A Comparative Simulation Study of MIMO System with ZF, MMSE and ML Equalization

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***Abstract*—In the last decade, to cope with the increasing demand for high data rates, MIMO technology emerged as a trend availing the use of multiple transmitters and receivers to aid the transfer of data without requiring higher bandwidth. It makes use of the multipath phenomenon where the waves rebound off from different objects and arrive at the receiver at receiving antenna but at different times. Spatial diversity and pre-coding makes the usage of MIMO to yield better performance. In this paper equal number of transmitting antennas and receiving antennas are considered. The channel of consideration here will be the Rayleigh multipath channel with binary phase shift keying(BPSK). The major issue in such a channel is the inter symbol interference. In order to minimize the ISI, various equalization techniques have been proposed such as zero forcing(ZF), minimum mean square error(MMSE), zero forcing with successive interference cancellation and optimal ordering(ZF-SIC), Minimum mean square error with successive interference cancellation(MMSE-SIC) and optimal ordering and maximum likelihood(ML). The simulation result when more number of transmitting and receiving antennas are used are measured for zero-forcing equalization scheme. We will design and simulate the various equalization techniques on a flat fading multipath Rayleigh channel and study their corresponding plots. The bit error rate corresponding to each scheme is delivered and the optimal method is identified..**

***Index Terms*— bit error rate, MIMO, minimum mean square, maximum likelihood, zero forcing.**

I. INTRODUCTION

Communications utilizing multiple antennas have emerged as a new trend in the wireless field. With this advent of wireless communications, the future technology needs to sustain services demanding high data rates such as quality voice, pictures and streaming video. The objective for attaining such high speed data transmission at the same time satisfying the reliability needs through wireless media in a dynamic fashion has been the motivation for a plethora of research in this signal processing domain. A primary challenge due to fading is the variation of the channel with time. In order to meet the demands for increased data rates, the consequences of multipath propagation and inter symbol interference(ISI) have aggravated the needs for complex equalizers. It becomes essential to clean up the interference from the signal. This is facilitated by the use of equalization techniques. Equalization techniques are basically employed to reduce the error between the actual output and the desired output. The filters at both the transmitter and the receiver are responsible for the interference that occurs in the transmitted signals along with the channel interference. The rest of the paper is organized as follows. In section II the description of the MIMO model which is being used in the paper has been elucidated along with the equalization schemes which are being simulated. The schemes include zero-forcing equalization, minimum mean square equalization with successive interference cancellation and optimal ordering and maximum likelihood equalization. Section III contains the evaluation done for these schemes. Section IV describes the related work on this domain. Section V includes the summary and conclusion of the paper and references in section VI.

II. DESCRIPTION

In this paper a M*N MIMO system is considered, where there are M transmitting antennas and N receiving antennas. The signals obtained at the receiving end act as the input to the equalizers. A flat multipath fading Rayleigh channel with binary phase shift keying modulation is considered.

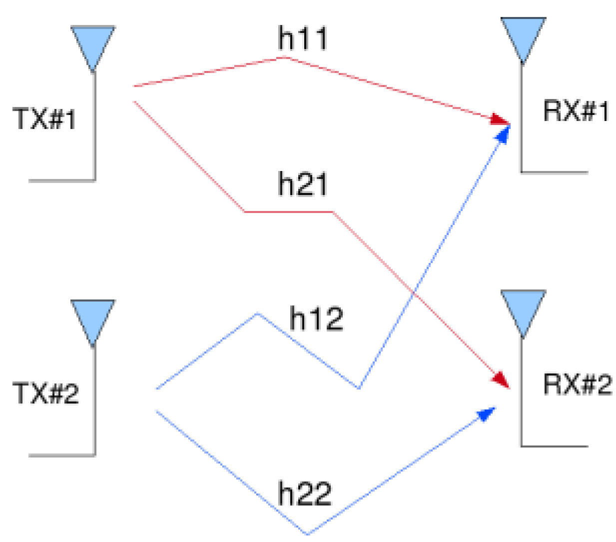

Fig. 1. 2*2 MIMO channel

For an MIMO channel comprising of two transmitters and two receivers, the normal working of the channel comprises of transmitting a sequence $(x_1, x_2, .. x_n)$ with each signal being sent in a particular slot of time. In this paper, as there are two receivers being considered, the symbols are grouped into sets of two. The first transmitting antenna transmits x_1 in the first time slot and x_3 in the second time slot allocated to it. Similarly, the second transmitting antenna transmits x_2 and x_4 in the first and second slot provided respectively. By transmitting in sets of two, the time slots required to send the data gets reduced to half, leading to doubling of the data rate. This is the skeleton on the working of a simple MIMO transmission system. The problem arises due to the interference which occurs between these transmitted signals. Equalizing these signals forms the remnant parts of the paper. The output obtained from the channel is written as

