SAT Student Manual 2

ASC English

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Chapter 1

Introduction

ASC's SAT Advanced course is designed to help students master the most difficult topics on the SATs. By focusing on and practicing these topics, advanced SAT students can improve their SAT scores.

The SAT is composed of ten sections - a 25-minute essay, six 25-minute sections, two 20-minute sections, and one 10-minute section. Total testing time is 3 hours and 45 minutes. The breakdown of each section is as follows:

Topic	Testing Time	Number of Questions	Skills Tested
Critical Reading	Two 25 minute sections and one 20 minute section	67 Questions in total: 19 sentence completions and 48 passage based questions	Vocabulary, sentence logic, answering questions and making inferences about a text
Math	Two 25 minute sections and one 20 minute section	54 Questions in total: 44 multiple choice and 10 student-produced responses	Integrating and applying mathematical concepts, including algebra, functions, geometry, probability, statistics, and data interpretation
Writing Multiple Choice	One 25 minute section and one 10 minute section	49 Questions in total: 25 Improving sentences, 18 identifying sentence errors, and 6 improving paragraph questions	Sentence structure and grammar, coherence and cohesion
Writing Essay	25 minutes	Write one essay on a given topic	Writing and analysis skills

You should also be aware that SAT test includes one 25-minute section called the experimental section. It can be in critical reading, math, or writing and is used by the testmakers to design and test questions for future exams. This section does not contribute to your SAT score, however, you won't know which section is the experimental section, so you should try your best on every part of the exam.

SAT Scoring

Each section (critical reading, math, and writing) is scored by giving you a raw score and then converting that to a scaled score. The raw score is the number of questions that you got correct minus one-fourth of the questions that you got wrong. Leaving a question blank does not affect your score. This equation can be seen as:

Raw Score: _____ correct − 0.25 (_____) incorrect = _____

The raw score is then converted to a scaled score between 200 and 800 points. It should be

noted that in the writing section, essay score is also factored into your scaled score. Additionally, in the math section, correct student-produced responses (grid ins) are worth one raw score point whereas incorrect student-produced responses (grid ins) do not affect your score.

This brings us to two very important questions:

1. If wrong answers lead to subtracting points, but a blank does not affect my score, should I guess?

The answer is, it depends. If you are able to eliminate at least one choice, then the long run average results in the same or greater raw score than if you didn't guess. This also depends on the individual test taker's personality. Someone that tends to be more cautious might be tempted to leave a lot more blank than one should. On the other hand, a person that is more risk-inclined may have a tendency of not leaving enough blank. Therefore, if you are unsure, then you should complete a practice section leaving a few blank and guessing on the majority of questions that you don't know for sure and find your raw score using the equation above. Then, calculate what your raw score would have been had you left more of the ones that you were unsure of blank. Use whatever strategy gives you the highest score.

2. What is a "good" SAT score?

Although many people know that the coveted 2400 is a perfect score on the SATs, many students and parents wonder what other scores are classified as "good". This question does not have a simple answer because a "good" SAT score for one college might not be "good" for a more competitive school. For example, the top schools in the country tend to look for scores at least in the 700s in each of the three sections (2100 total), whereas smaller, less competitive schools will accept lower scores. While it is true that the higher a student's SAT scores are, the more opportunities will be available to a student, there are schools for students with a large range of SAT scores. Fortunately, there are tools to help students figure out their SAT score goal and what is a "good" score for their ability level and the colleges that they are hoping to gain acceptance from.

So what is a good score on the SAT? The answer is: it depends on what schools and, in some cases, what programs of study a student is aiming for. Therefore, first step in deciding what a good score is would be to decide what colleges or universities interest you and come up with a few ideas of what you might want to study. Next, just check online what scores your ideal school is looking for and make it your goal to score a bit higher just so you stand out among all the other applicants. Oftentimes, the university's website will contain the average SAT score and GPA for admitted applicants. They might also give a 25% to 75% percentile scores. Someone in this range might be a good match for the school, whereas it might be more difficult for someone with SAT scores than the 25% percentile to be admitted. The College Board (the same company that makes the SATs) also has an online program called "My College Matches" to help students identify colleges that might be good for them sorted by individual factors such as SAT

score. Identifying potential areas of study could also help to put SAT scores in context. For example, a student who scores a 2100 by getting 800s on the verbal and reading section and a 500 on math might make it into a writing program at a top university but would not be considered by a high ranking technical institution. Every school and every student's situations are different.

About the Critical Reading Section

The critical reading section is composed of two parts, sentence completions and passage-based reading. The sentence completion focuses on vocabulary and sentence logic in order to select the word that best fits in the blank within the sentence. It is imperative that students learn to detect the types of sentence completions and the clues given in each of the sentences which will lead to the correct answer. For the passage-based reading, students will learn the types of passages and questions tested as well as strategies for detecting the correct answer and the reasons that incorrect answers are incorrect.

About the Math Section

The following topics are tested on the SAT math section: number and operations, algebra and functions, geometry and measurement, and data analysis, statistics, and probability questions.

Below is a list from the College Board of each topic tested in more detail:

Number and Operations ($20-25\%$ of the test)

- Arithmetic word problems (including percent, ratio, and proportion)
- Properties of integers (even, odd, prime numbers, divisibility, and so forth)
- Rational numbers
- Sets (union, intersection, elements)
- Counting techniques
- Sequences and series (including exponential growth)
- Elementary number theory

Algebra and functions questions ($35-40\%$ of the test)

- Substitution and simplifying algebraic expressions
- Properties of exponents
- Algebraic word problems
- Solutions of linear equations and inequalities
- Systems of equations and inequalities
- Quadratic equations
- Rational and radical equations
- Equations of lines

- Absolute value
- Direct and inverse variation
- Concepts of algebraic functions
- Newly defined symbols based on commonly used operations

Geometry and measurement questions ($25-30\%$ of the test)

- Area and perimeter of a polygon
- Area and circumference of a circle
- Volume of a box, cube, and cylinder
- Pythagorean theorem and special properties of isosceles, equilateral, and right triangles
- Properties of parallel and perpendicular lines
- Coordinate geometry
- Geometric visualization
- Slope
- Similarity
- Transformations

Data analysis, statistics, and probability questions ($10-15\%$ of the test)

- Data interpretation (tables and graphs)
- Descriptive statistics (mean, median, and mode)
- Probability

You will note that there is no pre-calculus or advanced trigonometry (sine, cosine, tangent, etc.), so if you haven't taken these classes, don't worry about it. However, you should be cognizant of when you took what classes and, consequently how much time that you will need to focus on each topic. For example, an 11th grader that took geometry in 9th grade may need to spend more time reviewing geometry than a 11th grader that is currently in a geometry class.

About the writing section

The writing section is composed of two sections, the 25 minute essay and multiple choice questions. Students will learn what the SAT graders are looking for and also practice with timing, brainstorming, and writing so that they can get a perfect score on the essay. Students will also be exposed to the three types of writing multiple choice questions– sentence improvements, sentence errors, and paragraph improvementsas well as the grammatical or other writing concepts taught in this section.

SAT Homework Agenda

Date Due	SAT Verbal	SAT Math

Part I

SAT Math

Chapter 2

General Strategies for SAT Math

Chapter 3

Numbers and Operations

1 Arithmetic Word Problems

General Equation: $\frac{a}{b} = \frac{c}{d}$

Example 1.?? If 3 gallons of water fill 2 fish bowls, how many gallons will fill 9 fish bowls?

Example 2. If y varies directly as x, and $y = 6$ when $x = 9$, what is the value of y when $x = 12$?

Example 3. Terri rides her bike every day on her way to school. On a sunny day, Terri can make the 2.6 mile trip in half of an hour at a constant pace. When it rains, Terri's speed drops by 1 mile per hour. If Terri's trip is the same distance in the rain, what total time it takes her to get to school?

MEDIUM

1. If the side length, s, of a square is increased by n, what is the ratio of the area of the new square to the old?

   Equation/Strategy: _____

   Solve:

   (a) $n^2 : 1$
   (b) $n^2 : s^2$
   (c) $s^2 : n^2$
   (d) $s^2 : (s+n)^2$
   (e) $(s+n)^2 : s^2$

---

2. From January to March, coats drop in price by 10% each month from the month before. Which ratio represents the cost of the jacket from January to March?

