Effect of Surface Area of a Spherical Bob on the Damping of a Simple Pendulum

18th April 2025

Word Count: 2978

Introduction

The simple pendulum is a commonly used tool to demonstrate simple harmonic motion. It is made of a mass hanging from a string or rod, known as a bob, swinging from a fixed point. When the bob is pulled so that it eliminates all slack on the string and causes the string to form an angle with the vertical, the bob can be released resulting in the string swinging back and forth. The pendulum swings, and as it does the bob experiences two forces; gravity and tension. The tension works as a centripetal force, pointing towards the fixed pivot point, while the gravity acts on the bob at various angles and magnitudes throughout its travel path. As the pendulum swings back and forth, it reaches a point where the string is perfectly parallel to the vertical, called the equilibrium point. After passing the equilibrium point with speed, the pendulum swings until it reaches a point at which it loses all of its speed, the point called the extreme position. The distance that a pendulum travels from its equilibrium point to its extreme position is known as amplitude.

An ideal pendulum is a theoretical simple pendulum in which the string or rod is massless, the pendulum faces no friction at the pivot or air resistance at the bob, and the acceleration due to gravity on the system is constant. In this ideal pendulum, the amplitude never decreases, and as such the pendulum theoretically swings indefinitely. In reality, the pendulum faces non-conservative forces such as friction and air resistance which results in some of the mechanical energy of the system being converted into thermal energy, reducing the amplitude of the pendulum and allowing the pendulum to eventually stop. The energy dissipation due to non-conservative forces of a pendulum is called damping.

The purpose of this experiment is to determine the effect of the surface area of a spherical pendulum bob on damping in a simple pendulum setup.

The independent variable in this experiment is the surface area of a spherical pendulum bob. The surface area can be measured by measuring the diameter of the pendulum bob, and using it in the following equation to find the surface area: $SA = 4\pi(\frac{d}{2})^2$. See Appendix A for a derivation of the equation.

The dependent variable in this experiment is the amplitude of the simple pendulum setup. The amplitude can be measured by finding the successive time it takes for the pendulum to return to equilibrium, and using it in the following equation: $\theta = 4\sqrt{\frac{\Delta t\sqrt{g}}{\pi\sqrt{l}} - 1}$. See Appendix B for a derivation for the equation.

The following materials are controlled:

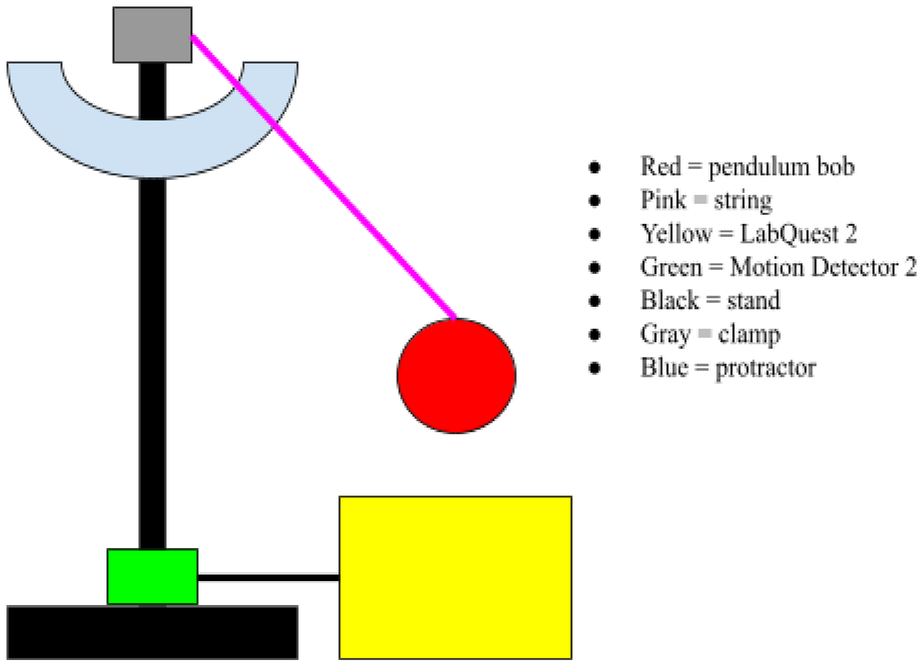
- Length of the string - 32 centimeters
- Mass of the pendulum bob - 56.16 grams
- Time of data collection - 120 seconds
- Intervals of data collection - 20 times/second
- Elevation of pendulum - 118 meters above sea level

