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Seismically Induced Water Level Fluctuations in the Wali Well, Beijing, China

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Water level records from a high-speed recorder in the Wali well have been compared with long-period seismograms to study the response of the well to seismic waves. Water level oscillations caused by two teleseismic events have been studied. The data show that the peak gain of well water level relative to aquifer pressure occurred at periods of 19-23 s for both events, in contrast with previous theory that predicted peak gain at periods of 35-39 s for a well with 565 m open to the aquifer. We present a more exact analysis of the vertical flow field in the open part of the well bore, which yields a theoretical response with peak gain much closer to the observed periods. The more exact analysis shows that during the seismically induced oscillations little flow takes place more than 200 m below the top of the aquifer.

INTRODUCTION

A well-aquifer system can be viewed as a volume strain meter in which sensitivity to strain is a function of the elastic and hydraulic properties of the aquifer and the geometry of the well [*Bodvarsson*, 1970]. Many observations have demonstrated that water levels in well-aquifer systems can fluctuate in response to earth tides, barometric pressure changes, fault creep, surface loading, and seismic waves [*Bredehoeft*, 1967; *Johnson et al.*, 1973; *Robinson and Bell*, 1971; *Rexin et al.*, 1962; *Roeloffs et al.*, 1989].

Certain wells display water level oscillations in response to the passage of surface waves from large distant earthquakes. *Blanchard and Byerly* [1935] conducted the earliest research on such oscillations, which they recorded on a seismograph drum, using a float as a water level sensor. After analyzing about 80 earthquakes recorded by a well on the island of Oahu, Hawaii, *Eaton and Takasaki* [1959] concluded that the largest water level fluctuations were caused by long-period Rayleigh waves. *Cooper et al.* [1965] carried out a theoretical analysis of seismically induced water level fluctuations in wells, which was shown by *Bredehoeft et al.* [1967] to predict accurately that the period of free oscillations in a well near Perry, Florida, would be 15 s. Since 1965, seismic water level oscillations have continued to be observed and reported [e.g., *Sterling and Smets*, 1971; *Wang*, 1985], but have not been compared with *Cooper et al.*' s [1965] theory, usually because the time resolution of the water level records is inadequate.

Hydrologists have also observed water level oscillations in wells as an underdamped response to slug or bailer tests, in which a known volume of water is rapidly added to or removed from a well. Such a test measures the water level response to a step function imbalance between reservoir head and well water level, and consequently, the test results can be synthesized from the response to an oscillating imbalance, using inverse Laplace transformation. *Kipp* [1985] derived an oscillatory response function based on a slightly different momentum balance from that of *Cooper et al.* [1965] and synthesized nondimensional step response functions that can be used to estimate transmissivity from underdamped slug or bailer tests. *Shapiro* [1988] has extended these results to allow for the possibility of fracture-dominated flow in the aquifer.

In this article we use expanded time scale recordings of two episodes of seismically induced water level fluctuations from a well near Beijing, together with long-period seismograms from Beijing Seismological Observatory at Baijiatan, to study the response of water level to the passage of seismic waves.

SEISMICALLY INDUCED WATER LEVEL FLUCTUATIONS

Consider a seismic wave with ground displacement, $w(t)$, having the form

$$w(t) = w_0 \exp(i \omega t) \tag{1}$$

where w_0 is a complex amplitude, ω is radian frequency, and t is time. We assume that the well-aquifer system behaves linearly, so that any field $f(r,z,t)$ set up by the passage of this wave can be expressed in the form

$$f(r,z,t) = f(r,z) \exp(i \omega t) \tag{2}$$

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where r is horizontal distance from the well axis, z is depth downward from the top of the aquifer (Figure 1), and $f(r,z)$ is in general complex. The passage of the seismic wave causes a pressure head fluctuation, $h_0 \exp(i\omega t)$, in the aquifer, which in turn produces a water level fluctuation, $x_0 \exp(i\omega t)$, in the well. The ratio x_0/h_0 is the amplification of water level in the well relative to pressure head in the aquifer. For a well of radius r_w that fully penetrates an aquifer of thickness d and transmissivity T, *Cooper et al.* [1965] determined that

$$|x_0/h_0| = \left[\left(1-\frac{\omega r_w^2}{2T}\,\mathrm{kei}\,(\alpha_w)-\frac{\omega^2 H_e}{g}\right)^2 + \left(\frac{\omega r_w^2}{2T}\,\mathrm{ker}\,(\alpha_w)\right)^2\right]^{-1/2} \quad (3)$$

in which g is acceleration due to gravity, and $H_e = H_w + 3d/8$ is the effective height of the water column in the well, where H_w is the height of the water column above the upper limit of the aquifer (Figure 1). Here kei and ker are Kelvin functions of order zero [*Abramowitz and Stegun*, 1972], and the argument α_w is related to the parameters of the well-aquifer system and the seismic wave by

$$\alpha_w = r_w(\omega S/T)^{1/2} \quad (4)$$

in which S is the storage coefficient of the aquifer. Equation (3) shows that x_0/h_0 is a function of the period of the seismic waves. *Cooper et al.* [1965] display curves based on equation (3) and show that for sufficiently high transmissivity there is a well-defined peak amplification at a frequency given by

$$\omega_w = (g/H_e)^{1/2} \quad (5)$$

Kipp [1985] solved the same equation as *Cooper et al.* [1965], except that the momentum balance included the entire open interval, whereas *Cooper et al.*'s [1965] analysis included only the upper half of the open interval. *Kipp*'s [1985] analysis consequently yields equation (3) for an oscillating source, except that the effective length is given by $H_e' = H_w + d/2$ for the case of equal screen and casing radii.

The data available to test equation (3) consist of water level records, $x(t)$, and seismograms, $a(t)$. In addition to equation (3), three more physical processes relate seismograms and water level oscillations. The ratio of seismically induced water level oscillations, x_0, to an oscillation at the same frequency in the seismograph record, a_0, can be written as

$$\frac{x_0}{a_0} = \frac{x_0}{h_0}\,\frac{h_0}{\theta_0}\,\frac{\theta_0}{w_0}\,\frac{w_0}{a_0} \quad (6)$$

where θ_0 is the complex amplitude of dilatation in the aquifer.

