Benchmarks for Discrete Fourier Transforms in R

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Abstract

The base DFT calculator in R, `stats::fft`, uses the Mixed-Radix algorithm of Singleton (1969). In this vignette we show how this calculator compares to `FFT` in the `fftw` package (Krey et al., 2011), which uses the FFTW algorithm of Frigo and Johnson (2005). For univariate DFT computations, the methods are nearly equivalent with two exceptions which are not mutually exclusive: (A) the series to be transformed is very long, and especially (B) when the series length is not highly composite. In both exceptions the algorithm `FFT` outperforms `fft`.

Contents

1 Benchmarking function

We use both functions in their default state, and ask them to transform the same univariate random series. Benchmark information comes from the `rbenchmark` program, and the versatile `plyr` and `reshape2` packages are used to manipulate the information for this presentation; `ggplot2` is used for plotting. First we load the libraries needed:

```
rm(list=ls())
library(fftw)
library(rbenchmark)
library(plyr)
library(reshape2)
library(ggplot2)
```

and create a benchmark function:

```r
reps <- 10
dftbm <- function(nd, repls=reps){
        set.seed(1234)
        x <- rnorm(nd, mean=0, sd=1)
        bmd <- benchmark(replications=repls, fftw::FFT(x), stats::fft(x))
        bmd$num_dat <- nd
        bmd$relative[is.na(bmd$relative)] <- 1   # NA happens.
        return(bmd)
}
```

2 Highly composite (HC) series

It's well known that DFT algorithms are most efficient for "Highly Composite Numbers"[1], specifically multiples of (2,3,5).

So, we create a vector of series lengths we wish to benchmark

```r
(nterms.even <- round(2**seq.int(from=4,to=20,by=1)))

##  [1]      16      32      64     128     256     512    1024    2048
##  [9]    4096    8192   16384   32768   65536  131072  262144  524288
## [17] 1048576
```

and use it with `lapply` and the benchmark function previously defined. These data are further distilled into a usable format with `ldply`:

```r
bench.even <- function(){
  benchdat.e <- plyr::ldply(lapply(X=nterms.even, FUN=dftbm))
  }
bench.even()
```

3 Non highly composite (NHC) series

DFT algorithms can have drastically reduced performance if the series length is not highly composite (NHC). We now test NHC series by adding one to the HC series-length vector (also restricting the total length for sanity's sake):

```r
nterms.odd <- nterms.even + 1
nterms.odd <- nterms.odd[nterms.odd < 50e3] # painfully long otherwise!
```

and performing the full set of benchmarks again:

[1] This is the reason for the `stats::nextn` function.

```
bench.odd <- function(){
  benchdat.o <- plyr::ldply(lapply(X=nterms.odd, FUN=dftbm))
  }
bench.odd() # FAIR WARNING: this can take a while!!
```

4 Visualization

In order to plot the results, we need to perform some map/reduce operations on the data (Wickham, 2011). We intend to show faceted `ggplot2`-based figures with row-wise summary information[2] so we can easily intercompare the benchmark data. The benchmark data we will show are `user.self`, `sys.self`, `elapsed`, and `relative`. The results are shown in Figure 1.

```
pltbench <- function(lentyp=c("even","odd")){
  benchdat <- switch(match.arg(lentyp), even=benchdat.e, odd=benchdat.o)
  stopifnot(exists("benchdat"))
  tests <- unique(benchdat$test)
  ## subset only information we care about
  allbench.df.drp <- subset(benchdat,
        select=c(test, num_dat, user.self, sys.self, elapsed, relative))
  ## reduce data.frame with melt
  allbench.df.mlt <- reshape2::melt(allbench.df.drp,
                                    id.vars=c("test","num_dat"))
  ## calculate the summary information to be plotted:
  tmpd <- plyr::ddply(allbench.df.mlt,
                      .(variable,  num_dat),
                      summarise,
                      summary="medians",
                      value=ggplot2::mean_cl_normal(value)[1,1])
  ## create copies for each test and map to data.frame
  allmeds <<- plyr::ldply(lapply(X=tests,
                                 FUN=function(x,df=tmpd){
                                       df$test <- x; return(df)
                                     }))
  ## plot the benchmark data
  # 1/sqrt(n) standard errors [assumes N(0,1)]
  g <- ggplot(data=allbench.df.mlt,
              aes(x=log10(num_dat),
                  y=log2(value),
                  ymin=log2(value*(1-1/sqrt(reps))),
                  ymax=log2(value*(1+1/sqrt(reps))),
                  colour=test,
                  group=test)) +
       scale_colour_discrete(guide="none") +
```

