# Simulating Gaussian tracers correlated with CMB lensing

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The goal is to generate simulations of large-scale-structure tracers which are appropriately correlated with a given simulation of the CMB lensing potential. The basic idea is to set up a linear system of equations from which it will be possible to solve for the coefficients involved granted we know the theoretical auto- and cross- spectra of the tracers.

Suppose we are to work with just three tracers: the CIB, LSST galaxies and an internal reconstruction, with spherical harmonic coefficients expressed as $I_{lm}, g_{lm}, \kappa^{rec}_{lm}$, respectively. Given a map of the true convergence with spherical harmonic coefficients $\kappa_{lm}$, the different tracers are correlated as

$$\kappa^{rec}_{lm} = \kappa_{lm} + n_{lm}, \tag{1}$$
$$g_{lm} = A^{g\kappa}_l \kappa_{lm} + u_{lm}, \tag{2}$$
$$I_{lm} = A^{I\kappa}_l \kappa_{lm} + A^{gI}_l u_{lm} + e_{lm}, \tag{3}$$
$$\tag{4}$$

where $n_{lm}, u_{lm}, e_{lm}$ are coefficients of the noise – and as such are presumed to each be uncorrelated with everything else – and $\kappa_{lm}$ are the coefficients of the true convergence for the particular realisation that we wish to generate correlated tracers for. Solving for the coefficients

$$A^{g\kappa}_l = \frac{C^{g\kappa}_l}{C^{\kappa\kappa}_l} \tag{5}$$
$$A^{I\kappa}_l = \frac{C^{I\kappa}_l}{C^{\kappa\kappa}_l} \tag{6}$$
$$A^{gI}_l = \frac{C^{gI}_l - A^{g\kappa}_l A^{I\kappa}_l C^{\kappa\kappa}_l}{C^{uu}_l} \tag{7}$$

and the noise spectra

$$C^{nn}_l = N^{\kappa\kappa}_l \tag{8}$$
$$C^{uu}_l = C^{gg}_l - (A^{g\kappa}_l)^2 C^{\kappa\kappa}_l \tag{9}$$
$$C^{ee}_l = C^{II}_l - (A^{I\kappa}_l)^2 C^{\kappa\kappa}_l - (A^{gI}_l)^2 C^{uu}_l, \tag{10}$$

where $N^{\kappa\kappa}_l$ is the internal reconstruction noise level.

From the above, it is clear that the problem is soluble if we know all the auto- and cross- spectra, in which case all we have to do is draw the coefficients $n_{lm}, u_{lm}$ and $e_{lm}$ from probability distributions with angular power spectra $N^{\kappa\kappa}_l, C^{uu}_l$ and $C^{ee}_l$.

The solution above can be generalised to any number of tracers as follows. Consider tracer $i$ with auto- spectrum $C^{ii}$ and cross-spectrum $C^{ij}$ with tracer $j$. Let it be described as a linear combination $\sum_p \sum_{lm} A^{ij}_l a^p(l,m)$ of harmonic coefficients $a^p$ with angular power spectra $C^{a^p a^p}_l$ and scaled with weights given by the square, off-diagonal matrix $A^{ij}_l$. In the case above, $a^p = \{\kappa_{lm}, u_{lm}, e_{lm}\}$, $C^{a^p a^p}_l = \{C^{\kappa\kappa}_l, C^{uu}_l, C^{ee}_l\}$. The general formula for the weights is

$$A^{ij}_l = \frac{1}{C^{a^j a^j}_l}\left(C^{ij}_l - \sum_{p=0}^{j-1} A^{jp}_l A^{ip}_l C^{a^p a^p}_l\right), \tag{11}$$

with auxiliary spectra given by:

$$C^{a^j a^j}_l = C^{jj}_l - \sum_{p=0}^{j-1} \left(A^{jp}_l\right)^2 C^{a^p a^p}_l. \tag{12}$$