Extreme events stress testing for capital allocation optimization of a multi-entity reinsurance company

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to see

Quel est le sens logique

Direct choquer le capital et voir un résultats

-¿ Trouver le graph optimal et deduire des propriété intéressantes

-¿ Est ce que choquer le capital requirements vaut l'emmerdement

choquer le capital et le capital requirement

inverse problem

https://www.les-crises.fr/la-vaste-blague-des-stress-tests-de-la-bce/

https://home.kpmg/xx/en/home/insights/2016/11/current-challenges-and-the-need-to-automate-stress-tests.html

https://www.moodysanalytics.com/risk-perspectives-magazine/integrated-risk-management/principles-and-practices/reducing-the-stress-in-stress-testing

apport du choc de required capital (demander à jérome si c'est standard) etude et approximation dans le cas du BMA qui est factor based

2) trouver comment est calculé la rétro de manière classique concept : une rétro paramétrique calculée suivant l'évènement et optimisant la solvabilité conception de la fonction de cout comment déterminer les paramètres ? cas d'usages et limites

to do :

Si on programme le problème direct, on peut réaliser des millions de simulations pour évaluer quels sotn les scénarios les plus dangereux. Et ensuite remonter à un exemple de cause. Il doit y avoir des relations entre les pertes sur les différentes entités sinon c'est du bullshit. Stochastic stress testing

jerome: les mesures de risque classiques. Les limites. Les avantages des stress test. Pourquoi les stress test sont limitées. Pourquoi les rendre automatiques et stochastiques

toujours articulation problème direct et et inverse.

PROGRAMMATION vérifier la formule du BCSR BMA diversisifcation regarder les livres de julien roland dans le concept, l'algoithme d'applique à une situation de perte particulière, cad pour une LOB specéfique, mais comme les traité peuvent être worder comme on veut, rien n'empèche d'utiliuser cet algo sur chaque LOB et d'écrire les traités en conséquencr programmer les modèles

MAISON definition du "premium risk" schema balance sheet considérée dessin graphe companie + balance sheet puis + exposure puis plus retrocession (3 en tout au fur et à mesure de l'introduction du modeling de la boite) figure choc sur la balance sheet reference BMA file reference swiss re model mortality

developpement de taylor du prix d'un bond shocké

ecrire le calcul de saturation du healthcare system trouver un cas où le groupe se porte bien mais les legal entities sont en breach trouver example financial crisis graphe pertes cat loss sur legal entities choisir les valeur des loss du scenario natural catastrophe faire le tableau des scenarios en fonction des modules de risque

trouver la définition des risques considérés regarder les differents other investments et donner les définitions

Les effets secondaire de chaque scénario sur les différentes lignes de business et certains éléments du bilan sont expliqués, quantifiés et pris en compte dans le calcul des pertes.

annexe donner les facteurs pour multiplier les risk driver du BMA et NAIC prendre les formules d'ihab pour shocker les bonds et regarder l'aspect risk free rate

calculer approximation linaire BMA et NAIC choisir une structure de boite pour faire les impacts de scenario

Contents

Chapter 1

Introduction

1.1 Introduction

FONGIBILITE ET DEPLACEMENT DE CAPITAL

Capital is the raw material and one of the main resource of a (re)insurance company. Capital is provided by shareholders to allow the company to underwrite risks and pay claims. The more capital a company owns, the more risk it can underwrite, and therefore grow its business as long as premiums paid by customers stay higher than the amount of claims paid. Knowing how and where to allocate this capital is a difficult task for any (re)insurance company and each of them have a unique way to do this allocation. Allocation can be done on several dimensions, including geographic or business lines. This multidimensional allocation is constrained by at least three forces whose inter-relations are complex and often opposites [**?**]:

- Solvency
- Profitability
- Growth

A simplified explanation of the interactions of these three forces is the following.

Investors and shareholders want the best return on their capital, and therefore, want to invest as low as possible of it in a given company. On the other hand, regulators wants to protect policyholders, imposing to companies certain levels of minimum capital, generally called ”risk based capital” or ”solvency capital requirement”. To sustain growth, capital should be allocated to the most profitable regions and business lines.

Unlike insurance companies which generally operates on a national level and on few business lines (except global insurer like AXA or Allianz), reinsurers are international and diversified by nature. They mitigate the risk of peak losses by diversifying their exposure on numerous geographic locations and business lines, ensuring minimum correlation in their portfolio. Due to their international nature, reinsurers are often formed of several legal entities, incorporated in countries in which the reinsurance market is dynamic, historic or both.

1.2 Capital Fungibility

Therefore, Capital should be allocated to the different legal entities of the re-insurance group in a way that both sustain growth and ensure a strong solvency. However, even moving capital from one legal entity to another is not straightforward. For instance, let consider a group formed of one entity incorporated in Europe with much of the capital, and a small entity incorporated in japan with significantly less capital but enough to meet regulatory requirements. From a group perspective, the solvency margin can be high. But if an extreme event happen in Japan, such as a strong earthquake, the Japanese entity could be strongly impacted and its losses could drive its capital below the minimum required capital, therefore triggering investigations or takeover from the local regulator. In this case, the solvency of the group doesn't prevent local accidents, and it is desirable to be able to transfer funds from the large European entity to the endangered Japanese one. But this fund transfer is not straightforward. Stated differently, Capital within an insurance group is not a highly **fungible** resource: having enough capital at group level doesn't ensure each legal entity can **access** this capital.

Two mechanisms can be used to transfer funds within a re-insurance group: moving capital and transferring losses from one legal entity to another. While moving capital is difficult and costly to put in practice (legal constrains, etc), transferring losses can be achieved by the use of insurance contracts, something that re-insurance companies are quite good at using. When used within the same re-insurance group, such insurance contracts are called "Internal Retrocession" and are a tool allowing to increase the fungibility of the whole group capital.

Even if Capital can be transfered from one entity to another, it should be allocated in the best way before any significant loss happen.

1.3 Capital Allocation : Internal Models versus Stress Tests

Capital allocation between business line or geographic allocation is a difficult task due to the numerous parameters playing to take into account such as :

- Currency
- Taxes
- Risk exposure
- Regulations
- Cash needs
- dynamism
- reinsurance cycle

To succeed in this allocation, reinsurance companies management teams can use several tools.

1.3.1 Internal Models

One of them is the internal capital model. It generally consists in a stochastic simulator providing losses probability distributions. Such tool is very useful, but cannot properly treat the case of extreme events. In facts, these models are calibrated on historic or simulated data, which implies the availability of large databases to properly estimate the risks distributions.

REF INNEFICACITE INTENAL MODLE SUR LES RISQUES EXTREMES

1.3.2 Scenario Stress Testing : Estimation of extreme events impacts

On the other hand, stress testing and scenarios have proved useful to study extreme events on which reliable probability estimates are not available REF ???. It consists in assuming the variation of a collection of variables, determined through a relevant scenario design, and study the impact of such variation on the company financial structure. Contrary to the stochastic nature of the internal capital model, a scenario is deterministic : it represents a single point on the loss distribution. The art of scenario design is to find the right balance between the extremeness of the scenario and its plausibility. A weak scenario would not add value compared to a standard result of an internal capital model, and on the contrary, a too strong scenario would just destroy the entire reinsurance industry and provide no insight on the company risk profile (the meteor fall scenario wiping out the human race is useless for re-insurers).

Why using scenarios to assess a company resiliency after an extreme event?

In soft market conditions (low prices and a lot of capacity available on the financial markets), the reinsurance business is highly competitive. Only the occurrence of catastrophic losses can trigger an increase in price : a lot of capacity will be used to pay claims, leading to a shortage of capacity supply on the market. Moreover, cedents will probably want to buy additional cover as the impact of catastrophic losses is not a remote event anymore, but a yesterday reality. This increase in demand and shortage of supply generally lead to an increase in prices. But for a reinsurer to take advantage of such interesting market conditions, it needs :

- First, to still have enough capital to operate after the catastrophic event (which is not an impossible event, like SCOR after the World Trade center terrorist attack [**?**]).
- Second, to be able to convince investors to inject capital in the company to very quickly underwrite risk and beat its competitors.
- Third, to be perceived as a solvent and stable business partner by cedents.

Scenarios are very useful tools to assess the state of a (re)insurance company after catastrophic events, and then examine and study its resiliency and its ability to make business after these extreme cases.

1.4 Report plan

Two studies are presented in this work. First we propose a way to model the loss ”spreading” in a reinsurance company financial structure on which an arbitrary stress test scenario is applied. This correspond to a ”direct problem”, on which an input (scenario and losses) is applied to a specified ”system” (group financial structure and retrocession agreements) from which the output (updated Capitals and capital requirement after losses) are computed for analysis. Financial structures consist in financial information of each legal entities and of the regulatory capital requirement of the country the legal entity is registered in. An interesting aspect of this work is the fact that the impact of a scenario is taken into account in the legal entity balance sheet but also in the computation of the regulatory capital requirement, leading to better solvency estimations. Several levels of approximation are used and their limitations are discussed.

Second, we propose a method for determining the optimal internal retrocession of a reinsurance group, considering a given scenario. The internal retrocession program is considered as optimal if it minimizes the decrease of solvency ratio of all entities of the group. This correspond to a different kind of problem”, where inputs (scenario, losses, group financial structure) are known but the system parameters (retrocession agreement) needs to be computed according to an optimization criteria, here one related to solvency. The final aim is study is to be able to define ”on-demand” retrocession agreements at group level allowing to optimize Capital Allocation on a per scenario basis.

Chapter 2

Deterministic modeling of scenario stress test an arbitrary reinsurance group

EXPLAIN SCENARIO AND APPROACH

One of the component of a scenario is the company on which shocks are applied. Given the same scenario inputs, the results can greatly vary from one company to another.

2.1 Modeling a reinsurance company financial structure

A reinsurance group est generally composed of several independent companies that are called ”legal entities”. These companies are subsidiaries of a mother company (or subsidiaries of subsidiaries), and not branches : each of them has regulatory requirements depending of the country in which they are incorporated. This is not the case of branches that are only extensions of their mother company.

2.1.1 Ownership structure

A hierarchical ownership structure is almost always chosen. (This allows to reduce fiscal losses during the dividends ”ascent” to the mother company and the shareholders). The word ”Group” is used to describe all companies directly or indirectly owned by the parent company including itself).

An example of ownership structure in given in 2.1. Black arrows represent the ownership links and a hierarchical structure is chosen in this example.

