

Recall that K is a kernel if and only if for all n and for all x_1, \dots, x_n , the matrix $(K(x_i, x_j))_{n \times n}$ is positive semidefinite.

The first half of the exercise follows from the fact that the sum of positive semidefinite matrices is pos. semidefinite.

For the second half we need to show that the **elemen^t-wise** product of two PSD matrices is PSD. In other words, if

$A = (a_{ij})_{n \times n}$ and $B = (b_{ij})_{n \times n}$ are PSD, then the "**Hadamard product**"

$A \circ B = (a_{ij} \cdot b_{ij})_{n \times n}$ is also PSD.

This is the "**Schur product theorem**", see, e.g., Wikipedia for several proofs.