Recall that

$$R_{k\text{-NN}} = \mathbb{E}\Big[\eta(X)\,\mathbb{P}\big(\mathrm{Bin}(k,\eta(X)) < \tfrac{k}{2} \,\big|\, X\big) + (1-\eta(X))\,\mathbb{P}\big(\mathrm{Bin}(k,\eta(X)) > \tfrac{k}{2} \,\big|\, X\big)\Big]$$

observe that for all $\eta \in [0,1]$,

$$\eta\,\mathbb{P}\big(\mathrm{Bin}(k,\eta) < \tfrac{k}{2}\big) + (1-\eta)\,\mathbb{P}\big(\mathrm{Bin}(k,\eta) > \tfrac{k}{2}\big)$$

$$= \min(\eta, 1-\eta) + |2\eta - 1|\,\mathbb{P}\big(\mathrm{Bin}(k, \min(\eta, 1-\eta)) > \tfrac{k}{2}\big)$$

(To see this, assume, without loss of generality, that $\eta < 1-\eta$ and compare the two sides.)

This implies that

$$R_{k\text{-NN}} - R^* = \mathbb{E}\Big[|2\eta(X) - 1| \cdot \mathbb{P}\big(\mathrm{Bin}(k, \min(\eta(X), 1-\eta(X))) > \tfrac{k}{2} \,\big|\, X\big)\Big]$$

$$\leq \sup_{0<p<\frac{1}{2}} (1-2p) \cdot \mathbb{P}\big(\mathrm{Bin}(k,p) > \tfrac{k}{2}\big)$$

$$= \sup_{0<p<\frac{1}{2}} (1-2p)\,\mathbb{P}\big(\mathrm{Bin}(k,p) - kp > k(\tfrac{1}{2} - p)\big)$$

$$\leq \sup_{0<p<\frac{1}{2}} \underbrace{(1-2p)}_{=q}\, e^{-2k(\frac{1}{2}-p)^2} \quad \text{(by Hoeffding)}$$

$$= \sup_{q\in[0,1]} q\, e^{-kq^2/2} = \frac{1}{\sqrt{ke}} \quad \text{(by differentiating with respect to } q\text{)}$$