Solar System Model

1 Introduction

A python script that simulates, in 2 dimensional cartesian coordinates, the orbits and the motion of 10 bodies in the solar system(the Sun, the 8 planets, and the dwarf planet Pluto). The orbits are generated by solving Newton's equations of gravity for each body. The position, velocity, and acceleration of the 10 bodies are represented by 10×2 matrices where the columns of the matrices are the x and y components of the bodies at any given time. The first five planets equation of motion ($Sun \rightarrow Jupiter$) is solved using Scipy's ODE integrator while the orbits of the outer planets ($Sun, Jupiter \rightarrow Pluto$) is solved using the velocity verlet algorithm.

2 Theory

Newton's law of gravity tells us that the acceleration of a body "i" is given by:

$$\frac{d^2\mathbf{r}_i}{dt^2} = G\sum_{j\neq i}\frac{M_j}{r_{ij}^3}\mathbf{r}_{ij} \tag{1}$$

Where M_j is the mass of body "j", G is the gravitational constant, and r_{ij} is the relative distance between the masses given by:

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j \tag{2}$$

The "kick-drift-kick" scheme of the velocity verlet algorithm is given by:

$$v_{i+\frac{1}{2}} = v_i + a_i\frac{\Delta t}{2} \tag{3}$$

$$x_{i+1} = x_i + v_{i+\frac{1}{2}}\Delta t \tag{4}$$

$$v_{i+1} = v_{i+\frac{1}{2}} + a_{i+1}\frac{\Delta t}{2} \tag{5}$$

This scheme is used in gravity simulations where the acceleration is dependent only on the positions of the masses and not on their velocities. There are two advantages of using this algorithm over other algorithms for orbital dynamics:

1. Time Reversal Invariant:

   Since Newton's equations of gravitational motion are invariant under time reversal. It is desirable to have a numerical approximation that mimics this symmetry.

2. Symplectic:

   The advantage of symplectic algorithms is that they posses global stability, meaning that the energy of the system will not diverge at sufficiently long periods of time. This is important for the simulation of the outer planets as we are numerically approximating the solution over a very long time period(the orbital period of Pluto!)

3 Assumptions

We will start the planets along the y-axis and separate them by the distance between their orbits in meters. The orbits will be shown on a 2 dimensional grid with no consideration of the eccentricity of the planetary orbits or their effect on the acceleration. Due to plotting limitations, the motion of all the planets can not be shown on the same plot. This is mainly due to the vast distances between the outer and the inner planets, the size of the markers can not be shown to scale.