

(17) Observe that for all  $w \in \mathbb{R}^d$  and for all data points  $(x_i, y_i)$ , by the definition of the margin  $f(w)$ ,

$$y_i \frac{w^T x_i}{\|w\| \cdot f(w)} \geq 1$$

but  $\underline{v} = \frac{w}{\|w\| \cdot f(w)}$ . Maximizing  $f(w)$  is equivalent to minimizing  $\|\underline{v}\| = \frac{1}{f(w)}$ . The optimization problem becomes

$$\min \|v\| \text{ subject to } y_i \cdot v^T x_i \geq 1 \quad \forall i=1, \dots, n.$$

This is a convex optimization problem with linear constraints. Equivalent (and more practical) formulation.

$$\min \|v\|^2 \text{ subject to } y_i \cdot v^T x_i \geq 1 \quad \forall i=1, \dots, n.$$

To show that the optimal  $\underline{v}$  lies in the subspace spanned by those  $x_i$  for which  $y_i \cdot v^T x_i = 1$  (these are the support vectors), suppose it is not true. Then (just like in the proof of the representer theorem)  $\underline{v}$  has a component in the orthogonal complement of the support vectors.

By projecting orthogonally to the subspace spanned by the support vectors, the projection  $\tilde{v}$  satisfies  $\tilde{v}^T x_i = v^T x_i$  for all support vectors — and therefore has the same margin — but  $\|\tilde{v}\| < \|v\|$ , contradicting the optimality of  $\underline{v}$ .