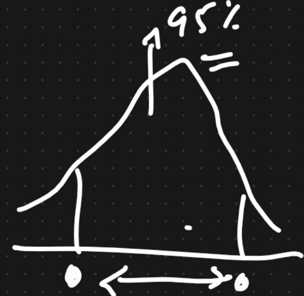
Hypothesis Testing

① Confidence Interval



$\alpha = 0.05$

Point Estimate ÷ The value of any statistic that estimate the value of a parameter is called a Point Estimate

Sample mean

$\boxed{\bar{x}} \rightarrow \boxed{\mu} \rightarrow$ parameter

$\approx$

Average height of the university 180cm

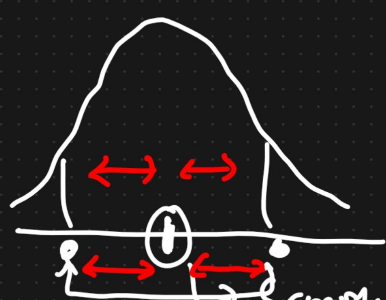
⇓

Sample data

⇓

CLASS ROOM

Confidence Interval



Sample Mean

[Point Estimate ± Margin of Error]

Derived $\rightarrow$

$$\boxed{\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}}$$

$\boxed{C \cdot I}$

when population Standard deviation is given

Z-score

n = sample size

σ = population std

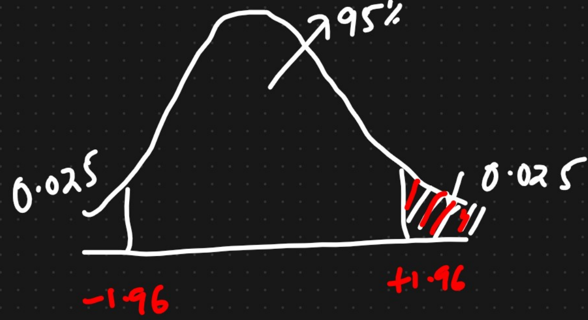
Significance value

95%

$Z_{0.05/2}$ $\quad Z_{0.025}$

$\alpha = 0.05$ 95%

1



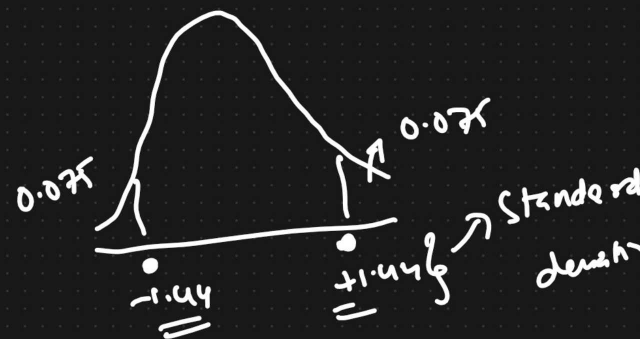
$1 - 0.025$
$= 0.975$

85%

Z table {Area under the body}

$1 - 0.025 = 0.975$

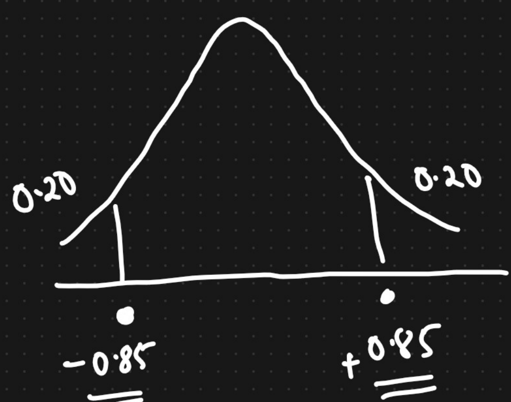
$1 - 0.075 = 0.925$



$C.I = 60\%$

$\alpha = 0.20$

$1 - 0.20 = 0.80$



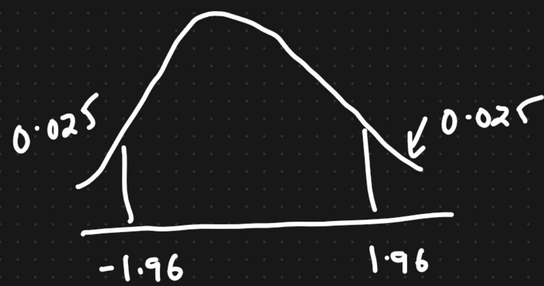
① On the verbal section of the CAT exam; the standard deviation is known to be 100. A sample of 25 test takers has a mean of 520. Construct a 95% C.I about the mean.

