



The  
University  
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Sheffield.

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# MEC 208

# Fluids Engineering

## Exam Format & Revision



# Fluids Engineering Exam 21<sup>st</sup> of January -9-11:30 am in Octagon Centre !



# Fluids Engineering Exam<sup>3</sup>

- Part A – 40 marks out of 80
  - Multiple choice
  - 20 questions (one correct answer, wrong answers are -1 mark)
  - 2 marks each
    - one correct answer
    - wrong answers are -1 mark
- Part B – 40 marks out of 80
  - 2 questions
  - 20 marks each

# Past papers

- Available on Mole
- Numerical answers available – at the end of the paper
- Worked solutions NOT available
- Three years ago format changed (so early papers slightly different)



# FAQ's

- Part A questions are similar in style to MOLE quizzes
- Everything I taught you is examinable (EXCEPT NAVIER-STOKES)
- Most data and equations are in the LBOTF

# Where to Get Help

- All filled lecture notes are now on Mole
- I won't be answering questions by email during the Christmas break
- Vacation Week 4 (11<sup>th</sup> January): all week in office (PLB C+09)





## 1. Fluids in Equilibrium

Pressure variation within a stationary fluid  $\frac{\partial p}{\partial z} = -\rho g$   $p + \rho g z = \text{constant}$

Fluids in Uniform Linear Acceleration  $\frac{\partial p}{\partial x} = -\rho a_x$   $\frac{\partial p}{\partial y} = -\rho a_y$   $\frac{\partial p}{\partial z} = -\rho(g + a_z)$

Fluids in Rigid Body Rotation  $\frac{\partial p}{\partial r} = \rho r \omega^2$   $\frac{\partial p}{\partial z} = -\rho g$   $\frac{\partial p}{\partial \theta} = 0$

$$d p = \frac{\partial p}{\partial x} d x + \frac{\partial p}{\partial y} d y + \frac{\partial p}{\partial z} d z$$

Total differential

Lines of constant pressure (i.e. free surface of the fluid) are given by  $dp=0$



## 2. Control Volume Analysis

Draw a CV so that the boundary crosses regions where you either know the properties or you want to know them.

The Mass Conservation Equation, MCE  $\sum \dot{m}_{in} = \sum \dot{m}_{out}$

1D flow  $\dot{m} = \rho_{in} A_{in} u_{in} = \rho_{out} A_{out} u_{out}$

The Force Momentum Equation, FME  $\sum F_x = \left( \sum \dot{M}_{out} - \sum \dot{M}_{in} \right)_x$

steady 1D incompressible flow  $\sum F_x = \dot{m}(u_{out} - u_{in})_x$  in LBT

The Torque Angular Momentum Equation, TAME  $\sum T_A = \left( \sum \dot{O}_{out} - \sum \dot{O}_{in} \right)_A$  in LBT

Steady 1D incompressible flow  $\sum T_A = \dot{m}(r_2 u_2 - r_1 u_1)$  in LBT

The Steady Flow Energy Equation  $\frac{\dot{Q} - \dot{W}}{\dot{m}} = \left( e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2} + gz_2 \right) - \left( e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} + gz_1 \right)$

WRONG in LBT, provided at exam





### 3. Laminar Flow

For laminar flow  $\dot{M}_{out} \approx \dot{M}_{in}$  so  $\sum F \approx 0$

Basic concepts  $Re = \frac{\rho u d}{\mu}$  : Axisymmetric flow  $\dot{q} = \int_0^R u 2\pi r dr$  : 2D flow  $\dot{q} = \int_{-L/2}^{L/2} u B dy$   
in LBT

Poiseuille's equation  $\dot{q} = -\frac{\pi R^4}{8\mu} \frac{dp}{dx}$  and  $\frac{\dot{q}}{b} = -\frac{h^3}{12\mu} \frac{dp}{dx}$  in LBT, but ....

Head loss and pressure loss  $\Delta p_L = h_f \rho g$

1. draw a fluid element
2. balance the forces (pressure and shear stress)
3. substitute  $\tau = \mu \frac{\partial u}{\partial y}$
4. integrate to get velocity profile  $u=f(y)$
5. apply boundary conditions.



