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What we are covering in Topic 1:

- **1. Fluids in Equilibrium**
  - **1.1 Introduction**
  - **1.2 Fluid Statics**

    1.2.1. Pressure variation within a fluid

    1.2.2. Hydrostatic Forces on Submerged Surfaces
  - **1.3. Fluids in Rigid Body Motion**

    1.3.1. Forces on a Fluid Element

    1.3.2. Fluids in Uniform Linear Acceleration

    1.3.3. Fluids in Rigid Body Rotation

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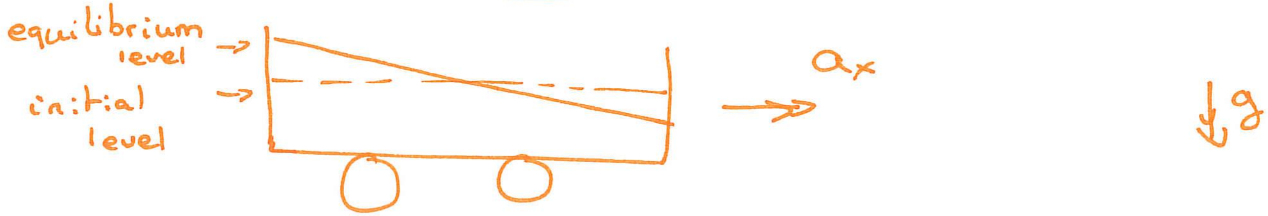
LAST TIME

$$\frac{\partial p}{\partial x} = -\rho a_x \qquad \frac{\partial p}{\partial y} = -\rho a_y \qquad \frac{\partial p}{\partial z} = -\rho (g + a_z)$$



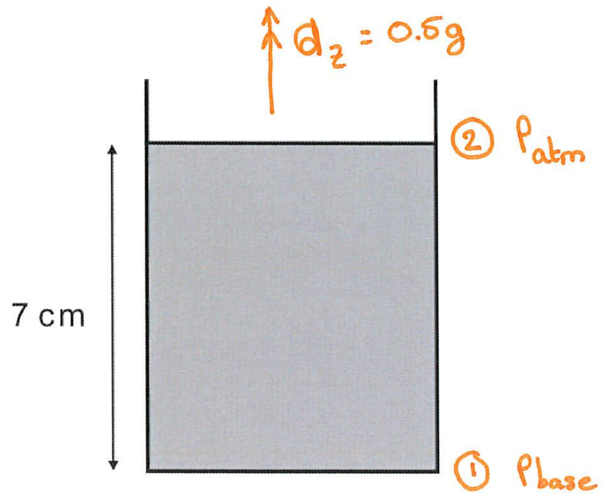
$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz$$

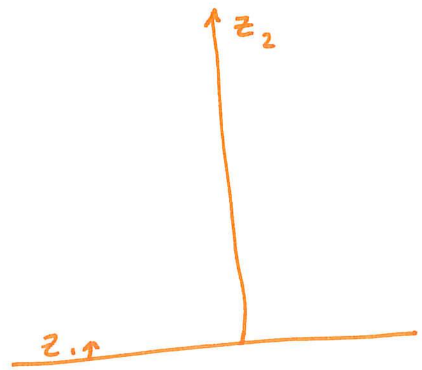
here $dp = -\rho a_x dx - \rho g \, dz$

to find slope ($dp = 0$) $\quad \dfrac{dz}{dx} = -\dfrac{a_x}{g}$

Example 1 - Drinking Coffee in a Lift

A coffee cup is carried in a lift which accelerates upwards at 0.5g. What pressure is experienced at the bottom of the cup if the liquid is 7cm deep (the density of coffee is 1010 kg/m^3).





We use equation (1.7)

$$\frac{dp}{dz} = -\rho(g + a_z)$$

$$= -\rho(g + 0.5g)$$

$$\partial p = -1.5g\rho\, dz$$

$$\int_1^2 dp = -1.5g\rho \int_1^2 dz \rightarrow (p_2 - p_1) = -1.5g\rho(z_2 - z_1) = 1.5g\rho(z_1 - z_2)$$

$$-p_1 = 1.5g\rho(0 - 0.07)$$

$$p_1 = 1.5(9.81)(1010)(0.07) = 1040 \text{ N/m}^2$$

$$= 0.0104 \text{ bar}$$

this is gauge pressure for absolute pressure add p_{atm}.

