Harris-Todaro Model in the Real Business Cycle Framework

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Motivation

- The classical Harris-Todaro model explains rural-urban migration through expected wage differentials but operates in a static framework.
- It lacks the ability to capture how migration decisions interact with macroeconomic dynamics, such as capital accumulation and productivity shocks.
- This project embeds the Harris-Todaro mechanism into a Real Business Cycle (RBC) framework to study labor reallocation in a dynamic, general equilibrium setting.
- The integrated model offers insights into how institutional wage rigidities and sectoral productivity affect migration patterns and macroeconomic outcomes—key to understanding structural transformation in developing economies.

Literature Review

- Harris and Todaro (1970) introduced a seminal two-sector model that explains rural-urban migration in developing countries. Their key contribution lies in the concept that migration decisions are driven by expected income differentials, rather than actual wages. The model shows how persistent urban unemployment can arise as a rational outcome when individuals migrate in response to potential higher urban earnings. This framework laid the foundation for understanding dual labor markets and continues to be relevant in examining structural unemployment. Its insights are particularly useful for integrating informal labor dynamics within a Real Business Cycle (RBC) framework, especially in the context of urban labor market frictions and informal sector absorption.

Literature Review

- Fernández and Meza (2015) examine the dynamics of informal employment in relation to business cycles in emerging economies, focusing on the case of Mexico. Contrary to the traditional view that informal employment acts as a countercyclical buffer during downturns, their findings suggest that informal employment in Mexico is procyclical—it increases during economic expansions and contracts during recessions. The authors argue that this behavior reflects an endogenous allocation of labor between the formal and informal sectors, driven by productivity shocks rather than a simple lack of formal jobs. This insight is particularly relevant for modeling labor flows in a dual-sector framework, as it implies that workers respond to changing economic incentives rather than rigid sectoral boundaries. Their results support the incorporation of informality into Real Business Cycle models and align with the Harris-Todaro type mechanisms, where labor mobility is influenced by expected income differentials across sectors.

Literature Review

- Aleksandar Vasilev (2017) developed a Real Business Cycle (RBC) model tailored to Bulgaria, incorporating both efficiency wages and a government sector. The model highlights how rigidities in the labor market, specifically through efficiency wage mechanisms, contribute to persistent unemployment—a key element relevant to dual labor market theories like Harris-Todaro. Vasilev's findings stress the importance of including government behavior in RBC models, as fiscal policy and wage rigidity significantly influence labor market outcomes and macroeconomic dynamics. This work underlines the value of modeling segmented labor markets and institutional features when analyzing cyclical behavior in developing or transitional economies.

Literature Review

- Van Solomon Ntassey's (2023) research investigates how labour market informality and labour mobility shape macroeconomic fluctuations in Sub-Saharan Africa. The study employs a Real Business Cycle (RBC) framework enriched with dual labour markets (formal and informal) and emphasizes the frictions that arise due to informality. Key insights include:

Labour Informality: Reduces the effectiveness of fiscal policies and creates asymmetric responses to shocks.

Labour Mobility: Plays a stabilizing role by allowing smoother transitions between sectors.

Model Contribution: Extends the RBC model by incorporating both informality and mobility in a dynamic setting calibrated for a typical African economy.

Policy Implication: Calls for formalization strategies and mobility-enhancing policies to strengthen macroeconomic resilience.

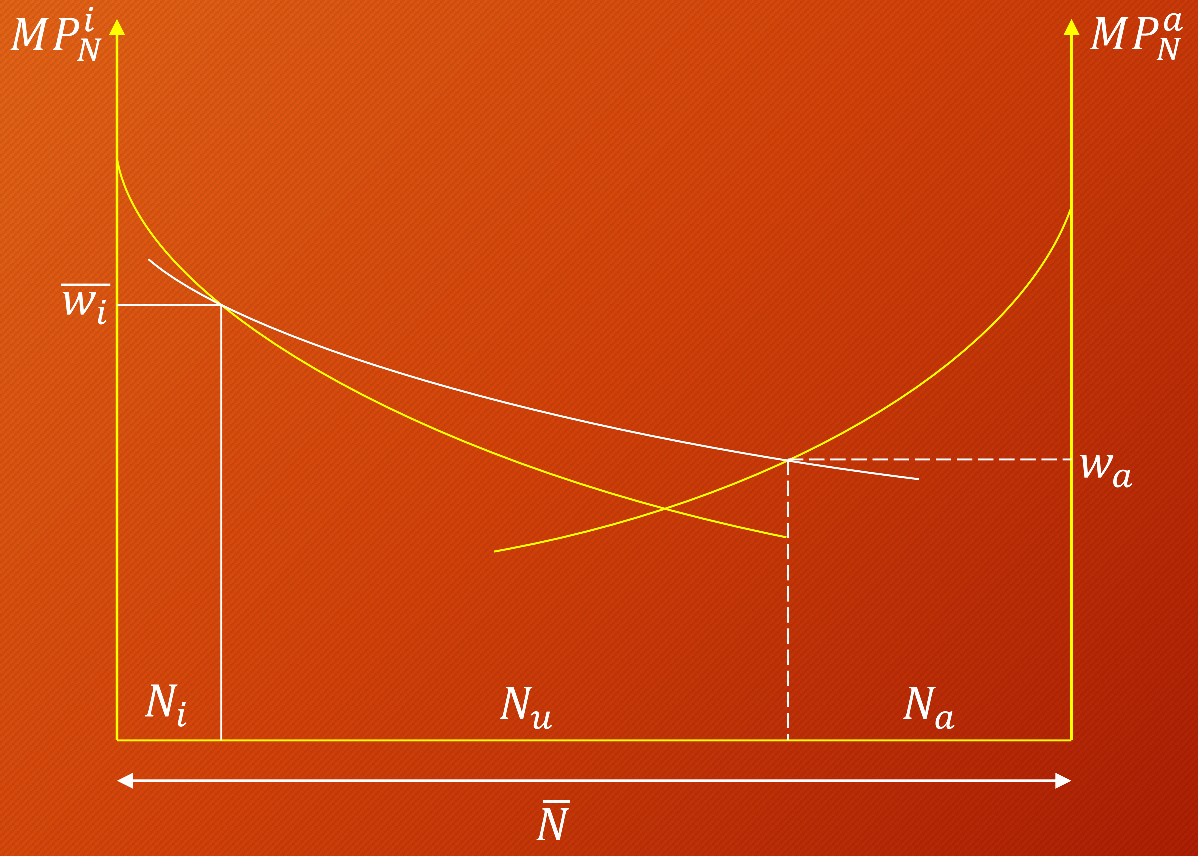
Assumptions

- Two sectors in the economy with rural-urban migration
- Industrial sector (subscript 'i') and agricultural sector (subscript 'a') with representative firm in each
- All firms are profit-maximisers and factor-price takers
- Agricultural sector hires labor up to the point where rural wages equates the marginal product of labor as they have profit-maximizing firms
- Capital accumulation takes place over time at with the constant rate of depreciation
- Only the industrial sector uses both labor and capital and agricultural sector only uses labor

