4. External Flow

4.1. Introduction - Boundary Layers

(Massey §8.1-8.2, White §7.1)

This section describes how we analyse flows around bodies immersed in a fluid stream (e.g. a flag pole bending in the wind, an aircraft in flight, or a bowled cricket ball).

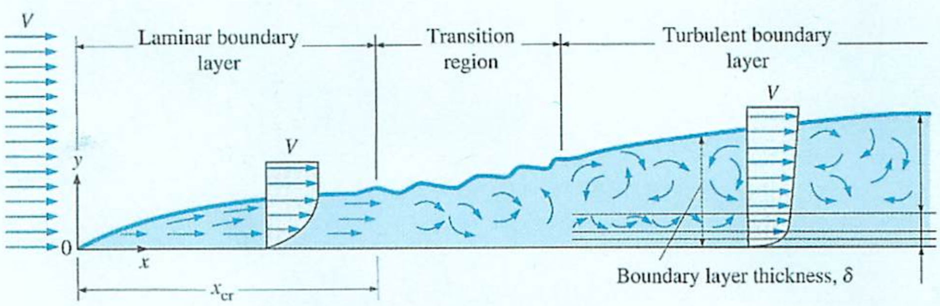
Imagine a body stationary in a fluid stream of velocity U. The fluid at the surface of the body must be stationary. There will be a layer of fluid close to the body travelling slowly. Here the viscous effects are important - we call this the *boundary layer*. Far from the body viscous effects are negligible (because the fluid is all travelling at the same speed).

It is important for engineers to determine the forces on bodies in fluid streams (e.g. for the aerodynamic design of vehicles, wind loading on a structure, or drag on a ship's hull). The force or *drag* is controlled by the shear stress at the walls (skin friction drag) and the pressure difference between the front face and the wake of the body (pressure or 'form' drag).

We will approach this in two ways. Firstly for *flat plates* with laminar flow we will derive and use an expression relating shear stress to velocity profile in the boundary layer (this will involve the FME). This will give equations giving the drag in terms of the Reynolds number. For turbulent flow on flat plates we will use empirical solutions. For *curved surfaces* we will use empirical drag coefficients to determine drag.

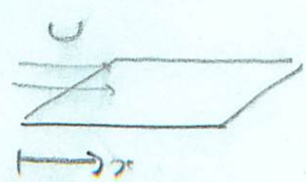
Boundary layers are also important in heat transfer. Convection depends on the properties of a thermal boundary layer. We will look at the thermal boundary layer and relationships for the heat transfer coefficient.

Consider a flat plate in a fluid flowing at U.



The fluid is slowed down by the presence of the plate (at the wall the fluid must be stationary). A laminar layer of slow moving fluid is formed. As more fluid is slowed down the boundary layer thickens. With increasing thickness the layer becomes unstable and turbulence develops. The turbulent layer thickness increases at a greater rate. Within the turbulent boundary layer there is a small viscous sub-layer near the wall. The layer is so thin that it does not effect the displacement and pressure of the bulk fluid flow.

We define a Reynolds number using the distance travelled along the plate;



$$Re_x = \frac{\rho U x}{\mu}$$

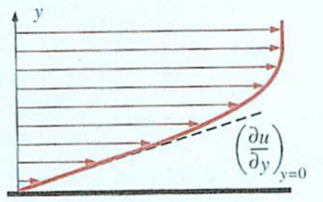
(4.1)

The transition from laminar to turbulent flow depends on the Reynolds number, plate roughness and main stream turbulence.

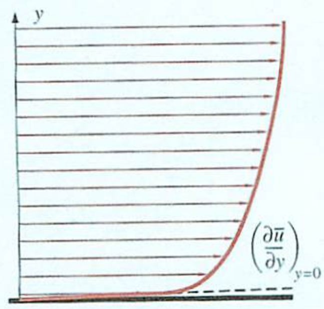
Typically Re< 10^5 laminar boundary layer
Re> 2×10^6 turbulent boundary layer

4.3. The Velocity Profile within a Boundary Layer.

In a turbulent boundary layer there is more mixing of the fluid particles. So the velocity distribution across the layer is more uniform. The velocity at the wall must be zero.



Laminar flow



Turbulent flow

The shear stress on the wall is given by;

$$\tau_w = \mu\left(\frac{du}{dy}\right)_{y=0}$$

So for the turbulent layer the velocity gradient at the wall will be greater. Hence a turbulent layer has greater wall shear stresses and drag.

4.4. The Thickness of a Boundary Layer

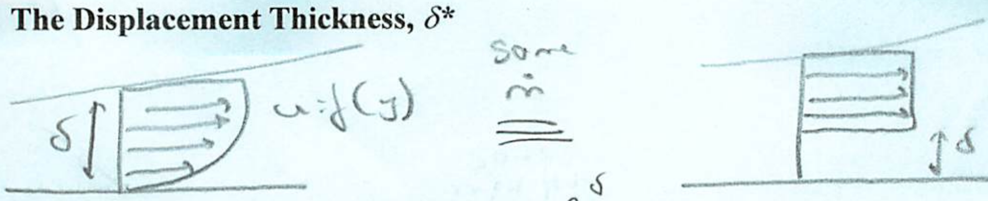
(Massey §8.3)

The boundary merges into the main flow - there is no clear division between the main stream and the fluid slowed by the plate. We define the thickness of a boundary layer, δ to be the distance from the plate at which;

$u =$ $0.99U$ (4.2)

We also define other thicknesses based on the effects of the boundary layer on the flow.

