

THE IMPACT OF EXTRAGALACTIC FOREGROUNDS ON INTERNAL DELENSING OF CMB B-MODE POLARIZATION

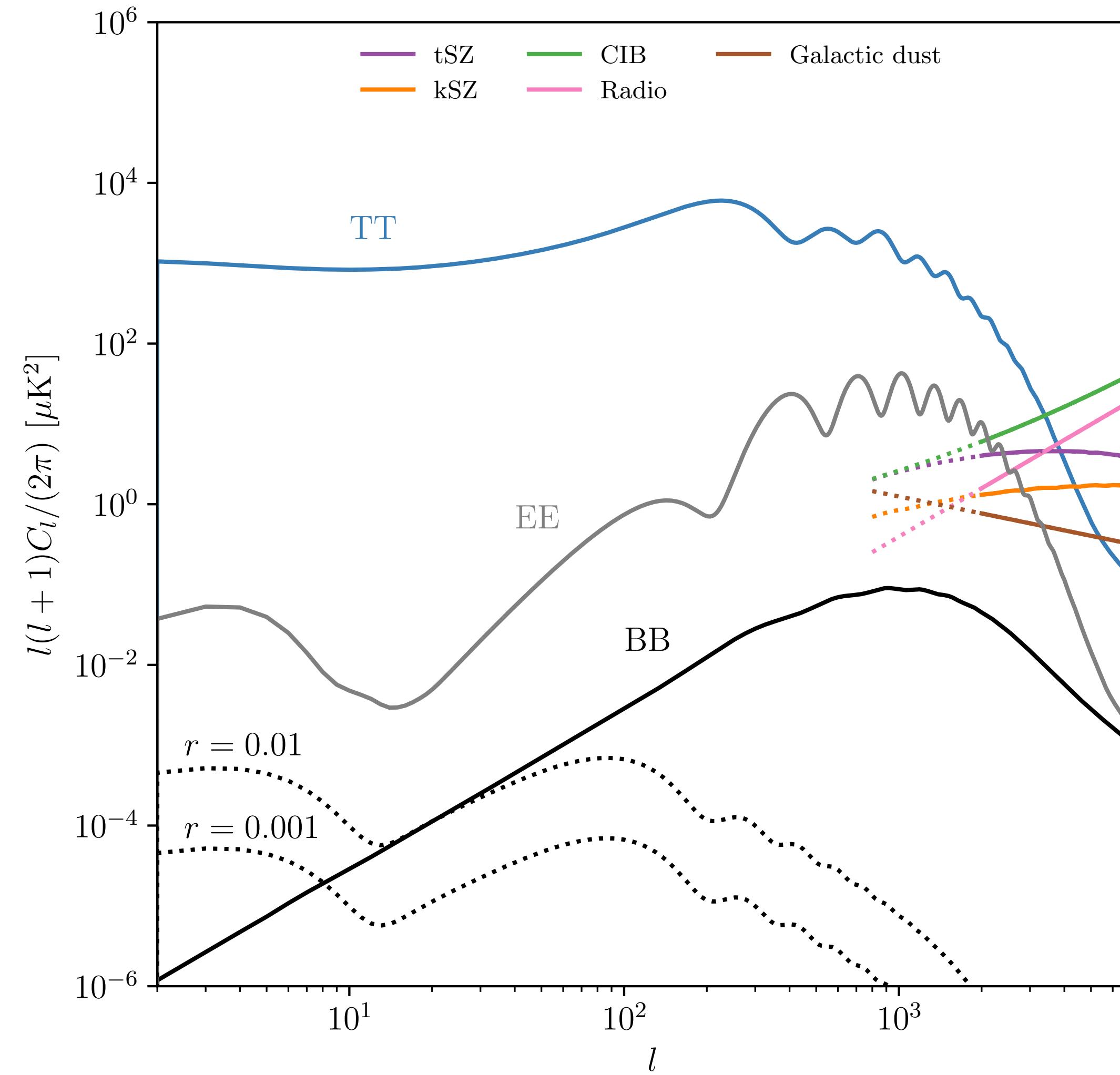
arXiv:2205.09000

L3.3 update

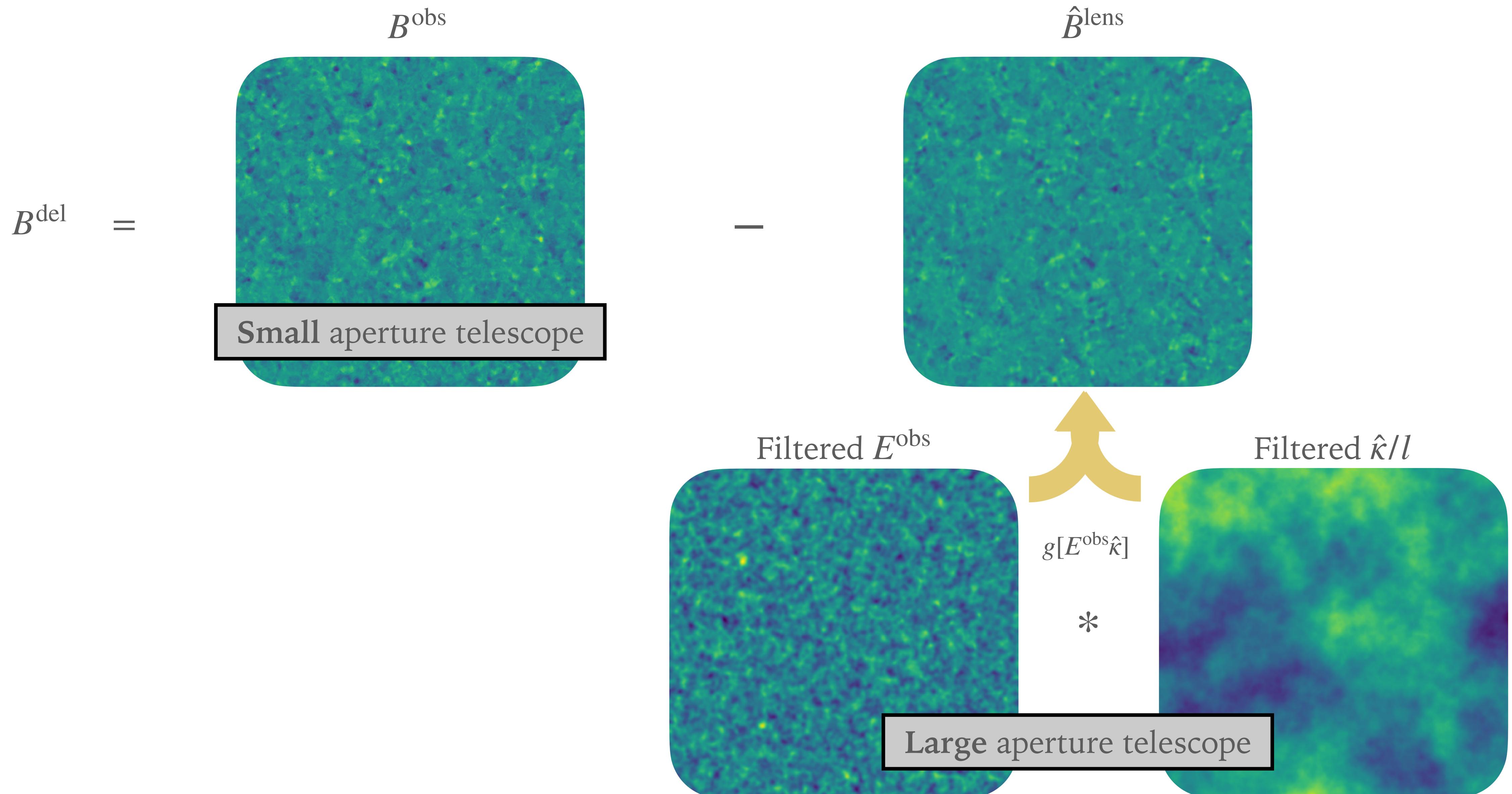
6/3/2022

Anton Baleato Lizancos (with Simone Ferraro)

CONTEXT

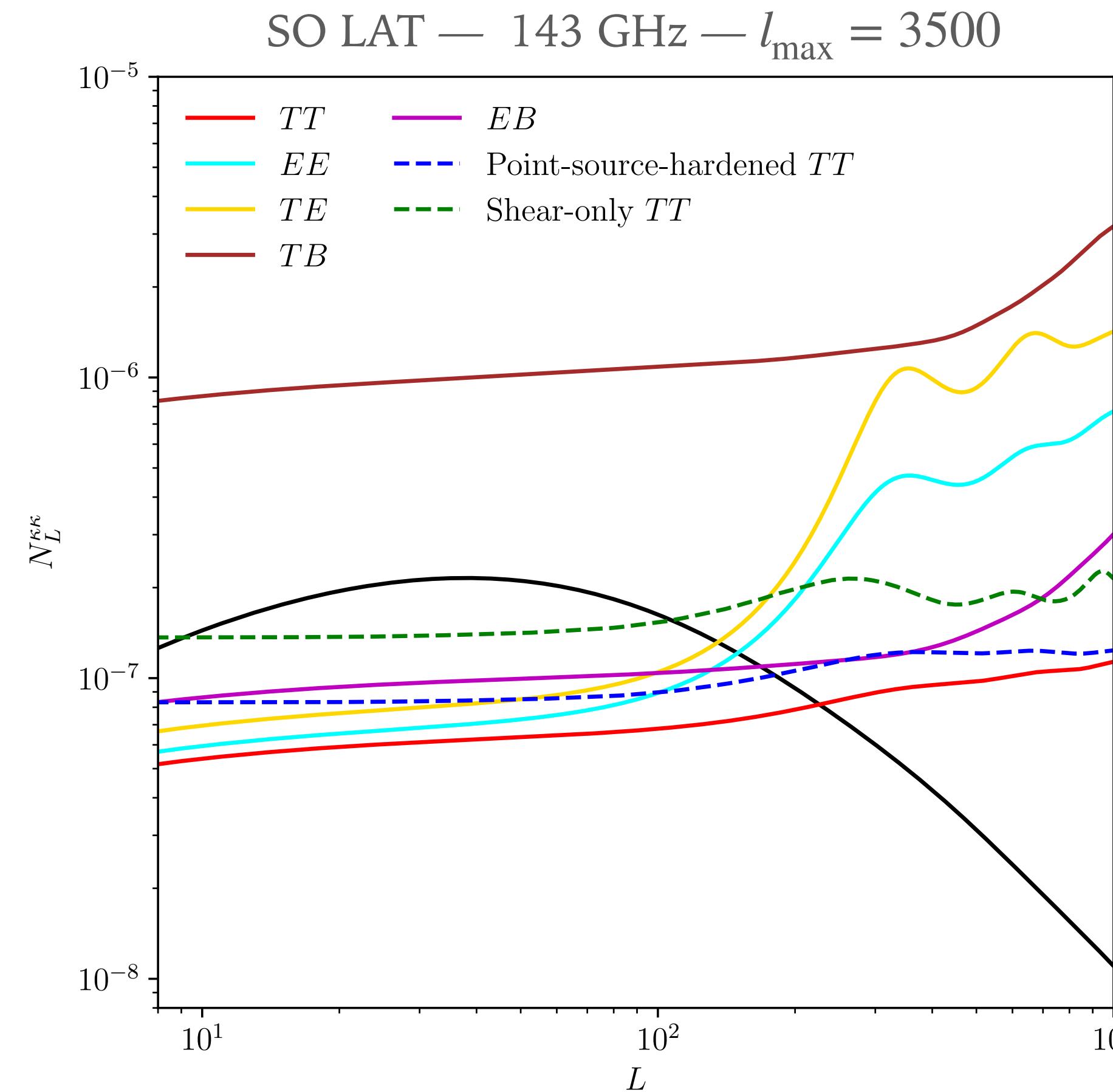


B-MODE TEMPLATE DELENSING



Patches of 8° on a side. Colour scales differ across panels.

