Brownian Motion

1 Introduction

A python script that simulates, in 2 dimensional Cartesian Coordinates, Brownian motion of a pollen grain surrounded by point-like molecules of water. The mass of the grain is 20 times that of each molecule. The grain starts at the center of the plot and the molecules are given random velocities using a numpy package random integer generator.

2 Theory

A real simulation of Brownian motion would be computationally expensive due to the large number of collisions that occur between the pollen grain and the water molecules surrounding it. The collisions would be inelastic and calculating the loss of energy and momentum of every collision would be beyond the scope of this project. In this simulation, we will treat the collisions as inelastic with no loss of energy or momentum.

For every collision, the grain will gain a momentum:

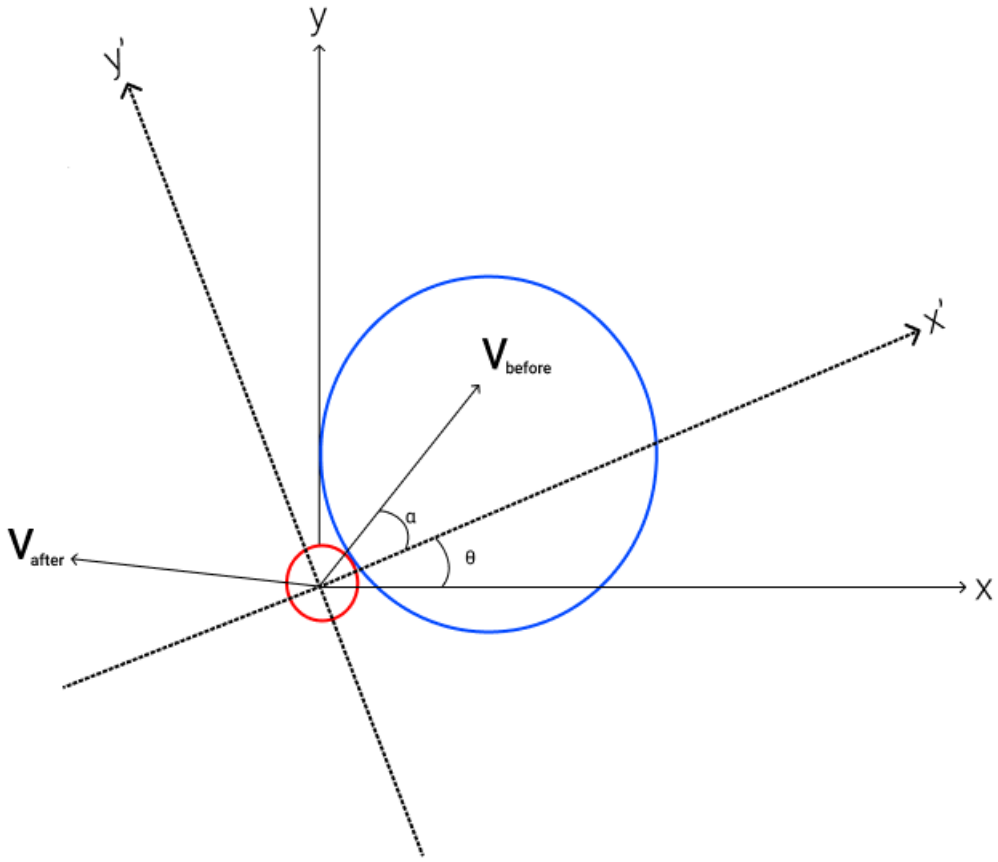
$$\Delta p = m(-2v_x) \tag{1}$$

Where m is the mass of the molecule, and v_x is the horizontal component of the molecule's initial velocity. We will assume that the vertical component of the molecule's velocity v_x will remain unchanged. The grain will gain a velocity increment:

$$\frac{\Delta p}{M} = \frac{m(-2v_x)}{M} \tag{2}$$

Where $M = 20 \times m$ is the mass of the grain. This is also another simplification where we assume that the total change in velocity of the grain will be added to its existing velocity and summed over the total number of collisions. This is physically incorrect, because it violates the conservation of momentum; but due to the mass of the grain being 20 times the mass of the particle it might be a good approximation.

Since the grain is circular there will be a coordinate change that we will need to apply to calculate the change in momentum and velocity of the grain. This must be done for every collision and it depends on the angle of the collision. A picture of the coordinates for one collision is shown in the following figure.



First we resolve the velocity of the molecule on the prime coordinates:

$$v_{x'} = v \times \cos(\alpha) \tag{3}$$

$$v_{y'} = v \times \sin(\alpha) \tag{4}$$

Then we resolve the velocity components on the non-prime coordinate axis:

$$v_x = (v_{x'} \times \cos(\theta)) + (v_{y'} \times \sin(\theta)) \tag{5}$$

$$v_y = (v_{x'} \times \sin(\theta)) + (v_{y'} \times \cos(\theta)) \tag{6}$$

3 Assumptions

We will simulate 100 molecules and run the animation over 800 frames. These values can be changed on line 10 and 11 of the script. The grain will start at the center of the graph and have a velocity 0 until the advent of the first collision. The positions and the velocities of the water molecules will be random and we will ensure that no molecules is generated within the boundaries of the grain.