Model Description

- The model economy is populated by three sets of agents:
  - A large number of identical families – each of which contains a continuum of workers/consumers
  - A large number of perfectly competitive agricultural firms
  - A large number of perfectly competitive industrial firms
- We assume a fixed mass of workers/consumers in every family given by $\bar{N} = 1$, which implies that total labor force available in the economy is fixed in all time periods

Harris-Todaro’s Static model



Families and individuals

- In order to maintain a representative agent framework in the economy with positive rates of urban unemployment (as in the Harris-Todaro model), we make two assumptions:
    1. Families own all the capital goods, K_t, in the economy. They use the proceeds from capital investments, $r_t K_t$, in period t to provide a stream of consumption to their members, C_{ft} and purchase new investment goods.
    2. Individuals cannot transfer assets between periods.
- Families distribute C_{ft} to "each member" *before* they know who will be hired by either of the firms (agricultural or industrial) during that period. (following Alexopoulos, 2004)

Families and individuals

- Individuals cannot directly engage in investment, they may increase their *period's consumption* "over" the amount provided by their family by obtaining employment.
- Firms (in both sectors) hire workers by offering them one-period contract that specifies the fixed units of time for which the worker must work (we assume it to be 1 for each worker employed).
- The wage rate in the industrial sector is institutionally fixed enforced by minimum wage given by $\overline{w_i}$
- The equilibrium agricultural employment and hence, the wage rate (w_a) are determined by the Harris-Todaro equilibrium condition for each time period.

Individuals

- The family classifies its members as
  - Employed in agricultural firms, N_{at},
  - Employed in industrial firms, N_{it} or
  - Unemployed, N_{ut}
- Thus, $N_{at} + N_{it} + N_{ut} = \bar{N} = 1$ holds in every time period
- Employed workers consume $C_{ft} + w_{jt}$ $\qquad j = i, a$
- Unemployed workers consume C_{ft}, i.e. the amount of "family-purchased" consumption

Family’s problem

- **The family’s problem**

$$\max_{C_{ft},K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t [N_{it}u(C_{it}) + N_{ut}u(C_{ut}) + N_{at}u(C_{at})]$$

subject to

$$C_{it} = C_{ft} + w_{it}$$
$$C_{ut} = C_{ft}$$
$$C_{at} = C_{ft} + w_{at}$$
$$C_{ft} \leq r_t K_t - \{K_{t+1} + (1-\delta)K_t\}$$

We also assume the felicity function to be $u(c_t) = \ln c_t$

Here, overall labor supply in the economy is fixed by assumption and thus, we do not have the labor supply decision problem of the workers/consumers

Firms

- At time t, the final agricultural and the final industrial goods, Y_{at} and Y_{it}, respectively, are produced by large numbers of perfectly competitive firms in each of the sectors having the following technologies:
- $Y_{it} = A_{it}K_t^{\gamma}N_{it}^{1-\gamma}$ (industrial) and $Y_{at} = A_{at}N_{at}^{1-\alpha}$ (agricultural)

where A_{jt} are the levels of technology (or Total factor Productivity (TFP) shocks), $j = i, a$

- The levels of technology follows the following processes:

(We assume the steady-state levels of TFP shocks as 1)

$$lnA_{it} = \rho_i lnA_{i(t-1)} + \varepsilon_{it}$$
$$lnA_{at} = \rho_a lnA_{a(t-1)} + \varepsilon_{at}$$

Firms' problems

- **Firms - Industrial sector and Agricultural Sector**

In equilibrium, firms maximize profits by choosing their respective factor inputs, and they are price-takers in terms of factor prices.

Industrial sector:

$$\max_{K_{it},N_{it}} Y_{it} - \overline{w_i}\, N_{it} - r_t K_t$$

$$\text{subject to } Y_{it} = A_{it} K_t^{\gamma} N_{it}^{1-\gamma}$$

Agricultural sector:

$$\max_{N_{at}} Y_{at} - w_{at}\, N_{at}$$

$$\text{subject to } Y_{at} = A_{at} N_{at}^{1-\alpha}$$

Optimality Conditions

- C_{ft}: $\frac{N_{it}}{C_{it}} + \frac{N_{at}}{C_{at}} + \frac{N_{ut}}{C_{ut}} = \lambda_t$
- K_{t+1}: $\lambda_t = \beta E_t \lambda_{t+1}(r_{t+1} + 1 - \delta)$
- K_t: $\frac{\gamma Y_{it}}{K_t} = r_t$
- N_{it}: $\frac{(1-\gamma)Y_{it}}{N_{it}} = \overline{w_i}$
- N_{at}: $\frac{(1-\alpha)Y_{at}}{N_{at}} = w_a$

Economy-wide Equilibrium conditions

Harris-Todaro migration Equilibrium requires $\frac{\partial Y_{at}}{\partial N_{at}} = w_a = \frac{\overline{w_i} N_{it}}{\bar{N} - N_{at}}$

$$C_{it} = C_{ft} + w_{it}$$
$$w_{it} = \overline{w_i}$$
$$C_{ut} = C_{ft}$$
$$C_{at} = C_{ft} + w_{at}$$
$$C_{ft} = r_t K_t - \{K_{t+1} + (1-\delta)K_t\}$$
$$N_{at} + N_{it} + N_{ut} = \bar{N} = 1$$
$$Y_{it} = A_{it} K_t^{\gamma} N_{it}^{1-\gamma}$$
$$Y_{at} = A_{at} N_{at}^{1-\alpha}$$
$$C_{ft} + I_t + w_{it} N_{it} + w_{at} N_{at} = r_t K_t + Y_{at} + Y_{it}$$

