

Circle geometry theorems

Source: <http://topdrawer.aamt.edu.au/Geometric-reasoning/Big-ideas/Circle-geometry/Angle-and-chord-properties>

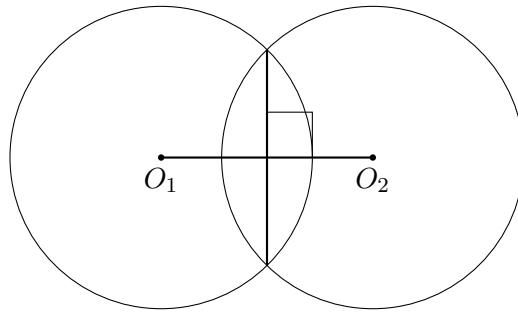
Circle geometry theorems

1. Intersecting circles: centres bisect common chord

Theorem. When two circles intersect, the line joining their centres bisects their common chord at right angles.

Suggested abbreviation. centres of touching circles

Diagram.

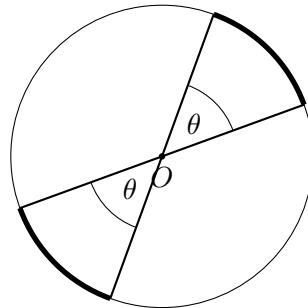


2. Equal arcs in equal circles \Leftrightarrow equal central angles

Theorem. Equal arcs on circles of equal radii subtend equal angles at the centre, and conversely.

Suggested abbreviation. equal arcs, equal angles

Diagram.

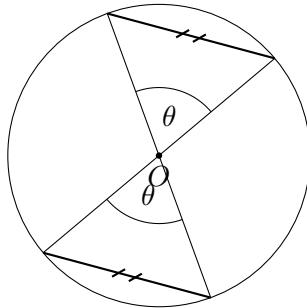


3. Equal central angles \Leftrightarrow equal chords

Theorem. Equal angles at the centre stand on equal chords, and conversely.

Suggested abbreviation. equal chords, equal angles OR angles standing on equal chords
OR angles standing on equal arcs

Diagram.

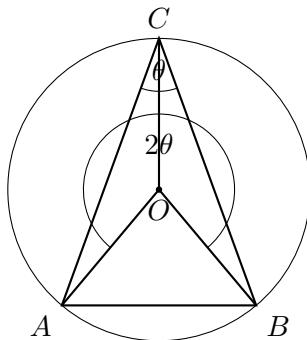


4. Central angle is twice circumferential angle

Theorem. The angle at the centre is twice the angle at the circumference subtended by the same arc.

Suggested abbreviation. angles at the centre and circumference

Diagram.

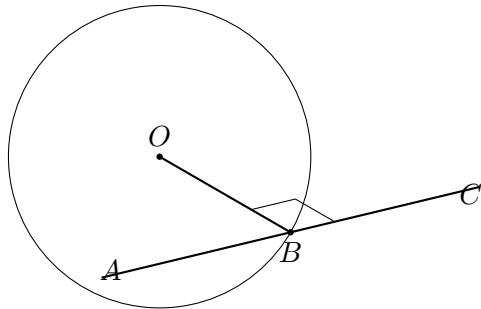


5. Tangent \perp radius at point of contact

Theorem. The tangent to a circle is perpendicular to the radius drawn to the point of contact and conversely.

Suggested abbreviation. tangent perpendicular to radius

Diagram.

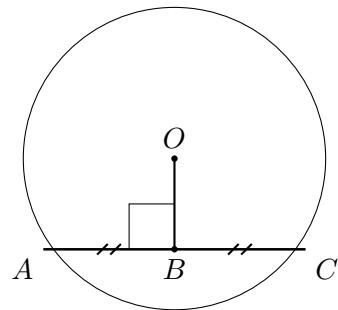


6. Perpendicular from centre to chord bisects chord

Theorem. The perpendicular from the centre of a circle to a chord bisects the chord.

Suggested abbreviation. perpendicular from the centre

Diagram.

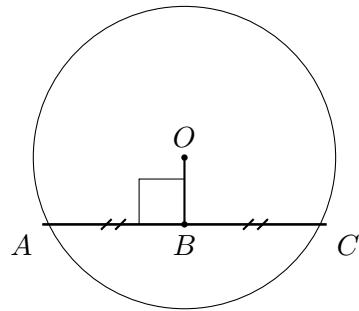


7. Centre to midpoint of chord is perpendicular to chord

Theorem. The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.

Suggested abbreviation. line joining centre to midpoint of chord

Diagram.