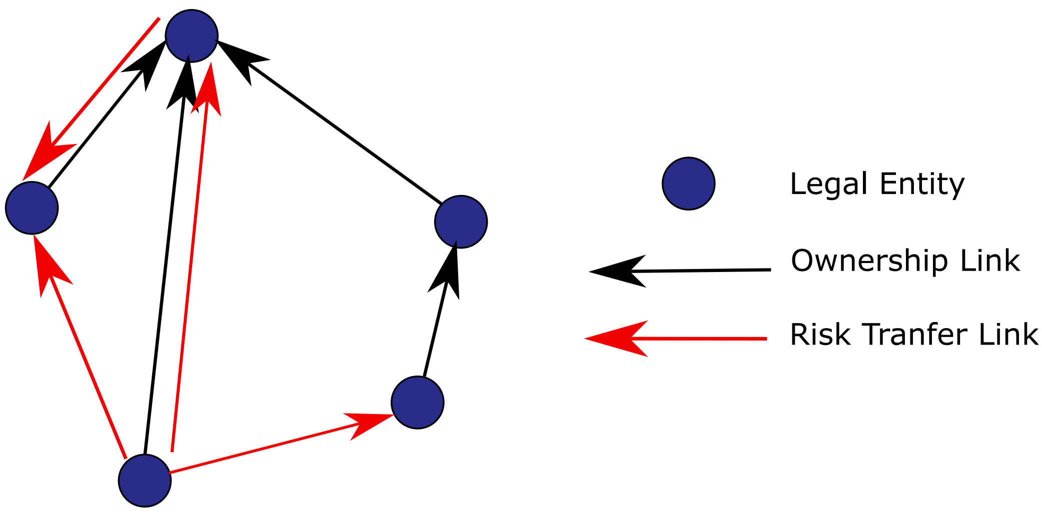

Figure 2.1: Representation of the (black arrow) ownership structure (arrow means ”belong to”) and (red arrow) risk transfer structure (arrow means ”transfer risk to”) of a (re)insurance group. Legal entities are represented as blue disks.

Financial statements representation

Each legal entity must disclose financial statements to its regulator. These financial statements contain at least :

- a balance sheet,
- a profit and loss statement,

(what about the cash flow statement?). The balance sheet display the assets and liabilities of the company, which are always equal at any given time. For the purpose of stress testing scenarios, only the balance sheet is considered. Indeed, the balance sheet is a representation of the company patrimony at a given time, and the solvency of an insurance company is assessed by comparing the capital on the balance sheet with a regulatory required capital. But this do not allow to easily visualize changes in premium income or charges.

This representation of the balance sheet is a simplified one, but these elements will be enough for the next step of the report to understand the dynamic of operation of a reinsurance company. It is important to understand that the wealth of the shareholders of the company correspond to the company ”Own Funds”, also called ”Capital”. More information on the variation of balance sheet values due to a shock are given in section **??**.

Moreover, each legal entity is characterized by its own balance sheet, and in this work, we will assume that the group balance sheet is the sum of all legal entities balance sheets.

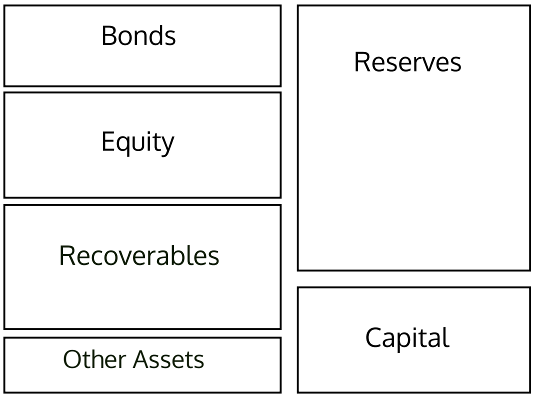

Figure 2.2: Simplified representation of a balance sheet.

JUSTIFY THIS ASSUMPTION

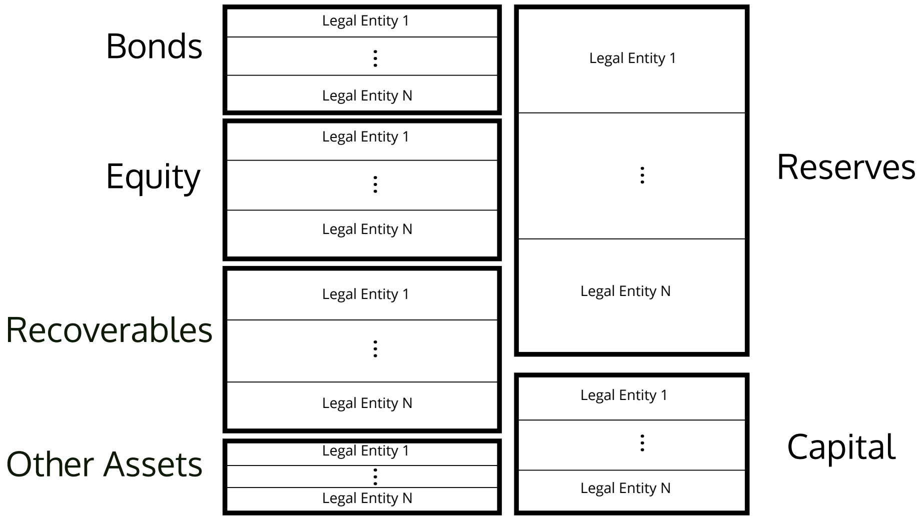

Figure 2.3: Aggregation of legal entities balance sheets into the group balance sheet.

From a mathematical point of view, a re-insurance group can composed of N legal entities, each of them having a C_i, i being the ith entity. All legal entities capitals are summarized by the vector:

$$C = (C_i)_{i \in [1,N]} \tag{2.1}$$

This vector can be splitted into several components related to the balance sheet :

$$C = FI + E + Recov + OA - Res \tag{2.2}$$

with FI the fixed Income (Bonds) vector, E the equity vector, $Recov$ the recoverable vector, OA the other asset vector and Res the reserve vector.

Exposure of legal entities

Each legal entity has its own financial position and risk profile, determined by the nature of the risks it underwrites. The exposure of a company is defined by the level of its liabilities toward policyholders per geographic zones and lines of businesses. For instance, a company underwriting natural catastrophe business in the USA is highly exposed to a hurricane on Florida but absolutely not to a deviation of life expectancy in China. This notion of exposure is important as it represents the ”entry points” of losses within the company. One can visualize the exposure as a target whose area proportional to the liabilities of a company. Within a reinsurance group, each legal entity can have very different exposures, and ultimately, each legal entity is liable to the risks it underwrites.

From a mathematical point of view, each legal entity can suffer a loss L_i, i being the ith entity. Losses on all legal entities are summarized by the vector:

$$L = (L_i)_{i\in[1,N]} \tag{2.3}$$

2.2 Modeling of risk transfer mechanisms

2.2.1 Risk Transfer structure

Sometime, the financial position of a legal entity is inadequate towards its risk profile. The company can be under-capitalized compared to the level of risks it underwrote, which is dangerous for shareholders and policyholders. On the other end, a company can be over-capitalized. This can be the case if the legal entity is newly opened and the regulator required a very strong capitalization to grant the reinsurance license. This case is not optimal for shareholders, whose return on capital is sub-optimal. Such cases lead to the desire of finding means to re-balance the risk profile and the financial position of companies.

To that end, two means are available. The first mean is capital transfer (called capital injection) from one company to another (within the group). In section 2.1.1, the ownership structure of a reinsurance group was presented as a first type of relation between companies. This structure is important to study the flow of dividends from subsidiaries to the mother company. But this ownership structure also constrains the capacities of capital transfers from one company to another. Transferring capital from one company to another, operation called ”Capital injection”, is a complex legal operation that cannot be performed easily and quickly. Instead of transferring capital between companies, another way is to transfer risks from one company to another. This kind of risk transfer is the core of the insurer and re-insurer skill. In the context of a risk transfer from one re-insurer to another, the agreement is called a retrocession agreement. The use of retrocession is much easier than capital injection as it is exactly the product sold by a re-insurer. Moreover, it can be used within a group (internal retrocession) or with other companies (external retrocession). With internal retrocession, profits stay inside the group but the risk too : it is just transferred from one legal entity to another.

An example of risk transfer structure through internal retrocession is presented in Figure 2.1. Risk transfer link are the red arrows.

A reinsurance company is generally re-insured through external retrocessions against extreme losses on several of its lines of businesses. It is a way to spread the risk through the entire industry and limit the probability of default of re-insurers. Each company is free to choose its own external retrocession program, but the more it retrocede risk, the more its profitability is reduced. Therefore, a re-insurer must find the good balance by according its retrocession program with its risk appetite.

To understand the concept of retrocession, let's assume that a legal entity have a retrocession program with an external reinsurance company. This retrocession program state that $\frac{1}{10}$ of all losses will be transferred from the company to the retrocessionaire. If the company bear a ”gross” loss of \$ 100 M, \$ 10 M will be paid by the retrocessionaire and the ”net” loss of the company is \$ 90 M. The ”gross” loss is the initial loss that the underwriting company must pay to its policyholders. After deduction of the contribution of the retrocessionaires, the final loss bore by the company is called the ”net” loss. The difference between the ”gross” loss and the ”net loss” is the part of the loss that is paid by the retrocessionaires (and can be called ”the retrocession”).

TODO : METTRE FIGURE

These terms can be confusing because there is no word difference between an external retrocession and an internal retrocession. A net loss can be after external retrocession, internal retrocession, or after both external and internal retrocession.

2.2.2 Proportional and Non proportional reinsurance

Reinsurance treaties can be proportional or non proportional. An example of proportional treaty is the quota-share (QS) treaty. A quota-share treaty consists in the transfer of an agreed share of risk α from the ceding company (the cedent) to a retrocessionaire. As the share α of the risk is ceded, a share α of premium are also transfered to the retrocessionaire (in practice a commission is paid back by the retrocessionaire to compensate the reinsurer costs) (see 2.4).

Two examples of non proportional treaties are provided in 2.4. The first type of non proportional treaty is the "Excess of Loss" treaty. In this type of treaty, the retrocessionaire must pay losses exceeding a certain limit called "Priority" (or "Deductible") up to a limit called the "Limit" (or "Coverage"). If the Limit is infinite, the treaty is considered as a "Stop Loss".

Treaties can be associated to a risk or an event. In the case of a risk trigger, the treaty is activated for every risk occurring. For example, if a treaty is protecting a portfolio of people with death insurance, the treaty will be activated each time a person dies. To activate an event treaty, the must be at least two risk occurring simultaneously. This type of treaty is generally used to protect against natural catastrophes or event where aggregation of risk of very likely.

