AUDITORNA (VEKT PROSTORI):

1. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad W = \{ X \in M_2(\mathbb{R}) \mid AX = XA \}$

DOKAŽITE DA JE W POTPROSTOR OD $M_2(\mathbb{R})$ I ODREDITE MU DIMENZIJU.

KRITERIJ POTPROSTORA:
NEKA JE V VEKTORSKI PROSTOR NAD POLJEM $\mathbb{F}$, I NEKA JE $M \subseteq V$
TADA JE M POTPROSTOR OD V
AKO I SAMO AKO
$(\forall x, y \in M)(\forall \alpha, \beta \in \mathbb{F})\ \alpha x + \beta y \in M$

NEKA SU $X, Y \in W$ I $\alpha, \beta \in \mathbb{R}$ PROIZVOLJNI

$$\left[\begin{array}{l} \alpha X + \beta Y \overset{?}{\in} W \\ \iff AZ \overset{?}{=} ZA \end{array} \right]$$

RAČUNAMO,

$$A(\alpha X + \beta Y) = \alpha \underbrace{AX}_{= XA} + \beta \underbrace{AY}_{= YA} = (\alpha X + \beta Y)A$$

$\Rightarrow \alpha X + \beta Y \in W \Rightarrow W$ POTPROSTOR OD $M_2(\mathbb{R})$

NEKA JE $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in W$ PROIZVOLJNA

$X \in W \Rightarrow AX = XA$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix} = \begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix}$$

$\Rightarrow c = 0,\ a = d$

DAKLE, SKUP W ČINE SVE MATRICE OBLIKA:

$$X = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$= \underbrace{a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\text{RAZAPINJU PROSTOR OD } W}, \quad a, b \in \mathbb{R}$$

$$\Rightarrow W = L\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ; \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right)$$

SKUP $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ JE LINEARNO NEZAVISAN

U $M_2(\mathbb{R})$ (DZ.) PA JE TAJ SKUP BAZA

ZA W.

$\Rightarrow \dim W = 2$

(3) DOKAŽITE DA JE $V = \{ p \in P_4 \mid p'(1) = 0 \}$ POTPROSTOR VEKTORSKOG PROSTORA P_4 TE MU NAĐITE NEKU BAZU I DIMENZIJU. NADOPUNITE BAZU DO BAZE VEKT. PROSTORA P_4.

NEKA SU $p, g \in V$ i $\alpha, \beta \in \mathbb{R}$ PROIZVOLJNI

$$[\, \alpha p + \beta g \in V \Leftrightarrow (\alpha p + \beta g)'(1) \overset{?}{=} 0 \,]$$

RAČUNAMO

$$(\alpha p + \beta g)'(1) = \alpha \underbrace{p'(1)}_{=0} + \beta \underbrace{g'(1)}_{=0} = 0$$

$$\Rightarrow \alpha p + \beta g \in V \Rightarrow V \text{ POTPROSTOR OD } P_4$$

NEKA JE $p(t) = at^4 + bt^3 + ct^2 + dt + e \in V$ PROIZVOLJAN

$p \in V \Rightarrow p'(1) = 0$

$\Rightarrow 4a + 3b + 2c + d = 0$

$\Rightarrow d = -4a - 3b - 2c$

DAKLE, V ČINE SVI POLINOMI OBLIKA,

$$p(t) = at^4 + bt^3 + ct^2 + (-4a-3b-2c)t + e$$
$$= a(t^4-4t) + b(t^3-3t) + c(t^2-2t) + e \cdot 1$$
$$\Rightarrow V = L(t^4-4t, t^3-3t, t^2-2t, 1)$$

SKUP $\{t^4-4t, t^3-3t, t^2-2t, 1\}$ JE LIN NEZAVISAN U $\mathcal{P}_4$ (DZ) PA JE TO BAZA ZA V.