(a) The Displacement Thickness, δ^*



Mass flow rate through the boundary layer $= \int_0^\delta \rho b u \, dy$

Mass flow rate if there had been no boundary layer $= \int_0^\delta \rho b U \, dy$

Reduction in mass flow by boundary layer $= \int_0^\delta \rho b (U-u) \, dy$

We represent this loss of flow as a missing layer of fluid with a thickness δ^* - the *displacement thickness*. Then;

$$\rho b U \delta^* = \int_0^\delta \rho b (U-u) \, dy$$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy} \qquad (4.3)$$

(b) The Momentum Thickness, θ.

The boundary layer will also reduce the overall momentum flow. We can define a *momentum thickness*, θ to represent this loss of momentum.

Momentum of the boundary layer $= \int_0^\delta \rho b u u \, dy$

Momentum of fluid with no boundary layer present $= \int_0^\delta \rho b u U \, dy$

Reduction in momentum caused by boundary layer $= \int_0^\delta \rho b u (U-u) \, dy$

Again we represent this as a missing layer of fluid of thickness, θ

$$(\rho b U \theta) U = \int_0^\delta \rho b (U-u) u \, dy$$

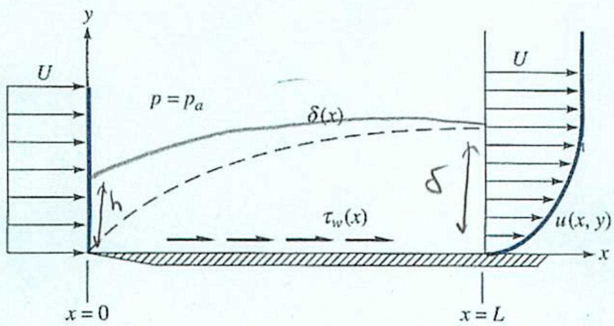
$$\boxed{\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy} \qquad (4.4)$$

4.5. The Boundary Layer Equation

(Massey §8.4, White §7.2 & Ex 3.11)

We will use the FME to derive an expression for the drag on a flat plate. The boundary layer starts at $x=0$ and we consider the portion up to $x=L$. This part of the boundary layer will exert some drag force, D on the flat plate. The larger the length of the plate the greater the drag force, so D=f(x). We define a CV using a streamline which is just outside the boundary layer. Remember that no fluid crosses a streamline.

There are no shear stresses acting on the top face of the CV (since we chose a streamline outside the viscous layer). We will assume the pressure does not vary along the plate. This assumption is ok for flat plates but not for curved bodies (see later).



The sum of the forces in the x direction $\xrightarrow{+ve}$

$$\sum F_x = -D \quad \text{(drag force on plate)}$$

Now consider the momentum flow rates in the x direction.

$$\sum \dot{M}_{in} = \int_0^h \rho U^2 b \, dy = \rho U^2 b h$$

$$\sum \dot{M}_{out} = \int_0^\delta \rho u^2 b \, dy$$

Assembling the FME gives us;

$$D = \rho U^2 b h - \rho b \int_0^\delta u^2 \, dy \tag{4.5}$$

This gives us the drag on the plate, but we do not know the distance, h. Apply MCE. Remember fluid does not cross a streamline.

$$\dot{m}_{in} = \rho U h b$$

$$\dot{m}_{out} = \int_0^\delta \rho u b \, dy$$

Equating these;

$$\rho U h b = \int_0^\delta \rho u b \, dy \Rightarrow h = \frac{1}{U}\int_0^\delta u \, dy \tag{4.6}$$

Substituting for h using equation (4.7) into (4.6) gives;

$$D = \rho U b \int_0^\delta u \, dy - \rho b \int_0^\delta u^2 \, dy$$

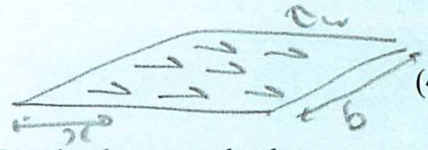
$$= \rho b \int_0^\delta u (U - u) \, dy$$

$$D(x) = \rho b \int_0^{\delta(x)} u(U-u) \, \mathrm{d} y \tag{4.7}$$

Using this we can determine the drag on a flat plat if we know the velocity, $u(y)$ distribution in the boundary layer.

We can present this in a tidier way using the momentum thickness equation (4.4);

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$



$$D(x) = \rho b U^2 \theta \qquad (4.8)$$

Momentum thickness is thus a measure of the drag on the plate. But the drag can also be obtained from the shear stress at the plate (or wall);

$$D(x) = b\int_0^{x} \tau_w(x)\,dx$$

or by differentiating wrt x

$$\frac{dD}{dx} = b\,\tau_w$$

We compare this with the differential of (4.8) wrt x

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

So we can get an expression for the wall shear stress;

$$\boxed{\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx}} \qquad (4.9)$$

This is the usual form of the *boundary layer equation*. It is the basis of all analysis of boundary layers. It is applicable to both laminar and turbulent flows.

