

Index of notation

Symbols	
$[-\infty, +\infty]$,	380
A	
$\mathcal{A} \times \mathcal{B}$,	143
aB, Ba ,	280
\mathcal{A}_C ,	36
$(A I)$,	163
A^o ,	389
\mathcal{A}_* ,	264
B	
B^{-1} ,	280
$\mathcal{B}_0(X)$,	197
BC ,	280
$\mathcal{B}(\mathbb{C})$,	74
$\mathcal{B}(\mathbb{R})$,	4
$\mathcal{B}(\mathbb{R})$,	74
$\mathcal{B}(\mathbb{R}^d)$,	4
$\mathcal{B}(X)$,	189
$B(X, \mathcal{A}, \mathbb{C})$,	120
$B(X, \mathcal{A}, \mathbb{R})$,	120
$B(x, r)$,	385
C	
c (cardinality of the continuum),	376
c_0 ,	89
$C_0^{\mathbb{C}}(X)$,	199
$C_0(X)$,	199
$C[a, b]$,	85
$C(X)$,	188, 197
D	
Δ (symmetric difference of sets),	373
Δ (modular function),	294
δ_x ,	8
\det ,	156
$\text{diam}(A)$,	393
$\text{diff}(A)$,	28
$D\mu$,	166
$\overline{D}\mu$,	166
$\underline{D}\mu$,	166
$\frac{dv}{d\mu}$,	126
E	
$e(C)$,	164
$e(U)$,	166
$E(X)$,	308
E_x ,	144
$E(X \mathcal{B})$,	342
$E(X Y = y_j)$,	341
E^y ,	144
F	
\mathcal{F} ,	5
$\langle f \rangle$,	96
$F'(x_0)$,	157
f^+ ,	46
f^- ,	46
$\mathcal{F}_\alpha(X)$,	255
\check{f} ,	285
$[f, g]$,	229
$f \prec U$,	192
$f \vee g$,	43
$f \wedge g$,	43
F_μ ,	19
\mathcal{F}_σ ,	5
$f * g$,	153, 298
$\{\mathcal{F}_n\}_{n=0}^\infty$,	345
$\{\mathcal{F}_t\}_{t \in T}$,	345

\mathcal{F}_τ	354	$L^p(X, \mathcal{A}, \mu)$	96
f_x	144, 285	\mathcal{L}^1	56
xf	285	$L^1(G)$	300
$f: X \rightarrow Y$	374	$\mathcal{L}^1(X, \mathcal{A}, \mu, E)$	400
f^y	144	$\mathcal{L}^1(X, \mathcal{A}, \mu, \mathbb{R})$	56
		ℓ^2	90
		λ	18
G		λ_d	18
\mathcal{G}	5	λ^*	13, 14
G_3	418	λ_d^*	14
$\mathcal{G}_\alpha(X)$	255	L_\bullet	233
$g \cdot x$	417	L^\bullet	233
$GL(d, \mathbb{R})$	284	$l(f, \mathcal{P})$	67
$\text{gr}(f)$	244	$\lim_n f_n$	43
\mathcal{G}_δ	5	$\liminf_n f_n$	43
		$\liminf_n x_n$	381
		$\underline{\lim}_n x_n$	381
H		$\lim_n s_n$	86
$(H) \int_a^b f$	430	$\limsup_n f_n$	43
$(H) \int_a^b f(x) dx$	430	$\limsup_n x_n$	381
		$\overline{\lim}_n x_n$	381
		ℓ^∞	89
I		$(L) \int_a^b f$	56
\mathcal{I}	229, 243	$(L) \int_a^b f(x) dx$	56
$\Im(f)$	75	$\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbb{C})$	91
$\inf(A)$	381	$\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbb{R})$	91
$\inf_n f_n$	43	$\mathcal{L}^\infty(X, \mathcal{A}, \mu, \mathbb{C})$	92
$\int f d\mu$	53, 55, 56, 120, 399	$\mathcal{L}^\infty(X, \mathcal{A}, \mu, \mathbb{R})$	92
$\int f(x) \mu(dx)$	56		
$\int f(x) d\mu(x)$	56		
$\int_A f d\mu$	56		
$\int_a^b f$	56, 68		
$\int_a^b f(x) dx$	56, 68		
$\int_a^b f$	67		
$\overline{\int}_a^b f$	67		
(\cdot, \cdot)	89		
$\Im(z)$	382		
J			
J_F	158		
K			
$\mathcal{K}^\mathbb{C}(X)$	184		
$\mathcal{K}(X)$	184		
L			
L_1	234		
$\underline{L}(f)$	234		
$\overline{L}(f)$	234		
$L^\infty(X, \mathcal{A}, \mu)$	96		

