

Proof. (for 5)

$$\begin{aligned}\mathbb{E}[h(X, Y) | \mathcal{G}] &= \mathbb{E}[h(x, Y) | \mathcal{G}]|_{x=X} \\ &= \mathbb{E}[h(x, Y)]|_{x=X} \\ &=: H(x)|_{x=X}\end{aligned}$$

(taking out what is known)
(Y is independent of \mathcal{G})

■