   Equation/Strategy: _____

   Solve:

   (a) 100 : 81
   (b) 10 : 81
   (c) 10 : 9
   (d) 9 : 1
   (e) 11 : 9

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3. If P varies jointly as T and inversely as V, and $P = 5$ when $V = 3$, what is the value of V when P doubles and T remains unchanged?

   Equation/Strategy: _____

   Solve:

   (a) 1.5
   (b) 2.5
   (c) 6
   (d) 10
   (e) 12

---

4. A jar contains D jellybeans. If there are A red jellybeans, B green jellybeans, and C orange jellybeans. What proportion of the jellybeans are red or orange?

   Equation/Strategy: _____

   Solve:

   (a) $\dfrac{A+C}{A+B+C}$
   (b) $\dfrac{A+C}{D}$
   (c) $\dfrac{D-(A+C)}{D}$
   (d) $\dfrac{D-B}{A+B+C}$
   (e) $\dfrac{D-B}{D}$

2 Rational Numbers

General Equation: $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Example 1. If $\frac{x}{3} = \frac{y}{5}$, write an inequality stating the relationship between x and y.

Example 2. If the number of microbes in a petri dish is reduced by a factor of one-third after each day, how many days will it take for the population to be less than 10% of the original amount?

Example 3. The batting average of a baseball player is given by the proportion of hits versus the number of times at bat. If Stanley has a batting average of 0.32 and had 24 hits in one season, what is the number of times Stanley was at bat?

MEDIUM

1. If the odds of choosing a red marble from a bag of marbles is 0.3. If the odds of choosing n red marbles is 0.027, what is the value of n?

   Equation/Strategy: _____

   Solve:

   (a) 2
   (b) 3
   (c) 6
   (d) 7
   (e) 9

---

2. One-third of Ms. Boyd's class takes French while four-fifths takes Spanish. How many students are taking both French and Spanish?

   Equation/Strategy: _____

   Solve:

   (a) 1/15
   (b) 2/15
   (c) 1/5
   (d) 1/3
   (e) 2/3

ADVANCED

3. Bob is two-thirds the age of his sister Sally. If Sally is four-thirds times the age of their sister, Patricia. If Patricia is 3 years older than Bob, what is Sally's age?

   Equation/Strategy: _____

   Solve:

   (a) 20
   (b) 24
   (c) 28
   (d) 32
   (e) 36

---

4. From 1970 to 2010, the population of US living on the coast has increased by approximately 40%. If approximately 40% of the total population of the USA lives on the coast in 2010 and the population in 2010 is 308 million people, what is the population (in millions) of the US in 1970?

   Equation/Strategy: _____

   Solve:

   (a) 49
   (b) 77
   (c) 88
   (d) 193
   (e) 220

3 Sequences and Series

General Equation:

Arithmetic Sequence

$$a_n = a_n + nd$$

Geometric Sequence

$$a_n = a_1 r^n$$

Arithmetic Series

$$S = \frac{n(a_1 + a_n)}{2}$$

Geometric Series

$$S = \frac{a_1(1 - r^n)}{1 - r}$$

Example 1. Shelly is preparing to run for a marathon. If Shelly starts running on the first week with 1 kilometer, and doubles the number of kilometers every week, how many miles will she run on the sixth week?

Example 2. Jenny receives a weekly allowance of $15 and is saving up to buy a new laptop that costs $450. If she has $100 saved up already, how many days will it take her to have enough for the laptop?

Example 3. A ball is dropped from a height of 1 meter and bounces a height of two-thirds of the previous bounce with every consecutive bounce. What is the total distance the ball has traveled after 5 bounces?

MEDIUM

1. Tom is doing a cross country bike ride. On the first day, Tom rode 5 miles. If he decides to ride an additional 20% every day, approximately how long will he have ridden on day 8?

Equation/Strategy: _____

(a) 13 miles

(b) 14 miles

(c) 17 miles

(d) 19 miles

(e) 20 miles

Month	2	5	7
Height	5	9.5	11.5

2. Eduardo purchases a potted plant to grow at home. The height is recorded (in inches) every month. If the rate of growth is constant, at what height did Eduardo buy the plant?

Equation/Strategy: _____

(a) 2 in

(b) 3 in

(c) 3.5 in

(d) 4 in

(e) 4.5 in

ADVANCED

3. A spherical balloon deflates at a rate such that the volume is cut in half every 2 minutes. If r is the initial radius, which expression represents the radius after 8 minutes?

Equation/Strategy: _____

Solve:

(a) $r/2$

(b) $r/4$

(c) $r/8$

(d) $r/16$

(e) $r/32$

---

4. A guitar string is plucked and the distance between the highest point and the lowest point of the first oscillation is 256 millimeters. If the distance the string travels is reduced by one-quarter with each oscillation, how many oscillations will it take for the string to have traveled a total distance of 700 millimeters?

Equation/Strategy: _____

Solve:

(a) 2.5

(b) 3

(c) 3.5

(d) 4

(e) 5

4 Elementary Number Theory

General Equation: $a|b = n$ if and only if $a = b \cdot n$

Example 1. Gumdrops come in bags of 60. Mr. Lee wants to buy enough bags so that he has a perfect square number. What is the least number of bags Mr. Lee will need to buy?

Example 2. The sum of the angles of a regular polygon is given by the equation $S(n) = 180(n-2)$ where n is the number of sides. How many sides does the smallest regular polygon have if is the smallest regular polygon possible such that each individual angle is over 120°?

Example 3. A group of n students participate in a math competition. The students are required to shake hands with all other participates in the competition. If each handshake between two people is counted only once, how many distinct handshakes are there overall?

MEDIUM

1. There are n jellybeans in a jar. The number of jellybeans can be separated into groups of 24 or groups of 42. What is the least possible number of jellybeans in the jar?

   Equation/Strategy: _____

   Solve:

   (a) 126
   (b) 168
   (c) 252
   (d) 504
   (e) 1008

---

2. A full revolution of the hour hand around the face of a clock corresponds to 12 hours. If the hour hand starts at 12:00 AM and completes 6.75 revolutions, what is the current time?

   Equation/Strategy: _____

   Solve:

   (a) 3:00 AM
   (b) 9:00 AM
   (c) 3:00 PM
   (d) 6:00 PM
   (e) 9:00 PM

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3. For regular polygons of n sides, if f is the number of faces, v is the number of vertices, and e is the number of edges, what is the value of $v - e + f$?

   Equation/Strategy: _____

   Solve:

   (a) 0
   (b) 1
   (c) 2
   (d) n-1
   (e) n

---

4. Ms. Rizzo has a box of play sand in her classroom. If the volume is a positive integer greater than 1 in^3, and is also a perfect square, what is the smallest possible length of the diagonal from one vertex to the other passing through the center of the cube?

   Equation/Strategy: _____

   Solve:

   (a) $\sqrt{2}$
   (b) $\sqrt{3}$
   (c) $2\sqrt{2}$
   (d) $2\sqrt{3}$
   (e) $4\sqrt{3}$

5 Sets

General Equation:

Union	Intersection
$p_{P \text{ or } Q} = p_P + p_Q - p_P \cdot p_Q$	$p_{P \text{ and } Q} = p_P \cdot p_Q$

Example 1. Liang is playing a game with a standard deck of cards. He wants the first card he picks up to be either a red card or a face card (Jack, Queen, or King). What is the probability he will choose either a red card or a face card?

Example 2. In the set of integers from 1 to 100 (inclusively), how many numbers are divisible by 3 or 5 but not both?

Example 3. Mr. Dropal has 16 students in his class. There are 10 male students. Half of all students are taking only Chinese and one-quarter are taking Chinese and French. If the class must take either French or Chinese, what proportion represents the maximum number of females taking only French?

MEDIUM

1. Mr. Carter has a garden of yellow, white, and red rose bushes. If the proportion of red rose bushes is 0.4. When he buys an additional 4 red rose bushes, the proportion increases to 0.5. How many rose bushes did Mr. Carter originally have?