$\mu * \nu$, 301	S
μ_X or μ , 309	$\text{sgn}(z)$, 108
$M(X, \mathcal{A}, \mathbb{C})$, 119	$\sigma(\mathcal{F})$, 3
$M(X, \mathcal{A}, \mathbb{R})$, 119	σ_X^2 , 309
	σ_X , 309
	$\sigma(X_i, i \in I)$, 309
N	$\sigma(X_1, X_2, \dots)$, 309
n , 243	$SO(d)$, 418
N , 243	$\sum_\alpha X_\alpha$, 241
$\mathcal{N}(n_1, \dots, n_k)$, 243	$\sup(A)$, 381
$N(0, 1)$, 311	$\sup_n f_n$, 43
$N(\mu, \sigma^2)$, 311	$\text{supp}(f)$, 184
v_a , 130	$\text{supp}(\mu)$, 207
v_f , 204	
$v \ll \mu$, 122, 125	
v_s , 130	
$v_1 * v_2$, 314	

O	U
$O(d)$, 284	$u(f, \mathcal{P})$, 67

P	V
$P(A B)$, 340	$\text{var}(X)$, 309
ϕ_μ , 331	V_\bullet , 233
$\prod_n \mathcal{A}_n$, 243	V^\bullet , 233
$\mathcal{P}(X)$, 374	$V_F[a, b]$, 133
	V^* , 106

R	X
$\overline{\mathbb{R}}$, 380	$x \vee y$, 380
$\Re(f)$, 75	$x \wedge y$, 380
$\mathcal{R}(f, \mathcal{P})$, 70, 429, 433	
$(R) \int_a^b f$, 68	
$(R) \int_a^b f(x) dx$, 68	
$\Re(z)$, 382	Z
	\bar{z} , 108

Index

A

\mathcal{A} -measurable
function, 42
set, 2
a.e., 50
a.e. [μ], 50
a.s., 319
absolute continuity, 122
for functions from \mathbb{R} to \mathbb{R} , 135
for signed and complex measures, 125
uniform, 129
absolutely continuous part of a measure, 130
absolutely convergent series, 88
act, 417
action, 417
adapted, 345
algebra, 300
Banach, 300
of functions, 392
of sets, 1
algebraic dual space, 106
almost everywhere, 50
almost everywhere differentiability
of finite Borel measures, 167
of functions of finite variation, 171
of monotone functions, 171
almost surely, 319
analytic
measurable space, 270
set, 248
measurability, 262
that is not a Borel set, 254
ancestor, 419
approximate identity, 305
atom of a σ -algebra, 272
axiom of choice, 27, 377

B

Baire
category theorem, 395
measure, 197
set, 197
 σ -algebra, 197, 226
Banach
algebra, 300
space, 87
Banach–Tarski paradox, 417, 419
base
for a family of neighborhoods, 280
for a topological space, 390
basis
Hamel, 30
Beppo Levi's theorem, 62
Bernoulli distribution, 315
bijection, bijective function, 375
binary expansion, 315–316, 382
binomial distribution, 318
Blackwell's theorem on analytic measurable spaces, 272
Bochner
integrable function, 399
integral, 399
Borel
function, 42
isomorphism, 259
measurability of the image of a Borel set
under an injective Borel function, 260
measurable function, 42, 189, 397
measure, 11, 189
product, regular, 222
 σ -algebra, 4, 189
subsets, 4, 189
Borel–Cantelli lemmas, 320

- bound for a linear operator, 106
- bounded
 - linear operator, 106
 - set, 385, 393
 - variation, function of, 133
- Bourbaki's treatment of integration, 215–218
- Brownian motion, 357
 - existence, 357–363
 - nowhere differentiable paths, 361

