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MEC 208 Fluids Engineering

Dr. Cécile M. Perrault

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Before we start

- No lecture next week !
- You should be almost done with Tutorial Sheet 3
- Remember quizzes on MOLE

Responses 46

Polling Closed

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SESSION 751641

Of the material covered in this lecture, which did you understand least ?

A. Flow between Infinitely Wide Parallel Plates- the basics

B. Flow between Infinitely Wide Parallel Plates- pressure driven (Poiseuille)

C. Flow between Infinitely Wide Parallel Plates- moving plate (Couette)

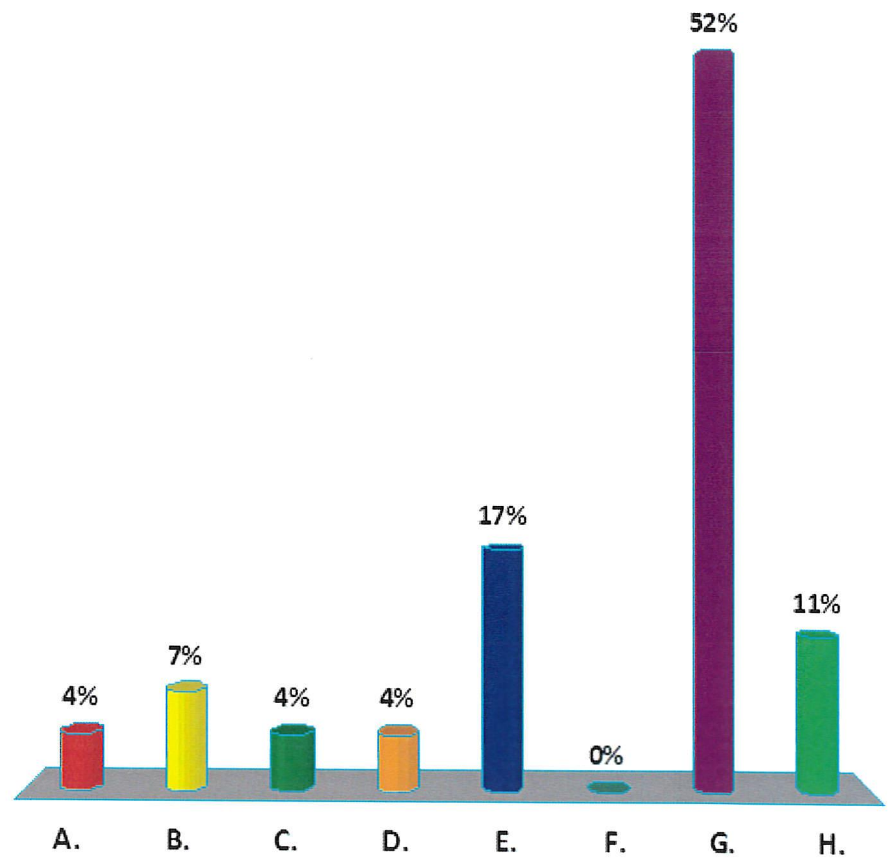
D. Flow between Infinitely Wide Parallel Plates- pressure driven and moving plate

E. Flow between Concentric Cylinders

F. Laminar Film on a Vertical Wall

G. The Cone on Plate Viscometer

H. Nothing



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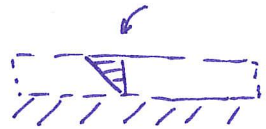
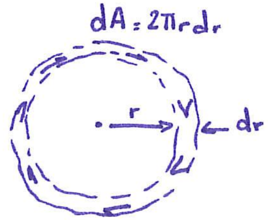
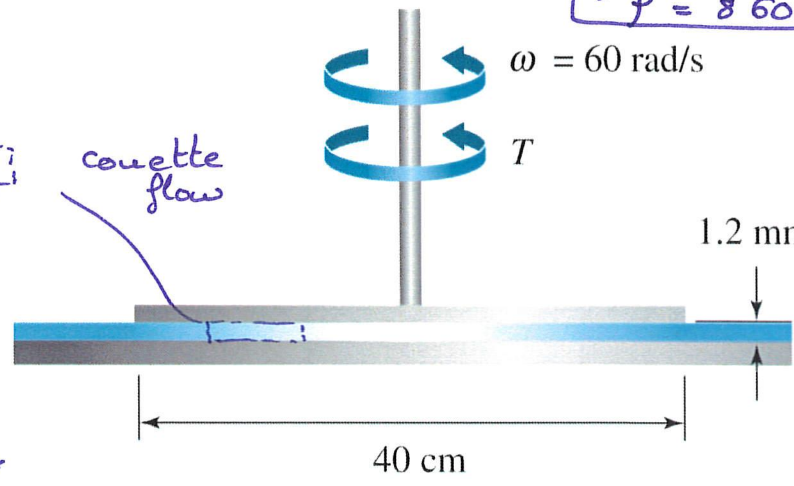
$\mu = 0.01$ Pa.s
$\rho = 860$ kg/m³





couette flow



in Couette flow, velocity profile

$$u = \frac{U}{a} y = \frac{r\omega}{a} y$$

$$\Rightarrow \tau = \mu \frac{du}{dy} = \mu \frac{r\omega}{a} = \frac{0.01 \times 60}{0.0012} r$$

$$= 500 r$$

$$T = \int \tau \, dA \, r = \int_0^{0.2} 500 r \, (2\pi r) \, r \, dr = 1000\pi \int_0^{0.2} r^3 dr = \frac{1000\pi \times .2^4}{4} = 1.26 \text{ N.m}$$

The largest Re will occur at $r = 0.2$ m $\Rightarrow Re_{max} = \frac{\rho R \omega a}{\mu} = \frac{860 \times 0.02 \times 60 \times 0.0012}{0.01}$

$= 1240$

$\Rightarrow$ laminar

4.1. Introduction - Boundary Layers

This section describes how we analyse flows around bodies immersed in a fluid stream (e.g. a flag pole bending in the wind, an aircraft in flight, or a bowled cricket ball).

