

## Homework 2. Group 3.

Deadline: January 14, 23:59

- Files should be submitted in Jupiter-readable format .ipynb.
- All cells should not produce any errors.
- Code should be clear, important comments are necessary. Clarity and transparency will be assessed, not only correct answers.
- Answer should be visible. It means that when grader executes cell he should see answer clearly as cell output.

### Problem 1 [50 pts]

1.1) Write base class *figure* with methods *area()*, *perimeter()* and *center\_distance(x,y)*. What first two methods do is clear. The last method *center\_distance(x,y)* calculates distance from center of the figure to point with coordinates (x,y).

Constructor of figure class takes two values (x0, y0) – center of figure. Decide which methods implementations should be left empty and which could be implemented on base class.

Outputs:

- Create object of *figure* class with center coordinates (1,0)
- Output values x0 and y0

1.2) Write child class *rectangular* and implement methods from base class.

Outputs:

- Create object of *rectangular* class with center coordinates (2,3) and sides 3 and 5.
- Output methods *area()*, *perimeter()* and *center\_distance(10,10)*.

1.3) Write child class *circle* and implement methods from base class.

Outputs:

- Create object of *circle* class with center coordinates (5,5) and radius 4.
- Output methods *area()*, *perimeter()* and *center\_distance(10,10)*.

### Problem 2 [50 pts] Pricing Forward in **Binomial model**.

- Consider Binomial model for evolution of stock price after  $n$  days  $S_n = S_0 X_1 X_2 \dots X_n$ , where  $X_1, \dots, X_n$  are iid random variables with distribution

$X$	1+u	1+d
$P$	$p$	$1-p$

$p = \frac{\tilde{r}-d}{u-d}$ , where  $\tilde{r}$  is risk-free interest rate earned for a day. Assume continuously accrued interest rate i.e. money grow via  $B_t = B_0 e^{rt}$ ,  $r$  – yearly interest rate.

- Consider forward contract with payoff  $f(S_T) = S_T - K$  ( $T$  – maturity,  $K$  – strike). Fair price of forward can be computer via

$$PV = E\left(\frac{S_T - K}{e^{rT}}\right)$$

Parameters for our task:  $S_0 = 100$ ,  $u = 0.01$ ,  $d = -0.006$ ,  $r = 7\%$  (yearly),

For forward contract:  $K = 110$ ,  $T = 3$  months.

Tasks:

- 1) Design and create class *forward*.

Output: payoff on given data

- 2) Design generic class pricer and two derived classes `analytic_pricer` and `mc_pricer`.

Outputs:

- price of given forward via Monte-Carlo method. Price via Monte-Carlo is sample mean of discounted payoffs
- analytic price of forward. Use the formula

$$PV = S_0 - \frac{K}{e^{rT}}$$

+10 points to the course: in separate markdown cell derive PV formula i.e. prove

$$E\left(\frac{S_T - K}{e^{rT}}\right) = S_0 - \frac{K}{e^{rT}}$$