$$y = Hx + wH \;\epsilon\; R^{m*n} \tag{1}$$

Where,

$H \in R^{m*n}$ is the MIMO channel matrix,

$x \in A^n$ is the transmitted symbol matrix,

$W \in R^n$ is an additive noise matrix.

The performance of MIMO communication system is measures in terms of binary success rate(BER). In this paper a 2*2 MIMO channel has been utilized to simulate the equalization schemes.

A few assumptions are made for the MIMO system considered in this paper, which are as follows.

a. The channel considered in this study is a flat fading channel in which the coherence bandwidth of the channel is greater than the bandwidth of the signal which ensures a uniform fading for all the frequency components.
b. Each of the transmitters and the receivers are assumed to be independent and randomly varying in time.
c. The receive antenna is found to have a noise with a Gaussian probability density function $p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(n-\mu)^2/2\sigma^2}$ with $\mu = 0$ and $\sigma^2 = \frac{N_o}{2}$.
d. The transmitted symbols get multiplied with a random varying complex number $h_{j,i}$, which will be known at the receiver. The real and imaginary parts of $h_{j,i}$ are also Gaussian distributed with a mean of $\mu_{h_{j,i}} = 0$ and variance of $\sigma^2_{h_{j,i}} = \frac{1}{2}$.

1. ZERO FORCING (ZF) EQUALIZER FOR MIMO CHANNEL:

The signal received on the first antenna is represented as

$$\boldsymbol{y_1} = \boldsymbol{h_{1,1}x_1} + \boldsymbol{h_{1,2}x_2} + \boldsymbol{n_1} = [\boldsymbol{h_{1,1}} \;\; \boldsymbol{h_{1,2}}] \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \end{bmatrix} + \boldsymbol{n_1} \tag{2}$$

The signal received on the second antenna is represented as

$$\boldsymbol{y_2} = \boldsymbol{h_{2,1}x_1} + \boldsymbol{h_{2,2}x_2} + \boldsymbol{n_2} = [\boldsymbol{h_{2,1}} \;\; \boldsymbol{h_{2,2}}] \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \end{bmatrix} + \boldsymbol{n_2} \tag{3}$$

Where ,

y_1, y_2 are the received symbols on the first and second antennas respectively.

$\boldsymbol{h_{1,1}}$ is the channel from the first transmit antenna to the first receive antenna,

$\boldsymbol{h_{1,2}}$ is the channel from the second transmit antenna to the second receive antenna.

$\boldsymbol{h_{2,1}}$ is the channel from the first transmit antenna to the second receive antenna,

$\boldsymbol{h_{2,2}}$ is the channel from the second transmit antenna to the second receive antenna

$\boldsymbol{x_1}$, $\boldsymbol{x_2}$ are the transmitted symbols,

$\boldsymbol{n_1}, \boldsymbol{n_2}$ are the noise on the 1st and 2nd receive antennas. $\boldsymbol{n_1}, \boldsymbol{n_2}$ are additive white Gaussian noise with a mean value of 0 and unit variance.

$\boldsymbol{h_{1,1}}, \boldsymbol{h_{1,2}}$ are the channel impulse response coefficients between receiver 1 and the two transmitters. $\boldsymbol{h_{2,1}}, \boldsymbol{h_{2,2}}$ are the channel impulse response coefficients between receiver 2 and the two transmitters. It is assumed that the receiver knows $\boldsymbol{h_{1,1}}, \boldsymbol{h_{1,2}}, \boldsymbol{h_{2,1}}, \boldsymbol{h_{2,2}}$. The receiver also knows $\boldsymbol{y_1}$ and $\boldsymbol{y_2}$. The unknown are $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$.