Ans). $\sigma = 100$ $\quad \bar{x} = 520$ $\quad n = 25$ $\quad \alpha = 0.05$

$$C.I = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

① Population σ

t test



$Z_{\alpha/2} \Rightarrow Z_{0.025}$

0.9750

$Z_{\alpha/2} = -1.96$ or 1.96

$$= \bar{x} + (1.96) \frac{100}{\sqrt{5}} = 559.2$$

95%

$$= \bar{x} - (1.96) \frac{100}{\sqrt{25}} = 480.8$$

$\rightarrow 520$

$\alpha = 0.05$ 95%

$1 - 0.95 = 0.05$

(F) On the verbal section of the CAT exam, a sample of 25 test takers has a mean of 520 with a standard deviation of 80. Construct a 95% C.I about the mean

Ans) $\bar{x} = 520$ $n = 25$ $S = 80$ $\sigma = 80$

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$ $\rightarrow$ Z-test $\rightarrow$ population sd.

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \Big\} \rightarrow t\text{-test} \Big\} \neq \text{Population sd}$$

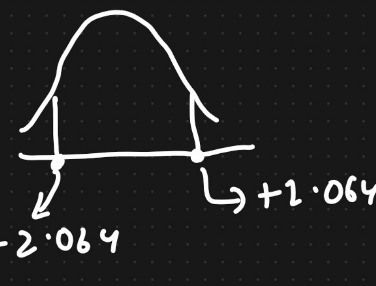
$\alpha = 0.05$ 95%

$$\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$\bar{x} + t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$\left\{ \begin{array}{l} t_{0.05/2} \Rightarrow t_{\underline{0.025}} \\ 1 - 0.025 = \boxed{0.975} \end{array} \right.$$

$df = n - 1 = 25 - 1 = 24$



Appl

$$\left. \begin{array}{l} 520 - 2.064 * \frac{80}{\sqrt{25}} = 486.978 \\ 520 + 2.064 * \frac{80}{\sqrt{25}} = 553.022 \end{array} \right\}$$

What is the CI of the size of the shark

$\bar{x}, \sigma, n$

① Hypothesis Testing

① population standard deviation

$\boxed{n \geq 30}$

Ⓐ { One sample Z-test }

Z-test { Measure or Compare Mean }

t-test { Measure or Compare Mean }

F-test ← ANOVA { Analysis of Variance }

CHI SQUARE { Measure or Compare between Categorical variable }

Popula $\left. \begin{array}{l} \\ n \geq 30 \end{array} \right\} \rightarrow$ Z test

$n \geq 30 \} \rightarrow$ F-test

① In the population, the average IQ is 100 with a standard deviation of 16. A team of scientists want to test a new medication to see whether it has a positive or negative effect on intelligence or no effect at all. A sample of 30 participants who have taken the medication has a mean IQ of 140. Did the medication affect intelligence?

C.I = 95%

① $\mu = 100$ $\sigma = 15$ $\bar{x} = 140$ $n = 30$

① Define Null Hypothesis

$H_0 \Rightarrow \mu = 100$

$H_1 \Rightarrow \mu \neq 100$ ✓

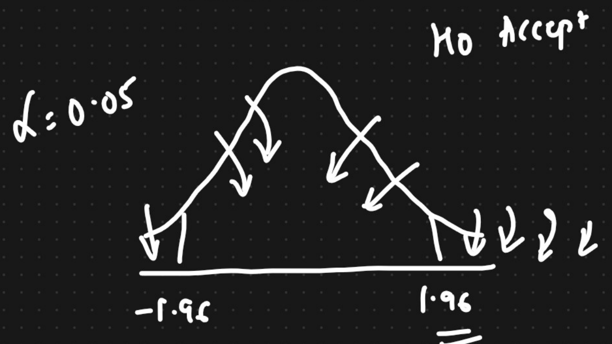
Accepted

② State Alpha

$\alpha = 0.05$

③ State Decision Rule

If Z is less than -1.96 or greater than 1.96, reject null hypothesis



$\boxed{-10}$

④ Calculate Z Test Statistic

↑

Does medication affect intelligence??

