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MEC 208 Fluids Engineering

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4.7. Turbulent Boundary Layer on a Flat Plate

In most practical situations the laminar part of the boundary layer is usually small and may be neglected. The turbulent part of the layer is more important since it gives a greater overall contribution to the drag forces.

Analysis depends on experimental data. Assumptions about the velocity profile may be made in the same way as for laminar flow (see for example White p.400).

The most commonly used formulae for boundary layer thickness and skin-friction coefficient are:

$$\frac{\delta}{x} \approx \frac{0.16}{\mathrm{Re}_x^{1/7}} \tag{4.23}$$

turbulent flow.

empirical station

$$c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}} \tag{4.24}$$

$$C_D \approx \frac{0.031}{\mathrm{Re}_L^{1/7}} \tag{4.25}$$

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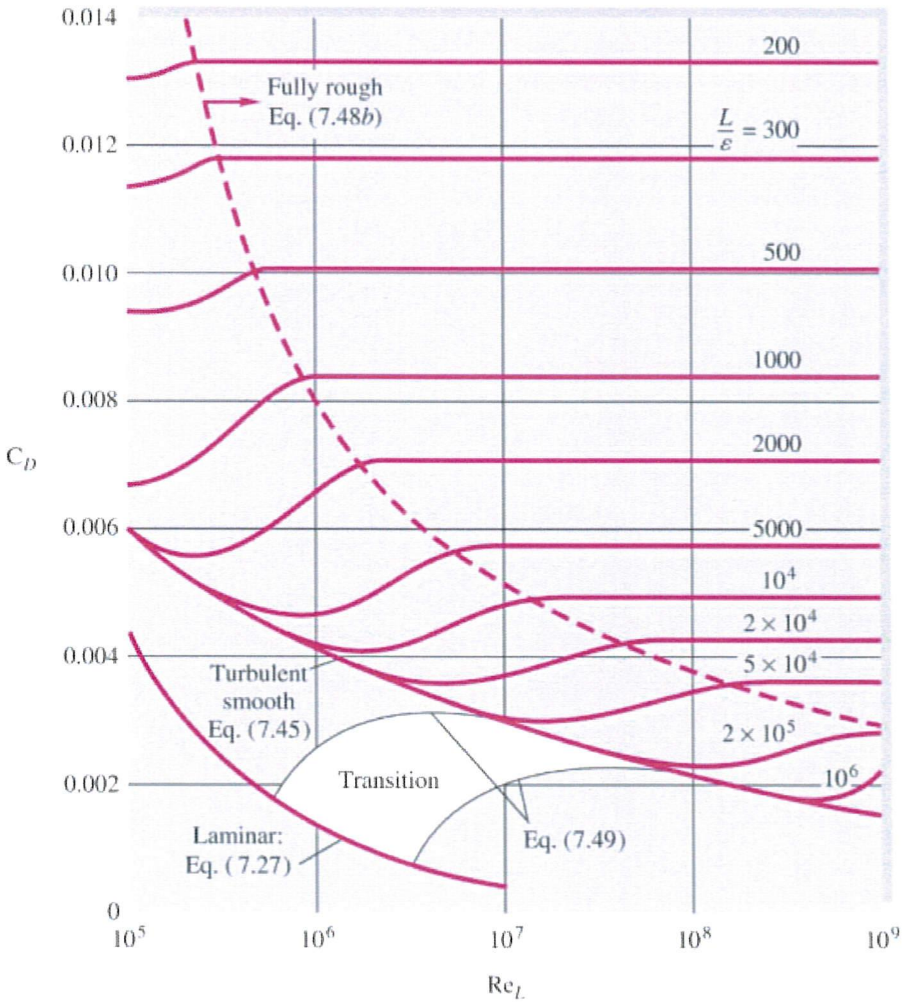
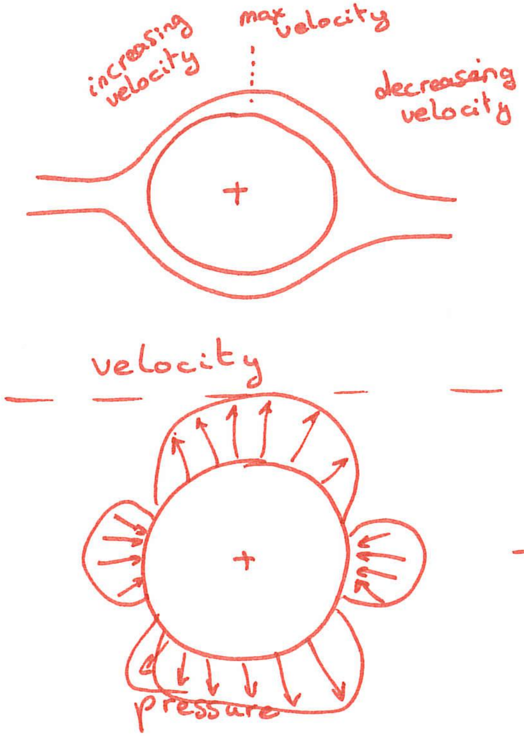
The above equations are for smooth flat plates. When roughness is involved the drag increases. The figure below shows flat plate drag coefficients, C_D for a laminar and turbulent flow. Notice how similar it is to the Moody chart (friction factors in pipes.).