The matrix notation for the equation is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{4}$$

Equation "(4)" can be represented as,

$$y = Hx + n \tag{5}$$

Where,

y is the received signal,

H is the Rayleigh fading channel with independent, identically distributed,

x is the transmitted signal which comprises of real and imaginary parts,

n is the additive white noise (AWGN).

The transmitted signal gets corrupted by both the Gaussian noise which is appended as well as the channel noise.

To solve for x. we need to find a matrix W which satisfies

WH = I, which is the zero forcing(ZF) linear detector for meeting the constraint is given by,

$$W = (H^H H)^{-1} H^H \tag{6}$$

This matrix is also known as the pseudo inverse for a general M*N matrix.

It is represented as ,

$$H^H H = \begin{bmatrix} \ddot{h}_{1,1} & \ddot{h}_{2,1} \\ \ddot{h}_{1,2} & \ddot{h}_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{1,2} + h_{2,1}h_{2,2} \\ h_{1,2}h_{1,1} + h_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \tag{8}$$

The zero forcing equalizer tries to null the interfering terms while performing the equalization, i.e. when solving for x_1 the interference from x_2 is tried to be nullified and vice versa. The zero forcing equalizer is known for its simplicity and ease for implementation

2. ZERO FORCING WITH SUCCESSIVE INTERFERENCE CANCELLATION :

By utilizing zero forcing (ZF) equalization, the receiver obtains the estimates of 2 transmitted symbols x_1, x_2. ie

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{9}$$

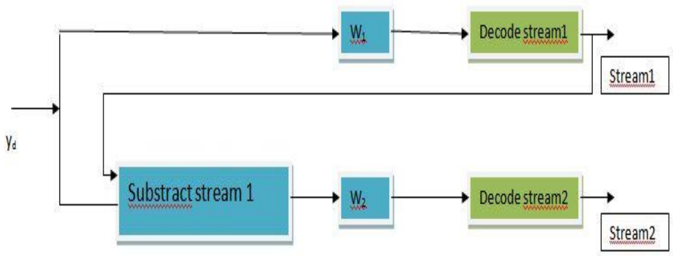

Fig. 2. Successive interference cancellation technique(SIC)

Successive interference cancellation (SIC) is done by decoding the signals from the strongest interferers and subtracts the re-encoded signal, effectively increasing SIR(signal to interference ratio). This is iterated for the first k interferers called k-SIC.

3. OPTIMAL ORDERING:

In 2*2 MIMO channel, the effect of $\hat{x}_1$ or $\hat{x}_2$ is chosen based on the received power at both the antennas. The power corresponding to $\hat{x}_1$ is given by

$$\boldsymbol{Px_1} = |\boldsymbol{h_{1,1}}|^2 + |\boldsymbol{h_{2,1}}|^2 \tag{10}$$

The received power corresponding to $\hat{x}_2$ is given by

$$\boldsymbol{Px_2} = |\boldsymbol{h_{1,2}}|^2 + |\boldsymbol{h_{2,2}}|^2 \tag{11}$$

If $\boldsymbol{Px_1} > \boldsymbol{Px_2}$ then the receiver decides to remove the effect of $\hat{x}_1$ from the received vectors y_1 and y_2 and then re-estimates $\hat{x}_2$.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} == \begin{bmatrix} y_1 - h_{1,1} & \hat{x}_1 \\ y_2 - h_{2,1} & \hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & x_2 + n_1 \\ h_{2,1} & x_2 + n_2 \end{bmatrix} \tag{12}$$

In matrix form,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,2} \\ h_{2,2} \end{bmatrix} x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{13}$$

$$r = hx_2 + n \tag{14}$$

The optimal way of combing the information from multiple copies of the received symbols in receive diversity is by applying maximal ratio combining (MRC).

Additional gain can be achieved by employing optimal ordering to successive interference cancellation. The problem of propagation of error is handled by assuming optimal ordering. The purpose optimal ordering is to ensure that the signal with lower error probability is selected, thus eliminating the probability of faulty interference cancellation and providing a minimal rate of error when compared against the successive interference cancellation

The symbol after equalization is

$$\hat{x} = \frac{hH_r}{hH_h} \tag{15}$$

The zero forcing equalizer with optimal ordering is obtained as above.