Both the observations of *Eaton and Takasaki* [1959] and the theory of *Cooper et al.* [1965] show that water wells several hundred meters deep respond primarily to seismic surface waves with periods of 10 s or more. Surface waves consist of Love waves with *SH* vibration and Rayleigh waves with regressive ellipsoidal vibration in a vertical plane containing the direction of propagation. Because *SH* vibrations have no associated dilatation, only Rayleigh waves are expected to cause water level oscillations in wells. For Rayleigh waves, the ratio of dilatation to ground displacement is given by

$$\frac{\theta_0}{w_0} = C\ \omega/v_R \quad (7)$$

in which C depends on Poisson's ratio ν, v_R is the Rayleigh wave phase velocity, and w_0 is the vertical component of displacement, upward taken positive [*Ewing et al.*, 1957; *Bredehoeft et al.*, 1965]. As ν varies from 0.1 to 0.45, C takes on values from 0.665 to 0.009. The aquifer penetrated by the Wali well is composed of unconsolidated sand and gravel, for which a Poisson's ratio of 0.38 or greater is appropriate [*Bourbié et al.*, 1987]. We assume that appropriate values for v_R are the phase velocities for continental Rayleigh waves given by *Oliver* [1962].

The ratio h_0/θ_0 is the response of pressure head to volume strain in the aquifer. From the amplitudes of water level tides in the Wali well, it has been determined that

$$\frac{h_0}{\theta_0} = -0.2 \times 10^7\ \mathrm{m} \quad (8)$$

[*Liu and Zheng*, 1985*a,b*]. The minus sign is required because pressure head declines with increasing dilatation. We assume that equation (8) is accurate to within 10%.

In order to determine the vertical ground motion from the seismogram, the seismograph magnification curve must be known. For the Kirnos seismograph operating at Baijiatan, the magnification is closely approximated by

$$\frac{a_0}{w_0} = \begin{cases} A & \text{if } \tau < 12.6\ \mathrm{s} \\ A(\tau/\tau_0)^{-3} & \text{if } \tau > 12.6\ \mathrm{s} \end{cases} \quad (9)$$

where τ is the wave period [*State Seismological Bureau*, 1986]. The magnification A as determined from the published parameters is about 900. However, in our calculations we have used the magnifications stamped on the seismograms, which are 907 and 858 for the 1984 and 1985 events, respectively (Table 1).

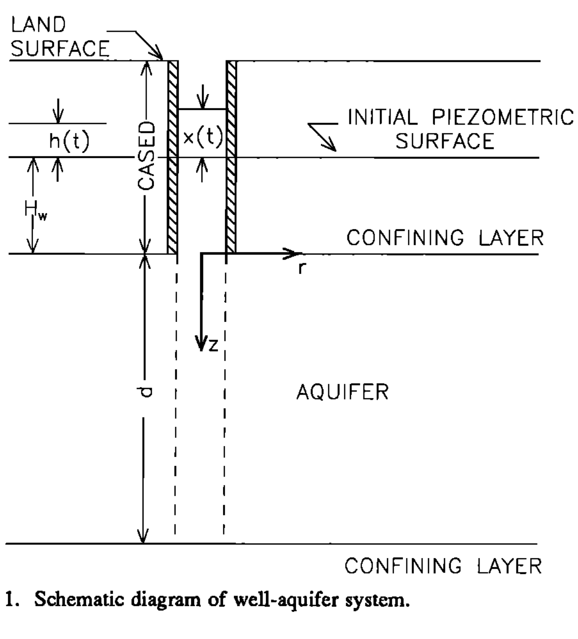

Fig. 1. Schematic diagram of well-aquifer system.

WALI WELL

Wali well is located in a northern suburb of Beijing. It was originally drilled to a depth of 657 m. The water level is presently about 3 m below the land surface, although the well has overflowed in the past. Core logs show that the thickness of the Quaternary overburden is 92 m, with its lower part consisting of highly permeable gravel with cobble size grains and

TABLE 1. Earthquake Parameters

Depth	Time, UT	Latitude, deg	Longitude, deg	Depth, km	Region	Magnitude M_s
March 24, 1984	0944:02.3	44.04N	148.12E	42	Kurile Islands	7.3
November 17, 1985	0940:21.2	1.639S	134.91E	10	West Irian Region	7.1

sand. There is no identifiable impermeable layer between the overburden and the bedrock, which consists of three layers of Cretaceous and Jurassic gravel. The main aquifer is formed by the layers of Quaternary gravel and sand together with the upper and the intermediate Cretaceous and Jurassic gravels. The well is cased to a depth of 94 m and is open below the casing. The inner diameter of the casing and the open hole are both 117 mm.

Water levels have been recorded continuously in the Wali well since the early 1970s, using a float hydrograph with a precision of about 2 mm. Water level tides with amplitudes of up to 12 cm, as well as barometric pressure fluctuations, have been recorded throughout the observation history. The barometric coefficient of the Wali well is about 5.0 mm/mbar. A typical half month of water level observations is shown in Figure 2.

In the past few years, an improved recording system has been used to record water level in the Wali well. When a seismic wave of sufficiently large amplitude causes water level oscillations in the well, the chart speed increases by a factor of 60. The resulting expanded time scale records of seismically induced water level fluctuations can be analyzed using spectral analysis.

QUANTITATIVE ANALYSIS OF SEISMICALLY INDUCED WATER LEVEL OSCILLATIONS

Figures 3 and 4 show hydrographs from the Wali well and vertical component seismograms from Baijiatan for two earth-

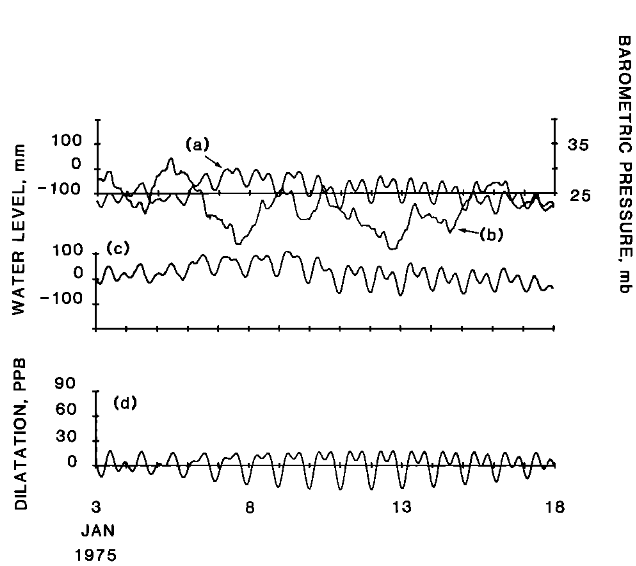

Fig. 2. Hydrograph records from Wali well. *(a)* Raw record of water level oscillations. *(b)* Barometric pressure record. *(c)* Water level oscillation after barometric correction. *(d)* Tidal dilatation calculated for elastic earth model.

quakes. The origin times, locations, and magnitudes of these two earthquakes are listed in Table 1. Because the distance between Wali and Baijiatan is less than 20 km, it is negligible compared with the distances between Wali and the earthquake epicenters. Consequently, we assume that the spectrum of ground motion at Wali is the same as at Baijiatan. The vertical seismograms and the hydrographs were digitized at one sample per second.

