

**Definition 0.0.1** (standard Brownian motion). A real-valued process  $W = (W_t)_{t \in [0,T]}$ , with  $W_0 = 0$ , defined on a filtered probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ , is called a standard  $\mathcal{F}$ -Brownian motion if:

1.  $W$  is  $\mathcal{F}$ -adapted.
2. For any  $0 \leq u \leq t \leq T$ ,  $W_t - W_u$  is independent of  $\mathcal{F}_u$ .
3. For any  $0 \leq u \leq t \leq T$ ,  $W_t - W_u \sim \mathcal{N}(0, t - u)$ .
4.  $W$  is sample-paths continuous, i.e. almost all sample paths of  $W$  are continuous.