If we know the velocity profile $u=f(y)$ then we can determine U and θ and hence get the thickness of the boundary layer and the shear stress τ on the plate.

Boundary Layer Equation with a Pressure Gradient.

This relationship was developed for the case where the pressure does not vary in the x-direction. The more general case is given by Massey §8.4 and gives a more general form of the boundary layer equation;

$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) - \frac{dU}{dx}U\delta^* \qquad \text{for } \frac{dp}{dx} \neq 0 \qquad (4.10)$$

4.6. Laminar Boundary Layer on a Flat Plate

(White §7.2)

We will now use the boundary layer equation to determine the size of the laminar boundary layer and the drag on a flat plate.

First we assume the velocity profile has a parabolic shape

$$u = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \quad \text{— an assumption by Karman 1921} \tag{4.11}$$

We can now find the momentum thickness from equation (4.4);

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy = \frac{2\delta}{15} \tag{4.12}$$

The wall shear stress can be determined from Newton's law of viscosity;

$$\tau_w = \mu\left(\frac{du}{dy}\right)_{y=0} \qquad \frac{du}{dy} = \frac{2U}{\delta} - \frac{2Uy}{\delta^2}$$

$$\tau_w = \frac{2\mu U}{\delta} \tag{4.13}$$

Then putting (4.12) and (4.13) into the boundary layer equation (zero pressure gradient)

$$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx}$$

$$\frac{2\mu U}{\delta \rho U^2} = \frac{d}{dx}\left(\frac{2\delta}{15}\right)$$

Separate the variables;

$$\delta \, d\delta = \frac{15\mu}{\rho U} dx$$

Integrating from 0 to x then the thickness varies from 0 to δ.

$$\int_0^\delta \delta \, d\delta = \int_0^x 15 \frac{\mu}{\rho U} dx$$

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U} \qquad \frac{\delta}{x} = 5.5\left(\frac{\mu}{\rho U x}\right)^{1/2}$$

Rewriting in terms of Reynolds number gives;

$$\frac{\delta}{x} = \frac{5.5}{\mathrm{Re}_x^{\ 1/2}} \tag{4.14}$$

We can now put this into (4.13) to determine the wall shear stress

$$\tau_w = \frac{2\mu U}{\delta} = \frac{2\mu U}{5.5x}\sqrt{Re_x}$$

It is convenient to express this wall shear stress as a non-dimensional number - called the *skin-friction coefficient,* c_f:

$$c_f \equiv \frac{2\tau_w}{\rho U^2} \tag{4.15}$$

Replacing the wall shear stress, τ_w gives;

$$c_f = \frac{0.73}{\mathrm{Re}_x^{\;1/2}} \tag{4.16}$$

We had to assume a velocity profile to obtain these solutions. The exact solution (obtained by Blasius in 1908 using an improved velocity profile analysis) is given by;

$$\frac{\delta}{x} = \frac{5}{\mathrm{Re}_x^{\;1/2}} \tag{4.17}$$

$$c_f = \frac{0.664}{\mathrm{Re}_x^{\;1/2}} \tag{4.18}$$

These expressions (4.17) and (4.18) give an estimate of the thickness of the layer, δ and the wall shear stress, τ_w at a distance x along the plate. These values vary with distance down the plate, $\delta = \delta(x)$ and $\tau_w = \tau_w(x)$. ($c_f = c_f(x)$

The total drag force, D for the whole plate is determined from the integral of the wall shear stress, i.e.

$$D = b\int_0^L \tau_w(x)\,dx \tag{4.19}$$

$$D = b\int_0^L \rho\frac{U^2}{2} c_f\, dx$$

Substituting (4.18) gives

$$D = b\int_0^L \rho\frac{U^2}{2}\, 0.664\sqrt{\frac{\mu}{\rho U x}}\, dx$$

$$D = 0.664\, b\sqrt{\rho\mu U^3 L} \tag{4.20}$$

Again it is convenient to define a non-dimensional *Drag Coefficient*, C_D

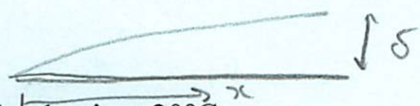
$$C_D \equiv \frac{D}{\frac{1}{2}\rho U^2 bL} \tag{4.21}$$

So for laminar zero pressure gradient flow over a plate;

$$C_D = \frac{1.328}{\mathrm{Re}_L^{\;1/2}} \tag{4.22}$$

Example 4.1. Laminar Drag on A Flat Plate

A long thin flat plate is placed parallel to a 20 m/s stream of air at 20°C. At what distance down the plate will the boundary layer be 1 mm thick? What is the shear stress at the wall at this location?



