

**Lemma 0.0.1** (Doob's measurability theorem). *Let  $X : \Omega \rightarrow \Psi$  be a mapping and  $(\Psi, \mathcal{G})$  a measurable space. A function  $Y : \Omega \rightarrow \mathbb{R}$  is  $\sigma(X)$ -measurable if and only if there exists a  $\mathcal{G}$ -measurable function  $h : \Psi \rightarrow \mathbb{R}$  s.t.  $Y = h(X)$ .*