Automata and Logics Over Nested Data

Adriana Baldacchino
Supervised by Prof. Andrzej Murawski

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UNIVERSITY OF OXFORD

Automata over a finite alphabet

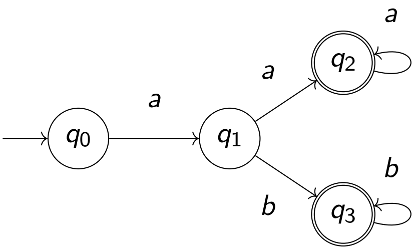
$a_i \in \Sigma$, where Σ is finite.

$$a_1 a_2 \ldots a_k \longrightarrow \boxed{\begin{array}{c}\text{Automaton}\\ \mathcal{A}\end{array}} \longrightarrow \text{accept/reject}$$

$\mathcal{L}(\mathcal{A}) = \{w \in \Sigma^* : \mathcal{A} \text{ accepts } w\}$.

Transitions: Automata over a finite alphabet

Each $a \in \Sigma$ determines the next move



Data Automata

- $a_i \in \Sigma$, where Σ is a finite set of *tags*;
- $d_i \in D$, where D is an infinite set of *data values*.

$$(a_1, d_1)(a_2, d_2)\ldots(a_k, d_k) \longrightarrow \boxed{\begin{array}{c}\text{Data Automaton}\\ \mathcal{A}\end{array}} \longrightarrow \quad \text{accept/reject}$$

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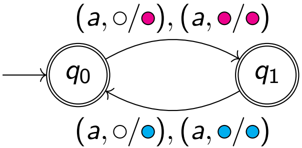
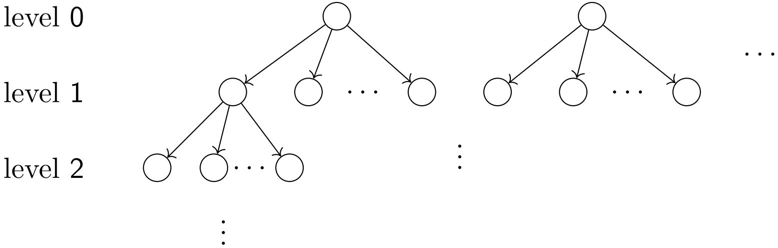
We can't simply specify a transition for each $(a, d) \in \Sigma \times D$ as this set is infinite.

Transitions: Class Memory Automata

- One way to handle infinite data is to use a finite set of colours (labels) $\mathcal{M} = \{{\color{magenta}\bullet}, {\color{cyan}\bullet}, {\color{yellow}\bullet}\}$.
- Each data value starts off as unlabelled ($\circ$), and is given a colour when encountered.
- These models are called (weak) Class Memory Automata.



Nested Data



Transitions for Automata over Nested Data

The most straightforward way to define transitions on nested data is to specify the labels of all the ancestors of a data value. For example, to transition on a level 2 data value, we need to specify its label, the label of its parent and its parent's parent.

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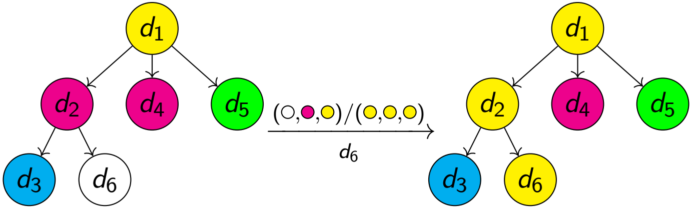

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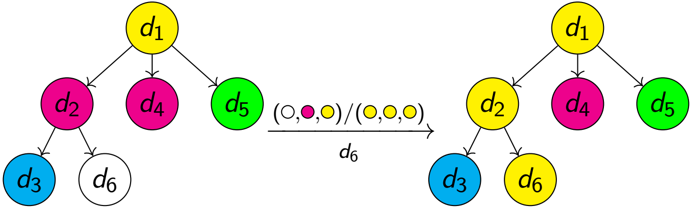

These automata are called *Nested-Data Class Memory Automata (NDCMA)*, and were introduced in [2].

