Algorithms for Programming Contests SS20 - Week 12

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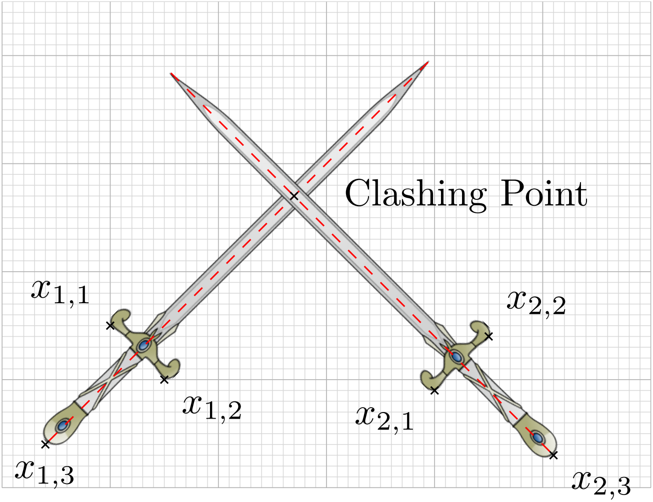
A: Swordfighting - Sample Solution

- Given three points specifying one sword:
- Construct the blade
  - Connect both points on the hilt $((x_{11}, y_{11})$ and $(x_{12}, y_{12}))$.
  - Construct orthogonal line through (x_{13}, y_{13}).
- Do this for both swords, find clashing point by intersecting both blades.
- Ensure that the clashing point is on the correct side of both hilt lines.

A: Swordfighting - Sample Solution



B: Euler Line - Sample Solution

- Homogenous coordinates of vertices a, b, c
- Centroid: normalize,

$$\left(\frac{1}{2}(a+b) \times c\right) \times \left(\frac{1}{2}(b+c) \times a\right)$$

- Ortocenter:

$$\left((a \times b \times e_3)^{\perp} \times c\right) \times \left((b \times c \times e_3)^{\perp} \times a\right)$$

- Circumcenter: normalize,

$$\left((a \times b \times e_3)^{\perp} \times \frac{1}{2}(a+b)\right) \times \left((b \times c \times e_3)^{\perp} \times \frac{1}{2}(b+c)\right)$$

C: Fallingwater - Sample Solution

- General idea: follow the water flow downwards, recurse if it splits.
- For a starting point, use the line straight down (intersect (x, y) with $(x, 0)$) and intersect it with all line segments.
- Find the first line segment hit.
- Depending on the slope, continue from left or right or both end points of the line segment.
- Take care when recursing: There may be many splits which later on come together again.
- To avoid exponential running time, process every line segment once. If you hit it again, stop the current recursion branch.

D: Family Pictures - Sample Solution

Problem

Given the projection of an image, restore the original image to determine relative height difference between two points.

Solution

- Projection preserves lines, is a projective transformation.
- Uniquely determined by preimage and image of a, b, c, d.
- Compute the projective transformation matrix M mapping point a to $(0, 0)$, b to $(1, 0)$, c to $(1, 1)$ and d to $(0, 1)$.
- Compute $e' = M \cdot e$ and $f' = M \cdot f$.
- Normalize e' and f' and compute height as e'_y / f'_y.

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E: Fractals - Sample Solution

- Use productions of the letters recursively.
- Afterwards, just simulate Lea walking the path.
- Translating $(0,0)$ to (x,y) in projective geometry: Multiply points with

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$$

- Rotating in projective geometry:

$$\begin{pmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Play around with GNUplot, it's worth it!