$$g^*(x) = 1 \iff q_1 f_1(x) \geq q_0 f_0(x)$$

$$\iff \frac{q_1}{\sqrt{\det \Sigma_1^{-1}}} \exp\left(-\frac{1}{2}(x-m_1)^T \Sigma_1^{-1} (x-m_1)\right) \geq \frac{q_0}{\sqrt{\det \Sigma_0^{-1}}} \exp\left(-\frac{1}{2}(x-m_0)^T \Sigma_0^{-1} (x-m_0)\right)$$

$$\iff (x-m_1)^T \Sigma_1^{-1} (x-m_1) - (x-m_0)^T \Sigma_0^{-1} (x-m_0) \leq 2 \log\left(\frac{q_1}{q_0}\right) + \log \frac{\det \Sigma_0}{\det \Sigma_1}$$

$$\iff \underbrace{x^T \Sigma_1^{-1} x - x^T \Sigma_0^{-1} x}_{=0 \text{ if } \Sigma_0^{-1} = \Sigma_1^{-1}} - 2\left(x^T \Sigma_1^{-1} m_1 + x^T \Sigma_0^{-1} m_0\right) \leq 2 \log \frac{q_1}{q_0} + \log \frac{\det \Sigma_0}{\det \Sigma_1} - m_1^T \Sigma_1 m_1 + m_0^T \Sigma_0 m_0$$

The Bayes decision is linear if and only if $\Sigma_0^{-1} = \Sigma_1^{-1}$