

# Algebra & Functions

## 1 Substitution and Simplifying Algebraic Expressions

**General Equation:**

Distributive Property

$$a(m + n) = am + an$$

**Example 1:** If Sally has 6 boxes and each box has 12 books, how many books does Sally have?

**Example 2:** Chase and Jill both collect stamps. If Jill has six less than twice the number of stamps than Chase does, how many stamps does Jill have if Chase has 24 stamps?

**Example 3:** Townsville High School has 30 classrooms, each with 15 desks. If 80% of desks are occupied in 90% and the other 10% of the classrooms are 100% full, how many students attend the school?

## MEDIUM

1. Chris has a collection of 120 records which consists of jazz, blues, and classical records. If one-fourth of the records are jazz and one-third of the records are blues, how many of the records are classical?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 30
  - (B) 40
  - (C) 50
  - (D) 60
  - (E) 70
- 

2. The density,  $d$  of an object is the ratio of its mass to its volume. If the volume of an object is halved and the mass is doubled, which expression represents the new density in terms of  $d$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $1/4d$
- (B)  $1/2d$
- (C)  $d$
- (D)  $2d$
- (E)  $4d$

## ADVANCED

3. Tommy is on the outer edge of a merry-go-round that moves at a constant speed. If  $r$  is Tommy's distance from the center, and the merry-go-round makes  $m$  full cycles during  $n$  minutes, which expression represents Tommy's distance traveled in 1 hour?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $\frac{mr\pi}{30n}$
  - (B)  $\frac{mr^2\pi}{30n}$
  - (C)  $\frac{120r\pi}{mn}$
  - (D)  $\frac{120mr^2\pi}{n}$
  - (E)  $\frac{120mr\pi}{n}$
- 

4. Bunny Slopes Co. is having a sale on winter wear of 20% off. Tracy buys a pair of ski goggles and receives an additional 10% off of the sale price. If the final cost of the goggles is \$90, how much did Tracy save?

**Equation/Strategy:**

**Solve:**

- (A) 22
- (B) 25
- (C) 35
- (D) 38
- (E) 125



## 2 Solutions of Linear Equations and Inequalities

### General Equation:

Point-Slope Form of a Line

$$y = mx + b$$

**Example 1:** If the cost of a cab has a base fare of \$2.50 and \$0.40 per mile, how much does a 10 mile ride cost?

**Example 2:** The cost of tablet devices has dropped an average of 3% of the original price every quarter. After how long will the cost of a tablet be less than half of the original cost?

**Example 3:** The population of Summerville increases at a constant annual rate. The population was recorded in January 2008 as 362,000 and again in July 2010 as 384,000. What is the annual rate of growth of the population?

## MEDIUM

1. The speed of a minute hand moves at a constant speed of  $1/60$  rpm (revolutions per minute). If the current time is 4:00 pm and the minute hand has made 1.75 revolutions, what time was it first recorded at?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 2:15 pm
- (B) 2:25 pm
- (C) 2:45 pm
- (D) 3:15 pm
- (E) 3:25 pm

2. Both Terri and Sam are shorter than Sissy, and Sissy and Hector is shorter than Roy. Which of the following must be true?
- I. Terri is shorter than Sam
  - II. Sissy is shorter than Hector
  - III. Sam is shorter than Roy

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) II and III

## ADVANCED

3. A chemical reaction results in the release the constant release of 70 joules of energy over a 14 minute period. If the total initial amount of energy in the system was 370 joules, how long will it take for the system to release all of its energy?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 1 hour 10 minutes
- (B) 1 hour 14 minutes
- (C) 1 hour 17 minutes
- (D) 1 hour 24 minutes
- (E) 5 hours 17 minutes

4. The number of students at Mainsville School with cellphones increases at a constant rate of  $n$  students per year. If the number of students with cellphones in January 2010 is 200 and the population of the student body is  $p$ , which expression represents the proportion of students with cellphones after  $m$  months?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $\frac{mn}{p + 200}$
- (B)  $\frac{mn}{p - 200}$
- (C)  $\frac{mn + 200}{p}$
- (D)  $\frac{mn + 2400}{p}$
- (E)  $\frac{mn + 2400}{12p}$

### 3 Properties of Exponents

#### General Equation:

The Laws of Exponents

$$a^m \cdot a^n = a^{m+n}$$

$$(ab)^m = a^m \cdot b^m$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

**Example 1.** There are 1000 millimeters in one meter and 1000 meters in one kilometer. How many times larger is a kilometer than a millimeter?