Steady-State of the model

$$A_a^* = 1 = A_i^*$$

$$\frac{Y_i^*}{K^*} = \frac{\beta^{-1}-1+\delta}{\gamma}$$

$$\frac{C_f^*}{Y_i^*} = \frac{\gamma(\beta^{-1}-1)}{\beta^{-1}-1+\delta}$$

$$\frac{N_i^*}{K_i^*} = (\beta^{-1} - 1 + \delta)^{\frac{1}{1-\gamma}}$$

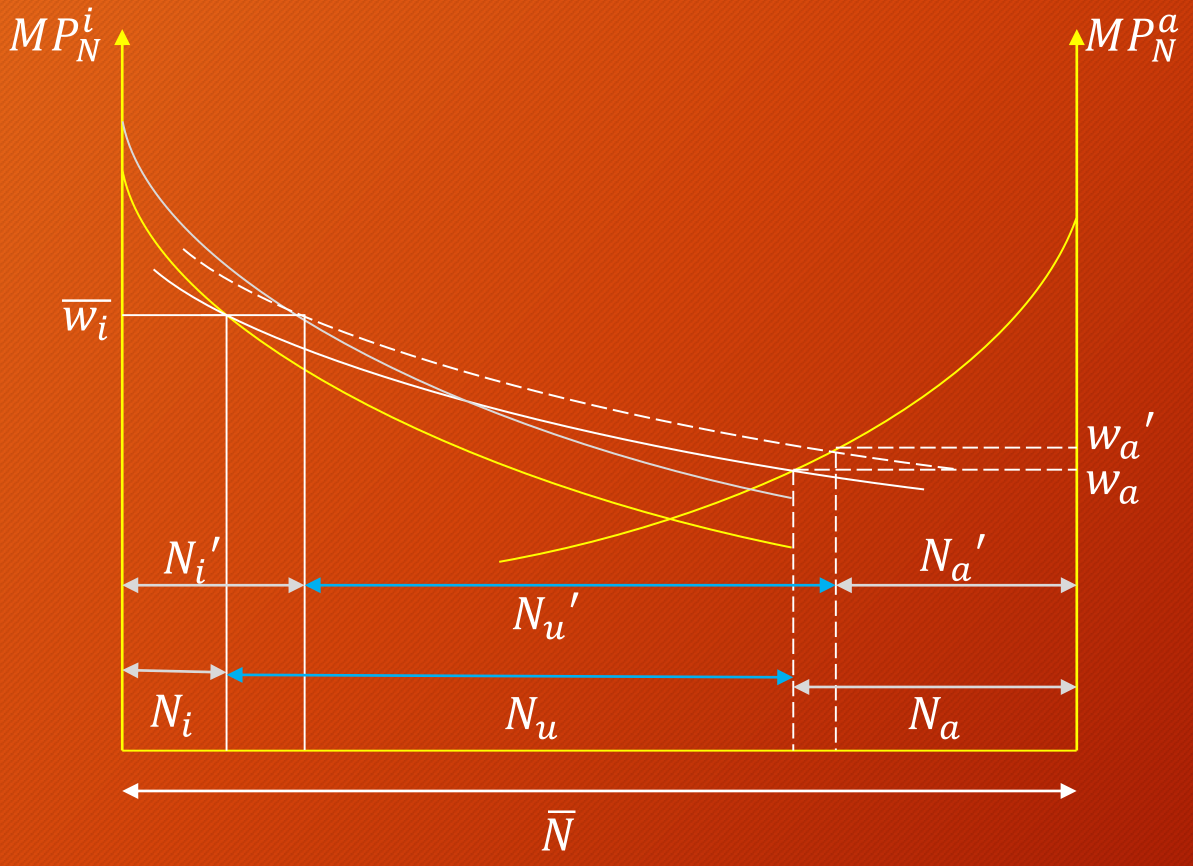
$$r^* = \beta^{-1} - 1 + \delta$$

$$\frac{C_f^*}{K^*} = \beta^{-1} - 1$$

$$\frac{Y_i^*}{Y_a^*} = \frac{\left(\frac{1}{N_a^*}-1\right)}{1-\gamma} > 1$$

$$\frac{N_i^*}{Y_i^*} = \frac{\left\{\gamma(\beta^{-1}-1+\delta)^{\frac{1}{1-\gamma}}\right\}}{\beta^{-1}-1+\delta}$$

