$$R^* = \mathbb{E}\left[\min(\eta(X), 1-\eta(X))\right]$$

$$= \mathbb{E}\left[\min\left(\frac{P(Y=1)\cdot f_1(X)}{f(X)}, \frac{P(Y=0)\, f_0(X)}{f(X)}\right)\right]$$

$$= \frac{1}{2}\int \min\left(f_1(x), f_0(x)\right) dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\underbrace{\int \left(f_0(x)+f_1(x)\right)dx}_{=2} - \frac{1}{2}\int |f_0(x)-f_1(x)|\,dx\right]$$

since $\min(a,b) = \frac{a+b}{2} - \frac{|a-b|}{2}$

$$= \frac{1}{2} - \frac{1}{4}\int |f_0(x)-f_1(x)|\,dx$$