Benchmarks for Discrete Fourier Transforms in R

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March 11, 2013

Abstract

The base DFT calculator in R, stats::fft, uses the Mixed-Radix algorithm of Singleton (1969). In this vignette we show how this calculator compares to FFT in the fftw package (Krey et al., 2011), which uses the FFTW algorithm of Frigo and Johnson (2005). For univariate DFT computations, the methods are nearly equivalent with two exceptions which are not mutually exclusive: (A) the series to be transformed is very long, and especially (B) when the series length is not highly composite. In both exceptions the algorithm FFT outperforms fft.

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1 Benchmarking function

We use both functions in their default state, and ask them to transform the same univariate random series. Benchmark information comes from the rbenchmark program, and the versatile plyr and reshape2 packages are used to manipulate the information for this presentation; ggplot2 is used for plotting. First we load the libraries needed:

```
rm(list = ls())
library(fftw)
library(rbenchmark)
library(plyr)
library(reshape2)
library(ggplot2)
```

and create a benchmark function:

```
reps <- 10
dftbm <- function(nd, repls = reps) {
    set.seed(1234)
    x <- rnorm(nd, mean = 0, sd = 1)
    bmd <- benchmark(replications = repls, fftw::FFT(x), stats::fft(x))
    bmd$num_dat <- nd
    bmd$relative[is.na(bmd$relative)] <- 1 # NA happens.
    return(bmd)
}</pre>
```

2 Highly composite (HC) series

It's well known that DFT algorithms are most efficient for "Highly Composite Numbers" 1, specifically multiples of (2,3,5).

So, we create a vector of series lengths we wish to benchmark

```
(nterms.even \leftarrow round(2^seq.int(from = 4, to = 20, by = 1)))
##
    [1]
              16
                       32
                                64
                                        128
                                                 256
                                                          512
                                                                  1024
                                                                           2048
    [9]
                     8192
                                                      131072 262144
            4096
                             16384
                                      32768
                                               65536
                                                                        524288
## [17] 1048576
```

and use it with lapply and the benchmark function previously defined. These data are further distilled into a usable format with ldply:

¹ This is the reason for the stats::nextn function.

```
bench.even <- function() {
   benchdat.e <- plyr::ldply(lapply(X = nterms.even, FUN = dftbm))
}
bench.even()</pre>
```

3 Non highly composite (NHC) series

DFT algorithms can have drastically reduced performance if the series length is not highly composite (NHC). We now test NHC series by adding one to the HC series-length vector (also restricting the total length for sanity's sake):

```
nterms.odd <- nterms.even + 1
nterms.odd <- nterms.odd[nterms.odd < 50000] # painfully long otherwise!</pre>
```

and performing the full set of benchmarks again:

```
bench.odd <- function() {
   benchdat.o <- plyr::ldply(lapply(X = nterms.odd, FUN = dftbm))
}
bench.odd() # FAIR WARNING: this can take a while!!</pre>
```

4 Visualization

In order to plot the results, we need to perform some map/reduce operations on the data (Wickham, 2011). We intend to show faceted ggplot2-based figures with row-wise summary information² so we can easily intercompare the benchmark data. The benchmark data we will show are user.self, sys.self, elapsed, and relative. The results are shown in Figure 1.

```
pltbench <- function(lentyp = c("even", "odd")) {
   benchdat <- switch(match.arg(lentyp), even = benchdat.e, odd = benchdat.o)
   stopifnot(exists("benchdat"))
   tests <- unique(benchdat$test)</pre>
```

http://geokook.wordpress.com/2012/12/29/row-wise-summary-curves-in-faceted-ggplot2-figures/

