

**Definition 0.0.1.** Consider two probability measures  $\mathbb{P}$  and  $\mathbb{Q}$  defined on  $(\Omega, \mathcal{A})$ .

- (equivalence)  $\mathbb{P}$  and  $\mathbb{Q}$  are said to be **equivalent** if

$$\mathbb{P}(A) = 0 \iff \mathbb{Q}(A) = 0$$

for all  $A \in \mathcal{A}$ ; i.e. they have the same set of null events in the  $\sigma$ -algebra  $\mathcal{A}$ .

- (absolute continuity)  $\mathbb{Q}$  is said to be **absolutely continuous** with respect to  $\mathbb{P}$ , written  $\mathbb{Q} \ll \mathbb{P}$ , if

$$\mathbb{P}(A) = 0 \implies \mathbb{Q}(A) = 0$$

for all  $A \in \mathcal{A}$ .