Stochastic Modeling and Optimization Problemset 3

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Problem 2

Given definitions:

- x_k the state vector,
- u_k the control vector, and
- w_k is the disturbance vector

In order for the terminal state $x_N = f_{N-1}(x_{N-1}, u_{N-1}) + g_{N-1}(w_{N-1})$ to be in the target set X_N, it is necessary and sufficient for $f_{N-1}(x_{N-1}, u_{N-1})$ belong to the effective target set E_N as defined below.

Part a

Given a prescribed target set $X_N \in \mathbb{R}^N$, we recursively solve for x_k in this target set. To do this, we can define the effective target set:

$$E_N = \left\{ z \in \mathbb{R}^n : z + g_{N-1}(w_{N-1}) \in X_N \forall w_{N-1} \in W_{N-1} \right\}$$

which will only be defined given the updated target set T_{N-1}, defined by:

$$T_{N-1} = \left\{ z \in \mathbb{R}^n : f_{N-1}(z, u_{N-1}) \in E_N \text{ for some } u_{N-1} \in U_{N-1} \right\}$$

The DP recursion becomes:

$$E_{k+1} = \left\{ z \in \mathbb{R}^n : z + g_k(w_k) \in T_{k+1} \forall w_k \in W_k \right\}$$

Which is used to update the target set:

$$T_k = \left\{ z \in \mathbb{R}^n : f_k(z, u_k) \in E_{k+1} \right\}$$

$$T_N = X_N$$

What follows is that for X_N to be reachable from set X_k of x_k if and only if $X_k \in T_k$

Part b

X_{k+1} is defined as before but must be contained in X_k for all $w_k \in W_k$

The target tube is defined as:

$$\left\{ \big(X_k, k\big), k = 1, ..., N \right\}$$

If we consider all sets of X_k but X_N to be in $\mathbb{R}^n$, the problem is the same as stated in part 1 with the additional requirement that all $x_k \in X_k$

To initialize the recursion, define the modeified target set:

$$X^*_{N-1} = T_{N-1} \cap X_{N-1}$$

it is necessary and sufficient that

$$x_{N-1} \in X^*_{N-1} \text{ and } T_{N-1}$$

e.g. x_{N-1} is from the modified target set.

The DP recursion is:

$$E^*_{k+1} = \left\{ z \in \mathbb{R}^n : z + g_k(w_k) \in X^*_{k+1} \forall w_k \in W_k \right\}$$

$$T^*_k = \left\{ z \in \mathbb{R}^n : f_k(z, u_k) \in E_{k+1} \text{ for some } u_k \in U_k \right\}$$

$$X^*_k = T^*_k \cap X_k$$

$$X^*_N = X_N$$

The target tube $\left\{ X_j, j; j = k+1, ..., N \right\}$ is reachable at time k if and only if $x_k \in T^*_k$ e.g. the target tube is reachable if and only if $X_0 \subset T^*_0$