Limitations of NDCMA

- Since the number of transitions is *finite* and NDCMA require us to label *all ancestors*, this formulation clearly limits us to process data values up to some bounded level/depth k.

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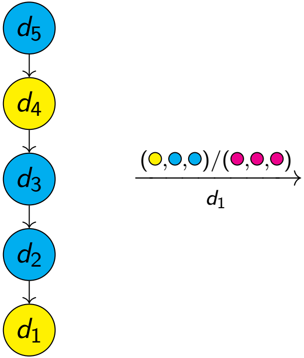
- As they have a decidable emptiness problem and nice closure properties, deterministic NDCMA were successfully applied to solve some decidability problems for fragments of ML [3, 1].
- Our goal is to extend this model to handle unbounded depth data values in a way that preserves these properties. We hope that this can help us extend the previous work on programming languages to handle programs with nested loops.

Memoryless Unbounded NDCMA

One way we can handle unbounded data is by matching our labelling, say (, ,) to a *sublabelling* of the ancestors of d:



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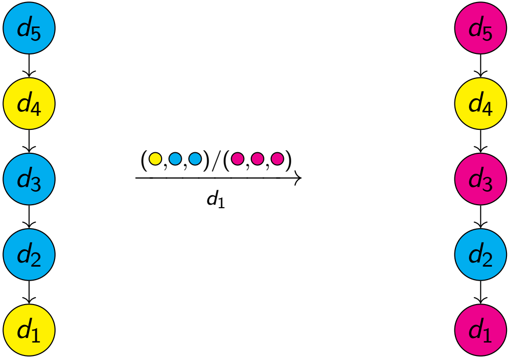

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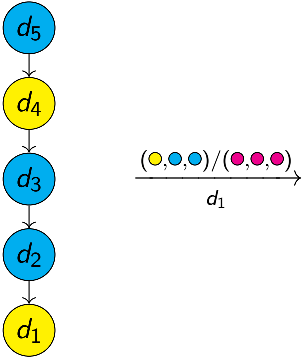

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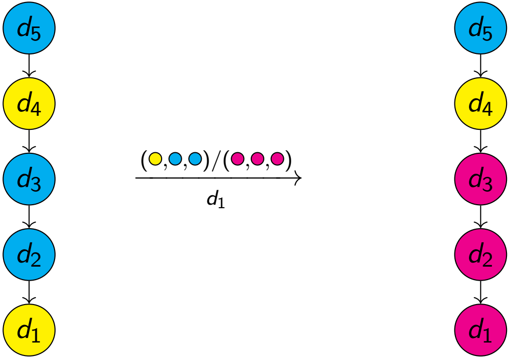
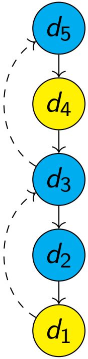


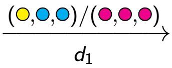
k-Memory Unbounded NDCMA

A more controlled way is to maintain a bounded memory $m(d)$ for each data value.

$m(d_3) = (d_5)$

$m(d_1) = (d_3, d_5)$





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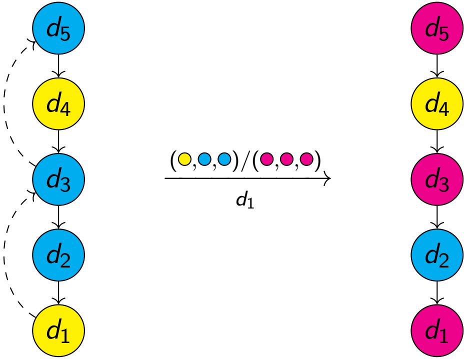


