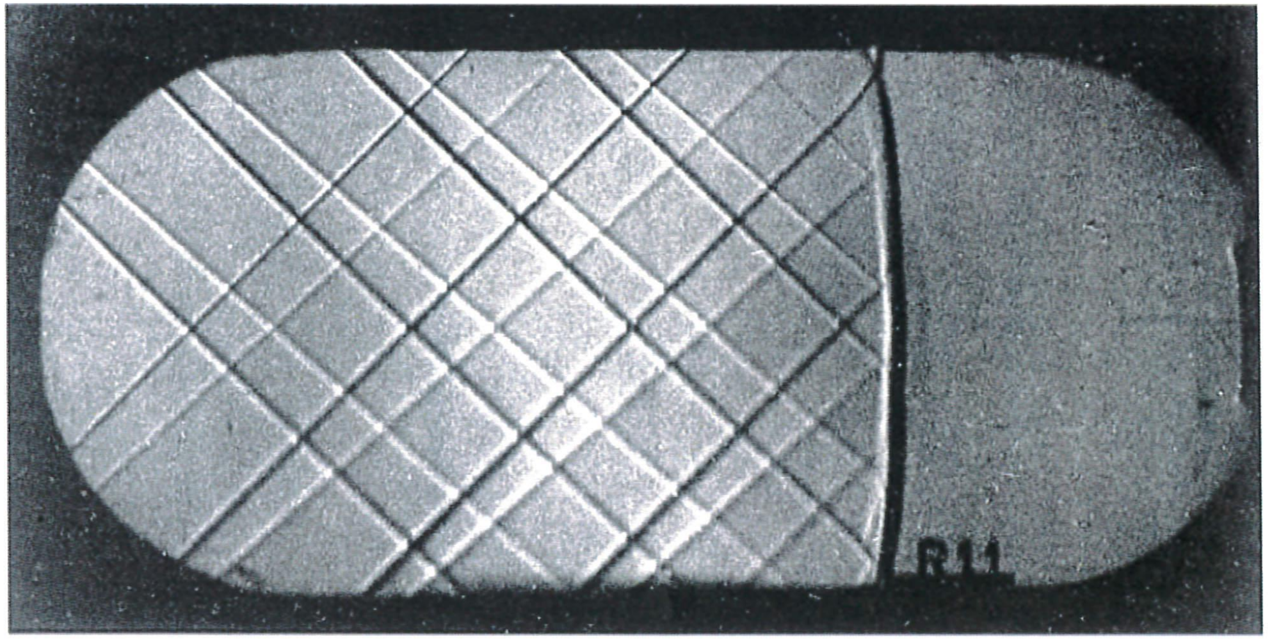
These relationships are also expressed in table form – known as *shock tables*. See the Little Book of Thermodynamics. The tables are presented for increasing values of M_1 (for a gas where γ=1.4). The downstream properties can then be calculated.



For shock waves we find:

- The upstream flow is supersonic – the downstream flow is subsonic
- Rarefaction (expansion) shocks are not possible. Only compression shocks (where the pressure increase) occur.
- The entropy increases across a shock. Stagnation pressure and stagnation density decrease.

The figure shows supersonic flow (left) and a shock wave forming in a tube. The criss-cross lines are caused by roughness and effects on the pipe wall – they are called Mach waves (they are not shock waves). The shock wave forms and past this the flow is sub-sonic – no Mach waves.



