Algorithms for Programming Contests SS20 - Week 07

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A: Street Lights - Sample Solution

Problem

How many street lights do you have to switch on to illuminate the whole street?

A: Street Lights - Sample Solution

- Sort positions of the street lights
- Starting from 0, go right to the last street light that still illuminates that spot
- Switch on that street light
- Repeat, starting from that position
- Afterwards, check if the last street light illuminates the end of the street

B: Soup Delivery - Sample Solution

Problem

- Given facility costs
- Given delivery costs from each facility to each customer
- Choose which facilities to use
- Supply all the customers
- Approximately (up to $4\times$) minimise cost

B: Soup Delivery - Sample Solution

Solution

- Local search
- Pick some valid strategy
  e.g. a single facility with minimal delivery costs
- While (non-negligible) improvement exists, apply it

- Add facility: each customer considers new facility
- Remove facility: all *its* customers choose new facility
  Make sure some facility is always selected
- Move facility: add+remove

B: Soup Delivery - Sample Solution

Solution: optimisations

- Addition cheaper than removal
  find the best addition and just use it (if exists)
- Facility priority queue for each customer
  better of worse, depending on the instance

C: Customs - Sample Solution

Problem

How can you divide up a country so that you get the maximum amount of tolls?

- $\frac{1}{2}$-approximation accepted

C: Customs - Sample Solution

Local search

- Max-Cut-Problem, can be approximated via greedy solution.
- Given any partition (A, B) of cities, repeatedly choose a city c, where more edges go to cities in the same partition than in the other partition and move it.
- Repeat until no such node exists.
- Runtime: Every step improves solution by at least 1. Solution is at most $|E|$.
- Correctness: at least half of all edges are between the countries.

C: Customs - Sample Solution

Directly greedy approach

- Keep assigned cities for two parts, and queue
- For each city, consider it edges to assigned cities
- Assign the city to the side it is less connected
- Each edge is considered once
- Most considered edges are cut
- We also check each edge once without considering, runtime $O(|V| + |E|)$

D: Woodchucking - Sample Solution

Problem

Which saws should approximately saw which block of wood?

D: Woodchucking - Sample Solution

- Approximation algorithm
- Use a greedy algorithm
- Sort lengths in decreasing order
- Always use the saw that will finish first
- $\frac{4}{3}$-approximation
- Needed here: $\frac{3}{2}$-approximation
- Care for integer overflows

E: Dolphins - Sample Solution

Problem

How close can you make the DNA strands of dolphins and mice?[1]

[1] This is real science.

E: Dolphins - Sample Solution

- For each pair (x, y) of letters, count how often it will be scored. Count (x, y) and (y, x) as the same pair.
- Assuming you put 0 on all diagonals and -10 everywhere else, the sum would be -120. Thus, starting from there, you have to distribute exactly 120 across the matrix (this is your budget).
- Distribute the budget greedily setting entries to 10. If you set an entry on the diagonal to 10, you spend 10, otherwise you spend 40 (because you need to also increase the symmetric entry!).
- Care about special cases! (see next slide)
- Compute the score.

E: Dolphins - Sample Solution

- Be careful with the last entry you set, you might not have enough budget to set it to 10.
- You only pay once for a diagonal but twice for a non-diagonal. Thus you get twice as much for the same investment. This has to be reflected in your choice when greeding.