**Example 2.** A pallet contains 5 rows of 5 columns of boxes, each column 5 boxes high. How many boxes does the pallet contain?

**Example 3.** The intensity of a sound is inversely proportional to the square of your distance from the source of the sound. If you are 6 times further away from a set of speakers at a concert as your friend, what is the ratio of the intensity of you to your friend?

## MEDIUM

1. A wooden block occupies a volume of  $2a$ . If the side length is an integer, what is the smallest possible value of  $a$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 2
  - (B) 3
  - (C) 4
  - (D) 8
  - (E) 16
- 

2. The sum of the squares of two numbers,  $a$  and  $b$ , is equal to the square of the sum. Which of the following must be true?

- I.  $a \cdot b = 0$
- II.  $a = b$
- III.  $(a + b)^3 = a^3 + b^3$

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) I is true
- (B) II is true
- (C) III is true
- (D) I and II is true
- (E) I, II, and III are true

## ADVANCED

3. The distance of an object falling from height is proportional to the square of time of the fall. If Betty drops a rock from the top of a building, the rock travels a distance of  $d^3$  meters after time  $t$  seconds. If both the distance and the time are whole numbers and  $d \neq t$ , what is the least distance traveled by the rock?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 2 m
  - (B) 4 m
  - (C) 8 m
  - (D) 32 m
  - (E) 64 m
- 

4. A coat goes on sale  $x$  percent off of the original sale price every month. Which of the following expressions represents the amount taken off after  $m$  months?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $n \left( \frac{x}{100} \right)^m$
- (B)  $n \left[ 1 - \left( \frac{x}{100} \right)^m \right]$
- (C)  $1 - \left( \frac{x}{100} \right)^m$
- (D)  $n \left[ 1 - \left( \frac{x^m}{100} \right) \right]$
- (E)  $n(1 - x^m)$

## 4 Systems of Equations and Inequalities

### General Equation:

Parallel Lines

$$m_1 = m_2$$

Perpendicular Lines

$$m_1 \cdot m_2 = -1$$

**Example 1.** Plane A and plane B are flying parallel to one another. If after 20 minutes, plane A has risen an altitude of 2000 m, how much has the altitude of plane B has risen in 30 minutes?

**Example 2.** When Johnny has  $x$  nickels and  $y$  dimes, his total is \$3. When he has  $y$  nickels and  $x$  dimes, his total increases by \$0.75. How many nickel and dimes did Johnny start off with?

**Example 3.** Two cars starting at the same intersection begin traveling perpendicular to one another. If the first car travels north west at a  $25^\circ$  angle of the point of intersection, what is the direction of the second car?

### MEDIUM

1. A coffee costs \$2.5 and a muffin costs \$3. If Tasha has \$11 and makes a purchase, what is the least amount of change she can receive?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) \$0.00
  - (B) \$0.50
  - (C) \$1.00
  - (D) \$1.50
  - (E) \$2.00
- 

2. Stacy and Ann enter a relay race that consists of 3 events, each worth 10 points. If Stacey earned 8 points and 9 points in the first two events, how what is the smallest number of points she will need to earn in the third event to win if Ann received received a 23 points total?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 6
- (B) 7
- (C) 10
- (D) 23
- (E) 24

### ADVANCED

3. Two planes leave from parallel terminals. If plane A travels north west at 400 mph, and plane B travels north west 600 mph 4 hours later, by what time will the second plane have equal distance traveled as the first plane?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 2
  - (B) 8
  - (C) 4
  - (D) 12
  - (E) 48
- 