[2] Based on this post:
`http://geokook.wordpress.com/2012/12/29/row-wise-summary-curves-in-faceted-ggplot2-figures/`

```
      theme_bw()+
      ggtitle(sprintf("DFT benchmarks of %s length series",toupper(lentyp))) +
      ylim(c(-11,11))+
      xlim(c(0.5,6.5))

  ## add previous summary curves if exist
  if (exists("allmeds.prev")){
     g <- g + geom_path(size=1.5, colour="dark grey", data=allmeds.prev,
                        aes(group=test))
                        }
  ## create a facetted version
  g2 <- g + facet_grid(variable~test) #, scales="free_y")
  ## add the summary data as a line
  g3 <- g2 + geom_path(colour="black", data=allmeds, aes(group=test))
  ## and finally the data
  print(g4 <<- g3 + geom_pointrange())
}
```

```
pltbench("even")
allmeds.prev <- allmeds
pltbench("odd")
```

5 Conclusion

Figure 1 compares the DFT calculations for HC and NHC length series. For univariate DFT computations, the methods are nearly equivalent with two exceptions which are not mutually exclusive: (A) the series to be transformed is very long, and especially (B) when the series length is not highly composite. In both exceptions the algorithm `FFT` outperforms `fft`. In the case of exception (B), both methods have drastically increased computation times; hence, zero padding should be done to ensure the length does not adversely affect the efficiency of the DFT calculator.

Session Info

```
utils::sessionInfo()

## R version 3.1.2 (2014-10-31)
## Platform: x86_64-apple-darwin13.4.0 (64-bit)
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
```

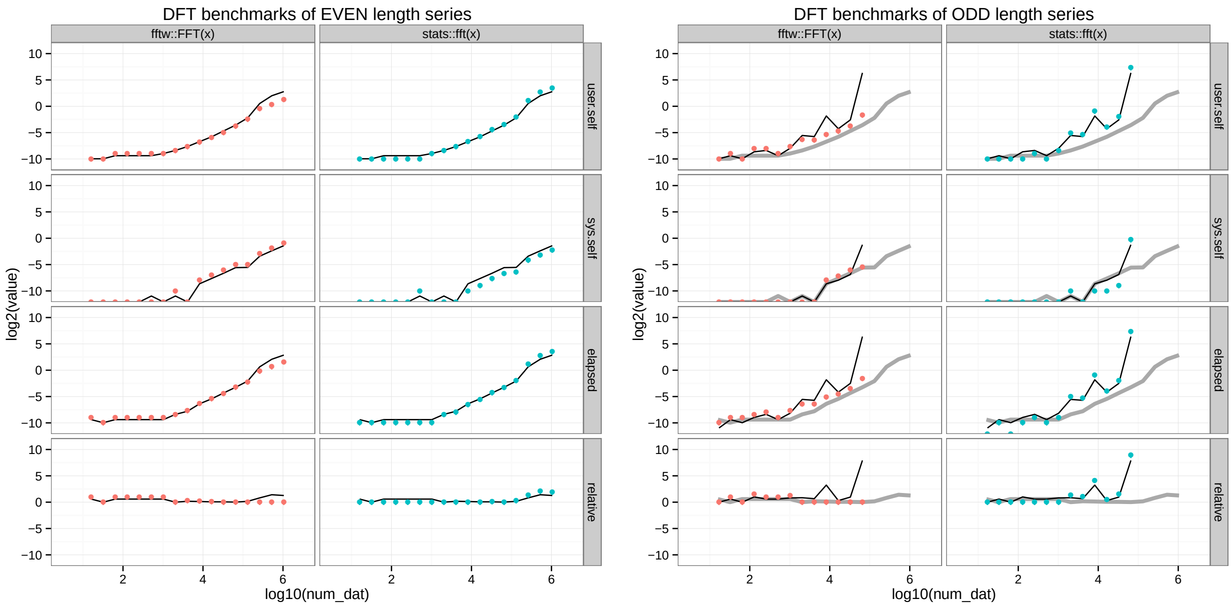

Figure 1: DFT benchmark results for HC series lengths (left), and NHC series lengths (right) as a function of logarithmic series length. In each figure, the left facet-column is for results from `fftw::FFT` and the right column is for `stats::fft`. We also show the summary curves from the HC results in the NHC frames (thick grey curve) to highlight the drastic degradation in performance.

```
##
## other attached packages:
## [1] knitr_1.9
##
## loaded via a namespace (and not attached):
## [1] evaluate_0.5.5 formatR_1.0    highr_0.4      stringr_0.6.2
## [5] tools_3.1.2
```

References

Frigo, M. and Johnson, S. G. (2005). The design and implementation of FFTW3. *Proceedings of the IEEE*, 93(2):216–231. Special issue on "Program Generation, Optimization, and Platform Adaptation".

Krey, S., Ligges, U., and Mersmann, O. (2011). *fftw: Fast FFT and DCT based on FFTW*. R package version 1.0-3.

Singleton, R. C. (1969). An Algorithm for Computing the Mixed Radix Fast Fourier Transform. *IEEE Transactions on Audio and Electroacoustics*, AU-17(2):93–103.

Wickham, H. (2011). The split-apply-combine strategy for data analysis. *Journal of Statistical Software*, 40(1):1–29.