Imagine a body stationary in a fluid stream of velocity U. The fluid at the surface of the body must be stationary. There will be a layer of fluid close to the body travelling slowly. Here the viscous effects are important - we call this the *boundary layer*. Far from the body viscous effects are negligible (because the fluid is all travelling at the same speed).

It is important for engineers to determine the forces on bodies in fluid streams (e.g. for the aerodynamic design of vehicles, wind loading on a structure, or drag on a ship's hull). The force or *drag* is controlled by the shear stress at the walls (skin friction drag) and the pressure difference between the front face and the wake of the body (pressure or 'form' drag).

We will approach this in two ways. Firstly for *flat plates* with laminar flow we will derive and use an expression relating shear stress to velocity profile in the boundary layer (this will involve the FME). This will give equations giving the drag in terms of the Reynolds number. For turbulent flow on flat plates we will use empirical solutions. For *curved surfaces* we will use empirical drag coefficients to determine drag.

Boundary layers are also important in heat transfer. Convection depends on the properties of a thermal boundary layer. We will look at the thermal boundary layer and relationships for the heat transfer coefficient.

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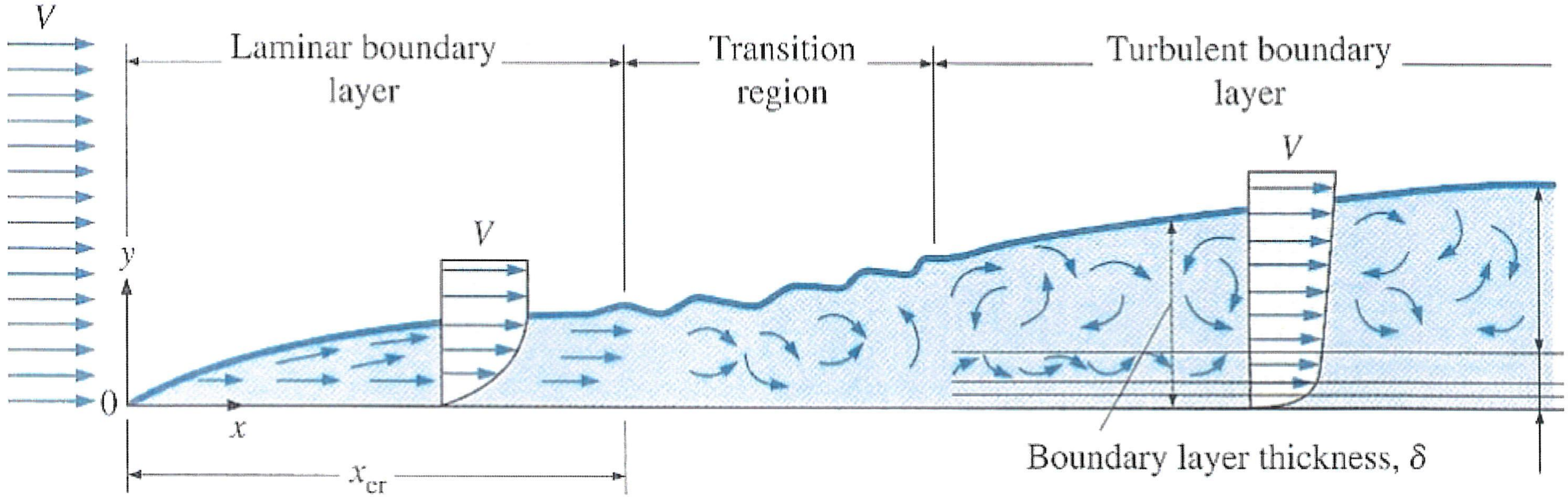
Consider a flat plate in a fluid flowing at U.



The fluid is slowed down by the presence of the plate (at the wall the fluid must be stationary). A laminar layer of slow moving fluid is formed. As more fluid is slowed down the boundary layer thickens. With increasing thickness the layer becomes unstable and turbulence develops. The turbulent layer thickness increases at a greater rate. Within the turbulent boundary layer there is a small viscous sub-layer near the wall. The layer is so thin that it does not effect the displacement and pressure of the bulk fluid flow.

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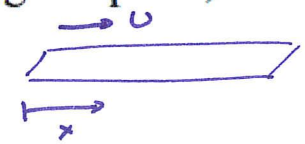
$$Re = \frac{\rho u d}{\mu}$$

