

# Implementation of the curved-sky $\tilde{B}$ -mode template

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We describe here the construction of a map-level template for lensing B-modes. We work to leading order in lensing and on the curved-sky formalism, a combination that has been shown to be a very good approximation to the true B-modes on large angular scales<sup>1</sup>. We arrive at a fast position-space implementation which is made publicly-available on [GitHub](#)<sup>2</sup>. These lensed B-modes can be approximated in harmonic space by<sup>3</sup>

$$\begin{aligned}\tilde{B}_{lm} &= \phi_{(lm)_1} E_{(lm)_2} \frac{1}{2i} [{}_2I_{ll_1l_2}^{mm_1m_2} - {}_{-2}I_{ll_1l_2}^{mm_1m_2}] \\ &= \frac{-i}{2} \frac{1}{2} (-1)^m \phi_{(lm)_1} E_{(lm)_2} \sqrt{(2l+1)(2l_1+1)(2l_2+1)/4\pi} [l_1(l_1+1) + l_2(l_2+1) - l(l+1)] \\ &\quad \times \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & -m \end{pmatrix} \left[ \begin{pmatrix} l_1 & l_2 & l \\ 0 & -2 & 2 \end{pmatrix} - \begin{pmatrix} l_1 & l_2 & l \\ 0 & 2 & -2 \end{pmatrix} \right]\end{aligned}\quad (1)$$

Inspired by the implementation of the quadratic estimators for lensing reconstruction in the publicly-available code [QuickLens](#)<sup>4</sup>, we write

$$\hat{\tilde{B}}_{lm} = \frac{(-1)^m}{2} \sum_{(lm)_1} \sum_{(lm)_2} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & -m \end{pmatrix} W_{l_1l_2l} \hat{\phi}_{(lm)_1} \hat{E}_{(lm)_2}, \quad (2)$$

where the weights

$$W_{l_1l_2l} = \frac{-i}{2} \sqrt{(2l+1)(2l_1+1)(2l_2+1)/4\pi} [l_1(l_1+1) + l_2(l_2+1) - l(l+1)] \left[ \begin{pmatrix} l_1 & l_2 & l \\ 0 & -2 & 2 \end{pmatrix} - \begin{pmatrix} l_1 & l_2 & l \\ 0 & 2 & -2 \end{pmatrix} \right] \quad (3)$$

can be cast in separable form as

$$W_{l_1l_2l} = \sum_i W_{l_1l_2l}^i, \quad (4)$$

with

$$W_{l_1l_2l}^i = \sqrt{(2l+1)(2l_1+1)(2l_2+1)/4\pi} \begin{pmatrix} l_1 & l_2 & l \\ -s_1^i & -s_2^i & s \end{pmatrix} w_{l_1}^i w_{l_2}^i w_l^i. \quad (5)$$

The value for the separable weights  $w_{l_j}^i$  can be found in [Table 1](#).

<sup>1</sup>Challinor & Lewis (2005), [astro-ph:0502425](#)

<sup>2</sup>[https://github.com/abaleato/curved\\_sky\\_B\\_template](https://github.com/abaleato/curved_sky_B_template)

<sup>3</sup>In this notation summation is implicit over matching pairs of indices.

<sup>4</sup><https://github.com/dhanson/quicklens>

$i$	$s_1^i$	$s_2^i$	$s^i$	$w_{l_1}^i$	$w_{l_2}^i$	$w_l^i$
1	0	2	2	$l_1(l_1+1)$	$-1/2$	$i$
2	0	2	2	$-1/2$	$l_2(l_2+1)$	$i$
3	0	2	2	$1/2$	$i$	$l(l+1)$
4	0	-2	-2	$l_1(l_1+1)$	$1/2$	$i$
5	0	-2	-2	$1/2$	$l_2(l_2+1)$	$i$
6	0	-2	-2	$-1/2$	$i$	$l(l+1)$

Table 1: Weights for a fast separable implementation of the lensed  $B$ -mode template.