$\rightarrow \dim V = 4$

$\dim P_4 = 5$

PROMOTRIMO UNIJU DOBIVENE BAZE S KANONSKOM BAZOM ZA $\mathcal{P}_4$ $\{1, t, t^2, t^3, t^4\}$

1 ✗

t ✓ REDUKCIJA JE ZAVRŠENA JER TREBA NADOPUNITI S SAMO JEDNIM ELEMENTOM

SKUP $\{t^4-4t, t^3-3t, t^2-2t, 1, t\}$ JE LIN NEZAVISAN U $\mathcal{P}_4$ (DZ) PA JE TO (JEDNA) NADOPUNA DO BAZE ZA $\mathcal{P}_4$

A injekcija $\Longleftrightarrow d(A)=0$ (defekt) $\quad A: V \to W \quad \boxed{r(A)+d(A)=\dim V}$

VJEŽBA (LIN. OPERATORI):

① $A: \mathbb{R}^3 \to \mathbb{R}^4$

$$A(x_1, x_2, x_3) = (x_1 - x_2 + x_3,\ 2x_1 - x_3,\ x_1 - 5x_2 + 2x_3,\ 3x_1 - x_2)$$

1> DOKAŽITE DA JE A LIN. OPERATOR

[PROVJERI $A(\alpha x + \beta y) = \alpha A x + \beta A y$]

NEKA SU $\alpha, \beta \in \mathbb{R}$ TE $x = (x_1, x_2, x_3)$ i $y = (y_1, y_2, y_3) \in \mathbb{R}^3$ PROIZVOLJNI

PREMA DEFINICIJI RAČUNAMO

$$A(\alpha x + \beta y) = A(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \alpha x_3 + \beta y_3) =$$

$$= ((\alpha x_1 + \beta y_1) - (\alpha x_2 + \beta y_2) + (\alpha x_3 + \beta y_3),\ 2(\alpha x_1 + \beta y_1) - (\alpha x_3 + \beta y_3),$$

$$(\alpha x_1 + \beta y_1) - 5(\alpha x_2 + \beta y_2) + 2(\alpha x_3 + \beta y_3),\ 3(\alpha x_1 + \beta y_1) - (\alpha x_2 + \beta y_2)) =$$

$$= \alpha(x_1 - x_2 + x_3,\ 2x_1 - x_3,\ x_1 - 5x_2 + 2x_3,\ 3x_1 - x_2) +$$

$$\beta\underbrace{(y_1 - y_2 + y_3,\ 2y_1 - y_3,\ y_1 - 5y_2 + 2y_3,\ 3y_1 - y_2)}_{A(y_1, y_2, y_3)} =$$

$= \alpha A(x) + \beta A(y) \Rightarrow A$ JE LIN. OPERATOR

2> $\operatorname{Ker} A = ?$

$(0,0,0,0) \in \mathbb{R}^4$

$$\operatorname{Ker} A = \{x \in \mathbb{R}^3 : A(x) = 0\}$$

$$= \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 - 2x_2 + x_3 = 0,\ 2x_1 - x_3 = 0,\ x_1 - 5x_2 + 2x_3 = 0,\ 3x_1 - x_2 = 0\}$$

$= \{(0,0,0)\} \Rightarrow d(A) = \dim(\operatorname{Ker}(A)) = 0$

DEFEKT

$\Rightarrow r(A) = \dim \mathbb{R}^3 - d(A) = 3 - 0 = 3$

TM O RANGU I DEFEKTU

b) BAZA ZA $\mathrm{Im}A$

SKUP $\mathrm{Im}A$ ČINE SVI VEKTORI IZ $\mathbb{R}^4$ OBLIKA

$(x_1 - x_2 + x_3, \ 2x_1 - x_3, \ x_1 - 5x_2 + 2x_3, \ 3x_1 - x_2) =$

$= (x_1, 2x_1, x_1, 3x_1) + (-x_2, 0, -5x_2, -x_2) +$

$(x_3, -x_3, 2x_3, 0) = x_1(1,2,1,3) + x_2(-1,0,-5,-1) +$

$x_3(1,-1,2,0) \quad , x_1, x_2, x_3 \in \mathbb{R}$

$\Rightarrow \mathrm{Im}A = L\big((1,2,1,3), (-1,0,-5,-1), (1,-1,2,0)\big)$

ZBOG LIN. NEZAVISNOSTI (DZ) GORNJI SKUP JE I BAZA ZA $\mathrm{Im}A$.