   Equation/Strategy: _____

   Solve:

   (a) 8
   (b) 12
   (c) 20
   (d) 24
   (e) 36

---

2. The average (arithmetic mean) of a set of 5 positive integers is 10 and the median is 10. What is the largest possible value of the largest member of the set?

   Equation/Strategy: _____

   Solve:

   (a) 10
   (b) 50
   (c) 22
   (d) 37
   (e) 40

ADVANCED

3. Helen has a box of 40 chocolates. The proportion of choosing a cherry filled or coconut is 0.3, whereas the proportion of choosing a coconut or creme filled is 0.4. If the number of creme filled is twice the number of cherry filled chocolates and there is at least one cherry filled chocolate, what proportion of coconut chocolates are there?

   Equation/Strategy: _____

   Solve:

   (a) 0.075
   (b) 0.1
   (c) 0.15
   (d) 0.2
   (e) 0.25

---

4. Robbie has a equal chances of getting into College A and College B. If the proportion of getting into either college is 0.84, what proportion represents his chances of getting into one of the two schools?

   Equation/Strategy: _____

   Solve:

   (a) 0.4
   (b) 0.42
   (c) 0.5
   (d) 0.6
   (e) 0.84

6 Counting Techniques

General Equation:

Permutations	**Combinations**
Without Repetition	*Without Repetition*
$_nP_r = \frac{n!}{(n-r)!}$	$_nC_r = \frac{n!}{r!(n-r)!}$
With Repetition	*With Repetition*
n^r	$\frac{(n+r-1)!}{r!(n-1)!}$

Example 1. Mandy is throwing an ice cream party and has 3 flavors of ice cream and 4 different toppings. How many different combinations can her guests make?

Example 2. Jerry is making a sundae with 3 scoops of ice cream. If he has 5 different flavors of ice cream, how many different combinations can Jerry make?

Example 3. If a four-digit pin number contains the digits 0 to 9 where no digit can be repeated more than twice, how many different combinations for pin numbers are possible?

MEDIUM

1. If set A contains all even integers under twenty and set B contains all even prime numbers, then the set of common elements between set A and set B is

Equation/Strategy: _____

Solve:

(a) {}
(b) {0}
(c) {2}
(d) {0, 2}
(e) All even numbers

---

2. If a four point star has 8 vertices, and an eight point star has 16 vertices, how many vertices does a 10 point star have?

Equation/Strategy: _____

Solve:

(a) 12
(b) 16
(c) 20
(d) 24
(e) 32

ADVANCED

3. A number of volleyballs compete in a tournament. If each team must play one another, and there are a total of 120 matches, how many teams competed?

Equation/Strategy: _____

Solve:

(a) 5 teams
(b) 6 teams
(c) 7 teams
(d) 8 teams
(e) 12 teams

---

4. An equilateral triangle is divided so that the midpoint of each line is the vertex of an inscribed triangle. If the process continues, how many triangles will there be after n divisions?

Equation/Strategy: _____

Solve:

(a) 2^n
(b) 3^n
(c) 4^n
(d) $4n$
(e) $8n$

7 SAT Worksheet: Basic Numbers and Operations Problems

8 SAT Worksheet: Medium Numbers and Operations Problems

9 SAT Worksheet: Advanced Numbers and Operations Problems

Chapter 4

Algebra and Functions: Part I

1 Substitution and Simplifying Algebraic Expressions

General Equation:

Distributive Property

$$a(m+n) = am + an$$

Example 1: If Sally has 6 boxes and each box has 12 books, how many books does Sally have?

Example 2: Chase and Jill both collect stamps. If Jill has six less than twice the number of stamps than Chase does, how many stamps does Jill have if Chase has 24 stamps?

Example 3: Townsville High School has 30 classrooms, each with 15 desks. If 80% of desks are occupied in 90% and the other 10% of the classrooms are 100% full, how many students attend the school?

MEDIUM

1. Chris has a collection of 120 records which consists of jazz, blues, and classical records. If one-fourth of the records are jazz and one-third of the records are blues, how many of the records are classical?

Equation/Strategy: _____

Solve:

(a) 30
(b) 40
(c) 50
(d) 60
(e) 70

---

2. The density, d, of an object is the ratio of its mass to its volume. If the volume of an object is halved and the mass is doubled, which expression represents the new density in terms of d?

Equation/Strategy: _____

Solve:

(a) $1/4d$
(b) $1/2d$
(c) d
(d) $2d$
(e) $4d$

ADVANCED

3. Tommy is on the outer edge of a merry-go-round that moves at a constant speed. If r is Tommy's distance from the center, and the merry-go-round makes m full cycles during n minutes, which expression represents Tommy's distance traveled in 1 hour?

Equation/Strategy: _____

Solve:

(a) $\dfrac{mr\pi}{30n}$
(b) $\dfrac{mr^2\pi}{30n}$
(c) $\dfrac{120r\pi}{mn}$
(d) $\dfrac{120mr^2\pi}{n}$
(e) $\dfrac{120mr\pi}{n}$

---

4. Bunny Slopes Co. is having a sale on winter wear of 20% off. Tracy buys a pair of ski goggles and receives an additional 10% off of the sale price. If the final cost of the goggles is \$90, how much did Tracy save?

Equation/Strategy:

Solve:

(A) 22
(B) 25
(C) 35
(D) 38
(E) 125

2 Solutions of Linear Equations and Inequalities

General Equation:

Point-Slope Form of a Line

$$y = mx + b$$

Example 1: If the cost of a cab has a base fare of $2.50 and $0.40 per mile, how much does a 10 mile ride cost?

Example 2: The cost of tablet devices has dropped an average of 3% of the original price every quarter. After how long will the cost of a tablet be less than half of the original cost?

Example 3: The population of Summerville increases at a constant annual rate. The population was recorded in January 2008 as 362,000 and again in July 2010 as 384,000. What is the annual rate of growth of the population?

MEDIUM

1. The speed of a minute hand moves at a constant speed of 1/60 rpm (revolutions per minute). If the current time is 4:00 pm and the minute hand has made 1.75 revolutions, what time was it first recorded at?

Equation/Strategy: _____

Solve:

(a) 2:15 pm
(b) 2:25 pm
(c) 2:45 pm
(d) 3:15 pm
(e) 3:25 pm

---

2. Both Terri and Sam are shorter than Sissy, and Sissy and Hector is shorter than Roy. Which of the following must be true?

I. Terri is shorter than Sam
II. Sissy is shorter than Hector
III. Sam is shorter than Roy

Equation/Strategy: _____

Solve:

(a) I only
(b) II only
(c) III only
(d) I and II
(e) II and III

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3. A chemical reaction results in the release the constant release of 70 joules of energy over a 14 minute period. If the total initial amount of energy in the system was 370 joules, how long will it take for the system to release all of its energy?

Equation/Strategy: _____

Solve:

(a) 1 hour 10 minutes
(b) 1 hour 14 minutes
(c) 1 hour 17 minutes
(d) 1 hour 24 minutes
(e) 5 hours 17 minutes

---

4. The number of students at Mainsville School with cellphones increases at a constant rate of n students per year. If the number of students with cellphones in January 2010 is 200 and the population of the student body is p, which expression represents the proportion of students with cellphones after m months?

Equation/Strategy: _____

Solve:

(A) $\frac{mn}{p+200}$ (B) $\frac{mn}{p-200}$

(C) $\frac{mn+200}{p}$ (D) $\frac{mn+2400}{p}$

(E) $\frac{mn+2400}{12p}$

3 Properties of Exponents

General Equation:

The Laws of Exponents

$$a^m \cdot a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$(ab)^m = a^m \cdot b^m \qquad (a^m)^n = a^{mn}$$

Example 1. There are 1000 milimeters in one meter and 1000 meters in one kilometer. How many times larger is a kilometer than a millimeter?

Example 2. A pallet contains 5 rows of 5 columns of boxes, each column 5 boxes high. How many boxes does the pallet contain?

