

Definition 0.0.1 (Ito process). An \mathcal{F} -adapted continuous process X is called an Ito process if it admits a representation

$$X_t = X_0 + \int_0^t \alpha_u \, du + \int_0^t \beta_u \cdot dW_u, \quad \text{for all } t \in [0, T]$$

for some \mathcal{F} -adapted processes α and β that are defined on $(\Omega, \mathcal{F}, \mathbb{P})$ and satisfy suitable integrability conditions. It is customary to write the integral formula above using differential notation as

$$dX_t = \alpha_t \, dt + \beta_t \cdot dW_t.$$