We define a Reynolds number using the distance travelled along the plate;

$$Re_x = \frac{\rho U x}{\mu} \tag{4.1}$$

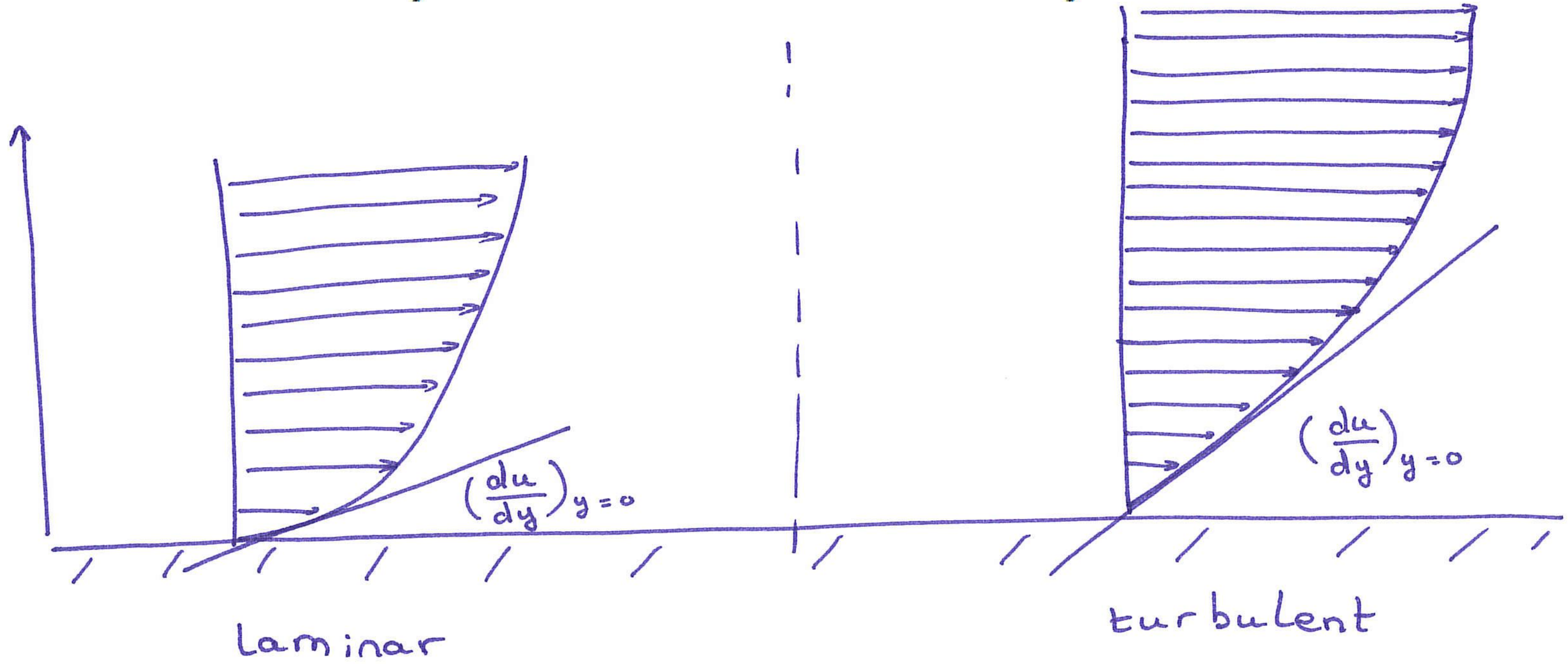


The transition from laminar to turbulent flow depends on the Reynolds number, plate roughness and main stream turbulence.

Typically	Re<	10^5	laminar boundary layer
	Re>	2×10^6	turbulent boundary layer

4.3. The Velocity Profile within a Boundary Layer.

In a turbulent boundary layer there is more mixing of the fluid particles. So the velocity distribution across the layer is more uniform. The velocity at the wall must be zero.



The shear stress on the wall is given by;

$$\tau_w = \mu \left(\frac{du}{dy}\right)_{y=0}$$

So for the turbulent layer the velocity gradient at the wall will be greater. Hence a turbulent layer has greater wall shear stresses and drag.

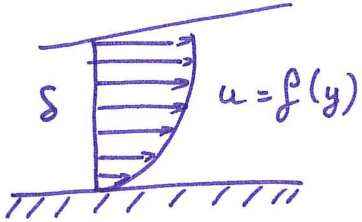
4.4. The Thickness of a Boundary Layer

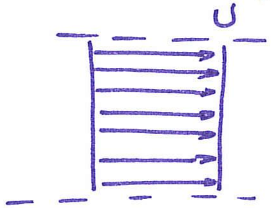
The boundary merges into the main flow - there is no clear division between the main stream and the fluid slowed by the plate. We define the thickness of a boundary layer, δ to be the distance from the plate at which;

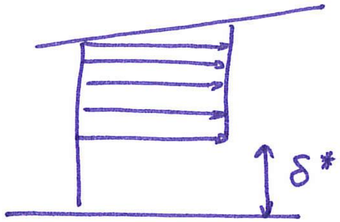
$$u = 0.99\ U \tag{4.2}$$

We also define other thicknesses based on the effects of the boundary layer on the flow.

(a) The Displacement Thickness, δ^*







Mass flow rate through the boundary layer $= \int_0^\delta \rho b u \, dy$

Mass flow rate if there had been no boundary layer $\int_0^\delta \rho b U \, dy$

Reduction in mass flow by boundary layer $\int_0^\delta \rho b (U - u) \, dy$

We represent this loss of flow as a missing layer of fluid with a thickness δ^* - the *displacement thickness*. Then;

$$\rho b U \delta^* = \int_0^\delta \rho b (U - u) \, dy$$

$$\delta^* = \int_o^\delta \left(1 - \frac{u}{U} \right) dy \qquad (4.3)$$

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(b) The Momentum Thickness, θ.

The boundary layer will also reduce the overall momentum flow. We can define a *momentum thickness*, θ to represent this loss of momentum.