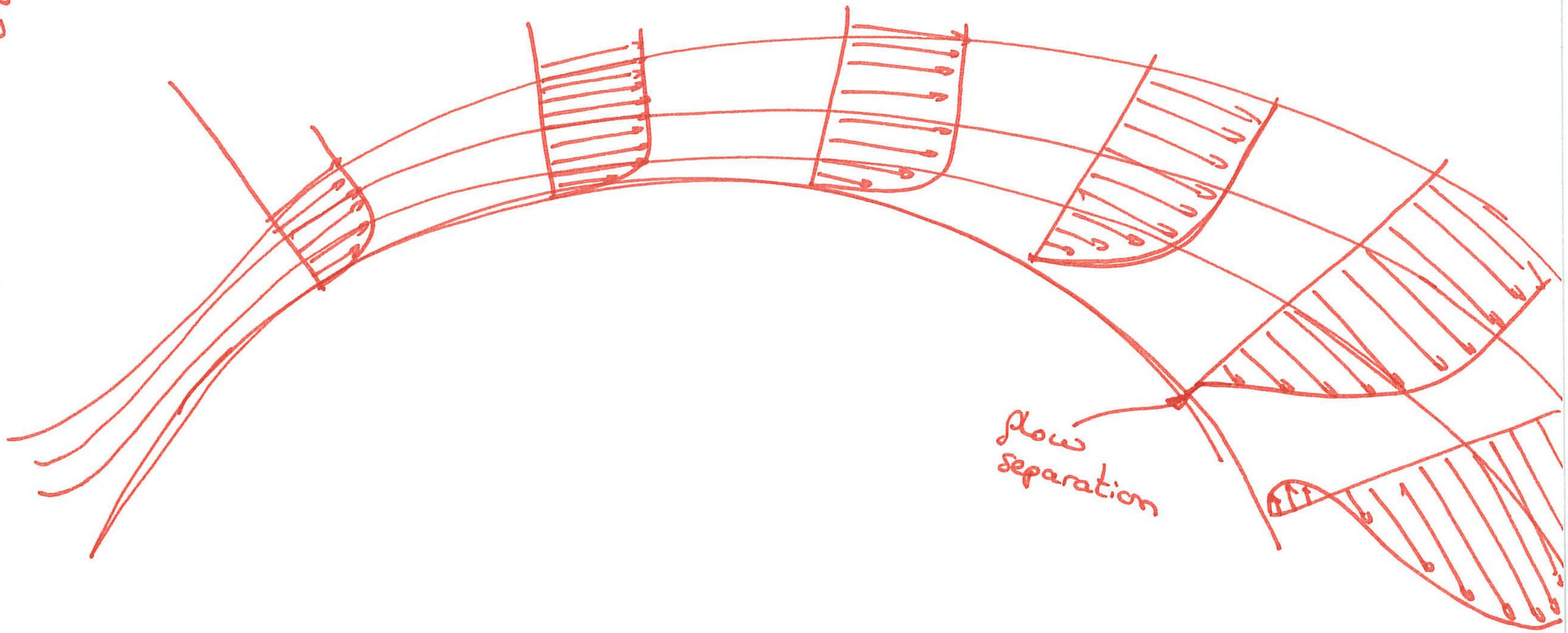


4.8. Pressure Gradient and Separation

The analysis so far has been for zero pressure gradient $\frac{dp}{dx} = 0$. This is suitable for flat bodies.

