

**Definition 0.0.1** (expectation). Let  $X$  be a r.v. on a probability space  $(\Omega, \mathcal{A}, \mathbb{P})$ . The **expectation** of  $X$  is

$$\mathbb{E}(X) = \int_{\Omega} X(\omega) \, d\mathbb{P}(\omega).$$

When  $X$  is a simple random variable, i.e. it takes on only finitely many values, it can be written as

$$X = \sum_{i=1}^n x_i \mathbf{1}_{A_i} \iff X(\omega) = \sum_{i=1}^n x_i \mathbf{1}_{A_i}(\omega), \quad \omega \in \Omega$$

where  $x_i \in \mathbb{R}$  and  $A_i = \{\omega : X(\omega) = x_i\} \in \mathcal{A}$ . Its expectation is given by

$$\mathbb{E}(X) = \sum_{i=1}^n x_i \mathbb{P}(A_i) = \sum_{i=1}^n x_i \mathbb{P}\{X = x_i\}.$$