Introduction

This textbook covers two topics involving convexity

- optimization
- sampling

> Given a multivariate real function f,
>
> 1. How fast can we find a point that minimizes f?
> 2. How fast can we sample a point according to the distribution with density defined by f?

These problems are very hard to solve in the general case, so we will restrict our analysis to convex sets and convex functions. There are some main benefits of assuming convexity:

- local minimum of a convex function is the global minimum
- natural operations like intersection and addition maintain convexity
- convex sets can be approximated by ellipsoids
    - for any convex set, consider the minimal enclosing ellipsoid.
    - an ellipsoid is represented by $(\mathbf{x}-\mathbf{c})^\top \mathbf{A}(\mathbf{x}-\mathbf{c})=1$, where $\mathbf{c}$ is the center of the ellipsoid and $\mathbf{A}$ is a positive semi-definite matrix.
    - an ellipsoid can be thought of as a "smooth" approximation of a convex set, in the sense that it captures the overall shape of the set, but with a smooth boundary.
    - in machine learning and statistics, when working with high-dimensional data, it's useful to approximate a convex set with an ellipsoid to make the computation more efficient