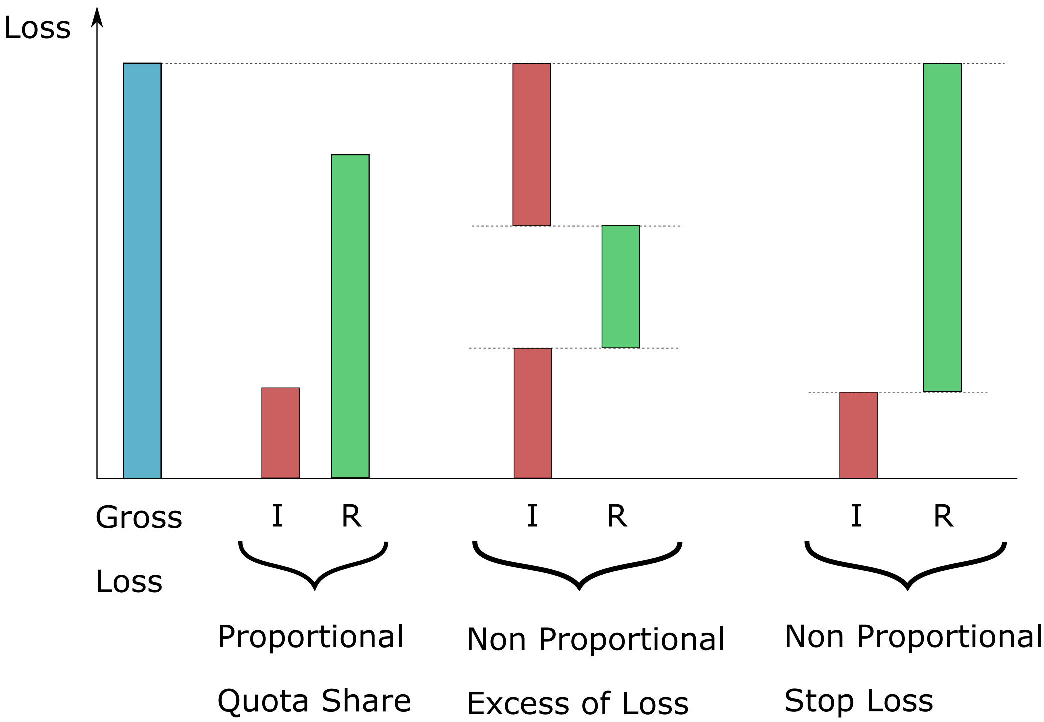

Figure 2.4: Several type of retrocession agreement. (blue) Gross loss of the insurer, (red) share of the loss attributable to the insurer, (green) loss attributable to the reinsurer (or retrocessionaire). The cession share of the quota share agreement is 80%.

2.2.3 Modeling of external retrocession

TODO : COMMENT EST GERE DANS UNE BOITE ?

2.2.4 Modeling internal Retrocession

Before any shock (loss) applied on legal entities, losses are null, meaning that :

$$L = (0)_{i \in [1,N]} \tag{2.4}$$

After a shock is applied, each entity suffers a loss of L:

$$L^0 = (L_i^0)_{i \in [1,N]} \tag{2.5}$$

After external retrocession is applied, the loss vector is denoted:

$$L = (L_i)_{i \in [1,N]} \tag{2.6}$$

However, it is assumed that internal retrocession agreements exist between entities. The loss vector after use of these retrocessions is denoted L^U, the superscript U meaning "updated":

$$L^U = (L_i^U)_{i \in [1,N]} \tag{2.7}$$

We propose to model the retrocession agreement between two legal entities by a function $r_{i \leftarrow j}$ taking the full loss vector L as an input and outputting the amount of loss transferred from entity j to entity i. i and j belong to $[1, N]$

$$r_{i \leftarrow j} : L \in \mathbb{R}^N \to \mathbb{R} \tag{2.8}$$

For simplicity, it is assumed that the values of $r_{i \leftarrow j}$ are always **positive**.

For instance, a proportional retrocession from entity A to entity B is written :

$$r_{B \leftarrow A}^{proportional}(L) = \alpha_{B \leftarrow A} L_A \tag{2.9}$$

with $\alpha_{B \leftarrow A} \in [0, 1]$ and L_A the loss of entity A.

While a proportional retrocession from entity B to entity A is written :

$$r_{A \leftarrow B}^{proportional}(L) = \alpha_{A \leftarrow B} L_A \tag{2.10}$$

An excess of loss agreement from A to B without deductible and limited at M_{BA} is written:

$$r_{B \leftarrow A}^{excessloss}(L) = min(L_B, M_{BA}) \tag{2.11}$$

Considering this definition of retrocession loss transfer, how to computer L^U from L^0 and the collection of $(r_{i \leftarrow j})_{(i,j)}$?

We define the operator $R[]$ by the following relation :

$$L^U = R[L] \tag{2.12}$$

L and L^U are vector belonging to $\mathbb{R}^N$ and the operator $R[]$ applies the following operation :

$$R[L] = \left(\sum_j r_{i \leftarrow j}(L) \right)_{(i \in [1,N])} \tag{2.13}$$

The brackets [] of $R[]$ denote the sum over the columns of R, while R is a matrix of functions, containing the retrocession functions (incomplete representation at this stage of the model presentation) :

$$R = \begin{pmatrix} ? & r_{1\leftarrow 2} & r_{1\leftarrow 2} & \cdots & r_{1\leftarrow N} \\ r_{2\leftarrow 1} & ? & r_{2\leftarrow 3} & \cdots & r_{2\leftarrow N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{N\leftarrow 1} & r_{N\leftarrow 3} & \cdots & r_{N\leftarrow (N-1)} & ? \end{pmatrix} \tag{2.14}$$

The matrix R is called the "Retrocession Matrix" and is a N by N square matrix. Each line of the matrix represent a loss receiving entity and each column a loss giving entity.

Expression 2.13 means that the ith element of the updated loss L^U is the sum of all losses given by other entities. For $i \neq j$, the retrocession function $r_{i\leftarrow j}$ was defined before. The case of $r_{i\leftarrow i}$ is more complex.

If no retrocession agreement exist between the legal entities, then R must be the identify matrix, so that $L^U = L^0$.

Then the R matrix expression can be refined by writing :

$$R = \begin{pmatrix} 1+? & r_{1\leftarrow 2} & r_{1\leftarrow 2} & \cdots & r_{1\leftarrow N} \\ r_{2\leftarrow 1} & 1+? & r_{2\leftarrow 3} & \cdots & r_{2\leftarrow N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{N\leftarrow 1} & r_{N\leftarrow 3} & \cdots & r_{N\leftarrow (N-1)} & 1+? \end{pmatrix} \tag{2.15}$$

Internal retrocession is a zero-sum game. This means that amount given by entity ji to j, are received by j. This can be accounted by adding an additional terms to the diagonal elements of R:

$$R = \begin{pmatrix} 1-(\sum_{i=2}^N r_{i\leftarrow 1}) & r_{1\leftarrow 2} & r_{1\leftarrow 2} & \cdots & r_{1\leftarrow N} \\ r_{2\leftarrow 1} & 1-(\sum_{i=1,3,4}^N r_{i\leftarrow 2}) & r_{2\leftarrow 3} & \cdots & r_{2\leftarrow N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{N\leftarrow 1} & r_{N\leftarrow 3} & \cdots & r_{N\leftarrow (N-1)} & 1-(\sum_{i=1}^{N-1} r_{i\leftarrow N}) \end{pmatrix} \tag{2.16}$$

This additional terms compensate every given loss by its corresponding received one on the relevant entity.

According to these definitions, one can demonstrate that $\sum_i L_i^U = \sum_i L_i^0$, accounting for the fact that money is transferred between entities and no money flows in or out of the insurance group.

Here are sime exemple of Retrocession matrices in simple cases.

Lets assume two entities names 1 and 2, with crossed proportional retrocession agreements.

The loss vector is :

$$L = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \tag{2.17}$$

The R matrix is :

$$R = \begin{pmatrix} 1 - r_{2\leftarrow 1} & r_{1\leftarrow 2} \\ r_{2\leftarrow 1} & 1 - r_{1\leftarrow 2} \end{pmatrix} \tag{2.18}$$

with :

$$r_{1\leftarrow 2} = \alpha_{1\leftarrow 2} L_2 \tag{2.19}$$

$$r_{2\leftarrow 1} = \alpha_{2\leftarrow 1} L_1 \tag{2.20}$$

The full loss expression is then :

$$L^U = R[L] = \begin{pmatrix} L_1(1 - \alpha_{2\leftarrow 1}) + L_2\alpha_{1\leftarrow 2} \\ L_1\alpha_{2\leftarrow 1} + L_2(1 - \alpha_{1\leftarrow 2}) \end{pmatrix} \tag{2.21}$$

This matrix make sense: entity 1's updated loss L_1^U is the loss L_1 at time 0 minus the loss given to 2 plus the loss received from 2.

In the case of excess of loss without deductible whose maximum amount are $M_{1\leftarrow 2}$ and $M_{2\leftarrow 1}$:

$$r_{1\leftarrow 2} = min(L_2, M_{1\leftarrow 2}) \tag{2.22}$$

$$r_{2\leftarrow 1} = min(L_1, M_{2\leftarrow 1}) \tag{2.23}$$

The full loss expression is :

$$L^U = R[L] = \begin{pmatrix} L_1 - min(L_1, M_{2\leftarrow 1}) + min(L_2, M_{1\leftarrow 2} \\ L_2 + min(L_1, M_{2\leftarrow 1}) - min(L_2, M_{1\leftarrow 2} \end{pmatrix} \tag{2.24}$$

The flexibility of on the definition of $r_{i\leftarrow j}$ allows for function composition, for instance a quota share followed by and excess of loss.

Moreover, in this model, all retrocession operation are realized at the exact same time. Amount are all "transferred" at the same moment and received funds are not cyclically taken into account. Cases of multistep retrocessions can be taken into account by compounding the Retrocession matrix with another one representing the next step of retrocessions. A single complex Retrocession matrix is obtained by compounding the several retrocessions steps.

Cp = Ci - R * L

Hypothèse : RC = RCi SR = Cp / RCi SR = (Ci - R * L) / RCi SR = SRi - R * (L / RCi) deltaSR = R * (L / RCi)

objectif : minimser deltaSR

2.3 Modeling of regulatory required capital

2.3.1 General presentation of regulatory capital requirements

Section 2.1, presented the balance sheet and the legal entities exposure as tool allowing to assess a reinsurance company risk profile in the context of stress testing. These information are provided by the company for investors and internal purposes.

