Force

Gravtional force $F = \frac{G*m_1*m_2}{R^2}\ (N)$ Gravtional field = $\frac{G*m_1}{R^2} * -\widehat{R}\quad (N/kg)$

Electrical force $F = \frac{q_1*q_2}{4*\pi*\varepsilon*R^2} * \hat{R}\ = E*q\ \ (N)$ Electrical field $E = \frac{q_1}{4*\pi*\varepsilon*R^2}\hat{R}\ \ (V/m)$

$\varepsilon = \ 8.854*10^{-12}$ $\mu = 4*\pi*10^{-7}\ H/m$

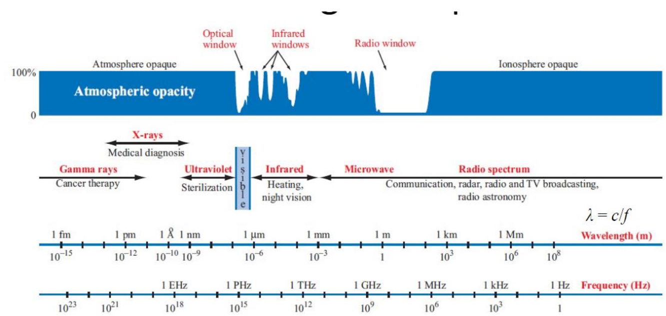
Waves

$y(x,t) = A*\cos\left(\frac{2*\pi*t}{T} - \frac{2*\pi*x}{\lambda} + \phi_0\right) \rightarrow y(x,t) = A*\cos(wt - \beta x)$

<Opposite sign positve diection> <Same sign negative direction>

$Phase\ velocity = u = \frac{\lambda}{T}\ = \lambda * f = \frac{w}{\beta}$

Lossy media $y(x,t) = A*e^{-\alpha x} * \cos\left(wt - \beta x + \phi\right)$



AC/DC circuit

Real battery terminal voltage $\Delta V = V_{emf} - I*r$

Kirchhoff current node law and voltage loop rule

Power $I*\Delta V = I*V_{emf} = I^2*R = \frac{V^2}{R}$ (W)

$Average\ power = I_{rms}^2 * R$

$\sin(\theta) = \cos(90-\theta) = \cos\left(\theta - 90\right)$ $\cos(-\theta) = \cos\left(\theta\right)$

AC voltage $\Delta V = V_{max} * \sin(wt)$ $w = 2\pi f$ $V_{rms} = \frac{V_{max}}{\sqrt{2}}$

Resistor in AC: $i = = \frac{V_{max}}{R} * \sin(wt)$ no phase difference $I_{max} = \frac{V_{max}}{R}$ $I_{rms} = \frac{I_{max}}{\sqrt{2}}$

Capacitor in AC: $I_C = w*C*\Delta V_{max} * \sin\left(wt + \frac{\pi}{2}\right)$ Current leads voltage by 90º $I_{max} = w*C*V_{max}$ $X_C = \frac{1}{w*C*j} = \frac{V_{max}}{I_{max}}$

