

Theorem 0.0.1. *The following results hold (pp.41–44):*

- (**reflection principle**) $\mathbb{P}(T_b < t) = 2\mathbb{P}(B_t > b)$. More precisely, T_b follows a Lévy distribution with parameters $(0, b^2)$ (or equivalently, an inverse Gamma distribution with shape 1/2 and scale b^2).

• We can therefore show that $\mathbb{P}(T_b < \infty) = 1$ but $\mathbb{E}(T_b) = \infty$.

• The moment generating function of T_b is $e^{-\sqrt{2\lambda}b}$ for $b, \lambda > 0$.

• For any $t \geq 0$, the running supremum $B_t^* = \sup_{s \leq t} B_s$ and $|B_t|$ have the same distribution.

• The joint distribution of $B_t^* = \sup_{s \leq t} B_s$ and B_t is given by

$$\mathbb{P}(B_t \leq x, B_t^* > y) = \mathbb{P}(B_t > 2y - x) = \mathbb{P}(B_t < x - 2y)$$

for every $t \geq 0$, $y \geq 0$ and $x \leq y$.

• The joint density of $B_t^* = \sup_{s \leq t} B_s$ and B_t is given by

$$\frac{2(2y - x)}{t} \cdot \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(2y - x)^2}{2t}\right)$$

for $x \leq y$, $y \geq 0$.

• The drawdown $B_t^* - B_t$ has the same distribution as $|B_t|$.