INTERNAL RECONSTRUCTIONS OF CMB LENSING

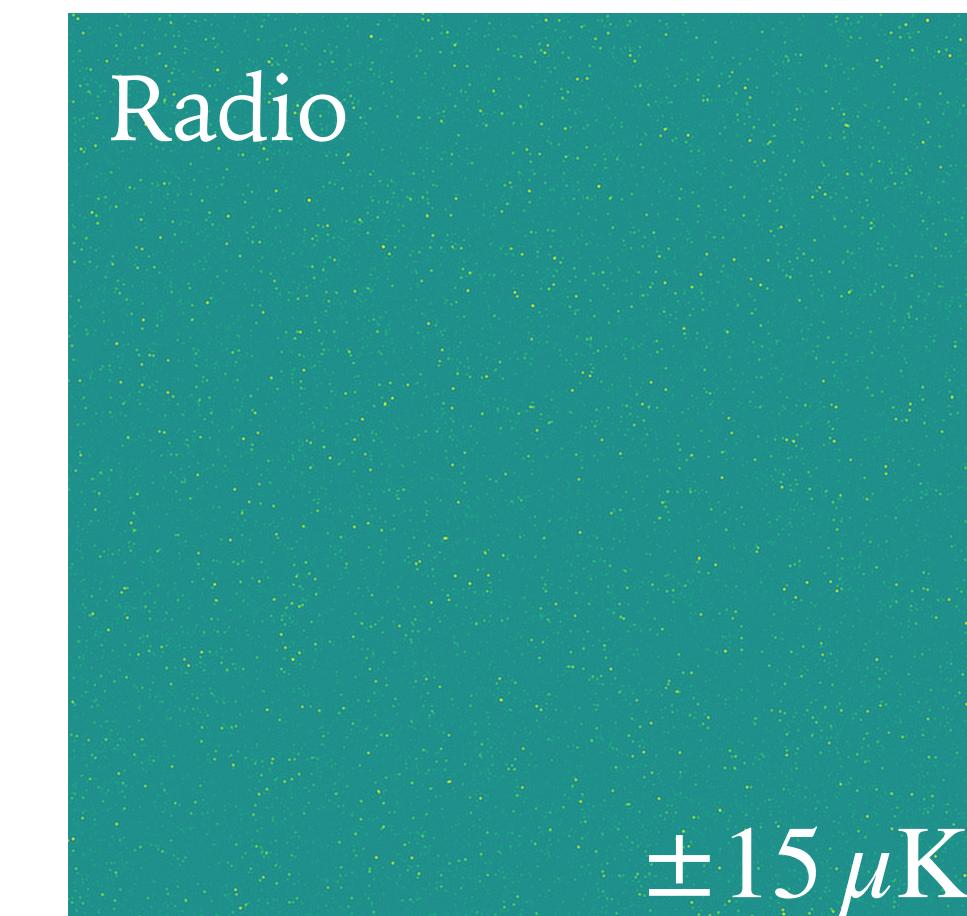
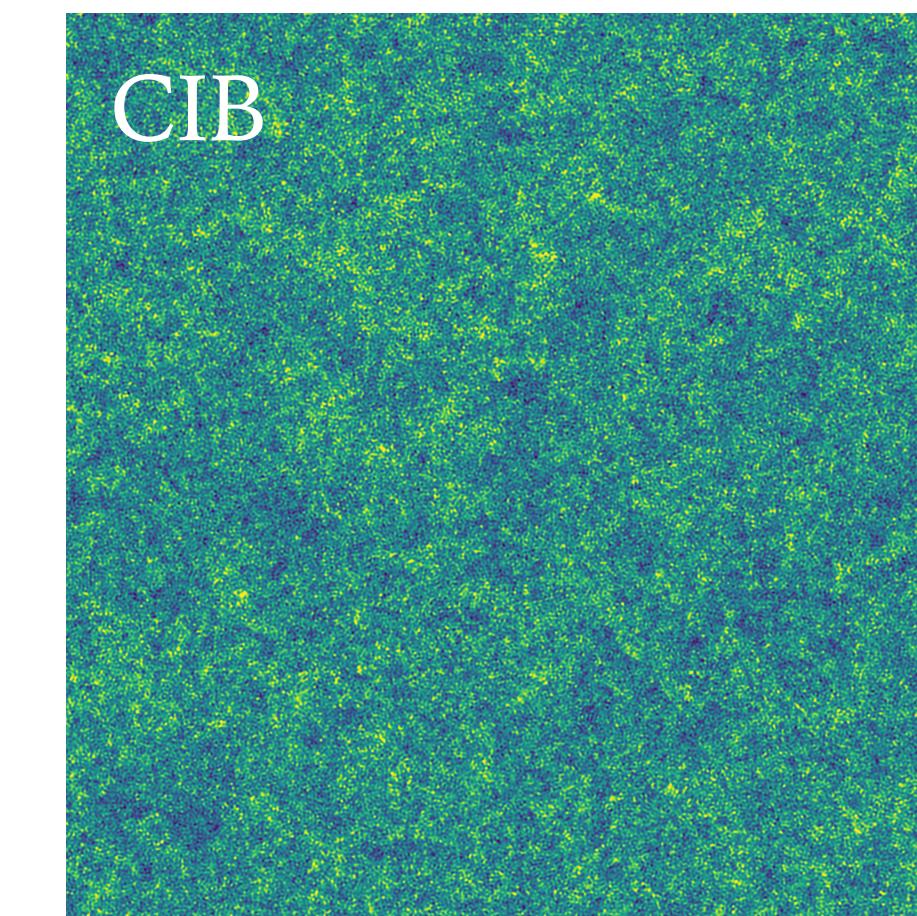
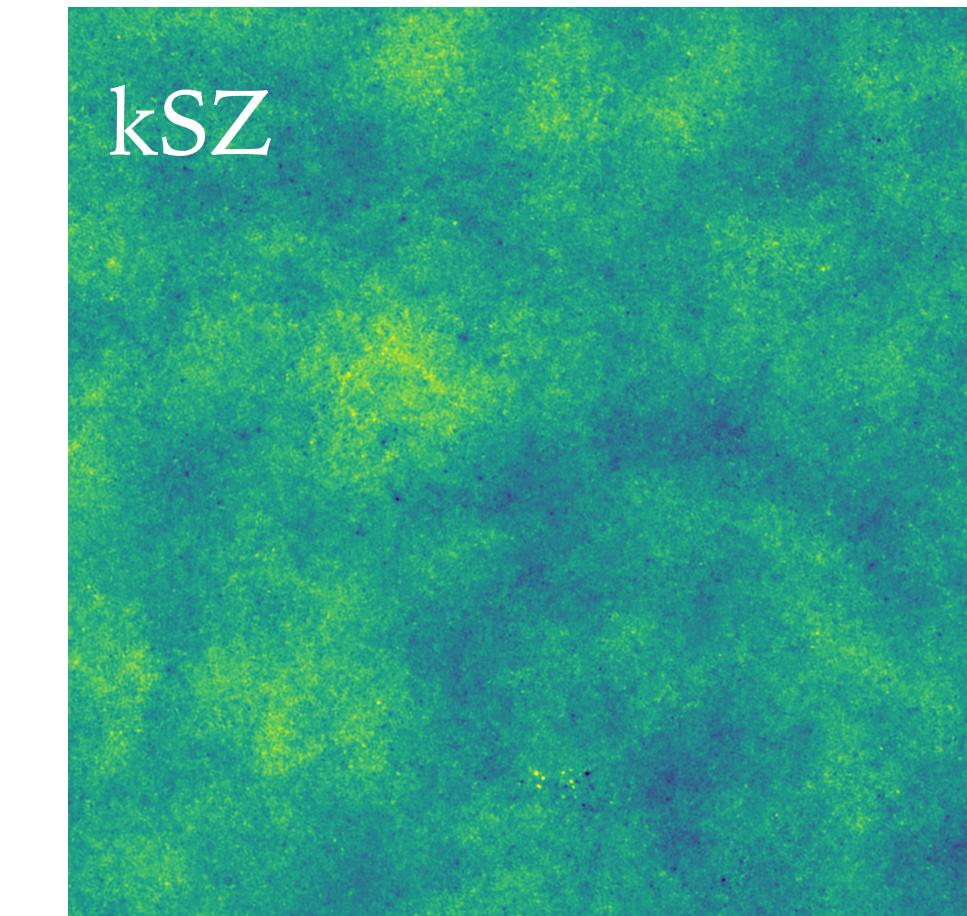
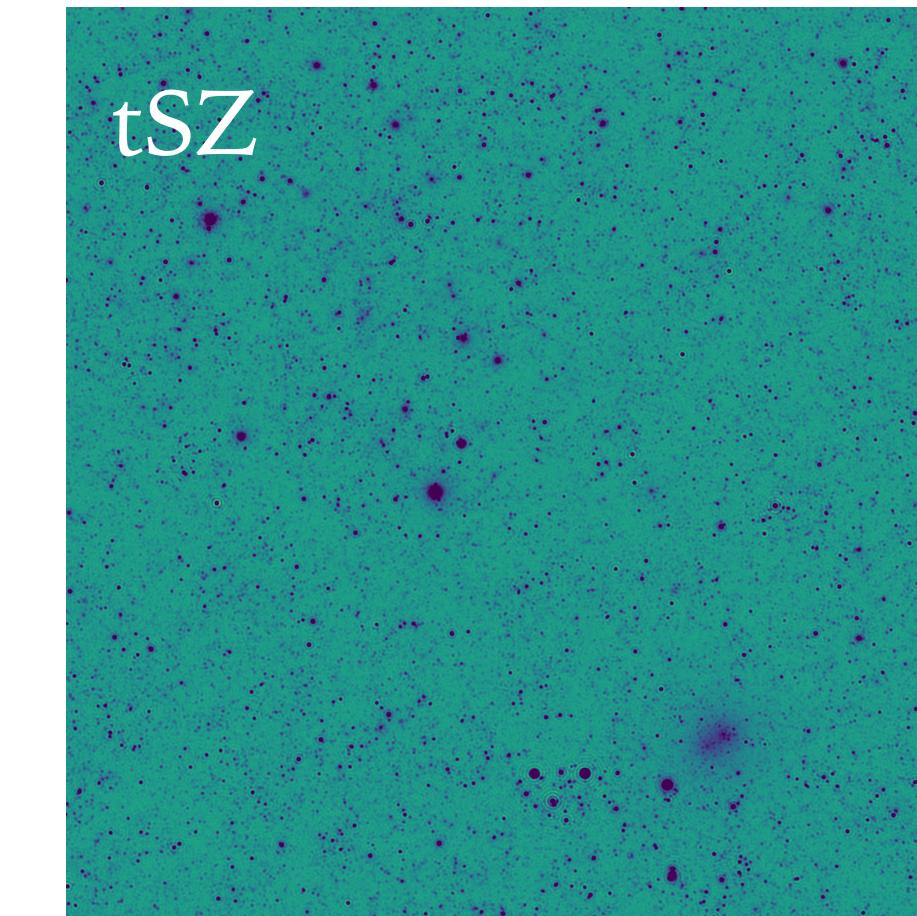
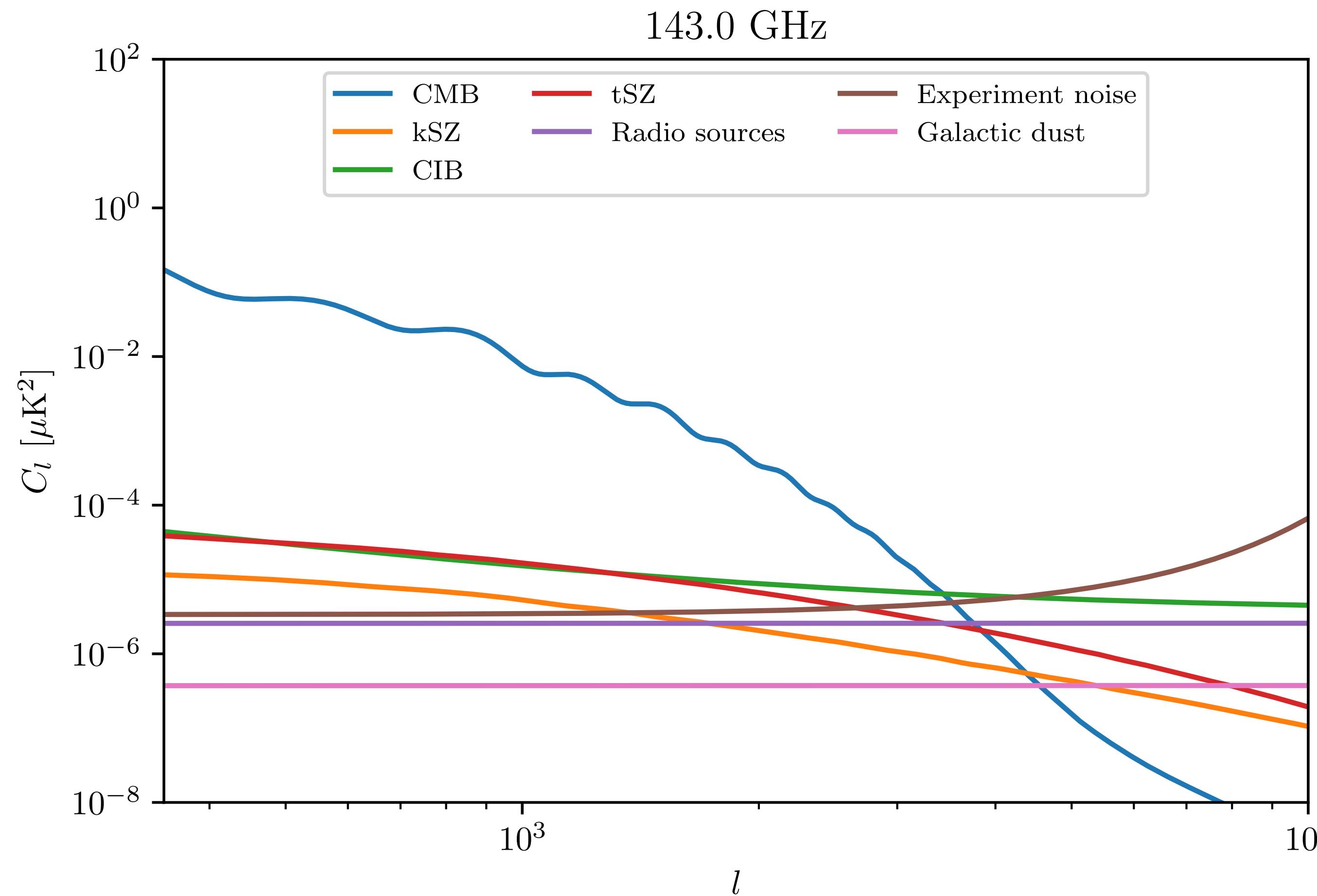


Our delensing efforts hinge on lensing reconstructions derived from temperature anisotropies

EXTRAGALACTIC FOREGROUNDS



In intensity, microwave sky contaminated by extragalactic emission: tSZ, kSZ, CIB, radio sources.