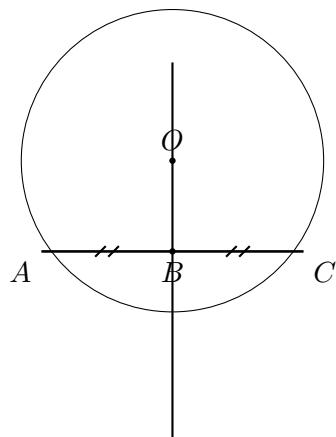


8. Perpendicular bisector of a chord passes through centre

Theorem. The perpendicular bisector of a chord passes through the centre of the circle.

Suggested abbreviation. perpendicular bisector of chord

Diagram.

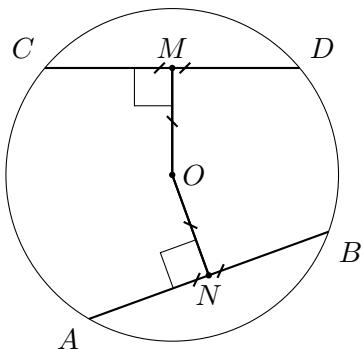


9. Equal chords in equal circles are equidistant from centre

Theorem. Equal chords in equal circles are equidistant from the centres.

Suggested abbreviation. equal chords equidistant from centre

Diagram.

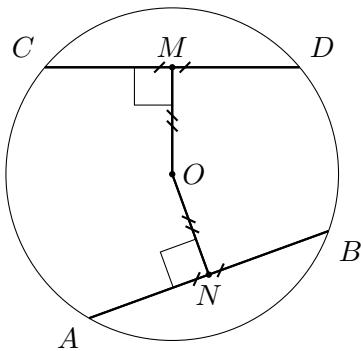


10. Chords equidistant from centre are equal

Theorem. Chords in a circle which are equidistant from the centre are equal.

Suggested abbreviation. equal chords equidistant from centre

Diagram.

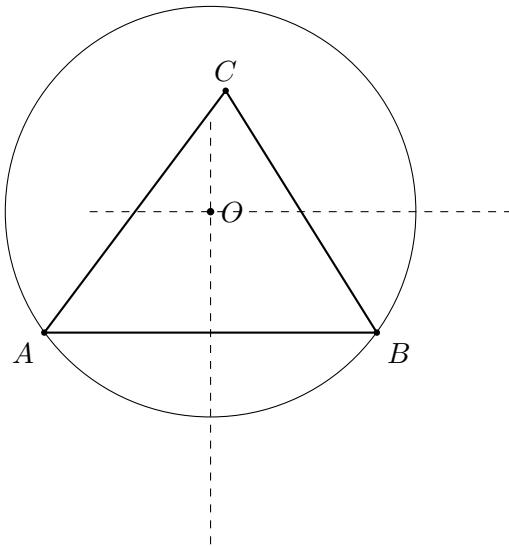


11. Three non-collinear points determine a unique circle (circumcentre)

Theorem. Any three non-collinear points lie on a unique circle, whose centre is the point of concurrency of the perpendicular bisectors of the intervals joining the points.

Suggested abbreviation. perpendicular bisector of chord passes through the centre

Diagram.

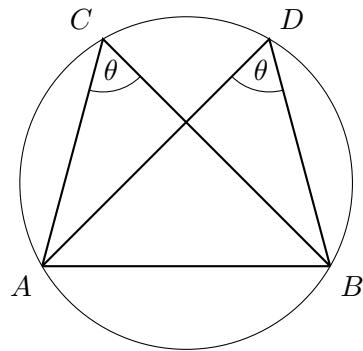


12. Angles in the same segment are equal

Theorem. Angles in the same segment are equal.

Suggested abbreviation. angles in the same segment

Diagram.

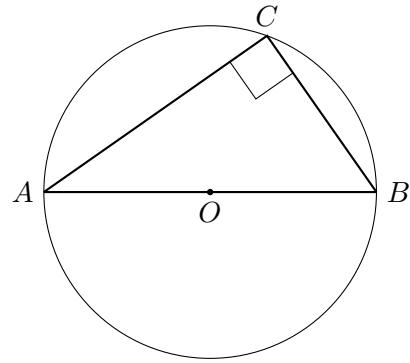


13. Angle in a semi-circle is a right angle

Theorem. The angle in a semi-circle is a right angle.

Suggested abbreviation. angle in a semi-circle

Diagram.