## 4. External Flow

The *displacement thickness* and *momentum thickness* - 'missing' layers of fluid in LBT

The *boundary layer equation* describes the variation of  $p$  and  $u$  within a boundary layer

The *skin-friction coefficient*,  $c_f$  and *drag coefficient*,  $C_D$  - convenient ways of ~~writing~~ <sup>writing</sup> wall shear stress and drag force;

$$c_f \equiv \frac{2\tau_w}{\rho U^2} \quad C_D \equiv \frac{D}{\frac{1}{2}\rho U^2 bL}$$

in LBT

Boundary layer on a flat plate (zero pressure gradient):

$$\text{Laminar } \frac{\delta}{x} = \frac{5}{\text{Re}_x^{1/2}} \quad c_f = \frac{0.664}{\text{Re}_x^{1/2}} \quad C_D = \frac{1.328}{\text{Re}_L^{1/2}}$$

$$\text{Turbulent } \frac{\delta}{x} \approx \frac{0.16}{\text{Re}_x^{1/7}} \quad c_f \approx \frac{0.027}{\text{Re}_x^{1/7}} \quad C_D \approx \frac{0.031}{\text{Re}_L^{1/7}}$$

Separation and drag on 2D and 3D bodies - qualitative description.

Empirical data for the drag coefficient for various shaped bodies  $C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$



Nusselt and Prandtl

Thermal boundary layer

Flow over a flat plate the relative

Laminar:  $Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$  when  $Re < 5 \times 10^5$

Turbulent:  $Nu_x = 0.023 Re_x^{1/2} Pr^{1/3} Nu_L$   $0.6 < Pr < 60$  &  $5 \times 10^5 < Re < 10^7$



## 5. Compressible Flow

$$h = e + \frac{p}{\rho} \quad de = c_v dT \quad \& \quad dh = c_p dT \quad \gamma = \frac{c_p}{c_v} \quad \& \quad R = c_p - c_v$$

Isentropic process  $\frac{T_1}{T_2} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{\rho_1}{\rho_2} \right)^{\gamma-1}$

Stagnation conditions  $h_0$ ,  $T_0$ ,  $p_0$  and  $\rho_0$  represent the pressure and density which would be achieved if flow were brought isentropically to rest.

$$h_0 = h + \frac{u^2}{2} \quad \frac{T_0}{T} = 1 + \frac{u^2}{2c_p T} \quad \frac{p_0}{p} = \left( 1 + \frac{u^2}{2c_p T} \right)^{\frac{\gamma}{\gamma-1}} \quad \frac{\rho_0}{\rho} = \left( 1 + \frac{u^2}{2c_p T} \right)^{\frac{1}{\gamma-1}}$$

If a process from 1 to 2 is *isentropic*

$$h_{01} = h_{02}, \quad T_{01} = T_{02}, \quad \rho_{01} = \rho_{02}, \quad p_{01} = p_{02}.$$



The Mach Number,  $M = \frac{u}{a}$       Velocity of sound  $a = \sqrt{\gamma RT}$

in LBT      in LBT      in LBT

$$\frac{T_0}{T} = 1 + \left( \frac{\gamma - 1}{2} \right) M^2 \quad \frac{p_0}{p} = \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M^2 \right]^{\frac{\gamma}{\gamma - 1}} \quad \frac{\rho_0}{\rho} = \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M^2 \right]^{\frac{1}{\gamma - 1}}$$

*Critical Conditions*, the fluid properties when the flow is sonic (i.e.  $M=1$ )

*Subsonic*      gas accelerates through converging duct & decelerates through diverging duct.

*Supersonic*      gas accelerates through diverging duct & decelerates through converging duct.

*Choking* - flow is sonic at the throat of the nozzle;  $M_t=1$ , then  $\dot{m}_{\max} = \rho_c a_c A_t$

*Shock wave* – sudden reduction in velocity and increase in static pressure. Use shock tables to determine properties before and after shock



*That's all Folks!*