4. MMSE (MINIMUM MEAN SQUARE ERROR) EQUALIZER:

The minimum mean square error equalizer is a linear equalizer, which directs at minimizing the variance of the difference between the transmitted signal and the signal which is obtained at the equalizer output. Apart from equalizing the channel, MMSE equalizer also subdues the noise in the channel and the ISI components in the output.

This method tries to find the coefficient W which can minimize the criterion below.

$$E\{[Wy - x][W_y - x]^H\} \tag{16}$$

Solving,

$$W = [H^H H + N_0 I]^{-1} H^H \tag{17}$$

An estimate corresponding to the signals transmitted is obtained for x_1 and x_2 as follows

$$\begin{bmatrix} \widehat{x_1} \\ \widehat{x_2} \end{bmatrix} = (H^H + N_0 I)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{18}$$

Successive interference cancellation and optimal ordering are allowed in a similar way as the zero-forcing equalizer. When the effect of one of the input signals is removed, the system acts like a one transmitter 2 receiver system, which is equalized by the maximal ratio combining(MRC) scheme. The MMSE equalizer helps in achieving as minimum MSE as possible. When the noise term N_o turns 0, MMSE equalizer becomes a zero-forcing equalizer.

5. MAXIMAL RATIO COMBINING:

Maximal ratio combining (MRC) involves multiplying the signal with a weight in proportion with its amplitude. So, signals that are strong are further amplified, while the weaker ones get weakened. It combines the signals in such a weighted manner while maintaining in phase to achieve highest signal to noise ratio possible. It can be defined as a method of combining such that when the signals are added, the gain of each channel will be proportional to the RMS value and inversely proportional to the square of the noise in that channel. It is also popularly called the pre detection or ratio squared combining.

6. MAXIMUM LIKELIHOOD EQUALIZATION:

In this equalization technique, the receiver tries to find the $\hat{x}$ to reduce $J = |y - Hx|^2$

$$J = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\|^2 \tag{19}$$

As it uses a BPSK modulation, x_1 and x_2 can take combinations of either +1 or -1.

$$J_{+1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right\|^2 \tag{20}$$

$$J_{+1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ -1 \end{bmatrix} \right\|^2 \tag{21}$$

$$J_{-1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\|^2 \tag{22}$$

$$J_{-1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ +1 \end{bmatrix} \right\|^2 \tag{23}$$

The symbol which is transmitted gets its estimate from the four values obtained above.

If the minimum is $J_{+1,+1}$ => [1 1] else
$J_{+1,-1}$ => [1 0] else
$J_{-1,+1}$ => [0 1] and
$J_{-1,-1}$ => [0 0].

III. EVALUATION

To demonstrate the performance of the MIMO system with the various equalization schemes, simulations were performed.

SIMULATION PROCEDURE:

a. The simulation is done by generating a random binary sequence of +1's and -1's with equal probability.
b. It is then modulated using binary phase shift keying.
c. The generated input is then coupled in pairs and sent in the time slot.
d. The signals are sent through a multipath fading Rayleigh channel with noise. Gaussian white noise is added to the channel after multiplication with the generated symbols. Various equalization techniques are adopted for channel equalization on the received symbols(at the receiver).
e. The bit errors are evaluated by employing hard decision decoding.
f. The procedure is repeated for different values of E_b/N_0 and compared with the theoretical results.

The flowchart for the simulation is shown below.

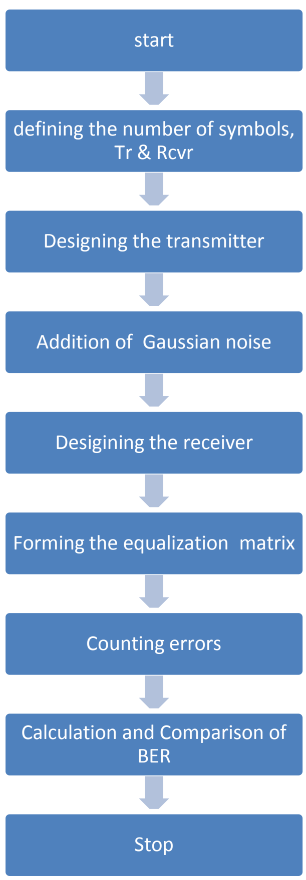

Fig. 3. Simulation procedure for simulating the equalization schemes.

