amplitude spectrum
phase spectrum

Normalization of Spectral Analyses

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Abstract

Units matter, and having correct ones is crucial to interpretation of spectral analyses. Here we outline the normalization used by `rlpSpec`, namely the power spectral density, and compare it to other quantities commonly encountered in spectral analysis.

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1 Background

There can often be confusion about the different quantities used in spectral analysis [1], partly due to myriad nomenclature within the incredibly vast literature on the subject. Regarding nomenclature: Phrases including "amplitude spectrum", "energy spectral density", "power", "power spectra", and even "spectra" all mean *something*, but are rarely equivalent.

Let us, for the sake of brevity, assume we are in the time domain, and we are considering a stationary signal f having Fourier Transform $\mathfrak{F}\{f\}$ which is complex, represented by F. The **amplitude spectrum** of this transform pair is simply the amplitude of F, or $\text{mod}\{F\}$; we will denote this as ${}^{(\mathtt{A})}S$. The corresponding **phase spectrum** is thus $\arg\{F\}$, denoted ${}^{(\phi)}S$.

How do we interpret the quantities

Table with equivalent expressions

The latter assumes that the spectrum has the normalization used in power – which is that the Nyquist is assumed to be 1,0, always (power doesn't ask for a sample interval). But psd assumes that the interval is 1 (unless you specify otherwise) so that the Nyquist is 0.5.

[1] This post to `R-help` very eloquently describes the problem, and provides some guidance: `http://r.789695.n4.nabble.com/Re-How-do-I-normalize-a-PSD-td792902.html`

Table 1: A comparison of functions comparable to `rlpSpec`, excluding raw periodogram estimators.

Function	Namespace	Sine m.t.?	Adaptive?	Reference
`mtapspec`	`RSEIS`	Yes	No	Lees and Park (1995)
`spectrum`	`stats`	No	No	R Core Team (2012)
`spec.mtm`	`multitaper`	Yes	Yes	Rahim and Burr (2012)
`SDF`	`sapa`	Yes	No	Percival and Walden (1993)

So say you have a white noise with variance 1: power will return a flat spectrum with level 0 db (=1) but psd will return a level of 2 (=3 db), both so that the level times the Nyquist will be 1. But logsmoo, told that the interval is 1, will multiply the spectrum by 2 to covert from lc**2/Nyquist to lc**2/Hz: giving 3 dB for the spectrum from power, but 6 dB (=4) for the spectrum from psd.

A straightforward way to understand normalization in spectral analysis is to generate a stationary series, normally distributed with variance 1, and expectation 0.

```
> no <- ne <- 128
> x <- rnorm(no, mean = 0, sd = 1)
> X <- fft(x)
> class(X)
```

```
[1] "complex"
```

```
> length(X)
```

```
[1] 128
```

Let us first find the amplitude and phase response:

```
> Sa <- Mod(X) # Amplitude spectrum
> Sp <- Arg(X) # Phase spectrum
```

followed by the energy spectral densities, noting the equivalence between

```
> XC <- Conj(X)
> all.equal(Se <- Sa**2, Se_2 <- Mod(XC * X), Se_2R <- Mod(X * XC))
```

```
[1] TRUE
```

```
> dB <- function(y) 10*log10(y)
> S0 <- dB(2)
```

1.1 stats::spectrum

Included in the core distribution of R is `stats::spectrum`, which accesses `stats::spec.ar` or `stats::spec.pgram` for either parametric and non-parametric estimation, respectively. The user can optionally apply a single cosine taper, and/or a smoothing kernel. Our method is non-parametric; hence, we will compare to the latter.

```
> spec.pgram(X, pad=1, taper=0.2, detrend=FALSE, demean=FALSE, plot=FALSE)
```

However, the logical arguments `detrend` and `demean` to `psdcore` are passed to `spec.pgram`; they are, by default, both TRUE.

As a matter of bookkeeping, we must deal with the working environment accessed by `rlpSpec` functions. Specifically, we should ensure `psdcore` does not access any inappropriate information by setting `refresh=TRUE`. We can then re-calculate the multitaper PSD and the raw periodogram with `plotpsd=TRUE`. The results are shown in Figure **??**.

1.2 multitaper::spec.mtm

1.3 SDF::sapa

References

Lees, J. M. and Park, J. (1995). Multiple-taper spectral analysis: A stand-alone C-subroutine. *Computers & Geosciences*, 21(2):199–236.

Percival, D. and Walden, A. (1993). *Spectral analysis for physical applications*. Cambridge University Press.

R Core Team (2012). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0.

Rahim, K. and Burr, W. (2012). *multitaper: Multitaper Spectral Analysis*. R package version 1.0-2.

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