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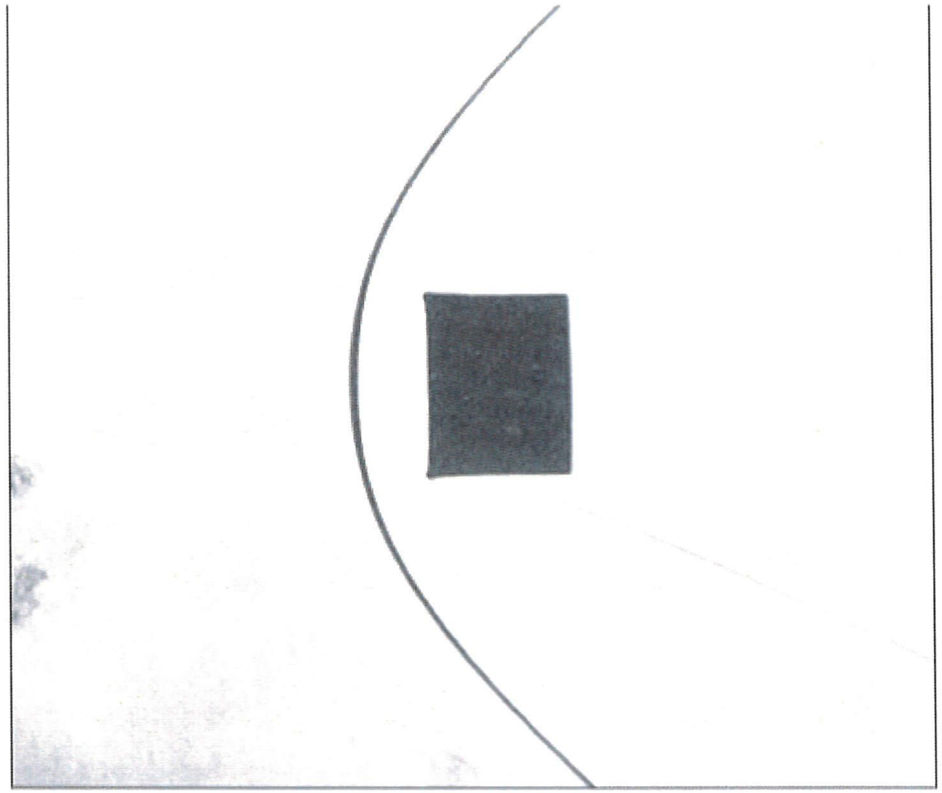
Shock waves also form in external flow. The figure shows a shock forming ahead of a bluff body in a supersonic air stream. The flow inside the shock is then sub-sonic. As the flow passes the corners of the body it speeds up to supersonic again and another shock forms on the sides of the body (this is called an oblique shock).



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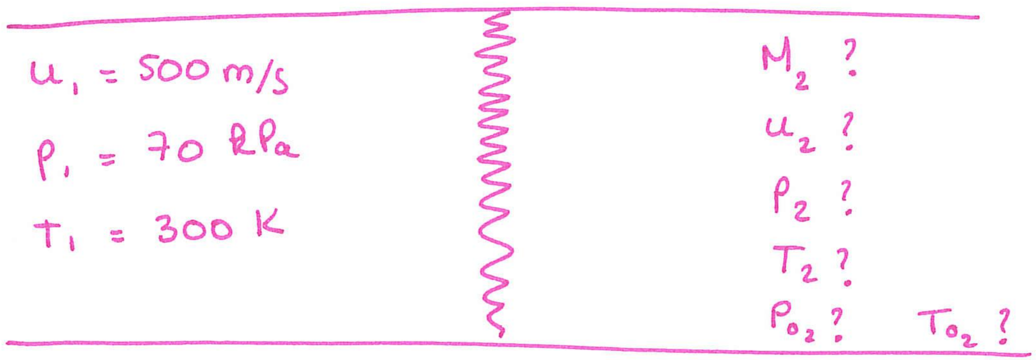
Example 5.7 – Property change across a shock

Air flowing at a velocity of 500 ms^{-1}, a static pressure of 70 kPa and a static temperature of 300 K undergoes a normal shock. For the location after the normal shock wave determine (a) Mach number, (b) velocity, (c) static pressure and temperature, and (d) stagnation pressure and temperature.



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Answer:

We are being asked to find M_2 , V_2, P_2, T_2, P_{02}, T_{02}

First find M_1 is required, hence find a_1

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 300} = 341.7 \text{ m/s} \qquad M_1 = \frac{u_1}{a_1} = 1.46$$

We can proceed in two ways, **either** using equations for property changes across a shock (5.38) to (5.42) – **or** use the shock tables.

We'll do this by using the shock tables

Looking in the tables for when M_1=1.4~~4~~6 gives:

$$\frac{p_2}{p_1} = 2.32 \qquad \frac{\rho_2}{\rho_1} = 1.793 \qquad \frac{T_2}{T_1} = 1.294 \qquad \frac{p_{02}}{p_{01}} = 0.942$$

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We can use mass continuity to find the velocity, u_2

$$\rho_1 u_1 = \rho_2 u_2 \qquad \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1}$$

$$u_2 = \frac{500}{1.793} = 278.86 \text{ m/s}$$

For the static temperature

$$\frac{T_2}{T_1} = \overset{1.294}{\cancel{1.793}} \qquad T_2 = 300 \times \overset{1.294}{\cancel{1.793}} = 537.9 \text{ K}$$

For the static pressure

$$\frac{p_2}{p_1} = 2.32 \qquad p_2 = 70 \times 2.32 = 162.4 \text{ kPa}$$

We now need the stagnation pressure and temperature. This is easily obtained from the static pressure and temperature and the Mach number using the *isentropic* equations (5.20) and (5.21) but this time lets use the isentropic flow tables.