There is clearly a resemblance between the water level oscillations and the vertical seismograms. This resemblance is enhanced by applying to each seismogram a band-pass filter with low- and high-frequency cutoff periods of 15 and 50 s (Figures 3*b* and 4*b*).

To test equation (3), we must determine the transfer function x_0/h_0 between pressure head in the aquifer and water level in the well. First, we used cross-spectral analysis to determine the transfer function between the vertical seismogram and the hydrograph, x_0/a_0. We then corrected the gain using equations (7), (8), and (9) to obtain x_0/h_0.

To estimate the autospectra and crossspectra of the hydrograms and corresponding seismograms, we applied smoothing operators to the discrete Fourier transforms of the data series. The discrete Fourier transform $F_x(\omega_j)$ of the hydrogram is

$$F_x(\omega_j) = \Delta t \sum_{l=1}^{N-1} x(l\,\Delta t)\exp(-i\;l\,\omega_j) \qquad j=0,\ldots,N-1 \tag{10}$$

where N is the number of samples, Δt is the sampling interval, and

$$\omega_j = 2\pi j/N$$

The discrete Fourier transform $F_a(\omega_j)$ of the seismogram is defined in an analogous manner. The modified Daniell smoothing operator [*Bloomfield*, 1976] is a moving average given by

$$D_j(y_k) = \frac{1}{2J}\sum_{j=-J}^{J} c_j y_{k-j} \tag{11a}$$

where

$$c_j = \begin{cases} 1/2J, & j=-J \\ 1/J, & j=-J+1,\ldots,J-1 \\ 1/2J, & j=J \end{cases} \tag{11b}$$

Each pass of modified Daniell smoothing reduces the variance of the spectral estimate by a factor of

$$b_J = (\sum_{j=-J}^{J} c_j^2)^{1/2} \tag{12}$$

The spectral estimates we used were

$$G_{xx}(\omega_k) = \frac{1}{N\,\Delta t} D_1(|\,F_x(\omega_k)|^2) \tag{13a}$$

$$G_{aa}(\omega_k) = \frac{1}{N\,\Delta t} D_1(|\,F_a(\omega_k)|^2) \tag{13b}$$

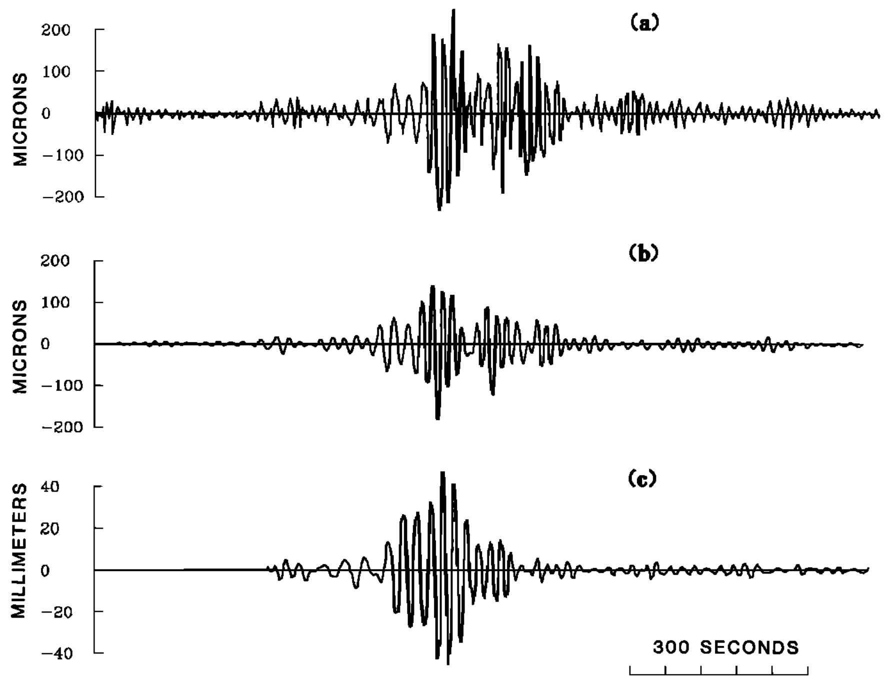

Fig. 3. Seismograph and hydrograph records for the event of March 24, 1984. *(a)* Vertical channel seismogram, corrected to nominal ground displacement by dividing by $A = 907$. *(b)* Vertical channel seismogram after band-pass filtering with passband 15-50 s. *(c)* Hydrograph record.

The cross spectrum was calculated as

$$G_{ax}(\omega_k) = \frac{1}{N\,\Delta t} D_1(F_x^*(\omega_k) F_a(\omega_k)) \qquad (13c)$$

where $F_x^*(\omega_k)$ is the complex conjugate of $F_x(\omega_k)$. If we assume that the seismogram is noise-free and that the hydrogram consists purely of the response to the seismic wave, plus uncorrelated noise, then the transfer function between the seismogram and hydrograph record at radian frequency ω_k can be estimated as