Materials and Method

Materials

The following equipment was used in the experiment:

- Vernier LabQuest 2
- Vernier Motion Detector 2 - uncertainty of .005 seconds
- Yardstick - uncertainty of .05 cm
- Protractor - uncertainty of .05 degrees
- Lab Stand with Steel rod, 64 centimeters tall
- String
- Pendulum clamp
- Electronic scale - uncertainty of .005 g
- Tissue paper
- Steel ball bearings
- Clear ball ornaments
- Duct tape
- String
- Scissors
- Small hook



Procedure

Stand Setup

1. Tape the motion detector to the base of the stand so that it points up
2. Connect the labquest to the motion detector, and place it out of the way of the pendulum
3. Set data collection intervals to 20 times per second, and a selected time of data collection, this experiment uses 120 seconds
4. Attach the clamp to the top of the stand
5. Tape the protractor just underneath the clamp, oriented so that the flat end is on top and parallel to the ground
6. Cut a length of string and record the length, this experiment uses 32 centimeters
7. Tie the small hook to one end of the string
8. Tape the free end of the string to the clamp

Pendulum bob Setup

1. Measure the diameter of each pendulum bob
2. Zero the digital scale
3. Weigh the first of your clear ornaments
4. Pick a mass for all of your pendulum bobs, this experiment uses 56.16 grams
5. Fill the pendulum bobs with ball bearings to get to 56.16 grams, then remove a small amount
6. Pack the ball bearings into tissue paper to form a sphere with the core holding the majority of the mass
7. Fill the pendulum bob with the tissue-paper/ball bearing sphere, adjusting to ensure the mass is at the desired value
8. Repeat for all pendulum bobs

Data collection

1. Attach the pendulum bob to the small hook at the end of the string
2. Hold out the pendulum bob at a 90 degree angle from the vertical using the protractor, making sure there is no slack on the string
3. Start data collection on the labquest and let go of the pendulum
4. Let the pendulum swing for the duration of data collection
5. Repeat for three trials for each pendulum bob, this experiment uses the following diameters: 1.0 cm, 2.5 cm, 3.0 cm, 3.5 cm

Design Choices

A string of length 32 centimeters was used due to the height of the stand being 64 centimeters. Using a string of half of the length of the stand ensures that the pendulum bob does interact with any surfaces during its swing.

The following diameters, in centimeters, of pendulum bobs were used: 1.0, 2.5, 3.0, 3.5. The diameters were used due to their commercial availability, increasing experiment reproducibility. The pendulum bobs were made of clear ornaments which were hollow and cut in half, allowing them to be filled with tissue paper and steel ball bearings. The lowest size was kept at 1.0 centimeter with an uncertainty of .05 centimeters to prevent the use of a measurement which is affected by uncertainty. At 1.0 centimeter, the uncertainty is at 5%, and as the size increases the uncertainty only decreases, reaching a minimum at 1.4%. The size was limited to 3.5 centimeters in order to ensure the integrity of the string-clamp connection throughout the experiment, as a greater size may have had a mass that would require raising the collective mass of the pendulum bobs to a value which could lead to problems in the string-clamp connection and error.

The mass of the bobs was homogenized to 56.16 grams due to the ease of fulfillment. The steel ball bearings used each .05 grams with an uncertainty of .005 grams, which allows for filling to a multiple of .05 for accuracy, allowing the researcher to count the number of steel ball bearings at each size, and making measurement more precise. The slight extension past a multiple of .05 allows the researcher to use tissue paper to surround the steel ball bearings, making them the core of the pendulum rather than allowing them to move around during the pendulum bob’s swing. Additionally, the mass was set at 56.16 grams to account for the weight of the heaviest ornament on its own, which is 22.14 grams with an uncertainty of .005 grams, ultimately allowing for all of the pendulum bobs to be filled. The mass was limited to 56.16 grams to ensure the integrity of the string and its connection to the clamp throughout the experiment.

The pendulum bobs were released at a 90 degree angle from the vertical, with an uncertainty of .5 degrees, in order to increase experiment reproducibility. Additionally, the behavior of a pendulum swinging through small angles differs from that of a pendulum swinging through large angles, meaning that the large angle of release requires a correction factor. Using the correction factor allows this research to commit to the understanding of a pendulum past its simple behavior.