Websky sims Stein+, Li+

BIASES TO LENSING SPECTRA FROM EXTRAGALACTIC FOREGROUNDS

$$\hat{\kappa} = \hat{\kappa}[\tilde{T} + s^{\text{NG}}, \tilde{T} + s^{\text{NG}}]$$

Lensing Foreground (correlated with κ)

Fgs are non-Gaussian and correlated with lensing. They can bias the power spectrum of lensing reconstructions:

$$\langle \hat{\kappa} \hat{\kappa} \rangle = \langle \hat{\kappa}[\tilde{T}, \tilde{T}] \hat{\kappa}[\tilde{T}, \tilde{T}] \rangle + 2 \langle \hat{\kappa}[\tilde{T}, \tilde{T}] \hat{\kappa}[s^{\text{NG}}, s^{\text{NG}}] \rangle + 4 \langle \hat{\kappa}[\tilde{T}, s^{\text{NG}}] \hat{\kappa}[\tilde{T}, s^{\text{NG}}] \rangle + \langle \hat{\kappa}[s^{\text{NG}}, s^{\text{NG}}] \hat{\kappa}[s^{\text{NG}}, s^{\text{NG}}] \rangle$$

“Primary bispectrum bias”

“Secondary bispectrum bias”

“Trispectrum bias”

as well as cross-correlations with low-z matter tracers

$$\langle g[\kappa] \hat{\kappa} \rangle = \langle g[\kappa] \hat{\kappa}[\tilde{T}, \tilde{T}] \rangle + \langle g[\kappa] \hat{\kappa}[s^{\text{NG}}, s^{\text{NG}}] \rangle$$

↑
“Bispectrum bias”

Amblard + 04

Van Engelen + 14

Osborne + 14

Ferraro & Hill 18

Sailer + 21

COULD THIS ALSO BIAS B-MODE DELENSING?

$$\Delta C_l^{BB, \text{del}} = -2 \Delta C_l^{\tilde{B} \times \hat{B}^{\text{lens}}} + \Delta C_l^{\hat{B}^{\text{lens}} \times \hat{B}^{\text{lens}}}$$

$$\Delta C_l^{\tilde{B} \times \hat{B}^{\text{lens}}} \supset g_l \left[\langle \tilde{B} \tilde{E} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c \right]$$

$C^{EE} \langle \kappa \hat{\kappa} [s^{\text{NG}}, s^{\text{NG}}] \rangle$

primary bispectrum bias

$$\begin{aligned} \Delta C_l^{\hat{B}^{\text{lens}} \times \hat{B}^{\text{lens}}} &\supset \\ &\supset 2 h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, \tilde{T}] E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c} \right] \\ &+ 4 h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}] \rangle_c} \right] \\ &+ h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c} \right] \end{aligned}$$

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$C^{EE, \text{obs}} \langle \kappa \hat{\kappa} [s^{\text{NG}}, s^{\text{NG}}] \rangle$

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$$C^{EE, \text{obs}} \langle \hat{\kappa}[\tilde{T}, s^{\text{NG}}] \hat{\kappa}[\tilde{T}, s^{\text{NG}}] \rangle$$

secondary bispectrum bias

$$\begin{aligned} \Delta C_l^{\hat{B}^{\text{lens}} \times \hat{B}^{\text{lens}}} &\supset \\ &\supset 2 h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, \tilde{T}] E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c} \right] \\ &+ 4 h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}] \rangle_c} \right] \\ &+ h_l \left[\langle E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c \right] \end{aligned}$$

COULD THIS ALSO BIAS B-MODE DELENSING?

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$C^{EE, \text{obs}} \langle \hat{\kappa}[s^{\text{NG}}, s^{\text{NG}}] \hat{\kappa}[s^{\text{NG}}, s^{\text{NG}}] \rangle$

trispectrum bias

$$\begin{aligned} \Delta C_l^{\hat{B}^{\text{lens}} \times \hat{B}^{\text{lens}}} &\supset \\ &\supset 2 h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, \tilde{T}] E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c} \right] \\ &+ 4 h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}] \rangle_c} \right] \\ &+ h_l \left[\underbrace{\langle E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c} \right] \end{aligned}$$

COULD THIS ALSO BIAS B-MODE DELENSING?

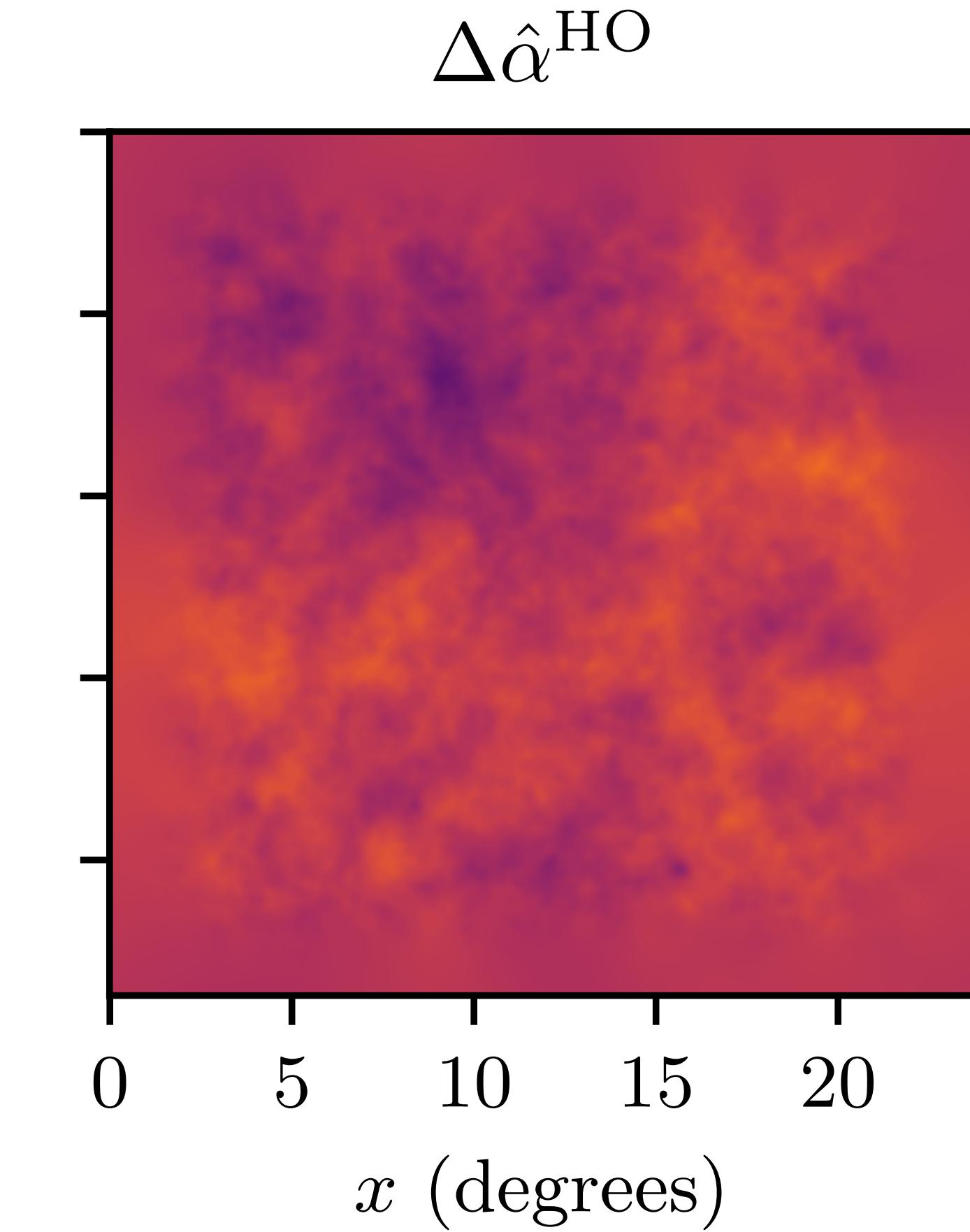
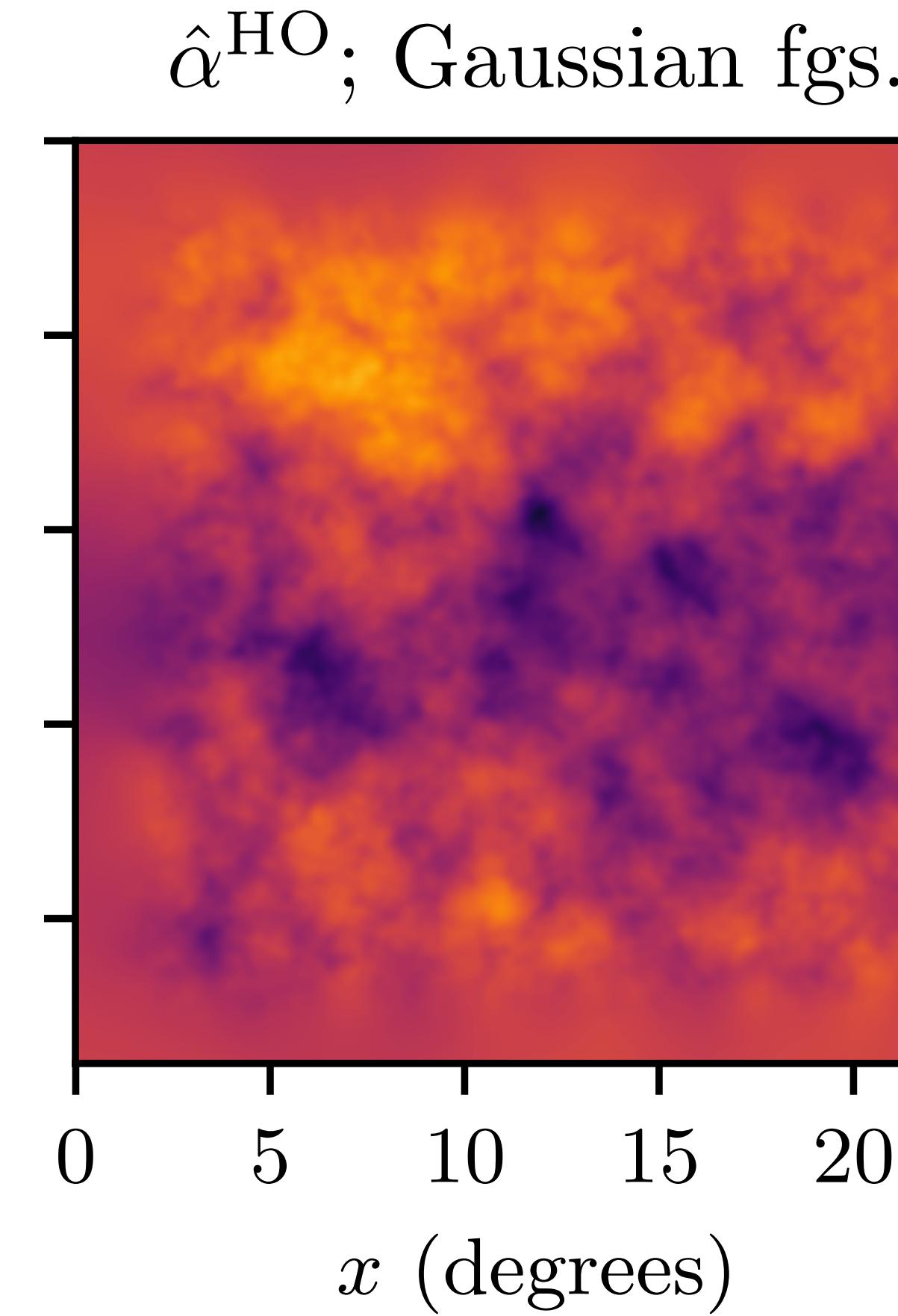
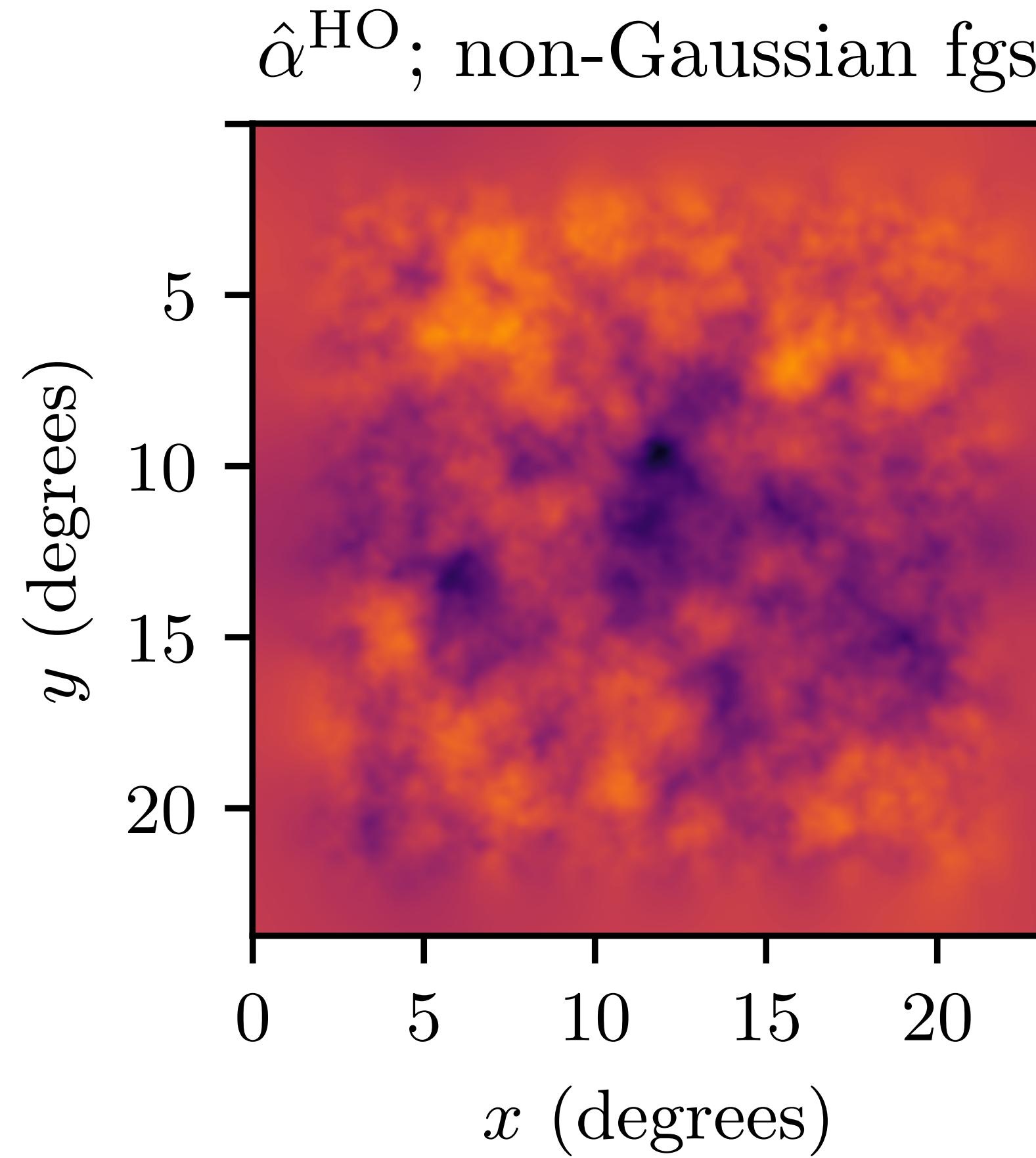
Using the Websky simulations, we measure

$$\begin{aligned}\Delta C_l^{BB, \text{del}} = & \langle |\tilde{B} - g_l [\tilde{E} \hat{\kappa} [f + s^{\text{NG}}, f + s^{\text{NG}}]]|^2 \rangle \\ & - \langle |\tilde{B} - g_l [\tilde{E} \hat{\kappa} [f + s^{\text{G}}, f + s^{\text{G}}]]|^2 \rangle.\end{aligned}$$

We take the foregrounds to be unpolarized.