$$\frac{x_0}{h_0} = \frac{G_{ax}(\omega_k)}{G_{xx}(\omega_k)} \qquad (14)$$

[*Bendat and Piersol*, 1986] The standard deviation of the gain estimate is

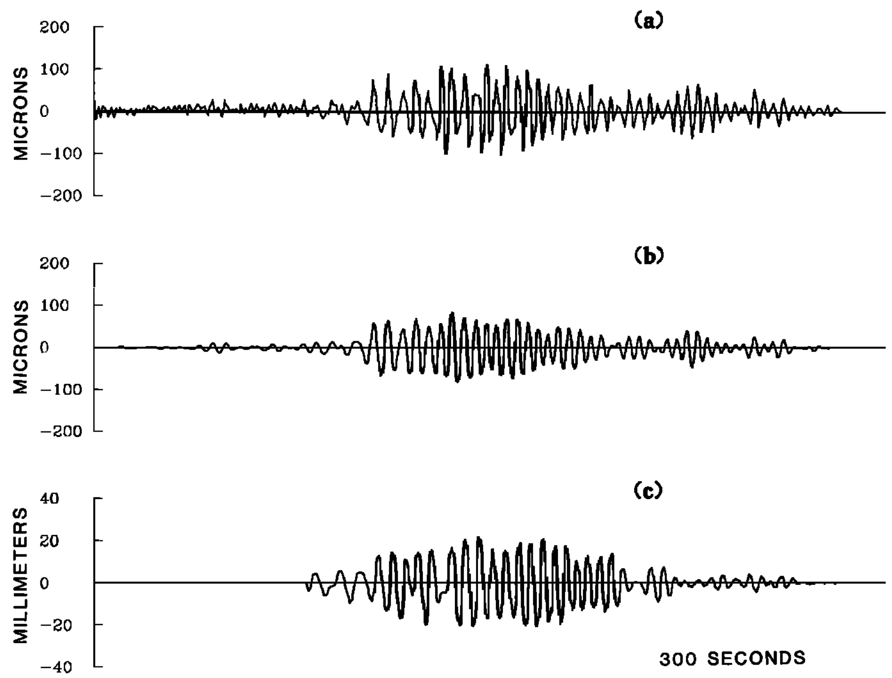

Fig. 4. Seismograph and hydrograph records for the event of November 17, 1985. *(a),(b),(c)* As in Figure 3, except that A = 858.

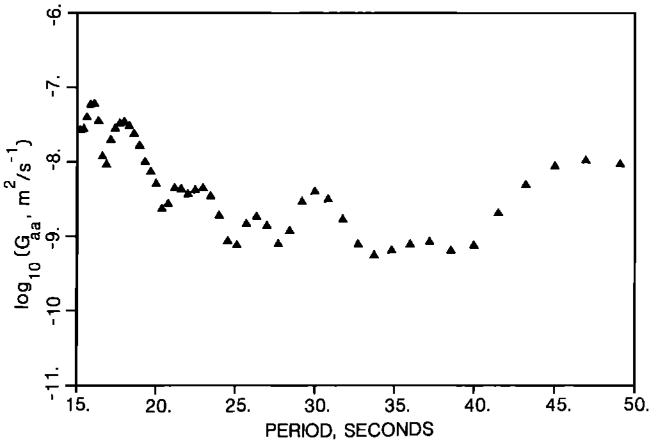

Fig. 5a

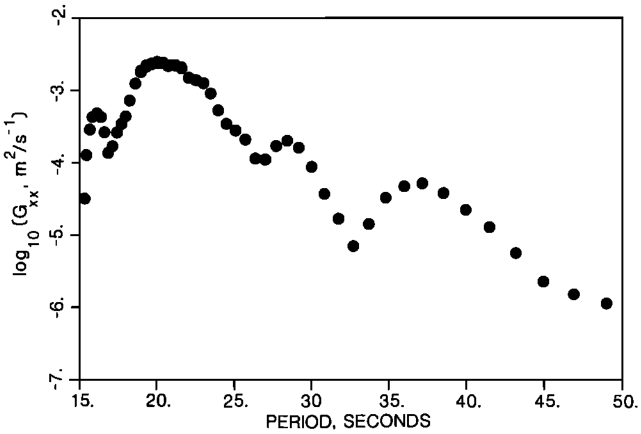

Fig. 5b

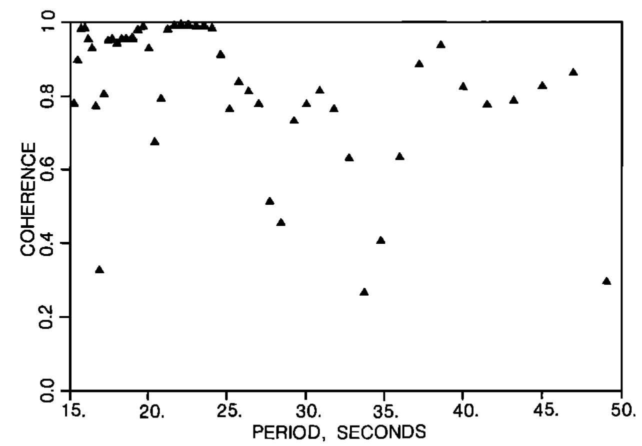

Fig. 5c

Fig. 5. Autospectra and coherence for the March 24, 1984, earthquake. *(a)* Autospectrum G_{aa} of the seismograph record. *(b)* Autospectrum G_{xx} of the hydrograph record. *(c)* Coherence between hydrograph and seismogram.

$$s.d.\ |x_0/a_0| = \frac{(1-\gamma_{ax}^2)^{1/2}}{\sqrt{2}\,|\gamma_{ax}|}\, b_1 |x_0/a_0| \tag{15}$$

where γ_{ax} is the coherence between the hydrograph record and seismogram, and $b_1 = 0.375$.

For the cross-spectrum calculation, the time bases of each seismogram and the corresponding hydrograph record were aligned at the lag for which their cross correlation was most negative. Figures 5*a* and 6*a* show the autospectra G_{aa} of the seismograph records during the two earthquakes. Figures 5*b* and 6*b* show the autospectra G_{xx} of the hydrograph records. Figures 5*c* and 6*c* show the coherence between each hydrograph record and the corresponding seismogram. Both hydrograph records have spectral peaks at periods near 20 s, while the peak spectral values for the seismograms records occur at 15-17 s. The coherence in the frequency band containing the seismogram and hydrograph record spectral peaks is relatively high for both

