

Examples 0.0.1 (Itô's lemma for semimartingales). We again consider the drawdown process $X_t = W_t^* - W_t$ where W_t is a standard Brownian motion and $W_t^* = \sup_{s \leq t} W_s$ is the running supremum.

We note that X is not an Itô process, but is a continuous semimartingale with $M_t = -W_t$, $A_t = W_t^*$ in its canonical decomposition. By Itô's lemma, we can compute, e.g.

$$\begin{aligned} dX_t^2 &= 2X_t dX_t + \frac{1}{2} \cdot 2 d\langle X \rangle_t \\ &= -2X_t dW_t + 2X_t dW_t^* + d\langle -W \rangle_t \\ &= -2X_t dW_t + 2X_t dW_t^* + dt. \end{aligned}$$