- C**
- C^1 function, 158
- Cantor
 - function, 48–49, 52, 130, 137, 178
 - set, 26–27, 47–49
 - singular function, 48
- Cantor's nested set theorem, 394
- capacitable, 266
- capacity, 266
- cardinality, 375
 - of the continuum, 376
- Cartesian product, 375
- Cauchy criterion
 - for Henstock–Kurzweil integrability, 431
 - for McShane integrability, 437
- Cauchy sequence, 86, 394
- Cauchy–Schwarz inequality, 90
- central limit theorem, 338
- chain rule, 157
- change of variable, 155–162
- characteristic function, 331, 375
 - continuity of, 331
 - derivatives of, 332
 - of binomial distribution, 338
 - of convolution, 333
 - of normal distribution, 333
 - of Poisson distribution, 339
 - of uniform distribution, 339
 - uniform continuity of, 339
- characterization
 - of absolutely continuous functions, 135–137, 173
 - of compact metric spaces, 395
- classification of Borel sets, 255–257
- closed
 - ball, 393
 - set, 385, 389
- closure
 - of a set, 385, 389
 - under an operation, 1
- compact
 - group, 279

- set, 387, 391
- topological space, 391
- complete
 - measure or measure space, 30
 - metric space, 87, 394
 - ordered field, 379
- completeness
 - of $C[a, b]$, 87
 - of L^p , 99
 - of $C_0(X)$ and $C_0^c(X)$, 199
 - of $M(X, \mathcal{A}, \mathbb{R})$ and $M(X, \mathcal{A}, \mathbb{C})$, 119
- completion of a σ -algebra or measure, 31
- complex
 - conjugate, 108
 - measure, 118
 - numbers, 74, 382
 - valued functions, 74
- concentration of a measure on a set, 130
- condensation point, 252
- conditional expectation of X given \mathcal{B} , 341
 - basic properties, 342–344
- conditional expectation of X , given that $Y = y_j$, 341
- conditional probability of A given B , 340
- conjugate
 - complex, 108
 - exponents, 93, 108
 - space, 106
- construction of random variables, 316, 365
- continuous
 - function, 386, 390
 - linear operator, 105
- continuous measure, 11
- continuum hypothesis, 376
- convergence
 - almost everywhere, 80
 - almost sure, 319
 - almost uniform, 82
 - in L^p -norm, 96
 - in p^{th} mean, 96
 - in distribution, 328
 - and characteristic functions, 337
 - in mean, 82
 - in measure, 79
 - in probability, 319
 - in \mathbb{R}^d , 386
 - of binomial distribution to Poisson, 339
 - of random series, 355
 - weak, 140, 328
- convergent
 - sequence in a metric space, 86, 394
 - sequence in \mathbb{R}^d , 386
- convergent series, 88

- converse to strong law of large numbers, 324
 - convex
 - function, 98
 - set, 384
 - convolution, 153, 298, 301, 314
 - coset, 96, 283
 - countability of the set of rational numbers, 376
 - countable
 - additivity, 7, 113, 118
 - set, 375
 - subadditivity, 9, 13
 - countably
 - additive, 7
 - measure, 7
 - generated σ -algebra or measurable space, 102, 270
 - separated σ -algebra or measurable space, 271
 - subadditive, 13
 - counting measure, 8
 - Cousin's lemma, 430
 - covering
 - Vitali, 164
 - cross sections, 267–270
 - cube
 - closed, 164
 - half open, 24
 - open, 166
- D**
- d -system, 37
 - Daniell, P. J., 226
 - De Morgan's laws, 374
 - decreasing sequence of sets, 5
 - defined piecewise, 418
 - dense set, 86, 390
 - density
 - of a distribution, 310
 - of a random variable, 310
 - density in L^p of subspace determined by
 - continuous functions, 101
 - simple functions, 100
 - step functions, 101
 - density of $\mathcal{H}(X)$ in $C_0(X)$, 199
 - derivate
 - lower, 166
 - upper, 166
 - derivative, 157, 166
 - determinant
 - of a linear operator, 156
 - of a matrix, 155
 - determines, 100
- E**
- Egoroff's theorem, 81
 - elementary
 - integral, 227
 - outcome, 307
 - empirical distribution function, 326
 - enumeration, 375
 - equidecomposable, 418
 - equivalence
 - classes of functions, 96
 - of McShane and Lebesgue integrals, 437–438
 - relation, 376
 - essentially bounded function, 92
 - event, 307
 - existence
 - of sequences of independent random variables, 317, 365
 - expectation, 308
 - expected value, 308
 - experiment, 307

extended
 real numbers, 380
 real-valued function, 46
 extremal subset, 407
 extreme point, 407