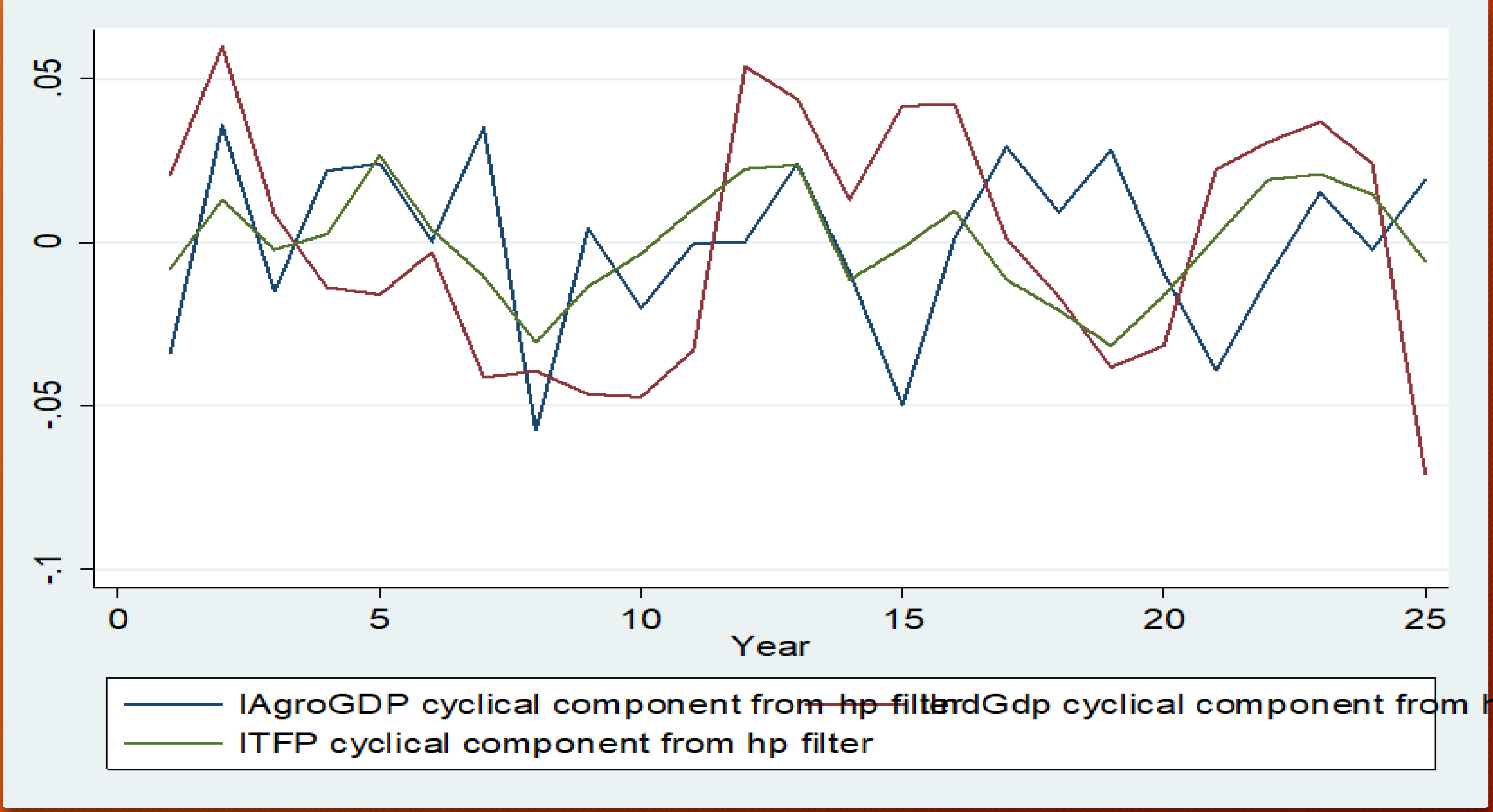
Expected Dynamics



Cyclical Fluctuations

- We use the HP filter (Hodrick & Prescott, 1997) to separate cyclical fluctuations from its smooth trend for each of the variables used in our analysis . The cyclical fluctuations help in giving a clear picture of the actual behavior of the variables under the dynamic economic environment.
- Our analysis has been conducted for the time period 1995-2019. The graphs show the logarithmic forms of the variables.
- Fluctuations above 0 for GDP in both the sectors indicate economic expansion or growth while values below 0 indicate economic recession.
- The HP filter's smoothing parameter (λ) influences the length of the cycles captured. Here we are working with annual data so we have taken λ = 100.

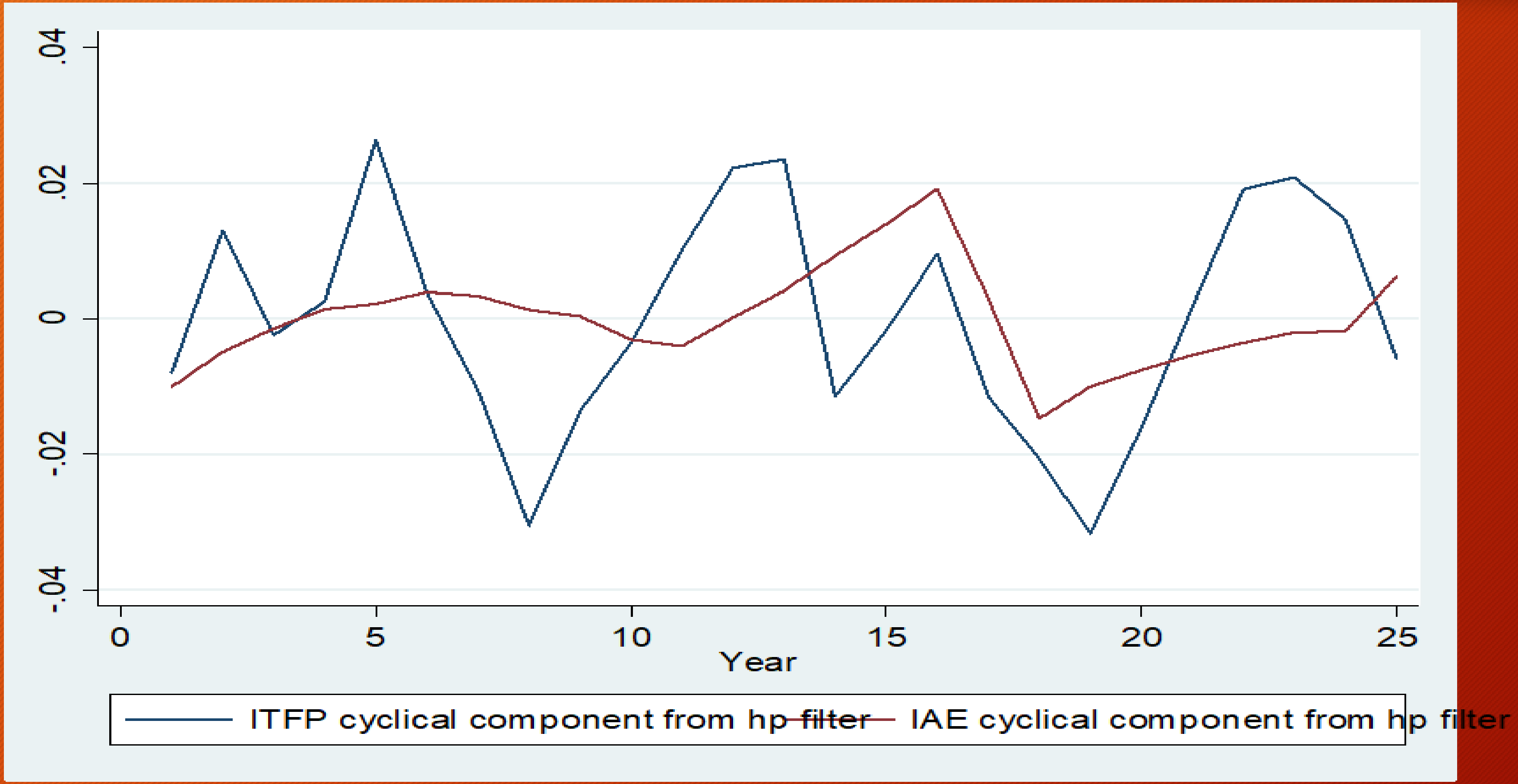
Fluctuations of output of both sectors with TFP