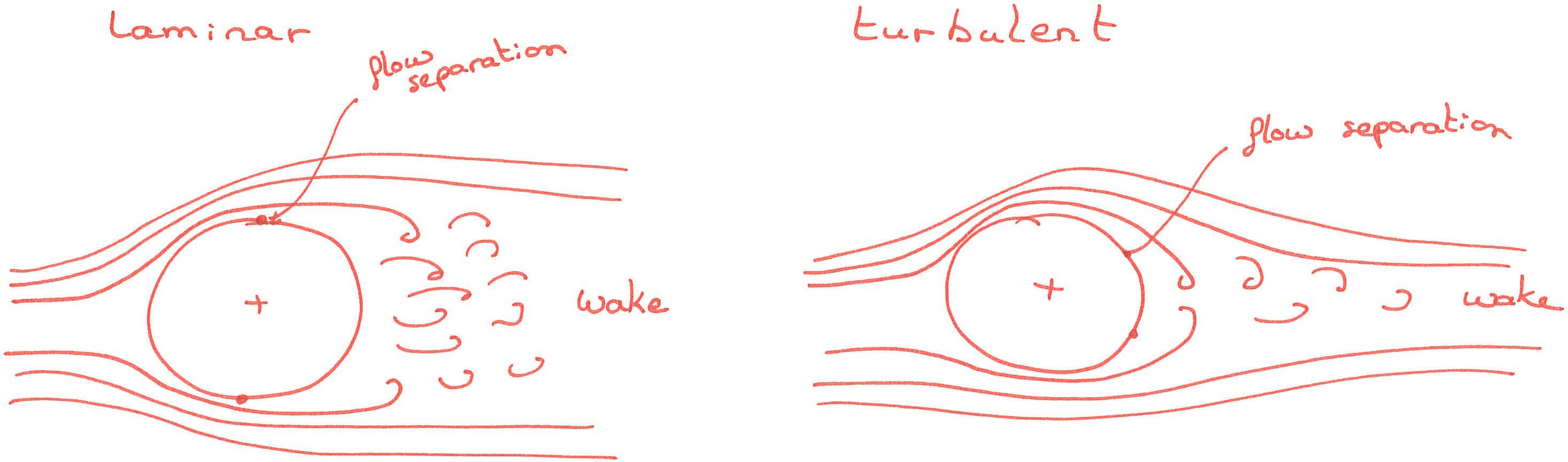
Consider the boundary layer in a curved body. There will be a pressure gradient as the fluid accelerates and decelerates over the body surface.





As the boundary layer grows the 'momentum loss' increases, (i.e. $\dot{M}_{out} - \dot{M}_{in}$ increases). An increasing pressure gradient acts to oppose this (i.e. F_x is negative). This causes the flow to reverse.

The point at which the flow first starts to reverse (is $\tau_w = 0$) is called the separation point. Separation causes the formation of a wake of disturbed fluid behind the body.



The pressure in the wake is approximately constant and less than that at the front of the body.Laminar flows are much more prone to separation. This is because the increase in velocity with distance along the body is much less rapid than turbulent flows. The adverse pressure gradient can then more easily halt the flow of fluid near the surface.

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4.9. Friction Drag and Pressure Drag

The drag forces we have been discussing so far are all associated with skin friction (i.e. τ_w acting over the surface). Drag also arises from the pressure difference between the front and back of the body.

