

Examples 0.0.1 (Ito's integral). We can show that $\int_0^t 2W_s dW_s$ is a martingale, by noting that

$$\mathbb{E} \left[\int_0^T |2W_u|^2 du \right] = \int_0^T \mathbb{E}[4W_t^2] dt = \int_0^T 4t dt = 2T^2 < \infty$$

where the first equality uses Fubini's Theorem.