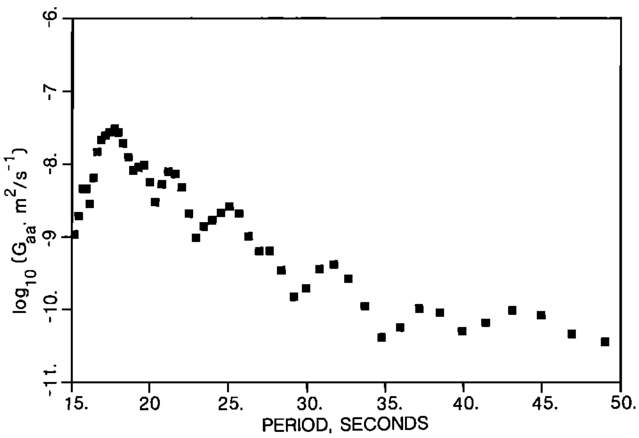

Fig. 6a

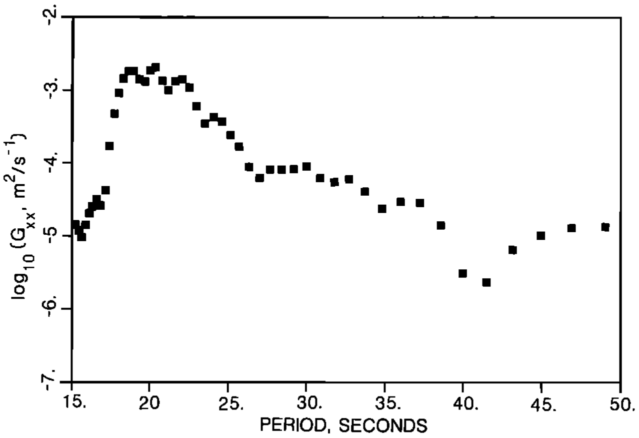

Fig. 6b

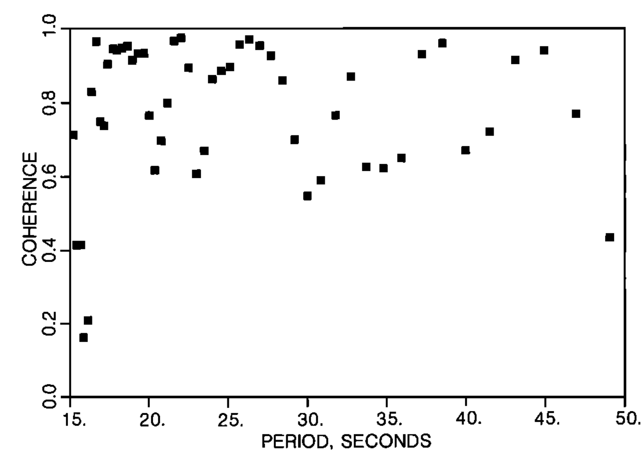

Fig. 6c

Fig. 6. Autospectra and coherence for the November 17, 1985, earthquake. *(a)*, *(b)*, and *(c)* As in Figure 5.

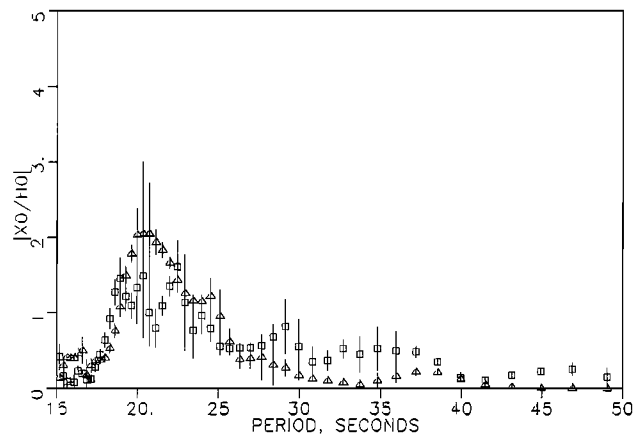

Fig. 7. The $|x_0/h_0|$ determined from data for the 1984 earthquake (triangles) and the 1985 earthquake (squares). Error bars are given by equation (15).

earthquakes. However, because the Daniell operator in equations (13*a*), (13*b*), and (13*c*) smooths the spectra only slightly, the coherences shown in Figures 5*c* and 6*c* may be artificially high.

Figure 7 shows estimates of $|x_0/h_0|$ with error bars equal to the standard deviation shown in equation (15). For some periods near the peak gain, the differences in amplitude between the responses to the 1984 and 1985 events appear to be significant. However, for both events, the peak gain occurs at periods of 19-23 s.

The corrected gain curves resemble the type curves given by *Cooper et al.* [1965] in that they have high amplification in a limited frequency band. However, unlike the type curves, both gain curves fall off to values below 1 with increasing period. Theoretically, as the period increases, the gain should fall to unity, but not below. A second discrepancy with the theory is that the period of peak gain, which is very well determined by the data, is much shorter than the values of 35 s or 39 s predicted by equation (5) using H_e as defined by *Cooper et al.* [1965] or H_e' as defined by *Kipp* [1985].

Modified Well Response

In order to explain the discrepancy between the period of peak response predicted by equation (3) and the observed period of peak response for the Wali well, we carried out a more exact analysis of the flow field set up by the water level oscillations in the part of the well that is open to the aquifer (see the appendix). The vertical flow field in the uncased part of the well is given by equation (A15) and the modified response by (A20). Figures 8*a* and 8*b* compare equations (3) and (A20) for $T/r_w^2 = 160\ s^{-1}$, $H_w = 100$ m, $S = 10^{-4}$, and several values of the aquifer thickness d. Like equation (3), equation (A20) predicts that there is a period of maximum gain for sufficiently high transmissivity. But although both equations predict that the period of peak gain should increase as d increases, equation (A20) predicts a much smaller increase. The reason is that the flow field given by equation (A15) becomes concentrated in the upper part of the open interval of the well as d increases. Figure 8*c* shows that *Cooper et al.*'s [1965] and *Kipp*'s [1985] assumption of linear variation of flow rate is a good one for small values of d but cannot be extended to larger values. Figure 8*b* also indicates that the peak gain should fall with increasing aquifer thickness. The reason is that an increase of aquifer thickness with constant transmissivity implies a decrease of hydraulic conductivity. Since the thickness of the section of aquifer from which flow occurs does not increase much with increasing d, the decreasing hydraulic conductivity translates to a lower effective transmissivity for the aquifer interval affected by the oscillations.

