

**Definition 0.0.1** (probability measure). Let  $\Omega$  be a nonempty set, and let  $\mathcal{A}$  be a  $\sigma$ -algebra of subsets of  $\Omega$ . A probability measure  $\mathbb{P}$  is a function that, to every set  $A \in \mathcal{A}$ , assigns a number in  $[0, 1]$ . We require:

- (i)  $\mathbb{P}(\Omega) = 1$ .
- (ii) (countable additivity) whenever  $A_1, A_2, \dots$  is a countable sequence of disjoint sets in  $\mathcal{A}$ , then

$$\mathbb{P}\left(\bigcup_{i=1}^\infty A_i\right) = \sum_{i=1}^\infty \mathbb{P}(A_i).$$

The triple  $(\Omega, \mathcal{A}, \mathbb{P})$  is called a **probability space**.