Algorithms for Programming Contests SS20 - Week 10

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A: Meteorite - Sample Solution

Problem

Check if given point is in polygon

Solution

Use, for example, raycasting algorithm

B: Fence Posts - Sample Solution

- Fence posts can be removed if they are not vertices of the convex hull.
- Use any convex hull algorithm and output those indices that belong to the vertices.

C: Surveillance - Sample Solution

- Let let R be the radius of the first camera and let r be the radius of the other cameras.
- We need to compute $(n-1)\pi r^2 + \pi R^2$.
- R is the minimum distance of the first camera to another camera minus r.
- r is between 0 and the minimum of
  - the minimum distance between cameras with the same radius divided by 2 and
  - the minimum distance between the first camera and another camera.
- Insight: This function is convex, so the maximum is achieved when r is one of the boundaries mentioned above.
- Check both, take the better one.

D: Azrieli - Sample Solution

- Given a set of $7 \leq n \leq 10$ points in a two-dimensional plane, decide whether a rectangle and triangle can be placed w.r.t. these points, such that the rectangle and triangle do not overlap or are enclosed in one another.

D: Azrieli - Sample Solution

- There are up to $n! \leq 10! \leq 4 \cdot 10^6$ possibilities to place a rectangle and triangle w.r.t. n points. Iterate through all of them:

$$\underbrace{x_0\ x_1\ x_2\ x_3}_{\text{rectangle}}\ \underbrace{x_4\ x_5\ x_6}_{\text{triangle}}\ \underbrace{x_7\ x_8\ x_9}_{\text{unused}}$$

- Check if first four points form a rectangle (check with scalar product whether there are 3 right angles)
- Check for valid assignment:
  - Use CCW method to check if line segments of triangle and rectangle overlap.
  - Use raycasting to check whether triangle/rectangle are fully enclosed within the other.
- Only checking one of both conditions is NOT sufficient, see examples!

D: Azrieli - Sample Solution

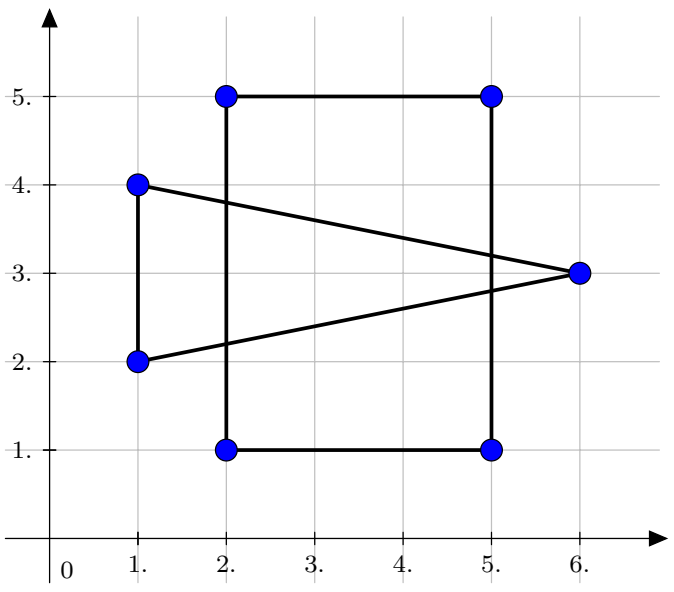

Figure: Raycasting fails.

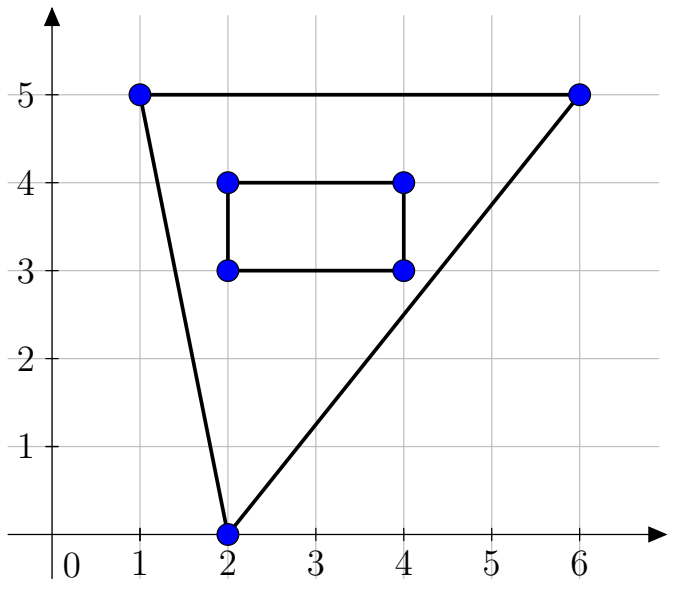

Figure: CCW fails.

E: Fragile Letters - Sample Solution

Problem

For a given polygon, compute the number of stable sides.

E: Fragile Letters - Sample Solution

- Do not rotate the polygon
- Consider every edge of the polygon as “floor”.
- An edge is *stable* iff
  - all other vertices lie on the same side and
  - the line perpendicular to the edge through the center of mass lies on the edge

E: Fragile Letters - Sample Solution

- For each edge of the letter check whether all non-incident vertices are on the same side.
- To do so, represent the edges as $y = m \cdot x + n$ or use the *CCW* function.
- Center of mass:

$$c_x = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$
$$c_y = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

- Check whether the projection of the center of mass to the line is on the edge.

F: Pathing - Sample Solution

- Key idea: The optimal path always moves in straight lines between corners of impassable locations or the start/target point.
- Build a graph with
  - nodes: corners of each impassable location as well as the start and target.
  - edges: if it is possible to move in a straight line between two nodes.
- Segment-hits-box needs care
  - Quick check: box and segment separated by the box edge
    no multiplications!
  - Weird edge case (in some approaches): diagonal cut only hits ends
- Do Dijkstra's Algorithm.