Momentum of the boundary layer $= \int_0^\delta \rho b u u \, dy$

Momentum of fluid with no boundary layer present $= \int_0^\delta \rho b u U \, dy$

Reduction in momentum caused by boundary layer $= \int_0^\delta \rho b u (U - u) \, dy$

Again we represent this as a missing layer of fluid of thickness, θ

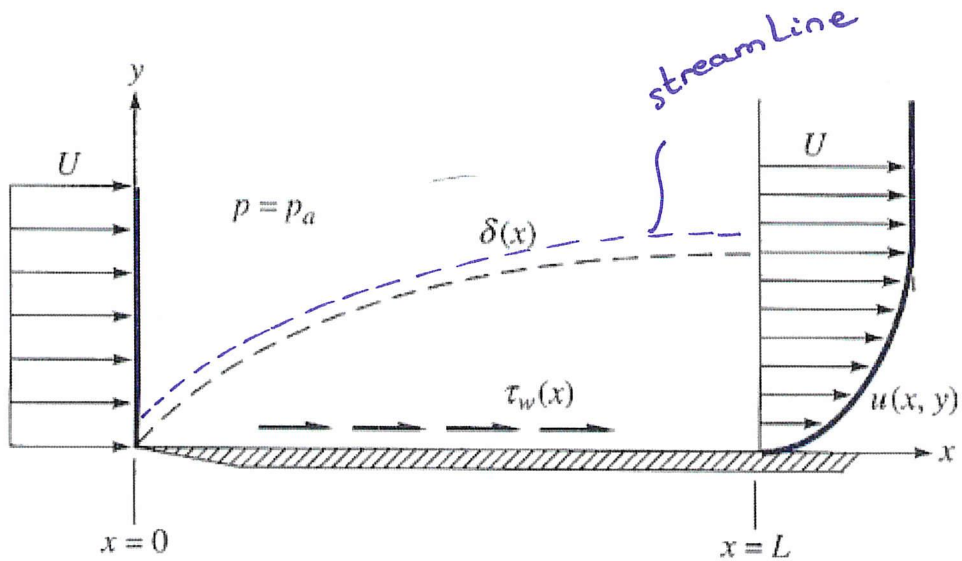
$$(\rho b U \theta) U = \int_0^\delta \rho b u (U - u) \, dy$$

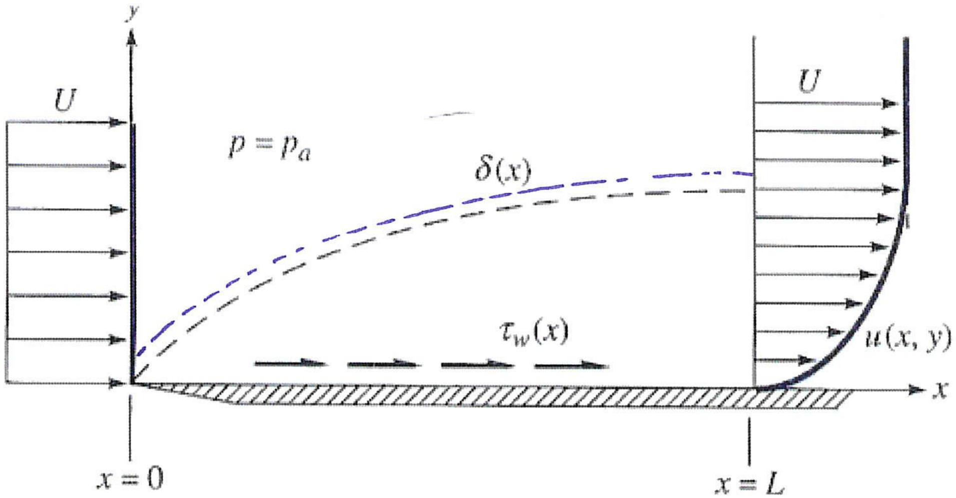
$$\theta = \int_o^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad (4.4)$$

4.5. The Boundary Layer Equation

We will use the FME to derive an expression for the drag on a flat plate. The boundary layer starts at $x=0$ and we consider the portion up to $x=L$. This part of the boundary layer will exert some drag force, D on the flat plate. The larger the length of the plate the greater the drag force, so $D=f(x)$. We define a CV using a streamline which is just outside the boundary layer. Remember that no fluid crosses a streamline.

There are no shear stresses acting on the top face of the CV (since we chose a streamline outside the viscous layer). We will assume the pressure does not vary along the plate. This assumption is ok for flat plates but not for curved bodies (see later).





The sum of the forces in the x direction

$$\sum F_x = -D$$ (drag force on plate)

Now consider the momentum flow rates in the x direction.

$$\sum \dot{M}_{in} = \int_0^h \rho U^2 b\,dy = \rho U^2 bh$$

$$\sum \dot{M}_{out} = \int_0^\delta \rho u^2 b\,dy$$

Assembling the FME gives us;

$$D = \rho U^2 bh - \rho b \int_0^\delta u^2\,dy \tag{4.5}$$

This gives us the drag on the plate, but we do not know the distance, h. Apply MCE. Remember fluid does not cross a streamline.

$$\dot{m}_{in} = \rho U h b$$

$$\dot{m}_{out} = \int_0^\delta \rho u b \, dy$$

Equating these;

$$\rho U h b = \int_0^\delta \rho u b \, dy \quad \Rightarrow \quad h = \frac{1}{U} \int_0^\delta u \, dy \tag{4.6}$$

Substituting for h using equation (4.7) into (4.6) gives;

$$D = \rho U b \int_0^\delta u \, dy - \rho b \int_0^\delta u^2 \, dy$$

$$= \rho b \int_0^\delta u \, (U - u) \, dy$$

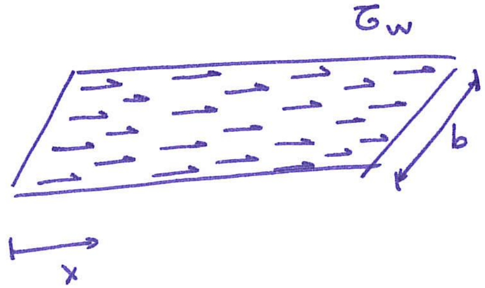
$$D(x) = \rho b \int_0^{\delta(x)} u(U - u) \, \mathrm{d}y \tag{4.7}$$

Using this we can determine the drag on a flat plat if we know the velocity, $u(y)$ distribution in the boundary layer.

