

**Lemma 0.0.1** (a discrete time result). *Let  $M = (M_n)_{n=0,1,\dots,N}$  be an  $\mathcal{F}$ -adapted and integrable stochastic process. Then  $M$  is an  $\mathcal{F}$ -martingale if and only if  $\mathbb{E}(M_\tau) = \mathbb{E}(M_0)$  for any stopping time  $\tau$  with values in  $\{0, 1, \dots, N\}$ .*