But regulators also have views on companies risk profile and financial position. Their aim is to protect customers from non-adequate and reckless behaviors from insurance and reinsurance companies by monitoring companies through several metrics. Here are the name several regulators and the country in which they are operating :

Regulator	**County**	**Regulation**
ACPR (Autorité de Contrôle Prudentiel et de Résolution)	France	Solvency 2
BMA (Bermuda Monetary Authority)	Bermuda	BSCR
FINMA (Eidgenössische Finanzmarktaufsicht)	Switzerland	?
MAS (Monetary Authority of Singapore)	Singapore	?
NAIC (National Association of Insurance Commissioners)	United States	RBC

To show their ability to pay their liabilities toward policyholders, (re)insurance companies are, among other aspects, assessed by regulators on their solvency. The solvency is generally defined by a ratio between the "Own funds" or the shareholder's equity of the company compared to a minimum Required Capital (RC) computed from its risk profile. The shareholder's equity is a buffer that the company will use to pay its claims in case of shortage of reserves : the regulator wants this buffer to be sufficient compared to the risk profile of the company. The solvency ratio is almost always expressed in the form:

$$Solvency\ Ratio = \frac{Company's\ Own\ Funds}{Required\ Risk\ Capital} \tag{2.25}$$

The required risk capital is generally defined as the capital needed by the company to limit its insolvency probability below a certain level α, which correspond to the formula [**?**] :

$$P(Asset - Liability \leq 0) \leq \alpha \tag{2.26}$$

With α generally equals to 99.5 %

The solvency assessment, ie the way of computing the required risk capital, is different between each countries (see table 2.3.1).

All these capital models are different in their computation but they relies on the same principle, which is to reflect the risk profile of a company in the form of a minimum solvency capital need.

TODO : IS IT TRUE ?

From a mathematical point of view, a legal entity have a capital requirement RC_i, i being the ith entity. All Capital requirements are summarized by the vector

$$RC = (RC_i)_{i \in [1,N]} \tag{2.27}$$

If the company's Capital falls below the required solvency capital, the regulator can impose sanctions, fine the company, takeover operations or even declare bankruptcy. Therefore, it is of paramount importance for a company management to keep a level of Own funds above the required solvency capital.

In certain jurisdictions, a company can ask to be evaluated based on its own assessment of its solvency, and not according to the standard solvency capital requirement formula. This is generally done through an internal capital model, and it must be validated by the regulator before being used to regulatory reporting.

2.3.2 Principles of "Risk Factor Based" Regulatory Required Capitals computation

This part explain the basis of computation of "factor based" regulatory required capitals and is based on the behavior the Bermuda Solvency Capital Requirement and the NAIC Insurance Risk Based Capital. No real required capital calculation would exactly correspond to this explanation but we consider that this is a good first approach to understand the dynamic of such computation. Adjustments for BMA BSCR and NAIC RBC are given in sections **??** and A.4. Currently, some required capital formulas are more complex, as the Solvency 2 standard formula. This formula is not studied in this work but could be an interesting subject to explore using the tools presented below.

The regulatory required capital is a quantity expressed in a given currency. In factor-based models, this capital RC is a function of several specialized sub-capitals SC_i :

$$RC = f^{\beta}(SC_1, ..., SC_i, ..., SC_N) \tag{2.28}$$

where SC_i is the sub capital associated with the i^{th} risk, N the number of risks taken into account for the calculation of RC and β is the correlation matrix between sub-capital used for diversification..

This diversification function can be of the form :

$$f^{\beta}(x_1, x_2, ..., x_n) = \sqrt{\sum_{i,j=1}^{N} \beta_{i,j} x_i x_j} \tag{2.29}$$

with $\beta_{i,j}$ denoting the correlation factors between sub-capital SC_i and sub-capital SC_i in the diversification.

To compute sub-capitals, risk drivers RD relevant to these sub-capitals are defined (generally from the financial statements of the company) and their value is multiplied by a risk factor representing the views of the regulatory agency on this risk. These risk factors are denoted F_{risk}, with $risk$ being the risk addressed by this factor.

The computation of a capital requirement is then :

$$RC = f^{\beta}(RD_1 \times F_1, ..., RD_i \times F_i, RD_N \times F_N) \tag{2.30}$$

TODO : SCHEMA : RELATION BALANCE SHEET VERS RISK BASED CAPITAL

To give a concrete example of Risk Factor Based required capital computation, the example of the BMA BSCR is presented below. The Bermuda Monetary Authority (BMA) was established in 1969 and regulates, inspects and supervises financial institutions operating in Bermuda. Insurance companies must report their calculation of Bermuda Solvency Capital Requirement (BSCR) annually to the BMA. The BSCR calculation is factor-based and relies on several risk drivers presented in table 2.3.2.

Risk modules sub-capitals are computed and a covariance adjustment is applied to the sum of these modules to lead to the BSCR, as seen in 2.3.2.

The diversification function is :

$$\begin{aligned} BSCR = \sqrt{C_{fi}^2 + C_{eq}^2 + C_{cur}^2 + C_{conc}^2 + C_{intrate}^2 + C_{prem}^2 + (1/2C_{cred} + C_{rsrv})^2} \\ \overline{+(1/2C_{cred})^2 + C_{cat}^2 + C_{life}^2} + C_{op} + C_{adj} \end{aligned} \tag{2.31}$$

The BMA adds to the BSCR the contribution of an operational risk and a risk adjustments to the risks presented in table A.4.2. This formula is really close to

Risk Name and Abbreviation	BMA	Risk Driver
Fixed income (fi)	x	Volume of Fixed Income instrument
Equity (eq)	x	Volume of Equity
Interest rate (intrate)	x	
Currency (cur)	x	
Concentration (conc)	x	
Premium (prem)	x	Volume of Premium
Reserve (rsrv)	x	Volume of Non-Life Reserves
Credit / Counterparty (cred)	x	Volume of recoverables
Catastrophe (cat)	x	Probable Maximum Loss
Life	x	Volume of Life Reserves

Table 2.1: List of risk and risk drivers for several solvency capital requirement.

the general regulatory required capital formula proposed in section 2.3.2, with a slight change on the treatment of the reserve and credit risk.

The computation of the sub-capitals is done by multiplying the risk driver by a factor, whose are summarized in appendix **??**.

The company capital used for the computation of the solvency ratio is the Statutory Economic Capital, which is the Bermudian economic vision of the company Own's funds. This Statutory Economic Capital differs from the Statutory Capital computed from the Statutory balance sheet. Main differences involve some intangible assets counting and the computation method of reserves.

Sub-capitals are computed by multiplying a risk driver by a certain factor. This part present the computation of these factors for Reserve and Credit risk. The reference document used to this computation is available at [**?**] and presents other risk module computation.

2.3.3 Risks modeled in regulatory required capital formulas and expression in BSCR

As the nature of the (re)insurance business is almost the same in every country, the risks modeled in the regulatory capital requirements are quite similar from one jurisdiction to another. Only the complexity of the implementation of the risk model differs. These risks are the building blocks of regulatory required capital model and the main ones are presented below. Moreover, for each risk, the corresponding computation in the BSCR is presented to highlight the computation mechanism.

Reserve risk - Non Life & Life

In solvency 2, the reserve risk is defined as : "The risk that the current reserves are insufficient to cover their run-off over a 12 month time horizon". This is the risk of a deviation of the reserves after the premiums have been earned. It models the uncertainty on the claims amount and timing. Most of Non-Life lines of businesses are included in this risk. Moreover, it is worth noting that Health business is generally included with Non-Life business in this module.

The commonly chosen risk driver by the regulator is the amount of gross or net reserves.

The reserve risk sub-capital is computed using the formula :

$$C_{resrv} = Net\ Reserve \times F^{Diversification} \times F^{LOB} \times F^{concentration} \tag{2.32}$$

with $Net\ Reserve$ the reserve net of external retrocession. The diversification factor formula is :

$$F^{Diversification} = 0.75 + 0.25 \times \frac{\sum_{area}^{NC} Res_{area}^2}{(\sum_{area}^{NC} Res_{area})^2} \tag{2.33}$$

with Res_{area} the reserve on a certain area and NC is the number of possible geographic area (18 for the BMA). If reserves are fully diversified on all countries (meaning that business is done in a maximum of countries), then the diversification factor equals $0.75 + 0.25 \times \frac{1}{NC} = 0.764$. On the contrary, if the company operate in a single country, then the diversification factor equals 1. In case of an additional loss, this factor's value should change as the reserve mix is slightly different before and after the loss. For simplicity, the value of this factor is not modified because of an additional loss and is seen as a characteristics of the company.

The Line of business factor F^{LOB} depends on the line of business affected by the loss, and varies from 13% to 51%.

The concentration factor $F^{concentration}$ depends on the business diversification of the company and is computed as :

$$F^{concentration} = 0.6 + 0.4 \times \left[\frac{Max\ Res}{Total\ Res}\right] \tag{2.34}$$

with $Max\ Res$ the maximum amount of reserve on one line of business and $Total\ Res$ the total amount of reserves. The concentration factor will be 1 for a mono line company and $0.6 + 0.4 \times \frac{1}{23} = 0.6174$ for a fully diversified and balanced company. Like for the diversification factor $F^{Diversification}$, it is assumed that the concentration factor is not modified by an additional loss.

Premium risk

The risk driver generally chosen by the regulator is the amount of gross or net premiums.

CALCUIL PERTINENT ?

Catastrophe Risk

This is the risk of financial losses due to high severity and low probability events like earthquake and hurricanes. This sub-capital is computed for risks lying outside of traditional "actuarial" property risk assessment, and requires specific method and software to be computed (AIR [?] or RMS [?]). The risk driver generally chosen by the regulator is the (net or gross) Probable Maximum Loss (PML). The PML is the maximum loss that can occur given a certain return period (generally 100 years or 250 years), but it must be noted that no definition seems to be widely accepted by the actuarial community [?].

Credit/Counterparty/Default risk

This is the risk of default of a counterparty owning some assets of the company. These assets include receivables and reinsurance recoverables. The counterparty risk is assessed by rating agency examining the risk profile of the asset and assigning it a rate dependent on its ability to meet its debt payments. A credit risk exist at the moment when a asset is transfered from one entity to another with just a promise to repay it. The more probable the repayment is, the smaller the credit risk is. The risk driver generally chosen by the regulator is the amount of recoverables.

The additional credit risk sub-capital ΔC_{cred} is computed by multiplying the additional recoverables $\Delta Recov$ by a factor F_{cred} depending on the counterparty rating.

$$\Delta C_{cred} = \Delta Recov \times F_{cred} \tag{2.35}$$

with $\Delta Recov$ the additional recoverables and F_{cred} the credit risk factor.