F
 Fatou's lemma, 63
 field, 379
 of sets, 2
 ordered, 379
 filtration, 345
 finer, 67
 finite
 additivity, 7, 113
 intersection property, 391
 measure or measurable space, 9
 signed measure, 114
 variation, function of, 133

finitely additive, 7
 measure, 7, 111

Fourier
 inversion formula, 334
 transform, 331

free group, 421

freely generated, 421

freely tagged partition, 436
 δ -fine, 436

F_σ , 5, 183

Fubini's theorem
 on iterated integrals, 148, 224
 on the differentiation of series, 171

function
 continuous, 386, 390
 lower semicontinuous, 175, 209
 uniformly continuous, 386
 upper semicontinuous, 175

G
 G -equidecomposable, 418

G -paradoxical, 420

gambling, 347, 348

gauge, 430

Gaussian
 distribution, 310

random variable, 311

G_δ , 5, 183

general linear group, 284

generalized Riemann integral, 429

generated, 421

freely, 421

Glivenko–Cantelli theorem, 326
 graph of a function, 244
 group, 384
 abelian, 384
 commutative, 384
 compact, 279
 general linear, 284
 locally compact, 279
 orthogonal, 284
 topological, 279

H

Haar measure, 285
 examples, 285, 292, 297
 existence, 286
 left, 285
 right, 285
 uniqueness, 290

Hahn decomposition, 116
 theorem, 116

Hahn–Banach theorem, 401

Hamel basis, 30

has

a finite expected value, 308
 an expected value, 308

Hausdorff space, 391

Heine–Borel theorem, 387

Henstock–Kurzweil

integrability, 430, 432
 of characteristic function of rationals, 432
 integral, 429, 430, 432
 extension of Lebesgue integral, 434–435

Hilbert space, 90

Hölder's inequality, 93

homeomorphic, 390

homeomorphism, 390

homogeneity, 85

homomorphism of groups, 384

I

I -capacitable, 266
 i.i.d., 320
 i.o., 320
 ideal, 302, 414
 identically distributed, 320
 identification of functions that agree (locally)
 almost everywhere, 96
 image of a set, 374
 imaginary part, 74–75, 382

- increasing sequence of sets, 5
 independent events, 312
 random variables, 312
 and product measures, 313
 σ -algebras, 312
 index set, 375
 indicator function, 331, 375
 infimum, 379
 infinitely often, 320
 injection, injective function, 375
 inner measure, 33
 inner product, 89
 space, 90
 inner regularity, 190
 integrable function, 56, 399
 over A , 56
 uniformly, 129
 integral basic properties, 53–60
 Bochner, 399
 convergence theorems, 61–64
 definition, 53–56
 improper, 71
 of f over A , 56
 with respect to finite signed or complex measure, 120
 integration by parts, 151, 173
 interior of a set, 389
 intermediate value theorem, 387
 interval, d -dimensional, 14
 inverse Fourier transform, 334
 inverse image of a set, 374
 irrational numbers, set of as a Polish space, 243
 homeomorphic to \mathcal{N} , 255
 isometric isomorphism, 106
 isometry, 106
 isomorphic, 259
 isomorphism Borel, 259
 isometric, 106
 measurable, 259
 of groups, 384
 theorem for Borel sets, 259, 261
 iterated integrals, 147
- J**
- Jacobian, 158
 Jensen's inequality, 98
 for conditional expectations, 354
- Jordan decomposition of complex measure, 118
 of signed measure, 117
 theorem, 117
- K**
- kernel, 65
 Kindler, J., 226
 Kolmogorov's consistency theorem, 368
 inequality, 322
 zero-one law, 321
- L**
- L -almost everywhere, 235
 L -measurable, 235
 L -negligible, 235
 L -null, 235
 L -summable, 234
 L^1 -bounded set, 129
 Lebesgue density theorem, 169
 integrability, 56
 integral, 56
 measurable function, 42
 measurable set, 15
 measure, 18
 outer measure, 13, 14
 point, 174
 set, 174
 Lebesgue decomposition of a measure, 130
 theorem, 130
- left uniformly continuous, 281
- left Haar measure, 285
- Lévy's metric, 339
- lifting linear, 406
 of \mathcal{L}^∞ , 405
 of a σ -algebra, 413
- limit inferior, 381
 of a sequence in \mathbb{R}^d , 386
 of a sequence in \mathbb{R} , 381
 of a sequence in a metric space, 86, 394
 point, 385
 superior, 381
- line segment, 384
- linear function, 383
 functional, 106