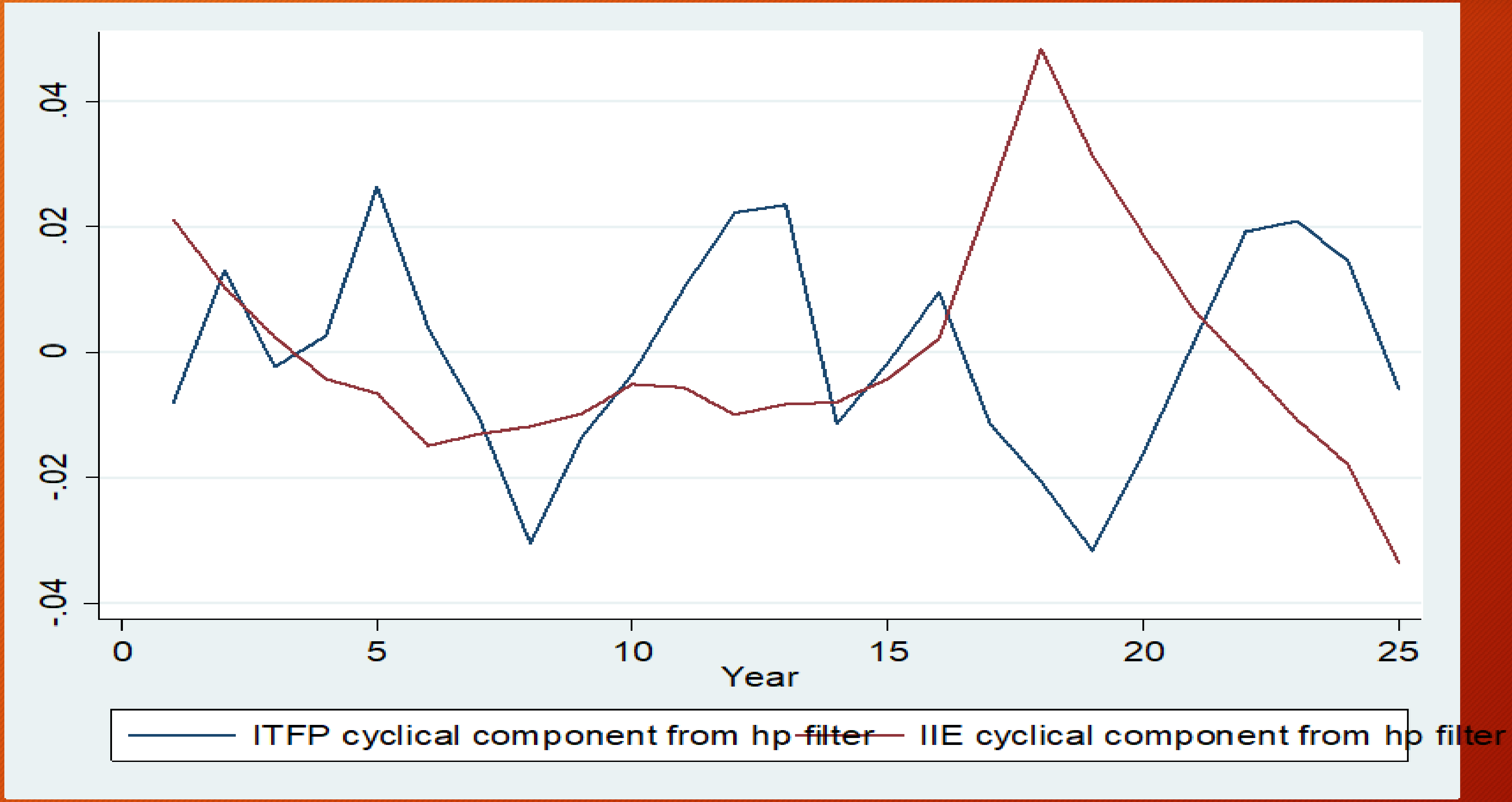
Empirical Results

- The blue and green lines indicate cyclical fluctuations in logarithmic forms of agriculture and industrial outputs respectively. The brown line indicates cyclical fluctuations in logarithmic form of Total Factor Productivity (TFP).
- It is observed that for most of the time period, a positive TFP shock boosts output in both sector while a negative shock contracts output in both sectors. (TFP shocks are procyclical). The correlation between agriculture output and TFP is 0.9870. The correlation between industrial output and TFP is 0.9771.
- The time period in our analysis accounts for one of the greatest recessions in the world economy which is the Global Financial Crisis of 2008. It is observed during this period employment, output and TFP follow a declining trend.