4. A coyote is chasing roadrunner in a parallel path. If the roadrunner and coyote are running at a constant rate of 30 mph and the roadrunner has a 20 mile gain on the coyote, how much faster will the coyote need to run if he is going to catch up to the roadrunner in 2 hours?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 10 mph
- (B) 20 mph
- (C) 25 mph
- (D) 30 mph
- (E) 40 mph





## 5 Equations of Lines

### General Equation:

Slope-Intercept Form of a Line

$$y = mx + b$$

**Example 1.** Sunshine taxi charges a base fare of \$2.60 and \$0.40 for every quarter mile. If Elle's ride is 5 miles, how much is her ride?

**Example 2.** A parking lot charges \$10 for the first 4 hours and \$2 up to every additional hour. If George leaves his car for 8 and a half hours, how much is he charged?

**Example 3.** Moe's dad will give him \$1 for every  $x$  points over 50 on his math test, where  $x$  is a whole number of points. Moe received 86 points on his math test and earned \$12. How many points does Moe need to earn \$1?

## MEDIUM

1. Travis is a car salesman and earns \$10 an hour plus a flat commission fee for each car he sells. If Travis works 30 hours and has earned \$1,000 in a week, how much does Travis earn in commission per car if he sells 4 cars?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 80
- (B) 160
- (C) 320
- (D) 360
- (E) 640

2. Cherry is setting up a can drive at her school. For every 50 cans after 100 she collects, the donation center will give her one ticket to an amusement park. If Cherry wants a total of 10 tickets, what is the least number of cans she will need to collect?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 500
- (B) 600
- (C) 1050
- (D) 1500
- (E) 5100

## ADVANCED

3. The value of a car depreciates every year at a constant rate of  $p\%$  of the total value. If the initial value of the car is  $d$  dollars, what is the current value of the car after  $m$  months?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $d - pm$
- (B)  $d - \frac{pm}{12}$
- (C)  $d - \frac{pm}{100}$
- (D)  $d - \frac{pm}{1200}$
- (E)  $d\left(1 - \frac{pm}{1200}\right)$

4. The value of an interior angle of a regular  $n$ -gon increases as a linear function of  $n$ . If an interior angle of a 4-gon is  $90^\circ$  and a 6-gon is  $120^\circ$ , what is the sum of all interior angles of an  $n$ -gon whose interior angles are each  $144^\circ$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 10
- (B) 12
- (C) 144
- (D) 576
- (E) 1440

## 6 Absolute Value

**General Equation:**

$$|a - b| = |b - a|$$

**Example 1.** Dallas walks to her friends house 6 blocks north of her house. She then walks 3 blocks south to visit Francis. What is the total distance traveled by Dallas?

**Example 2.** A train travels east  $x$  miles, then travels west  $y$  miles. What expression gives the net distance traveled by the train?

**Example 3.** A ball thrown from the ground travels a distance of  $n$  meters and bounces with a height of half of the previous height. If the ball bounces 5 times, what is the total distance traveled by the ball in terms of  $n$ ?

## MEDIUM

1. A toy train travels on a circular path with a diameter of 10 feet. If Ellen runs the train forward 2.5 revolutions, then in reverse for 1.5 revolutions, what is the total distance traveled by the train?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $5\pi$  meters
- (B)  $10\pi$  meters
- (C)  $40\pi$  meters
- (D)  $20\pi$  meters
- (E)  $100\pi$  meters

2. If two objects are falling at the same speed,  $s$  meters per second, what is the total distance over  $t$  seconds traveled by both objects if the distance between them is  $d$  units?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $|st|$
- (B)  $|st - d|$
- (C)  $|2sd|$
- (D)  $|2st|$
- (E)  $|2st - d|$