The pendulum setup was established at 118 meters above sea level. This was done due to the geographic location of the experiment. All data was collected at the same surface within an hour, meaning all data was collected at the same elevation above sea level. This was done in order to eliminate any error that could arise out of a changing acceleration due to gravity.

The data was collected 20 times per second, or once every .05 seconds. This was done to ensure that the motion sensor would catch the pendulum bob as it crossed its equilibrium point. In the event of the motion sensor being triggered multiple times in the same swing, an equilibrium position can be established by comparing the positions with those of different swings with only one position. Additionally, the limit increases experiment reproducibility by allowing the data to be collected using outdated versions of the Vernier Motion Detector.

The data was collected over 120.00 seconds with an uncertainty of .005 seconds. This was done to improve experiment reproducibility. At 20 points per second for 120.00 seconds, data collection per experiment involves collecting 2400 data points of position and time. For four experimental conditions over three trials results in the collection and required analysis of 28,800 data points. Additionally, the time was set to a time interval during which none of the pendulums would stop moving completely.

The experiment was kept at three trials to improve experiment reproducibility and adhere to the time constraint included in the agreement made when sourcing equipment. Three trials allows for the experiment to be reproduced on lower-end Vernier Labquest systems which lack the ability to collect and store such quantities of data.

Results and Conclusion

In order to relate the collected motion sensor data back to the original research question, significant processing must be done. Firstly, the raw motion sensor data points must be sorted into one of three categories: detecting the pendulum bob swinging through its equilibrium, detecting the bob just before or after swinging through its equilibrium, and detecting the stand only. This was done by generalizing two values: the value of the stand which would be picked up, and the value of the equilibrium position. The first of the two values was consistent between all trials and experimental conditions at $.58m \ \pm .01m$. The uncertainty value does not contribute to the uncertainty of the results, but rather allows the data points to be generalized and removed to improve data readability. Next, the value of the equilibrium position was found. This differed between experimental conditions due to the differing radii of the pendulum bobs, with the larger radii reducing the distance between the bob and the motion detector at the equilibrium position. The values that fit within .57 and 59 meters can be classified as the motion detector detecting the stand only during the swing of the pendulum, and can subsequently be ignored. The values that do not fit within either established value can be classified as the motion detector detecting the pendulum bob swinging just after or just before the equilibrium position, and can subsequently be ignored. Raw motion detector data tables have been excluded to improve readability.

Pendulum Bob Diameter	Motion Detector Equilibrium Position
1.0 cm	$.21m \pm .01m$
2.5 cm	$.20m \pm .01m$
3.0 cm	$.17m \pm .01m$
3.5 cm	$.17m \pm .01m$

Following the sorting of data, the useful data, the data categorized as detecting the pendulum bob swinging through equilibrium, must be processed. This was done by establishing the time that each swing through the equilibrium occurred and labeling it. Take the example of the first swing, A1. The time at A1 is noted as the time at which the swing started, then the time at A2, the subsequent swing through the equilibrium, is noted as the time at which the swing ended. A2 is the end of the first swing, but is the beginning of a second swing, so it is marked as two points, one beginning and one end. The subsequent swing data point, A3, is marked as the end of the second swing. Between three points, two time differences or Δt values are found, which are used to find the amplitude for each swing.

After finding the amplitude for each swing in each trial of an experimental condition, the trials are compared to each other, based on the number of swings. The trial with the lowest number of swings defines a number of points which need to be averaged, and from that average the amplitude over the number of swings within a finite amount of time is defined for a function. Processed amplitude tables have been excluded to improve readability.

The rate at which damping occurs, or the rate at which the amplitude decreases, can be modeled in a graph with the natural log of $\frac{A}{A_0}$ along the y axis and number of swings along the x axis. The slope of this graph divided by negative 1 yields the amplitude decay constant. See Appendix C for the derivation. Using a linear regression to determine the slope of the graph, and dividing it by negative 1, the following values can be found for the Amplitude Decay Constant:

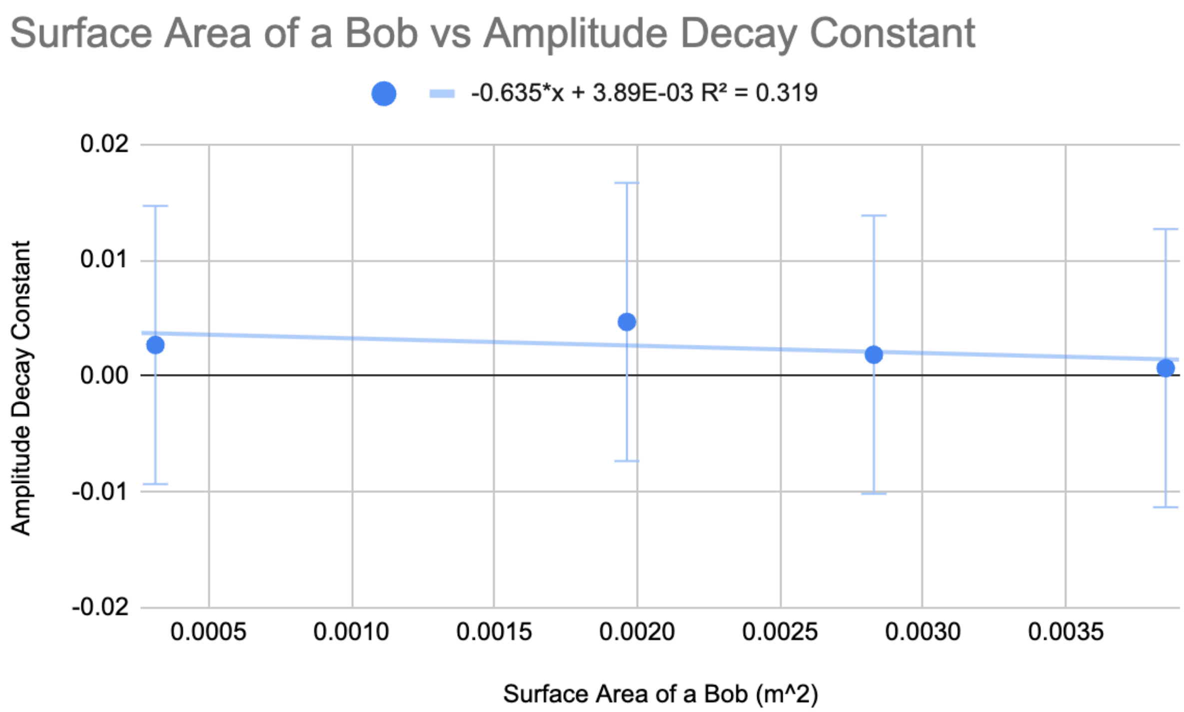
Surface Area of Pendulum Bob	Amplitude Decay Constant
$.0001\pi\, m^2$	$2.68 * 10^{-3}$
$.000625\pi\, m^2$	$4.67 * 10^{-3}$
$.0009\pi\, m^2$	$1.84 * 10^{-3}$
$.001225\pi\, m^2$	$6.78 * 10^{-4}$

See Appendix D for the regression plots

Uncertainty

Surface Area	Amplitude Decay Constant
$\pm$ 0.00338185941 m^2	$\pm\ 0.01200570721$

See Appendix E for the uncertainty calculation.



Due to the high uncertainty values of the data points, a correlation cannot be found between surface area of a spherical bob and the amplitude decay constant of the pendulum system, which directly correlates to the rate at which damping occurs. The best fit line suggests a negative correlation, however it's R^2 value is below .5 and the high uncertainty of the data points allows for many different configurations in which the correlation could be positive, nonexistent, or negative to a differing extent.

Evaluation

While currently research doesn't have exact values for my experiment, the correlation has been explained. From a classical physics standpoint, the correlation is explained by the drag forces that affect the pendulum bob. The drag forces that affect the bob are caused by interaction with air molecules that slow the bob, which is affected by the roughness of the surface of the bob and its geometry. For a spherical pendulum bob, one can consider that a hemisphere of the bob faces the impact of drag forces during swing. As surface area increases, the area of the hemisphere increases, meaning a greater area for more air molecules to impact and slow.

The methodology has a few strengths which improve from the validity of the findings. Error was significantly reduced in multiple design choices. The elevation of the pendulum setup was kept constant throughout data collection, which allowed for a constant value of g, the acceleration due to gravity. Additionally, confounding variables which could have affected the impact of drag forces, such as mass and material on the surface of the pendulum bob, were accounted for.

The methodology has a few weaknesses which could be improved to make the findings more credible. The pendulum bob surface areas used do not increase in regular intervals, which could be fixed by utilizing different pendulum bobs of increasing sizes. Additionally, the mass of the pendulum bobs was not perfectly centered in each bob, which may have allowed the mass to shift during swing, introducing an opposing force on upswings and an assisting force on downswings. This can be fixed by utilizing solid metal pendulum bobs, which differ in their densities, allowing for mass to be held constant while volume is varied. The experiment only carried out three trials due to equipment use constraints, which makes the data statistically insignificant. This can be rectified by carrying out additional trials. Additionally, the constraints required the motion sensor to be limited to 20 data points collected per second, which underutilized the Vernier Motion Detector 2's ability to collect 100 data points per second. This change could improve the statistical significance of the data and reduce the uncertainty due to the measurement of time. Finally, the time of data collection may not have allowed for amplitude decay to be observed and interpreted qualitatively. Rather than collecting data for a set interval of time and recording the number of swings, which differed significantly, the pendulums could be allowed to swing until a specific amplitude, which presents a specific point for application in the amplitude decay equation. The small interval of data collection, paired with the low number of data points collected per second adds to the uncertainty. The Uncertainty of the results originates from two values: the calculation of time between subsequent strings and the measurement of the length of the string. Firstly, the uncertainty due to the measurement of the length of the string can be reduced by using more precise measuring tools such as the Vernier Caliper, which would decrease the uncertainty due to the length significantly. The Vernier Caliper can also be used in calculating the surface area of the bob, thus allowing for the uncertainty of the surface area of the bob to be reduced. Most importantly, the uncertainty due to the measurement of time can be reduced by increasing the data recording interval of the Vernier Motion Detector to as high as it will go, which is 1000 times per second. Finally, the effect of the initial amplitude can account for the uncertainty. Simple harmonic motion has been modeled in classical physics with relatively small initial release angles, less than $\frac{\pi}{4}$. This may limit the assumptions on which the amplitude was derived, thereby adding error to the uncertainty. This can be rectified by utilizing small release angles, which would add uncertainty of the release angles to the uncertainty calculation, but would guarantee the validity of the assumptions on which the equation for amplitude is defined.

Appendices

Appendix A - Derivation of Surface Area

$SA_{sphere} = 4\pi r^2$ $r = \frac{d}{2}$

Substituting $\frac{d}{2}$ for r $SA_{sphere} = 4\pi(\frac{d}{2})^2$

Where d = diameter of the sphere

Appendix B - Derivation of Amplitude

$T = 2\pi\sqrt{\frac{l}{g}}$ for small angle values

Where T = period, l = length of the string, and g = acceleration due to gravity

Adding a correction factor which allows for the use of angles of release greater than 1 radian

$T = 2\pi\sqrt{\frac{l}{g}}(1 + \frac{\theta^2}{16})$

Where θ = Amplitude in radians

Between successive swings through equilibrium, the pendulum travels through half of a period

$\Delta t = \frac{T}{2}$ Where Δt = time between subsequent equilibrium swings

$\Delta t = \pi\sqrt{\frac{l}{g}}(1 + \frac{\theta^2}{16})$ substituting out period

$\frac{\Delta t}{\pi}\sqrt{\frac{g}{l}} = 1 + \frac{\theta^2}{16}$ solving for Amplitude

$\frac{\Delta t}{\pi}\sqrt{\frac{g}{l}} - 1 = \frac{\theta^2}{16}$

$\theta^2 = 16(\frac{\Delta t}{\pi}\sqrt{\frac{g}{l}} - 1)$

$\theta = \sqrt{16(\frac{\Delta t}{\pi}\sqrt{\frac{g}{l}} - 1)}$

$\theta = 4\sqrt{\frac{\Delta t\sqrt{g}}{\pi\sqrt{l}} - 1}$ simplifying

Appendix C - Decay Constant Derivation

$$A = A_0 e^{-\lambda t}$$

Where

A = amplitude

A_o = initial amplitude

λ = decay constant

t = time

$$\frac{A}{A_0} = e^{-\lambda t}$$

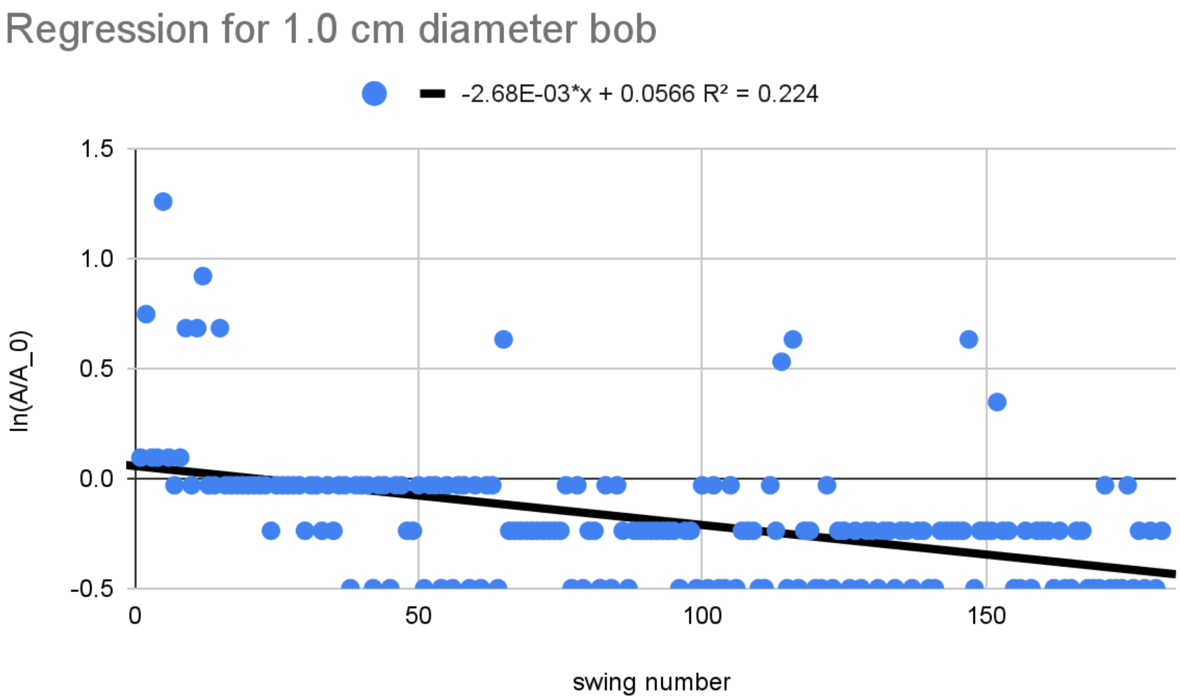
$$ln(\frac{A}{A_0}) = -\lambda t$$

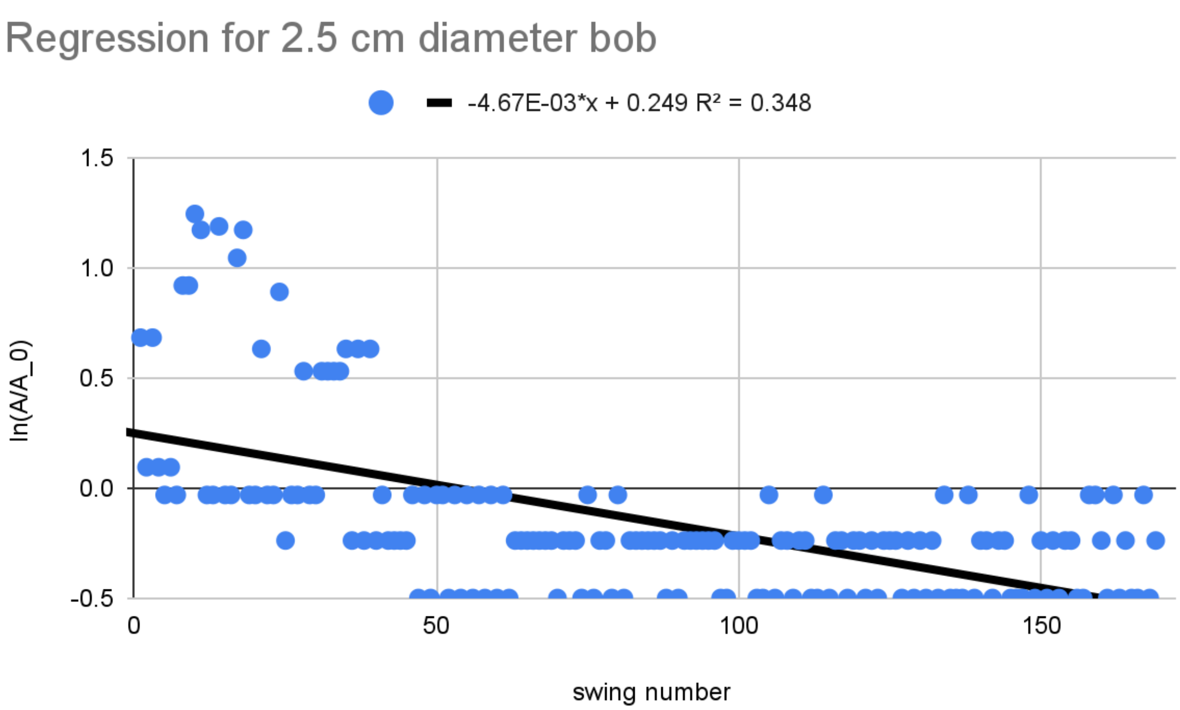
$$\lambda = \frac{ln(\frac{A}{A_0})}{-t}$$

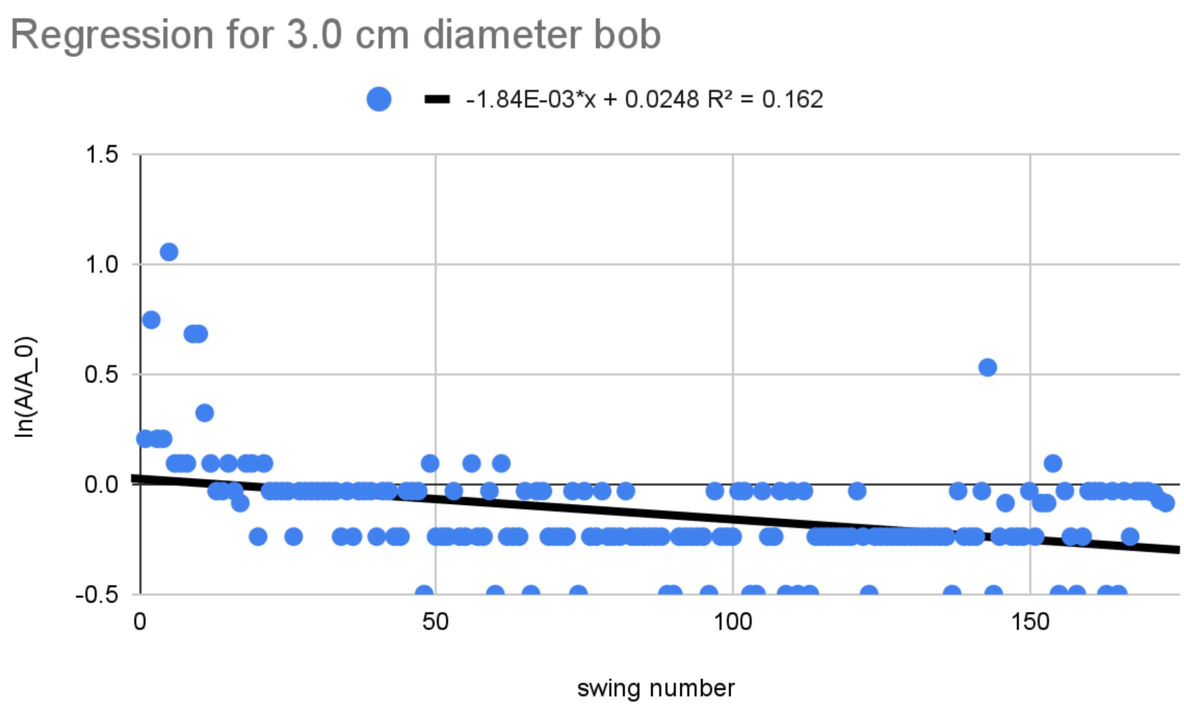
The swing number can be substituted for time as the total swing number is the number of swings that occur within a 120 second interval

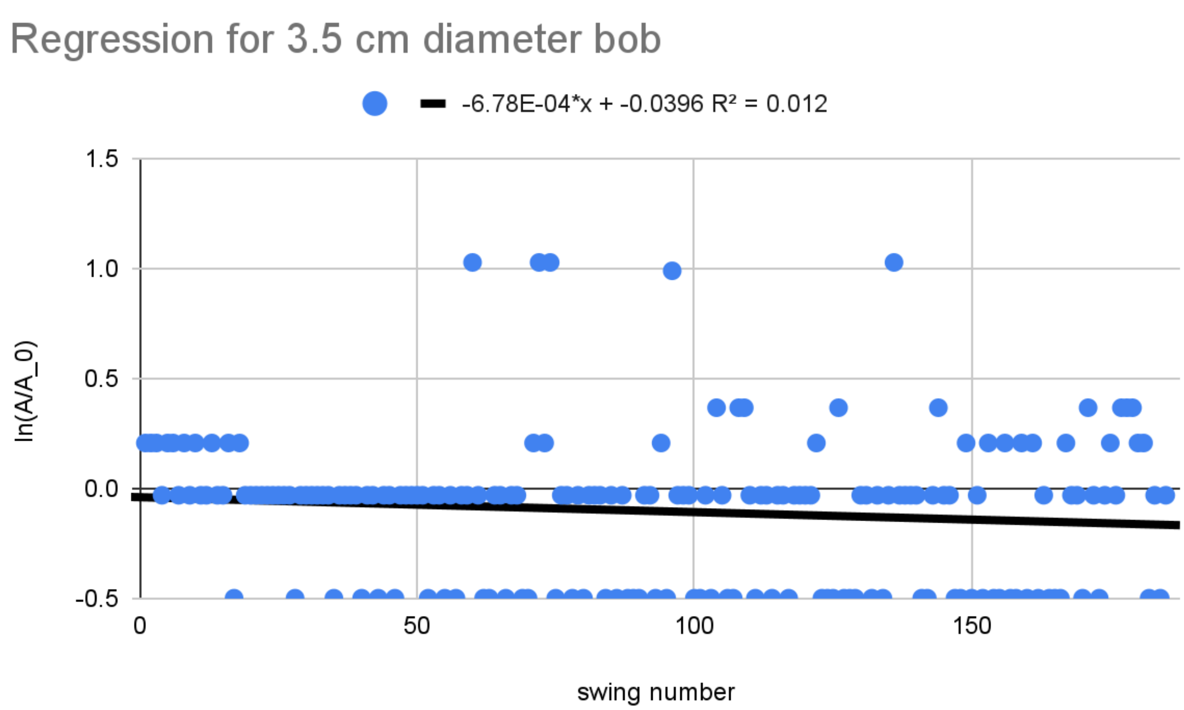
The initial amplitude is 90 degrees with an uncertainty of .5 degrees, or $\frac{\pi}{2}$ radians.

Appendix D - Regression Plots






Regression for 3.0 cm diameter bob
-1.84E-03*x + 0.0248 R² = 0.162
1.5
1.0
0.5
0.0
-0.5
ln(A/A_0)
0
50
100
150
swing number


Regression for 3.5 cm diameter bob
-6.78E-04*x + -0.0396 R² = 0.012
1.5
1.0
0.5
0.0
-0.5
ln(A/A_0)
0
50
100
150
swing number

Appendix E - Uncertainty Calculation

For Surface Area:

The uncertainty comes from the use of the yardstick, which is .0005 meters. It is squared because the measured diameter is used to find radius, which is squared to find surface area.

diameter	radius	radius uncertainty	uncertainty squared	Average of Uncertainty Squared
0.01	0.005	0.1	0.01	0.00338185941
0.025	0.0125	0.04	0.0016	
0.03	0.015	0.03333333333	0.001111111111	
0.035	0.0175	0.02857142857	0.0008163265306	

For Amplitude Decay Constant:

The uncertainty is used from the uncertainty of $\frac{A}{A_0}$, as it is the same as the natural log of the fraction, and factoring in the uncertainty of time, The uncertainty was found by using the equation derived for amplitude in Appendix B and focusing on the two measured terms, delta t, and l, and using their uncertainties. Ignoring all constants in the equation, the delta t term is taken to the ½ power while the l term is taken to the ¼ power. The uncertainty for the l term, the length of the string, is fixed, and results due to use of a yardstick. It is $\frac{.05\ centimeters}{32\ centimeters}$, or 0.0015625. The uncertainty for the delta t results due to the use of a Vernier Motion Detector 2. It is .005 divided by the time values. The average uncertainty in delta t across all of the data is 0.007743388143. The total uncertainty for the amplitude fraction, then, is ¼ * the uncertainty for l + ½ * the uncertainty for delta t. However, since the amplitude decay constant was derived from the slope of the graph which adds another time uncertainty, the uncertainty is ¼ * the uncertainty for l + 3/2* the uncertainty for delta t. This results in an uncertainty of 0.01200570721.