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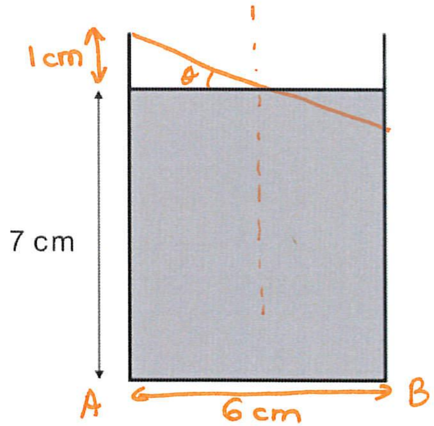
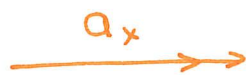
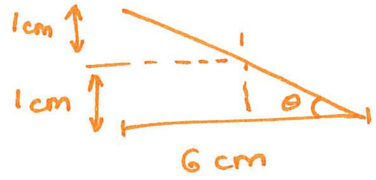
Example 2 - Drinking Coffee in a Racing Car

The coffee cup is now resting on a horizontal surface in a racing car. At what rate can the car accelerate if the coffee is not to spill? Sketch the gauge pressure distribution acting at the bottom of the cup.





If the mug is symmetric about the centre axis the tilted surface must intersect the axis as shown (the volume of coffee must be conserved)



The slope of the surface is given by (1.11);

for $dp = 0$ $$\frac{dz}{dx} = -\frac{a_x}{g}$$

so; $$\tan\theta = \frac{a_x}{g}$$

From the geometry of the cup; θ=18.4° for the liquid just to spill over. Hence;

$$\tan 18.4 = \frac{a_x}{9.81}$$

maximum acceleration, a_x= $3.26\ m/s^2$

The pressure at the bottom of the cup may be determined from (1.7) noting that a_z=0;

$$\frac{\partial p}{\partial z} = -\rho(a_z + g)$$

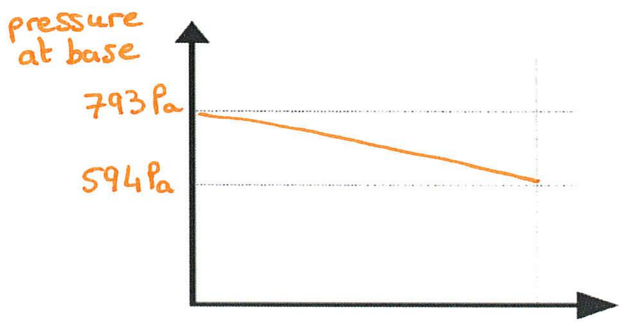
$$\frac{\partial p}{\partial z} = -\rho g \qquad \int_1^2 \partial p = -\rho g \int_1^2 dz$$

Integrating gives;

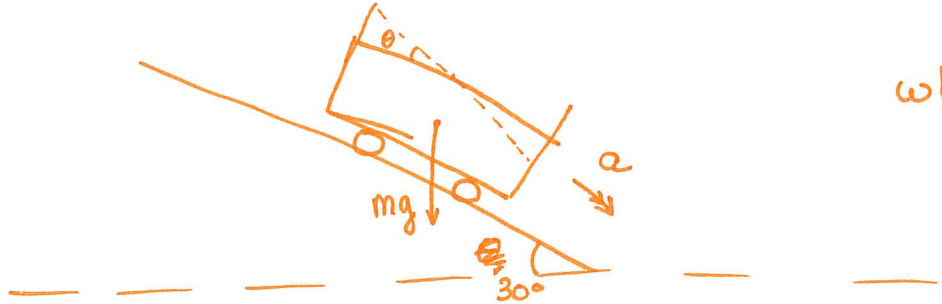
$$P_2 - P_1 = -\rho g (z_2 - z_1)$$

So gauge pressure at A= $1010\,(0.08)\,9.81 = 793$ Pa

So gauge pressure at B= $1010\,(0.06)\,9.81 = 594$ Pa



Extra Example



what is θ?