From the data book; air at 20°C

$\rho = 1.2$ kg/m^3 and $\mu = 18.15 \times 10^{-6}$ Pas

Using expression (4.17); $\frac{\delta}{x} = 5\sqrt{\frac{\mu}{\rho U x}}$

$$x = \frac{\delta^2 U \rho}{25\mu} = \frac{(10^{-3})^2 . 20 . 1.2}{25 . 18.15 \times 10^{-6}} = 53\text{mm}$$

Since this is only valid for laminar flow we should check the Reynolds number at this location, *x*;

$$Re_x = \rho\frac{Ux}{\mu} = \frac{1.2 . 20 . 53\times10^{-6}}{18.15\times10^{-6}} = 7\times10^4$$

laminar assumption ok (just)

The wall shear stress at this location is given by (4.16);

$$c_f = \frac{0.664}{Re_x^{1/2}} = 0.0025$$

$$\text{then } \tau_w = \frac{\rho U^2 c_f}{2} = 0.6\ \text{N/m}^2$$

4.7. Turbulent Boundary Layer on a Flat Plate

(Massey §8.6, White §7.4)

In most practical situations the laminar part of the boundary layer is usually small and may be neglected. The turbulent part of the layer is more important since it gives a greater overall contribution to the drag forces.

Analysis depends on experimental data. Assumptions about the velocity profile may be made in the same way as for laminar flow (see for example White p.400).

The most commonly used formulae for boundary layer thickness and skin-friction coefficient are:

$$\frac{\delta}{x} \approx \frac{0.16}{Re_x^{1/7}} \tag{4.23}$$

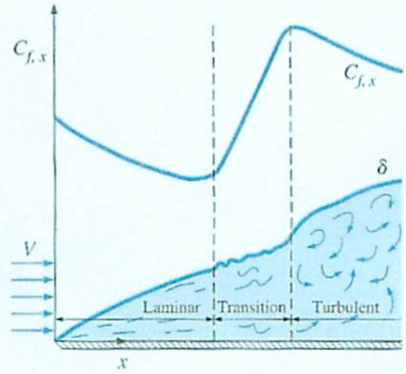
turbulent flow

– empirical solutions

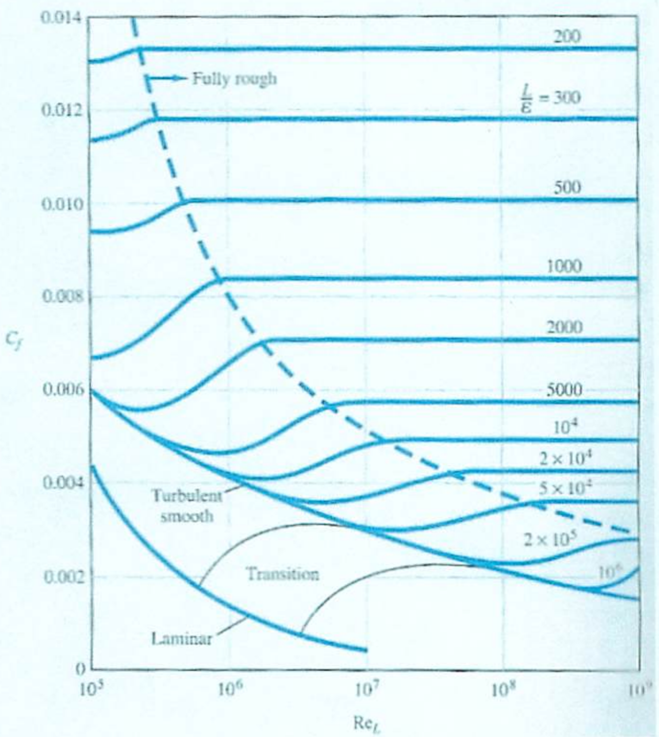
$$c_f \approx \frac{0.027}{Re_x^{1/7}} \tag{4.24}$$

$$C_D \approx \frac{0.031}{Re_L^{1/7}} \tag{4.25}$$

The skin friction coefficient c_f thus varies down a plate from the laminar region (where it is low) to the turbulent region (where it is higher). Equations (4.18) and (4.24) look as follows:



The above equations are for smooth flat plates. When roughness is involved the drag increases. The figure below shows flat plate drag coefficients, C_D for a laminar and turbulent flow. Notice how similar it is to the Moody chart (friction factors in pipes.).