Fluctuations of agricultural employment with TFP



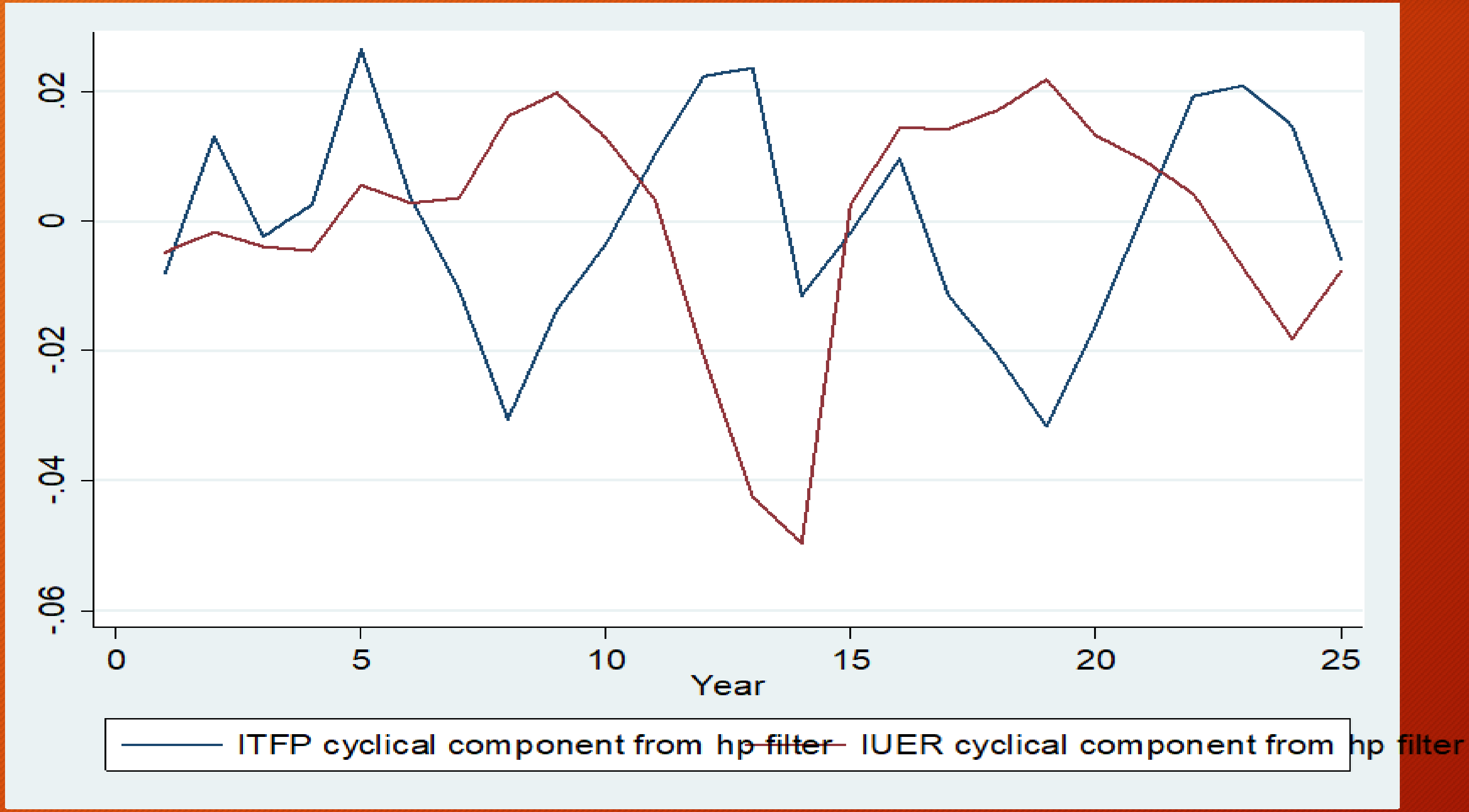
Fluctuations of industrial employment with TFP



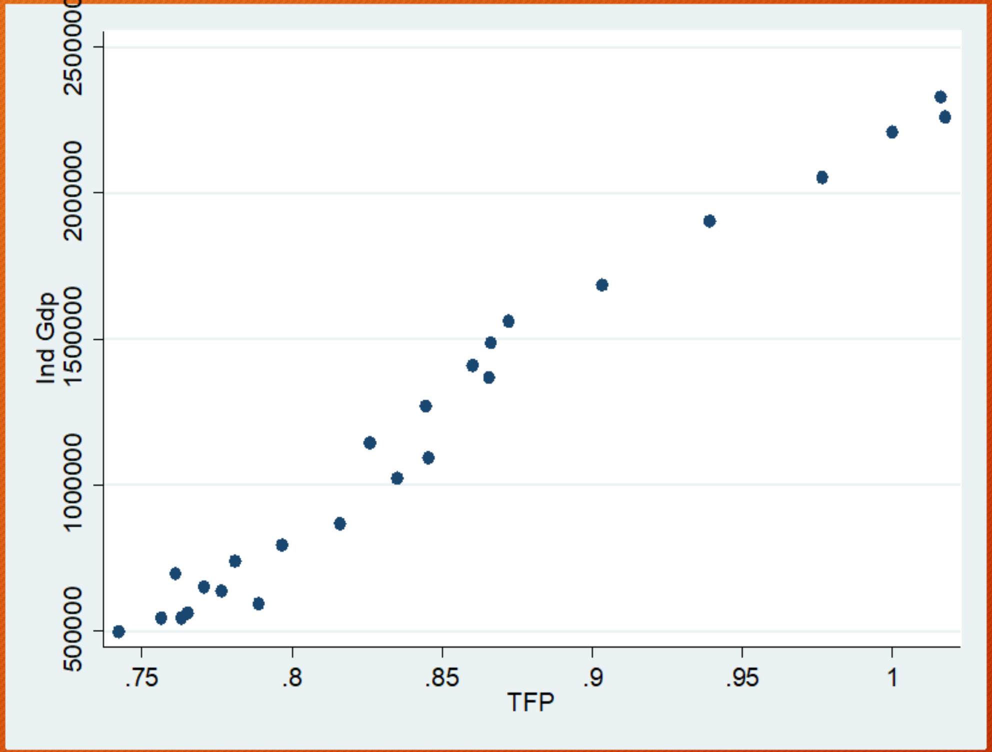
Empirical Results

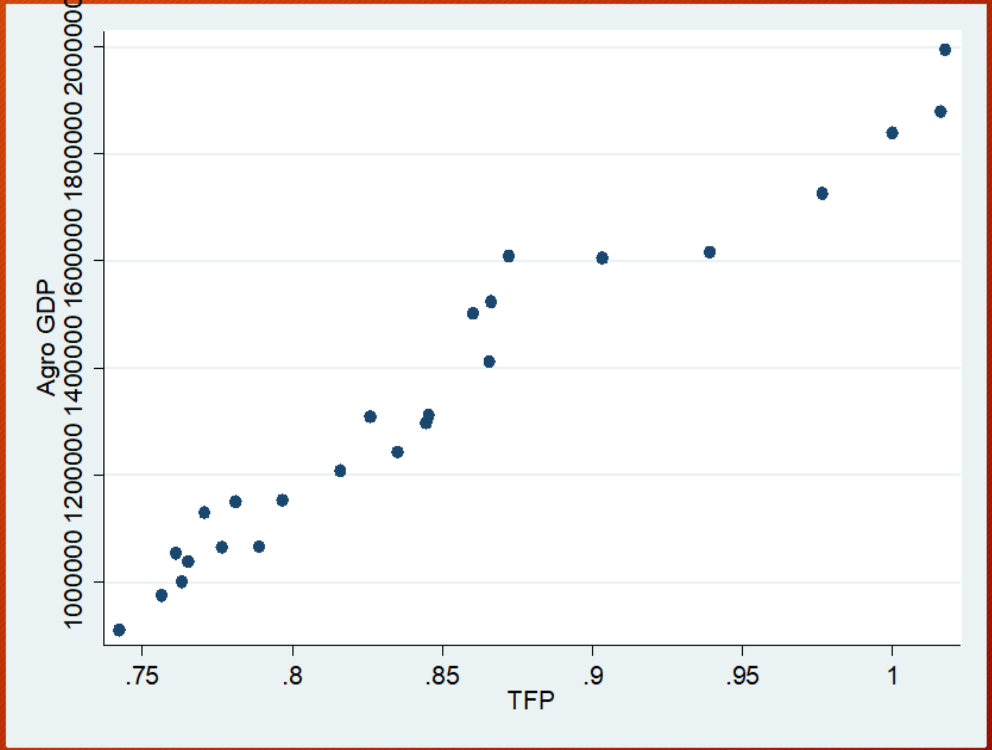
- A positive TFP shock enhances labour employment in both sectors. Correlation for employment in agriculture sector with TFP is **-0.9598** while that for industrial sector is 0.9065. (Labourers migrate from rural to urban sector as TFP is having a negative impact on agriculture employment).
- Employment in agriculture sector is the least volatile while that in the industrial sector is highly volatile.
- The output of both sectors is more volatile than TFP.
- In the following graph, we show fluctuation of TFP with unemployment.
- Unemployment and TFP should ideally be negatively related which is observed in the graph. Very high TFP shock results in very low levels of unemployment.
- The correlation between the two was computed as -0.6120. The correlation between agriculture output with unemployment rate is -0.5507 while that for industrial output is -0.5676.