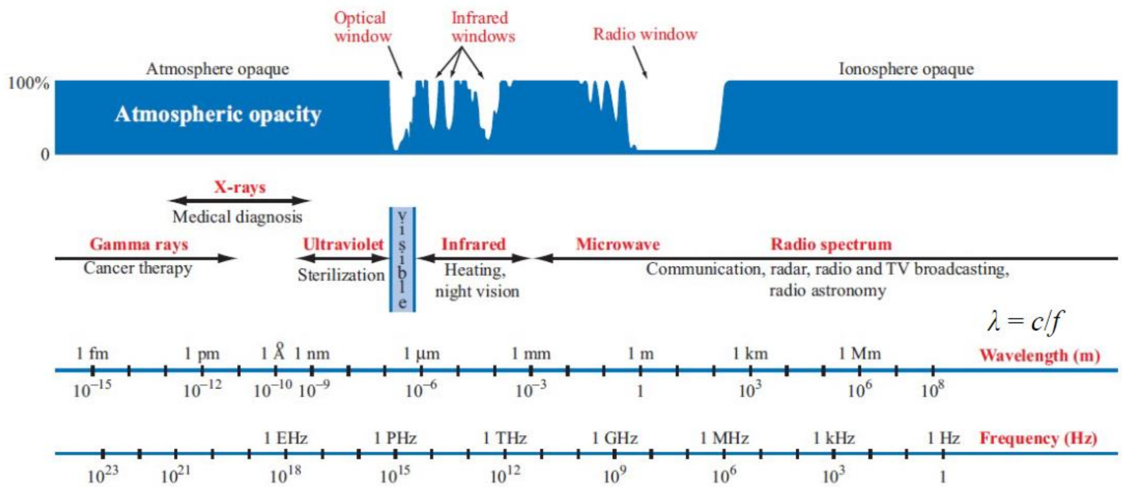
Inductor in AC: $I_L = \frac{V_{max}}{w*L} * \sin\left(wt - \frac{\pi}{2}\right)$ Current lags voltage by 90º $X_L = w*L*j$

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$	
$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$	$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
$\mathbf{z} = x + jy = \lvert\mathbf{z}\rvert e^{j\theta}$	$\mathbf{z}^* = x - jy = \lvert\mathbf{z}\rvert e^{-j\theta}$
$x = \Re\mathfrak{e}(\mathbf{z}) = \lvert\mathbf{z}\rvert\cos\theta$	$\lvert\mathbf{z}\rvert = \sqrt[+]{\mathbf{z}\mathbf{z}^*} = \sqrt[+]{x^2 + y^2}$
$y = \Im\mathfrak{m}(\mathbf{z}) = \lvert\mathbf{z}\rvert\sin\theta$	$\theta = \tan^{-1}(y/x)$
$\mathbf{z}^n = \lvert\mathbf{z}\rvert^n e^{jn\theta}$	$\mathbf{z}^{1/2} = \pm\lvert\mathbf{z}\rvert^{1/2} e^{j\theta/2}$
$\mathbf{z}_1 = x_1 + jy_1$	$\mathbf{z}_2 = x_2 + jy_2$
$\mathbf{z}_1 = \mathbf{z}_2$ iff $x_1 = x_2$ and $y_1 = y_2$	$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$
$\mathbf{z}_1\mathbf{z}_2 = \lvert\mathbf{z}_1\rvert\lvert\mathbf{z}_2\rvert e^{j(\theta_1+\theta_2)}$	$\dfrac{\mathbf{z}_1}{\mathbf{z}_2} = \dfrac{\lvert\mathbf{z}_1\rvert}{\lvert\mathbf{z}_2\rvert} e^{j(\theta_1-\theta_2)}$
$-1 = e^{j\pi} = e^{-j\pi} = 1\angle\pm180^\circ$	
$j = e^{j\pi/2} = 1\angle 90^\circ$	$-j = e^{-j\pi/2} = 1\angle -90^\circ$
$\sqrt{j} = \pm e^{j\pi/4} = \pm\dfrac{(1+j)}{\sqrt{2}}$	$\sqrt{-j} = \pm e^{-j\pi/4} = \pm\dfrac{(1-j)}{\sqrt{2}}$

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	⟷	A
$A\cos(\omega t + \phi_0)$	⟷	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	⟷	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	⟷	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	⟷	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	⟷	$Ae^{j(\phi_0-\pi/2)}$
$\dfrac{d}{dt}(z(t))$	⟷	$j\omega\widetilde{Z}$
$\dfrac{d}{dt}[A\cos(\omega t + \phi_0)]$	⟷	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	⟷	$\dfrac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	⟷	$\dfrac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

