

**Theorem 0.0.1.** *The following results hold (pp.41–44):*

- **(reflection principle)**  $\mathbb{P}(T_b < t) = 2\mathbb{P}(B_t > b)$ . More precisely,  $T_b$  follows a Lévy distribution with parameters  $(0, b^2)$  (or equivalently, an inverse Gamma distribution with shape  $1/2$  and scale  $b^2$ ).
- We can therefore show that  $\mathbb{P}(T_b < \infty) = 1$  but  $\mathbb{E}(T_b) = \infty$ .
- The moment generating function of  $T_b$  is  $e^{-\sqrt{2\lambda}b}$  for  $b, \lambda > 0$ .
- For any  $t \geq 0$ , the running supremum  $B_t^* = \sup_{s \leq t} B_s$  and  $|B_t|$  have the same distribution.
- The joint distribution of  $B_t^* = \sup_{s \leq t} B_s$  and  $B_t$  is given by

$$\mathbb{P}(B_t \leq x, B_t^* > y) = \mathbb{P}(B_t > 2y - x) = \mathbb{P}(B_t < x - 2y)$$

for every  $t \geq 0$ ,  $y \geq 0$  and  $x \leq y$ .

- The joint density of  $B_t^* = \sup_{s \leq t} B_s$  and  $B_t$  is given by

$$\frac{2(2y - x)}{t} \cdot \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(2y - x)^2}{2t}\right)$$

for  $x \leq y$ ,  $y \geq 0$ .

- The drawdown  $B_t^* - B_t$  has the same distribution as  $|B_t|$ .