- linear (*cont.*)
 - lifting, 406
 - operator, 105
 - order, 377
 - transformation, 105
- locally
 - almost everywhere, 92
 - null set, 92
- locally compact
 - group, 279
 - topological space, 182
- lower
 - bound, 379
 - derivate, 166
 - integral, 67
 - semicontinuous, 175, 209
 - sum, 67
- Lusin space, 274
- Lusin's theorem, 208
- M**
- martingale, 346
 - convergence theorem, 348
 - related to differentiation, 347–348, 353
 - relative to $\{\mathcal{F}_n\}$, 346
 - reverse, 356
- maximum, 43, 380
- McShane
 - integrability, 436
 - integral, 436
 - equivalent to Lebesgue integral, 437–438
- mean value theorem, 387
- measurability of analytic sets, 262
- measurable
 - function, 42, 73, 235
 - set, 15
 - space, 8
 - analytic, 270
 - standard, 270
- measure, 7
 - absolutely continuous, 122, 125
 - Borel, 11
 - complex, 118
 - continuous, 11
 - countably additive, 7
 - counting, 8
 - discrete, 11
 - finitely additive, 7, 111
 - Haar, 285
 - inner, 33
 - on (X, \mathcal{A}) or on X , 8
- outer, 33
- positive, 114
- product, 145
- Radon, 215–218
- space, 8
 - translation-invariant, 25, 285
- mesh, 70, 429
- metric, 86, 393
 - space, 86, 393
- metrizability
 - of second countable compact Hausdorff spaces, 186
 - of second countable locally compact Hausdorff spaces, 187
- metrizable space, 393
- metrize, 393
- minimum, 43, 380
- Minkowski's inequality, 94
- modular function, 294
- monotone class, 40
 - theorem, 40
- monotone convergence theorem, 61
 - for conditional expectations, 344
 - for Henstock–Kurzweil integral, 434
 - for McShane integral, 437
- monotonicity, 13
- μ -a.e., 50
- μ -almost everywhere, 50
 - on E , 50
- μ -integrable function, 56
- μ -measurable, 31
- μ -negligible, 30
- μ -null, 30
- μ^* -measurable sets, 15, 212–218
- N**
- negative
 - part, 46, 117
 - set, 115
- neighborhood, open, 389
- net, 305
- non-measurable set, 27–29
- non-regular Borel measure, 197
- norm, 84, 385
 - associated to an inner product, 90
 - of a linear operator, 106
 - of a partition, 70, 429
- normal
 - distribution, 310
 - number, 325
 - random variable, 311
 - to base b , 325
- normal topological space, 183

- normed
 linear space, 85
 vector space, 85
nowhere dense set, 395
- O**
one-point compactification, 185
open
 ball, 385, 393
 cover, 386, 391
 neighborhood, 389
 set, 385, 389
optional time, 345
order topology, 393
ordinal numbers
 spaces of, 189
orthogonal, 91
 group, 284
 matrix, 284, 418
 operator, 426
 vectors, 426
orthonormal basis, 426
outer measure, 12, 33
outer regularity, 190
- P**
paradoxical, 420
parallelogram law, 90
partial order, 377
partially ordered set, 377
partition, 67, 377, 429
path, 357
 π -system, 37
point
 of density, 169
 of dispersion, 169
point at infinity, 185
point mass, 8
Poisson distribution, 319
polar coordinates, 162
Polish space, 239
positive
 linear functional, 107, 181, 192, 202
 measure, 114
 part, 46, 117
 set, 115
power set, 374
probability, 307
 space, 307
product
 measure, 145
 of Borel σ -algebras, 219–220, 243
- of countably many probability measures,
 365
of indexed family of sets, 375
of infinite sequence of measurable spaces,
 243
of sets, 375
of uncountably many probability measures,
 370
 σ -algebra, 143, 243
topology, 392
- Q**
quotient space, 96
- R**
Radon measure, 215–218
Radon–Nikodym
 derivative, 126
 theorem, 123, 125, 129, 404
random variable, 308
 continuous, 309
 discrete, 309
 real-valued, 308
real
 numbers, field of, 379
 part, 74–75, 382
rectangle with measurable sides, 143
reduced word, 421
refinement, 67
regular
 Borel measure, 190
 Borel product, 222
 finite signed or complex measure, 200
 measure, 23, 34, 189
regularity
 of finite Borel measures on Polish spaces,
 245
 of finite Borel measures on \mathbb{R}^d , 34
 of finite Borel measures on Souslin spaces,
 275
 of Lebesgue measure, 23
relation, 376
relatively compact set, 263
reverse martingale, 356
Riemann
 integrability, 67, 430
 integral, 68, 430
 sum, 70, 429, 433
Riesz Representation Theorem, 192
Riesz, F, 164
right
 uniformly continuous, 281

