

**Lemma 0.0.1** (convex transform of a martingale). *Let  $M$  be an  $\mathcal{F}$ -martingale and let  $h : \mathbb{R} \rightarrow \mathbb{R}$  be a convex function. If the process  $X = h(M)$  is integrable, then it is an  $\mathcal{F}$ -submartingale.*