Looking in the tables for when M_1=1.4$\cancel{4}$6 gives:

$$\frac{p_0}{p} = 3.465 \qquad \frac{T_0}{T} = 1.426$$

Starting with the temperatures:

$$\frac{T_{01}}{T_1} = 1.426 \qquad T_{0_1} = 300 \times 1.426 = 427.8 \text{ K}$$

But remember that flow across a shock is adiabatic so; $T_{0_1} = T_{0_2} = 427.8$ K

Now to find p_{02}. For the isentropic flow 01 to 1:

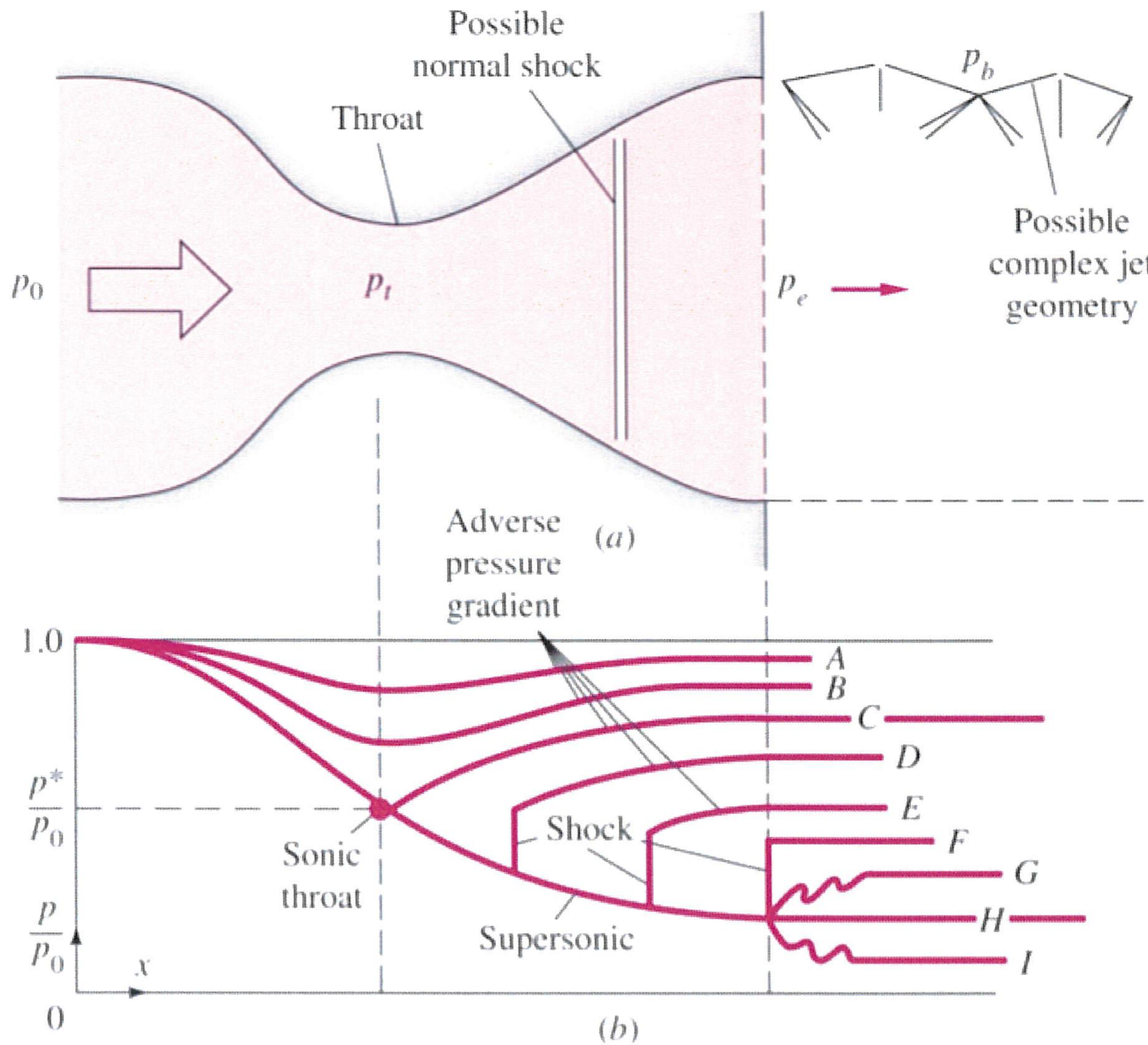
$$\frac{p_{01}}{p_1} = 3.465 \qquad P_{0_1} = 3.465 \times 70 = 242.55 \text{ kPa}$$

