

(from Theorem 3.3.3) If M is a continuous, square integrable \mathcal{F} -martingale, then

$$M^2 - \langle M \rangle$$

is also an \mathcal{F} -martingale. In particular, if $M_0 = 0$, then $\mathbb{E}[M_t^2 - \langle M \rangle_t] = M_0^2 - \langle M \rangle_0 = 0$ and therefore

$$\text{Var}(M_t) = \mathbb{E}(M_t^2) = \mathbb{E}(\langle M \rangle_t).$$

+end_src
** Ref