## ADVANCED

3. Lucky and Sunshine are two horses that are running in opposite directions of each other. If Lucky's velocity is  $p$  mph and is twice the velocity of Sunshine, what is the distance between the horses after  $t$  minutes?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $\left| \frac{pt}{60} \right| + \left| \frac{pt}{120} \right|$
- (B)  $\left| \frac{pt}{60} + \frac{pt}{120} \right|$
- (C)  $\left| \frac{pt}{30} \right| + \left| \frac{pt}{60} \right|$
- (D)  $\left| \frac{pt}{30} + \frac{pt}{60} \right|$
- (E)  $\left| \frac{pt}{30} \right| + \left| \frac{pt}{120} \right|$

4. A ping pong ball travels a constant velocity of  $h$  inches per  $s$  seconds with every successive hit. What is the ping pong speed if it travels  $g$  feet in  $m$  minutes?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- |                                     |                                     |
|-------------------------------------|-------------------------------------|
| (A) $\left  \frac{5gh}{ms} \right $ | (C) $\left  \frac{5hm}{gs} \right $ |
| (B) $\left  \frac{gh}{5ms} \right $ | (D) $\left  \frac{hm}{5gs} \right $ |
|                                     | (E) $\left  \frac{gs}{5hm} \right $ |

## 7 Direct and Inverse Variation

### General Equation:

Direct Variation

$$y = kx$$

Joint Variation

$$xy = k$$

**Example 1.** If there are approximately 30 centimeters in one foot, how many centimeters are there in 18 inches? (1 foot = 12 inches)

**Example 2.** When a delivery truck carries heavier packages, the speed of the truck decreases. If the truck is able to go 60 mpg when it carries 400 lbs, how much time will it take the truck to carry 600 lbs and travel 100 miles?

**Example 3.** The speed of a vehicle increases as the amount of fuel used increases, and the amount of time spent traveling over a fixed distance decreases. If a vehicle goes 30 miles an hour for 60 minutes using one gallon of gasoline, how much gasoline will be required to speed the vehicle by 30 miles per hour for 45 minutes?

## MEDIUM

1. The area of a square is proportional to the length of the diagonal. If  $d$  is the area, which of the following describes the area in terms of  $d$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $d^2$
  - (B)  $(d\sqrt{2})^2$
  - (C)  $(d/\sqrt{2})^2$
  - (D)  $d^2\sqrt{2}$
  - (E)  $\sqrt{2}d^2$
- 

2. The measure of acidity, pH, of a compound is proportional inversely proportional to the compound's hydroxide concentration,  $[\text{OH}^-]$ . If the pH is 4 when  $[\text{OH}^-]$  is  $10^{-10}$ , what is the pH decreased by when the  $[\text{OH}^-]$  is decreased by a factor of 10?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 1
- (B) 3
- (C) 4
- (D) 30
- (E) 40

## ADVANCED

3. The odds of guessing a number correctly are proportional to the number of terms to guess from. If the odds of choosing  $p$  numbers out of  $q$  terms is 0.4, what are the least odds of choosing  $p$  out of  $q + 1$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 0.3
  - (B) 0.33
  - (C) 0.36
  - (D) 0.375
  - (E) 0.38
- 

4. The volume,  $V$ , of a right cylinder is directly proportional to its radius and height. If the radius is doubled and the height is halved, what is the new volume in terms of the old volume,  $V$ ? ( $V = \pi r^2 h$ )

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $V/4$
- (B)  $V/2$
- (C)  $V$
- (D)  $2V$
- (E)  $4V$

## 8 Quadratic Equations

**General Equation:** If  $f(x)$  is a quadratic functions with roots  $r$  and  $s$ , then the coordinate of the vertex (maximum or minimum) is

$$\left(\frac{s-r}{2}, f\left(\frac{s-r}{2}\right)\right)$$

**Example 1.** Reese jumps, starting from the ground, and reaches a maximum height of 6 feet at 3 seconds. How long does the trip take from when she first jumped until she returned back to the ground?

**Example 2.** A cannonball is fired from ground-level and hits the ground after  $t$  seconds. If the maximum height is  $h$  ft, write the coordinate that expresses the maximum of the cannonball's trajectory.

