

*Proof.* We only need to show the converse. Since  $X$  and  $Y$  are modifications of each other,  $\mathbb{P}(X_t = Y_t) = 1$  for all  $t$  in some countable set  $I$ . Since countable unions of null sets are again null sets<sup>1</sup>, countable intersections of sets with full measure, have again full measure. Hence,  $\mathbb{P}(\bigcap_{t \in I} \{X_t = Y_t\}) = 1$ . ■

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<sup>1</sup>Follows from countable additivity of probability measure (Definition 1.2.2).