The dependency of the factor toward the counterparty rating is presented in table 2.3.3

Investment risk : Spread Risk

A very classical investment of (re)insurance companies is government and corporate bonds. A bond is an instrument of indebtedness of the bond issuer to the holder. It can be viewed as a loan from the bond issuer to the holder. One key

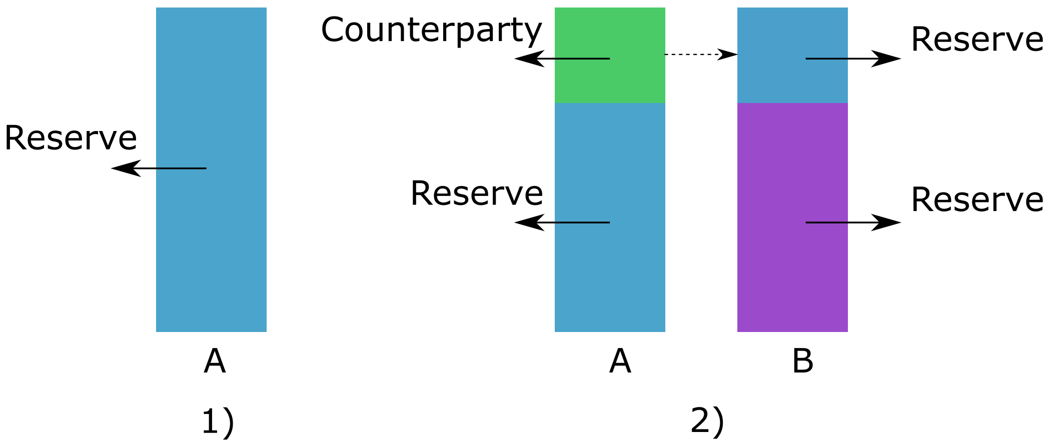

Figure 2.5: Comparison between reserve and counterparty risk. In case 1), the company hold all the risk, and this value is used to compute the reserve risk. In case 2), company A transferred a part of its risk to company B. The remaining share is used to compute reserve risk, whereas the transferred part (called recoverable) is used to compute a counterparty risk. Company B just add to its reserve risk the share received from A.

BMA Rating	**S & P rating**	**Credit Factor Fcred (%)**
0	AA government	0
1	AAA	0.7
2	AA+ to AA-	1.5
3	A+ to A-	3.5
4	BBB+ to BBB-	7
5	BB+ to BB-	12
6	B+ to B-	20
7	CCC+ to CCC-	27
8	below CCC-	35

Table 2.2: BMA Credit Risk factor by counterparty rating.

point is that a bond grants an ownership right (like equity) of the company to the holder : it is debt that a company has toward the holder. A bond can be modeled as a series of cash flows. First the holder bears a negative cash flow when it buys the bond to the government of company. After, the company pays coupon (or interest), which are computed as a percentage of the principal (the borrowed sum), over the duration of the bond. The principal is payed at bond maturity.

As bonds can be traded, the value of a bond depends on how much the market is willing to pays for it. When the bond is issued, the interest rate is set at, for example, 5% of the principal (sum payed to purchase the bond, which is the capital invested to get a 5% additional money). After, the bond can be traded on the market. The current yield of a bond is the effective annual interest rate, computed as the coupon value divided by the current market value of the bond. If its market value decreases, it's yield increases (as the coupon value is set at the issuance of the bond).

A bond can be characterized by its credit/yield spread. This is the difference of yield between a given bond and a risk-free bond (US government bond for example). This yield reflect the riskiness of the bond compared to a risk-free one : if the probability of default of the bond issuer is higher than a risk free one, the investor wants a premium for this default risk. The more risky a bond, the higher its yield spread.

One risk is the decreases of bond value after an interest rate move. If bond spread increases, as its coupon value is set, it mean that its price (ie the value at which it can be sold) decreases. This is a bad news for the insurance company which count on the bonds to pay its claims and provide additional investment revenues. The risk driver generally chosen by the regulator is the amount of fixed income instrument hold by the company.

TODO : CALCUL ?

Investment risk : Equity investment Risk

The Equity investment risk is the risk of a deviation of the value of equity hold as an investment by the (re)insurance company. As public listed companies' stocks can be easily traded, an economic downturn, the mismanagement of a company or political troubles can lead to a decrease of stocks value. Therefore, the equity portfolio value of the (re)insurance company can fluctuate and undergo an important decrease. This is particularly true in the case of a financial crisis like the 2007/2008 one.

The risk driver generally chosen by the regulator is the amount of equity hold by the company.

TODO : CALCUL ?

Investment risk : Other investment Risk

The Other investments risk is the risk of a deviation of the value of other investments hold by the (re)insurance company. Other investments can be of numerous types, as explained in section 2.1.1. For each type of investment the drivers of value fluctuation must be understood and assessed independently.

The value of real estate investments can decrease in the case of a mortgage crisis for example. As people cannot repay their mortgages, they are forced to sell their houses at a discounted price, leading to a decrease of housing prices. Private equity value can decrease if the company's performance is under expectations and if it goes bankrupt.

The risk driver generally chosen by the regulator is the amount of each investments hold (property, private equity, etc) by the company.

TODO : CALCUL OU NEGLIGEABLE ?

Concentration risks

The concentration risk represent the risk of aggregation of losses on assets that share common characteristics. For example, holding equity from only one company, operating on one business and in one country is a situation of high concentration risk. A way to decrease this risk is to invest in diversified companies, from a geographic and business line point of view.

2.4 How to shock the balance sheet items ?

Previous sections presented the components of the model representing the insurance company and several risk transfer mechanisms. This section present how to shock these components to assess the effect of an event on the company's financial structure.

The core principle of stress testing is to apply a shock on the (re)insurance company and assess the impact on its structure. This impact can be assessed on at least two aspects that are studied after : the financial structure, represented by the financial statements, and its required capital. A bottom-up approach was chosen : shocks are applied on each legal entity, whose consequences can be summed to give the impact at group level.

2.4.1 Different kind of shocks and modeling

Depending on the designed scenario, shocks can be of different types and any mix of them is a priori relevant. Most shocks fall into the following categories:

- Loss related to an insurance claim
- Invested Asset price variation
- Underlying distribution modification

Shocks related to change in the underlying distribution will not be considered in this report. Shocks related to losses and assets price variation are summarized by the ensemble of vectors defined by equation 2.36:

$$S = \left\{L^U; Recov = L^U - L; \Delta B; \Delta E_q, \Delta E_r\right\} \tag{2.36}$$

with L^U the loss vector after internal retrocession, L the loss vector before internal retrocession, $Recov$ is the recoverable vector defined as the difference in losses before and after use of the internal retrocession,

assumption, the internal credit risk is the same within the group. Simplification to make calculation easier, can be improved later.

B Bond vector, ΔE_q the equity vector containing the variation of equity in percentage,

E_r external retrocession vector. The vector depend on the scenario granularity. Here we keep a simplified view.

2.4.2 Shocked elements on the balance sheet

Our model is a simplification of reality, therefore, only the following assets are shocked. More items could be added in later developments. The balance sheet's shocked items in Assets are :

- Investments : assets invested in different kind of securities :
  - Fixed income (Bonds, Mortgage Backed Securities, etc)
  - Equity (stocks of public listed companies) + Alternative investments (real estate, private equity, etc)
- Recoverable : amount of reinsurance retrocession owed to the company by its retrocessionaires.

Shocked items in Liabilities are :

- Non-Life Provisions : reserves associated to Non-Life liabilities.

TODO : EXPLAIN RELATIONS IN BALANCE SHEET
The updated capital vector is denoted C^U and computed as:

$$C^U = FI^U + E^U + Recov^U + OA^U - Res^U \tag{2.37}$$

Next sections explains how the components are modified by the application of shocks of several natures.

2.4.3 Effect of a loss on the balance sheet

The Reserves of a company are its views at a given time of its future liabilities. Methods used to compute reserves are prospective and based on actuarial principles. Just after an event leading to a loss, the existence of a liability become certain but not its amount. As claims are received by the company the final amount of the loss become more and more accurate. In the context of stress testing, the scenario is considered all-knowing and the final loss amount of known immediately after the event. In this vision, the reserves must be increased of the gross loss amount.

Increase of Reserves by the gross loss

When a loss occurs, the amount of reserves of the legal entity increases to account for these new liabilities to pay. The reserve vector is updated by the loss as:

$$Res^U = Res + L \tag{2.38}$$

Why adding the gross loss and not the net loss ?

The net loss is the loss that is ultimately bore by the company. It is the gross loss minus the recoverables (paid by other reinsurers). As recoverables are taken into account in the Asset side of the balance sheet, the gross loss must be added to the Liability side. In that way, the own fund variation is equal to the net loss if all recoverables are actually recovered.

Use of External retrocession and increase of recoverables

When a (re)company bears a loss and has not purchased any retrocession, the loss is totally impacted on the company and its reserves are increased at the level of the loss. If it purchased some retrocession, its retrocessionaires will pay their share of the loss. As this payment is not instantaneous, the reinsurance recoverable item is increased of the amount of retrocession that is payable to the company. When this payment is made, the recoverable item is converted into cash in the balance sheet. Reinsurance recoverables can be considered as receivables, but the likelihood of payment is less than receivables as the retrocessionaires might default. Moreover, receivables can be of numerous types (premiums, etc), whereas reinsurance recoverables only relate to reinsurance sums.

For example, if the company bears a \$ 50 M loss and its retrocession agreement states that 50% of this sum is payed by retrocessionaires, than the recoverable item in the balance sheet is increased of \$ 50 M $\times$ 50 % = \$ 25 M.

The following section present the impact of shocks on Assets and Liabilities. The Asset item shocked are investments and Recoverables. The liability item shocked are Non Life and Life Reserves.

$$Recov^U = Recov + (L - L^0) + (L^U - L) \tag{2.39}$$
$$= Recov + external_retro + internal_retro \tag{2.40}$$

TODO : CASE OF INTERNAL RETOCESSION

2.4.4 Shocking Fixed Income Investments (Bonds)

Source wikipedia and investopedia

Taylor development at order 2

$$P(r) = P(r_0) + P'(r_0)(r - r_0) + \frac{P^{(2)}(r_0)}{2}(r - r_0)^2 + R_n \tag{2.41}$$

Variational expression :

$$dP(r) = P'(r)dr + \frac{P^{(2)}(r)}{2}dr^2 \tag{2.42}$$

Using the classical definition of modified duration S and convexity C :

$$S = -\frac{P'(r)}{P(r)} \tag{2.43}$$

$$C = \frac{P^{(2)}(r)}{P(r)} \tag{2.44}$$

The quantities S and C are computed by external providers.

$$dP(r) = P(r)\left[-Sdr + \frac{C}{2}dr^2\right] + R_n \tag{2.45}$$

In practice, the remainder is neglected and the formula applied on finite rate variation :

$$\Delta P(r) = P(r)\left[-S\Delta r + \frac{C}{2}(\Delta r)^2\right] \tag{2.46}$$

EXPRESSION AVEC LE VECTEUR DE SHOCK

2.4.5 Shocking Equity

The Equity item on the balance sheet is the market value of the stocks held by the company. A first and simple approach consist in modifying the stocks market value by a given percentage defined in the scenario specification. For example, let's assume that a financial crisis leads to a decreases of -10% of CAC40 stocks value, then the market value of the stock held by the company is also decreased of -10%. Detailed calculations can be done considering that each company stock have a unique behavior. For example, in case of a pandemic, it is likely that stock price of airlines will decreases, and that the stock price of pharmaceutical companies will increase. The overall impact of such shock on the company equity investment must be computed considering the weight of each industry (even companies) has on the investment portfolio.

