

Theorem 0.0.1 (existence and uniqueness of conditional expectation). *Let \mathcal{G} be a sub- σ -algebra of \mathcal{A} . Then*

- (existence) there exists a conditional expectation $\mathbb{E}[X|\mathcal{G}]$ for any $X \in L^1(\Omega, \mathbb{P})$.*
- (uniqueness) any two conditional expectations of $X \in L^1(\Omega, \mathbb{P})$ respective to \mathcal{G} are equal \mathbb{P} -a.s.*