

1. For the mechanical system shown in the figure below:

a) Properly label the displacements, springs, dampers and mass. None of the springs or dampers are the same.

b) State your assumptions about the displacements and velocities.

c) Draw the three free-body diagrams that are necessary for analysing this system. The forces shown on these diagrams must be consistent with your assumptions from part (b).

d) For each of your free-body-diagrams, derive the corresponding dynamic equation.

The use of Laplace transforms is not required.

b)

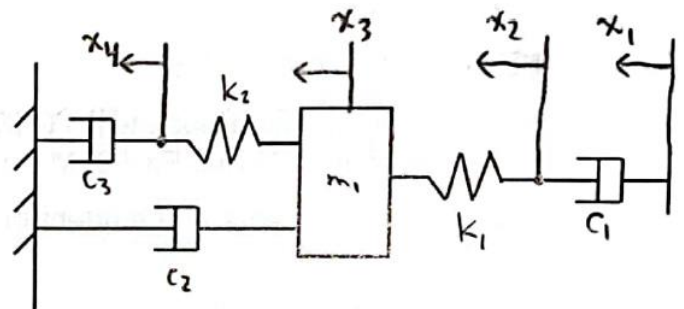
$$\begin{aligned} x_1 > x_2 & \quad \dot{x}_1 > \dot{x}_2 \\ x_2 > x_3 & \quad \dot{x}_2 > \dot{x}_3 \\ x_3 > x_4 & \quad \dot{x}_3 > \dot{x}_4 \end{aligned}$$

c)

$$k_1(x_2 - x_3) \rightarrow \leftarrow c_1(\dot{x}_1 - \dot{x}_2)$$

$$\begin{aligned} k_2(x_3 - x_4) & \rightarrow \\ c_2 \dot{x}_3 & \rightarrow \end{aligned} \quad \boxed{m_1} \quad \leftarrow k_1(x_2 - x_3)$$

$$c_3 \dot{x}_4 \rightarrow \leftarrow k_2(x_3 - x_4)$$



d)