$$C_D = C_{D_{pressure}} + C_{D_{friction}} \qquad (4.26)$$

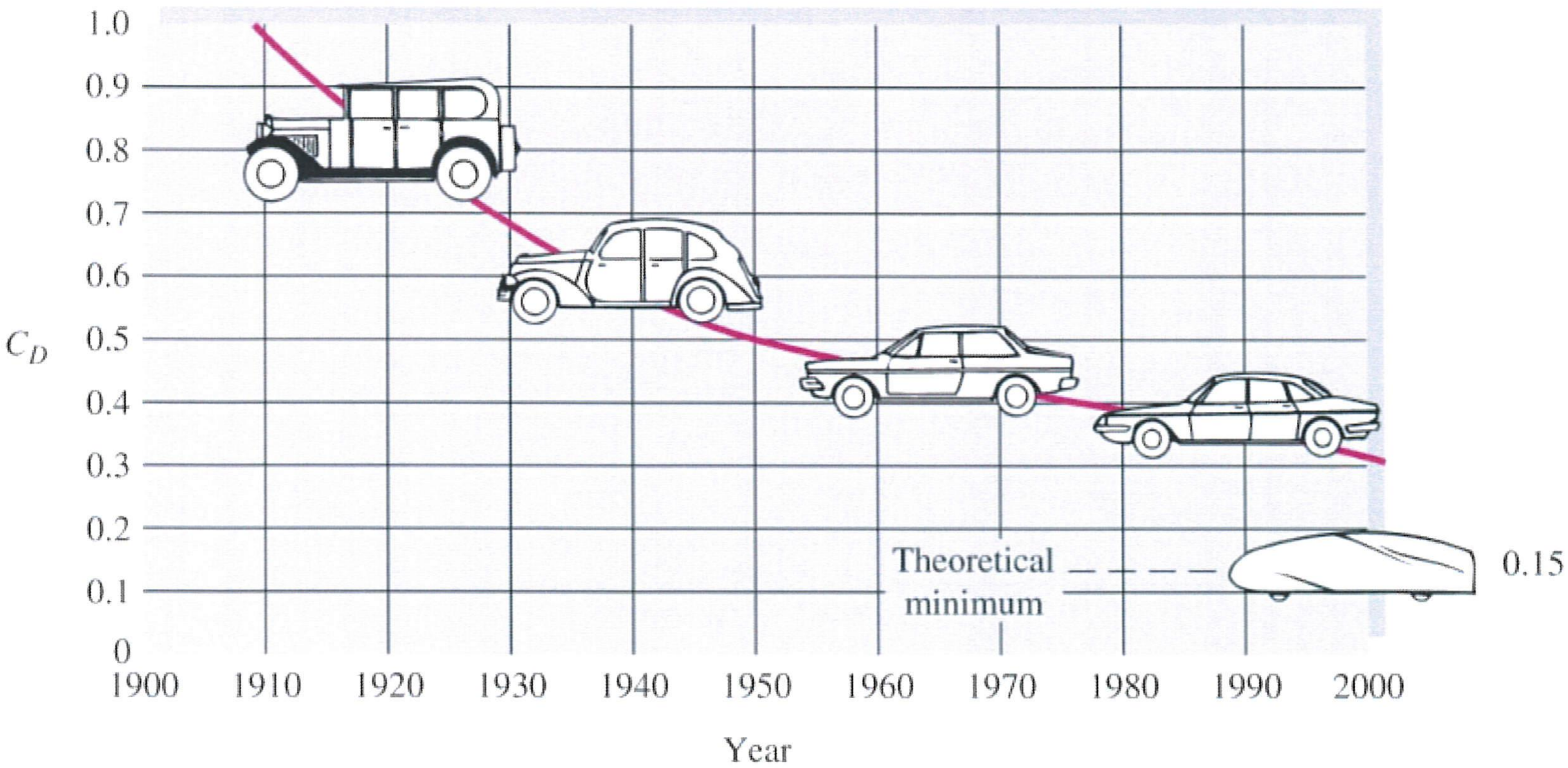
And often $C_{D_{pressure}} \gg C_{D_{friction}}$

For a body of arbitrary shape the definition of C_D is usually given in terms of the projected frontal area in the direction of the fluid flow,

$$\boxed{C_D = \frac{D}{\frac{1}{2}\rho U^2 A}} \qquad (4.27)$$

(Note for a flat plate equation 4.21, we used the plan area $A = bl$ and not the projected area - definitions may vary.)

The pressure drag is highly dependent on the wake (and thus the separation point). Bodies where the flow separates earlier show much higher pressure drag. Since the separation point and the properties of a wake are difficult to analyse, Drag coefficients are frequently determined empirically. The data is presented in the form of charts and tables.