In practice, the updated equity component is computed as :

$$E^U = E \times \Delta E_q \tag{2.47}$$

2.5 How to shock the regulatory required capital ?

The following list presents and describe the risks components of risk factor based capital requirements we propose to consider:

- Reserve risk: The reserve risk depends on the amount of provision.
- Credit risk: Retrocession recoverable are increased as retrocessions are activated by a loss. The amount of recoverables is used in the credit risk module
- Fixed income investment risk : a loss of value of fixed income instruments
- Equity and other investments : a loss of value of equity

Most risk drivers presented in A.4.2 are elements of the balance sheet. The representation of the balance sheet presented in section A.3 is, according to us, at the right granularity to capture most of the risks involved in solvency capital model but still simple enough to allow easy interpretation and readability.

A general introduction on the computation of regulatory required capital was given in section 2.3.2. This section explains how to shock such regulatory required capital during a scenario stress test.

A general method to compute the regulatory required capital is presented.

Linear approximations allowing quick computation of regulatory required capital are presented.

We think that this approach is of interest for companies wanting to perform quick updates of their solvency ratio anytime during a year, while complete computations are generally done on a yearly or quarterly basis. Practical implementations are presented for the BMA BSCR.

2.5.1 Shocking a regulatory capital requirement - Exact formulation

Following the introduction on regulatory required capital computation presented in section 2.3.2, one interesting aspect of scenarios is to apply shock on the regulatory capital computation. Before the shock, the required capital will be denoted RC and after the shock RC^U (U standing for Updated).

After the shock, final risk drivers RD_i^U are modified but some (final) risk factors F_i^U can also be modified. Although not common, we will keep this dependence the the sake of generality.

These two required capitals can be written as :

$$RC = f^{\beta}(RD_1 \times F_1, ..., RD_i \times F_i, RD_N \times F_N) \tag{2.48}$$

and

$$RC^U = f^{\beta}(RD_1^U \times F_1^U, ..., RD_i^U \times F_i^U, RD_N^U \times F_N^U) \tag{2.49}$$

The difference in required capital after and before the shock is simply :

$$\Delta RC = RC^U - RC \tag{2.50}$$

For numerical computation, this formulation is accurate, exact and usable.

2.5.2 Shocking a regulatory capital requirement - Approximate formulation

But to understand the dynamics of the modification of the required capital during the shock, an approximate expression is desirable. Considering that even large losses will lead to relatively small sub-capital variations compared to the overall required capital, the diversification function 2.29 can be simplified using its Taylor expansion as :

TODO : JUSTIFY THIS ASSUMPTION

$$RC^U = RC + \frac{1}{RC} \sum_{i,j=1}^{N} \beta_{i,j} \left[C_i \Delta C_j + C_j \Delta C_i \right] \tag{2.51}$$

with $\Delta C_i = RD_i^U \times F_i^U - RD_i \times F_i$ the variation of risk i sub-capital.
Finally, the required capital variation is :

$$\Delta RC = \frac{1}{RC} \sum_{i,j=1}^{N} \beta_{i,j} \left[C_i \Delta C_j + C_j \Delta C_i \right] \tag{2.52}$$

In order to simplify the calculations, the cross-correlation terms $\beta_{i,j}$ where $i \neq j$ will be set to 0. The BMA BSCR contains some cross correlation terms but only concerning Life risks. As there risk are not be considered in the following calculation, there terms are neglected. Full expressions could be used in the case of numerical development of these relations.

2.5.3 Linear approximation of BMA BSCR

The diversification function of the BMA BSCR is :

$$BSCR = \sqrt{C_{fi}^2 + C_{eq}^2 + C_{cur}^2 + C_{conc}^2 + C_{intrate}^2 + C_{prem}^2 + (1/2C_{cred} + C_{rsrv})^2 + (1/2C_{cred})^2 + C_{cat}^2 + C_{life}^2} + C_{op} + C_{adj} \tag{2.53}$$

The linear approximation of this function is :
TODO : COMPUTE LINEAR APROXIMATION

2.5.4 Case study - Evolution of Required Capital in function of the use of external retrocession

In this part, the reinsurance group consist in a single company, incorporated in a country imposing the computation of a required capital through the use of a risk factor based formula.

This company hold a certain amount of capital and report a certain amount of required capital. We assume this company will experience a determined, certain loss of L millions $.

This company have to choices to mitigate this loss L. The first option is to bear the entire risk and pay the L millions. For this, this company needs to hold enough funds and this loss will directly translate into a loss of value of the equity of the shareholders. This will considered as the base case. Another possibility is to buy external (to the group) retrocession to another reinsurer, which will reduce the loss but also the profitability of the company. In real life, a company never knows when it will be impacted by a loss, otherwise it will buy retrocession to cover its loss and just pay a ”small” premium compared to the loss.

What happen from a required capital point of view. The first option translate in an increase of reserves before the claims are paid, which result in an increase of required capital for reserve risk. In the second option, as a part of the loss is transfered to the retrocessionaire, the reserve risk increases less. But this retrocessionaire may not pays it's share of the loss, which results in an increase of credit risk.

Which option is the best for the group ?

In the option 1, the additional reserves is L, the loss amount. The required sub-capital for reserve risk is then $L \times F^R$.

The shocked required capital is :

$$RC_1^U = RC + \frac{C_{reserve}}{RC} L \times F^R \tag{2.54}$$

The subscript 1 means "option 1".

In option 2, a share α of the loss if sent to an external retrocession and a share $(1 - \alpha)$ is kept in the group. The reserve risk sub-capital variation is then $(1 - \alpha)L \times F_{reserve}$. The retrocessionaire must pay αL to the group, creating a counterparty risk of $\alpha L \times F_{counterparty}$.

The shocked required capital is :

$$RC_2^U = RC + \frac{C_{reserve}}{RC}(1 - \alpha)L \times F^R + \frac{C_{counterparty}}{RC}\alpha L \times F^C \tag{2.55}$$

If no risk is ceded, corresponding to $\alpha = 0$, then $RC_1^U = RC_2^U$.

The variation of required capital associated to the transfer of risk is computed as :

$$RC_2^U - RC_1^U = \frac{\alpha L}{RC}(SC_{counterparty}F_{counterparty} - SC_{reserve}F_{reserve}) \tag{2.56}$$

The amplitude of this variation is proportional with the share of risk ceded α, but its sign depend only of the current risk profile of the company (its sub-capitals) and the regulatory environment (risk factors).

To reduce the capital requirement, this variation needs to be negative, which is the case if the following relation is met :

$$\frac{SC_{counterparty}}{SC_{reserve}} \leq \frac{F_{reserve}}{F_{counterparty}} \tag{2.57}$$

The interpretation of this relation is simple : a company can reduce its required capital by the use of external retrocession only if its current counterparty risk sub-capital over reserve risk sub-capital ratio is below a certain level imposed by the regulator formula. From a pure regulatory required capital point of view, the less counterparty risk a company has, the more interesting it is to transfer its risks.

BMA BSCR reserve risk

Contrary to the expression given before, the BMA BSCR computation uses a different definition of reserve risk and counterparty risk.

Considering the classic definition of counterparty risk sub-capital C^C and reserve risk sub-capital C^R, two derived sub-capitals are defined as :

$$\begin{cases} C_{cred,resv} = \frac{1}{2}C^C + C^R \\ C_{cred} = \frac{1}{2}C^C \end{cases}$$

A similar calculation to the previous one leads to the following condition of required capital with the use of retrocession :

$$\frac{C^C + C^R}{C^C + 2C^R} \leq \frac{F^R}{F^C} \tag{2.58}$$

In the definition of the BMA BSCR, the relation $F^R \geq F^C$ is always verified. This means that equation 2.58 is always true : purchasing external retrocession will always reduce the BSCR.

TODO : VERIFY CALCULATIONS

2.5.5 Case study : Effect of internal retrocession on required capital for a dual entity group

FIGURE 2 entities

In that case, the Group is composed of two independent entities, incorporated in the same country and using the same currency. At group level, the options in the case of a loss are the same than in section 2.5.4 : pay the loss or buy retrocession before.

At a legal entity level, another option is possible : transferring the risk from one entity to another. What are the effect of such transfer at group and legal entity level ?

The effect is first studied at legal entity level and the group effect is derived from the sum of the effects on legal entities.

The two entities are called 1 and 2. Entity 1 bears a loss L^1 and entity 2 a loss L^2. We assume that 2 transfer a share α of its risk to 1.

The variations of required capital of both entities are :

$$\begin{cases} \Delta RC^1 : \frac{SC^{R,1}}{RC^1} L^1 F_{reserve} + \frac{SC^{R,1}}{RC^1} \alpha L^2 F_{reserve} \\ \Delta RC^2 : \frac{SC^{R,2}}{RC^2} (1-\alpha) L^2 F_{reserve} + \frac{SC^{C,2}}{RC^2} \alpha L^2 F_{counterparty} \end{cases}$$

The group required capital level is :

$$\Delta RC^1 + \Delta RC^2 = \alpha L^2 \left[\frac{SC^{R,1}}{RC^1} F_R + \left(\frac{SC^{C,2}}{RC^2} F_C - \frac{SC^{R,2}}{RC^2} F_R \right) \right] + \left(\frac{SC^{R,1}}{RC^1} L^1 F_R + \frac{SC^{R,2}}{RC^2} L^2 F_R \right) \tag{2.59}$$

Results are similar to the case of one entity. Increasing the ceded share of risk α can decrease the overall required capital if and only if $L^2 \left[\frac{SC^{R,1}}{RC^1} F_R + \left(\frac{SC^{C,2}}{RC^2} F_C - \frac{SC^{R,2}}{RC^2} F_R \right) \right]$ is negative. For this, entity 1 must have low current reserves and entity 1 low current counterparty risk and high current reserve risk. One conclusion we can draw is that to reduce its required capital requirement, a company should consider modifying its current risk profile toward a more balanced one. If a company has a very biased risk profile, introducing internal retrocession can both increase or decrease its required capital.

