$$R_n(A \cup B) = \mathbb{E} \sup_{a \in A \cup B} \left| \frac{1}{n} \sum_i \sigma_i a_i \right|$$

If, for a given $\sigma = (\sigma_1, \ldots, \sigma_n)$, the supremum is achieved for some $a^* \in A$, then

$$\sup_{a \in A \cup B} \left| \frac{1}{n} \sum_i \sigma_i a_i \right| = \sup_{a \in A} \left| \frac{1}{n} \sum_i \sigma_i a_i \right|$$

otherwise

$$\sup_{a \in A \cup B} \left| \frac{1}{n} \sum_i \sigma_i a_i \right| = \sup_{a \in B} \left| \frac{1}{n} \sum_i \sigma_i a_i \right|$$

so in all cases,

$$\sup_{a \in A \cup B} \left| \frac{1}{n} \sum_i \sigma_i a_i \right| \le \sup_{a \in A} \left| \frac{1}{n} \sum_i \sigma_i a_i \right| + \sup_{a \in B} \left| \frac{1}{n} \sum_i \sigma_i a_i \right|$$

Take expectation of both sides to obtain the first inequality. The second equality is obvious.

To prove the third, for a given $\sigma$, let $c^* \in A \oplus B$ be such that

$$\sup_{c \in A \oplus B} \left| \frac{1}{n} \sum_i \sigma_i c_i \right| = \left| \frac{1}{n} \sum_i \sigma_i c_i^* \right|.$$

Then $c^* = a^* + b^*$ for some $a^* \in A$, $b^* \in B$.

But then

$$\sup_{c \in A \oplus B} \left| \frac{1}{n} \sum_i c_i \sigma_i \right| = \left| \frac{1}{n} \sum_i (a_i^* + b_i^*) \sigma_i \right|$$

$$\le \left| \frac{1}{n} \sum_i a_i^* \sigma_i \right| + \left| \frac{1}{n} \sum_i b_i^* \sigma_i \right| \le \sup_{a \in A} \left| \frac{1}{n} \sum_i a_i \sigma_i \right| + \sup_{b \in B} \left| \frac{1}{n} \sum_i b_i \sigma_i \right|.$$

Take expectations on both sides to conclude.

For the last equality, simply notice that since a linear function over a convex polytope always achieves its maximum on one of the vertices,

$$\sup_{a \in \text{absconv}(A)} \left| \frac{1}{n} \sum_i \sigma_i \cdot a_i \right| = \left| \frac{1}{n} \sum_i \sigma_i \cdot a_i^* \right| \quad \text{for some } a^* \in A$$

$$\leq \sup_{a \in A} \left| \frac{1}{n} \sum_i \sigma_i \cdot a_i \right|$$

On the other hand, since $A \subset \text{absconv}(A)$,

$$\sup_{a \in A} \left| \frac{1}{n} \sum_i \sigma_i \cdot a_i \right| \leq \sup_{a \in \text{absconv}(A)} \left| \frac{1}{n} \sum_i \sigma_i \cdot a_i \right|$$

and therefore the two quantities must be equal.