RLC circuit maximum frequency $w = \frac{1}{\sqrt{L*C}}$

Attenuating wave: $Ae^{-\alpha x}\cos(wt - \beta x + \phi_0) \rightarrow A*e^{-\alpha x} * e^{j(-\beta x+\phi_0)}$



Transmission lines

Lines affects $\phi = \frac{2\pi f * l}{c} = \frac{2\pi l}{\lambda}$ $\lambda = \frac{c}{f}$ $R_s = \sqrt{\frac{\pi * f * \mu}{\sigma}} \rightarrow surface\ resistance\ of\ conductor$

- When $\frac{l}{\lambda}$ is very small ignore effects $c = 3 * 10^8\ m/s$
- When $\frac{l}{\lambda} > 0.01$, need to account for phase delay and possibly reflection
- When $\frac{l}{\lambda} > 0..25$, definitely need to account for phase delay and possibly reflection

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

$\varepsilon_0 \times \varepsilon_r = \varepsilon$

$L' * C' = \mu\varepsilon$

$\frac{G'}{C'} = \frac{\sigma}{\varepsilon}$

Air line: $\varepsilon = \varepsilon_0 = 8.854 * \frac{10^{-12} F}{m}$

$\mu = \mu_0 = 4 * \pi * 10^{-7}$

$\sigma = 0$

$G' = 0$

Dispersion → Distorts signals because different frequency components → Proportional to the length of the transmission line

Telegraphers equations (time domain):

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

Telegraphers equations (phasor domain):

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

Derive the *wave equations* by separating variables

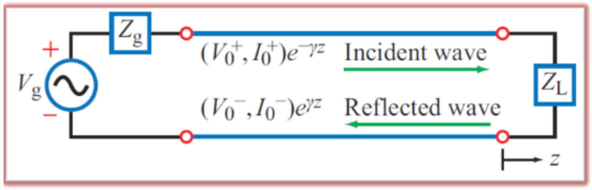
$$\frac{d^2\tilde{V}}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

$$Complex\ prepagation\ constant = \gamma = \sqrt{(R' + j * w * L')(G' + j * w * C')} = \alpha + \beta j$$

$Attenuation\ constant = \alpha = Re(\gamma)\ \ Np/m$ $\qquad$ $Phase\ constant = \beta = Im(\gamma)\quad rad/m$

$Characteristic\ Impedance = Z_0 = \frac{R' + jwL'}{\gamma} = \frac{\sqrt{R' + jwL'}}{\sqrt{G' + jwC'}} = \frac{V_0^+}{I_0^+} = \frac{V_0^-}{I_0^-}$ $\qquad$ $Phase\ velocity = u_p = \frac{w}{\beta} = f * \lambda$



$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{-\gamma z}$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}$$

$L' = \frac{\beta Z_0}{w}$ $\qquad$ $\alpha = \sqrt{R'G'} = R'\sqrt{\frac{C'}{L'}}$ $\qquad$ $\beta = w * \sqrt{L' * C'} = \frac{w * \sqrt{\varepsilon_r}}{c}$

$$R'C' = L'G'$$

Lossless transmission line

R' and G' are negligible → $\alpha = 0$ $\qquad$ $Phase\ velocity = u_p = \frac{c}{\sqrt{\varepsilon_r}}$ $\qquad$ $Z_0 = \sqrt{\frac{L'}{C'}}$

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})\,(h/w)$

$$\lambda = \frac{c}{f * \sqrt{\varepsilon_r}}$$

$$\beta = \frac{w * \sqrt{\varepsilon_r}}{c}$$

$$Normalized\ load\ impedance = z_L = \frac{Z_L}{Z_0}$$

$$voltage\ reflection\ coefficient = \Gamma = \frac{z_L - 1}{z_L + 1} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^+}{V_0^-} = -\frac{I_0^+}{I_0^-} = |\Gamma| * e^{j\theta_r}$$

$$Wave\ impdeance = Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)} \rightarrow ratio\ of\ total\ voltage\ to\ total\ current$$