(2) ODREDITE MATRICU OPERATORA $T: \mathbb{R}^2 \to \mathbb{R}^3$,

$T(x,y) = (x+y, \ x-y, \ x)$ U PARU KANONSKIH BAZA

$(e) = \{\overset{e_1}{(1,0)}, \overset{e_2}{(0,1)}\}$

$(f) = \{\overset{f_1}{(1,0,0)}, \overset{f_2}{(0,1,0)}, \overset{f_3}{(0,0,1)}\}$

$$T(f,e) = \begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array} \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} \in M_{32}$$

Te_2

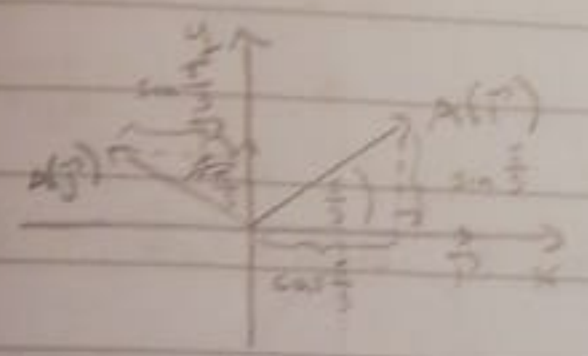
$Te_1 = (1,1,1) = \underline{\underline{1}} \cdot f_1 + \underline{\underline{1}} \cdot f_2 + \underline{\underline{1}}\, f_3$

AUDITORNA VJEŽBA (9.1)

(LINEARNI OPERATORI U RAVNINI I PROSTORU)

1

a)



ROTACIJA U POZITIVNOM SMJERU (SUPROTNO OD KAZALJKE NA SATU)

$$A(\vec{i}) = \left(\cos\frac{\pi}{3}\right)\vec{i} + \left(\sin\frac{\pi}{3}\right)\vec{j} = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$A(\vec{j}) = \underbrace{\left(-\sin\frac{\pi}{3}\right)\vec{i} + \left(\cos\frac{\pi}{3}\right)\vec{j}} = -\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}$$

(b)

$$A(e) = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

(c)

$$(A(e))^T = \begin{bmatrix} \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix} = \begin{bmatrix} \cos\left(-\frac{\pi}{3}\right) & -\sin\left(-\frac{\pi}{3}\right) \\ \sin\left(-\frac{\pi}{3}\right) & \cos\left(-\frac{\pi}{3}\right) \end{bmatrix}$$

ROTACIJA ZA $-\frac{\pi}{3}$ //

$$= (A^{-1})(e) = (A(e))^{-1}$$

$A(e)^T$ JE MATRICA ROTACIJE OKO ISHODIŠTA ZA $-\frac{\pi}{3}$

I VRIJEDI

$$A(e)^T = (A(e))^{-1}$$

[$A(e)$ JE ORTOGONALNA MATRICA]

(d)

$\boxed{A \text{ REGULARAN (BIJEKCIJA)} \iff A(e) \text{ REGULARNA MATRICA}}$

$$\det(A(e)) = \begin{vmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} + \frac{3}{4} = 1 \neq 0 \; //$$

$\Rightarrow A$ REGULARAN OPERATOR

$$\boxed{r(A) = r(A(e))}$$

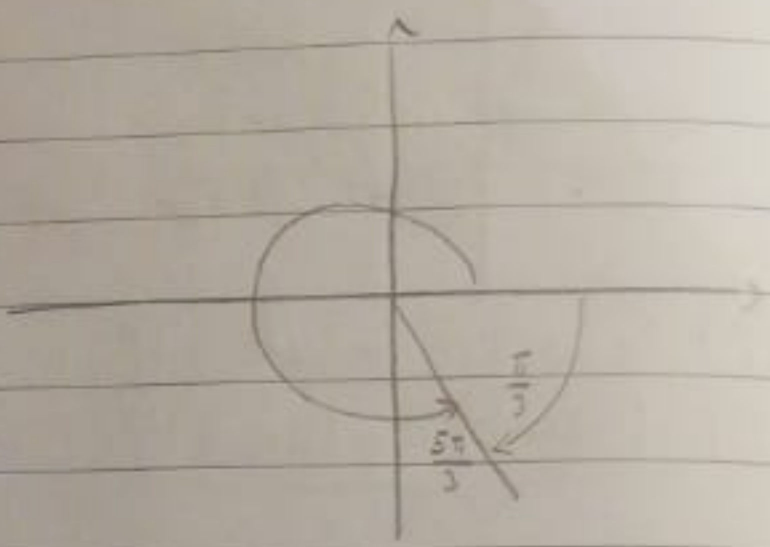
AKO SU LIN. OPERATORU DOMENA I KODOMENA ISTE DIMENZIJE $\longrightarrow$ INJEKTIVNOST $\Leftrightarrow$ SURJEKTIVNOST $\Leftrightarrow$ BIJEKTIVNOST