² Based on this post:

```
## subset only information we care about
allbench.df.drp <- subset(benchdat, select = c(test, num_dat, user.self,</pre>
    sys.self, elapsed, relative))
## reduce data.frame with melt
allbench.df.mlt <- reshape2::melt(allbench.df.drp, id.vars = c("test", "num_dat"))
## calculate the summary information to be plotted:
tmpd <- plyr::ddply(allbench.df.mlt, .(variable, num_dat), summarise, summary = "med</pre>
    value = ggplot2::mean_cl_normal(value)[1, 1])
## create copies for each test and map to data.frame
allmeds <<- plyr::ldply(lapply(X = tests, FUN = function(x, df = tmpd) {
    df$test <- x
    return(df)
}))
## plot the benchmark data 1/sqrt(n) standard errors [assumes N(0,1)]
g \leftarrow ggplot(data = allbench.df.mlt, aes(x = log10(num_dat), y = log2(value),
    ymin = log2(value * (1 - 1/sqrt(reps))), ymax = log2(value * (1 + 1/sqrt(reps)))
    colour = test, group = test)) + scale_colour_discrete(guide = "none") +
    theme_bw() + ylim(c(-11, 11)) + xlim(c(0.5, 6.5)) + ggtitle(sprintf("DFT benchma
    toupper(lentyp)))
## add previous summary curves if exist
if (exists("allmeds.prev")) {
    g <- g + geom_path(size = 1.5, colour = "dark grey", data = allmeds.prev,
        aes(group = test))
## create a facetted version
g2 <- g + facet_grid(variable ~ test)</pre>
## add the summary data as a line
g3 <- g2 + geom_path(colour = "black", data = allmeds, aes(group = test))
## and finally the data
print(g4 <<- g3 + geom_pointrange())</pre>
```

```
pltbench("even")
allmeds.prev <- allmeds
pltbench("odd")</pre>
```

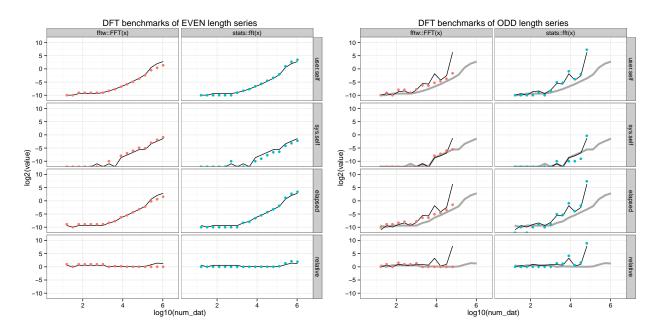


Figure 1: DFT benchmark results for HC series lengths (left), and NHC series lengths (right) as a function of logarithmic series length. In each figure, the left facet-column is for results from fftw::FFT and the right column is for stats::fft. We also show the summary curves from the HC results in the NHC frames (thick grey curve) to highlight the drastic degradation in performance.

5 Conclusion

Figure 1 compares the DFT calculations for HC and NHC length series. For univariate DFT computations, the methods are nearly equivalent with two exceptions which are not mutually exclusive: (A) the series to be transformed is very long, and especially (B) when the series length is not highly composite. In both exceptions the algorithm FFT outperforms fft. In the case of exception (B), both methods have drastically increased computation times; hence, zero padding should be done to ensure the length does not adversely affect the efficiency of the DFT calculator.

Session Info

```
sessionInfo()
## R version 2.15.3 (2013-03-01)
## Platform: x86_64-apple-darwin9.8.0/x86_64 (64-bit)
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
    [1] parallel datasets
                            grDevices grid
                                                 graphics tools
                                                                     stats
   [8] utils
##
                  methods
                            base
##
## loaded via a namespace (and not attached):
## [1] digest_0.6.3
                      evaluate_0.4.3 formatR_0.7
                                                     knitr_1.1
## [5] stringr_0.6.2
```

References

- Frigo, M. and Johnson, S. G. (2005). The design and implementation of FFTW3. *Proceedings of the IEEE*, 93(2):216-231. Special issue on "Program Generation, Optimization, and Platform Adaptation".
- Krey, S., Ligges, U., and Mersmann, O. (2011). fftw: Fast FFT and DCT based on FFTW. R package version 1.0-3.
- Singleton, R. C. (1969). An Algorithm for Computing the Mixed Radix Fast Fourier Transform. *IEEE Transactions on Audio and Electroacoustics*, AU-17(2):93–103.
- Wickham, H. (2011). The split-apply-combine strategy for data analysis. *Journal of Statistical Software*, 40(1):1–29.