$$\Sigma F = m \ddot{x}_2 = 0$$

$$0 = c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_2 - x_3)$$

$$\Sigma F = m_1 \ddot{x}_3$$

$$m_1 \ddot{x}_3 = k_1(x_2 - x_3) - k_2(x_3 - x_4) - c_2 \dot{x}_3$$

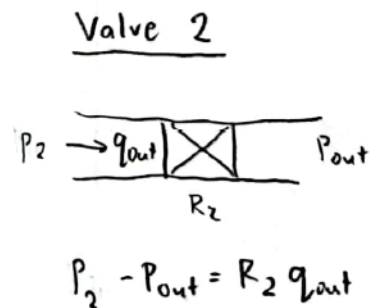
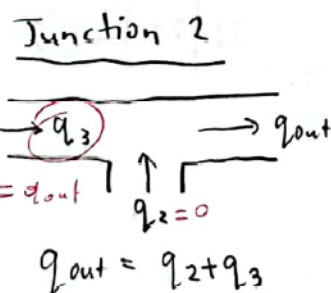
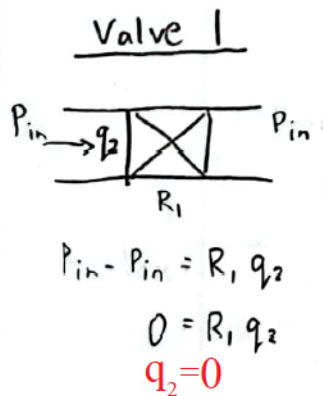
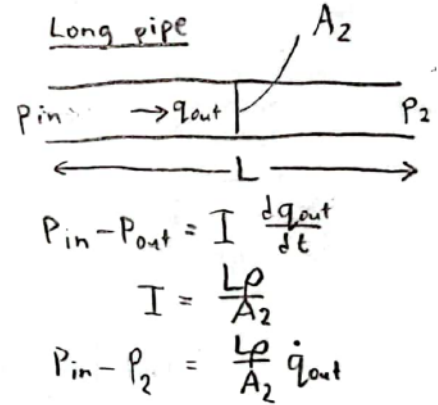
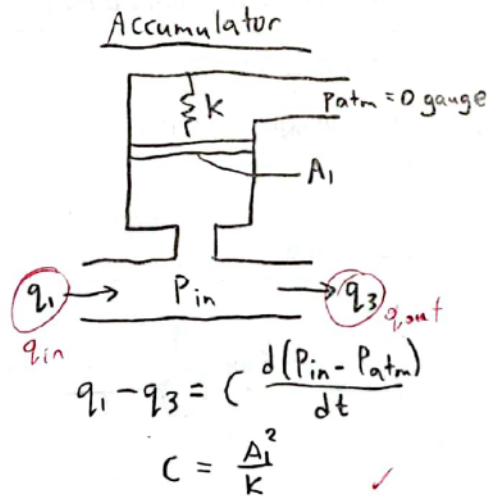
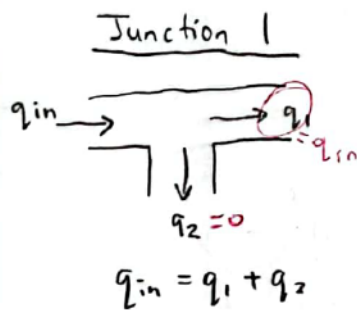
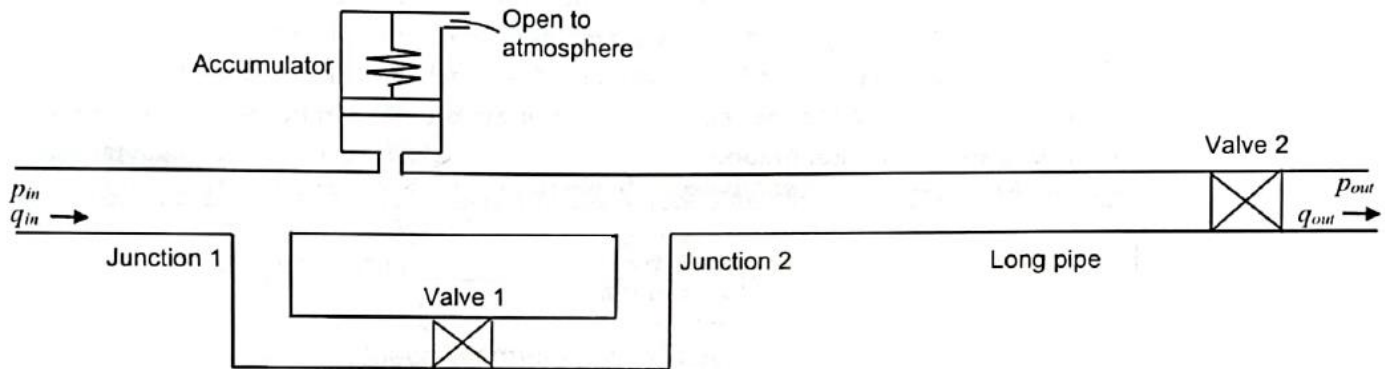
$$\Sigma F = m \ddot{x}_4 = 0$$

$$0 = k_2(x_3 - x_4) - c_3 \dot{x}_4$$

**Note about question 2:** Because the pressures before and after Valve 1 both equal  $p_{in}$  there is no flow through this valve. Since  $q_2=0$  the variables  $q_1$  and  $q_3$  used in the answer below are redundant. We did not deduct marks for this mistake.