ZERO-FORCING EQUALIZER:

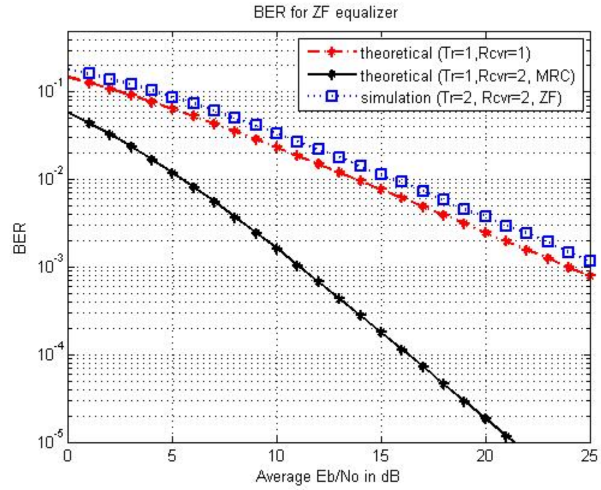

Fig. 4. Simulation for zero forcing equalizer

The simulated results for zero forcing equalization in 2*2 MIMO system is found to match with the theoretical 1*1 system. But it can be observed that it gives no significant improvement despite having 2 antennas. It does not provide the enhancement of data rates . The receivers are the essential parts when it comes to improvising the data rate gains. Alternative equalization schemes are adopted to move the the curve towards a maximal ratio combining curve obtained using one transmitter and two receivers.

ZERO-FORCING EQUALIZER WITH SUCCESSIVE INTERFERENCE CANCELLATION:

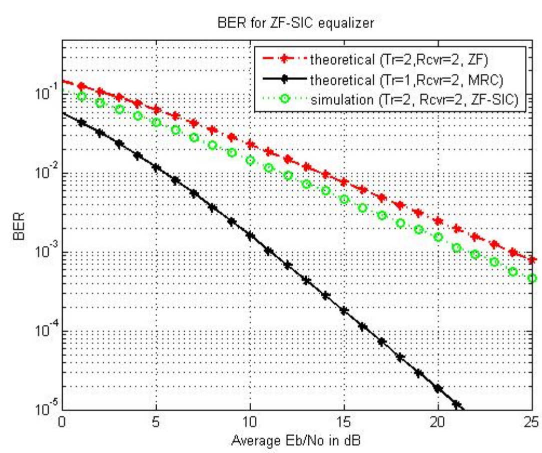

Fig. 5. Simulation for zero forcing equalizer with successive interference cancellation

The zero-forcing detectors have lesser computational calculations as the majority of operations include matrix operations. It has diversity in the order of N-M+1, where M is the number of transmitting antennas and N is the number of receiving antennas. The additional implementation of successive interference cancellation to the zero forcing equalizer results in the improvement of around 2.2 dB.

ZERO-FORCING EQUALIZER WITH SUCCESSIVE INTERFERENCE CANCELLATION AND OPTIMAL ORDERING:

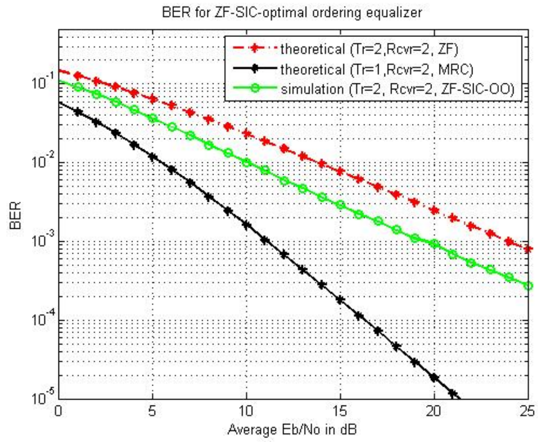

Fig. 6. Simulation of zero forcing equalizer with SIC and optimal ordering

It can be seen that by using optimal ordering along with SIC results in an additional 2dB improvement over the SIC method. The ordered variant of zero-forcing equalization shows better performance over both the normal and SIC variant of zero-forcing equalizer.

