Scalar Valued Functions

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Abstract

Whilst the author believes the raison d'être of this manuscript is obvious, they do not believe that the scope is.

The *Theory of Functions* is rich and central to Mathematics. As such, we limit our scope here to definitions and graphs of univariate functions $f : \mathbb{R} \to \mathbb{R}$. Whilst we include common equalities between different functions - say circular and exponential - what you will not find here are derivations of any sort. You will **not** find proofs **nor** set theoretic discussions of "jectivities", binary relations, etc. Furthermore there is a purposeful lack of rigour in this / catalogue/; theorems are asserted as is, with no warranty and no proof. Finally, analytic concerns of limits and convergence are also dutifuly ignored.

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1. Elementary

These such functions are continuous on their domains and include taking **sums**, **products**, **roots** and **compositions** of finitely many algebraic or transcendental functions.

1.1. Algebraic

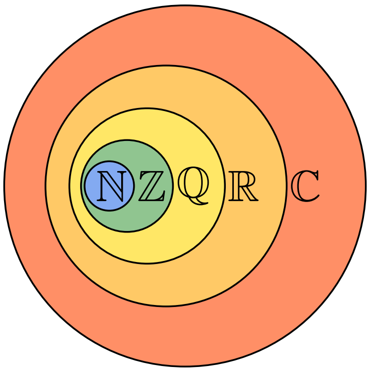
1.1.1. Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_0 = \sum_{k=0}^{n} a_k x^k \tag{1}$$

todo polynomial subplot grid from x^0 to x^7.

1.1.2. Rational

Much in the same way that $\mathbb{Q}$ is defined as any element $\frac{a}{b}$ where $a, b \in \mathbb{Z}$:



a function f is called a rational function if it can be written in the form:

$$f(x) = \frac{P(x)}{Q(x)} \tag{2}$$

where $P(x)$ and $Q(x)$ are polynomial functions of x and Q is not the zero function.

1.2. Transcendental

1.2.1. Exponential

1.2.2. Logarithm

1.2.3. Trigonometric

1.2.4. Inverse Trigonetric

1.2.5. Hyperbolic

1.2.6. Inverse Hyperbolic

2. Non-Elementary

2.1. Gamma

2.2. Error

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\Pi}} \int_0^x e^{(-t)^2} \,\mathrm{d}t \tag{3}$$

2.3. Elliptic

2.4. Bessel

2.5. Riemann Zeta

2.6. Fresnel

$$\mathrm{S}(x) = \int_0^x \sin(t^2) \,\mathrm{d}t,\ \mathrm{C}(x) = \int_0^x \cos(t^2) \,\mathrm{d}t \tag{4}$$

3. Discontinuous

3.1. Absolute Value

3.2. Step

3.2.1. Heaviside

3.2.2. Floor

3.2.3. Ceiling

3.2.4. Square Wave