Numbers and Operations

1 Arithmetic word problems (including percent, ratio, and proportion)

General Equation: $\frac{a}{b} = \frac{c}{d}$

Example 1. If 3 gallons of water fill 2 fish bowls, how many gallons will fill 9 fish bowls?

Example 2. If y varies directly as x, and $y = 6$ when $x = 9$, what is the value of y when $x = 12$?

Example 3. Terri rides her bike every day on her way to school. On a sunny day, Terri can make the 2.6 mile trip in half of an hour at a constant pace. When it rains, Terri's speed drops by 1 mile per hour. If Terri's trip is the same distance in the rain, what total time it takes her to get to school?

MEDIUM

1. If the side length, s, of a square is increased by n, what is the ratio of the area of the new square to the old?

Equation/Strategy: _____

Solve:

(A) $n^2 : 1$

(B) $n^2 : s^2$

(C) $s^2 : n^2$

(D) $s^2 : (s+n)^2$

(E) $(s+n)^2 : s^2$

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2. From January to March, coats drop in price by 10% each month from the month before. Which ratio represents the cost of the jacket from January to March?

Equation/Strategy: _____

Solve:

(A) $100 : 81$

(B) $10 : 81$

(C) $10 : 9$

(D) $9 : 1$

(E) $11 : 9$

ADVANCED

3. If P varies jointly as T and inversely as V, and $P = 5$ when $V = 3$, what is the value of V when P doubles and T remains unchanged?

Equation/Strategy: _____

Solve:

(A) 1.5

(B) 2.5

(C) 6

(D) 10

(E) 12

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4. A jar contains D jellybeans. If there are A red jellybeans, B green jellybeans, and C orange jellybeans. What proportion of the jellybeans are red or orange?

Equation/Strategy: _____

Solve:

(A) $\dfrac{A+C}{A+B+C}$

(B) $\dfrac{A+C}{D}$

(C) $\dfrac{D-(A+C)}{D}$

(D) $\dfrac{D-B}{A+B+C}$

(E) $\dfrac{D-B}{D}$

2 Properties of integers (even, odd, prime numbers, divisibility,and so forth)

General Equation: $a = qk + r$, q = quotient, r = remainder

Example 1. If $10 < n < 20$ and has a remainder of 5 when divided by 6, what is the value of n?

Example 2. What is the smallest three digit integer who has an even number of prime factors?

Example 3. If the quotient of $6n$ and 5 is a with a remainder of n and the quotient of $5n$ and 6 is b with a remainder of $\frac{n}{2}$, find the value of n in terms of a and b.

MEDIUM

1. There are d number of students in a class. When broken into n groups of 3 with three groups having one extra student in each. If the number of students in the class is greater than 15, what is the minimum number of groups?

   Equation/Strategy: _____

   Solve:

   (A) 4
   (B) 5
   (C) 6
   (D) 15
   (E) 18

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2. Set S contains n terms which has an even sum. What is the maximum number of odd terms that can be in S?

   Equation/Strategy: _____

   Solve:

   (A) n-2
   (B) n-1
   (C) n
   (D) n/2
   (E) 2

ADVANCED

3. For $n \neq d$, when n is divided by d, the remainder is 0. When d^2 is divided by n, the remainder is 0. If n has x prime factors, how many prime factors must d have?

   Equation/Strategy: _____

   Solve:

   (A) x-1
   (B) x
   (C) x+1
   (D) n
   (E) d

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4. A class is divided into quarters. The groups are then divided into thirds, with each group having 1 extra person. What is the minimum number of students in the class?

   Equation/Strategy: _____

   Solve:

   (A) 13
   (B) 14
   (C) 16
   (D) 21
   (E) 28

3 Rational Numbers

General Equation: $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Example 1. If $\frac{x}{3} = \frac{y}{5}$, write an inequality stating the relationship between x and y.

Example 2. If the number of microbes in a petri dish is reduced by a factor of one-third after each day, how many days will it take for the population to be less than 10% of the original amount?

Example 3. The batting average of a baseball player is given by the proportion of hits versus the number of times at bat. If Stanley has a batting average of 0.32 and had 24 hits in one season, what is the number of times Stanley was at bat?

MEDIUM

1. If the odds of choosing a red marble from a bag of marbles is 0.3. If the odds of choosing n red marbles is 0.027, what is the value of n?

Equation/Strategy: _____

Solve:

(A) 2
(B) 3
(C) 6
(D) 7
(E) 9

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2. One-third of Ms. Boyd's class takes French while four-fifths takes Spanish. How many students are taking both French and Spanish?