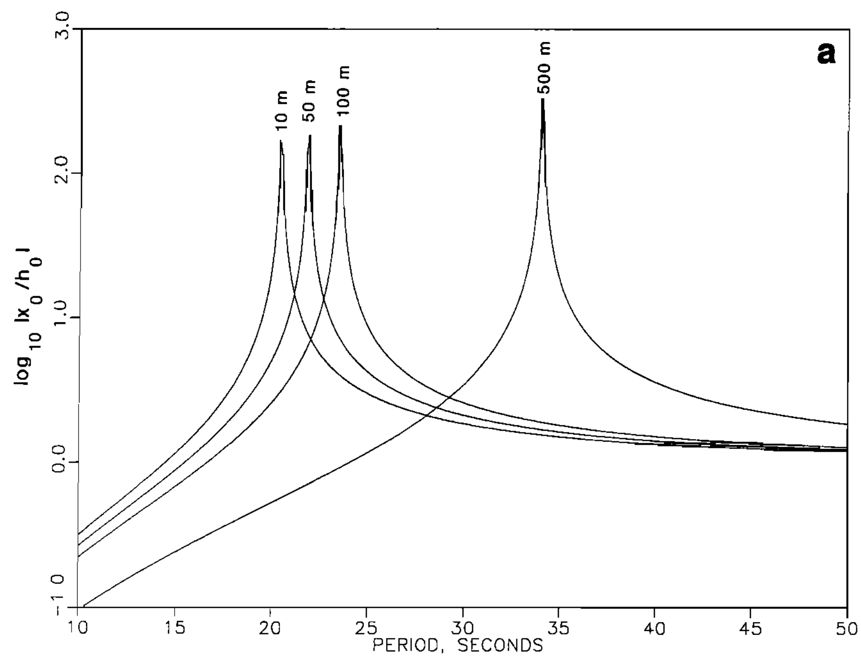

Fig. 8. The $|x_0/h_0|$ evaluated from equation (3), as functions of period, for several values of d, which are marked on each curve. Other output parameters are described in the text. *(b)* Same as Figure 8*a*, but for equation (A20). *(c)* Ratio of flow rate in the open part of the well bore to flow rate at the well water surface, from equation (A15), using the same parameters as in Figures 8*a* and 8*b*.

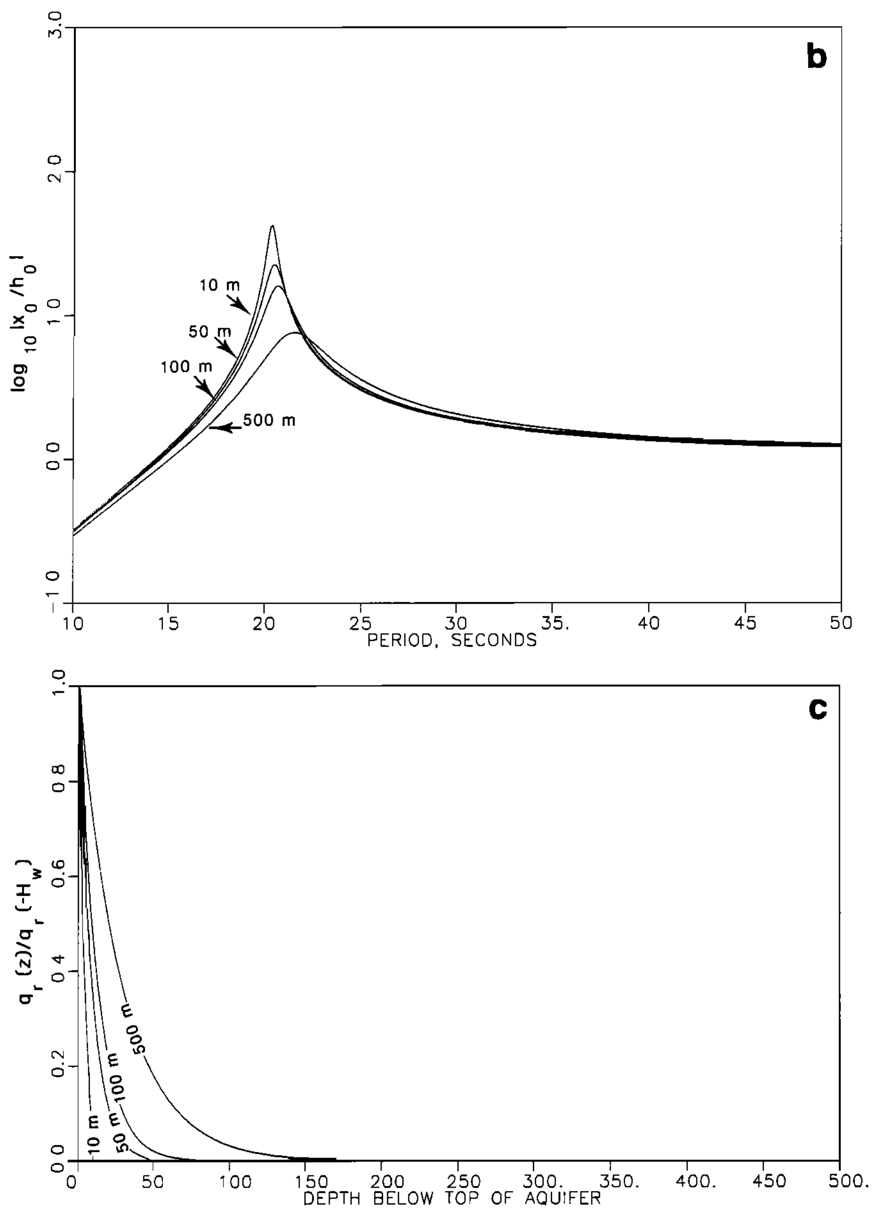

Fig. 8. (continued)

Figure 9 compares the gains and phases determined from the data with equation (A20), evaluated for H=92 m, d=565 m, and r_w=0.117 m. Equation (A20) is not very sensitive to the storage coefficient, for which a value of 5×10^{-4} was chosen. Error bars include the standard deviation from equation (15), as well as 5% uncertainty in Rayleigh wave velocity and 10% uncertainty in h_0/θ_0. Figure 9 shows that the period of peak gain determined from the data agrees much more closely with that predicted by (A20) than with ω_w from equation (5). The amplitude of the gain as determined from the data also agrees with (A20) for the 1984 event but is smaller than (A20) for the 1985 event. Because alignment of the time bases of the seismogram and hydrograph records has a large effect on the phase function as estimated from the data, we have not attempted to compare the observed and theoretical phases.

