For each x define $\mu(x)$ as the conditional median of Y, given $X=x$, that is any number such that

$$\mathbb{P}(Y \geq \mu(x) \mid X=x) \geq \frac{1}{2}$$

and

$$\mathbb{P}(Y \leq \mu(x) \mid X=x) \leq \frac{1}{2}.$$

(If Y has a strictly positive conditional density given $X=x$, then $\mu(x)$ is uniquely defined.)

For any $x \in \mathcal{X}$ and $f: \mathcal{X} \to \mathbb{R}$, if $f(x) > \mu(x)$,

$$E_x|Y - f(x)| - E_x|Y - \mu(x)|$$

denotes conditional expectation given $X=x$

$$= E_x\Big(\underbrace{|Y - f(x)| - |Y - \mu(x)|}\Big)$$

$= f(x) - \mu(x)$ if $Y \leq \mu(x)$

$\geq \mu(x) - f(x)$ if $Y > \mu(x)$

$$\geq (f(x) - \mu(x))\Big[\mathbb{P}_x(Y \leq \mu(x)) - \mathbb{P}(Y > \mu(x))\Big] \geq 0$$

The case when $f(x) < \mu(x)$ is handled similarly.

Thus, the conditional median $\mu(x)$ minimizes the absolute loss:

$$E|\mu(X) - Y| \leq E|f(X) - Y| \quad \text{for all } f: \mathcal{X} \to \mathbb{R}.$$