$Wave\ impedance\ Z(d) = Z_0 * \left[\frac{1 + \Gamma_d}{1 - \Gamma_d}\right], \Gamma_d = \Gamma * e^{-j*2\beta d}$ $\qquad$ $Input\ impedance = Z_{in} = Z_0 * [\frac{z_L + j * \tan(\beta l)}{1 + j * z_L * \tan(\beta l)}]$

$$Forward\ voltage = V_0^+ = \left(\frac{\tilde{V}_g * Z_{in}}{Z_g + Z_{in}}\right) * \left[\frac{1}{e^{j\beta l} + \Gamma * e^{-j\beta l}}\right] \rightarrow this\ part\ may\ not\ be\ used$$

Full final equation for phasor voltage and phasor current on the lossless line:

$$\tilde{V}(z) = |V_0^+| e^{j\phi^+}\left[e^{-j\beta z} + |\Gamma| e^{j\theta_r} e^{j\beta z}\right]$$

$$\tilde{I}(z) = \frac{|V_0^+| e^{j\phi^+}}{|Z_0| e^{j\phi_z}}\left[e^{-j\beta z} - |\Gamma| e^{j\theta_r} e^{j\beta z}\right]$$

phasor solutions

Full final equation for instantaneous voltage and current on the line:

$$v(z) = \mathrm{Re}\{\tilde{V}(z) e^{j\omega t}\} = |V_0^+|\{[\cos(\omega t - \beta z + \phi^+)] + |\Gamma|[\cos(\omega t + \beta z + \theta_r + \phi^-)]\}$$

$$i(z) = \mathrm{Re}\{\tilde{I}(z) e^{j\omega t}\} = \frac{|V_0^+|}{|Z_0|}\{\cos(\omega t - \beta z + \phi^+ - \phi_z) + |\Gamma|\cos(\omega t + \beta z + \phi^- - \phi_z)\}$$

time domain solutions

$$\boxed{V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left[\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right]}$$

Vector analysis

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\,\mathrm{co}$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\,\mathrm{si}$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\,\mathrm{si}$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\,\mathrm{co}$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2+A_y^2+A_z^2}$	$\sqrt[+]{A_r^2+A_\phi^2+A_z^2}$	$\sqrt[+]{A_R^2+A_\theta^2+A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

- Cylindrical <r, ϕ, z>
- Spherical <R, θ, ϕ>
- Gradient (∇)
  - $\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
- Divergence ($\nabla\cdot\vec{A}$)
  - $\nabla\cdot\vec{A} = div\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
- Curl ($\nabla \times \vec{A}$)
  - $\nabla \times \vec{A} = curl\,\vec{A} = (\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}) \times (Ax\hat{x} + Ay\hat{y} + Az\hat{z})$
- Stroke's theorem
  - $\int \nabla\times\vec{B}\cdot ds = \oint \vec{B}\cdot\overrightarrow{dl}$
- Divergence theorem
  - $\int_V \nabla\cdot\vec{E}dV = \int_S \vec{E}\cdot\overrightarrow{ds}$

Gradient of cylindrical coordinate → $\frac{\partial A}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial A}{\partial\phi}\hat{\phi} + \frac{\partial A}{\partial z}\hat{z}$

Gradient of Spherical coordinate → $\frac{\partial A}{\partial R}\hat{R} + \frac{1}{R} * \frac{\partial A}{\partial\theta}\hat{\theta} + \frac{1}{Rsin\theta}\frac{\partial A}{\partial\phi}\hat{\phi}$

Coulomb's Law (Find electric field given charge)

$\overrightarrow{E}(R) = \frac{q}{4\pi\varepsilon * R^2}\hat{R} = \frac{q}{4\pi\varepsilon * |R|^3}\vec{R}\ \ (V/m) \rightarrow$ $electric\ field\ at\ point\ P\ due\ to\ single\ charge$