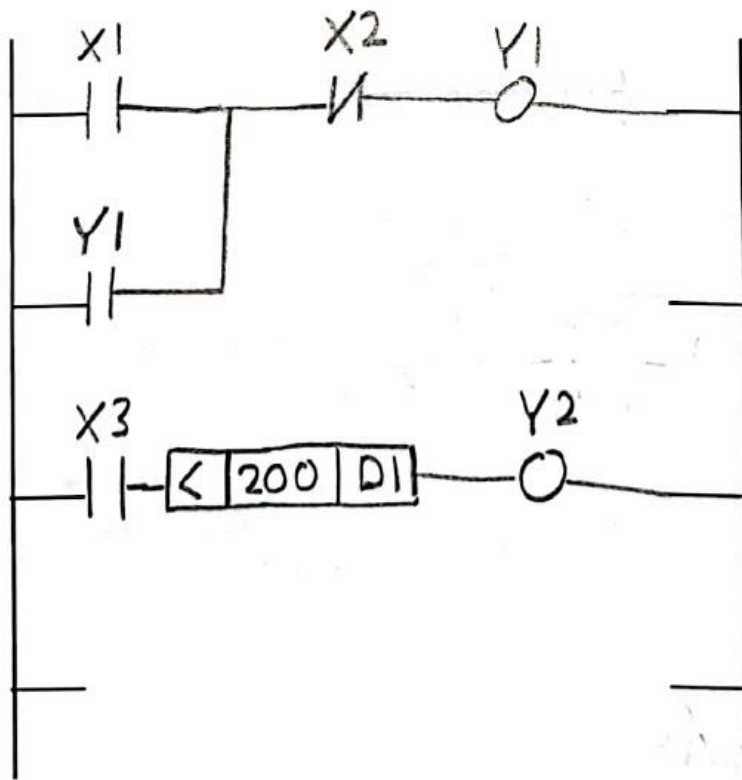
2. The circuit diagram for a hydraulic system is given below. It consists of two junctions, two valves, one accumulator, and one long pipe. The valves are different sizes. The pressures  $p_{in}$  and  $p_{out}$  are larger than atmospheric pressure.

Draw a separate, properly labelled, small diagram for each hydraulic element. Write the dynamic equation for each element next to each drawing. **Do not use redundant variables. For example, if  $A=B$  then write the equation in terms of  $A$  instead of creating another variable. The use of Laplace transforms is not required.**



3. A conveyor system is to be controlled by a PLC. A conveyor passes parts over a weight sensor. Pressing the start button should turn on the conveyor's motor. The motor should remain on after the button is no longer being pressed. The motor should be turned off (or not turn on initially) whenever the stop button is pressed. Both switches are normally open momentary pushbutton switches. The weight sensor is connected to the PLC's analog to digital convertor and the part's weight in grams is stored in data register D1. A proximity sensor's output is turned on when the part is over the weight sensor. Write a program to activate a solenoid to eject all parts that are over 200 grams. The solenoid does not have to be kept on for a specific duration of time. The motor does not have to be stopped for the part weighing and ejecting. Write the required ladder logic program in the space provided below. Use the signal definitions listed in the table below. Use only the instructions taught in this course in your answer. Do not use any Markers, RT or UDC instructions.

Signal	Description
X1	Start switch (normally open)
X2	Stop switch (normally open)
X3	Proximity sensor
D1	Part weight in grams
Y1	Motor
Y2	Solenoid



4. Three sensors are connected to a PLC. The first sensor changes PLC signal X1, the second changes X2 and the third changes X3. PLC signals Y1 and Y2 control actuators that are connected to the PLC. Your program should use counter C1 and timer T1. The timer resolution is 0.1 seconds.

- Assume that initially X1 is ON and X2 is OFF.
- When X1 is OFF and X2 is OFF the signal Y1 should be turned ON.
- Y1 should remain ON even if X1 returns to its initial state.
- When X2 is ON, both Y1 and Y2 should be turned OFF, and the timer and counter should be reset to zero.
- When Y1 is ON and X3 changes from ON to OFF three times the signal Y2 should be turned ON for a period of 6 seconds, and then turned OFF.
- When Y1 is OFF or Y2 is ON the counter should be reset to zero.

Write the required ladder logic program in the space provided below. Use the signal definitions listed in the table below. Use only the instructions taught in this course in your answer. Do not use any Markers, RT or UDC instructions.