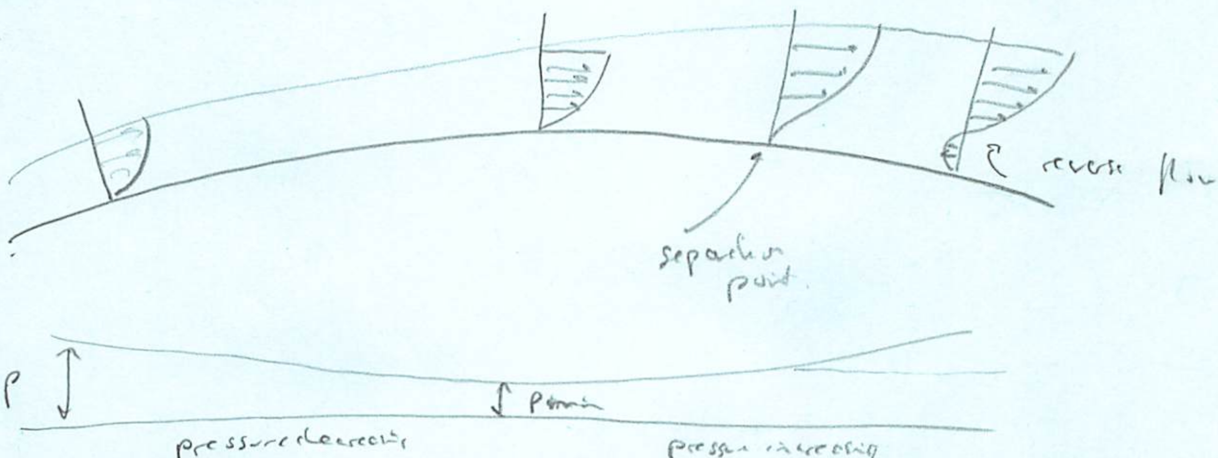
4.8. Pressure Gradient and Separation

(Massey §8.8, White §7.5)

The analysis so far has been for zero pressure gradient $\frac{dp}{dx} = 0$. This is suitable for flat bodies.

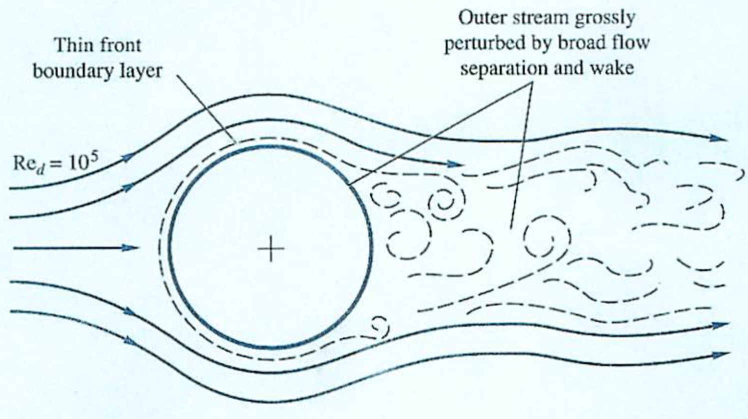
Consider the boundary layer in a curved body. There will be a pressure gradient as the fluid accelerates and decelerates over the body surface.

$(\dot{M}_{out} - \dot{M}_{in})$ increasing



As the boundary layer grows the 'momentum loss' increases, (i.e. $\dot{M}_{out} - \dot{M}_{in}$ increases). An increasing pressure gradient acts to oppose this (i.e. F_x is negative). This causes the flow to reverse.

The point at which the flow first starts to reverse (is $\tau_w = 0$) is called the separation point. Separation causes the formation of a wake of disturbed fluid behind the body.



The pressure in the wake is approximately constant and less than that at the front of the body.Laminar flows are much more prone to separation. This is because the increase in velocity with distance along the body is much less rapid than turbulent flows. The adverse pressure gradient can then more easily halt the flow of fluid near the surface.

4.9. Friction Drag and Pressure Drag

(Massey §8.7, White §7.6)

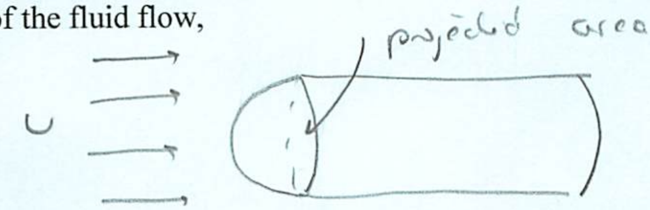
The drag forces we have been discussing so far are all associated with skin friction (i.e. τ_w acting over the surface). Drag also arises from the pressure difference between the front and back of the body.

$$C_D = C_{D_{pressure}} + C_{D_{friction}} \tag{4.26}$$

And often $C_{D\,pressure} >> C_{D\,friction}$

For a body of arbitrary shape the definition of C_D is usually given in terms of the projected frontal area in the direction of the fluid flow,

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A} \tag{4.27}$$

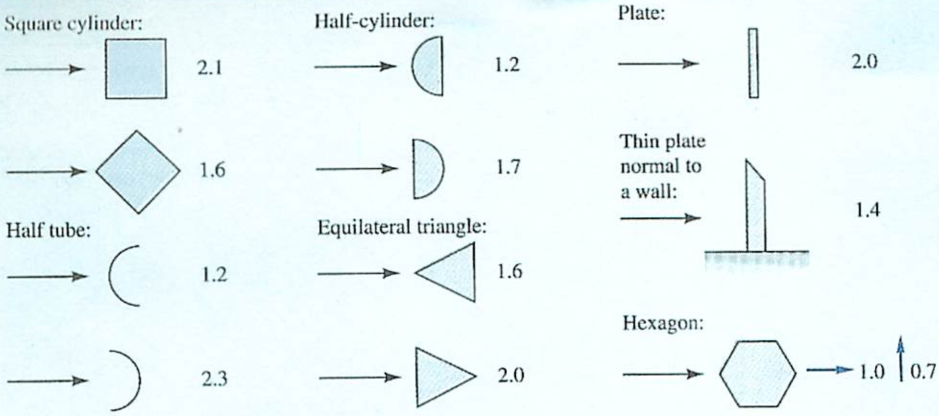


(Note for a flat plate equation 4.21, we used the plan area $A=bl$ and not the projected area - definitions may vary.)