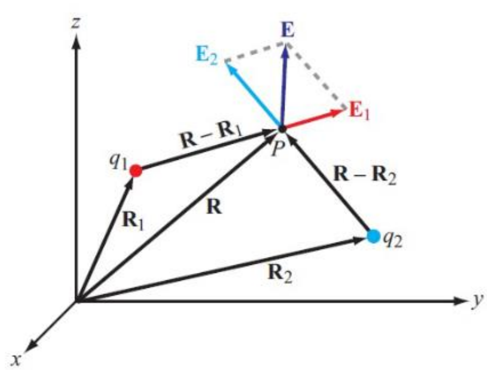
$\vec{F} = q' * \vec{E}\ (N) \rightarrow electric\ force\ on\ a\ charge\ placed\ at\ P$

$$\overrightarrow{E(R)} = \frac{1}{4\pi\varepsilon}\sum_{i=0}^{N}\frac{q_i * (\vec{R} - \vec{R_i})}{|\vec{R} - \vec{R_i}|^3}\ \ (V/m)$$

$$Q = \int_{v'} \rho_v(\vec{r'})dV' \rightarrow total\ charge\ in\ a\ volume, \rho_v\ is\ charge\ density$$

$\overrightarrow{E(R)} = \int_{v'} \frac{\rho_v(\vec{R'})(\vec{R} - \vec{R'})}{4\pi\varepsilon\,|\vec{R} - \vec{R'}|^3} dV' \rightarrow field\ at\ point\ P$

Infinite Plane (Disk) of charge $\vec{E} = \pm\hat{z}\frac{\rho_v}{2\varepsilon}$ Infinite line of charge $E = \frac{\rho_v}{2\pi\varepsilon r}\hat{r}$



$$\vec{E}(\vec{R}) = \int_{v'} \frac{\rho_v(\vec{R}')(\vec{R} - \vec{R}')}{4\pi\varepsilon|\vec{R} - \vec{R}'|^3} dV'$$ In a volume

$$\vec{E}(\vec{R}) = \int_{s'} \frac{\rho_s(\vec{R}')(\vec{R} - \vec{R}')}{4\pi\varepsilon|\vec{R} - \vec{R}'|^3} ds'$$ Over a surface

$$\vec{E}(\vec{R}) = \int_{l'} \frac{\rho_l(\vec{R}')(\vec{R} - \vec{R}')}{4\pi\varepsilon|\vec{R} - \vec{R}'|^3} dl'$$ On a line

Gauss's Law (Find charge given a field)

$\vec{D} = \varepsilon \vec{E}\ (C/m^2)$ $\quad \varepsilon = \varepsilon_r * 8.854 * 10^{-12}$ $\quad \vec{D} = \hat{R}\frac{q}{4*\pi*R^2}$

$\oint_S \vec{D} \cdot ds' = q = \int_{v'} \rho_v \, dV' \rightarrow determine\ electric\ flux\ density\ D$ $\quad \nabla \cdot \vec{D} = \rho_v(x,y,z) \rightarrow differential\ form$

Electric Potential

$V = -\int_{l'} \vec{E} \cdot \widehat{dl'}$ $\quad V_{21} = V_2 - V_1 = -\int_{P1}^{P2} \vec{E} \cdot \widehat{dl'} \rightarrow potential\ difference\ between\ P1\ and\ P2$

$\oint \vec{E} \cdot \widehat{dl'} = 0 \rightarrow for\ any\ closed\ path$ $\quad V = -\int_{\infty}^{P} \vec{E} \cdot \widehat{dl'} \rightarrow zero\ reference\ at\ infinity\ (free\ space\ and\ material\ media)$

$V(\overrightarrow{R}) = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i}{|\vec{R} - \vec{R_i}|}$ $\quad \vec{E} = \frac{V}{L}$ $\quad \vec{E} = -\nabla V$