(e) $A^{-1} = A^5$

$A^6 = I_{V^2} \quad / \circ A^{-1}$

ROTACIJA za $6 \cdot \frac{\pi}{3} = 2\pi$

$\Rightarrow A^5 = A^{-1}$

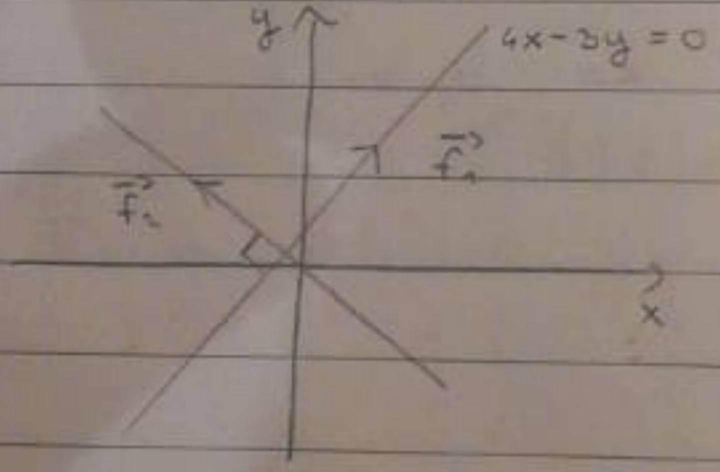


(2)

$B : V^2 \to V^2$ OPERATOR ZRCALJENJA

$p \ldots 4x - 3y = 0$

(a)



BIRAMO BAZU (f):

$\vec{f_1} = \frac{1}{5}(3\vec{i} + 4\vec{j})$

$\vec{f_2} = \frac{1}{5}(-4\vec{i} + 3\vec{j})$

$B(\vec{f_1}) = \vec{f_1}$

$B(\vec{f_2}) = -\vec{f_2}$ $\Rightarrow B(f) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\underbrace{I_{V^2}}_{\text{KODOMENA}} \overbrace{(e, f)}^{T} = \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$

STARA BAZA, NOVA BAZA, DOMENA

$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1$

$\Rightarrow \exists \varphi \in \mathbb{R}$ td $\cos\varphi = \frac{3}{5}$, $\sin\varphi = \frac{4}{5}$

$\Rightarrow I_{V^2}(e, f)$ JE MATRICA ROTACIJE pa $I_{V^2}(e, f)^T = I_{V^2}(e, f)^{-1}$

(a) $5x - 3y = 0$

$x = 1 \Rightarrow y = \frac{5}{3}$ $\qquad$ $x = 3 \Rightarrow y = 5$

$\vec{s} = \vec{i} + \frac{5}{3}\vec{j}$ $\qquad$ $\vec{s} = 3\vec{i} + 5\vec{j}$

VEKTOR SMJERA

PRAVAC I RAVNINA U PROSTORU!

(c) $B(e) = I_{V^2}(e,f)\, B(f) \cdot \underbrace{I_{V^2}(f,e)}_{I_{V^2}(e,f)^{-1} = I_{V^2}(e,f)^T}$

$= \frac{1}{25}\begin{bmatrix} -7 & 25 \\ 25 & 7 \end{bmatrix}$

$\Downarrow$

VRIJEDI OPĆENITO!

