

## DFSC 5340.02 Assignment 3

Due: Monday October 12 @ 11:59PM  
Total Points: 120 (30 points for each question)

1. You want to be 95% confident of estimating the population proportion to within a sampling error of  $\pm 0.03$ , what sample size is needed? Show your steps to reach the conclusion.

Here, the confidence level  $C = 95\%$ .

margin of error  $E \leq 0.03$

We know,  $E = \frac{\alpha}{2} \left( \sqrt{\frac{p(1-p)}{n}} \right)$

$$\begin{aligned}\alpha &= 1 - C \\ &= 1 - 0.95 \quad (\text{as } C = 95\%) \\ &= 0.05\end{aligned}$$

Let's assume  $p^* = 0.5$

The sample size,  $E \leq 0.03$

$$\begin{aligned}&\Rightarrow \frac{\alpha}{2} \left( \sqrt{\frac{p(1-p)}{n}} \right) \leq 0.03 \\ &\Rightarrow \frac{0.05}{2} \left( \sqrt{\frac{0.5(1-0.5)}{n}} \right) \leq 0.03 \\ &\Rightarrow \frac{(0.05)(0.5)}{\sqrt{n}} \leq 0.03 \\ &\Rightarrow \sqrt{n} \geq \frac{(0.05)(0.5)}{0.03} \\ &\Rightarrow n \geq 1067.11 \\ &\Rightarrow n \approx 1068\end{aligned}$$

The sample size is 1068 (Answer)

2. An IQ test is designed to have scores that have a standard deviation of 12. An SRS of students at a large university will be given the test in order to construct a 95% confidence interval for the mean IQ of all students at the university. How many students should be tested so that the margin of error will be equal to 3 points? Round your answer up to the nearest whole number (i.e. 19.2 rounds to 20).

changed  
it to  
3 points

Hence, the standard deviation = 12 (s)

margin of error = 3 (m)

empirical value of Z-score at 95%

confidence interval = 1.96 (z)

We know, the sample size for 95% confidence interval is given by

$$\frac{(Z \cdot s)^2}{m^2}$$

$$= \frac{(1.96 \cdot 12)^2}{3^2}$$

$$= \frac{(1.96 \cdot 12)^2}{9}$$

$$= 61.47$$

$$\approx 62$$

(Answer)

3. You want to construct a confidence interval for the proportion of all Penn State World Campus students who own a MacBook. You have no idea what the population proportion is. In order to construct a 95% confidence interval with a margin of error of 0.050, what is the minimum sample size that you should obtain?

Hint: For a 95% confidence interval for a proportion,  $z^* = 1.96$ .

$$\begin{aligned} \text{Here, the margin of error} &= 0.050 (E) \\ \text{confidence} &= 0.95 \\ z^* &= 1.96 \end{aligned}$$

The minimum sample size,

$$\begin{aligned} &\frac{(z_{\alpha/2})^2 \times p(1-p)}{E^2} \\ &= \frac{1.96^2 \times 0.5(1-0.5)}{0.050^2} \\ &= 384.16 \\ &\approx 385 \end{aligned}$$

The sample size is 385

4. Sales of a new line of athletic footwear are crucial to the success of a company. The company wishes to estimate the average weekly sales of the new footwear to within \$200 with 95% reliability. The initial sales indicate that the standard deviation of the weekly sales figures is approximately \$1,400. How many weeks of data must be sampled for the company to get the information it desires?

Hence,

$$\text{margin of error} = 200 (E)$$

$$\text{standard deviation} = 1400 (\sigma)$$

$$\text{Confidence level} = 0.95 (c)$$

$$\text{critical value} = 1.96$$

$$\begin{aligned}\text{Sample size} &= \frac{(Z_{\alpha/2})^2 + \alpha^2}{E^2} \\ &= \frac{1.96^2 \times 1400^2}{200^2}\end{aligned}$$

$$= 188.23$$

$$= 189$$

189 weeks of data must be sampled.

(Answer)