**Example 3.** The sum of two integers  $x$  and  $y$  is  $m$  and the product of the two integers is  $n$ . What is  $n$  in terms of  $x$ ?

## MEDIUM

1. Stacey is making a rectangular garden for her rose bushes. If the perimeter needs to be 100 cm, what is the maximum area she can enclose?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $25 \text{ cm}^2$
- (B)  $50 \text{ cm}^2$
- (C)  $100 \text{ cm}^2$
- (D)  $500 \text{ cm}^2$
- (E)  $625 \text{ cm}^2$

2. For two integers  $p$  and  $q$ , the sum of their squares is equal to the square of their sum. What is the value of  $pq$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

## ADVANCED

3. A sector of a circle is inscribed in a square. If the radius of the circle is equal to the side length of the square,  $s$ , what is the area of the inscribed sector in terms of  $s$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $\pi s^2/4$
- (B)  $\pi s^2/2$
- (C)  $s^2/4$
- (D)  $s^2/2 - \pi$
- (E)  $s^2 - \pi/4$

4. Andres has 1.2 kilometers of fencing he wishes to use to create two adjacent pens for his sheep and his goats. If he uses the fencing for the perimeter and a divider in the middle of the entire pen, what is the length, in meters of the shortest side? (1 kilometer = 1000 meters)

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 150 m
- (B) 300 m
- (C) 400 m
- (D) 600 m
- (E) 750 m



## 9 Rational and Radical Equations

### General Equation:

If  $\frac{a}{b} = \frac{c}{d}$ , then

- $ad = bc$

- $\frac{a}{c} = \frac{b}{d}$

**Example 1.** If the sequence  $x$ ,  $\_\$ ,  $y$  has a common ratio between each term, what is the value of the missing term?

**Example 2.** A three digit number is evenly divided by a two digit number such that the quotient is a perfect square. What is the smallest such three digit number and two digit number pair?

**Example 3.** For an integer  $n$ , the square root and cube root are both integers. If the square root and cube root of  $n$  are distinct, what is the smallest sum of both roots of such a number?

## MEDIUM

1. The odds of choosing a red marble is 1 out of  $p$  marbles and the odds of choosing a green marble is 1 out of  $q$  marbles. Which expression represents the odds of choosing a red or a green marble?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $(p + q)/pq$
  - (B)  $1/pq$
  - (C)  $2/pq$
  - (D)  $p + q$
  - (E)  $(1 - p)(1 - q)$
- 

2. Benjamin is missing cards in his deck of cards. If in his deck of 50 cards there are  $x$  kings and  $y$  queens. What are the odds of choosing a queen and a king?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $xy/50$
- (B)  $(x + y)/50$
- (C)  $2500/xy$
- (D)  $(x + y)/2500$
- (E)  $xy/2500$

## ADVANCED

3. The ratio of the sides of a rectangle is  $a : b$ . If one is added to both sides, the new ratio of sides is  $b : a$ . Which of the following must be true?

- I. The rectangle is a square
- II. The area is  $a^2$
- III. The side length is 1

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) I is true
  - (B) II is true
  - (C) III is true
  - (D) I and II are true
  - (E) I, II, and III are true
- 

4. A square is inscribed in a circle. If the radius of the circle double, by what factor does the side length of the square grow?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 2
- (B)  $\sqrt{2}$
- (C) 4
- (D)  $2\sqrt{2}$
- (E)  $4\sqrt{2}$

## 10 Concepts of Algebraic Functions

**General Equation:** If  $f(a) = b$ , then  $(a, b)$  is a coordinate of the graph of  $f$ .

**Example 1.** If Timmy sells less lemonade in week  $x$  than in week  $y$  and Timmy's sales has increased every week, what relationship describes  $y$  to  $x$ ?

**Example 2.** Luckystar is a horse that races in the Belmont racetrack. If Luckystar runs at  $x$  mph at time  $p$  and again at time  $q$ , what is his increase in average speed over the interval from  $p$  to  $q$ ?