First determine the accelerations in the x-direction and the z-direction

$$mg \sin\theta = ma \qquad \Rightarrow a = 0.5g$$

$$\Rightarrow a_x = a \cos 30 \quad \& \quad a_z = -a \sin 30$$

$$\frac{\partial p}{\partial x} = -\rho a_x \qquad \frac{\partial p}{\partial y} = -\rho a_y \qquad \frac{\partial p}{\partial z} = -\rho(g + a_z)$$

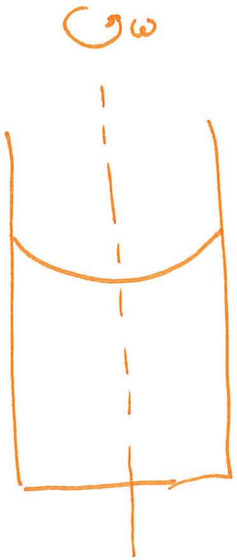
for $\partial p = 0$ $\qquad \dfrac{dz}{dx} = \dfrac{-a_x}{g + a_z}$

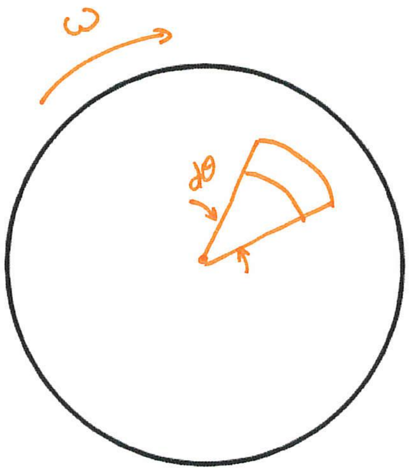
$$\tan\theta = -\frac{0.5g \cos 30}{g - 0.5g \sin 30} \qquad \Rightarrow \theta = 30°$$

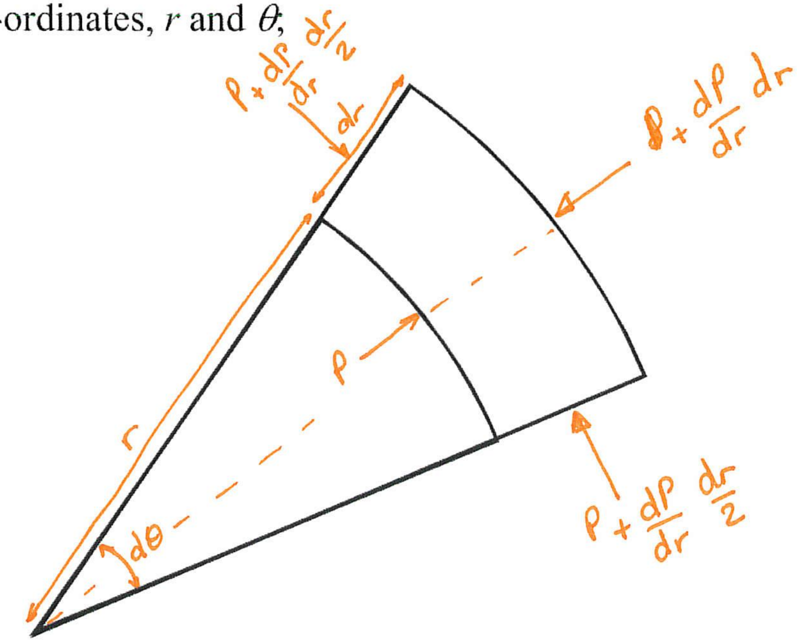
$\Rightarrow$ the slope is independant of the density of the fluid

1.3.3. Fluids in Rigid Body Rotation

Now consider an element of fluid in cylindrical co-ordinates, r and θ;







The fluid is rotating at a constant angular velocity, ω. So, there is an acceleration towards the centre of the circle of $r\omega^2$.

Newton's 2nd Law for the element (towards the centre);

$$-p\,(r\,d\theta)\,dz + \left(p + \frac{dp}{dr}dr\right)\left((r+dr)\,d\theta\right)dz - 2\left(p + \frac{dp}{dr}\frac{dr}{2}\right)\left(\sin\frac{d\theta}{2}\right)(dr)(dz) = \rho\, r\,d\theta\,dr\,dz\,(r\omega^2) \qquad (1.12)$$

simplifying and neglecting higher order terms, gives;

(as $d\theta \to 0$, $\sin\theta \to \theta$) and $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$$\boxed{\frac{\partial p}{\partial r} = \rho r \omega^2} \qquad (1.13)$$