COMPARISON OF ZERO-FORCING EQUALIZERS:

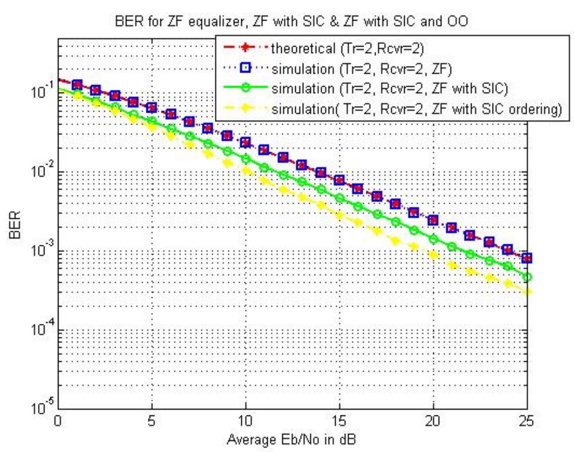

Fig. 7. Comparison of zero forcing equalizers

Figure 7 shows the comparison amongst zero-forcing, zero-forcing with SIC and zero-forcing with optimal ordering. It is seen that with optimal ordering, highest improvement can be observed.

MINIMUM MEAN SQUARE ERROR EQUALIZER:

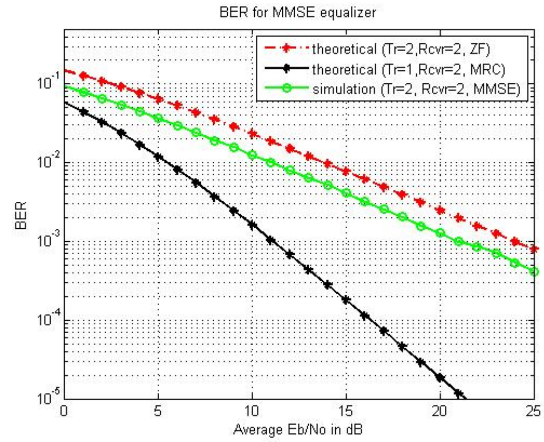

Fig. 8. Simulation of MMSE equalizer

MMSE equalizer is also popularly called the optimal detector owing to its ability to minimize both interference and noise. The simulation results in about a 3dB improvement over zero-forcing in a MIMO system with Rayleigh flat fading.

MINIMUM MEAN SQUARE ERROR WITH SUCCESSIVE INTERFERENCE CANCELLATION AND OPTIMAL ORDERING:

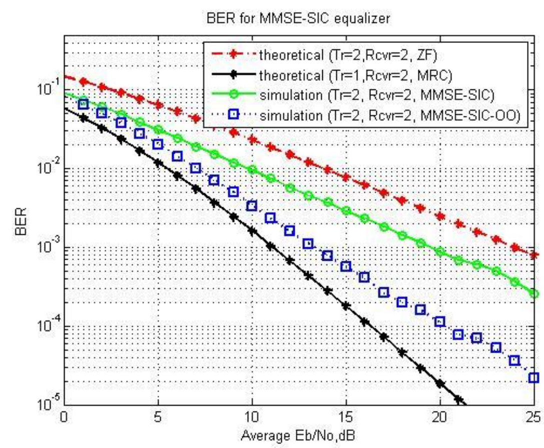

Fig. 9. Simulation of MMSE equalizer with SIC and optimal ordering

The simulation results for the MMSE equalizer with optimal ordering shows that there is a 5 dB improvement over the MMSE with successive interference cancellation. The significance of the successive interference cancellation for MMSE arises because of its ability to reduce the noise from being amplified. The performance is further incremented by ordering these process, increasing the reliability of the method by providing error free cancellation schemes. Thus making the optimally ordered successive interference cancellation superior than the simple SIC. But the results obtained are marginally less when compared to the maximum likelihood equalization.

