Machine Learning Topic 1

Introduction and Binary Classification

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Binary Classification

- Definitions μ and η : slide 4
- Definition of R(g), as function of expected loss: slide 5
- Definition of Bayes Classifier $g^*(x)$: slide 5
- Bayes Classifier Theorem: bottom of slide 5
- Proof Bayes classifier is optimal: slide 6
- Definition of plug-in classifier: slide 7
- $\bullet\,$ Data-based classifier and it's risk: slide 8
- Consistency, universal and strong: slide 9
- \bar{R} : best risk of family, $\hat{R_n}$: empirical risk, both being used to bound $R(g_n)$

Additional and Alternative Definitions

$$\eta(x) = \frac{\mathbb{P}\{Y=1\}f_1(x)}{f(x)}, \text{ where } f_1(x) \text{ is the conditional distribution of } x \text{ given } Y=1$$

$$f(x|Y=1) = \frac{f(x)\mathbb{P}(Y=1|X=x)}{\mathbb{P}(Y=1)}$$

$$\mathbb{P}\{Y=1\} = \mathbb{E}[\eta(x)]$$

Error Estimation

Error-counting estimator:

(8.1 of Book)

Given, m is the testing sequence:

$$\hat{L_{n,m}} = \frac{1}{m} \sum_{j=1} m \mathbb{1}_{g_n(X_{n+j}) \neq Y_{n+j}}$$

Clearly unbiased: $\mathbb{E}\{\hat{L_{n,m}}||D_n\} = L_n$

The conditional distribution of $\hat{L_{n,m}}$ given D_n is binomial with paramters m and L_n