DFSC 5340.02 Assignment 2

1. X is a normally normally distributed variable with mean μ = 30 and standard deviation σ = 4. Find

a) P(x < 40)

b) P(x > 21)

c) P(30 < x < 35)

(a) For $x = 40$, the z-value $z = (40-30)/4 = 2.5$

$P(x<40) = P(z<2.5) = $ (area to the left of 2.5) $= \boxed{0.9938}$ Answer

(b) For $x = 21$, $z = (21-30)/4 = -2.25$

$P(x>21) = P(z>-2.25) = $ total area − area to the left of z

$= 1 - .0122 = \boxed{0.9878}$ Answer

(c) For $x = 30$, $z = (30-30)/4 = 0$ and for $x = 35$,

$z = (35-30)/4 = 1.25$

$P(30<x<35) = P(0<z<1.25) = $ (area to the left of z − area to the left of 0) $= 0.8944 - 0.5 = \boxed{0.3944}$ Answer

2. A radar unit is used to measure speeds of cars on a motorway. Thespeeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

Here, $\mu = 90$ and $\sigma = 10$. We have to get the probability that $x > 100$ which is $P(x>100)$

For, $x = 100$, $z = (100-90)/10 = 1$ (We know $z = \frac{x-\mu}{\sigma}$)

$P(x>100) = P(z \pm 1)$

$=$ (total area) − (area to the left of $z = 1$)

$= 1 - 0.8413$

So, $P(x>100) = \boxed{0.1587}$ Answer

3. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

Here, $\mu = 500$, $\sigma = 100$. Tom got 585 marks. We have to find how many students get less than 585.

$x < 585$. $Z = (x-\mu)/\sigma$ (Here $x = 585$)

$= (585-500)/100 = 0.85$

$P(Z<0.85) = P(Z<0) + P(0<Z<0.85)$

$P(Z<0) = 0.5$

$P(0<Z<0.85) = 0.3023$

Total Probability $= 0.5 + 0.3023 = 0.8023$

Tom did better than most of the students and he will get chance.

4. The length of life of an instrument produced by a machine has a normal distribution with a mean of 12 months and standard deviation of 2 months. Find the probability that an instrument produced by this machine will last

a) less than 7 months.

b) between 7 and 12 months.

Here, mean $\mu = 12$ months, standard deviation $\sigma = 2$ months.

For, $X = 22$

$Z = (X-\mu)/\sigma = (7-12)/2 = -2.5$

a $P(X<7) = P(Z<-2.5)$

$= 0.0062$ (Answer)

b $P(7<X<Z) = P(-2.5<Z<0)$

$= 0.4938$ (Answer)

5. The annual salaries of employees in a large company are approximately normally distributed with a mean of $50,000 and a standard deviation of $20,000.

a) What percent of people earn less than $40,000?

b) What percent of people earn between $45,000 and $65,000?

c) What percent of people earn more than $70,000?

(a) For $X = 40{,}000$, $Z = -0.5$

Area to the left of $Z = 0.5$ is equal to $0.3085 = 30.85\%$ earn less than $40,000.

(b) For $X = 45000$, $Z = -0.25$ and for $X = 65000$, $Z = 0.75$

Area between $Z = -0.25$ and $Z = 0.75$ is equal to $0.3720 = 37.20$ earn between $45,000 and $65000

(c) For $X = 70000$, $Z = 1$

Area to the right of $Z = 1$ is equal to $0.1586 = 15.86\%$ earn more than $70,000.