Fluctuations of TFP and Unemployment Rates

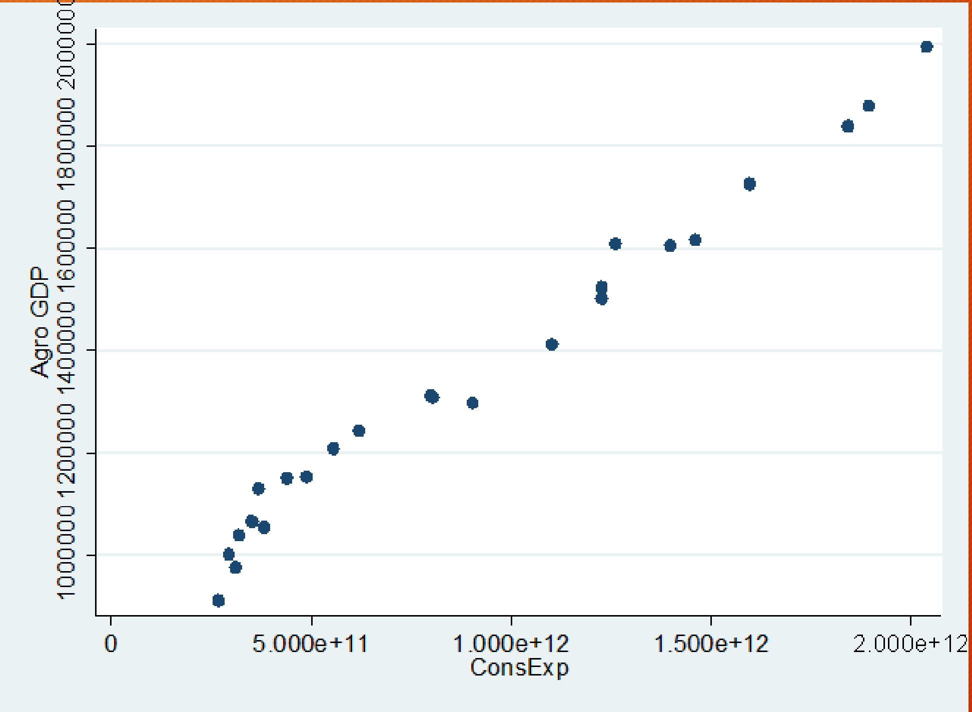


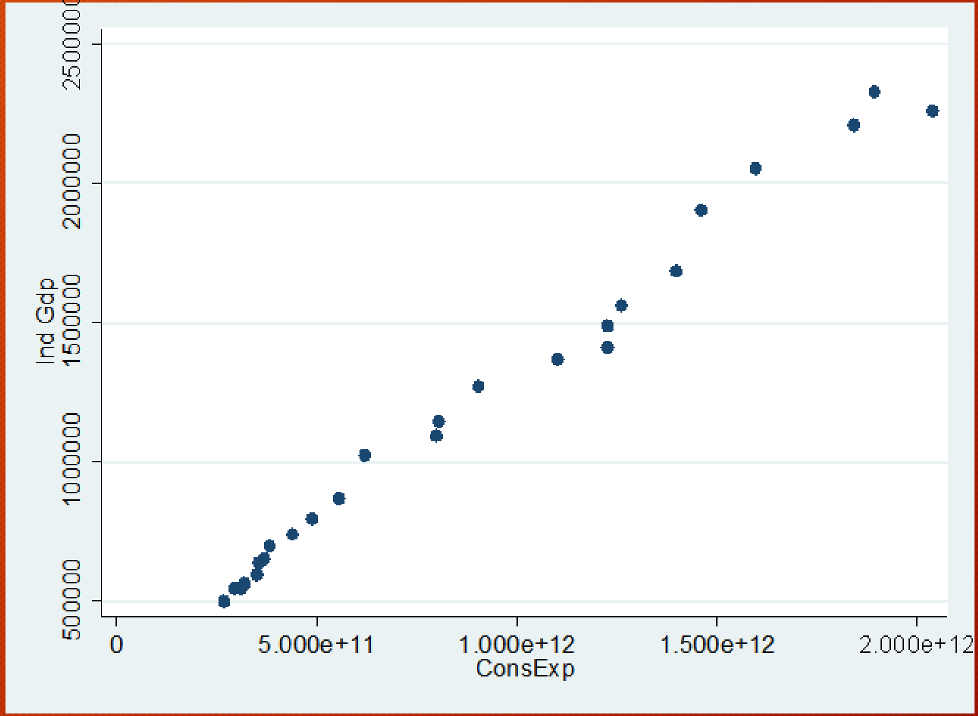
Relation of TFP with output of both sectors using scatter plots