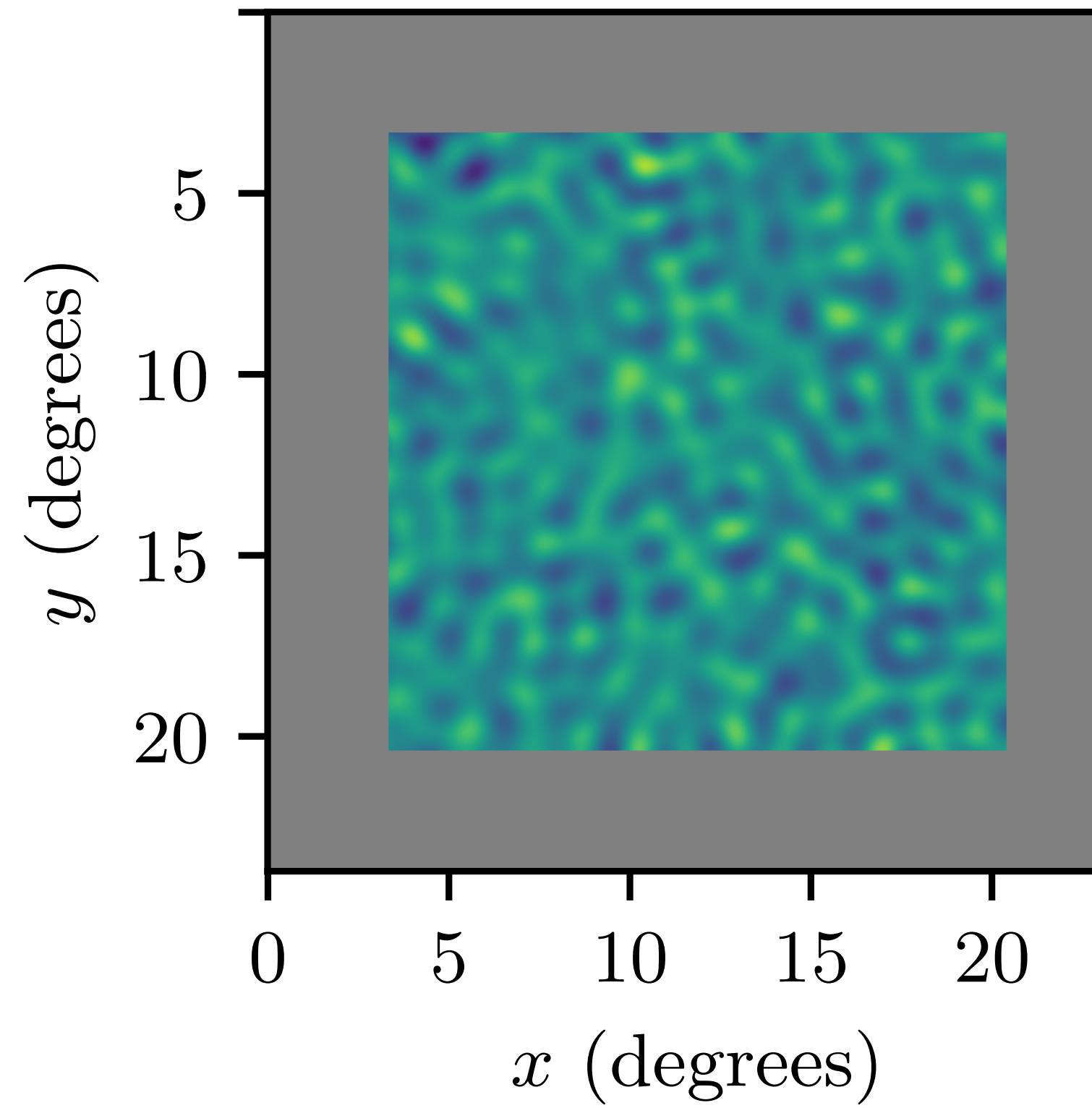
BIASES TO LENSING RECONSTRUCTIONS FROM EXTRAGALACTIC FOREGROUNDS

$$\alpha \equiv 2\kappa/l$$

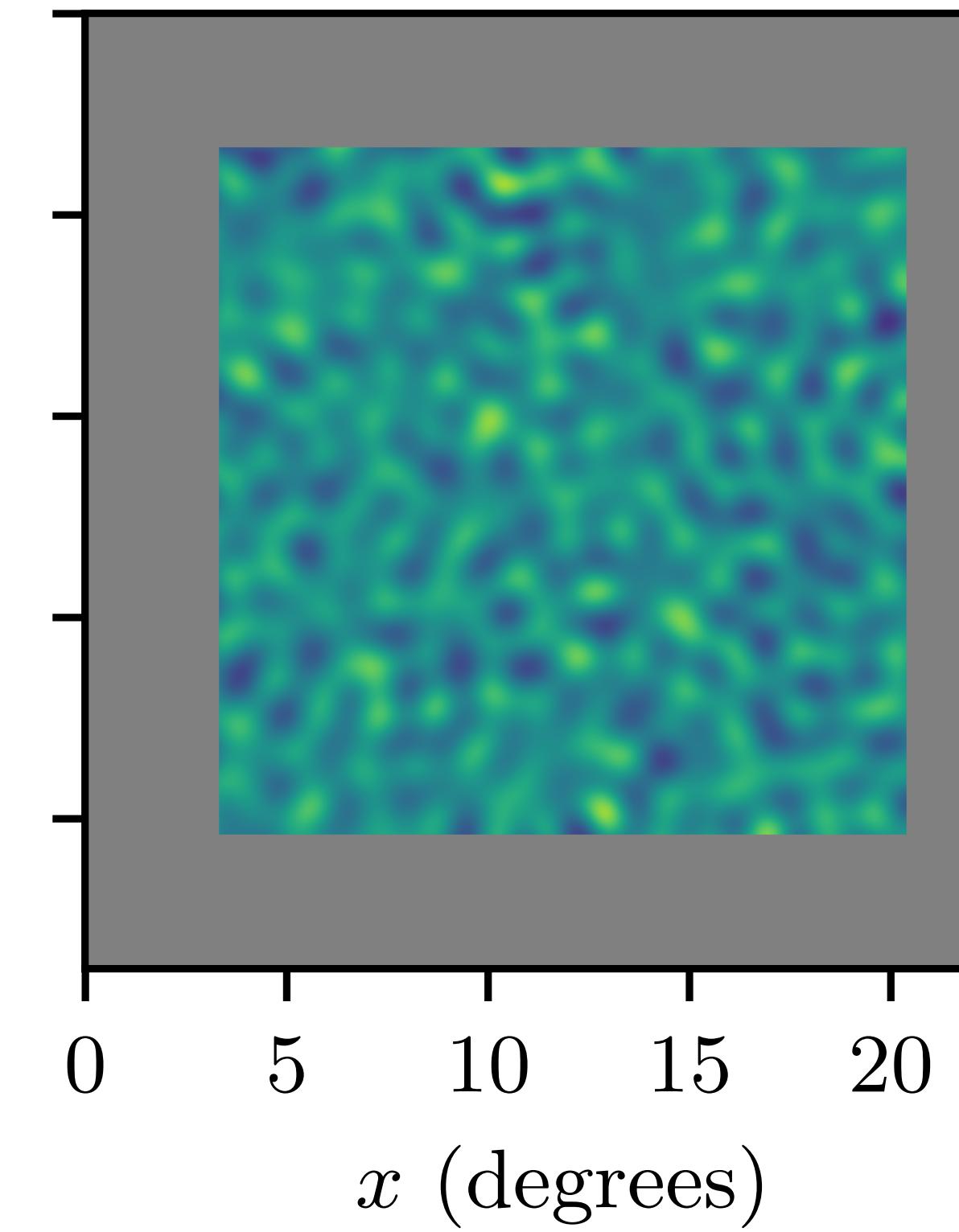


DELENSED B-MODES IN THE PRESENCE OF FOREGROUNDS

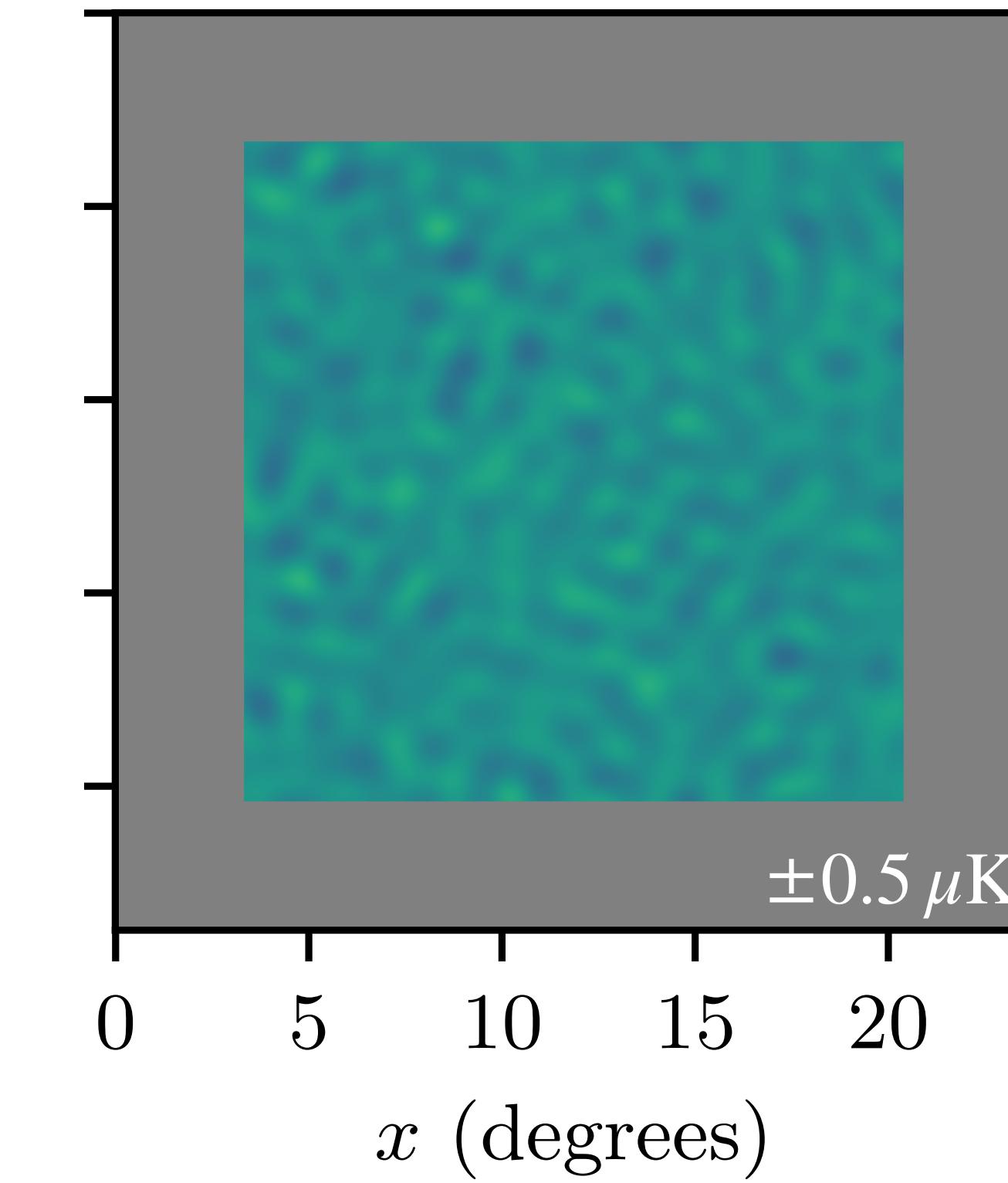
B^{del} ; non-Gaussian fgs.



B^{del} ; Gaussian fgs.