And for the flow across the shock:

$$\frac{p_{02}}{p_{01}} = 0.942 \qquad P_{0_2} = 0.942 \times 242.55 = 228.482 \text{ kPa}$$

5.13. Converging-Diverging Nozzles

We know that if we want to accelerate flow from sub-sonic to supersonic we have to have a converging-diverging duct. If the back pressure is low enough we will have super-sonic flow in the diverging part and a variety of shock wave conditions can occur.



Curves (0) no flow ($p_b=p_0$)

Curve (A, B) The back pressure is not low enough to cause sonic flow at the throat.

Curve (C) The velocity has reached the point that at the throat, $M_t=1$. The flow in the nozzle is sub-sonic throughout.

Curve (H) Here the back pressure p_b is such that the pressure ratio p_o/p_b exactly corresponds to the critical area ratio A_e/A_c for a supersonic M_e at the exit. The velocity has reached the point that at the throat, $M_t=1$. The flow in the diverging part of the nozzle is sonic throughout (and isentropic in the entire nozzle). This is called the *design pressure ratio* of the nozzle.

According to the isentropic flow equations it would not be possible for the p_b to lie between curves C & H.

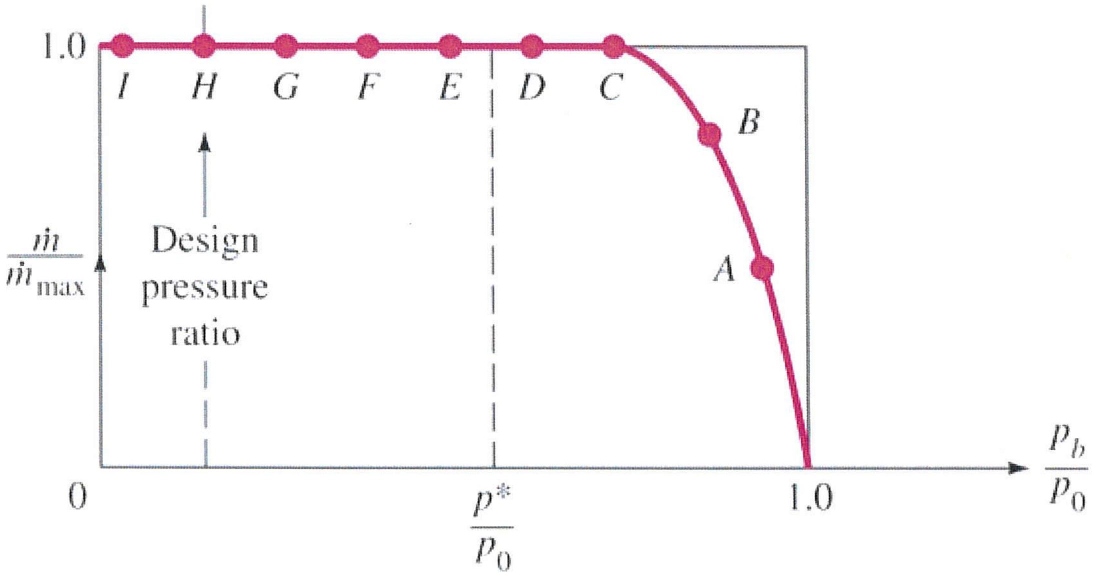
Curve (D,E) The throat remains choked at the sonic value, and a shock wave forms in the diverging part – this makes $p_e = p_b$. This means there is a sub-sonic diffuser flow to the back-pressure condition.

Curve (F) The shock wave stands right at the exit.

Curve (G) There is no place for a shock wave to form in the duct. Instead a series of oblique shocks forms outside the nozzle in the supersonic jet. The flow then compresses back up to p_b.

Curve (I) The back pressure is dropped further but the nozzle cannot respond. The exit flow expands in a complex pattern in the jet until it falls to p_b.

For all pressures below case C, the flow is choked and the mass flow has reached its maximum value.



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THAT'S IT !!