(d)

$(B(\vec{i}+\vec{j}))(e) = B(e)\cdot(\vec{i}+\vec{j})(e)$

$= \frac{1}{25}\begin{bmatrix} -7 & 25 \\ 25 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 17 \\ 31 \end{bmatrix}$

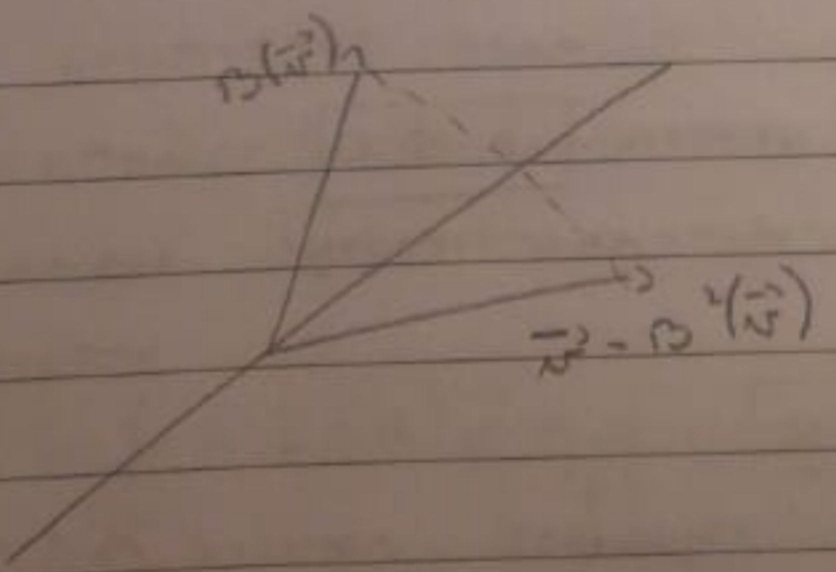
$\Rightarrow B(\vec{i}+\vec{j}) = \frac{17}{25}\vec{i} + \frac{31}{25}\vec{j}$

(e) $r(B(f)) = 2 \Rightarrow B$ REGULARAN

(f) $B \circ B = I_{V^2}$

$(B \circ B)(f) = B(f)\cdot B(f) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= (I_{V^2})(f) \Rightarrow B \circ B = I_{V^2}$



AUDITORNA VJEŽBA:

SVOJSTVENE VRIJEDNOSTI, DIJAGONALIZACIJA

(1)

$Z_n := \#$ ZEČEVA NAKON n GODINA

$L_n := \#$ LASICA NAKON n GODINA

$$\begin{cases} Z_{n+1} = 4Z_n - 2L_n \\ L_{n+1} = Z_n + L_n \end{cases}$$

SUSTAV 2 LINEARNE REKURZIJE PRVOG REDA

$$X_n := \begin{bmatrix} Z_n \\ L_n \end{bmatrix} \Rightarrow X_{n+1} = \underbrace{\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}}_{=A} X_n$$

$$\Rightarrow X_n = AX_{n-1} = A^2 X_{n-2} = A^3 X_{n-3} = \dots = A^n X_0$$

$$= A^n \begin{bmatrix} 100 \\ 10 \end{bmatrix}$$

DIJAGONALIZIRAMO MATRICU A:

$$k_A(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda - 4 & 2 \\ -1 & \lambda - 1 \end{vmatrix} =$$

$$= \lambda^2 - 5\lambda + 4 + 2 = \lambda^2 - 5\lambda + 6 = \lambda^2 - 2\lambda - 3\lambda + 6 =$$

$$= (\lambda - 2)(\lambda - 3)$$

$$\Rightarrow \lambda_1 = 2, \quad \lambda_2 = 3 \quad \text{SV. VR.}$$

TRAŽIMO PRIPADNE SV. VEKTORE:

$$A\vec{x} = \lambda\vec{x} \Leftrightarrow A\vec{x} = \lambda I \vec{x}$$
$$\Leftrightarrow (\lambda I - A)\vec{x} = \vec{0}$$
$$\vec{x} \neq \vec{0}$$