ΔB^{del}

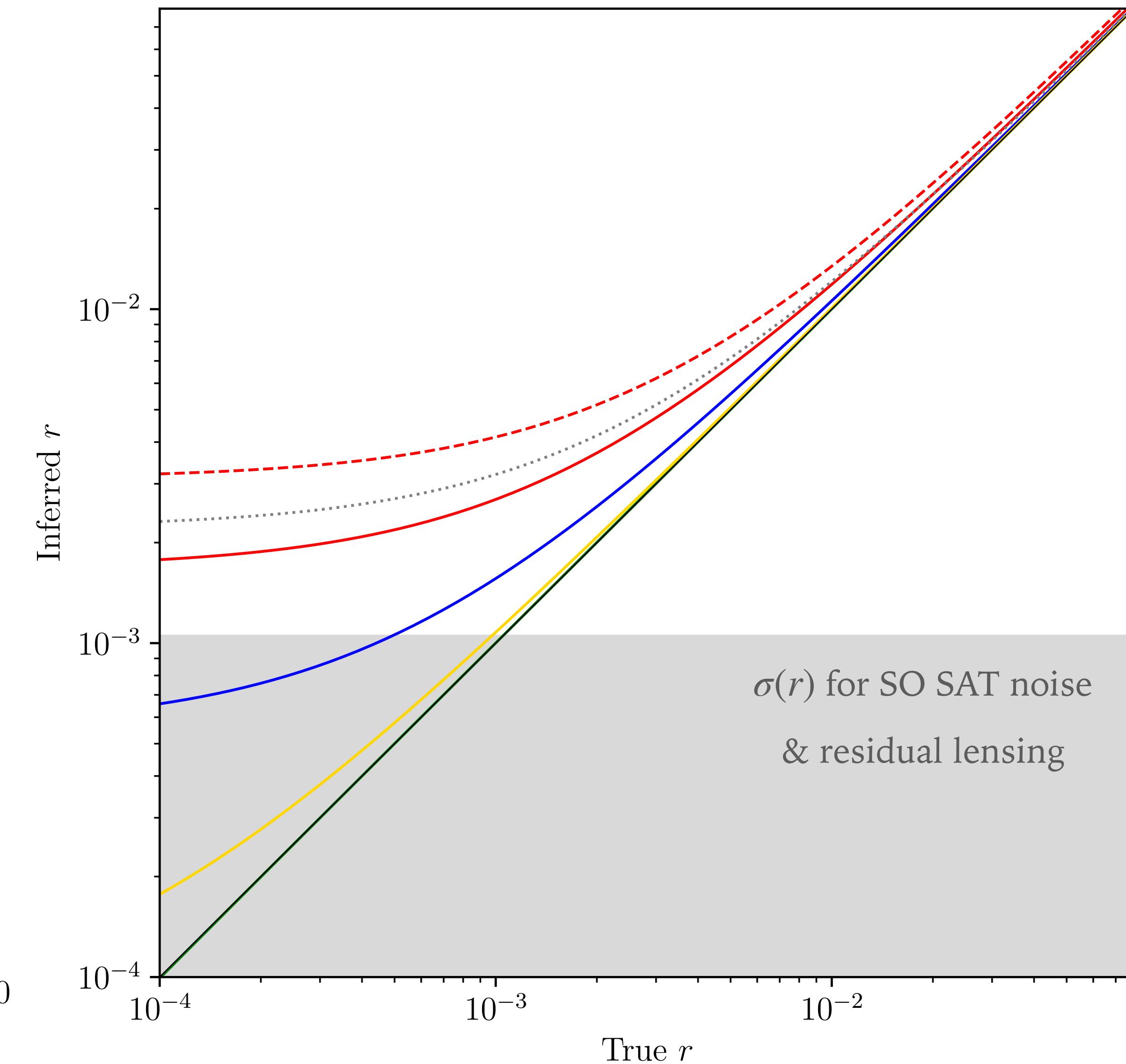
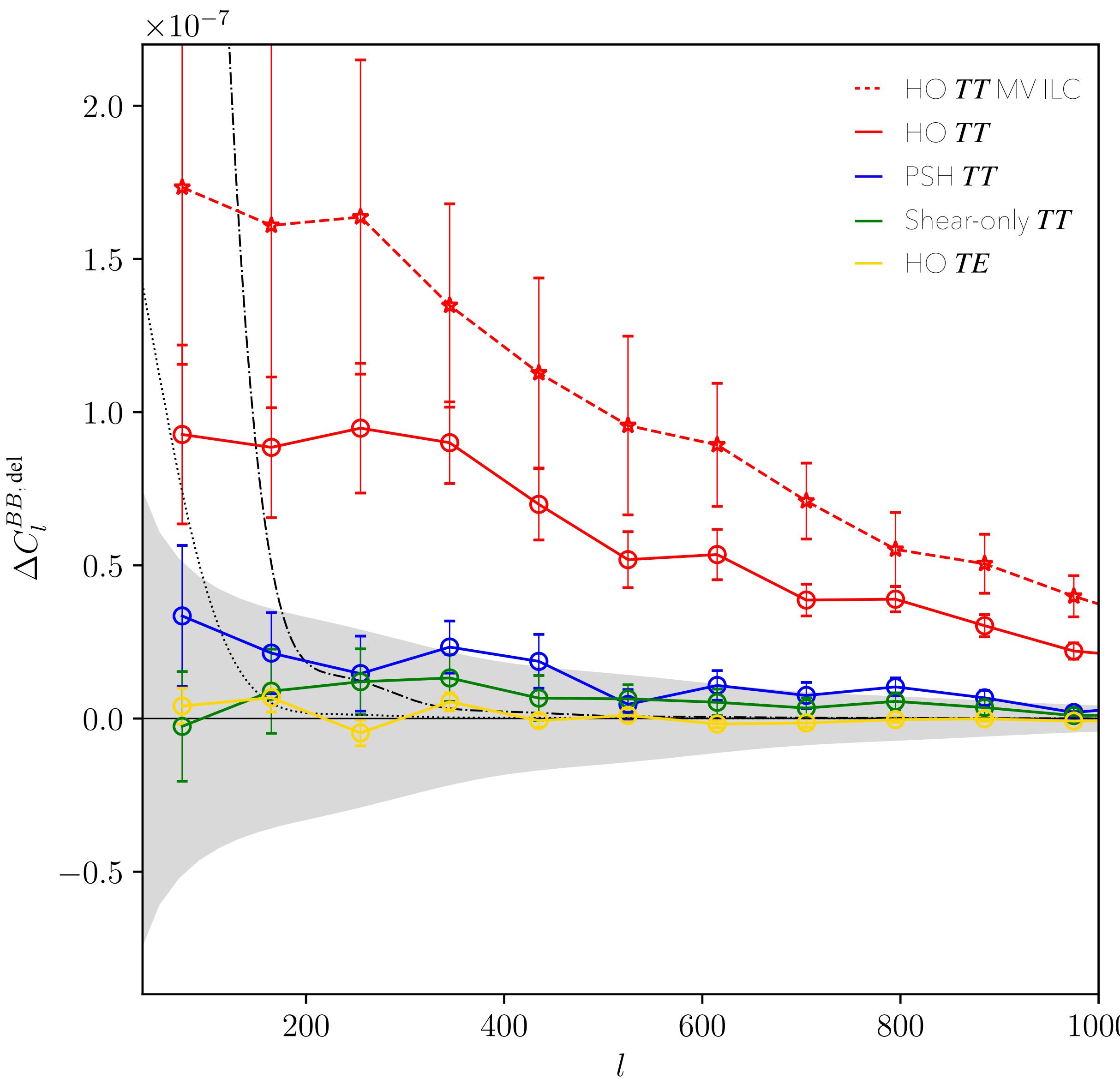


Noiseless B -modes, but SO-like template with $l_{\max,T} = 3500$ in reconstructions

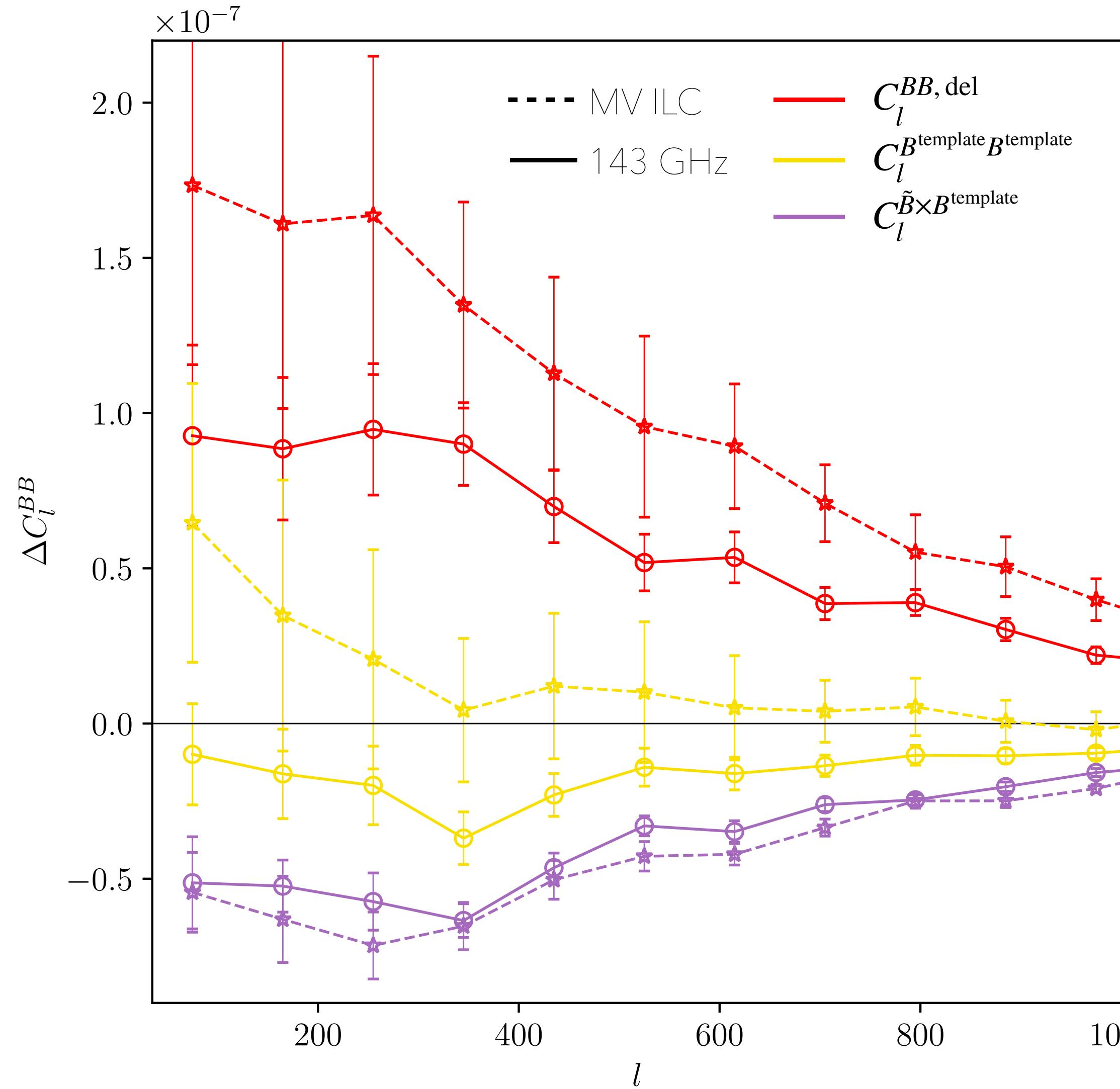
Filtered out $l > 300$ modes to highlight degree-scale signal

PATHWAYS TO MITIGATION

HO \equiv Hu-Okamoto
 PSH \equiv Point-source-hardened



MV ILC LEADS TO HIGHER BIAS



When both $\Delta C^{B \text{ template } B \text{ template}}$ and $\Delta C^{\tilde{B} \text{ template}}$ are dominated by bispectrum bias, they cancel each other out, and scatter also \downarrow

MV ILC boosts tSZ by a factor of $\sim 2^{1/2}$

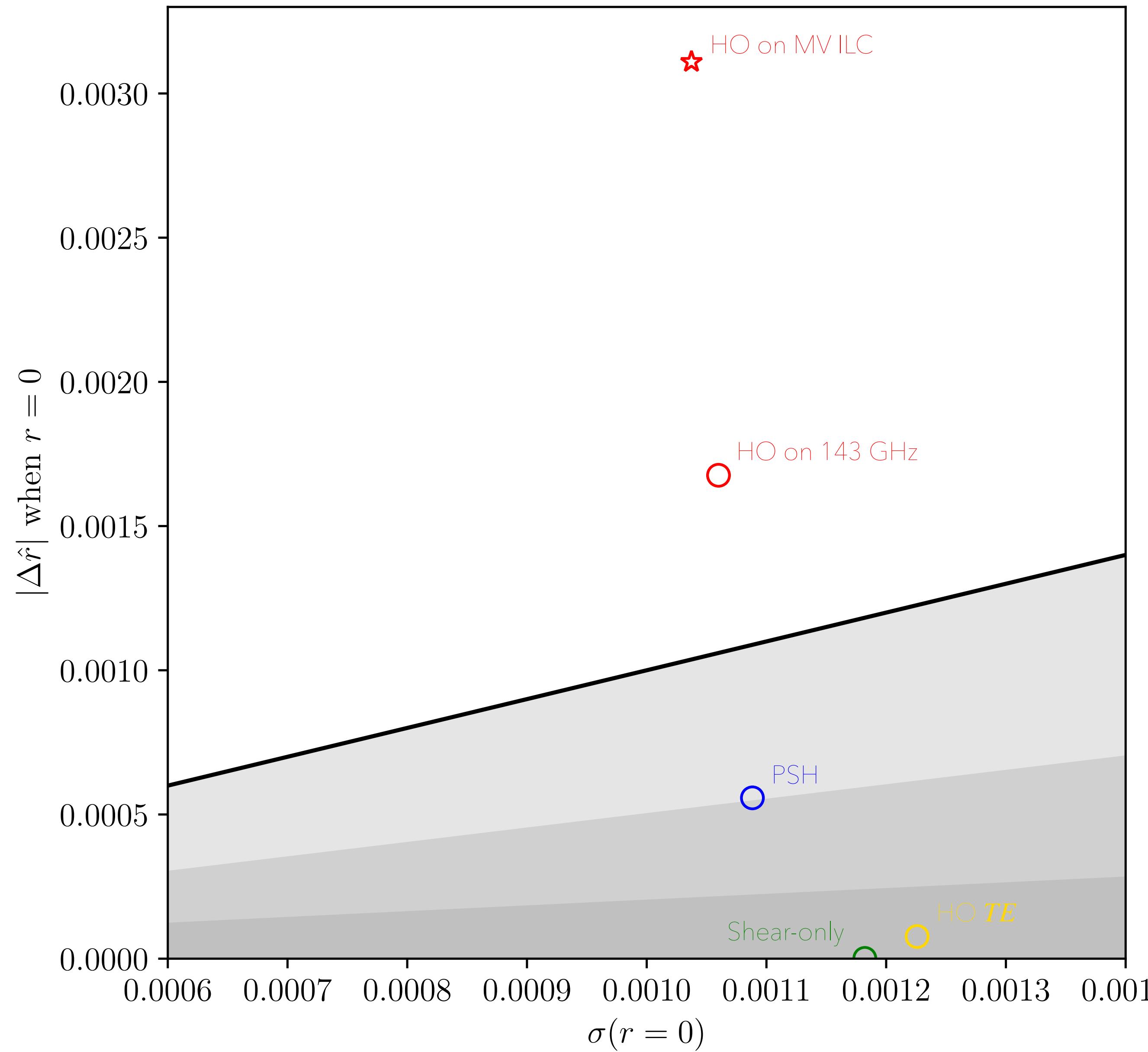
\Rightarrow trispectrum grows by $(2^{1/2})^4$, bispectrum less responsive