$V(\vec{\mathbf{R}}) = \int_{v'} \frac{\rho_v(\vec{\mathbf{R}}')}{4\pi\varepsilon|\vec{\mathbf{R}} - \vec{\mathbf{R}}'|} dV'$ In a volume

$V(\vec{\mathbf{R}}) = \int_{s'} \frac{\rho_s(\vec{\mathbf{R}}')}{4\pi\varepsilon|\vec{\mathbf{R}} - \vec{\mathbf{R}}'|} ds'$ Over a surface

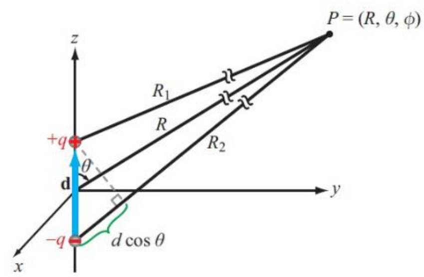
$V(\vec{\mathbf{R}}) = \int_{l'} \frac{\rho_l(\vec{\mathbf{R}}')}{4\pi\varepsilon|\vec{\mathbf{R}} - \vec{\mathbf{R}}'|} dl'$ On a line

Dielectrics

- An electric dipole consists of 2-point charges of equal magnitude but opposite polarity
  - Applications: Dielectrics, molecular bonds, antennas

$\vec{p} = q * \vec{d} \rightarrow dipole\ moment$ $\quad V(\overrightarrow{R}) = \frac{\vec{p}\cdot\hat{R}}{4\pi\varepsilon_0|\vec{R} - \vec{R_i}|^2} = \frac{q*d*cos\theta}{4\pi\varepsilon_0*R^2}$

$\vec{E}(\overrightarrow{R}) = \frac{qd}{4\pi\varepsilon_0|\vec{R} - \vec{R_i}|^3}\left(\hat{R}\ 2*cos\theta + \hat{\theta}\ sin\theta\right)\ V/m$ only when R>>d



Types of dipoles in matter

- Permanent
  - Molecule having atoms with different electronegativity
  - Polar molecule → water
- Instantaneous
  - Electrons happen to concentrate in one place
- Induced
  - A permanent dipole or applied electric field near another atom induces a dipole

In a dielectric material $\vec{D} = \varepsilon_0\vec{E} + \vec{P} \rightarrow P\ is\ the\ electric\ flux\ density\ induced\ by\ applied\ field\ E$

$\vec{P} = \varepsilon_0 * X_e * \vec{E}$ $\quad \vec{D} = \varepsilon_0\vec{E} + \varepsilon_0 * X_e * \vec{E} = \varepsilon_0(1 + X_e)\vec{E}$ $\quad (1 + X_e) = relative\ permittivity = \varepsilon_r$

Conductors & Resistors

- Conductors are materials in which some of the electrons are free electrons
  - Electrons can move relatively freely through the material
  - Copper, aluminum, and silver
  - Charge Carrier: A particle carrying charge that is free to move

$Total\ Current\ I = \int_S \vec{J} \cdot \overrightarrow{ds} \rightarrow \vec{J} = \rho_v * \vec{u}\ \left(\frac{A}{m^2}\right) \rightarrow u\ is\ velocity \rightarrow \rho_v = q * N, N\ is\ \#\ of\ charges\ per\ unit\ volume$

Drift velocity, u: Steady state average velocity of the electrons

Mobility μ: Accounts for the effective mass of charged particle and the average distance before stopped by colliding

$\overrightarrow{u_e} = -\mu_e\vec{E} \rightarrow drift\ velocity\ of\ electrons\ (m/s)$ $\quad \overrightarrow{u_h} = \mu_h\vec{E} \rightarrow drift\ velocity\ of\ holes\ (m/s)$

