

Definition 0.0.1 (probability measure). Let Ω be a nonempty set, and let \mathcal{A} be a σ -algebra of subsets of Ω . A probability measure \mathbb{P} is a function that, to every set $A \in \mathcal{A}$, assigns a number in $[0, 1]$. We require:

(i) $\mathbb{P}(\Omega) = 1$.

(ii) (countable additivity) whenever A_1, A_2, \dots is a countable sequence of disjoint sets in \mathcal{A} , then

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}(A_i).$$

The triple $(\Omega, \mathcal{A}, \mathbb{P})$ is called a **probability space**.