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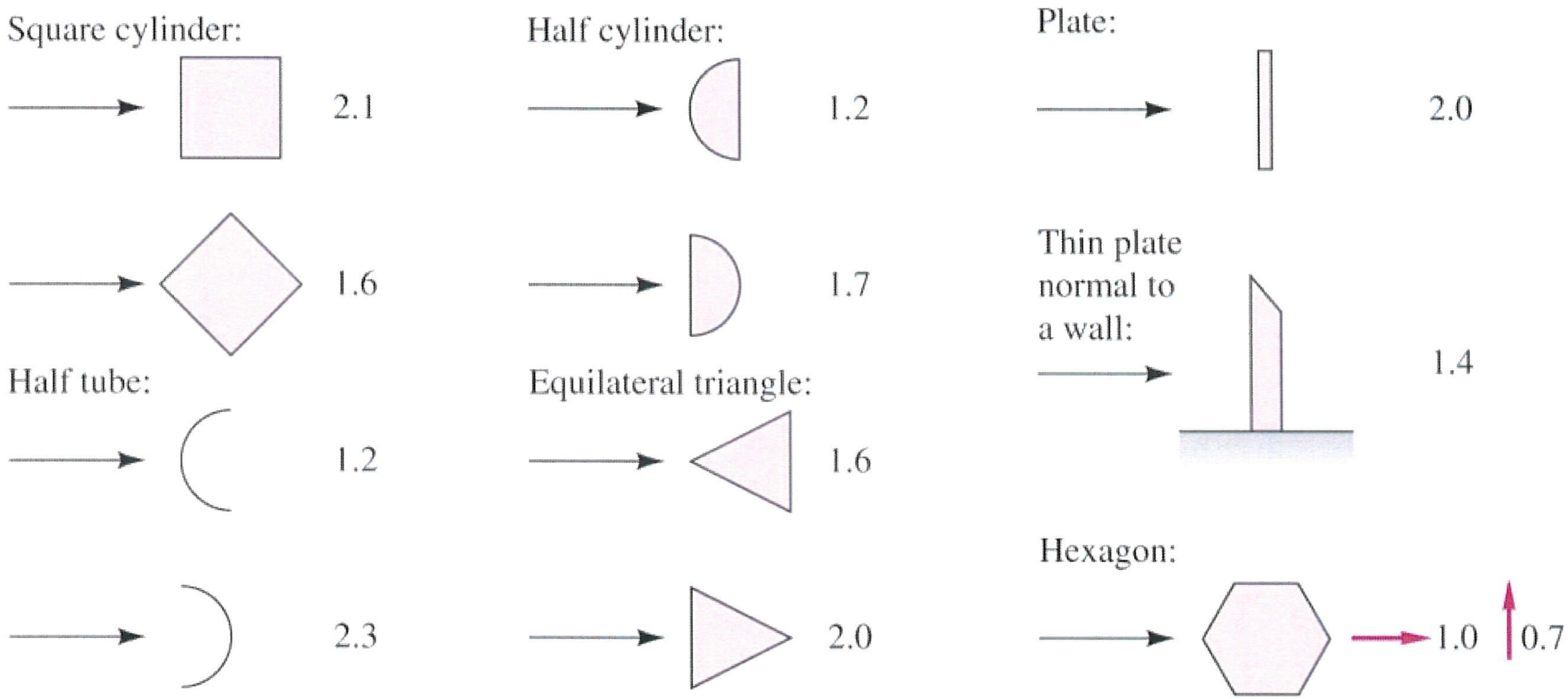
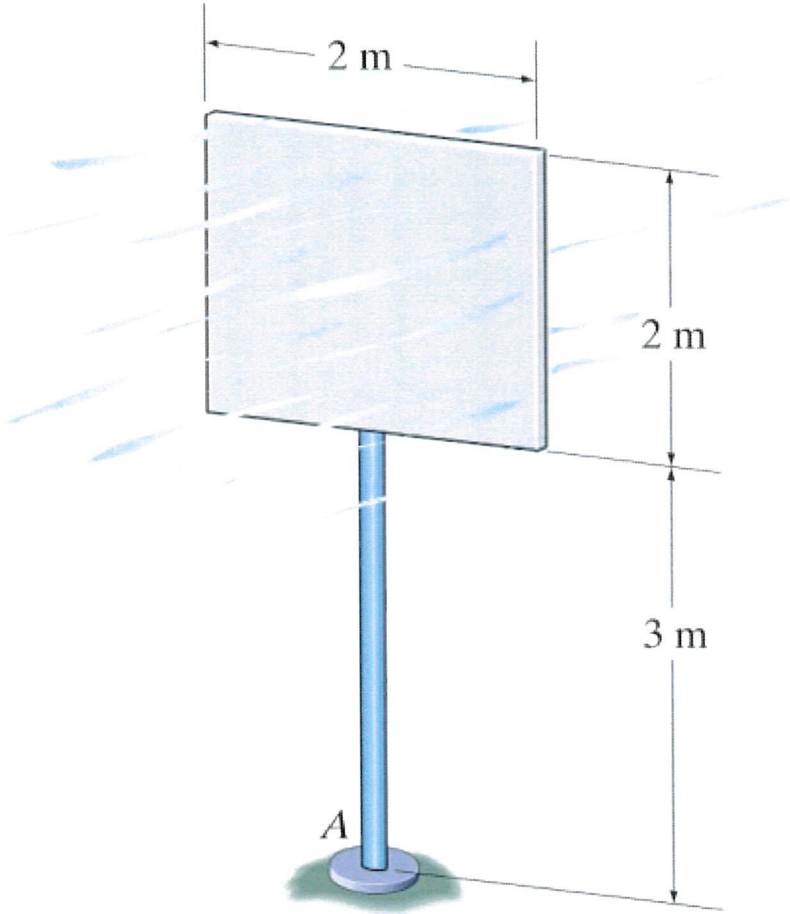


Example 4.2. Drag on a sign



Determine the moment developed at the base A of the square sign due to wind drag if the front of the signboard is subjected to a 16-m/s wind. The air is at 20°C. Neglect the drag on the pole.

The air is considered to be incompressible. The flow is steady.

$\nu = \frac{\mu}{\rho}$

From table in revision notes, $\rho = 1.202\ \mathrm{kg/m^3}$ and $\nu = 15.1(10^{-6})\ \mathrm{m^2/s}$ for air at $T = 20°C$.