We can present this in a tidier way using the momentum thickness equation (4.4);

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$D(x) = \rho b U^2 \theta \qquad (4.8)$$



Momentum thickness is thus a measure of the drag on the plate. But the drag can also be obtained from the shear stress at the plate (or wall);

$$D(x) = b \int_0^x \tau_w(x)\, dx$$

or by differentiating wrt x

$$\frac{dD}{dx} = b\,\tau_w$$

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We compare this with the differential of (4.8) wrt x

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

So we can get an expression for the wall shear stress;

$$\boxed{\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx}} \qquad (4.9)$$

This is the usual form of the *boundary layer equation*. It is the basis of all analysis of boundary layers. It is applicable to both laminar and turbulent flows.

If we know the velocity profile $u=f(y)$ then we can determine U and θ and hence get the thickness of the boundary layer and the shear stress τ on the plate.

Boundary Layer Equation with a Pressure Gradient.

This relationship was developed for the case where the pressure does not vary in the *x*-direction. The more general case is given by Massey §8.4 and gives a more general form of the boundary layer equation;

$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) - \frac{dU}{dx}U\delta^* \qquad \text{for } \frac{dP}{dx} \neq 0 \tag{4.10}$$

4.6. Laminar Boundary Layer on a Flat Plate

We will now use the boundary layer equation to determine the size of the laminar boundary layer and the drag on a flat plate.

First we assume the velocity profile has a parabolic shape

$$u = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \quad \text{— an assumption by Karman (1928)} \tag{4.11}$$

We can now find the momentum thickness from equation (4.4);

$$\theta = \int_o^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right) dy = \frac{2\delta}{15} \tag{4.12}$$

The wall shear stress can be determined from Newton's law of viscosity;

$$\tau_w = \mu\left(\frac{du}{dy}\right)_{y=0} \qquad \frac{du}{dy} = \frac{2U}{\delta} - \frac{2Uy}{\delta^2}$$

$$\tau_w = \frac{2\mu U}{\delta} \tag{4.13}$$

Then putting (4.12) and (4.13) into the boundary layer equation (zero pressure gradient)

$$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx}$$

$$\frac{2\mu U}{\delta \rho U^2} = \frac{d}{dx}\left(\frac{2\delta}{15}\right)$$

Separate the variables;

$$\delta \, d\delta = \frac{15\mu}{\rho U} dx$$

Integrating from 0 to x then the thickness varies from 0 to δ.

$$\int_o^\delta \delta \, d\delta = \int_o^x 15 \frac{\mu}{\rho U} dx$$

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U} \qquad \frac{\delta}{x} = 5.5 \left(\frac{\mu}{\rho U x}\right)^{1/2}$$

Rewriting in terms of Reynolds number gives;

$$\frac{\delta}{x} = \frac{5.5}{\mathrm{Re}_x^{\ 1/2}} \tag{4.14}$$

We can now put this into (4.13) to determine the wall shear stress

$$\tau_w = \frac{2\mu U}{\delta} = \frac{2\mu U}{5.5x}\sqrt{Re_x}$$

It is convenient to express this wall shear stress as a non-dimensional number - called the *skin-friction coefficient,* c_f:

$$c_f \equiv \frac{2\tau_w}{\rho U^2} \tag{4.15}$$

Replacing the wall shear stress, τ_w gives;

$$c_f = \frac{0.73}{\mathrm{Re}_x^{1/2}} \tag{4.16}$$

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We had to assume a velocity profile to obtain these solutions. The exact solution (obtained by Blasius in 1908 using an improved velocity profile analysis) is given by;

$$\boxed{\begin{aligned} \frac{\delta}{x} &= \frac{5}{\mathrm{Re}_x^{\;1/2}} \\ c_f &= \frac{0.664}{\mathrm{Re}_x^{\;1/2}} \end{aligned}}$$

(4.17)

(4.18)

These expressions (4.17) and (4.18) give an estimate of the thickness of the layer, δ and the wall shear stress, τ_w at a distance x along the plate. These values vary with distance down the plate, $\delta = \delta(x)$ and $\tau_w = \tau_w(x)$. $\therefore$ $C_f = C_f(x)$

The total drag force, D for the whole plate is determined from the integral of the wall shear stress, i.e.

