

Proof. We only need to show the converse. Since X and Y are modifications of each other, $\mathbb{P}(X_t = Y_t) = 1$ for all t in some countable set I . Since countable unions of null sets are again null sets¹, countable intersections of sets with full measure, have again full measure. Hence, $\mathbb{P}(\bigcap_{t \in I} \{X_t = Y_t\}) = 1$. ■

¹Follows from countable additivity of probability measure (Definition 1.2.2).