Example 3. The intensity of a sound is inversely proportional to the square of your distance from the source of the sound. If you are 6 times further away from a set of speakers at a concert as your friend, what is the ratio of the intensity of you to your friend?

MEDIUM

1. A wooden block occupies a volume of $2a$. If the side length is an integer, what is the smallest possible value of a?

Equation/Strategy: _____

Solve:

(a) 2
(b) 3
(c) 4
(d) 8
(e) 16

---

2. The sum of the squares of two numbers, a and b, is equal to to the square of the sum. Which of the following must be true?

I. $a \cdot b = 0$
II. $a = b$
III. $(a+b)^3 = a^3 + b^3$

Equation/Strategy: _____

Solve:

(a) I is true
(b) II is true
(c) III is true
(d) I and II is true
(e) I, II, and III are true

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3. The distance of an object falling from height is proportional to the square of time of the fall. If Betty drops a rock from the top of a building, the rock travels a distance of d^3 meters after time t seconds. If both the distance and the time are whole numbers and $d \neq t$, what is the least distance traveled by the rock?

Equation/Strategy: _____

Solve:

(a) 2 m
(b) 4 m
(c) 8 m
(d) 32 m
(e) 64 m

---

4. A coat goes on sale x percent off of the original sale price every month. Which of the following expressions represents the amount taken off after m months?

Equation/Strategy: _____

Solve:

(a) $n\left(\frac{x}{100}\right)^m$
(b) $n\left[1-\left(\frac{x}{100}\right)^m\right]$
(c) $1-\left(\frac{x}{100}\right)^m$
(d) $n\left[1-\left(\frac{x^m}{100}\right)\right]$
(e) $n(1-x^m)$

4 Systems of Equations and Inequalities

General Equation:

Parallel Lines	Perpendicular Lines
$m_1 = m_2$	$m_1 \cdot m_2 = -1$

Example 1. Plane A and plane B are flying parallel to one another. If after 20 minutes, plane A has risen an altitude of 2000 m, how much has the altitude of plane B has risen in 30 minutes?

Example 2. When Johnny has x nickels and y dimes, his total is \$3. When he has y nickels and x dimes, his total increases by \$0.75. How many nickel and dimes did Johnny start off with?

Example 3. Two cars starting at the same intersection begin traveling perpendicular to one another. If the first car travels north west at a 25° angle of the point of intersection, what is the direction of the second car?

MEDIUM

1. A coffee costs $2.5 and a muffin costs $3. If Tasha has $11 and makes a purchase, what is the least amount of change she can receive?

   Equation/Strategy: _____

   Solve:

   (a) $0.00
   (b) $0.50
   (c) $1.00
   (d) $1.50
   (e) $2.00

---

2. Stacy and Ann enter a relay race that consists of 3 events, each worth 10 points. If Stacey earned 8 points and 9 points in the first two events, what is the least number of points she will need to earn in the third event to win if Ann received received a 23 points total?

   Equation/Strategy: _____

   Solve:

   (a) 6
   (b) 7
   (c) 10
   (d) 23
   (e) 24

ADVANCED

3. Two planes leave from parallel terminals. If plane A travels northwest at 400 mph, and plane B travels northwest 600 mph 4 hours later, by what time will the second plane have equal distance traveled as the first plane?

   Equation/Strategy: _____

   Solve:

   (a) 2
   (b) 8
   (c) 4
   (d) 12
   (e) 48

---

4. A coyote is chasing roadrunner in a parallel path. If the roadrunner and coyote are running at a constant rate of 30 mph and the roadrunner has a 20 mile gain on the coyote, how much faster will the coyote need to run if he is going to catch up to the roadrunner in 2 hours?

   Equation/Strategy: _____

   Solve:

   (a) 10 mph
   (b) 20 mph
   (c) 25 mph
   (d) 30 mph
   (e) 40 mph

5 Equations of Lines

General Equation:

Slope-Intercept Form of a Line

$$y = mx + b$$

Example 1. Sunshine taxi charges a base fare of \$2.60 and \$0.40 for every quarter mile. If Elle's ride is 5 miles, how much is her ride?

Example 2. A parking lot charges \$10 for the first 4 hours and \$2 up to every additional hour. If George leaves his car for 8 and a half hours, how much is he charged?

Example 3. Moe's dad will give him \$1 for every x points over 50 on his math test, where x is a whole number of points. Moe received 86 points on his math test and earned \$12. How many points does Moe need to earn \$1?

MEDIUM

1. Travis is a car salesman and earns $10 an hour plus a flat commission fee for each car he sells. If Travis works 30 hours and has earned $1,000 in a week, how much does Travis earn in commission per car if he sells 4 cars?

   Equation/Strategy: _____

   Solve:

   (a) 80
   (b) 160
   (c) 320
   (d) 360
   (e) 640

---

2. Cherry is setting up a can drive at her school. For every 50 cans after 100 she collects, the donation center will give her one ticket to an amusement park. If Cherry wants a total of 10 tickets, what is the least number of cans she will need to collect?

   Equation/Strategy: _____

   Solve:

   (a) 500
   (b) 600
   (c) 1050
   (d) 1500
   (e) 5100

ADVANCED

3. The value of a car depreciates every year at a constant rate of $p\%$ of the total value. If the initial value of the car is d dollars, what is the current value of the car after m months?

   Equation/Strategy: _____

   Solve:

   (a) $d - pm$
   (b) $d - \frac{pm}{12}$
   (c) $d - \frac{pm}{100}$
   (d) $d - \frac{pm}{1200}$
   (e) $d\left(1 - \frac{pm}{1200}\right)$

---

4. The value of an interior angle of a regular n-gon increases as a linear function of n. If an interior angle of a 4-gon is $90°$ and a 6-gon is $120°$, what is the sum of all interior angles of an n-gon whose interior angles are each $144°$?

   Equation/Strategy: _____

   Solve:

   (a) 10
   (b) 12
   (c) 144
   (d) 576
   (e) 1440

Chapter 5

Algebra and Functions: Part II

1 Absolute Value

General Equation:

$$|a - b| = |b - a|$$

Example 1. Dallas walks to her friends house 6 blocks north of her house. She then walks 3 blocks south to visit Francis. What is the total distance traveled by Dallas?

Example 2. A train travels east x miles, then travels west y miles. What expression gives the net distance traveled by the train?

Example 3. A ball thrown from the ground travels a distance of n meters and bounces with a height of half of the previous height. If the ball bounces 5 times, what is the total distance traveled by the ball in terms of n?

MEDIUM

1. A toy train travels on a circular path with a diameter of 10 feet. If Ellen runs the train forward 2.5 revolutions, then in reverse for 1.5 revolutions, what is the total distance traveled by the train?

Equation/Strategy: _____

Solve:

(a) 5π meters
(b) 10π meters
(c) 40π meters
(d) 20π meters
(e) 100π meters

---

2. If two objects are falling at the same speed, s meters per second, what is the total distance over t seconds traveled by both objects if the distance between them is d units?

Equation/Strategy: _____

Solve:

(a) $|st|$
(b) $|st - d|$
(c) $|2sd|$
(d) $|2st|$
(e) $|2st - d|$

ADVANCED

3. Lucky and Sunshine are two horses that are running in opposite directions of each other. If Lucky's velocity is p mph and is twice the velocity of Sunshine, what is the distance between the horses after t minutes?

Equation/Strategy: _____

Solve:

(a) $\left|\frac{pt}{60}\right| + \left|\frac{pt}{120}\right|$
(b) $\left|\frac{pt}{60} + \frac{pt}{120}\right|$
(c) $\left|\frac{pt}{30}\right| + \left|\frac{pt}{60}\right|$
(d) $\left|\frac{pt}{30} + \frac{pt}{60}\right|$
(e) $\left|\frac{pt}{30}\right| + \left|\frac{pt}{120}\right|$

---

4. A ping pong ball travels a constant velocity of h inches per s seconds with every successive hit. What is the ping pong speed if it travels g feet in m minutes?

Equation/Strategy: _____

Solve:

(A) $\left|\frac{5gh}{ms}\right|$
(B) $\left|\frac{gh}{5ms}\right|$
(C) $\left|\frac{5hm}{gs}\right|$
(D) $\left|\frac{hm}{5gs}\right|$
(E) $\left|\frac{gs}{5hm}\right|$

2 Direct and Inverse Variation

General Equation:

Direct Variation	Joint Variation
$y = kx$	$xy = k$

Example 1. If there are approximately 30 centimeters in one foot, how many centimeters are there in 18 inches? (1 foot = 12 inches)

Example 2. When a delivery truck carries heavier packages, the speed of the truck decreases. If the truck is able to go 60 mpg when it carries 400 lbs, how much time will it take the truck to carry 600 lbs and travel 100 miles?

Example 3. The speed of a vehicle increases as the amount of fuel used increases, and the amount of time spent traveling over a fixed distance decreases. If a vehicle goes 30 miles an hour for 60 minutes using one gallon of gasoline, how much gasoline will be required to speed the vehicle by 30 miles per hour for 45 minutes?

MEDIUM

1. The area of a square is proportional to the length of the diagonal. If d is the area, which of the following describes the area in terms of d?

   Equation/Strategy: _____

   Solve:

   (a) d^2
   (b) $(d\sqrt{2})^2$
   (c) $(d/\sqrt{2})^2$
   (d) $d^2\sqrt{2}$
   (e) $\sqrt{2}d^2$

---

2. The measure of acidity, pH, of a compound is proportional inversely proportional to the compound's hydroxide concentration, $[OH^-]$. If the pH is 4 when $[OH^-]$ is 10^{-10}, what is the pH decreased by when the $[OH^-]$ is decreased by a factor of 10?

   Equation/Strategy: _____

   Solve:

   (a) 1
   (b) 3
   (c) 4
   (d) 30
   (e) 40

ADVANCED

3. The odds of guessing a number correctly are proportional to the number of terms to guess from. If the odds of choosing p numbers out of q terms is 0.4, what are the lease odds of choosing p out of $q + 1$?

   Equation/Strategy: _____

   Solve:

   (a) 0.3
   (b) 0.33
   (c) 0.36
   (d) 0.375
   (e) 0.38

---

4. The volume, V, of a right cylinder is directly proportional to its radius and height. If the radius is doubled and the height is halved, what is the new volume in terms of the old volume, V? ($V = \pi r^2 h$)

   Equation/Strategy: _____

   Solve:

   (a) $V/4$
   (b) $V/2$
   (c) V
   (d) $2V$
   (e) $4V$

3 Quadratic Equations

General Equation: If $f(x)$ is a quadratic functions with roots r and s, then the coordinate of the vertex (maximum or minimum) is

$$\left(\frac{s-r}{2},\ f\left(\frac{s-r}{2}\right)\right)$$

Example 1. Reese jumps, starting from the ground, and reaches a maximum height of 6 feet at 3 seconds. How long does the trip take from when she first jumped until she returned back to the ground?

Example 2. A cannonball is fired from ground-level and hits the ground after t seconds. If the maximum height is h ft, write the coordinate that expresses the maximum of the cannonball's trajectory.

Example 3. The sum of two integers x and y is m and the product of the two integers is n. What is n in terms of x?

MEDIUM

1. Stacey is making a rectangular garden for her rose bushes. If the perimeter needs to be 100 cm, what is the maximum area she can enclose?

   Equation/Strategy: _____

   Solve:

   (a) 25 cm^2
   (b) 50 cm^2
   (c) 100 cm^2
   (d) 500 cm^2
   (e) 625 cm^2

---

2. For two integers p and q, the sum of their squares is equal to the square of their sum. What is the value of pq?

   Equation/Strategy: _____

   Solve:

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

ADVANCED

3. A sector of a circle is inscribed in a square. If the radius of the circle is equal to the side length of the square, s, what is the area of the inscribed sector in terms of s?

   Equation/Strategy: _____

   Solve:

   (a) $\pi s^2/4$
   (b) $\pi s^2/2$
   (c) $s^2/4$
   (d) $s^2/2 - \pi$
   (e) $s^2 - \pi/4$

---

4. Andres has 1.2 kilometers of fencing he wishes to use to create two adjacent pens for his sheep and his goats. If he uses the fencing for the perimeter and a divider in the middle of the entire pen, what is the length, in meters of the shortest side? (1 kilometer = 1000 meters)

   Equation/Strategy: _____

   Solve:

   (a) 150 m
   (b) 300 m
   (c) 400 m
   (d) 600 m
   (e) 750 m

4 Rational and Radical Equations

General Equation:

If $\frac{a}{b} = \frac{c}{d}$, then

- $ad = bc$
- $\frac{a}{c} = \frac{b}{d}$

Example 1. If the sequence x, _____, y has a common ratio between each term, what is the value of the missing term?

Example 2. A three digit number is evenly divided by a two digit number such that the quotient is a perfect square. What is the smallest such three digit number and two digit number pair?

Example 3. For an integer n, the square root and cube root are both integers. If the square root and cube root of n are distinct, what is the smallest sum of both roots of such a number?

MEDIUM

1. The odds of choosing a red marble is 1 out of p marbles and the odds of choosing a green marble is 1 out of q marbles. Which expression represents the odds of choosing a red or a green marble?

Equation/Strategy: _____

Solve:

(a) $(p+q)/pq$
(b) $1/pq$
(c) $2/pq$
(d) $p+q$
(e) $(1-p)(1-q)$

---

2. Benjamin is missing cards in his deck of cards. If in his deck of 50 cards there are x kings and y queens. What are the odds of choosing a queen and a king?

Equation/Strategy: _____

Solve:

(a) $xy/50$
(b) $(x+y)/50$
(c) $2500/xy$
(d) $(x+y)/2500$
(e) $xy/2500$

ADVANCED

3. The ratio of the sides of a rectangle is $a : b$. If one is added to both sides, the new ratio of sides is $b : a$. Which of the following must be true?

I. The rectangle is a square
II. The area is a^2
III. The side length is 1

Equation/Strategy: _____

Solve:

(a) I is true
(b) II is true
(c) III is true
(d) I and II are true
(e) I, II, and III are true

---

4. A square is inscribed in a circle. If the radius of the circle double, by what factor does the side length of the square grow?

Equation/Strategy: _____

Solve:

(a) 2
(b) $\sqrt{2}$
(c) 4
(d) $2\sqrt{2}$
(e) $4\sqrt{2}$

5 Concepts of Algebraic Functions

General Equation: If $f(a) = b$, then (a, b) is a coordinate of the graph of f.

Example 1. If Timmy sells less lemonade in week x than in week y and Timmy's sales has increased every week, what relationship describes y to x?

Example 2. Luckystar is a horse that races in the Belmont racetrack. If Luckystar runs at x mph at time p and again at time q, what is his increase in average speed over the interval from p to q?

Example 3. If Arnold has $3\times3, 4\times4$, and 5×5 cubes, what is the least number of cubes Arnold will need to make a building that is 27 units high?

Example 3. Lindsey and Ricky have separate college tuition funds created on a Monday. Lindsey's account starts at an initial amount of \$0.50 and the total doubles every day, whereas Ricky's college tuition starts at \$1 initially and the total doubles every day. On what day will Lindsey begin having greater amounts in her account than Ricky will have in his account?

MEDIUM

1. If the amount of bacteria in a colony doubles every hour, which of the following cannot be a relative factor of the population of bacteria?

   Equation/Strategy: _____

   Solve:

   (a) 1/2
   (b) 0
   (c) 1
   (d) 2
   (e) 8

---

2. f has the property that $f(\square) = \clubsuit$ and $f(\clubsuit) = \square$ for all $\clubsuit$ and $\square$. Which of the following represents the equation for f in terms of x?

   Equation/Strategy: _____

   Solve:

   (a) $f(x) = x$
   (b) $f(x) = x^2$
   (c) $f(x) = 0$
   (d) $f(x) = 1$
   (e) $f(x) = \sqrt{x}$

ADVANCED

3. The odds of choosing a prime number out of x terms is m and the odds of choosing and even number out of the same x terms is n. What are the odds of choosing an even prime out of x^2 terms?