But it can be beneficial to increase the group required capital if it allows to highly reduce the required capital a small entity. Such result can be desirable for a newly created subsidiary whose priority is growth. Taking the example of entity 2, introducing his internal retrocession allow to replace a share α of the reserve risk $\frac{SC^{R,2}}{RC^2} \alpha L^2 F_{reserve}$ with a counterparty risk $\frac{SC^{C,2}}{RC^2} \alpha L^2 F_{counterparty}$. Assuming that $SC^{R,2} F_R$ is greater than $SC^{C,2} F_C$, the entity 2 required capital is reduced.

2.5.6 Perspectives

Case of different regulatory environment

In that case, risk factors and diversification weights are modified.

Interesting case to consider further.

2.6 How to shock the solvency ratio ?

In previous section 2.4 and 2.5, we demonstrated how to shock the capital and capital requirement of a legal entity. The final step consist in reuniting these two aspects in the assessment of shock effect on the company solvency. Considering the update capital C^U and capital requirement vector RC^U, the updated solvency ratio is computed as:

$$SR^U = \frac{C^U}{RC^U} \tag{2.60}$$

We present results on several levels of approximation.

2.6.1 Case with only capital shocked

$$SR^U = \frac{C^U}{RC^U} = \frac{C + \Delta C}{RC + \Delta RC} \approx \frac{C + \Delta C}{RC} \tag{2.61}$$

2.6.2 Case with capital and required capital shocked

$$SR^U = \frac{C^U}{RC^U} = \frac{C + \Delta C}{RC + \Delta RC} \tag{2.62}$$

2.7 Model Wrap-up

2.8 Impact of scenarios on a reinsurance company solvency ratios

put everything together on realistic scenarios

The impact of several scenarios on the financial structure and the required capital of a artificial reinsurance company is studied. The impact of the financial statement was presented in section **??** and on the regulatory required capital in section **??**.

Scenarios proposed are :

- Natural catastrophes
- Financial crisis
- Pandemic
- A reserve stress test based on natural catastrophes and financial crisis

Description of the company tested

!!!!!!!!!!!!!!!!!! nb companies, financial structure, retrocession

The companies belonging to the Group are :

Group in Bermuda P and C in BERmuda P and C in Us Life and Health in US

Natural catastrophe scenario

Most reinsurers underwrite risk associated with natural catastrophes. The occurrence of natural catastrophes (hurricanes, earthquakes, floods, wildfires, etc) poses huge problems for insurer as a large part of their portfolio can be hit by such event. This is due to their local implantation that prevents them to diversify risk on a worldwide basis, and their reluctance to bear such extreme risk. In the context of

Return Period	Florida Hurricane	California Earthquake
1-500	0	0

Table 2.3: Events defined in the Natural catastrophes scenario.

Legal entity	1	2
Net Loss (of external retrocession)	0	0
Net Loss (of external and internal retrocession)	0	0

Table 2.4: Losses bore by legal entities, net of external retrocessions and net of external and internal retrocessions.

stress testing, an reinsurer must know its exposure by type of peril and peril zone. This is generally done by dedicated catastrophe modeling teams, relying on catastrophe modeling software such as RMS or AIR. A lot of reinsurer were created after years marked with strong natural catastrophes, such as hurricane Andrews, which led to the creation of PartnerRe.

The probability distribution of losses per peril type and zone is computed by simulating the impact of hundred of thousand of catastrophe scenarios on the company portfolio. The gross loss of a natural catastrophe is associated with an occurrence probability ie. a return period : the higher the return period, the higher the severity of the event. This information is useful when designing scenarios : with proper catastrophe modeling, the loss applied on the company is chosen according to a desired return period. Some scenarios might involve whether numerous small catastrophes or few very severe catastrophes. The knowledge of returns period allow to apply shocks whose likelihood is known.

The scenario chosen is the case of a hurricane hitting the Florida coast and an earthquake arising in California. The loss distribution of these events are displayed in :

In order for this scenario to have a meaningful impact, a return period of 1 in 250 years is chosen. As these two events can be considered as independent, their respective return period can be chosen as 1 in 15 years.

The gross losses of these two events are : 1-15 y GROSS LOSS

After usage of the company external retrocession treaties, the net loss are : NET LOSSES

The net loss is then spread through the company's structure through the use of its internal retrocession agreements.

This lead to the following repartition of losses in the legal entities :

The required capital are modified through the mechanisms explained in section

Legal entity	**Group**	**1**
Initial Required Capital	0	0
Final Required Capital	0	0

Table 2.5: Evolution of the required capital of Group and legal entities before and after the scenario impact.

Legal entity	**Group**	**1**
Initial Solvency Ratio	0	0
Final Solvency Ratio	0	0

Table 2.6: Evolution of Group and legal entities solvency ratio before and after the scenario impact.

??, leading to the following values :

Using informations from table2.8 and table 2.8, the solvency ratio of Group and legal entities is displayed in table 2.8 :

GRAPHE PERTES

Financial crisis scenario

A financial crisis is a very dangerous event for a reinsurance company. In fact, contrary to services or industrial companies who will suffer from a decrease of activity in the future, a reinsurance company will also have difficulties to pay its past liabilities. The financial crisis may decrease the value of the company's assets : there is less money to pay claims of contracts signed previously, leading to an increased risk of insolvency.

A number of financial crisis arose in the XXth and XXIth century : EXAMPLE CRISIS

Their characteristics are always different and they are very difficult to forecast.

A financial crisis has an impact on asset value, which can be considered as a loss, and also on the regulatory required capital, as asset risk are generally taken into account. Methods to shock assets are presented in section **??**. The time aspect is key to understand the scenario impact. In fact, even if the asset value decreases a lot at the peak of the crisis, they might stabilize at a value which is not too far from their original value. But financial statements and the computation of regulatory required capital are done at a given date. The worst case is when the peak of the crisis happen at the closing date, which is the case in the scenarios considered here.

Legal entity	**Group**	**1**
Fixed income losses	0	0
Equity Losses	0	0
Other investments losses	0	0

Table 2.7: Evolution of Group and legal entities assets before and after the scenario impact.

Legal entity	**Group**	**1**
Initial Solvency Ratio	0	0
Final Solvency Ratio	0	0

Table 2.8: Evolution of Group and legal entities solvency ratio before and after the financial crisis scenario.

PROVIDE VALUE WITH JUSTIFICATION

The losses associated with such crisis are the following :

As no risk transfer or mitigation mechanism exists in the case of asset losses, the asset allocation must be done carefully to ensure the solvency of all legal entities.

Solvency ratios before and after the financial crisis are displayed in :

Some impacts are neglected but could be taken into account in other work. For example, a financial crisis would probably reduce the premium volume collected. This can be the case for aviation insurance where the premium volume is proportional to the air traffic. The bankruptcy of a big customer can also lead to a loss of premium income.

Pandemic scenario

The word pandemic is used to describe the case of the propagation of a disease in the whole planet. Numerous pathologies can be classified as pandemic : influenza, AIDS, smallpox and even obesity. The most important aspect of a pandemic the fact that the disease, whatever it is, is spread everywhere on the planet.

For extreme scenario, only the case of influenza will be considered. The reasons for this choice are :

- The speed of propagation of this disease : the virus is easily transmitted through respiration or contact, unlike AIDS.
- The fact that it is liked to a pathogen agent and to not hygiene conditions

(like obesity which is linked with food intake and genetic).

- The fact that new stems of virus are active and make such event probable.
- The high infection rate but low fatality rate of such disease, which can lead to strong economic disruption but is unlikely to eradicate humanity.
- The strong literature dealing with this disease.

The characteristics of the chosen scenario are :

- Return period : 1 in 200 years
- Excess Mortality : 1 ‰ to 2.5 ‰ depending on the country
- Case Mortality (number of infected who die) : 3 %
- Number of hospitalized people over 1 death : 4
- W-shaped mortality curve in function of people age : specific to pandemic contrary to classic epidemic

It is assumed that the pandemic is worldwide and last 3 month.

Mortality rates are computed using the Swiss Re pandemic model allowing to compute mortality rates per countries and age of a pandemic close to the 2019 Spanish Flu pandemic.

Such a pandemic might have strong economic impacts. Schools will probably be close to prevent to propagation of the virus, leading to large absenteeism at work. Companies being crippled by this shortage of workers might reduce their activities and maybe go bankrupt. A decrease of demand might be experienced in specific sectors involving face to face contact like tourism, entertainment, or the gathering of people in the same place, like airlines, public transportations and shopping malls. All these elements might lead to a global recession.

The impact of such pandemic on several Life and Non-Life business lines is presented below. Naturally the impact of a pandemic on Non-Life business lines is difficult to estimate. Nevertheless, the reader will at least find a list of potentially impacted business lines and the drivers behind the impact.

Mortality The Mortality business line is the most obvious business line impacted by a pandemic. As the pandemic develops, people dies, which trigger death policy. A simple way of computing the impact of a mortality increase is to multiply the sum at risk, which is the sum that should be paid if all policyholders dies, with the excess mortality, which is the additional fraction of people dying because of the pandemic. This sum at risk can be spitted by country and age to refine the calculation.

Longevity As more people die because of the pandemic, less pensions are paid. This phenomenon hedges to mortality loss but full calculations of this effect are out of the scope of this work. No hedging will be considered in this work for sake of conservatism.

Health As people hospitalized, the cost of care might increases dramatically. But humans can only receive a limited quantity of care before dying. Moreover, hospitals might be saturated by the number of patient, therefore limiting the final amount of heathcare costs. Thus, there is a non linear effect where the total amount of healthcare cost saturate at a limit which is the limit of the healthcare system.

Credit & Surety As people try to avoid shopping malls during a pandemic, large retailers might experience losses and difficulties to pay their suppliers. These suppliers being insured by trade credit insurers against default of payment, it might lead to loss to reinsurers reinsuring trade credit insurers. a method to compute the impact of a pandemic on a trade credit portfolio is to use the 2008 financial crisis a reference case. It consists in comparing the loss ratio of the 2008 or 2009 year to the average of "normal" years and then identifying the contribution of the financial crisis in terms of additional loss ratio. This contribution is then applied on the current portfolio. Even if this method is indirect, it can be considered as conservative as a pandemic economic disruption might be less severe the the 2008 financial crisis.

Business Interruption The shortage or supply and demand might lead some businesses to stop activity, and then trigger business interruption covers. Estimating the impact of the pandemic on business interruption of difficult as cover triggers can be very different from one portfolio to another. One key aspect is that most business interruption cover are triggered by physical damage to the "company" (building etc). The pandemic is not considerer as a "physical damage" which leads us to think that most covers would not be triggered.

Marine During a pandemic, harbors might forbid ships to dock or leave, fearing to bear the responsibility of the propagation of the virus. Loss to reinsurer could arise if perishable cargo is destroyed for example.

