

**Theorem 0.0.1** (existence and uniqueness of conditional expectation). *Let  $\mathcal{G}$  be a sub- $\sigma$ -algebra of  $\mathcal{A}$ . Then*

*1. (existence) there exists a conditional expectation  $\mathbb{E}[X|\mathcal{G}]$  for any  $X \in L^1(\Omega, \mathbb{P})$ .*

*2. (uniqueness) any two conditional expectations of  $X \in L^1(\Omega, \mathbb{P})$  respective to  $\mathcal{G}$  are equal  $\mathbb{P}$ -a.s.*