

# MAE 3120

## HW 04

Due 04/11

For full credit, show all your work.

### 1 Dynamic response of a Thermistor

A thermistor is immersed in a liquid to monitor its temperature fluctuations. The temperature (in Kelvin) varies in time as:

$$T(t) = 310 + 10 \cos(2\pi t)$$

The thermistor is connected to an electronic transducer system. Overall the system responds as a first-order dynamic system with static sensitivity of 1 mV/K. A step-input calibration of the system reveals that its time constant is  $\tau = 1$  s.

Assume that at  $t < 0$  the thermistor is immersed in the fluid, which has a constant temperature  $T = 310$  K. The sinusoidal forcing starts at  $t \geq 0$ .

- Determine the angular frequency,  $\omega$ , of the temperature fluctuations.
- Plot the output  $E(t)$  in mV for the first 30 s. On the same graph overlay the input  $T(t)$ .
- Once the system has reached steady state determine the time lag  $\beta$  in seconds between the output and the input.
- Remember the definition of the dynamic error (excluding the transient):

$$\epsilon_f(\omega) = 1 - \frac{1}{\sqrt{1 + \omega^2 \tau^2}}$$

Compute it.

- What should be a minimal value of the time constant for the thermistor faithfully measure the temperature in the steady-state regime? Here we will consider that the thermistor is adequately sized if the dynamic error is kept below 0.1%.

- The definition of the time constant of a thermistor is related to the time it takes to reach thermal-equilibrium

$$\tau = \frac{mC}{hA}$$

Assume the that the thermocouple is spherical, has constant density  $\rho$ , and both the specific heat  $C$  and heat transfer coefficient  $h$  are constant. By how much would you need to reduce the radius of the thermocouple to have a faithful response?

## 2 Pressure Measurement

A pressure transducer is connected to a digital data-acquisition board on a computer. The characteristics of the pressure transducer and the data-acquisition system (DAS) are given below. All uncertainties are given at 95% confidence level.

Additionally, you know:

The pressure transducer output ( $V_{out}$ ) is defined by its sensitivity,  $K$ , which is a function of supply voltage ( $V_s$ ) and pressure  $P$ :

$$V_{out} = K(1/\text{Pa}) \times P(\text{Pa}) \times V_s(\text{VDC})$$

Here the expected pressure is up to  $P = 1,000$  kPa.

### *Pressure Transducer Characteristics*

Sensitivity:  $K = 1 \times 10^{-7} \text{ Pa}^{-1}$

Supply voltage:  $V_s = 10$  VDC

Range: 0-2,000 kPa

Linearity:  $\pm 0.25\%$  of reading

Repeatability:  $\pm 0.06\%$  of reading

Hysteresis:  $\pm 0.1\%$  of reading

### *DAS Characteristics*

Number of bits: 16 bits

Input range: 0 – 10, 0 – 1, 0 – 0.1, 0 – 0.01 V

Gain error:  $\pm 1$  LSD

Linearity:  $\pm 1$  LSD

- What is expected range of the pressure transducer output (in Volts).
- Select the DAS input range that will give the best accuracy.
- Estimate the uncertainties (at 95% confidence level) from the pressure transducer,  $u_{PT}$  and the DAS,  $u_{DAS}$ .
- What is the overall uncertainty on pressure measurements made by this system.
- Which component contribute the most to uncertainty? Provide a scheme to reduce the overall uncertainty.

## 3 MTV accuracy

Molecular Tagging Velocimetry is an elegant technique to measure fluid velocity,  $U$ , in a time of flight manner:

$$U = M \frac{\delta x}{\delta t}$$

Where  $M$  is the magnification in pixel/m,  $\delta x$  the measured displacement in pixel, and  $\delta t$  the time interval over which the displacement is measured in second.

(a) How accurately can the velocity be determined if we have the following uncertainties:

$$u_M = 3\%, u_{\delta x} = 1\%, \text{ and } u_{\delta t} = 0.0001\%.$$

(b) Which term contributes the most to the uncertainty?

(b) Suppose now that the velocity must be measured with an accuracy of  $\pm 2\%$ . How accurate must the magnification be?

## 4 Taguchi Design Arrays

A company makes CMOS sensors that are to be used in scientific high-speed cameras. They want to maximize the sensor sensitivity and need to test four parameters ( $a$ ,  $b$ ,  $c$ , and  $d$ ). A three-level experiment (with each level of each parameter repeated 3 times) is chosen. Levels 1, 2, and 3 correspond to low, medium, and high values of each respective parameter. A standard Taguchi 9-run design array is used. For each experimental run, 50 CMOS sensors are tested. For a given light source intensity, the engineers measure the recorded intensity averaged over the sensor,  $X_i$ . The results are shown in the table below.

Run#	$a$	$b$	$c$	$d$	$X_i$
1	1	1	1	1	2146
2	1	2	2	2	2422
3	1	3	3	3	2539
4	2	1	2	3	2572
5	2	2	3	1	2664
6	2	3	1	2	2592
7	3	1	3	2	2715
8	3	2	1	3	2357
9	3	3	2	1	2307

(a) Calculate the 12 level averages, i.e.  $\bar{X}_{a1}, \bar{X}_{a2}, \dots, \bar{X}_{d3}$ .

(b) Generate 4 plots showing the dependence of  $\bar{X}$  on each parameter. Make sure to label your plots properly. For simplicity, you can set level  $a1$  as 1,  $a2$  as 2, etc.

(c) Based on these 9 experiments, which levels of each parameter do you recommend?

(d) Is a confirmatory experiment required? If so, what are the levels to be tested?

In [ ]:

