

Homework 1. Group 3.

- Files should be submitted in Jupiter-readable format .ipynb. If you strongly prefer another format please contact me and we agree.
- All cells should not produce any errors.
- Code should be clear, important comments are necessary. Clarity and transparency will be assessed, not only correct answers.
- Answer should be visible. It means that when grader executes cell he should see answer clearly as cell output.

Problem 1. Consider the *Brownian bridge* process $B_t = W_t - tW_1, 0 \leq t \leq 1$.

- a) Simulate 7 trajectories with 1000 points each of this process and plot them. During simulations you can use function `bm_simulations(n_paths, granularity)` which we wrote in class.
Outputs: list of trajectories (do not print, just get); plot with 7 trajectories.
- b) Create 1000 paths of Brownian bridge consisting 100 points each. Check expectation $EB_{0.5}$ and covariance $cov(B_{0.5}, B_{0.3})$. What is true value of this covariance?

b.1) In a separate cell derive $\gamma(t, s) = cov(B_t, B_s)$. You can use as a fact that $cov(W_t, W_s) = \min(t, s)$.

Outputs:

- 1) sample mean of $B_{0.5}$
 - 2) sample covariance of $B_{0.5}$ and $B_{0.3}$
 - 3) cell in markdown format with derivations.
- c) Check independence of increments of Brownian bridge. Namely, check via χ^2 test independence of $B_{0.5} - B_{0.3}$ and $B_{0.3}$.

Output: p-value of the test and conclusion about rejection of hypothesis on independence.

Problem 2. Create derived class from `rv_continuous` which generates distribution with pdf

$$f(x) = \frac{2}{\pi(e^{-x} + e^x)}$$

- a) Try to find mean, standard deviation and median via class functionality. Did you find all quantities?
- b) Simulate 1000 points, build histogram and plot pdf at the same graph. If it takes a lot of time to simulate 1000 points try 100 points.