$\vec{J} = \vec{J_e} + \vec{J_h} = \rho_e * \overrightarrow{u_e} + \rho_h * \overrightarrow{u_h} = (-\rho_e * \mu_e + \rho_h * \mu_h)\vec{E}$ $\quad \vec{J} = \sigma\vec{E}\ (A/m^2)$

Semiconductor / dielectric $\quad \sigma = (-\rho_e * \mu_e + \rho_h * \mu_h) = -N_e q\mu_e + N_h q\mu_h \left(\frac{S}{m}\right) \rightarrow e(N_e\mu_e + N_h\mu_h)$

Conductor $\sigma = -\rho_e * \mu_e = N_e e\mu_e$ $\quad$ For perfect dialectic: $N_e = 0, \sigma = 0, J = 0$

For perfect conductor $\mu_e = \infty, \sigma = \infty, E = D = 0$

$For\ any\ conductor\ R = \frac{V}{I} = \frac{-\int_{l'} \vec{E}\cdot\overrightarrow{dl'}}{\int_S \vec{J}\cdot\overrightarrow{ds}} = \frac{-\int_{l'} \vec{E}\cdot\overrightarrow{dl'}}{\int_S \sigma\vec{E}\cdot\overrightarrow{ds}}$ $\quad R = \frac{l}{\sigma_1 * A_1 + \sigma_2 * A_2} \rightarrow resistance\ coaxial\ cable$

Electric Boundary Conditions

Field Component	Any Two Media	Medium 1 Dielectric ε_1	Medium 2 Conductor
Tangential E	$\mathbf{E}_{1t} = \mathbf{E}_{2t}$	$\mathbf{E}_{1t} = \mathbf{E}_{2t} = 0$	
Tangential D	$\mathbf{D}_{1t}/\varepsilon_1 = \mathbf{D}_{2t}/\varepsilon_2$	$\mathbf{D}_{1t} = \mathbf{D}_{2t} = 0$	
Normal E	$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$	$E_{1n} = \rho_s/\varepsilon_1$	$E_{2n} = 0$
Normal D	$D_{1n} - D_{2n} = \rho_s$	$D_{1n} = \rho_s$	$D_{2n} = 0$

Notes: (1) ρ_s is the surface charge density at the boundary; (2) normal components of $\mathbf{E}_1$, $\mathbf{D}_1$, $\mathbf{E}_2$, and $\mathbf{D}_2$ are along $\hat{\mathbf{n}}_2$, the outward normal unit vector of medium 2.

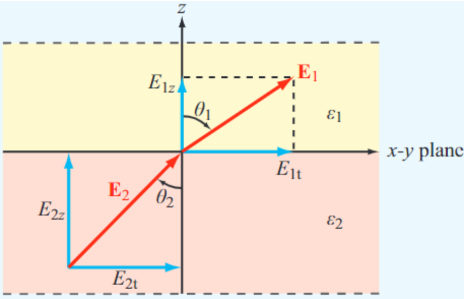
← General Boundary Conditions

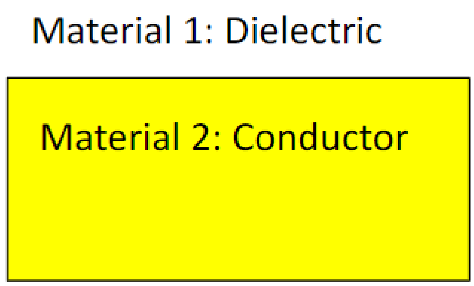
$$E = \frac{Q}{A\varepsilon}$$

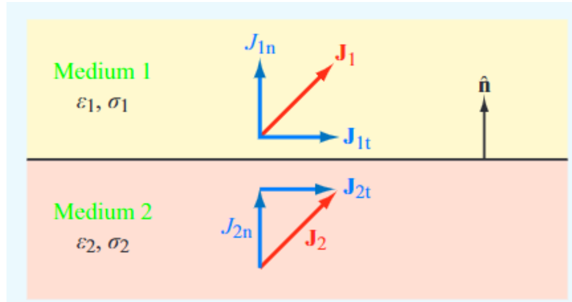
DD: $E_{1t} = E_{2t}$ & $E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n}$