   Equation/Strategy: _____

   Solve:

   (a) $1/mn$
   (b) $1/x$
   (c) $1/x^2$
   (d) $1/(m+n)$
   (e) Cannot be determined

---

4. If Kat is on a swing that starts at a height of h off the ground and reaches the ground after $t/3$ seconds, at what position will she be at $15t$ relative to her beginning position?

   Equation/Strategy: _____

   Solve:

   (a) h
   (b) $-h$
   (c) 0
   (d) $2h$
   (e) $-2h$

6 Newly Defined Symbols Based on Commonly Used Operations

General Equation: Use the notation as you would with variables

Example 1. If $f \otimes g$ is defined as $f \cdot g - (f + g)$, what must be true of f and g so that $f \otimes g = 0$?

Example 2. If $f\%g$ is defined as the remainder of f when divided by g, what is $(x - 4x + 4)\%(x - 2)$?

Example 3. S is a set with elements $s_1, s_2, \ldots s_n$. Let $S \bullet S$ be defined as $s_1 \cdot s_1 + s_2 \cdot s_2 + \ldots s_n \cdot s_n$. If $S \bullet S = 0$ what must be true of the elements of S?

MEDIUM

1. If $x^2 - 64 = 36$ and $x - 8 = 9$, what is the value of $x + 8$?

   Equation/Strategy: _____

   Solve:

   (a) 4
   (b) 18
   (c) 25
   (d) 64
   (e) 100

---

2. $i\#a$ is defined as i^a. If $i\#2 = -1$ and $i\#3 = -i$, what is the value of $i\#115$?

   Equation/Strategy: _____

   Solve:

   (a) 1
   (b) -1
   (c) i
   (d) $-i$
   (e) 0

ADVANCED

3. $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x whereas $\lceil x \rceil$ is defined as the least integer greater than or equal to x. What is the value of $\lfloor \lceil x \rceil \rfloor$

   Equation/Strategy: _____

   Solve:

   (a) 1
   (b) 0
   (c) x
   (d) x^2
   (e) Cannot be determined

---

4. The ternary operation $a@b@c$ is defined as $a = b$ when $a \geq 0$, and $a = -a$ when $a < 0$. What is the value of $a^2@ - \left|a^2\right| @\sqrt{(-a^2)^2}$?

   Equation/Strategy: _____

   Solve:

   (a) $-a^2$
   (b) a^2
   (c) $\left|-a^2\right|$
   (d) $\sqrt{a^4}$
   (e) $\sqrt{-a^4}$

Chapter 6

Geometry and Measurement: Part I

1 Area and Perimeter of a Polygon

General Equation:

Area	Perimeter
The measure of the surface of a two-dimensional shape	The measure of the distance around a two-dimensional shape

Example 1: The lengths of an n-sided figure are each doubled. If the perimeter of the original n-sided figure was P, what is the value of the new perimeter?

Example 2: A circle inscribed in a box has a radius equal to half of the diameter of the box. What area of the circle is not in the square if the radius is 1?

Example 3: A pizza box has a perimeter that is 1.5 times the circumference of the pizza. If the pizza has a diameter of 16 inches, what is the difference between the perimeter of the box and the circumference of the pizza?

MEDIUM

1. A square is inscribed in a circle. If the area of the square is 9 sq. in., what is the difference between the circumference of the circle and the perimeter of the square?

   Equation/Strategy: _____

   Solve:

   (a) $3\sqrt{2}\pi - 12$
   (b) $12 - 3\sqrt{2}\pi$
   (c) $6\sqrt{2}\pi - 12$
   (d) $12 - 6\sqrt{2}\pi$
   (e) 3

---

2. A right triangle has an area of b sq. units. If a rectangle has a height equivalent to the height of the right triangle, and a base equivalent to twice the base of the right triangle, what is the area of the rectangle in terms of a?

   Equation/Strategy: _____

   Solve:

   (a) a
   (b) $2a$
   (c) $4a$
   (d) a^2
   (e) $2a^2$

ADVANCED

3. A rectangle whose width is twice its height is inscribed in a semicircle whose radius is equal to width of the rectangle. If h is the height of the rectangle, which expression represents the area of the semicircle not in the area of the rectangle?

   Equation/Strategy: _____

   Solve:

   (a) $2w^2(\pi - 1)$
   (b) $2w^2(9w^2\pi - 1)$
   (c) $2w^2(2\pi - 1)$
   (d) $2w^2(2\pi - 1)$
   (e) $w^2(\pi - 2)$

---

4. Triangle A is inscribed in equilateral triangle B so that the vertices of A are the midpoints of the sides of B. If the area of B is 12 sq. units, what is the area of the B with A removed?

   Equation/Strategy:

   Solve:

   (a) 6 sq. units
   (b) 7 sq. units
   (c) 8 sq. units
   (d) 9 sq. units
   (e) 12 sq. units

2 Area and Circumference of a Circle

General Equation:

Area of a Circle	**Circumference of a Circle**
$A = \pi r^2$	$C = 2\pi r$

Example 1: Sally baked a chocolate cake that has a diameter of 10 inches. How much frosting is required to cover the top of the cake?

Example 2: A car's wheel travels at 10 revolutions before coming to a stop. If the diameter of the tire is 2 ft, what is the total distance the wheel has traveled?

Example 3: If a minute hand on a clock is 7cm longs and moves from 0 minutes to 25 minutes, what is the length of the distance traveled by the minute hand?

MEDIUM

1. The circumference of a circle is 6 cm. What is the area of one-sixth of the circle?

Equation/Strategy: _____

Solve:

(a) $\frac{3}{2\pi}$
(b) $\frac{3\pi}{2}$
(c) 9π
(d) $\frac{3}{2}$
(e) $\frac{2}{3}$

---

2. The area of the shadow of a basketball is directly proportional to the distance of the ball to the ground. If the area of the shadow of a basketball is 27 sq. in when the ball is 4 feet from the ground, what is the length of the radius when the ball is 2 feet from the ground?

Equation/Strategy: _____

Solve:

(a) $\frac{3\sqrt{6\pi}}{2\pi}$ sq. in.
(b) $\sqrt{\frac{27\pi}{2}}$ sq. in.
(c) $\sqrt{\frac{27}{\pi}}$ sq. in.
(d) 13.5 sq. in.
(e) 54 sq. in.

ADVANCED

3. A outdoor circular pool is drained and covered with a tarp at the end of the season. If the tarp is one foot wider on all sides than the pool, and the area of the pool is 16 square feet, how much longer is the circumference of the tarp than the circumference of the pool?

Equation/Strategy: _____

Solve:

(a) 2π
(b) $8\sqrt{\pi} + 2\pi$
(c) $8\sqrt{\pi} - 4\sqrt{\pi}$
(d) $8\sqrt{\pi} - 2\pi$
(e) $\frac{4\sqrt{\pi}}{\pi} + 1$

---

4. A circular track consists of two concentric circles. Runner A runs a lap on the inner most track in 15 minutes, while runner B runs on the outer most track in the same time. How much faster was the rate of runner B if the outer track is 2 meters further away from the center of the track?

Equation/Strategy:

Solve:

(a) $\frac{2\pi}{15}$
(b) $\frac{4\pi}{15}$
(c) $\frac{2}{15}$
(d) $\frac{4}{15}$
(e) $\frac{\pi}{15}$

3 Volume of a Box, Cube, and Cylinder

General Equation:

Volume of a Box	**Volume of a Cube**	**Volume of a Cylinder**
$V = l \times w \times h$	$V = s^3$	$V = \pi r^2 h$

Example 1: Kallie is filling her bookshelf with books that are $8 \times 5 \times 2$ in^3. If Kallie has 16 such books, how much volume do the books occupy?

Example 2: A cube is made by stacking smaller 4×4 smaller cubes. What is the total volume of the cube?

Example 3: Pete stacks pizza pies on each other in a freezer for later heating. The pizzas are 2 inches thick and have a diameter of 16 inches. What volume do 10 pizzas occupy?

