

By writing

$$K(x, y) = \sum_{s \in \{0, 1\}^m} \mathbb{1}_{s \text{ is a substring of } x} \cdot \mathbb{1}_{s \text{ is a substring of } y},$$

we see that if we define the 2^m -dimensional vector $\Phi(x)$ whose components correspond to the substrings $s \in \{0, 1\}^m$ and

$$(\Phi(x))_s = \mathbb{1}_{s \text{ is a substring of } x}.$$

Note that the dimension is *exponentially large* in m .