

[total variation, aka first-order variation] Let $\pi_n = \{t_0 = 0, t_1, \dots, t_{m_n} = T\}$ be a partition of $[0, T]$ with $n(\pi) = m_n$ subintervals. The maximum step size of the partition is denoted by

$$|\pi_n| := \max_{1 \leq k \leq m_n} (t_k - t_{k-1}).$$

For a given partition π_n , we define the corresponding measure of the oscillation of a function f by

$$V_n(f) = \sum_{k=1}^{m_n} |f(t_k) - f(t_{k-1})|.$$

The total variation of f is defined as

$$V(f) := \lim_{|\pi_n| \rightarrow 0} V_n(f) = \lim_{|\pi_n| \rightarrow 0} \sum_{k=1}^{m_n} |f(t_k) - f(t_{k-1})|$$

Lemma 0.0.1. *An*