$$D = b\int_o^L \tau_w(x)\,dx \tag{4.19}$$

$$D = b\int_0^L \rho \frac{U^2}{2} C_f \, dx$$

Substituting (4.18) gives

$$D = b\int_0^L \rho \frac{U^2}{2} (0.664)\sqrt{\frac{\mu}{\rho U x}}\, dx$$

$$D = 0.664 b\sqrt{\rho\mu U^3 L} \tag{4.20}$$

Again it is convenient to define a non-dimensional *Drag Coefficient, C_D*

$$C_D \equiv \frac{D}{\frac{1}{2}\rho U^2 bL} \tag{4.21}$$

So for laminar zero pressure gradient flow over a plate;

$$\boxed{C_D = \frac{1.328}{\mathrm{Re}_L^{1/2}}} \tag{4.22}$$

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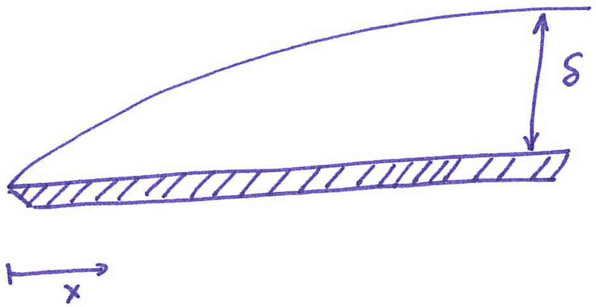
Example 4.1. Laminar Drag on A Flat Plate

A long thin flat plate is placed parallel to a 20 m/s stream of air at 20°C. At what distance down the plate will the boundary layer be 1 mm thick? What is the shear stress at the wall at this location?



From the data book; air at 20°C

$\rho = 1.2\ kg/m^3$ and $\mu = 18.15 \times 10^{-6}$ Pas

Using expression (4.17); $\frac{\delta}{x} = 5\sqrt{\frac{\mu}{\rho U x}}$

$$x = \frac{\delta^2 U \rho}{25\mu} = \frac{(10^{-3})^2 (20)(1.2)}{25\,(18.15 \times 10^{-6})} \approx 53\ mm$$

.

Since this is only valid for laminar flow we should check the Reynolds number at this location, x;

$$\mathrm{Re}_x = \frac{\rho U x}{\mu} = \frac{(1.2)(20)(53\times 10^{-3})}{18.15 \times 10^{-6}} = 7 \times 10^{4}$$

$\Rightarrow$ laminar (just)

The wall shear stress at this location is given by (4.16);

$$c_f = \frac{0.664}{\mathrm{Re}_x^{\ 1/2}} = 0.0025$$

then $\tau_w = \dfrac{\rho U^2 c_f}{2} = 0.6\ \mathrm{N/m^2}$

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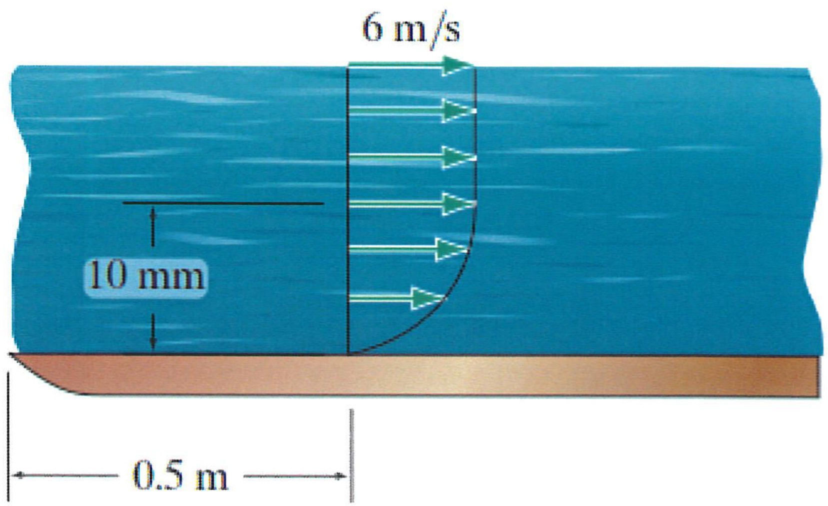
If thickness of the boundary layer at a distance of 0.5 m from the plate's edge is 10 mm, determine the boundary layer thickness at a distance of 1 m



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assume steady, incompressible flow

Reynolds number at $x = 0.5\,m$ and $1\,m$ are :

$$Re_{x\,(x=0.5\,m)} = \frac{\rho U x}{\mu} = 0.5\,U\left(\frac{\rho}{\mu}\right)$$

$$Re_{x\,(x=1\,m)} = \frac{\rho U x}{\mu} = U\left(\frac{\rho}{\mu}\right)$$

at $x = 0.5\,m$, $\delta = 0.01\,m$

$$\delta = \frac{5x}{\sqrt{Re_x}} \qquad 0.01\,m = \frac{5(0.5)}{\sqrt{0.5\,U\left(\frac{\rho}{\mu}\right)}} \implies U\frac{\rho}{\mu} = 125\,000$$

Thus, at $x = 1\,m$, $Re_x = U\frac{\rho}{\mu} = 125\,000$

then $$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5}{\sqrt{125000}} = 0.01414\,m = 14.1\,mm$$