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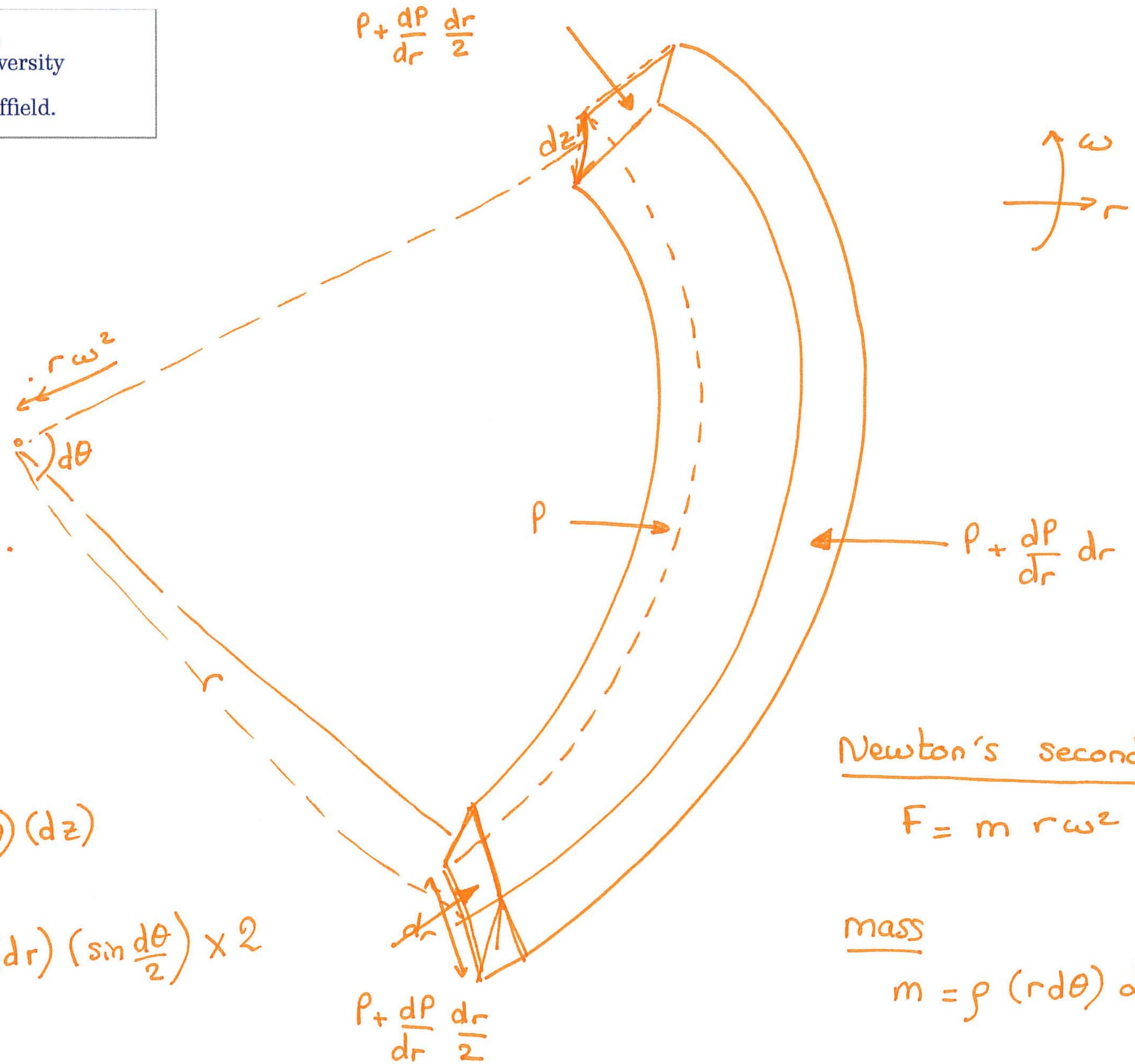

Forces

$\rightarrow P(dz)(r\,d\theta)$

$\leftarrow \left(P + \frac{dP}{dr} dr\right)\left((r+dr)\,d\theta\right)(dz)$

$\rightarrow \left(P + \frac{dP}{dr}\frac{dr}{2}\right)(dz)(dr)\left(\sin\frac{d\theta}{2}\right) \times 2$

Newton's second law

$F = m\,r\omega^2$

mass

$m = \rho\,(r\,d\theta)\,dz\,(dr)$