The pressure drag is highly dependent on the wake (and thus the separation point). Bodies where the flow separates earlier show much ~~lower~~ higher pressure drag. Since the separation point and the properties of a wake are difficult to analyse. Drag coefficients are frequently determined empirically. The data is presented in the form of charts and tables.

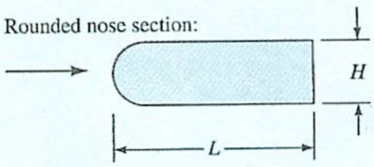
The drag coefficient for many bodies is approximately constant for $Re>10^4$. The tables below show drag coefficients for 2D and 3D bodies.

Shape	C_D based on frontal area	Shape	C_D based on frontal area



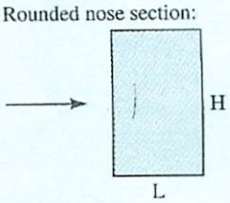
Shape	C_D	Shape	C_D	Shape	C_D
Square cylinder:	2.1	Half-cylinder:	1.2	Plate:	2.0
	1.6		1.7	Thin plate normal to a wall:	1.4
Half tube:	1.2	Equilateral triangle:	1.6		
	2.3		2.0	Hexagon:	→ 1.0 ↑ 0.7

Rounded nose section:



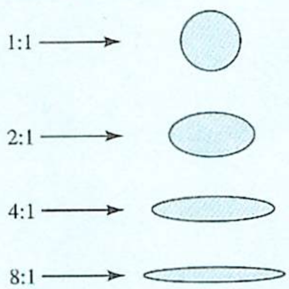
L/H:	0.5	1.0	2.0	4.0	6.0
C_D:	1.16	0.90	0.70	0.68	0.64

Rounded nose section:



L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9

Elliptical cylinder:



	Laminar	Turbulent
1:1	1.2	0.3
2:1	0.6	0.2
4:1	0.35	0.15
8:1	0.25	0.1

Table 7.3 Drag of Three-Dimensional Bodies at Re $\geq 10^4$

Body	C_D based on frontal area
Cube:	1.07
Cube (diagonal to flow)	0.81
Cup:	1.4
Cup (reversed)	0.4
Disk:	1.17
Parachute (Low porosity):	1.2

Body	C_D based on frontal area							
Cone:	θ:	10°	20°	30°	40°	60°	75°	90°
	C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

Body	C_D based on frontal area								
Short cylinder, laminar flow:	L/D:	1	2	3	5	10	20	40	∞
	C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

Body	C_D based on frontal area						
Porous parabolic dish [23]:	Porosity:	0	0.1	0.2	0.3	0.4	0.5
	← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
	→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

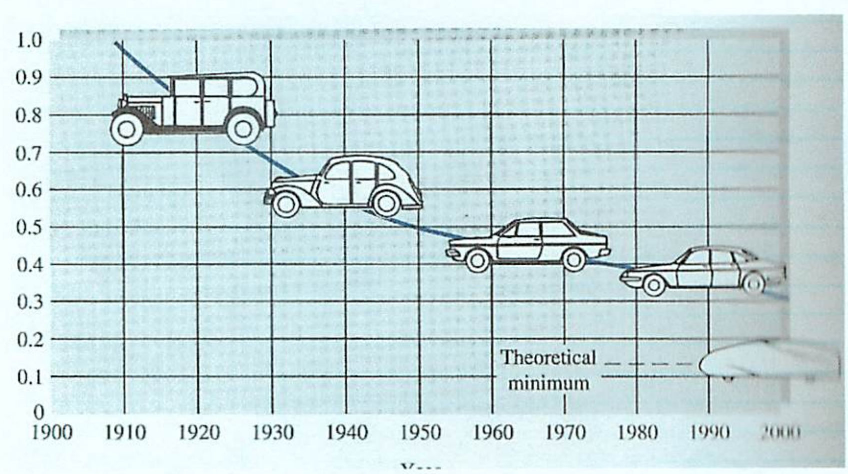
Body	C_D based on frontal area	
Average person:	→ $C_DA \approx 9$ ft^2	↑ $C_DA \approx 1.2$ ft^2

Body	C_D based on frontal area				
Pine and spruce trees [24]:	U, m/s:	10	20	30	40
	C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

Body	Ratio	C_D based on frontal area
Rectangular plate:	b/h 1	1.18
	5	1.2
	10	1.3
	20	1.5
	∞	2.0

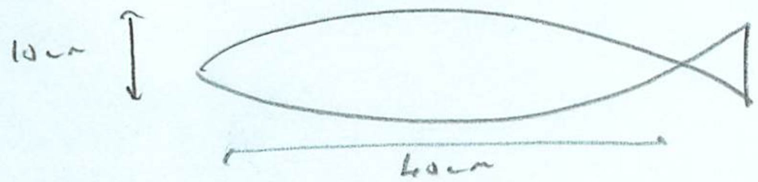
Body	Ratio	C_D based on frontal area
Flat-faced cylinder:	L/d 0.5	1.15
	1	0.90
	2	0.85
	4	0.87
	8	0.99

Body	Ratio	Laminar	Turbulent
Ellipsoid:	L/d 0.75	0.5	0.2
	1	0.47	0.2
	2	0.27	0.13
	4	0.25	0.1
	8	0.2	0.08



Example 4.2. Drag on a Salmon

Estimate the power a salmon uses to swim at 0.5 m/s.