(1) $\lambda_1 = 2$

$$(2I - A)\vec{x} = \vec{0}$$

$$\left[\begin{array}{cc|c} -2 & 2 & 0 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ -1 & 1 & 0 \end{array}\right] \Rightarrow -x_1 + x_2 = 0$$

$$x_1 = x_2$$

$$x_2 = \alpha, \ \alpha \in \mathbb{R} \Rightarrow x_1 = \alpha$$

$$\Rightarrow \vec{x} = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \alpha \neq 0$$

(2) $\lambda_2 = 3$

$$(3I - A)\vec{x} = \vec{0} \Rightarrow \vec{x} = \alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ \alpha \neq 0$$

VRIJEDI $A = SDS^{-1}$, GDJE $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $S = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

$$\Rightarrow A^n = (SD\underbrace{S^{-1})(S}_{I}D\underbrace{S^{-1}) \cdots}_{I} \underbrace{(S}_{I}DS^{-1}) = SD^nS^{-1}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2^n & 0 \\ 0 & 3^n \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}^{-1}}_{\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}} = \begin{bmatrix} -2^n + 2\cdot 3^n & 2^{n+1} - 2\cdot 3^n \\ -2^n + 3^n & 2^{n+1} - 3^n \end{bmatrix}$$

$$\rightarrow X_n = A^n X_0 = \begin{bmatrix} 170\cdot 3^n - 80\cdot 2^n \\ 90\cdot 3^n - 80\cdot 2^n \end{bmatrix} \begin{matrix} = Z_n \\ = L_n \end{matrix}$$

$$\lim_{n \to \infty} \frac{Z_n}{L_n} = \lim_{n \to \infty} \frac{170\cdot 3^n - \cancel{80\cdot 2^n}}{90\cdot 3^n - \cancel{80\cdot 2^n}} = \frac{2}{1} = 2$$

[LOTKA - VOLTERRIN MODEL $\rightarrow$ KORISTI SE U EKOLOGIJI]

(2)

$$\begin{cases} F_{n+1} = F_n + \underbrace{F_{n-1}}_{G_n} \\ G_{n+1} = F_n \end{cases}, \quad X_1 = \begin{bmatrix} F_1 \\ G_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$X_n = A^{n-1} X_1$$

DZ. //

(3)

$A: M_2 \to M_2$

$A(M) = M^T$

KANONSKA BAZA ZA M_2

$$E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A(e) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A(E_{11})$

$$k_A(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & -1 & \lambda & 0 \\ 0 & 0 & 0 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-1) \begin{vmatrix} \lambda-1 & 0 & 0 \\ -1 & \lambda & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix} = (\lambda-1)^2 \begin{vmatrix} \lambda & -1 \\ -1 & \lambda \end{vmatrix} =$$

$$= (\lambda-1)^2(\lambda^2-1) = (\lambda-1)^3(\lambda+1)$$

$\Rightarrow$ SV. VR.

$\lambda_1 = 1, \quad \lambda_2 = -1$

(KRATNOST 3) (KRATNOST 1)

$A(m) = \lambda m$

$M^T = M$

$A(m) = -m$

$M^T = -M$

(6)

$A \in M_n$

PRETP. DA JE ZBROJ EL. SVAKOG STUPCA c

RAČUNAMO

$$k_A(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda - a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & \lambda - a_{22} & \cdots & -a_{2n} \\ -a_{31} & -a_{32} & \cdots & -a_{3n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & \lambda - a_{nn} \end{vmatrix}$$

$$= \begin{bmatrix} \text{PRVOM RETKU} \\ \text{DODAMO SVAKI PREOSTALI} \end{bmatrix} = \begin{vmatrix} \lambda - \underbrace{\sum_{j=1}^{n} a_{j1}}_{c} & \lambda - \underbrace{\sum_{j=1}^{n} a_{j2}}_{c} & \cdots & \lambda - \underbrace{\sum_{j=1}^{n} a_{jn}}_{c} \\ & & & \\ & & & \end{vmatrix}$$

$$= (\lambda - c) \begin{vmatrix} 1 & \cdots & 1 \\ & \cdots & \end{vmatrix} \Rightarrow k_A(c) = 0$$

$\Rightarrow$ c JE SV. VR. OD A