\Rightarrow template auto- moves “up”

\Rightarrow $\Delta C^{BB, \text{del}}$ can \uparrow while $|\Delta C^{B \text{ template } B \text{ template}}|$ and $|\Delta C^{\tilde{B} \text{ template}}| \downarrow$ or stay =

BIAS VS NOISE

For all estimators, $l_{\max} = 3500$



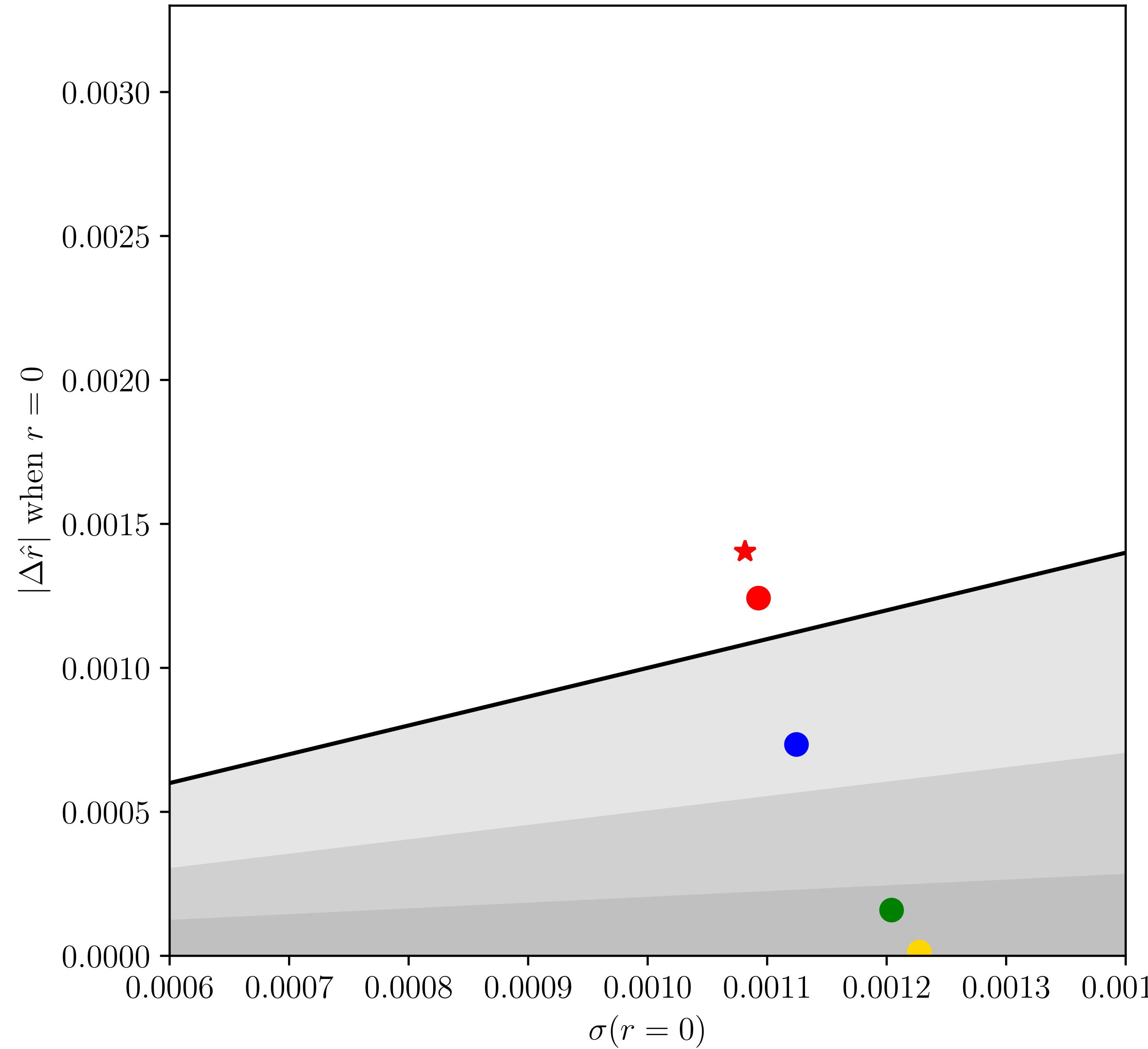
$$|\Delta \hat{r}|/\sigma(r) = 1$$

$$|\Delta \hat{r}|/\sigma(r) = 1/2$$

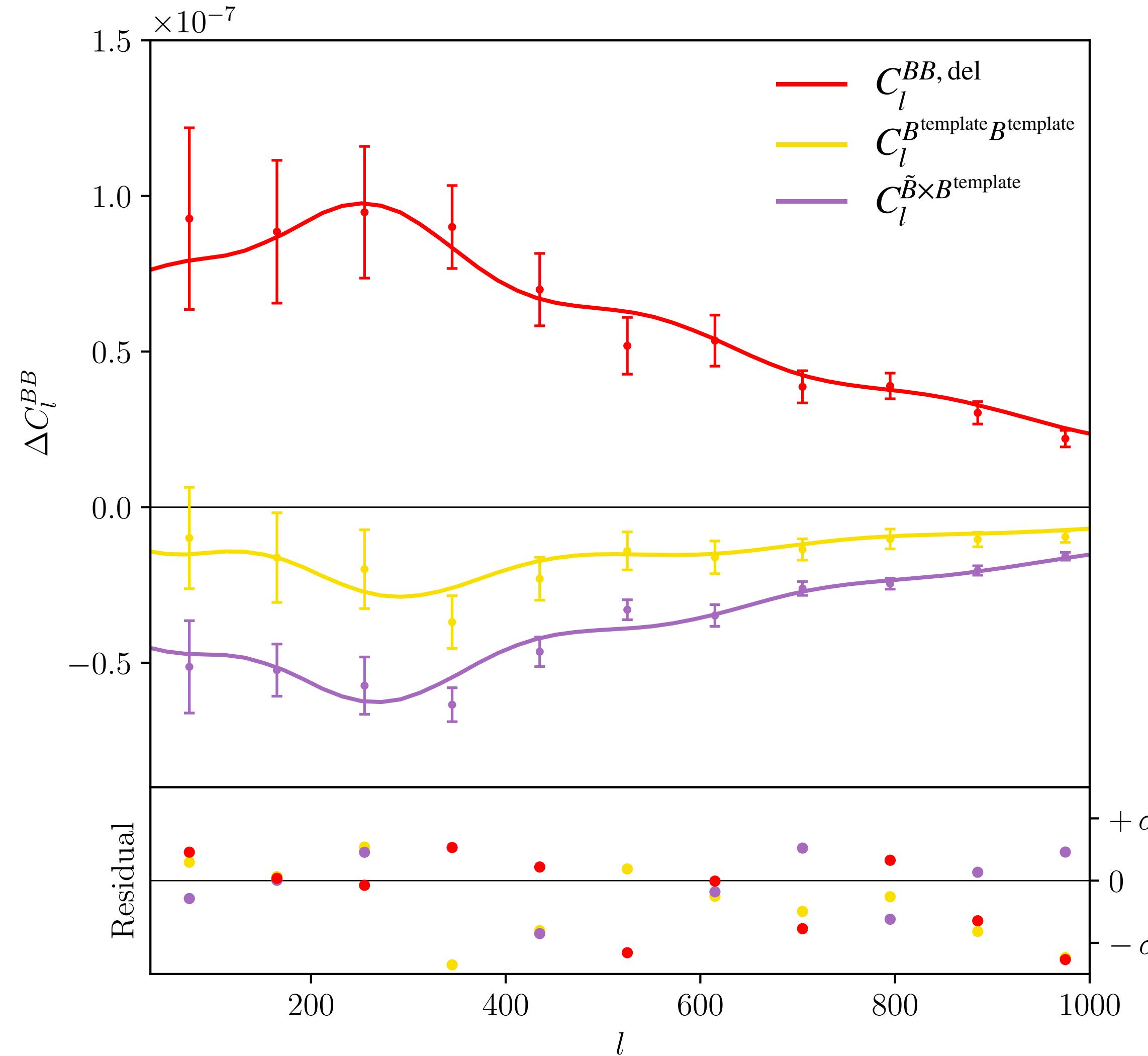
$$|\Delta \hat{r}|/\sigma(r) = 1/5$$

BIAS VS NOISE

For all estimators, $l_{\max} = 3000$



MODELING THE BIAS



The dominant terms can be modelled as

$$\Delta C_l^{\hat{B}^{\text{lens}} \times \hat{B}^{\text{lens}}} = \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} W^2(\mathbf{l}, \mathbf{l}') \left(\mathcal{W}_{l'}^E \mathcal{W}_{|\mathbf{l}-\mathbf{l}'|}^\kappa \right)^2 \times C_{l'}^{EE, \text{tot}} \Delta C_{\mathbf{l}-\mathbf{l}'}^{\hat{\kappa}\hat{\kappa}},$$

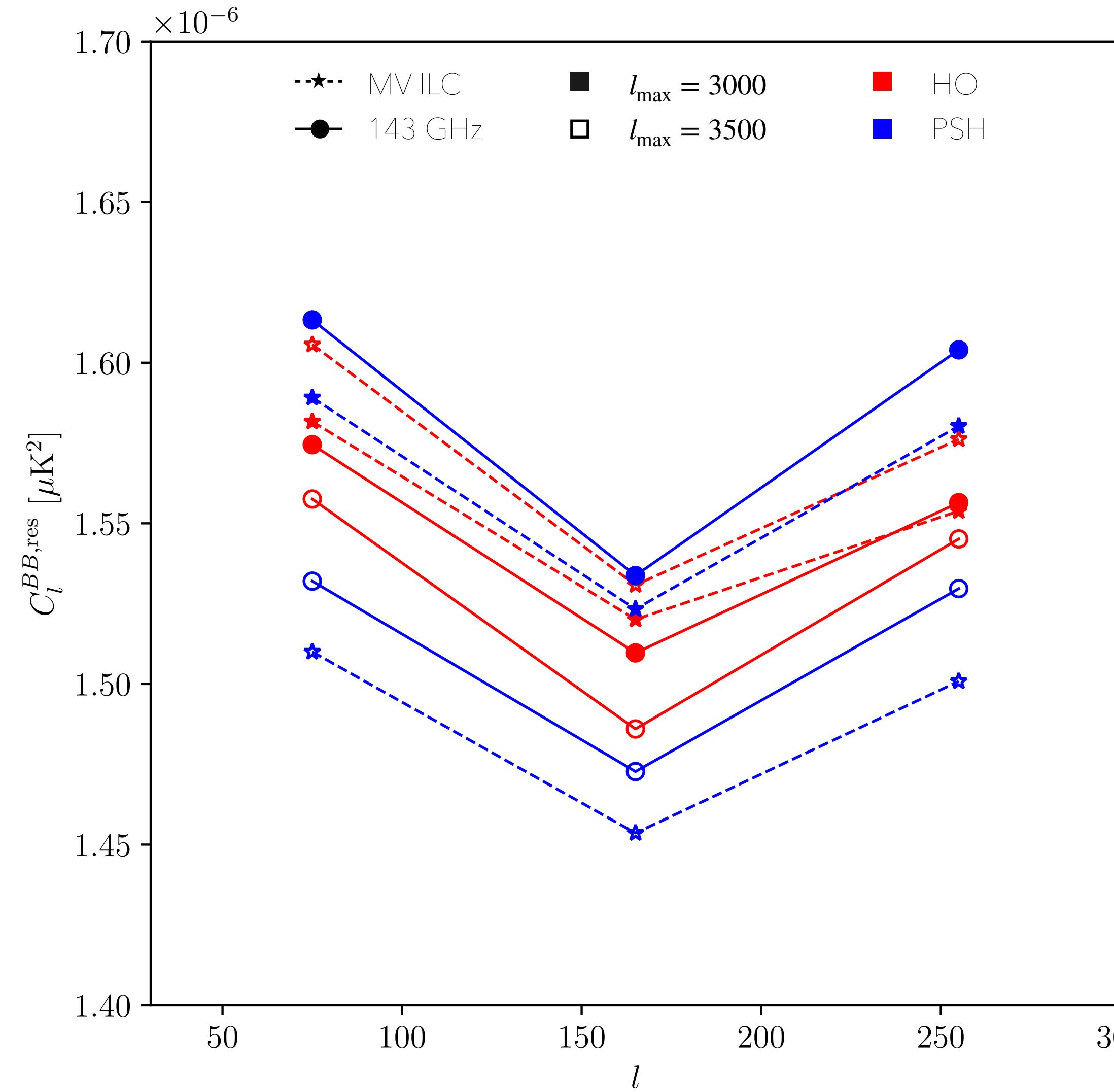
Smooth fit to auto-spectrum of a (biased) TT reconstruction

and

$$\Delta C_l^{\tilde{B} \times \hat{B}^{\text{lens}}} = \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} W^2(\mathbf{l}, \mathbf{l}') \mathcal{W}_{l'}^E \mathcal{W}_{|\mathbf{l}-\mathbf{l}'|}^\kappa \times C_{l'}^{EE} \Delta C_{\mathbf{l}-\mathbf{l}'}^{\kappa\hat{\kappa}}.$$

Ditto to cross-correlation between TT and pol-only reconstructions

WHAT ESTIMATOR IS BEST?



If the modeling approach works, we'll avoid bias

... BUT ...

