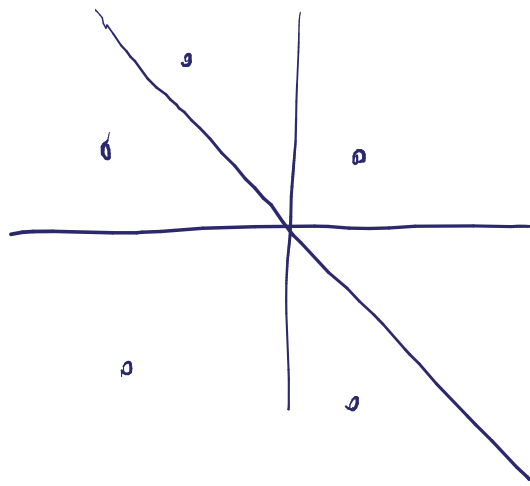


$\mathcal{H}_0$  contains all half planes whose boundary contains the origin. Each line



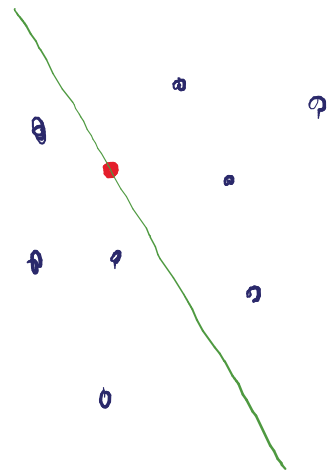
$ax + by = 0$  defines two dichotomies

(one for  $ax + by \geq 0$ ,  
another for  $ax + by \leq 0$ )

By rotating the line, the dichotomy only changes if a point is crossed. In total, one obtains  $2n$  dichotomies (if the points are in general position), so  $s_{\mathcal{H}_0}(n) = 2n$ .

To determine  $s_{\mathcal{H}}(n)$ , we proceed by induction.

Suppose we fix  $n-1$  points in general position so that  $s_{\mathcal{H}}(n-1)$  dichotomies are realized.



Now add the  $n$ -th point.

Each dichotomy of the  $n-1$  points gives rise to one on  $n$  points except for those that are realized by a line containing the new point.

By the first part of the exercise, there are  $2(n-1)$  of these. Thus,  $s_{\mathcal{H}}(n) = s_{\mathcal{H}}(n-1) + 2(n-1)$

Since  $s_{\mathcal{H}}(1) = 2$ , we obtain

$$s_{\mathcal{H}}(n) = s_{\mathcal{H}}(n-1) + 2(n-1) = s_{\mathcal{H}}(n-2) + 2((n-1) + (n-2)) = \dots = 2 + 2 \sum_{i=1}^{n-1} i = \underline{\underline{n^2 - n + 2}}$$