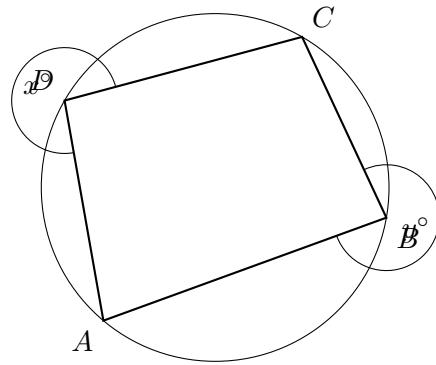


14. Opposite angles of a cyclic quadrilateral are supplementary

Theorem. Opposite angles of a cyclic quadrilateral are supplementary.

Suggested abbreviation. opposite angles in a cyclic quad $x + y = 180$

Diagram.



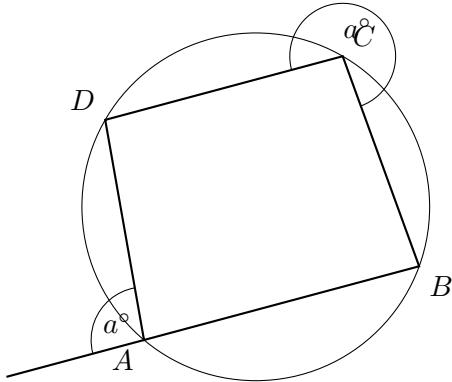
$$x + y = 180$$

15. Exterior angle of a cyclic quadrilateral

Theorem. The exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle.

Suggested abbreviation. exterior angle of cyclic quad

Diagram.



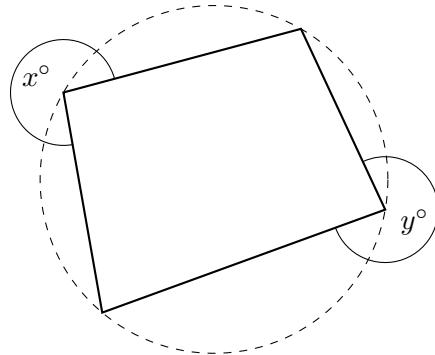
16. Converse: supplementary opposite angles \Rightarrow cyclic

Theorem. If the opposite angles in a quadrilateral are supplementary then the quadrilateral is cyclic.

Note: This theorem is also a test for four points to be concyclic.

Suggested abbreviation. converse of opposite angles in a cyclic quad If $x + y = 180$ then $ABCD$ is a cyclic quadrilateral.

Diagram.



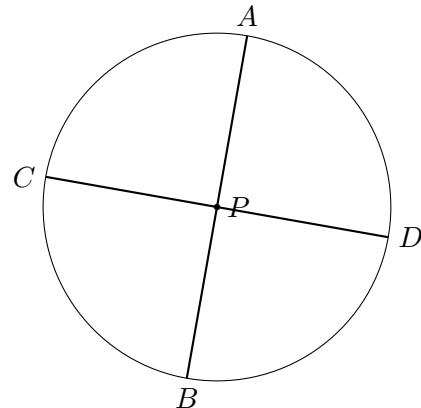
If $x + y = 180$ then $ABCD$ is a cyclic quadrilateral.

17. Intersecting chords (power of a point)

Theorem. The products of the intercepts of two intersecting chords are equal.

Suggested abbreviation. intersecting chords $AP \times BP = CP \times DP$

Diagram.



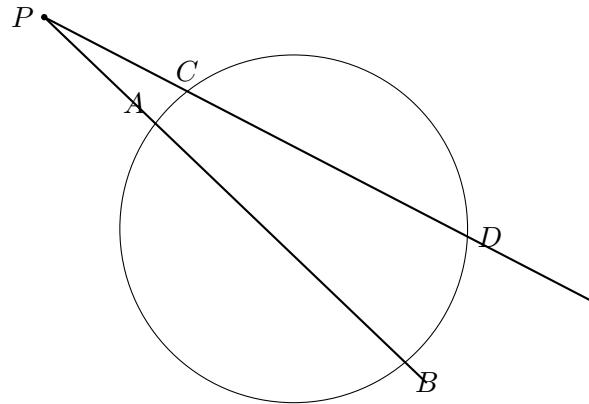
$$AP \times BP = CP \times DP$$

18. Intersecting secants from an external point

Theorem. The products of the intercepts of two intersecting secants to a circle from an external point.

Suggested abbreviation. intersecting secants $AP \times BP = CP \times DP$

Diagram.



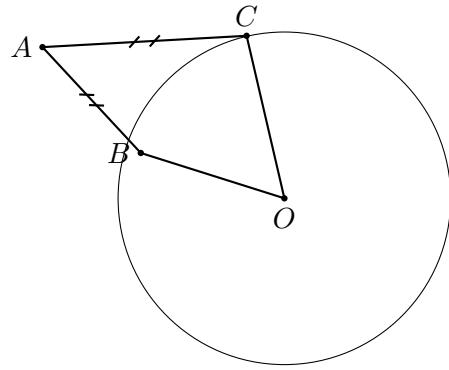
$$AP \times BP = CP \times DP$$

19. Tangents from an external point are equal

Theorem. Tangents to a circle from an external point are equal.