MAXIMUM LIKELIHOOD EQUALIZER:

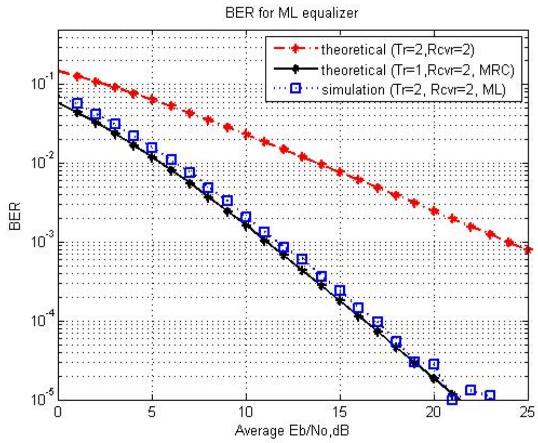

Fig. 10. Simulation of maximum likelihood equalizer

It can be observed that the simulation results obtained with maximum likelihood equalizer is close to the output obtained using maximal ratio combining of 1 transmitter and 2 receivers. This gives both the required throughput gain as well as the diversity gain which is desired. Maximum likelihood receiver carries out an optimal vector decoding by decreasing the probability of error. Though it has the disadvantage of exponential increase of complexity as per the modulation order and the increase in the number of transmitters.

Maximum likelihood equalizer shows the best performance among all the equalizers considered. The bit error rate values are found to be decreasing in the order of

ZF>MMSE>ZF-SIC>MMSE-SIC>ZF-SIC-OO>MMSE-SIC-OO>ML

ZERO-FORCING FOR A 4*4 MIMO CHANNEL:

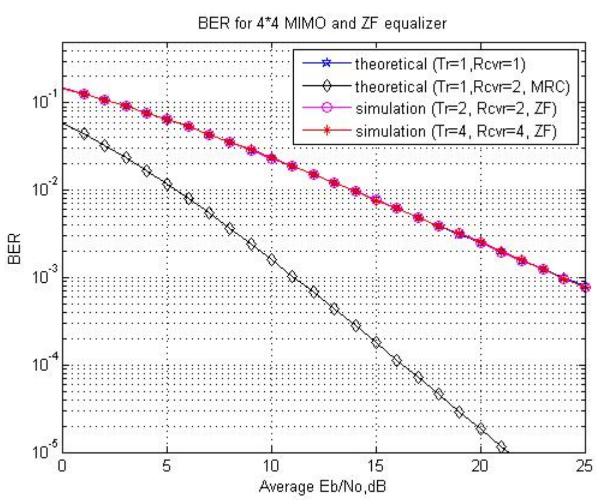

Fig. 10. Simulation of ZF for 4*4 MIMO system

It can be seen that there is no difference in the performance when the number of transmitters and receivers is doubled. It can be seen that having a 4 transmitter and 4 receiver antennas gives no better performance over the 2*2 MIMO system.

IV. RELATED WORK

Eun cheol et al, have demonstrated the co-channel interference(CCI) alleviation for co-operative communication systems and have plotted the BER vs SNR curve for the schemes[3]. CCI is responsible for determining the amount of resource that can be reutilized. Rohit et al, have discussed the minimum mean square error(MMSE) equalizer for a BPSK, QPSK and 16-QAM modulation[6]. It was observed that BPSK gave the best performance when compared to the other two schemes. It also gives insight into the performance of the linear equalizers which demand lower complexity when compared to the other available schemes. Satish Kumar et al, have compared and analyzed the MMSE equalizer for varying sizes of transmitters and receivers and obtained[7]. Low complexity techniques for equalization have their usage over complex schemes and their potential was demonstrated using a basis expansion model (BEM) with Gaussian noise by Leus et al. [8]

Future work can include evaluating the performance for other sizes of the receivers. Apart from that, blind processing strategy which uses the transmitted signal properties to equalize the received signal can be done.

V. SUMMARY AND CONCLUSION

Equalization schemes demand importance because of their continual usage in wireless systems requiring high data rates. In the cases of mobile fading channels, the inter-symbol interference is accounted for.

In this paper, the inter symbol interference which arises due to the multipath fading is reduced by employing various equalization techniques

Zero forcing equalizer is suitable and performs theoretically, but it cannot handle noisy environments. To combat this drawback, the Minimum Mean Square Error (MMSE) equalizer counterbalances the inter symbol interference that arises by eliminating the noise in the channel.

By observation it can be seen that by using the Maximum Likelihood equalization, interference can be handled.

VI. References

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