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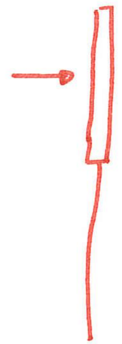
for $\frac{b}{h} = \frac{2\,m}{2\,m} = 1$, $C_D = 1.18$

here $A = 2\,m \times 2\,m = 4\,m$

from eq 4.27

$$F_D = C_D\left(\frac{1}{2}\rho U^2 A\right) = 1.18\left(\frac{1}{2}(1.202)(16)^2(4)\right) = 726,20\ N$$



here F_D will act through the center of the signboard.

Therefore, with respect to point A

$\curvearrowright + \Sigma M_A = 0$

$$M_A - 726.2\,(4\,m) = 0$$

$$M_A = 2.9 \times 10^3\ N.m$$

$$= 2.9\ kN.m$$

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5. Compressible Flow

(Massey §12, White §9, Potter §9)

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5.1. Introduction

So far we have considered incompressible flow (i.e. liquid or low speed gas flows). If fluid is moving close to the speed of sound density changes become significant.

The Mach Number, M - ratio of the fluid velocity to the speed of sound in the fluid;

$$M = \frac{u}{a} \qquad a = \text{velocity of sound} \tag{5.1}$$

For liquids it is difficult to cause a large density change (need pressures ~1000 atm). Liquids are virtually incompressible.

For gases even very low pressure ratios (2:1) can result in significant density changes. Typically for $M > 0.3$ we must consider compressibility effects.

In this course we will simplify the analysis by making three assumptions about the flow;

reversible - no losses due to friction

adiabatic - no heat transfer to / from fluid

perfect gas - use the perfect gas equation of state

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5.2. Thermodynamics Concepts

(a) **Perfect gas** (i.e. ideal gas). Ideal gas equation of state applies

$$p = \rho RT \qquad pV = mRT \tag{5.2}$$

E = internal energy

(b) **Enthalpy**. The combination of properties (E+pV) occurs so frequently that we define a new property; enthalpy, H, and specific enthalpy, $h = \dfrac{H}{m}$

$$H = E + pV \qquad h = e + \frac{p}{\rho} \tag{5.3}$$

units J/kg

(5.4)

(c) **Specific Heat Capacities.**

$$de = c_v dT \quad \& \quad dh = c_p dT \tag{5.5}$$

For a perfect gas the specific heats are constant so;

$$e_2 - e_1 = C_v (T_2 - T_1) \quad \& \quad h_2 - h_1 = C_p (T_2 - T_1) \tag{5.6}$$

units $J/kg.K$

$$\gamma = \frac{c_p}{c_v} \quad \& \quad R = c_p - c_v \tag{5.7}$$

(d) **Reversible Process**. If a process can be reversed by returning all the heat or work lost/gained during the process - known as a reversible process. Viscous (fluid friction) effects are a source of irreversibility.

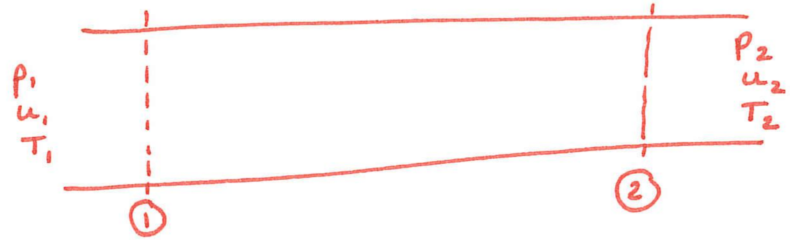
(e) **Isentropic Process**. A process where the entropy is constant. For this to happen $\Delta Q=0$ so the process must be adiabatic and frictionless (i.e. reversible). For an isentropic process from 1 to 2 (i.e. $s_1=s_2$). For proofs see 1st year thermodynamics.

$$\frac{p_1}{p_2}=\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}=\left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \qquad (5.8)$$

The analysis of compressible flow is greatly simplified if we assume isentropic flow.

5.3. Stagnation Conditions

Consider a compressible flow between states 1 and 2;



The SFEE gives;

$$\frac{\dot{Q}-\dot{W}}{\dot{m}} = \left(\frac{p_2}{\rho_2} + \frac{u_2^2}{2} + gz_2 + e_2\right) - \left(\frac{p_1}{\rho_1} + \frac{u_1^2}{2} + gz_1 + e_1\right)$$

If the process is adiabatic and we have no shaft work $\dot{Q} = \dot{W} = 0$. For a gas since the density is low then we can neglect the gravitational effects (neglect gz_1 and gz_2). Then the SFEE becomes;

$$\left(\frac{p_2}{\rho_2} + \frac{u_2^2}{2} + e_2\right) - \left(\frac{p_1}{\rho_1} + \frac{u_1^2}{2} + e_1\right) = 0$$

or substituting $h = \dfrac{p}{\rho} + e$ gives

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad (5.9)$$

thus at any point in the flow the sum $h + \dfrac{u^2}{2}$ is constant.