Lets assume the salmon is elliptical in shape. Estimate *L/d*=4

From table 2 C_D = *0.25* for laminar flow and 0.1 for turbulent flow.

Determine Reynolds number (note at 20°C)

Re= $\dfrac{1000 \cdot 0.5 \cdot 0.1}{1.003\times10^{-3}} = 5\times10^{4}$ probably laminar

Drag coefficient, C_D $= 0.25$

Drag force, *D*= $0.25 \cdot \dfrac{1000 \cdot 0.5^2}{2} \cdot \dfrac{\pi\, 0.1^2}{4} = 0.25\ N$

If the fish swims at 0.5 *m/s.*

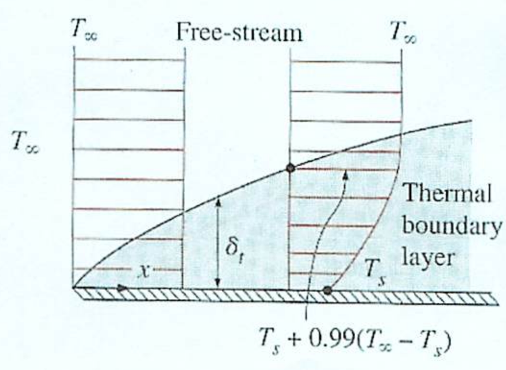
Power required = $D.U = 0.25 \times 0.5 \approx \frac{1}{8}\ W$

4.10. The Thermal Boundary Layer

(see Cengel, Heat & Mass Transfer chapter 6)

We have seen how a velocity boundary layer (where u varies from 0 to 0.99U) develops over a flat plat. Likewise a thermal boundary layers forms over a flat plaet where the plate is at a different temperature to the fluid flowing over it.

The fluid particles in contact with the plate reach the same temperature at the plate (Ts) they then exchange heat with their neighbours and so on. A temperature profile forms. The thickness of the boundary layer is defined as the place where the temperature difference T- T_s, equals 0.99(T_∞- T_s) this is called δ_t



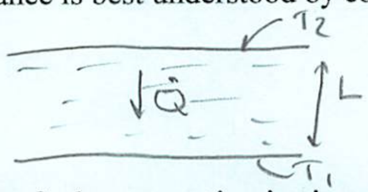
The convection heat transfer rate anywhere along the surface depends on the temperature gradient at that location.

4.10. Prandtl and Nusselt Numbers

A useful non-dimensional number in heat transfer is the **Nusselt number, Nu**.

$$Nu = \frac{hL}{k} \qquad (4.31)$$

Where h is the heat transfer coefficient (W/m^2.K) and L is a characteristic length. Its significance is best understood by considering heat transfer through layer of fluid:



$\Delta T_2 : T_2 - T_1$

Heat transfer by convection is given by:

$$\dot{q}_{conv} = h\Delta T$$

Heat transfer by conduction is given by:

$$\dot{q}_{cond} = \frac{k\Delta T}{L}$$

where k (W/m.K) is the thermal conductivity of the fluid.

Their ratio is:

the larger the the more efficient is convection

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

Another important parameter to describe thermal boundary layers is the **Prandtl number, Pr**. It defines the ratio of the diffusion of momentum to the diffusion of heat:

$$\Pr = \frac{\mu c_p}{k} = \frac{\text{diffusion of momentum}}{\text{diffusion of heat}} \qquad (4.28)$$

Where c_p is the specific heat capacity (at constant pressure). Typical values:

	Pr
Liquid metals	0.01
Gases	1
Water	10
Oil	100,000

Pr expresses the relation between δ & δ_t

This means that heat diffuses very quickly in liquid metals (Pr<<1) and very slowly in oils (Pr>>1). This means that the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity layer.

For *laminar flow over a flat plate* it is possible to determine analytically the thermal boundary layer thickness from the laminar velocity boundary layer (the Blasius solution):

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \tag{4.29}$$

Combining with equation 4.17 gives:

$$\delta_t = \frac{5x}{\mathrm{Re}_x^{\ 1/2}\ \mathrm{Pr}^{1/3}} \tag{4.30}$$

– Thickness of the thermal boundary layer δ_t.

4.11. Convective Heat Transfer and Boundary Layers

Heat transfer from/to various bodies is often represented by an equation of the form:

$$\mathrm{Nu} = C\,\mathrm{Re}_L^{\ m}\ \mathrm{Pr}^n$$

Where C, m, and n are constants.