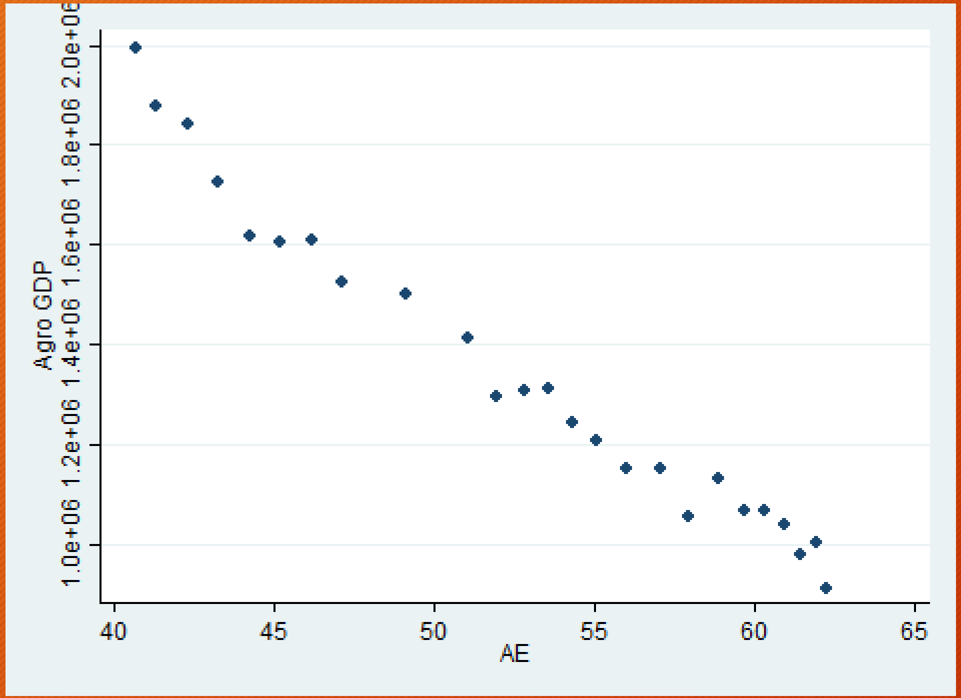
Relation of consumption expenditure with output of both sector using scatter plots

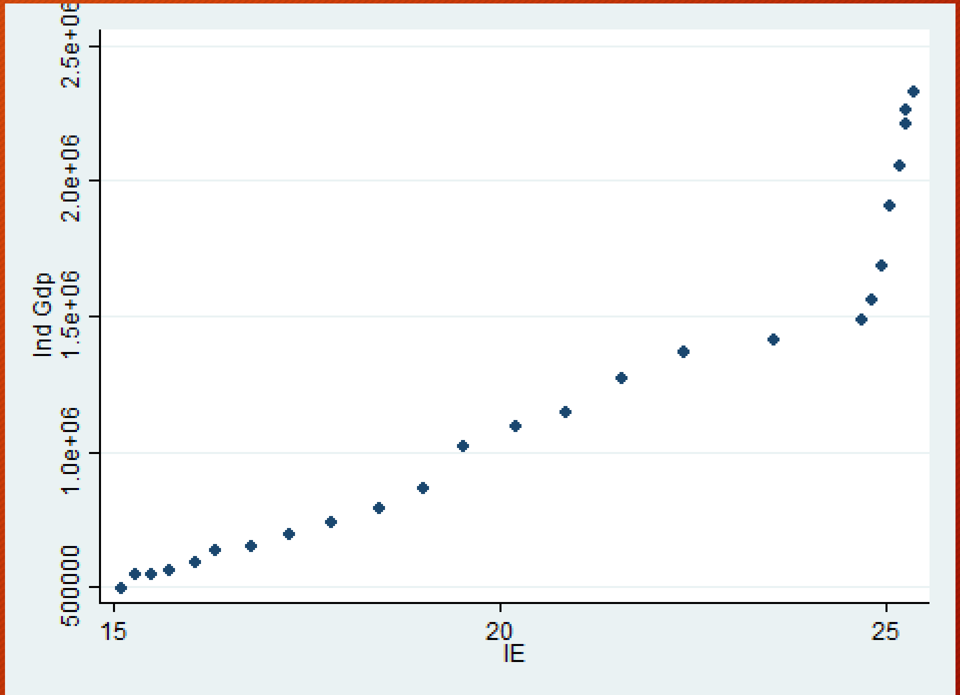




The correlation between agricultural output and consumption expenditure is **0.9917** while that between industrial output and consumption expenditure is **0.9939**.

Relation of (Agro GDP vs AE) & (In GDP vs IE) using scatter plots





The correlation between agricultural output and employment is **-0.9857** while that between industrial output and employment is **0.9532.**

References for Data Collection

Variable Name	Source	Log Form
Industrial Output (GDP)	RBI Handbook of Statistics on the Indian Economy	lIndGDP
Agriculture Output (GDP)	RBI Handbook of Statistics on the Indian Economy	lAgroGDP
Industrial Employment	World Bank	lIE
Agriculture Employment	World Bank	lAE
Unemployment Rate	World Bank	lUER
Total Factor Productivity (TFP)	FRED	lTFP
Consumption Expenditure	World Bank	lConExp
Real Interest Rate	YCharts	lintR

References for Data Collection

- https://data.rbi.org.in/BOE/OpenDocument/2409211840/OpenDocument/opendoc/openDocument.jsp?logonSuccessful=true&shareId=0
- https://data.worldbank.org/indicator/NE.CON.TOTL.KN?locations=IN
- https://data.worldbank.org/indicator/SL.UEM.TOTL.ZS?locations=IN-XN/1000
- https://fred.stlouisfed.org/series/RTFPNAINA632NRUG
- https://data.worldbank.org/indicator/SL.IND.EMPL.ZS?locations=IN
- https://data.worldbank.org/indicator/SL.IND.EMPL.ZS
- https://ycharts.com/indicators/india_real_interest_rate

Steady-State Results

```
%endogenous variables
var Ch, Ce, Cu, Ca, Ne, Na, Nu, Yi, Ya, Ai, Aa, K, r, w_a, u;
%exogenous variables
varexo e_i, e_a;

%parameters list
parameters w_i, delta, beta, gamma, alpha, rho_a, rho_i, Abar_a, Abar_i, Nbar;

w_i = 2;
delta = 0.05;
beta = 0.95;
gamma = 0.6;
alpha = 0.4;
rho_a = 0.8;
rho_i = 0.6;
Abar_i = 1;
Abar_a = 2;
Nbar = 10;
```

```
STEADY-STATE RESULTS:

Ch          48.4316
Ce          91.1216
Cu          48.4316
Ca          48.4632
Ne          21.345
Na          0.00739199
Nu          -11.3524
Yi          106.725
Ya          0.0526329
Ai          1
Aa          1
K           312.065
r           0.205197
w_a         4.27215
u           -1.13608
```