MEDIUM

1. A rectangular box has a volume of 36 in^3 If cubes of side length 2 are placed into the rectangular box, what is the least amount of volume of the box not filled by the cubes?

Equation/Strategy: _____

Solve:

(a) 0 in^3
(b) 2 in^3
(c) 4 in^3
(d) 6 in^3
(e) 8 in^3

---

2. A cylinder is placed in a rectangular box so that the diameter of the cylinder is the width and height of the rectangle. What is the ratio of volume of the cylinder to the volume of the box?

Equation/Strategy: _____

Solve:

(a) $4 - \pi$
(b) $\frac{4}{\pi}$
(c) $\frac{\pi}{4}$
(d) $\frac{8}{\pi}$
(e) $\frac{\pi}{8}$

ADVANCED

3. A cube of volume V of integer side length s is split into 6 smaller rectangular boxes of equal volume. What is the value of V if it is the smallest such cube?

Equation/Strategy: _____

Solve:

(a) 6
(b) 36
(c) 64
(d) 125
(e) 216

---

4. Cubic blocks are stored in a cylindrical container so that the diagonal length of the cube is the diameter length of the cylinder. If the height of the cylinder is 5 times the radius, what is the most number of cubes that fit into the cylinder?

Equation/Strategy:

Solve:

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

4 Pythagorean Theorem and Special Properties of Isosceles, Equilateral, and Right Triangles

General Equation:

The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Example 1: The perimeter of an equilateral triangle is 16 cm. What is the triangle's area?

Example 2: Gretchen and Samuel leave school. Gretchen walks 45° due south west and Samuel walks 45° due south east. If they each live a quarter mile away from school, what is the distance between their houses?

Example 3: A 13 ft ladder is propped up against a wall. If the base of the ladder to the base of the wall is equidistant of the top of the ladder to the base of the wall, what is the difference between the length of the ladder and the base of the ladder to the base of the wall?

MEDIUM

1. The area of an equilateral triangle is 16 cm^2. What is the triangle's perimeter to the nearest hundredth?

   Equation/Strategy: _____

   Solve:

   (a) 3.04
   (b) 6.07
   (c) 6.08
   (d) 18.23
   (e) 18.24

---

2. The ratio of the sides of a right triangle are in a proportion of $2 : 3 : x$ where x is the proportion of the longest side. If the hypotenuse is 27 units long, what the area of the triangle to the nearest hundredth?

   Equation/Strategy: _____

   Solve:

   (a) 168.23
   (b) 168.20
   (c) 243
   (d) 292.04
   (e) 292.05

ADVANCED

3. The hypotenuse of a right triangle is twice as long as the shortest side. If the length of the longer leg is 10 cm, what is the perimeter of the triangle?

   Equation/Strategy: _____

   Solve:

   (a) $10 + 10\sqrt{3}$
   (b) $20 + \sqrt{3}$
   (c) $20 + 10\sqrt{3}$
   (d) $30 + 10\sqrt{3}$
   (e) 30

---

4. The side lengths of a rectangle are in a proportion of $a : a + 4$. If the perimeter is 76 sq. units, what is the length of the hypotenuse?

   Equation/Strategy:

   Solve:

   (a) 12 units
   (b) 16 units
   (c) 24 units
   (d) 20 units
   (e) 40 units

Chapter 7

Geometry and Measurement: Part II

1 Properties of Parallel and Perpendicular lines

General Equation: For lines L_1 and L_2 with slopes m_1 and m_2 respectively

Parallel Lines	Perpendicular Lines
$m_1 = m_2$	$m_1 \cdot m_2 = -1$

Example 1: Two planes are traveling parallel to one another. If the first plane travels in a direction 30 miles north and 40 miles east of where it began, and the second plane travels 60 miles north of where it began, how far east must it travel in order to remain parallel to the first plane?

Example 2: Two cars starting at the same location are traveling perpendicular to one another. What is the distance between the two cars if the first car has traveled 30 miles in one hour and the second car has traveled 40 miles in one hour?

Example 3: A frog's tongue shoots straight up into the air to catch a fly that is flying horizontally. If the distance from the fly's speed is 6 meters per second and the speed between the frog and the fly is 20 meters per 2 seconds, how fast must the frog's tongue move in order to catch the fly?

MEDIUM

1. Roads A and B are parallel roads. If Road C runs perpendicular to road A, which of the following must be true?

   I. Road C is perpendicular to Road B

   II. Road A and C form a 90° angle

   III. The distance between Road A and Road B is uniform

   Equation/Strategy: _____

   Solve:

   (a) I only
   (b) II only
   (c) III only
   (d) I and II are true
   (e) I, II, and III are true

---

2. Two dogs are running perpendicular paths, starting at the same location. If the first dog runs m meters in 5 minutes, and the second dog runs m meters in 10 minutes, which of the following represents the distance between the dogs after 10 minutes?

   Equation/Strategy: _____

   Solve:

   (a) $m\sqrt{2}$
   (b) $m\sqrt{3}$
   (c) $\sqrt{2m^2 + m^2}$
   (d) $\sqrt{\frac{m^2}{2} + m^2}$
   (e) $\sqrt{4m^2 + m^2}$

ADVANCED

3. A set of parallel roads runs perpendicular to a second set of parallel roads. If the distance between each intersection along the road is at most m km, which of the following cannot be the distance between diagonal intersections?

   Equation/Strategy: _____

   Solve:

   (a) m
   (b) $2m$
   (c) $\frac{m}{2}$
   (d) $\frac{m\sqrt{2}}{2}$
   (e) $m\sqrt{2}$

---

4. A pair of parallel chords are inscribed in a circle, equidistant from the center. If the distance between the chords is equal to the radius, r, what is the length each chord in terms of r?

   Equation/Strategy:

   Solve:

   (a) $\frac{r\sqrt{3}}{2}$
   (b) $r\sqrt{3}$
   (c) $\frac{3r^2}{4}$
   (d) r
   (e) $2r$

2 Slope

General Equation: Slope indicates the steepness of a line. The slope is denoted as a ratio of the rise over the run of a line, or the change in the vertical distance over the change in the horizontal distance between any two points on a line. For two points (x_1, y_1) and (x_2, y_2), the slope, m, is

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Points $(2, 3)$ and $(5, k)$ lie on the line $3x - my = 6$. What is the value of k in terms of m?

Example 2: What is the slope of the line formed when connecting the minute hand and the hour hand on a clock at 9 am if the distance between them is twice the length of the hour hand?

Example 3: A toy rocket is launched from the ground at a 60° angle into the air. Assuming the rocket travels a straight path, what is the slope of its path from the ground to its maximum?

MEDIUM

1. A ski lift travels 1500 feet at a 60° angle of elevation to the top of a mountain. What is the slope of the path that the ski lift travels?

   Equation/Strategy: _____

   Solve:

   (a) $\sqrt{3}/3$
   (b) $1/2$
   (c) $\sqrt{3}/2$
   (d) $2\sqrt{3}/3$
   (e) $\sqrt{3}$

---

2. If the line $3x + ky = 8$ passes through point $(-4, 5)$, what is the value of k?

   Equation/Strategy: _____

   Solve:

   (a) $-4/5$
   (b) $7/4$
   (c) 4
   (d) 5
   (e) $23/4$

ADVANCED

3. An equilateral triangle is drawn in the first quadrant with coordinates $(0, 0)$ and $(6, 0)$. What is the slope of the line formed from the point at the origin to the third point?

   Equation/Strategy: _____

   Solve:

   (a) $1/2$
   (b) $\sqrt{3}$
   (c) 2
   (d) 3
   (e) 6

---

4. L_1 and L_2 are two lines with slopes m_1 and m_2 respectively. If $m_1 \cdot m_2 = 1$, which of the following cannot be true?

   Equation/Strategy:

   Solve:

   (a) L_1 and L_2 are parallel lines
   (b) L_1 and L_2 are intersecting lines
   (c) L_1 and L_2 are perpendicular lines
   (d) L_1 and L_2 are coinciding lines
   (e) m_1 and m_2 are reciprocals

3 Similarity

General Equation: Two polygons are similar if all corresponding sides are in proportion.