Sport, Leisure & Entertainments A pandemic could lead top the canceling of world cup or concerts.

Reverse stress test

A reverse stress test if a stress test that leads to business failure. A definition of ”business failure” must be chosen to calibrate the reverse stress test. Here, several definition are proposed and the reverse stress test is considered a ”success” if at least one definition if fulfilled.

- Group Solvency ratio falls below 100%
- At least one Legal Entity solvency ratio falls below 100%
- The Group Capital rating falls below BBB

The reverse stress test proposed is a combination of natural catastrophes and a financial crisis, whose characteristics are provided below :

- Europe Windstorm
- California Earthquake
- Florida Hurricane
- Global Financial Crisis

Chapter 3

Inverse problem : How to compute the retrocession agreements to optimize the reinsurance group solvability ?

3.1 State of the art

Article Philipovic

3.2 Problem Statement

Goal: assuming a given ”sinistre”, what is the internal retrocession structure allows to optimize a given solvency criteria ?

The aim of this section is to provide an algorithm allowing to compute the optimal internal retrocession of a reinsurance group.

Entity : definition

variables of entity i : capital, capital requirement, any other variable

risk tranfer mechanism between entities

function fij(parameters of the contract, entities internal variables

3.3 Optimal Solvency criterion proposition

The optimality criterion is the

$$Cost = \sum \frac{1}{SR^i - 1} \tag{3.1}$$

or infinity if at least 1 SR is less than 1

theoretical results on risk transfers and cost associated with the distribution of risks in terms of required capital.

The aim of the inverse problem is to determine a set of parameters of the R matrix (the internal retrocession structure) allowing to maximize the "health" of the insurance group. This optimization is done knowing the losses amounts and entities capitals and required capitals.

If the loss function is the sum of all entities capital, then the formulation is use less as it doesn't depend on the matrix

$$SR^U = \frac{C^U}{RC^U} = \frac{C - L^U}{RC^U} = \frac{C - T[L]}{RC^I + \Delta RC[L^U]} \tag{3.2}$$

Case where the loss function is the sum of solvency ratios

assumption, required capital is not shocked

$$L = SR_1 + SR_2 = SR_1^I + SR_2^I - \frac{L_1^U}{RC_1^0} - \frac{L_2^U}{RC_2^0} \tag{3.3}$$

The aim is to minimize the following quantity

$$L = \frac{L_1(1-\alpha_{2\leftarrow 1}) + L_2\alpha_{1\leftarrow 2}}{RC_1} + \frac{L_1\alpha_{2\leftarrow 1} + L_2(1-\alpha_{1\leftarrow 2})}{RC_2} \tag{3.4}$$

$$\begin{cases} \frac{\partial L}{\partial \alpha_{1\leftarrow 2}} = \ \ L_2(\frac{1}{RC_1} - \frac{1}{RC_2}) \\ \frac{\partial L}{\partial \alpha_{2\leftarrow 1}} = -L_1(\frac{1}{RC_1} - \frac{1}{RC_2}) \end{cases}$$

Depending on the sign of required capitals RC_1 a,d RC_2, two values correspond to a minimum of the loss function : (0 and 1) or (1 and 0)

this leads to total transfers for capital from the entity with the smaller RC to the one with the higher RC.

There is no equilibrium in SR and can lead to overloading the entity with the bigger RC.

What happen if delta RC is taken into account ? Whatr in 3D ? ND ?

For an arbitrary number of entitie:

problem can be expressed as linear programming solution is on the edges of the domain for proportional reinsurance, hypercube and alpha between 0 and 1 $http : //www.ens-lyon.fr/denif/data/concept_analyse_algo_x/2007/cours/cours4.pdf$ only valid if RC not shocked and for proportional reinsurance consequence : values of factors are either 0 or 1. Should use a weighting in the sum of SR and see what is the result.

Can we achieve better global solvency with a smarter loss function ?

Other aspect : what happen if RC is shocked ?

Case of harmonic mean of solvency ratios:

$$L = \frac{w_1}{SR_1} + \frac{w_2}{SR_2} \tag{3.5}$$

w_1 and w_2 are solvency ratio weights, the higher the value, the more importance gets the associated solvency ratio in in the optimization.

Such cost function will favor balanced solvency ratio, preventing full transfer of money observed with sum loss function.

PUT A GRAPH OF HARMONIC LOSS FUNCTION CHOWING VALUES

3.4 Optimal retrocession structure

on donne un nombre de filiales, elles ont chacunes un niveau de capital et un capital requirement à l'instant t on applique le choc (loss sur chacune de ces filiales)

on cherche par optimisation à déterminer la resereu et les taux de cession optimaux pour minimiser la hausse (voir faire baisser) le capital requirement

avantage d'un comportement linéaire du capital requirement : résolution probable avec une inversion de matrice. A etudier critère d'ptimisation : - capital requirement - solvency ratios

Contrainte sur le graphe : graphe orienté (retro unidirectionelle) acyclique directed acyclic graph

directed graph ssi lower triangular : je pense que oui

Des astuces et approximation de roll forward seront proposées afin de faciliter le calcul de l'impact de ces chocs sur les capital requirements. Leur précision sera étudiée afin d'en évaluer la pertinence.

Chapter 4

Conclusion

Appendix A

Appendix

A.1 Definitions

PML : Probable Maximum Loss equity investment, shareholder's equity Legal entity et branch Shareholder's Equity: Computed from the GAAP Balance Sheet Gross loss : loss before any use of retrocession Net loss : loss after use of external retrocession Internal net loss : loss after us of external and internal retrocession

A.2 Group Structure, Financial Information and regulatory requirements - a re-regarder

To define clear naming conventions, lets call the Artificial Reinsurance company the ARC Group. This Group is made of a mother company incorporated in Bermuda, called ARCG, and of several subsidiaries listed below :

- ARCBM : subsidiary incorporated in Bermuda
- ARCUS : subsidiary incorporated in the USA
- ARCSE : subsidiary incorporated in Europe
- ARCA : subsidiary incorporated in Singapore

These subsidiaries are fully owned by ARCG (ask about the status of PartnerRe Ltd, is it a holding ?, what are the relations with PRCL?). These companies are not branches of ARCG, each of them is a single stand alone company owned by ARCG.

A.3 Balance sheet representation

Each company part of ARC Group reports financial statements. The structure of their balance sheet is presented below.

A.4 NAIC Risk Based Capital (RBC)

The NAIC (National Association of Insurance Commissioners) is the US standard setting organization operating on US soil. The NAIC is not a regulatory agency but it authors and is responsible of the required capital formula provided blow. The NAIC required capital (Risk Based Capital or RBC) formula is similar to the BMA one, taking the risks detailed in table A.4.2 into account and using the following diversification formula :

$$RBC = \sqrt{C_{fi}^2 + C_{eq}^2 + C_{cred}^2 + C_{rsrv})^2 + C_{prem}^2 + C_{cat}^2} + C_{subsidiary} \tag{A.1}$$

The NAIC adds a subsidiary risk to the risks presented in table A.4.2. This subsidiary risk is the required capital for holding participation in (re)insurance companies, and this capital charge is computed by the RBC formula applied on the subsidiary.

A.4.1 Credit Risk

The additional credit risk sub-capital ΔC_{cred} is computed by multiplying half the additional recoverables $\Delta Recov$ by a factor F_{cred} equals to 10%.

$$\Delta C_{cred} = \frac{1}{2}\Delta Recov \times F_{cred} = \Delta Recov \times 5\% \tag{A.2}$$

with $\Delta Recov$ the additional recoverables and F_{cred} the credit risk factor.

A.4.2 Reserve risk

The additional reserve risk sub-capital ΔC_{rsrv} is computed by multiplying the additional reserves ΔRes by a concentration factor F_{con} and an adjustment factor F_{adj} :

$$\Delta C_{rsrv} = \Delta Res \times F_{adj} \times F_{con} \tag{A.3}$$

The concentration factor value is 0.815. The adjustment factor is computed through a complex formula that can be found in the NAIC reference document REF NAIC DOC. For simplicity this factor will be set to 1 on the following studies.

Risk Name and Abbreviation	**BMA**	**NAIC**	**Risk Driver**
Fixed income (fi)	x	x	Volume of Fixed Income instrument
Equity (eq)	x	x	Volume of Equity
Interest rate (intrate)	x	0	
Currency (cur)	x	0	
Concentration (conc)	x	0	
Premium (prem)	x	x	Volume of Premium
Reserve (rsrv)	x	x	Volume of Non-Life Reserves
Credit / Counterparty (cred)	x	x	Volume of recoverables
Catastrophe (cat)	x	x	Probable Maximum Loss
Life	x	0	Volume of Life Reserves

Table A.1: List of risk and risk drivers for several solvency capital requirement.

A.5 Rating Agency capital requirement : S&P Risk based Insurance Capital Model

In order to operate on the reinsurance market, a company must prove to its customers that it is solvent and will be able to pay its reinsurance liabilities. But a good return on equity must be provided to shareholders to convince them in invest in the company rather than in another. Contrary to regulators, whose aim is principally the defense of customers, Rating Agencies rates insurance companies with their proprietary risk based capital model. The rating Agency point of view is balanced, as solvency cannot be separated from profitability. The rate applied to an insurance company is very important because it represents an external view on the company economic situation thaty is accessible to customers. Several elements of the company are taken into account to compute the rate. One of these elements is the strength of the capital, which is assigned a specific rate computed with a risk based capital model. Other elements play a role in the computation of the final rate, such as ownership of the company or the management team.

Insurance companies are generally rated by several rating agencies, whose several examples are given below :

- Standard & Poor's (S&P)
- Moody's
- Fitch
- AM Best

In this work, only the S&P risk based insurance capital model will be used.

In their view, the shareholder's equity of the company is retreated to get rid of XXXXXXX and YYYYYY. This lead to the Total Available Capital (TAC), which is the S&P view of the company wealth. In front of this TAC, several Target Capitals are computed using a factor based deterministic formula, using balance sheet elements as an input. For each rate AAA, AA, A and BBB, a Target Capital is computed; Obviously, the values of these Target Capitals is decreasing from AAA to BBB. A AAA company has a very strong TAC in regards of its risk profile. This is not the case of a BBB company. To be granted a certain rate, the TAC must be above the Target Capital associated to this rate.

A.6 Practical implementation on NAIC RBC

The diversification function of the NAIC RBC is :

$$RBC = \sqrt{C_{fi}^2 + C_{eq}^2 + C_{cred}^2 + C_{rsrv})^2 + C_{prem}^2 + C_{cat}^2} + C_{subsidiary} \tag{A.4}$$

The linear approximation of this function is :