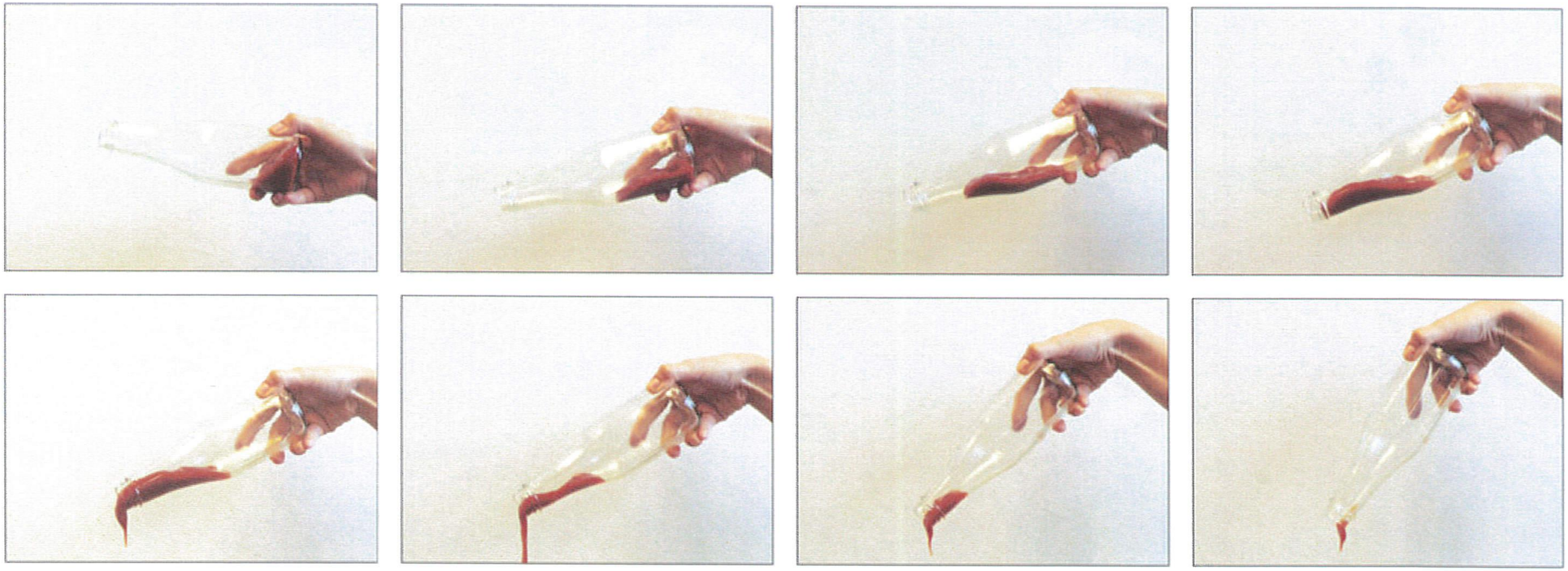
International New York Times

TRY TIME

SCIENCE

With New Nonstick Coating, the Wait, and Waste, Is Over

By KENNETH CHANG MARCH 23, 2015



A coating developed by a start-up company, LiquiGlide, makes the inside of a bottle permanently wet, allowing viscous fluids like ketchup to pour out easily. LiquiGlide

http://liquiglide.com/

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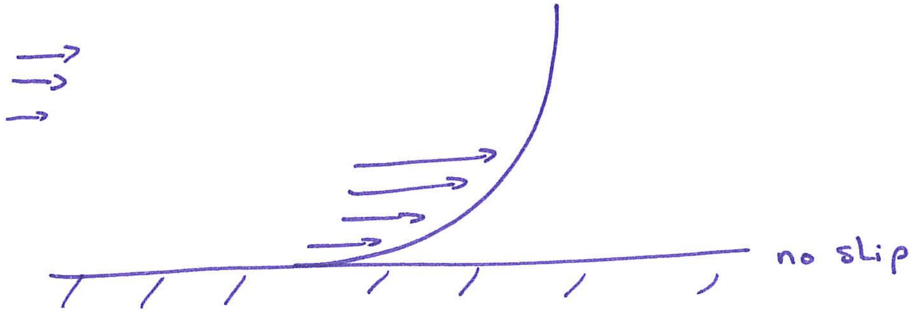
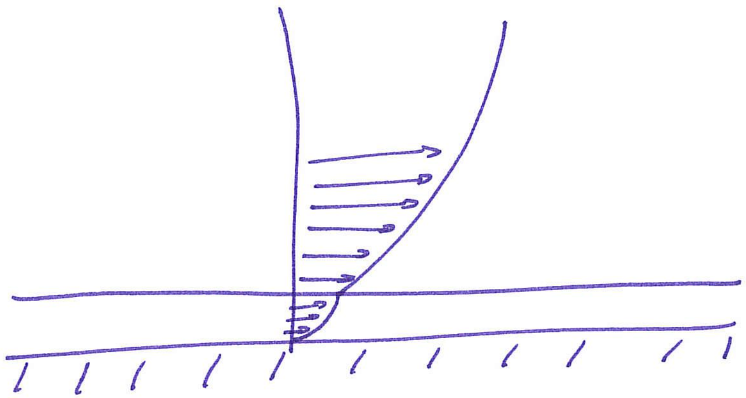


fluid flowing over solid surface



fluid flowing over a liquid coating

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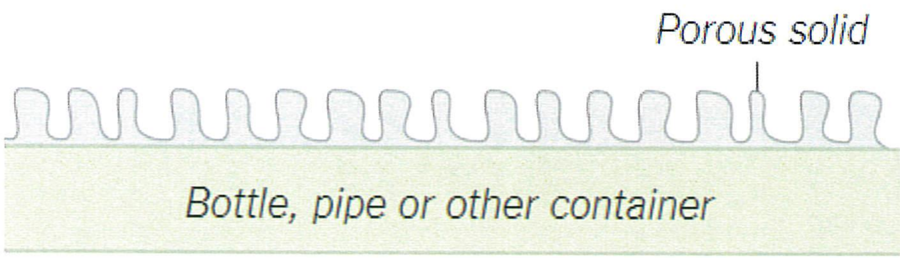
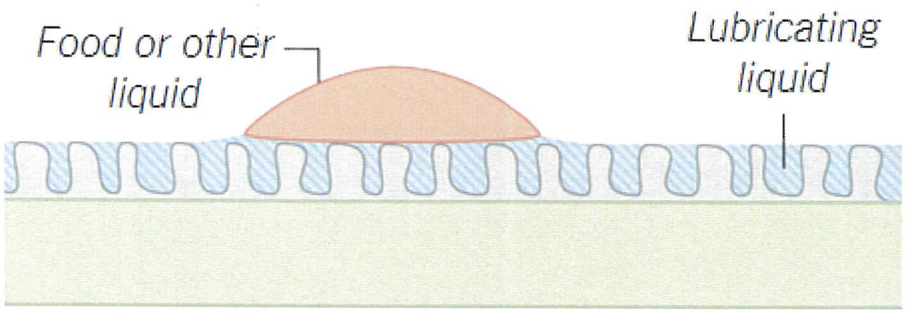
A Permanently Wet Surface