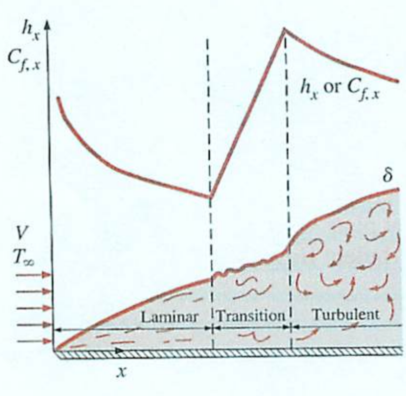
For laminar flow over a flat plate the relationship is:

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{\ 0.5}\ \mathrm{Pr}^{1/3} \quad \text{when} \quad \mathrm{Re}<5\mathrm{x}10^5 \tag{4.31}$$

And for turbulent flow:

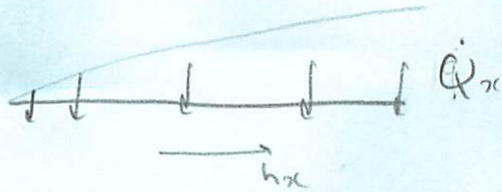
$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{\ 0.8}\ \mathrm{Pr}^{1/3} \quad \text{when} \quad 0.6<\mathrm{Pr}<60 \quad \text{and} \quad 5\mathrm{x}10^5<\mathrm{Re}<10^7 \tag{4.32}$$

The figure below shows how the heat rasnfer coeffciient varies over the length of the plate (note how similar it is the the way the skin friction coefficient varies)



The average Nusselt number can be found by integrating h down the length of the plate:

$$h = \frac{1}{L}\int_0^L h_x dx$$



This gives:

For laminar flow over a flat plate the relationship is:

$$\mathrm{Nu}_L = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{\;0.5}\,\mathrm{Pr}^{1/3} \qquad \text{when} \quad \mathrm{Re}<5\times10^5 \tag{4.33}$$

And for turbulent flow:

$$\mathrm{Nu}_L = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{\;0.8}\,\mathrm{Pr}^{1/3} \qquad \text{when} \quad 0.6<\mathrm{Pr}<60 \quad \text{and} \quad 5\times10^5<\mathrm{Re}<10^7 \tag{4.34}$$

The first equation gives the heat transfer for the entire plate when the flow is completely laminar. The second for when the flow is completely turbulent (i.e. the small laminar layer at the leading edge is neglected).

Example 4.3. Hot oil flowing over a plate.

Q. Engine oil at 60°C flows at 2 m/s over the top of a 5 m long flat plate at 20°C. Determine the heat transfer per unit width of the entire plate.

For engine oil at 40°C use:

c_p=1964 K/kg.K,
μ=0.2177 kg/m.s,
k=0.1444 W/m.K,
ρ=864 kg/m^3

A. First work out the Prandtl and Reynolds numbers

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{0.2177 \cdot 1964}{0.1444} = 2961$$

$$\mathrm{Re}_L = \frac{\rho u L}{\mu} = \frac{864 \cdot 2 \cdot 5}{0.2177} = 4.024\times10^4$$

This is less than Re_c (5×10^5) so the flow is laminar and we use (4.33)

$$\mathrm{Nu}_L = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{\;0.5}\,\mathrm{Pr}^{1/3} \quad 0.664 \cdot (4.024\times10^4)^{0.5}\,(2961)^{0.333} = 1913$$

So $h=$ $\frac{k}{L}\,\mathrm{Nu} = \frac{0.1444}{5} \cdot 1913 = 55.25$ W/m^{2}K

Heat transfer: $\dot{Q} = hA_s(T_\infty - T_s)$

$$\frac{\dot{Q}}{5} = 55.25 \cdot 5 \cdot (60-20) = 11{,}050 \text{ W/m}$$

4.12. Concluding Remarks

In this section we have looked at the flow past bodies immersed in a fluid stream. A layer of slow moving fluid forms around the body. It is only within this *boundary layer* that viscous effects are important. Outside the boundary layer the fluid may be considered inviscid since velocity variation between fluid layers is negligible (we used Bernouilli's equation to relate velocity to pressure in this region).

We derived an expression relating the shear stress to the velocity profile within the boundary layer. This *boundary layer equation* coupled with an assumed velocity distribution can be used to determine the wall shear stress and hence the *friction drag*. We looked at the simple case of the drag on a flat plate in both laminar and turbulent flow and determined *skin friction coefficients,* c_f.

The effects of an applied pressure gradient were considered qualitatively. In an adverse pressure gradient the flow can *separate* and a *wake* forms. The formation of the wake is important and controls the *pressure drag* on the body. Since the point of separation and flow within a wake are difficult to analyse we use empirical data for *drag coefficients,* C_D to determine the drag on engineering components.

When the surface is hotter or colder than the fluid stream a thermal boundary layer is formed. The heat transfer by convection to or from the plate can be expressed as the non-dimensional Nusselt number. Relationships for the Nusselt number were given in terms of the Prandtl and Reynolds numbers. These relationships can be used to determine convective heat transfer to and from flat plates.