right Haar measure, 285
rising sum lemma, 164

S

Saks–Henstock lemma
for Henstock–Kurzweil integral, 433
for McShane integral, 437
sample point, 307
Schröder–Bernstein theorem, 261, 376, 418
second countable topological space, 391
second moment, 309
sections, 144
semimetric, 86
seminorm, 85
separability of L^p , 102
separable space, 86, 390
separated σ -algebra or measurable space, 270
separation
of points
 by a family of functions, 392
 by a family of sets, 270
of sets
 by Borel sets, 257
 by open sets, 182
theorem for analytic sets, 257
set
Lebesgue measurable but not Borel measurable, 48
not Lebesgue measurable, 27–29
theory, basic concepts, 373–378
 σ -algebra, 2
 generated by a collection of sets, 3, 270
 σ -algebra or measurable space
 countably generated, 102, 270
 countably separated, 271
 separated, 270
 σ -compact topological space, 183
 σ -field, 2
 σ -finite
 measure or measurable space, 9
 set, 9
 σ -ring, 228
signed measure, 114
simple function, 42, 397
simulation
 of normal random variables, 319
 of random variables, 317
singular part of a measure, 130
singularity of measures, 130
Souslin space, 274
special orthogonal group, 418
standard deviation, 309

standard measurable space, 270
step function, 101, 102
stochastic process, 345
 continuous-time, 345
 discrete-time, 345
Stone's condition, 227
Stone, M. H., 226
Stone–Weierstrass theorem, 392
stopping time, 345
strong law of large numbers, 322
 derived from martingale convergence theorem, 356
strongly
 integrable function, 399
 measurable function, 397
subcover, 386, 391
submartingale, 347
subspace
 of a topological space, 390
 of a vector space, 383
summable, 234
summable function, 56
sup norm, 85
supermartingale, 347
support
 of a function, 184
 of a measure, 207
supremum, 379
surjection, surjective function, 375
symmetric
 difference, 373
 set, 280

T

tag, 70, 429
tagged partition, 70, 429
 δ -fine, 430
 subordinate to δ , 430
tagged subpartition, 432
 δ -fine, 433
 subordinate to δ , 433
tail
 σ -algebra, 321
 event, 321
three series theorem, 327
Tietze extension theorem, 188
tightness, 335
 uniform, 335
Tonelli's theorem, 147
topological
 dual space, 106
 group, 279
 space, 389

topology, 389
 generated by a metric, 393
 generated by family of functions,
 390
 generated by family of sets, 390
 induced by a metric, 393
 induced by another topology, 390
 inherited from a topological space,
 390
 metrizable, 393
 order, 393
 product, 392
 usual, 389
 weaker, 390
 total variation
 of complex measure, 119
 of signed measure, 117
 totally bounded space or set, 395
 trace
 of \mathcal{A} on C , 36
 of μ on C , 36
 translate, 25
 translation-invariant measure, 25, 285
 triangle inequality, 85, 86
 truncated random variable, 323
 Tychonoff's theorem, 392

U

unconditionally convergent series, 91
 uniform convergence of distribution
 functions, 326
 uniform distribution, 310
 uniform norm, 85
 uniformly
 absolutely continuous, 129
 dense family of functions, 392
 integrable, 129
 uniformly continuous
 function, 386
 left, 281
 right, 281
 unimodular, 294
 universal set, 253
 universally measurable set, 264
 upcrossing, 349
 inequality, 350

upper
 bound, 379
 derivate, 166
 integral, 67
 semicontinuous, 175
 sum, 67
 Urysohn's lemma, 184

V
 vanishes
 at $-\infty$, function that, 133
 at infinity, function that, 199
 variance, 309
 variation
 of a complex measure, 118
 of a signed measure, 117
 of a vector-valued measure, 404
 of F over $[a, b]$, 133
 vector
 lattice, 227
 space, 383
 vector-valued measure, 404
 version of a conditional expectation, 342
 Vitali
 covering, 164
 covering theorem, 164
 volume, 14

W

weak
 convergence, 140
 topology, 274
 weak law of large numbers, 320
 weak-* topology, 274
 Weierstrass approximation theorem, 325
 Wiener measure, 356
 existence, 357–361
 Wilson, Trevor, 419
 word, 421

Z

Zaanen, A. C., 226
 zero-dimensional space, 251
 Zorn's lemma, 377