Simulating Loops Example

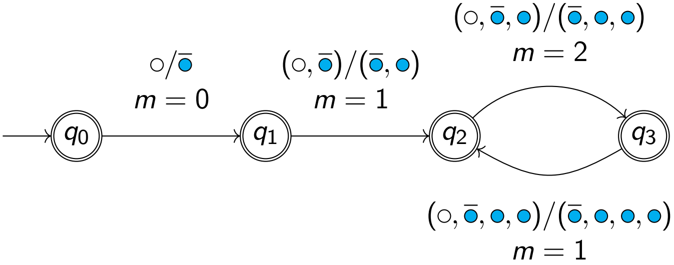
Suppose we had a very simple program,

$$d_0; \mathit{while}\ \mathrm{true}\ \mathit{do}\ (d_1; d_2)$$

and we want to accept all execution traces of this program.

Simulating Loops Example

We simulate this using an automaton with labels $\mathcal{M} = \{\bullet, \bar{\bullet}\}$, where $\bar{\bullet}$ represents the 'head' of the program.



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- It is clear that the words accepted by this k-memory NDCMA are of the form $d_1 d_2 d_3 \dots d_k$ where d_i is the parent of d_{i+1} in the underlying structure.

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- Even though the correct nesting properties are preserved, the language of this automaton is equivalent to the one that would be generated by say *while* true *do* d_1.
- This is to say the loops cannot be deduced from the language itself, allowing us to compare differently shaped programs with the same underlying traces.

Closure Properties

- Using the theory of well-structured transition systems (WSTS) we showed that the language emptiness problem is decidable for both these models.

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- Using the theory of well-structured transition systems (WSTS) we showed that the language emptiness problem is decidable for both these models.
- We further present closure and decidability properties for these languages.

	$L_1 \cup L_2$	$L_1 \cap L_2$	L^c	$L_1 \subseteq L_2$	$L_1 = L_2$
Memoryless NDCMA	✓	✓	X	X	X
k-memory NDCMA	✓	X	X	X	X
Det. k-memory NDCMA	X	X	✓	X	?
Det. Bounded NDCMA	✓	✓	✓	✓	✓

Further Considerations

- The question of whether equivalence is decidable for deterministic k-memory NDCMA is still open. It is still possibly decidable, as there exist automata with decidable equivalence/undecidable inclusion — DPDA.
- We hope to also extend the work in [1] to find an analogous translation of FOSC with loops to k-memory NDCMA.
- Logics for data words with unbounded data have not yet been investigated thoroughly. It would be interesting to see an analogous result as in [4] — where satisfiability of a logic over data words (NDLTL) was shown using an equivalent model to bounded NDCMA.

Thank you!

Slides can be found at https://abaldacchino.github.io/wilpresentation.pdf

References I

[1] Benedict Bunting and Andrzej Murawski. "Contextual Equivalence for State and Control via Nested Data". In: *Proceedings of the 39th Annual ACM/IEEE Symposium on Logic in Computer Science*. LICS '24. Tallinn, Estonia: Association for Computing Machinery, 2024. ISBN: 9798400706608. DOI: 10.1145/3661814.3662109.

[2] Conrad Cotton-Barratt, Andrzej Murawski, and C.-H. Luke Ong. "Weak and Nested Class Memory Automata". In: *Language and Automata Theory and Applications*. Ed. by Adrian-Horia Dediu et al. Cham: Springer International Publishing, 2015, pp. 188–199. ISBN: 978-3-319-15579-1.

[3] Conrad Cotton-Barratt et al. "Fragments of ML decidable by nested data class memory automata". In: *International Conference on Foundations of Software Science and Computation Structures*. Springer. 2015, pp. 249–263.

References II

[4] Normann Decker et al. “Ordered Navigation on Multi-attributed Data Words”. In: *CONCUR 2014 – Concurrency Theory*. Ed. by Paolo Baldan and Daniele Gorla. Berlin, Heidelberg: Springer Berlin Heidelberg, 2014, pp. 497–511. ISBN: 978-3-662-44584-6.