Let $p = \mathbb{P}(Y=1)$ and assume, without loss of generality, that $p \le \frac{1}{2}$.
Then the Bayes decision is 0 and $R^* = p$.

$$\mathbb{E}R(g_n) = \mathbb{P}(\mathrm{maj}(Y_1,\dots,Y_n)=0,\ Y=1) + \mathbb{P}(\mathrm{maj}(Y_1,\dots,Y_n)=1,\ Y=0)$$

$$= p\,\mathbb{P}\left(\mathrm{Bin}(n,p) < \tfrac{n}{2}\right) + (1-p)\,\mathbb{P}\left(\mathrm{Bin}(n,p) \ge \tfrac{n}{2}\right)$$

$$\le p + (1-p)\,\mathbb{P}\left(\mathrm{Bin}(n,p) - np \ge n\left(\tfrac{1}{2}-p\right)\right)$$

$$\le p + (1-p)\,e^{-n\left(\frac{1}{2}-p\right)^2}$$

If $p<\frac{1}{2}$, this very close to $p=R^*$.
If $p=\frac{1}{2}$, $\mathbb{E}R(g_n) = \frac{1}{2}$.

Let n be odd and suppose $\sum_{i=1}^{n} Y_i = k$.
If $k \ne \frac{n-1}{2}$, then $\mathbb{1}_{g_{n-1}(X_i, D_{n,i}) \ne Y_i} = \mathbb{1}_{\mathrm{maj}(Y_1,\dots,Y_n) \ne Y_i}$.
So $R_n^{(D)}(g_n) = \min(k, n-k)$. Since $\sum_i Y_i = \mathrm{Bin}(n,p)$,

in this case $R_n^{(D)}(g_n) = n$.

$$R_n^{(D)}(g_n) = \frac{1}{n} \underbrace{\min\left(\mathrm{Bin}(n,p),\ n-\mathrm{Bin}(n,p)\right)}_{\approx np} \cdot \mathbb{1}_{\sum_i Y_i \ne \frac{n-1}{2}} + \mathbb{1}_{\sum_i Y_i = \frac{n-1}{2}}$$

So $\mathbb{E}R_n^{(D)}(g_n) \approx p$

$$\mathrm{var}(R_n^{(D)}) \approx \mathbb{E}\left(R_n^{(D)}(g_n) - np\right)^2 \ge (1-p)^2\, \underbrace{\mathbb{P}\left(\sum_i Y_i = \tfrac{n-1}{2}\right)}_{\text{tiny if } p<\frac{1}{2} \text{ but } \ge \frac{\text{const}}{\sqrt{n}} \text{ if } p=\frac{1}{2}}$$