

By Sauer's lemma, for each $i = 1, \dots, k$,

$$s_{A_i}(n) \leq (n+1)^V$$

As seen in class, $s_{\bigcup_{i=1}^k A_i}(n) \leq \sum_{i=1}^k s_{A_i}(n) \leq k(n+1)^V$

If we find an integer n such that $k(n+1)^V < 2^n$, then n is an upper bound for the VC-dimension of $\bigcup_{i=1}^k A_i$.

But $k(n+1)^V < 2^n \Leftrightarrow n > \log_2 k + V \log_2(n+1)$

$$\Leftrightarrow n+1 > \log_2(2k) + V \log_2(n+1)$$

which, by the hint, is implied by

$$n+1 > 4V \log_2(2V) + 2 \log_2(2k)$$

Thus, the VC-dim is at most $4V \log_2(2V) + \underbrace{2 \log_2(2k)}_{< 4k} - 1$

The second part is similar, using the fact that the shatter coefficient is bounded by

$$(n+1)^{kV}$$