Equation/Strategy: _____

Solve:

(A) 1/15
(B) 2/15
(C) 1/5
(D) 1/3
(E) 2/3

ADVANCED

3. Bob is two-thirds the age of his sister Sally. If Sally is four-thirds times the age of their sister, Patricia. If Patricia is 3 years older than Bob, what is Sally's age?

Equation/Strategy: _____

Solve:

(A) 20
(B) 24
(C) 28
(D) 32
(E) 36

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4. From 1970 to 2010, the population of US living on the coast has increased by approximately 40%. If approximately 40% of the total population of the USA lives on the coast in 2010 and the population in 2010 is 308 million people, what is the population (in millions) of the US in 1970?

Equation/Strategy: _____

Solve:

(A) 49
(B) 77
(C) 88
(D) 193
(E) 220

4 Sequences and Series (including exponential growth)

General Equation:

Arithmetic Sequence

$$a_n = a_n + nd$$

Geometric Sequence

$$a_n = a_1 r^n$$

Arithmetic Series

$$S = \frac{n(a_1 + a_n)}{2}$$

Geometric Series

$$S = \frac{a_1(1 - r^n)}{1 - r}$$

Example 1. Shelly is preparing to run for a marathon. If Shelly starts running on the first week with 1 kilometer, and doubles the number of kilometers every week, how many miles will she run on the sixth week?

Example 2. Jenny receives a weekly allowance of $15 and is saving up to buy a new laptop that costs $450. If she has $100 saved up already, how many days will it take her to have enough for the laptop?

Example 3. A ball is dropped from a height of 1 meter and bounces a height of two-thirds of the previous bounce with every consecutive bounce. What is the total distance the ball has traveled after 5 bounces?

MEDIUM

1. Tom is doing a cross country bike ride. On the first day, Tom rode 5 miles. If he decides to ride an additional 20% every day, approximately how long will he have ridden on day 8?

Equation/Strategy: _____

(A) 13 miles

(B) 14 miles

(C) 17 miles

(D) 19 miles

(E) 20 miles

Month	2	5	7
Height	5	9.5	11.5

2. Eduardo purchases a potted plant to grow at home. The height is recorded (in inches) every month. If the rate of growth is constant, at what height did Eduardo buy the plant?

Equation/Strategy: _____

(A) 2 in

(B) 3 in

(C) 3.5 in

(D) 4 in

(E) 4.5 in

ADVANCED

3. A spherical balloon deflates at a rate such that the volume is cut in half every 2 minutes. If r is the initial radius, which expression represents the radius after 8 minutes?

Equation/Strategy: _____

Solve:

(A) $r/2$

(B) $r/4$

(C) $r/8$

(D) $r/16$

(E) $r/32$

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4. A guitar string is plucked and the distance between the highest point and the lowest point of the first oscillation is 256 millimeters. If the distance the string travels is reduced by one-quarter with each oscillation, how many oscillations will it take for the string to have traveled a total distance of 700 millimeters?

Equation/Strategy: _____

Solve:

(A) 2.5

(B) 3

(C) 3.5

(D) 4

(E) 5

5 Elementary Number Theory

General Equation: $a|b = n$ if and only if $a = b \cdot n$

Example 1. Gumdrops come in bags of 60. Mr. Lee wants to buy enough bags so that he has a perfect square number. What is the least number of bags Mr. Lee will need to buy?

Example 2. The sum of the angles of a regular polygon is given by the equation $S(n) = 180(n-2)$ where n is the number of sides. How many sides does the smallest regular polygon have if is the smallest regular polygon possible such that each individual angle is over 120°?

Example 3. A group of n students participate in a math competition. The students are required to shake hands with all other participates in the competition. If each handshake between two people is counted only once, how many distinct handshakes are there overall?

MEDIUM

1. There are n jellybeans in a jar. The number of jellybeans can be separated into groups of 24 or groups of 42. What is the least possible number of jellybeans in the jar?

   Equation/Strategy: _____

   Solve:

   (A) 126

   (B) 168

   (C) 252

   (D) 504

   (E) 1008

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2. A full revolution of the hour hand around the face of a clock corresponds to 12 hours. If the hour hand starts at 12:00 AM and completes 6.75 revolutions, what is the current time?

   Equation/Strategy: _____

   Solve:

   (A) 3:00 AM

   (B) 9:00 AM

   (C) 3:00 PM

   (D) 6:00 PM

   (E) 9:00 PM

ADVANCED

3. For regular polygons of n sides, if f is the number of faces, v is the number of vertices, and e is the number of edges, what is the value of $v - e + f$?

