$$K(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} x^n e^{-x^2/2} \cdot \frac{1}{\sqrt{n!}} y^n e^{-y^2/2}$$

$$= e^{-(x^2+y^2)/2} \cdot \underbrace{\sum_{n=0}^{\infty} \frac{1}{n!} (xy)^n}_{= e^{xy}} = e^{-(x-y)^2/2}$$

Gaussian kernel.

The natural generalization for $x, y \in \mathbb{R}^d$ is

$$K(x,y) = \exp\left(-\tfrac{1}{2}\|x-y\|^2\right)$$

By some "reverse engineering", we get that, if we define, for all n and all n-tuples $(i_1, \dots, i_n)$ with $i_1, \dots, i_n \in \{1, \dots, d\}$,

$$\left(\Phi(x)\right)_{n,(i_1,\dots,i_n)} = \frac{1}{\sqrt{n}} x_{i_1} \cdot \ldots \cdot x_{i_n} \cdot e^{-\|x\|^2/2}$$

then $$K(x,y) = \sum_{n=0}^{\infty} \sum_{i_1,\dots,i_n \in \{1,\dots,d\}} \left(\Phi(x)\right)_{n,i_1,\dots,i_d} \left(\Phi(y)\right)_{n,i_1,\dots,i_n}$$