

Proposition 0.0.1. *Let W be a one-dimensional standard Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. For a real number $\gamma \in \mathbb{R}$, we define the process \widetilde{W} by setting $\widetilde{W}_t = W_t - \gamma t$ for $t \in [0, T]$. Let the probability measure $\tilde{\mathbb{P}}$, equivalent to \mathbb{P} on (Ω, \mathcal{F}_T) , be defined through the formula (see Definition 6.4.1 for \mathcal{E})*

$$\eta_T = \frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} = \exp\left(\gamma W_T - \frac{1}{2}\gamma^2 T\right) = \mathcal{E}_T(\gamma W).$$

Then \widetilde{W} is a standard Brownian motion on the probability space $(\Omega, \mathcal{F}, \tilde{\mathbb{P}})$.