Figure 10 compares the vertical flow fields predicted by (A15) with *Cooper et al.*'s [1965] assumption that flow rate decreases linearly with depth below the top of the casing. The

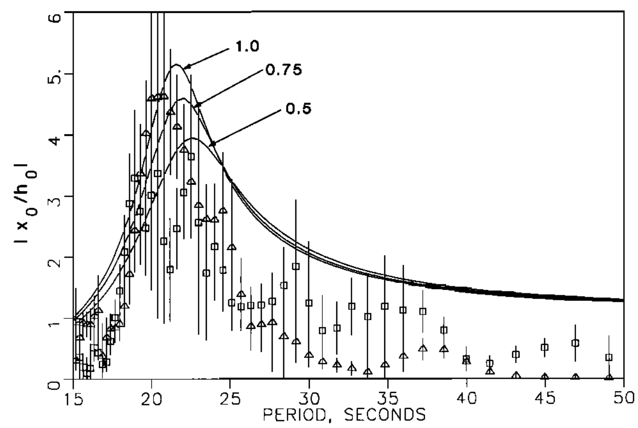

Fig. 9. Gain estimates from data for the 1984 earthquake (triangles) and the 1985 earthquake (squares), compared with the response calculated using equation (A20) (curves). Each curve is labeled with the assumed transmissivity in m^2/s. Other values used in equation (A20) are described in the text.

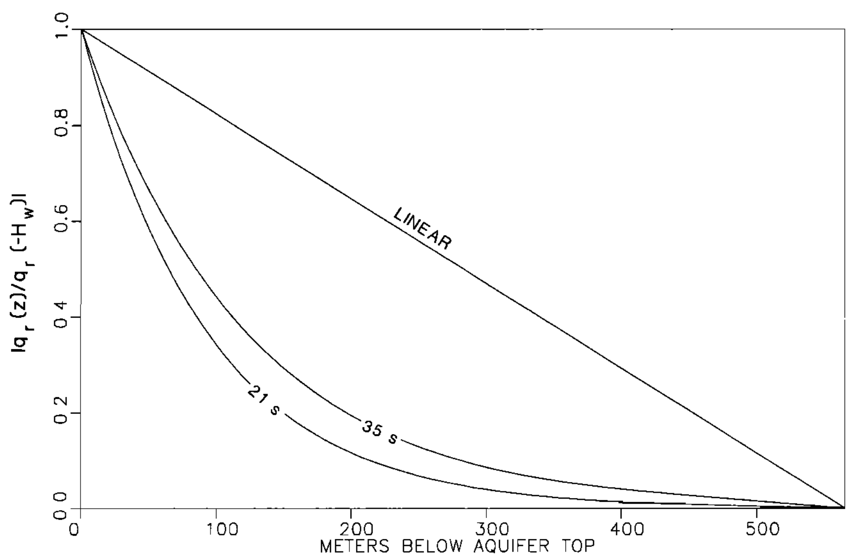

Fig. 10. Ratio of flow rate in the open part of the well to flow rate at the well water surface. The linear function is the one assumed by *Cooper et al.* [1965]. The other two curves were calculated from equation (A15) assuming $T = 0.2\ m^2/s$, with other parameters the same as those used to calculate the curves in Figure 9. Each of those curves is labeled with the period for which it was calculated.

more exact analysis shows that even at the longer resonant period predicted by equation (5), most of the vertical flow takes place in the upper 200 m of the aquifer. Consequently, the effective height of the water column as derived by *Cooper et al.* [1965] or *Kipp* [1985] is too large.

CONCLUSIONS

We have found that the period of peak gain for the Wali well is 19-23 s for two earthquakes. This period is very well determined by the data and is significantly shorter than the 35-s or 39-s period predicted by *Cooper et al.* [1965] or *Kipp* [1985], respectively. We find that a more exact analysis of vertical flow in the well bore is required to explain the fundamental period at Wali, which is open to the aquifer over a length of 565 m. This analysis shows that the free period of a well exposed to the aquifer over a considerable range of depth is relatively insensitive to the length of the exposed section of the aquifer.

Bredehoeft et al. [1965] found that equation (3) gave a good estimate of the fundamental period of a well near Perry, Florida, which is open to the formation over a length of 12 m. Figure 11 compares the predictions of equations (3) and (A20) for the Perry well, and it can be seen that the periods of peak gain are very close for the two equations although the gain from equa-

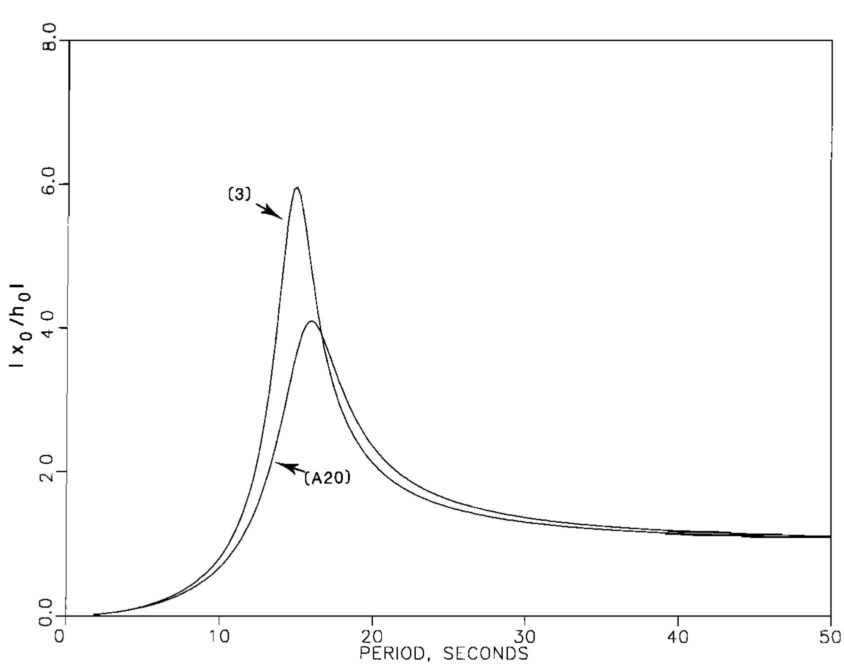

Fig. 11. The $|x_0/h_0|$ for the Perry, Florida, well described by *Bredehoeft et al.* [1965], using equations (3) and (A20).

tion (A20) is somewhat lower. Unfortunately, *Bredehoeft et al.* [1965] could not accurately estimate gain from the data they had available. We believe that either equation (A20) or equation (3) will correctly predict the period of peak gain if the well is open over only a short interval, but as the interval gets longer, equation (A20), based on a more exact analysis, should be used.