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We define a new term the stagnation enthalpy, h_0.

$$h_0 = h + \frac{u^2}{2} \tag{5.10}$$

Thus if a fluid flow is reduced to rest adiabatically it will have an enthalpy equal to h_0 the stagnation enthalpy.

If the compressible fluid is also a perfect gas then we can include equation (5.5)

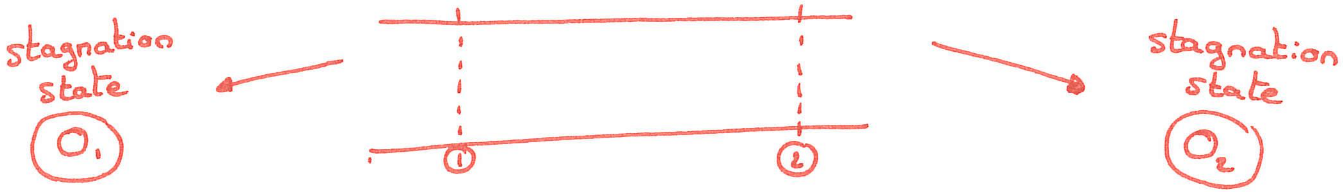
So, $$c_p T_0 = c_p T + \frac{u^2}{2} \quad \text{or} \quad T_0 = T + \frac{u^2}{2c_p}$$

Where T_0 is the *stagnation temperature*.

stagnation state:
imaginary process
where fluid is brought
to rest isentropically

$$\cancel{\frac{T_0}{T} = 1 + \frac{u^2}{2c_pT}} \qquad (5.11)$$

Thus for any adiabatic process between state 1 and 2 we can say;



and for a perfect gas (5.12)

$$T_{0_1} = T_{0_2}$$

$5.10 : h_0 = h - \frac{u^2}{2}$

$5.11 : \frac{T_0}{T} = 1 + \frac{u^2}{2 C_p T}$

$5.8 : \frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma}$

5.4. Isentropic Pressure and Density Relations

The expressions (5.10) and (5.11) requires the flow to be adiabatic. If the flow is also isentropic, then we can use the equation (5.8) to find relations for pressure and density;

For an isentropic process we can relate the temperature change to the pressure change (or density change using equation (5.8);

$$\left(\frac{P_1}{P_2}\right) = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \qquad \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

So for combining this with (5.11) gives a relationship between the static pressure and the stagnation pressure.

$$\boxed{\frac{p_0}{p} = \left(1 + \frac{u^2}{2c_p T}\right)^{\frac{\gamma}{\gamma-1}}} \tag{5.13}$$

$$\boxed{\frac{\rho_0}{\rho} = \left(1 + \frac{u^2}{2c_p T}\right)^{\frac{1}{\gamma-1}}} \tag{5.14}$$

The quantities p_0 and ρ_0 are the *stagnation pressure and density*. They represent the pressure and density which would be achieved if the flow were brought isentropically to rest.

The stagnation values (p_0, T_0, h_0 and ρ_0) are useful reference conditions in a flow.

Note: Don't get confused between static and stagnation. Static pressure & static temperature are the actual fluid properties (i.e. ordinary pressure and temperature). Symbols, p and T.

For any fluid state given by h, p, T, ρ there is a corresponding stagnation state h_0, p_0, T_0, ρ_0 which is achieved by an imaginary process where the fluid is reduced to rest isentropically.

The stagnation properties are simply another way of writing the static properties to incorporate the flow velocity.

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Example 5.1 Determine the Stagnation Conditions

Air flowing at 75 m/s is at a pressure of 140 kPa and 260°C. Determine the stagnation conditions.

air $\quad R = 287 \text{ J/kg K} \quad C_p = 997 \text{ J/kg K}$

The static conditions are;

$p = 140 \text{ kPa} \quad T = 260\ ^\circ\text{C}$

For the stagnation conditions p_0, T_0, and ρ_0. Imagine an *isentropic* process from static to stagnation conditions;

from (5.11) $\quad T_0 = T + \frac{u^2}{2 C_p} = 533 + \frac{75^2}{2(997)} = 535.8 \text{ K}$

from (5.13) $\quad P_0 = p\left(1 + \frac{u^2}{2 C_p T}\right)^{\frac{\gamma}{\gamma-1}} = 140 \times 10^3 \left(1 + \frac{75^2}{2(997)\,533}\right)^{\frac{1.4}{1.4-1}}$

$= 142.6 \text{ kPa}$

For the stagnation density;

<u>EITHER</u>: determine static density

from (5.2) $\rho = \frac{p}{RT} = \frac{140 \times 10^3}{287 \cdot 533} = 0.915 \ kg/m^3$

Then $\rho_0/\rho = \left(1 + \frac{u^2}{2 C_p T}\right)^{\frac{\gamma}{\gamma - 1}} = 1.013$

from (5.14)

$\rho_0 = 0.915\ (1.013) = 0.927 \ kg/m^3$

<u>OR</u>: quicker to determine the stagnation density by using the equation of state for stagnation conditions i.e.;

$$\rho_0 = \frac{p_0}{R T_0} = \frac{142.6 \times 10^3}{287 \cdot 535.8} = 0.927 \ kg/m^3$$

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Summary - Static and Stagnation Conditions

For the flow between two states 1 and 2;

(i) For any *adiabatic* compressible process, with no shaft work, Stagnation enthalpy is constant. The SFEE gives; $h_{01} = h_{02}$

(ii) If the fluid is a *perfect gas* then, $dh = c_p dT$, and stagnation temperature is constant $T_{01} = T_{02}$.

(iii) If the process from 1 to 2 is *reversible* (i.e. it is also *isentropic*), and both 01 to 1 and 2 to 02 are reversible, then;

$$\frac{T_{01}}{T_{02}} = \left(\frac{p_{01}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$

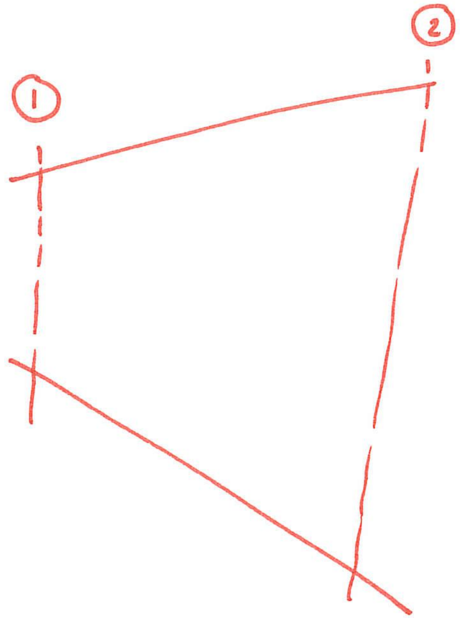
but since $T_{01} = T_{02}$ then $p_{01} = p_{02}$. So for a isentropic process the stagnation state does not change.

(iv) If the process from 1 to 2 is *irreversible* then p_0 and ρ_0 do not remain constant but vary throughout the flow as energy is lost to friction.

Example 5.2 Isentropic expansion of air

Air expands isentropically through a duct from p_1=125 kPa and T_1=100°C to p_2=80 kPa and u_2=325 m/s. Determine T_2, T_{01}, T_{02}, p_{01}, p_{02}, and u_1. Given that γ=1.4 and c_p=993 J/kg K.

$p_1 = 125$ kPa
$T_1 = 100$ °C
$u_1 = ?$



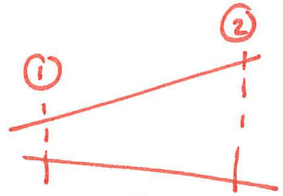
$p_2 = 80$ kPa
$u_2 = 325$ m/s
$T_2 = ?$

0_1

p_{0_1} ?
T_{0_1} ?

0_2

p_{0_2} ?
T_{0_2} ?



$P_1 = 125\,kPa$ $\quad$ $P_2 = 80\,kPa$

$T_1 = 100\,°C$ $\quad$ $T_2 = ?$

$u_1 = ?$ $\quad$ $u_2 = 325\,m/s$

For the *isentropic* process 1 to 2 we can use (5.8)

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad \frac{373}{T_2} = \left(\frac{125}{80}\right)^{\frac{0.4}{1.4}} \qquad T_2 = 328\,K$$

For the isentropic process 2 to 02 we can use (5.11)

$$\frac{T_{o_2}}{T_2} = 1 + \frac{u_2^2}{2\,C_p T_2} \qquad \frac{T_{o_2}}{328} = 1 + \frac{325^2}{2\,(993)\,328} \qquad T_{o_2} = 381\,K$$

Since 1 to 2 is isentropic $T_{01}=T_{02}=$

381 K

For the isentropic process 2 to 02 we can use (5.13)

$$\frac{p_{02}}{p_2} = \left(1 + \frac{u_2^{\,2}}{2c_p T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$p_{0_2} = 135.4$ kPa

Since 1 to 2 is isentropic $p_{01}=p_{02}=$ 135.4 kPa

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For the isentropic process 1 to 01 we can use (5.11)

$$\frac{T_{01}}{T_1} = 1 + \frac{u_1^2}{2c_p T_1}$$

$u_1 = 126$ m/s

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