Suggested abbreviation. tangents from external point

Diagram.

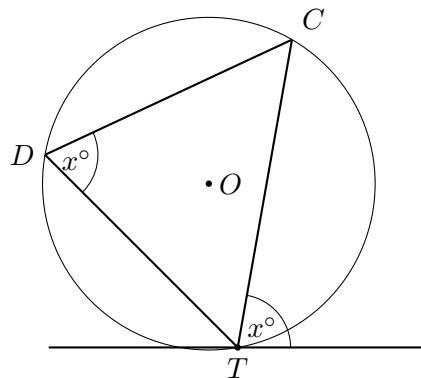


20. Alternate segment theorem (tangent–chord angle)

Theorem. The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

Suggested abbreviation. angle in alternate segment OR angle between tangent and chord

Diagram.

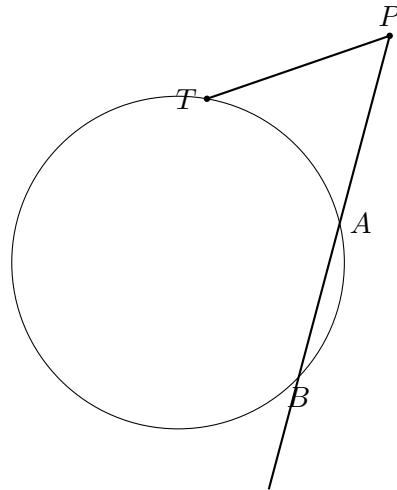


21. Tangent–secant theorem (power of a point)

Theorem. The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point.

Suggested abbreviation. square of the tangent OR intersecting tangent and secant OR tangent and secant $PT^2 = AP \times PB$

Diagram.



$$PT^2 = AP \times PB$$

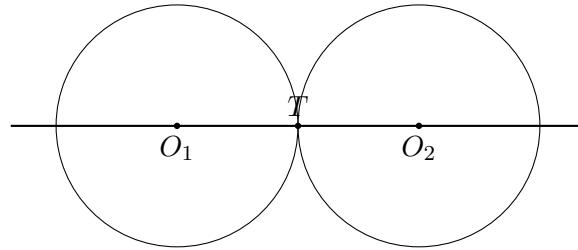
Supplementary theorems

S1. Touching circles share a common tangent at the contact point

Theorem. Two circles touch if they have a common tangent at the point of contact.

Suggested abbreviation. tangent of touching circles

Diagram.

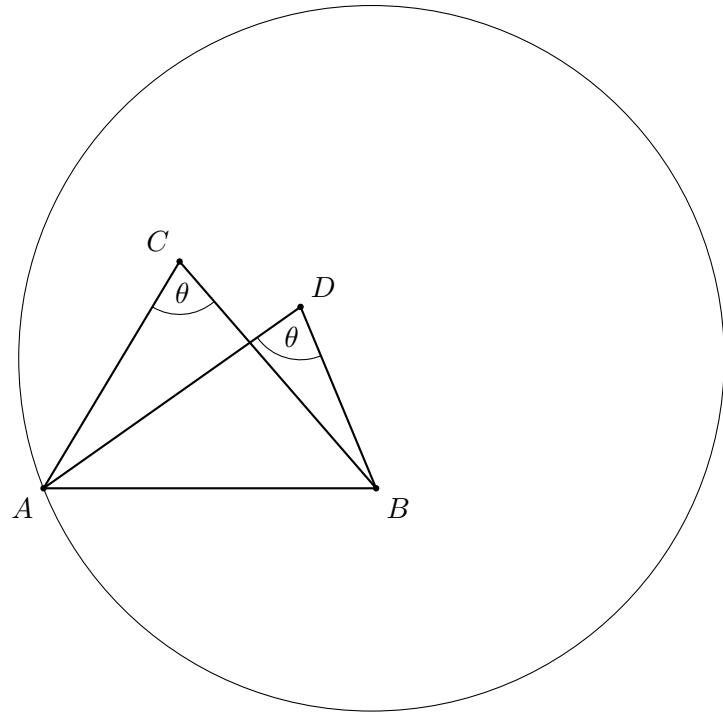


S2. Converse of “angles in the same segment”

Theorem. If an interval subtends equal angles at two points on the same side of it then the endpoints of the interval and the four points are concyclic.

Suggested abbreviation. converse of angles in the same segment

Diagram.



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