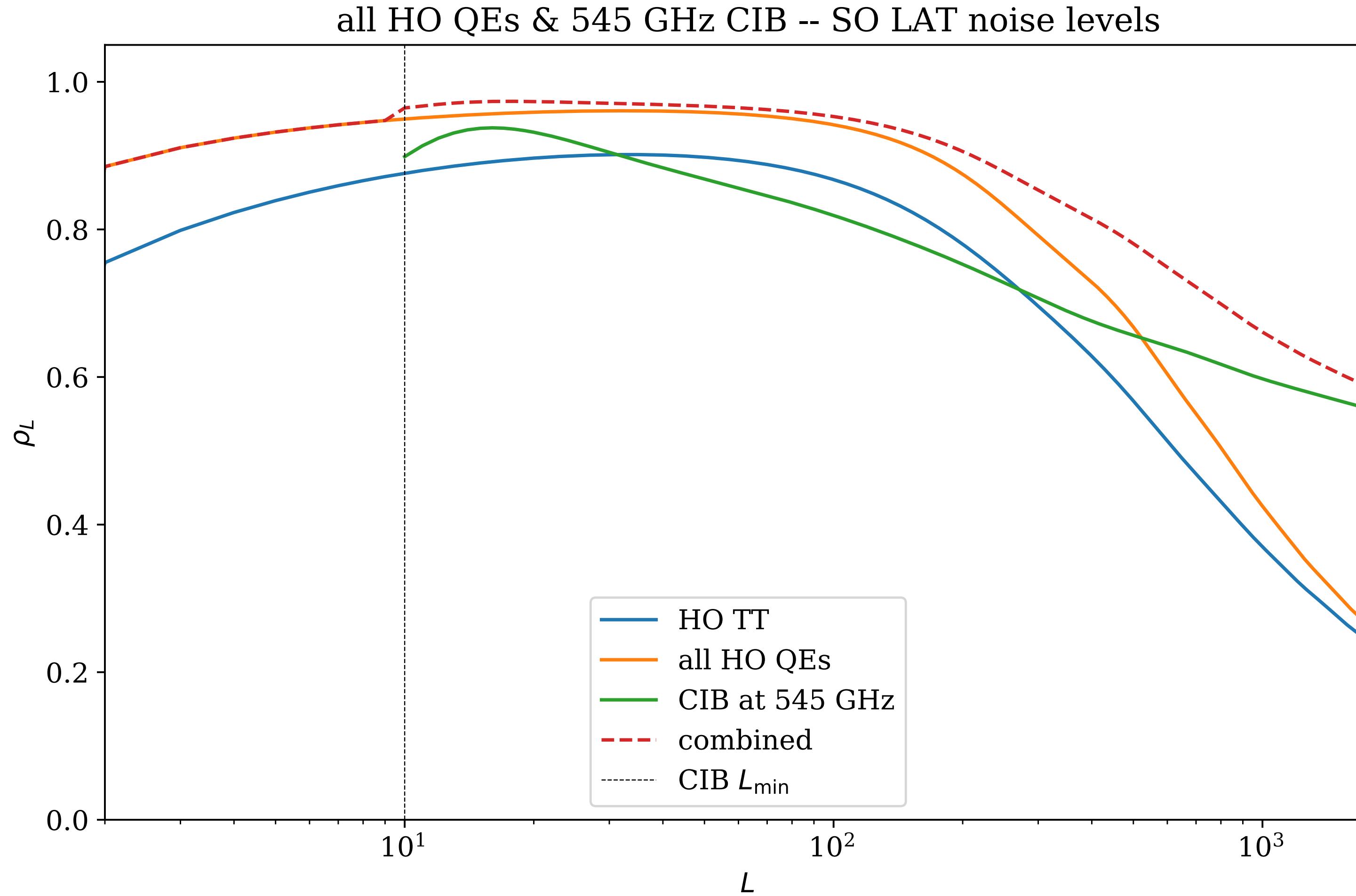
we still face a **tradeoff** between removing actual lensing B-modes and adding spurious power due to fg non-Gaussianity.

We must determine what choice of

- Lensing estimator
- l_{max}
- Foreground cleaning scheme
- Point-source masking protocol

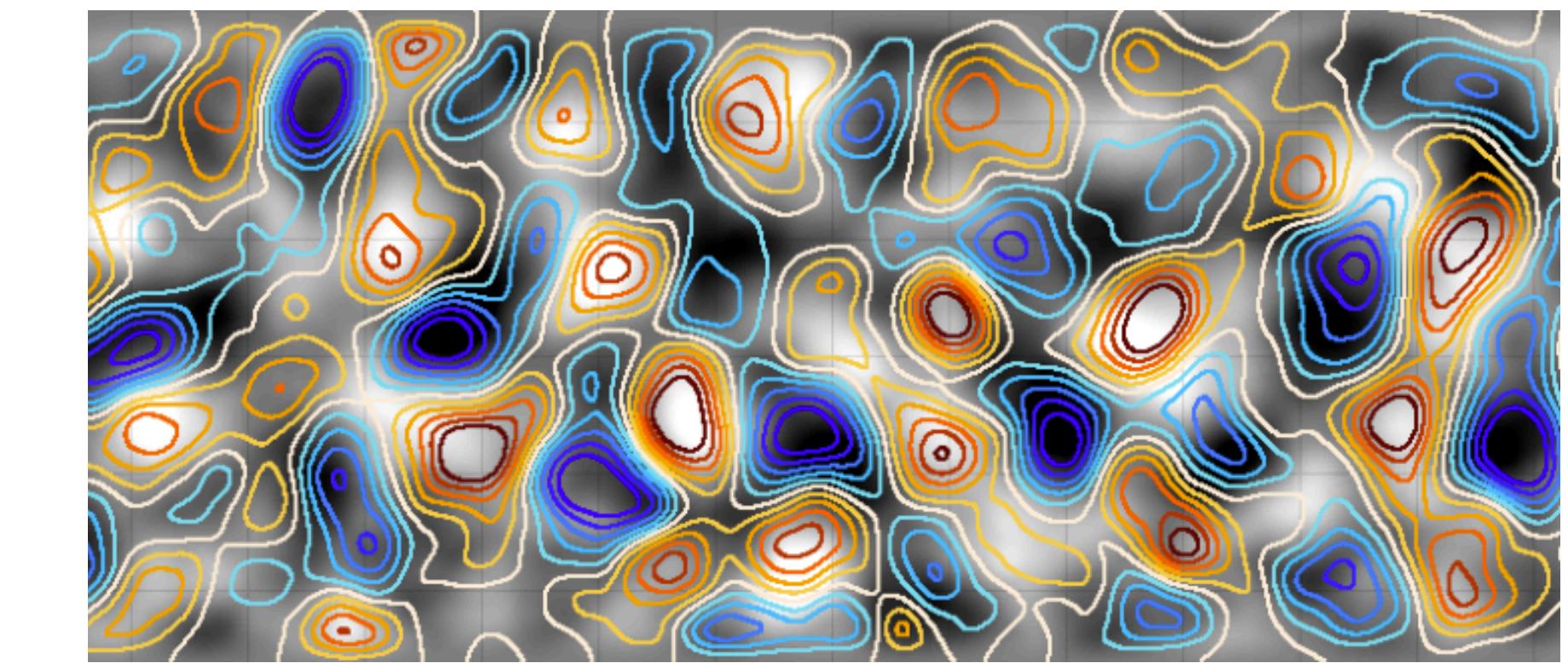
results in the least amount of B-mode power after delensing.

MULTI-TRACER DELENSING



$$\kappa_{LM}^{\text{comb}} = \sum_i c_L^i \hat{\kappa}_{LM}$$

e.g., the CIB:



ACT lensing vs. CIB

Darwish++ 20

MULTI-TRACER DELENSING BIAS

$$\begin{aligned}\Delta C_l^{BB, \text{del}} = & -2g_l \left[c^{TT} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] + h_l \left[(c^{TT})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\ & - 2g_l \left[c^{TE} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] + h_l \left[(c^{TE})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\ & + 2h_l \left[c^{TT} c^{TE} \langle E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\ & + 2h_l \left[c^{TT} \langle E^{\text{obs}} \sum_{i \neq TT, TE} c^i \hat{\kappa}^i E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] - (s^{\text{NG}} \rightarrow s^G).\end{aligned}$$

Dilution of TT bias

MULTI-TRACER DELENSING BIAS

$$\begin{aligned}\Delta C_l^{BB, \text{del}} = & -2g_l \left[c^{TT} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] + h_l \left[(c^{TT})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\ & - 2g_l \left[c^{TE} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] + h_l \left[(c^{TE})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\ & + 2h_l \left[c^{TT} c^{TE} \langle E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\ & + 2h_l \left[c^{TT} \langle E^{\text{obs}} \sum_{i \neq TT, TE} c^i \hat{\kappa}^i E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] - (s^{\text{NG}} \rightarrow s^G).\end{aligned}$$

Dilution of TE bias

MULTI-TRACER DELENSING BIAS

$$\begin{aligned}
\Delta C_l^{BB, \text{del}} = & -2g_l \left[c^{TT} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] + h_l \left[(c^{TT})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\
& - 2g_l \left[c^{TE} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] + h_l \left[(c^{TE})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\
& + 2h_l \left[c^{TT} c^{TE} \langle E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\
& + 2h_l \left[c^{TT} \langle E^{\text{obs}} \sum_{i \neq TT, TE} c^i \hat{\kappa}^i E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] - (s^{\text{NG}} \rightarrow s^G).
\end{aligned}$$

New term involving:

$$\begin{aligned}
\Delta C_l^{BB, \text{res}} \supset & \\
\supset & 2h_l \left[c^{TT} c^{TE} \langle \overbrace{E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}]} E^{\text{obs}} \hat{\kappa}^{TE} [\tilde{T}, \tilde{E}] \rangle_c \right] \\
& + 4h_l \left[c^{TT} c^{TE} \langle \overbrace{E^{\text{obs}} \hat{\kappa}^{TT} [\tilde{T}, s^{\text{NG}}]} E^{\text{obs}} \hat{\kappa}^{TE} [s^{\text{NG}}, \tilde{E}] \rangle_c \right]
\end{aligned}$$

\sim primary bispectrum bias

\sim secondary bispectrum bias

MULTI-TRACER DELENSING BIAS

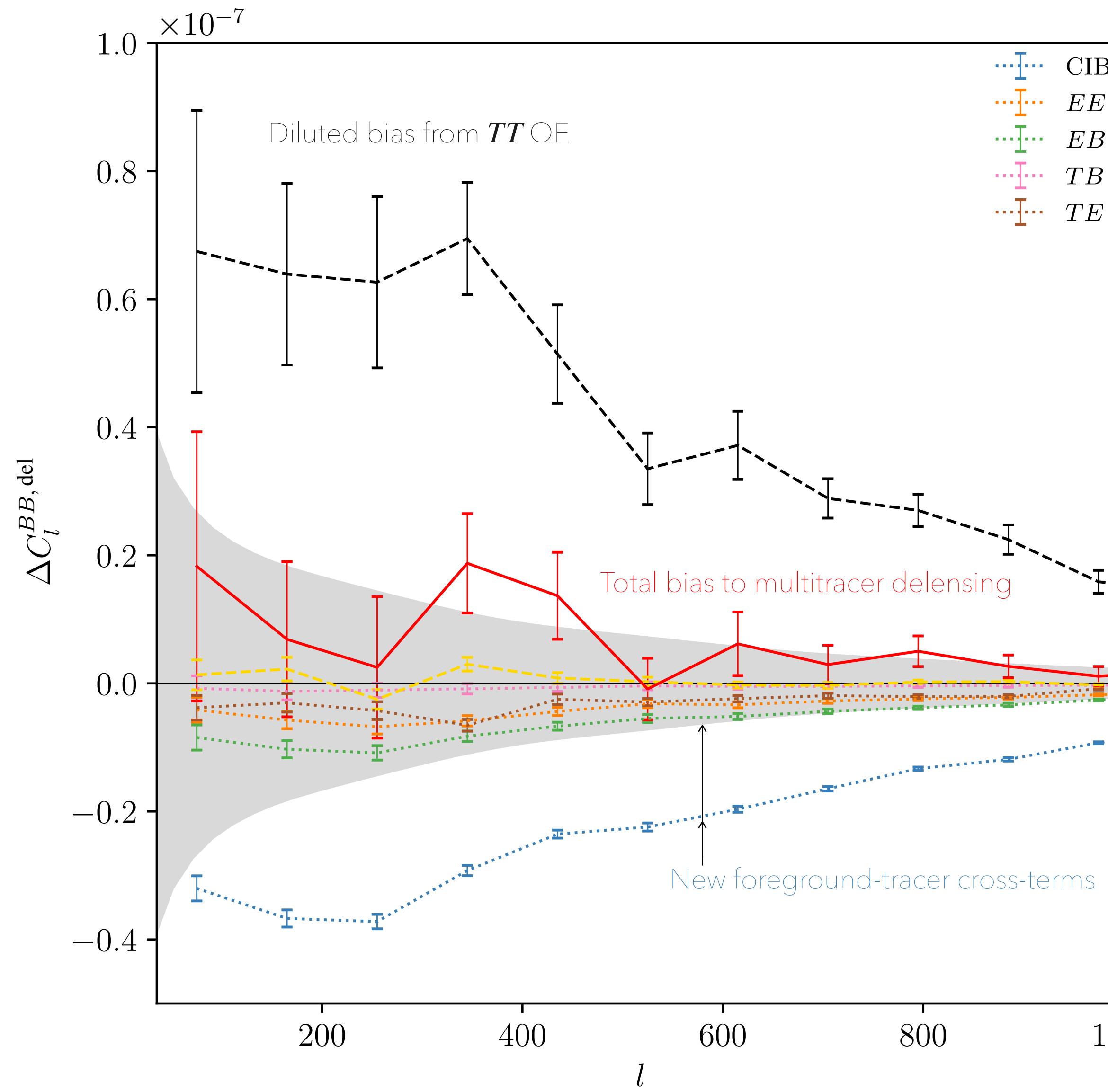
$$\begin{aligned}
\Delta C_l^{BB, \text{del}} = & -2g_l \left[c^{TT} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] + h_l \left[(c^{TT})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\
& - 2g_l \left[c^{TE} \langle \tilde{B} \tilde{E} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] + h_l \left[(c^{TE})^2 \langle |E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E]|^2 \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\
& + 2h_l \left[c^{TT} c^{TE} \langle E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] E^{\text{obs}} \hat{\kappa}^{TE} [f_T + s^{\text{NG}}, f_E] \rangle \right] - (s^{\text{NG}} \rightarrow s^G) \\
& + 2h_l \left[c^{TT} \langle E^{\text{obs}} \sum_{i \neq TT, TE} c^i \hat{\kappa}^i E^{\text{obs}} \hat{\kappa}^{TT} [f_T + s^{\text{NG}}, f_T + s^{\text{NG}}] \rangle \right] - (s^{\text{NG}} \rightarrow s^G).
\end{aligned}$$