   Equation/Strategy: _____

   Solve:

   (A) 0

   (B) 1

   (C) 2

   (D) n-1

   (E) n

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4. Ms. Rizzo has a box of play sand in her classroom. If the volume is a positive integer greater than 1 in^3, and is also a perfect square, what is the smallest possible length of the diagonal from one vertex to the other passing through the center of the cube?

   Equation/Strategy: _____

   Solve:

   (A) $\sqrt{2}$

   (B) $\sqrt{3}$

   (C) $2\sqrt{2}$

   (D) $2\sqrt{3}$

   (E) $4\sqrt{3}$

6 Sets (union, intersection, elements)

General Equation:

Union	Intersection
$p_{P \text{ or } Q} = p_P + p_Q - p_P \cdot p_Q$	$p_{P \text{ and } Q} = p_P \cdot p_Q$

Example 1. Liang is playing a game with a standard deck of cards. He wants the first card he picks up to be either a red card or a face card (Jack, Queen, or King). What is the probability he will choose either a red card or a face card?

Example 2. In the set of integers from 1 to 100 (inclusively), how many numbers are divisible by 3 or 5 but not both?

Example 3. Mr. Dropal has 16 students in his class. There are 10 male students. Half of all students are taking only Chinese and one-quarter are taking Chinese and French. If the class must take either French or Chinese, what proportion represents the maximum number of females taking only French?

MEDIUM

1. Mr. Carter has a garden of yellow, white, and red rose bushes. If the proportion of red rose bushes is 0.4. When he buys an additional 4 red rose bushes, the proportion increases to 0.5. How many rose bushes did Mr. Carter originally have?

   Equation/Strategy: _____

   Solve:

   (A) 8
   (B) 12
   (C) 20
   (D) 24
   (E) 36

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2. The average (arithmetic mean) of a set of 5 positive integers is 10 and the median is 10. What is the largest possible value of the largest member of the set?

   Equation/Strategy: _____

   Solve:

   (A) 10
   (B) 50
   (C) 22
   (D) 37
   (E) 40

ADVANCED

3. Helen has a box of 40 chocolates. The proportion of choosing a cherry filled or coconut is 0.3, whereas the proportion of choosing a coconut or creme filled is 0.4. If the number of creme filled is twice the number of cherry filled chocolates and there is at least one cherry filled chocolate, what proportion of coconut chocolates are there?

   Equation/Strategy: _____

   Solve:

   (A) 0.075
   (B) 0.1
   (C) 0.15
   (D) 0.2
   (E) 0.25

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4. Robbie has a equal chances of getting into College A and College B. If the proportion of getting into either college is 0.84, what proportion represents his chances of getting into one of the two schools?

   Equation/Strategy: _____

   Solve:

   (A) 0.4
   (B) 0.42
   (C) 0.5
   (D) 0.6
   (E) 0.84

7 Counting Techniques

General Equation:

Permutations	**Combinations**
Without Repetition	*Without Repetition*
$_nP_r = \frac{n!}{(n-r)!}$	$_nC_r = \frac{n!}{r!(n-r)!}$
With Repetition	*With Repetition*
n^r	$\frac{(n+r-1)!}{r!(n-1)!}$

Example 1. Mandy is throwing an ice cream party and has 3 flavors of ice cream and 4 different toppings. How many different combinations can her guests make?

Example 2. Jerry is making a sundae with 3 scoops of ice cream. If he has 5 different flavors of ice cream, how many different combinations can Jerry make?

Example 3. If a four-digit pin number contains the digits 0 to 9 where no digit can be repeated more than twice, how many different combinations for pin numbers are possible?

MEDIUM

1. If set A contains all even integers under twenty and set B contains all even prime numbers, then the set of common elements between set A and set B is

Equation/Strategy: _____

Solve:

(A) {}
(B) {0}
(C) {2}
(D) {0, 2}
(E) All even numbers

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2. If a four point star has 8 vertices, and an eight point star has 16 vertices, how many vertices does a 10 point star have?

Equation/Strategy: _____

Solve:

(A) 12
(B) 16
(C) 20
(D) 24
(E) 32

ADVANCED

3. A number of volleyballs compete in a tournament. If each team must play one another, and there are a total of 120 matches, how many teams competed?

Equation/Strategy: _____

Solve:

(A) 5 teams
(B) 6 teams
(C) 7 teams
(D) 8 teams
(E) 12 teams

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4. An equilateral triangle is divided so that the midpoint of each line is the vertex of an inscribed triangle. If the process continues, how many triangles will there be after n divisions?

Equation/Strategy: _____

Solve:

(A) 2^n
(B) 3^n
(C) 4^n
(D) $4n$
(E) $8n$