A new surface coating is designed to remain wet and slippery, allowing other liquids to slide across it easily.



A thin layer of porous material is sprayed onto the inside of a container.

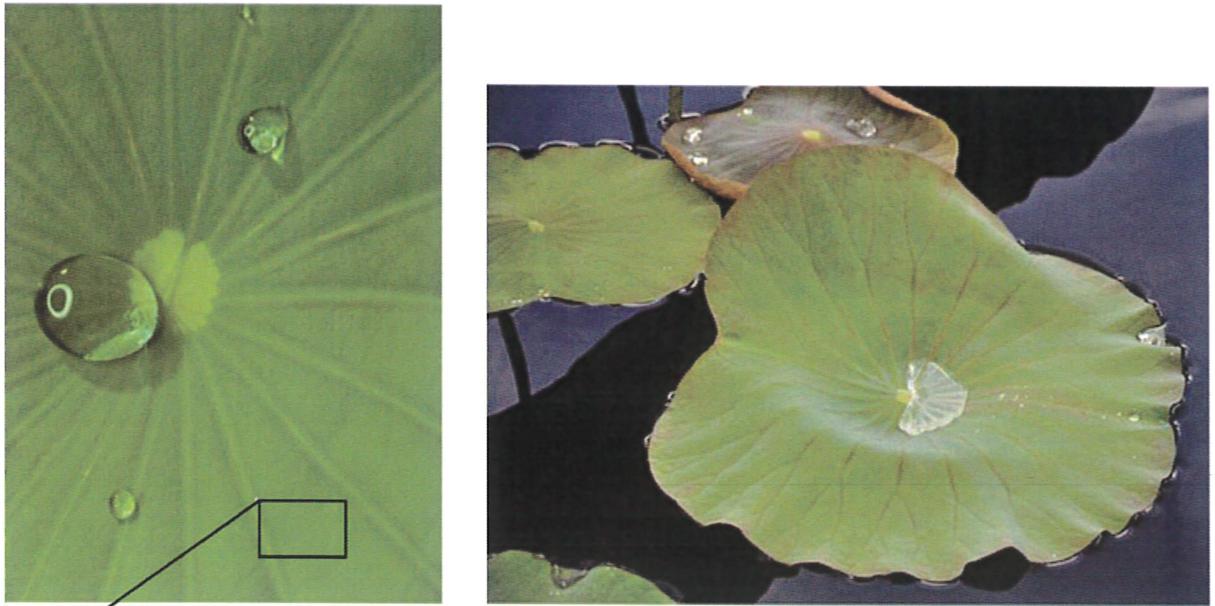


A lubricating liquid is sprayed onto the porous surface, filling the tiny gaps. The liquid is held in place by capillary forces and creates a slippery surface for food or other liquids.

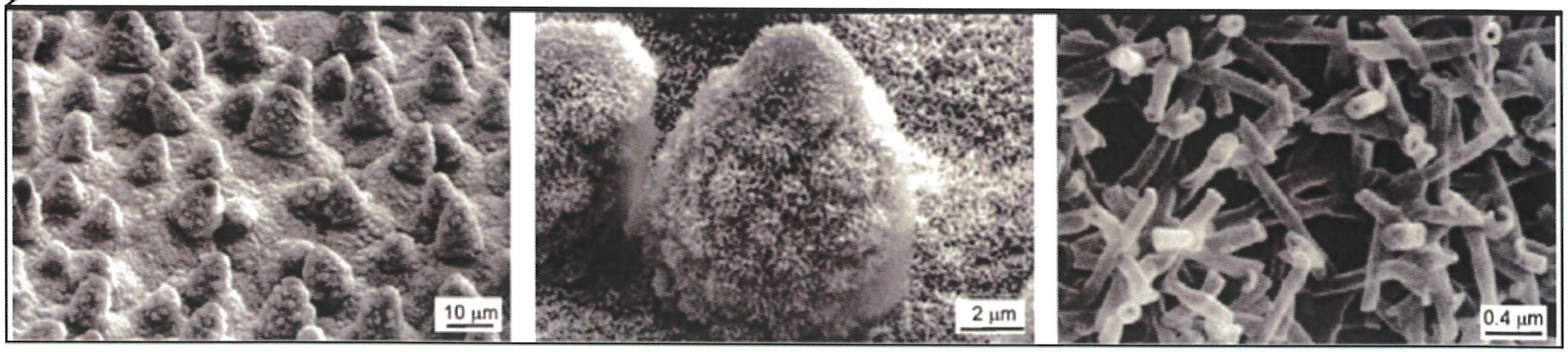
Source: LiquiGlide

By The New York Times

Lotus Leaf



(a)



(b)