New term involving:

$$\Delta C_l^{BB, \text{res}} \supset 2h_l \left[c^{TT} c^i \langle \overbrace{E^{\text{obs}} \hat{\kappa}^i}^{} E^{\text{obs}} \hat{\kappa}^{TT} [s^{\text{NG}}, s^{\text{NG}}] \rangle_c \right]$$

A function of $\langle sss \rangle$ or $\langle \kappa ss \rangle$

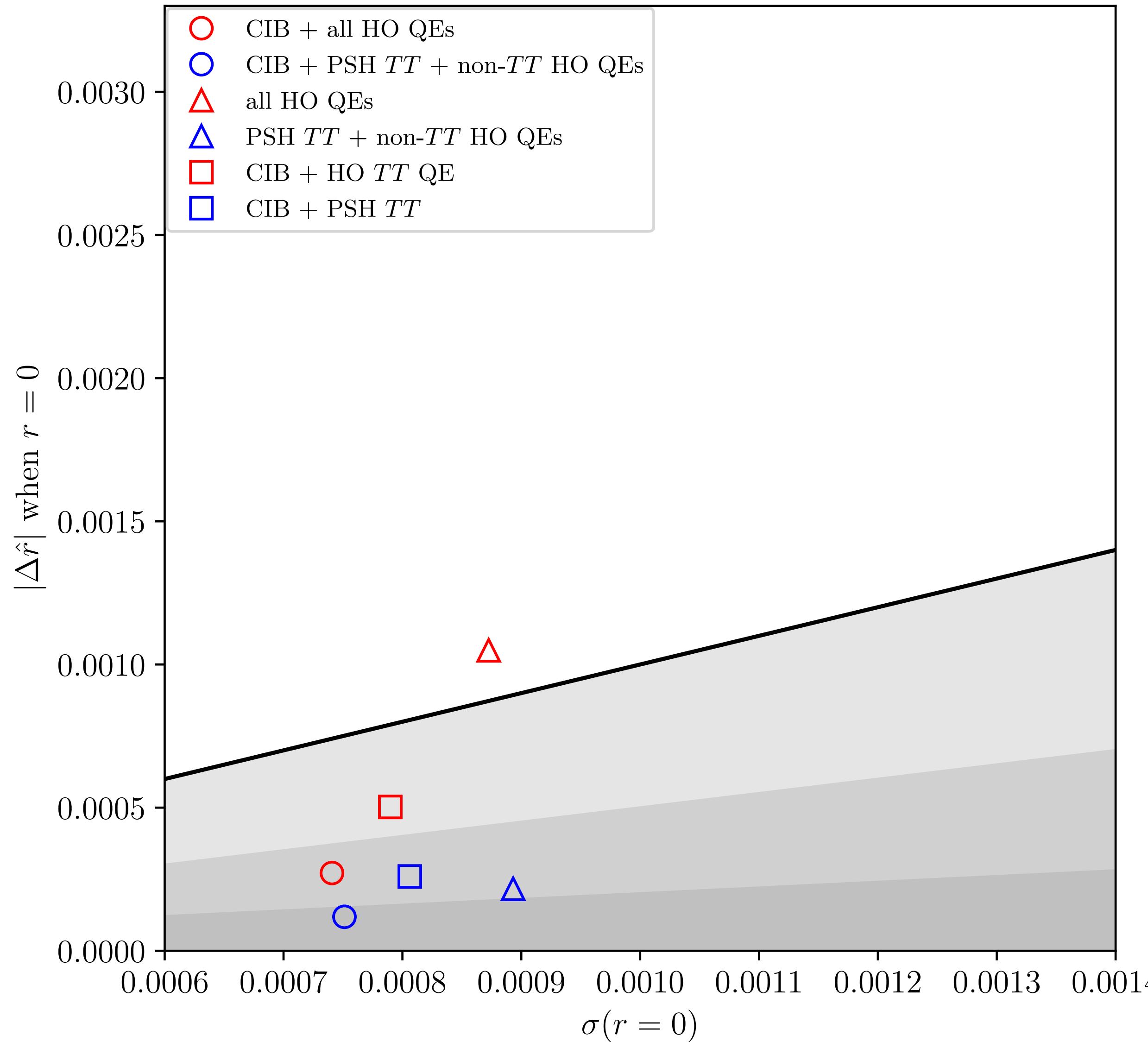
BIAS TO MULTI-TRACER DELENSING



The diluted bias from TT cancels extensively with new cross-terms involving internal and external tracers — especially when the CIB is used as a matter tracer

Here too, empirically-calibrated models can capture the residual effects of foreground non-Gaussianity

BIAS VS NOISE



$$|\Delta \hat{r}|/\sigma(r) = 1$$

$$|\Delta \hat{r}|/\sigma(r) = 1/2$$

$$|\Delta \hat{r}|/\sigma(r) = 1/5$$

CONCLUSIONS

Key points:

- Non-Gaussianity of extragalactic fgs must be taken into account when delensing with TT — otherwise risk $\sim 1.5\sigma$ bias on r (for $l_{\max} = 3500$)
- Naive fg cleaning could be detrimental — reconstructing from MV ILC leads to $\sim 3\sigma$ bias on r
- Bias can be modelled away using analytic expressions relying on empirically-calibrated $C^{\hat{k}\hat{k}}$ and $C^{k\hat{k}}$
- Non-trivial couplings appear when coadding TT with pol-only QEs or external tracers — they actually help us by cancelling diluted bias from TT
- Point-source-hardened or shear-only QEs are rather immune to bias while retaining much of delensing efficiency — they can lead to lower B-mode power after delensing than Hu-Okamoto QE

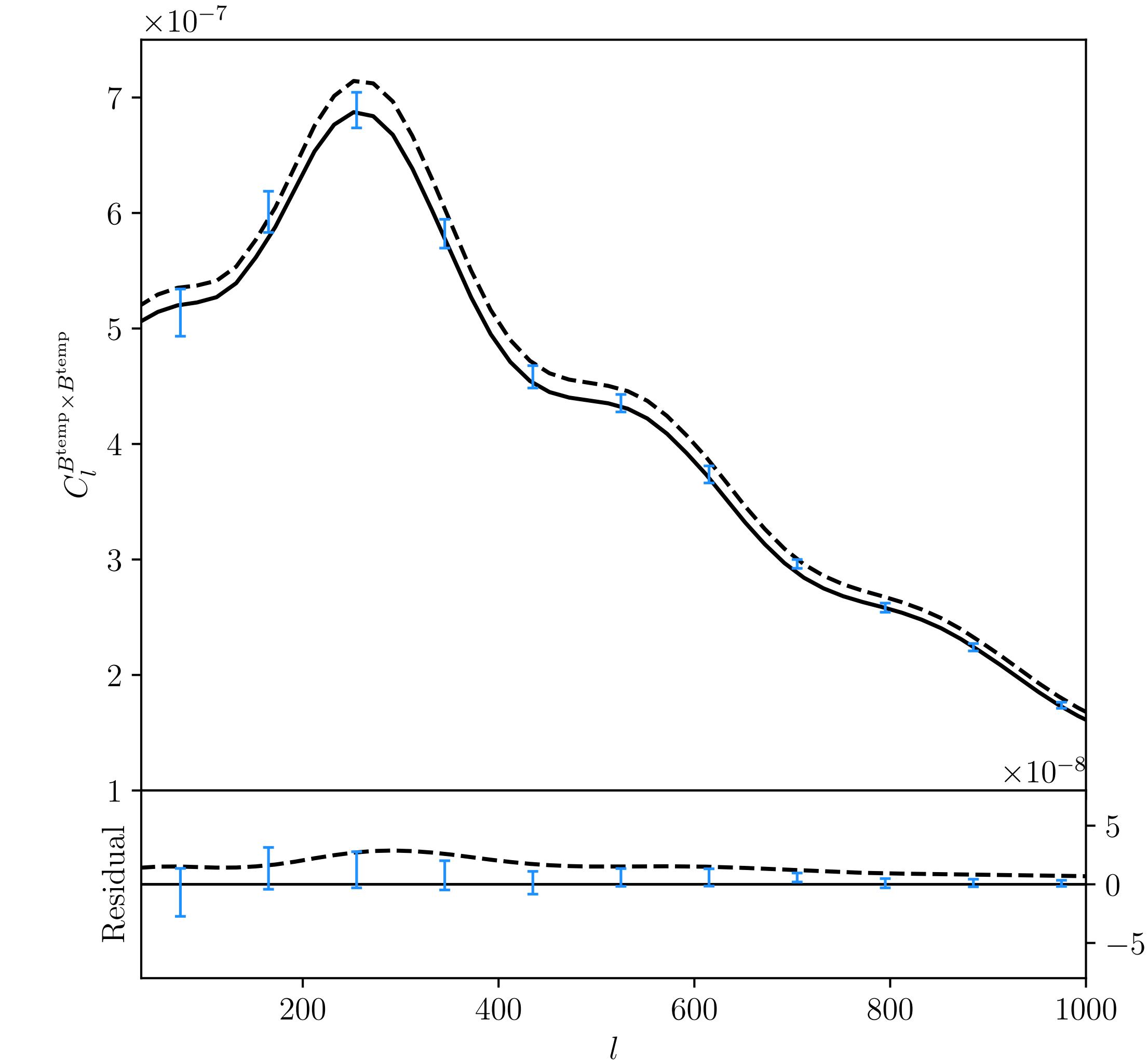
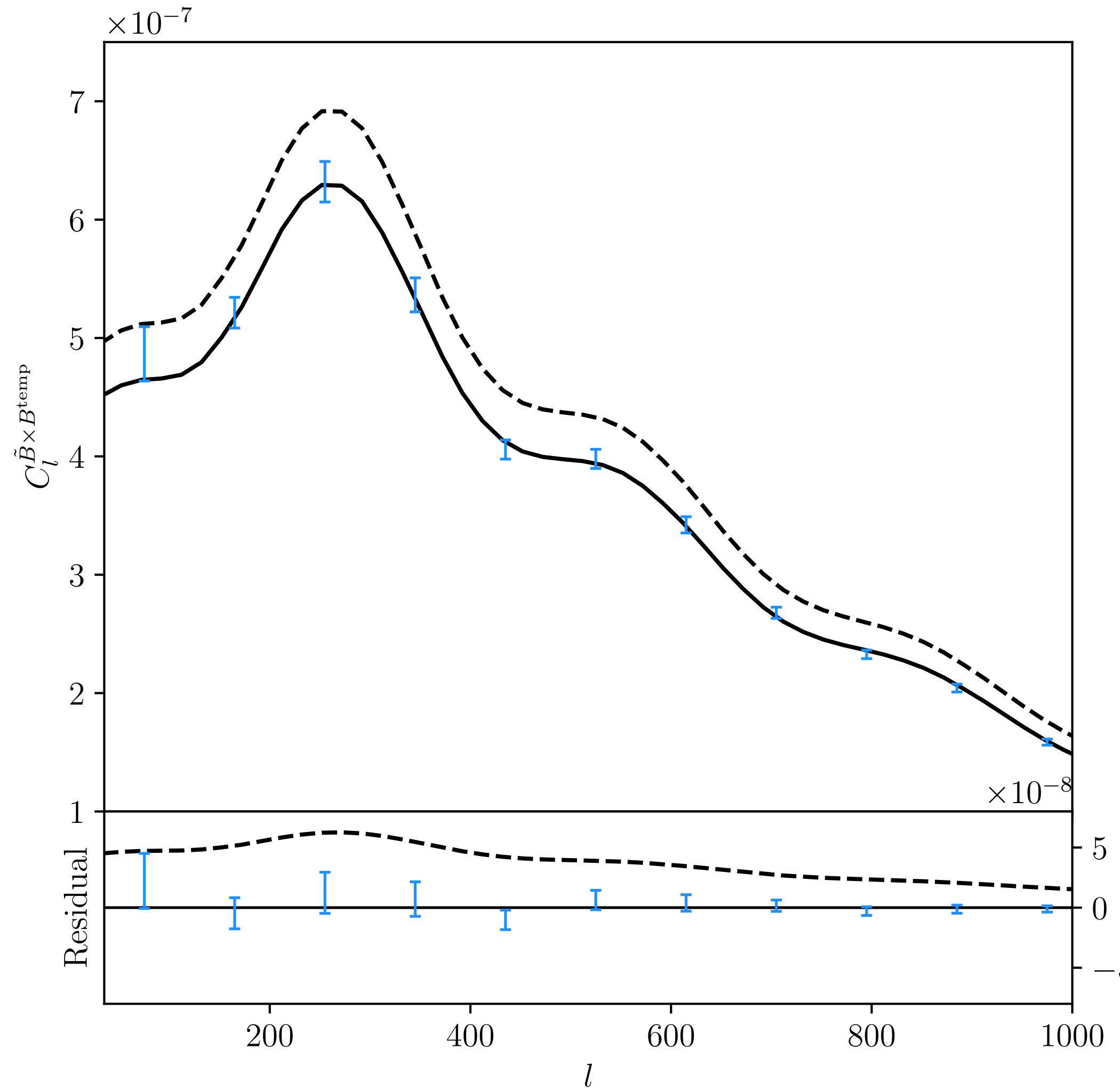
Future work for SO:

- Determine what pipeline choices lead to lowest B-mode power spectrum after delensing
- Assess cross-terms with LSST galaxies
- Incorporate empirical modeling into our likelihoods
- Extend to polarized foregrounds (point sources) [Noah, Simone]
- Gauge fg impact on delensing of acoustic peaks [Alex]

Thanks!

ADDITIONAL SLIDES

EMPIRICALLY-CALIBRATED MODELS OF BIASED B-MODE SPECTRA



Models were computed using $\langle C^{\hat{k}\hat{k}} \rangle$ and $\langle C^{\kappa\hat{k}} \rangle$ featuring {

Gaussian fgs
 non-Gaussian fgs