$$\Rightarrow \boxed{Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}$$

$$Z = \frac{140 - 100}{\frac{15}{\sqrt{30}}} = \frac{40}{2.74} = \boxed{14.60}$$

Decision Rule: Since $Z = 14.60$ is greater than 1.96 Reject the null hypothesis, Accept H1 → It affected intelligence

② C.I = 85% C.I = 75% {Homework} in the way

One Sample t-test

① In the population, the average IQ is 100. A team of scientists wants to test a new medication to see if it has a either +ve or -ve or no effect at all. A sample of 30 participants who have taken the medication has a mean of 140 with a standard deviation of 20. Did the medication affect intelligence?

$\alpha = 0.05$

95% C.I

Ans) $\bar{x} = 140$ $s = 20$ $\mu = 100$ $n = 30$

① Define H_0 & H_1

$H_0 \Rightarrow \mu = 100$

$H_1 \Rightarrow \mu \neq 100$

② $\alpha = 0.05$

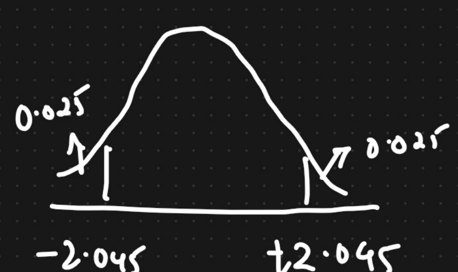
③ Degree of freedom

$df = n-1 = 30-1 = 29$

④ State Decision Rule

$1 - 0.025$

$= 0.9750$



If t is less than -2.045 or greater than 2.045, reject the null hypothesis

⑤ t test statistics

$\boxed{99}.6$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{140 - 100}{\frac{20}{\sqrt{30}}} = \frac{40}{3.65} = 10.96$$

⑥ ∵ 10·96 is greater than 2·045, reject the null hypothesis

Accept Alternate Hypothesis → +ve impact.

③ Chi-Square Test

Chi Square test tests claims about population proportions.

It is a non parametric test that is performed on categorical (nominal or ordinal) data.

Eg: In 2000 U.S Census, the ages of individuals in a small town were found to be the following

<18	18-35	>35
20%	30%	50%

In 2010, ages of n=500 individuals were sampled. Below are the results

<18	18-35	>35
121	288	91

Using $\alpha = 0.05$, would you conclude that the population distribution of ages has changed in the last 10 years?

Ans)

	Less than 18	18-35	>35
Expected	20%	30%	50%

n=500

	<18	18-35	>35
Observed	121	288	91
Expected	100	150	250

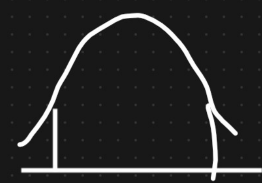
800 X 20%

① Define Null and Alternate Hyp

② $\alpha = 0.05$

$H_0 \rightarrow$ The data meet the expected distribution

$H_1 \rightarrow$ The data do not meet the " "



5.99

③ Calculate degree of freedom

$df = K-1 = 3-1 = 2$

④ Decision Rule

χ^2 is greater than $\boxed{5.99}$,

reject H_0

Accept H_1

⑤ Test Statistics

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = \left[\frac{(121-100)^2}{100} + \frac{(288-150)^2}{150} + \frac{(91-250)^2}{250} \right]$$

$$\chi^2 = 232.494$$

$\chi^2 > 5.99$

Reject H_0

Accept H_1

ANOVA

F-test

↳ Homework

↳ Saturday

1 hour

1. 2 proportion Z test
2. ANOVA
3. Practical python code

↳ Binomial Distr
Pareto Distrib

Permutation & Combination } 5 min