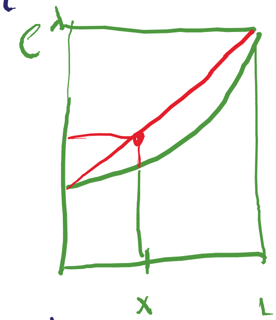
$$\mathbb{P}\left(\sum_{i=1}^{n} X_i \geq t\right) \underset{\lambda>0}{\leq} \frac{\mathbb{E}\left[e^{\lambda \sum_{i=1}^{n} X_i}\right]}{e^{\lambda t}} = \frac{\prod_{i=1}^{n} \mathbb{E}\left[e^{\lambda X_i}\right]}{e^{\lambda t}}$$

(Chernoff bound)

$$\leq \frac{\prod_{i=1}^{n} \mathbb{E}\left[X_i e^{\lambda} + 1 - X_i\right]}{e^{\lambda t}} = \frac{\prod_{i=1}^{n} \left(1 + m_i (e^{\lambda} - 1)\right)}{e^{\lambda t}}$$

$(m_i := EX_i)$

since $e^{\lambda x} \leq x e^{\lambda} + 1$. by convexity of $e^{\lambda x}$



$$\leq \frac{\prod e^{m_i (e^{\lambda} - 1)}}{e^{\lambda t}} = \exp\left(\underbrace{\sum_{i=1}^{n} m_i}_{=m} (e^{\lambda} - 1) - \lambda t\right)$$

use $1 + x \leq e^{x}$

$$= \exp\left(m (e^{\lambda} - 1) - \lambda t\right)$$

This is minimized if $m(e^{\lambda} - 1) - \lambda t$ is minimal.

By differentiating,

$$m e^{\lambda} - t = 0 \implies \lambda = \ln \frac{t}{m}$$

By resubstituting this value of λ, we obtain the desired inequality.