Example 1: Square ABCD and square EFGH have side lengths in a ratio of 1 : 2. What percentage increase is the area of EFGH to ABCD?

Example 2: Line segment DE is drawn in triangle ABC so that D is the midpoint of AB and E is the midpoint of AC. What is the ratio of the areas of ADE to EDBC?

Example 3: Two concentric circles are drawn such that the radius of the outer circle is twice the diameter of the inner circle. What percentage is the circumference of the outer circle to the circumference of the inner?

MEDIUM

1. The circumference of a children's basketball is 27.5 inches whereas the circumference of an NBA basketball is 29.5 inches. What is the ratio of their volumes?

   Equation/Strategy: _____

   Solve:

   (a) 55 : 59
   (b) 351 : 433
   (c) 351 : 434
   (d) 437 : 469
   (e) 438 : 470

---

2. An equilateral triangle is inscribed in another so that only the vertices of the inner triangle touch the edges of the outer. What is the greatest possible ratio of the area of the smaller triangle to the bigger one?

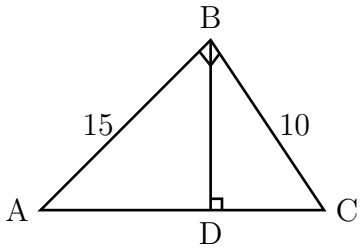
   Equation/Strategy: _____

   Solve:

   (a) 1 : 9
   (b) 1 : 4
   (c) 1 : 3
   (d) 1 : 2
   (e) 2 : 3

ADVANCED

3. Right triangle BDC is similar to triangle ABC as shown below.



   If AB = 15 and BC = 10, what is the length of BD?

   Equation/Strategy: _____

   Solve:

   (a) 7.5
   (b) 8
   (c) 12.5
   (d) $\frac{30\sqrt{13}}{13}$
   (e) $150 - \sqrt{325}$

---

4. What is the ratio of the diagonal of the a cube with a volume of 8 cm^3 to the diagonal of a cube with a volume of 64 cm^3?

   Equation/Strategy:

   Solve:

   (a)
   (b)
   (c)
   (d)
   (e)

4 Transformations

General Equation: Let $f(x)$ be a function. Then the function

$$g(x) = a \cdot f(b(x-h)) + k$$

is a transformation of $f(x)$ where

- a is the vertical stretch/compression
  - If $|a| < 1$, then g is a vertical stretch of f
  - If $|a| > 1$, then g is a vertical compression of f
  - If a is negative, then g is a reflection of f about the $x-$axis
- b is the horizontal stretch
  - If $|b| < 1$, then g is a horizontal stretch of f
  - If $|b| > 1$, then g is a horizontal compression of f
  - If b is negative, then g is a reflection of f about the $y-$axis
- h is the horizontal shift
  - If $h > 0$, then g is a horizontal shift of f by h units to the right
  - If $h < 0$, then g is a horizontal shift of f by h units to the left
- k is the vertical shift
  - If $k > 0$, then g is a vertical shift of f by k units up
  - If $k < 0$, then g is a vertical shift of f by k units down

Example 1: Describe the transformation of f by the function $2 \cdot g(x) + 3 = f(x)$.

Example 2: If $f(3) = -2$, and $g(x) = f(x-2)+3$, what coordinate must be on the graph of g?

Example 3: Let $g(x) = |f(x-1) - 2| - 3$. What is the vertical shift of $f(x)$ by $g(x)$?

MEDIUM

1. If $g(x) = |f(x)|$ and $(-x, y)$ is a coordinate of $f(x)$, which of the following is a coordinate of $g(x)$?

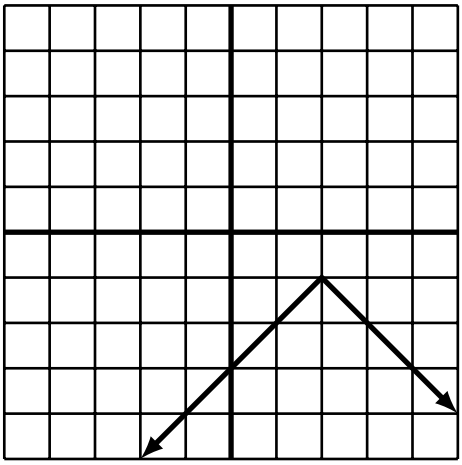
   Equation/Strategy: _____

   Solve:

   (a) (y, x)
   (b) (x, y)
   (c) $(x, -y)$
   (d) $(-x, y)$
   (e) $(-x, -y)$

---

2. The graph of $g(x)$ is shown below.



   If $g(x)$ is a transformation of $|x|$, which of the following is the equation of $g(x)$?

   Equation/Strategy: _____

   Solve:

   (a) $-|x-2|-1$
   (b) $|2-x|-1$
   (c) $-|x+2|-1$
   (d) $-|x+1|+2$
   (e) $-|x-1|+2$

ADVANCED

3. Which of the following can represent a transformation of the function $f(x)$?

   Equation/Strategy: _____

   Solve:

   (a) $\dfrac{1}{1/f(x)}$
   (b) $f(f^{-1}(f(x))$
   (c) $\sqrt[3]{f(x)^3}$
   (d) $\sqrt{f(x)^2}$
   (e) $f(x \cdot 0!)$

---

4. The function $f(x)$ has a coordinate $(m, -n)$. $f(x)$ is shifted k units down and g units right, then reflected across the line $y = x$. Which of the following is the resulting coordinate of the transformation?

   Equation/Strategy:

   Solve:

   (a) $(-m-g, n+k)$
   (b) $(-n+k, m-g)$
   (c) $(m-g, -n+k)$
   (d) $(-n-k, m-g)$
   (e) $(m-g, -n-k)$

Chapter 8

Geometry and Measurement: Part III

1 Coordinate Geometry

General Equation:

Example 1:

Example 2:

Example 3:

MEDIUM

1. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy: _____

   Solve:

2. (a)

   (b)

   (c)

   (d)

ADVANCED

(e)

3. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy:

   Solve:

4. (a)

   (b)

   (c)

   (d)

   (e)

2 Geometric Visualization

General Equation:

Example 1:

Example 2:

Example 3:

MEDIUM

1. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy: _____

   Solve:

2. (a)

   (b)

   (c)

   (d)

ADVANCED

(e)

3. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy:

   Solve:

4. (a)

   (b)

   (c)

   (d)

   (e)

Chapter 9

Data Analysis, Statistics, and Probability

1 Data Interpretation with Tables

General Equation:

Example 1:

Example 2:

Example 3:

MEDIUM

1. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy: _____

   Solve:

2. (a)

   (b)

   (c)

   (d)

ADVANCED

(e)

3. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy:

   Solve:

4. (a)

   (b)

   (c)

   (d)

   (e)

2 Data Interpretation with Graphs

General Equation:

Example 1:

Example 2:

Example 3:

MEDIUM

1. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy: _____

   Solve:

2. (a)

   (b)

   (c)

   (d)

ADVANCED

(e)

3. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy:

   Solve:

4. (a)

   (b)

   (c)

   (d)

   (e)

3 Descriptive Statistics: Mean, Median, and Mode

General Equation:

Example 1:

Example 2:

Example 3:

MEDIUM

1. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy: _____

   Solve:

2. (a)

   (b)

   (c)

   (d)

ADVANCED

(e)

3. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy:

   Solve:

4. (a)

   (b)

   (c)

   (d)

   (e)

4 Probability

General Equation:

Example 1:

Example 2:

Example 3:

MEDIUM

1. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy: _____

   Solve:

2. (a)

   (b)

   (c)

   (d)

ADVANCED

(e)

3. **Equation/Strategy:** _____

   Solve:

   (a)

   (b)

   (c)

   (d)

   (e)

   Equation/Strategy:

   Solve:

4. (a)

   (b)

   (c)

   (d)

   (e)

Part II

SAT Verbal

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The Six Most Frequently Missed Errors on SAT Writing Multiple Choice: Strategies for Sentence Improvements and Sentence Errors

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Sentence Improvements

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Chapter 13

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Chapter 16

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Chapter 17

Vocabulary