APPENDIX

Let z represent vertical distance below the top of the aquifer (Figure 1). Assume that the water level in the well varies as

$$X(t) = H_w + x(t) \tag{A1}$$

and that $x(t)$ can be expressed as

$$x(t) = x_0 \exp(i\omega t)$$

Since no fluid enters the well bore through the casing, the fluid velocity in the casing must be independent of depth and given by

$$q_z(z,t) = -i\omega x_0 \exp(i\omega t) \quad \text{for } z<0 \tag{A2}$$

where q_z is the volume rate of flow per unit area in the z direction (downward). The pressure at the bottom of the casing is equal to the weight of the water column plus the time rate of change of momentum of the part of the water column in the casing:

$$\rho(H_w+x)\frac{\partial^2 x}{\partial t^2} + \rho g(H_w+x) = p(0,t) \tag{A3}$$

where p is fluid pressure in the well bore, ρ is the fluid density, and g is the acceleration due to gravity. We make the approximation that

$$|x(t)| << H_w$$

The equation governing flow in the well bore is [*e.g., Vallentine,* 1969]

$$\frac{\partial q_z}{\partial t} + q_z\frac{\partial q_z}{\partial z} = g - \frac{1}{\rho}\frac{\partial p}{\partial z} \tag{A4}$$

We assume that the second, nonlinear term on the left side of (A4) is small enough to be neglected. Integrating (A4) from 0 to z and evaluating at $z = d$ gives

$$p(0,t) = \rho\frac{\partial}{\partial t}\int_0^d q_z(\zeta,t)d\zeta + p(d,t) - \rho g d \tag{A5}$$

Our goal is to substitute (A5) for the right side of (A3). First, the right side of (A5) should be expressed in terms of the aquifer properties and aquifer pressure disturbance.

The first term on the right side of (A5) is proportional to the rate of change of momentum of the fluid within the well bore below the bottom of the casing. *Cooper et al.* [1965] assumed that the velocity in this part of the well varied linearly with depth. Instead, we develop a more exact expression for the flow field in the water column. Specifically, we seek $q_z(z,t)$ of the form

$$q_z(z,t) = q_z(z)\exp(i\omega t) \tag{A6}$$

such that

$$q_z(0,t) = -\frac{dx}{dt} \tag{A7a}$$

$$q_z(d,t) = 0 \tag{A7b}$$

Conservation of mass dictates that

$$\frac{\partial}{\partial z}q_z(z,t) = \frac{2}{r_w}q_r(z,t) \tag{A8}$$

where $q_r(z,t)$ is the volume rate at which fluid leaves a unit area of aquifer and enters the well bore. Next, define the drawdown at the well boundary as the difference between the pressure in the well and the pressure in the aquifer at great distance from the well:

$$s(r_w,z,t) = z + H_w + h(t) - p(z,t)/\rho g \tag{A9}$$

We allow drawdown to vary with depth, but we neglect any vertical flow in the aquifer induced by the depth-varying drawdown. In this case, the drawdown at depth z due to flow out of the aquifer of the form

$$q_r(z,t) = q_r(z)\exp(i\omega t) \tag{A10}$$

is

$$s(r_w,z,t) = \frac{r_w d\, q_r(z)}{T} \times [\ker(\alpha_w) + i\,\mathrm{kei}(\alpha_w)]\exp(i\omega t) \tag{A11}$$

In deriving (A11), we have used the expression given by *Hsieh et al.* [1987] for drawdown in a well subject to a periodic discharge, for the case where $\alpha_w<0.1$. Equating the right sides of (A9) and (A11), differentiating with respect to z, and then substituting the result into (A4) yields

$$\frac{\partial q_z(z,t)}{\partial t} = r_w g U\frac{dq_r(z)}{dz}\exp(i\omega t) \tag{A12}$$

in which

$$U = (d/T)[\ker(\alpha_w) + i\,\mathrm{kei}(\alpha_w)] \tag{A13}$$

and nonlinear terms have been omitted. Substitute (A8) to get

$$\frac{d^2 q_z(z)}{dz^2} = \frac{2i\omega}{r_w^2 g U}q_z(z) \tag{A14}$$

which can be solved for $q_z(t)$ subject to the boundary conditions (A7a) and (A7b) to get

$$q_z(z) = -i\omega x_0\frac{-\exp[-\beta(d-z)] + \exp[\beta(d-z)]}{\exp[\beta d] - \exp[-\beta d]} \tag{A15}$$

where

$$\beta = (2i\omega/r_w^2 g U)^{1/2} \tag{A16}$$

From (A15) we determine that

$$i\omega\int_0^d q_z(\zeta)d\zeta = \frac{\omega^2}{\beta}\frac{1-\exp(-\beta d)}{1+\exp(-\beta d)}x_0 \tag{A17}$$

Evaluating (A9) at $z = d$ gives

$$p(d)\exp(i\omega t) = \rho g h_0\exp(i\omega t) + \rho g\,[\,d + H_w - s(r_w,d)\exp(i\omega t)\,] \tag{A18}$$

Substituting (A17) and (A18) into (A5) gives

$$p(0,t) = \rho g H_w + \rho g h_0 \exp(i\omega t) + \frac{\rho\omega^2}{\beta} \frac{1-\exp(-\beta d)}{1+\exp(-\beta d)} x_0 \exp(i\omega t) - \frac{\rho g r_w^2 U}{2} \frac{\partial q_z(z,t)}{\partial z}\Big|_{z=d} \tag{A19}$$

Finally, equating the right sides of (A19) and (A3) and solving for the ratio x_0/h_0 gives

$$\frac{x_0}{h_0} = \left[-\frac{\omega^2}{g}[H_w + \frac{1}{\beta}\frac{1-\exp(-\beta d)}{1+\exp(-\beta d)}] - i\omega U r_w^2 \frac{\beta \exp(-\beta d)}{1-\exp(-2\beta d)} + 1 \right]^{-1} \tag{A20}$$

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