**Example 3.** If Arnold has  $3 \times 3$ ,  $4 \times 4$ , and  $5 \times 5$  cubes, what is the least number of cubes Arnold will need to make a building that is 27 units high?

**Example 3.** Lindsey and Ricky have separate college tuition funds created on a Monday. Lindsey's account starts at an initial amount of \$0.50 and the total doubles every day, whereas Ricky's college tuition starts at \$1 initially and the total doubles every day. On what day will Lindsey begin having greater amounts in her account than Ricky will have in his account?

## MEDIUM

1. If the amount of bacteria in a colony doubles every hour, which of the following cannot be a relative factor of the population of bacteria?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $1/2$
- (B) 0
- (C) 1
- (D) 2
- (E) 8

2.  $f$  has the property that  $f(\square) = \clubsuit$  and  $f(\clubsuit) = \square$  for all  $\clubsuit$  and  $\square$ . Which of the following represents the equation for  $f$  in terms of  $x$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $f(x) = x$
- (B)  $f(x) = x^2$
- (C)  $f(x) = 0$
- (D)  $f(x) = 1$
- (E)  $f(x) = \sqrt{x}$

## ADVANCED

3. The odds of choosing a prime number out of  $x$  terms is  $m$  and the odds of choosing an even number out of the same  $x$  terms is  $n$ . What are the odds of choosing an even prime out of  $x^2$  terms?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $1/mn$
- (B)  $1/x$
- (C)  $1/x^2$
- (D)  $1/(m + n)$
- (E) Cannot be determined

4. If Kat is on a swing that starts at a height of  $h$  off the ground and reaches the ground after  $t/3$  seconds, at what position will she be at  $15t$  relative to her beginning position?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $h$
- (B)  $-h$
- (C) 0
- (D)  $2h$
- (E)  $-2h$

## 11 Newly Defined Symbols Based on Commonly Used Operations

**General Equation:** Use the notation as you would with variables

**Example 1.** If  $f \otimes g$  is defined as  $f \cdot g - (f + g)$ , what must be true of  $f$  and  $g$  so that  $f \otimes g = 0$ ?

**Example 2.** If  $f \% g$  is defined as the remainder of  $f$  when divided by  $g$ , what is  $(x - 4x + 4) \% (x - 2)$ ?

**Example 3.**  $S$  is a set with elements  $s_1, s_2, \dots, s_n$ . Let  $S \bullet S$  be defined as  $s_1 \cdot s_1 + s_2 \cdot s_2 + \dots + s_n \cdot s_n$ . If  $S \bullet S = 0$  what must be true of the elements of  $S$ ?

### MEDIUM

1. If  $x^2 - 64 = 36$  and  $x - 8 = 9$ , what is the value of  $x + 8$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 4
- (B) 18
- (C) 25
- (D) 64
- (E) 100

2.  $i\#a$  is defined as  $i^a$ . If  $i\#2 = -1$  and  $i\#3 = -i$ , what is the value of  $i\#115$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 1
- (B) -1
- (C)  $i$
- (D)  $-i$
- (E) 0

### ADVANCED

3.  $\lfloor x \rfloor$  is defined as the greatest integer less than or equal to  $x$  whereas  $\lceil x \rceil$  is defined as the least integer greater than or equal to  $x$ . What is the value of  $\lfloor \lceil x \rceil \rfloor$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A) 1
- (B) 0
- (C)  $x$
- (D)  $x^2$
- (E) Cannot be determined

4. The ternary operation  $a@b@c$  is defined as  $a = b$  when  $a \geq 0$ , and  $a = -a$  when  $a < 0$ . What is the value of  $a^2 @ -|a^2| @ \sqrt{(-a^2)^2}$ ?

**Equation/Strategy:** \_\_\_\_\_

**Solve:**

- (A)  $-a^2$
- (B)  $a^2$
- (C)  $|-a^2|$
- (D)  $\sqrt{a^4}$
- (E)  $\sqrt{-a^4}$