DC: $E = D = 0 \; in \; perfect \; conductor$

$D_{1t} = E_{1t} = 0$ & $D_{1n} = \varepsilon_1 E_{1n} = \rho_s$







Dielectric – dielectric

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

Dielectric – conductor

$$\overrightarrow{D_1} = \varepsilon_1 \overrightarrow{E_1} = \rho_s \hat{n}$$

Conductor – Conductor

$$\varepsilon_1 * \frac{J_{1n}}{\sigma_1} - \varepsilon_2 * \frac{J_{2n}}{\sigma_2} = \rho_s \rightarrow J_{1n} = J_{2n}$$

Capacitor

$$C = \frac{Q}{V} = \frac{\int_S \varepsilon \vec{E} \cdot \vec{ds}}{-\int_{l'} \vec{E} \cdot \vec{dl}} = \frac{\varepsilon * A}{d}$$

$$RC = \frac{V}{I} * \frac{Q}{V} = \frac{\varepsilon}{\sigma}$$

$$W = \int_0^Q \frac{q}{C} \, dq = \frac{1}{2} * \frac{Q^2}{C} = \frac{1}{2} * C * V^2 = \frac{1}{2} \varepsilon E^2 (Ad)$$

$$Q = \int_S \varepsilon \vec{E} \cdot \overrightarrow{ds}$$

$$F = Q * E = \frac{1}{2} \varepsilon * A * (E)^2 \; where \; E = \frac{V}{d}$$

$$W = \int_V \frac{1}{2} \varepsilon E^2 \, dV$$

Magnetic Forces and torques

$$Force = \frac{\mu I_1 I_2 a}{\pi a}$$

$$Magnetic\ force = \vec{F}_m = q\vec{u} \times \vec{B} \; (N)$$

$$Lorentz\ force = q\vec{u} \times \vec{B} + q\vec{E}$$

$$dF_m = I\vec{dl} \times \vec{B} = dQ\vec{u} \times \vec{B}$$

$$Force\ on\ any\ closed\ current\ loop\ in\ a\ uniform\ magnetic\ field = 0$$

$$W = \int F \cdot dl$$

$$Magnetic\ torque\ T = \vec{m} \times \vec{B} \; (N \cdot m)$$

$$|T| = N * I * A * B * \sin\theta$$

$$\vec{m} = \hat{n} N * I * A$$

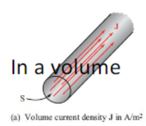
Biot-Savart Law

$$\vec{B} = \mu_0 \vec{H} \; (T)$$

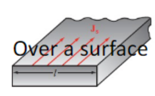
$$\overrightarrow{dH} = \frac{NI}{4\pi R^2} * \overrightarrow{dl} \times \hat{R} \left(\frac{A}{m}\right)$$

$$\overrightarrow{dB} = \frac{IN\,\mu_0}{4\pi R^2} * \overrightarrow{dl} \times \hat{R} \; (T)$$

$$\vec{\boldsymbol{B}}(\vec{\boldsymbol{R}}) = \int_{\mathcal{V}'} \frac{\mu \vec{\boldsymbol{J}}(\vec{\boldsymbol{R}}') \times (\vec{\boldsymbol{R}} - \vec{\boldsymbol{R}}')}{4\pi |\vec{\boldsymbol{R}} - \vec{\boldsymbol{R}}'|^3} d\mathcal{V}' = \frac{\mu}{4\pi} \int_{\mathcal{V}'} \frac{\vec{\boldsymbol{J}} \times \hat{\boldsymbol{R}}}{R^2} d\mathcal{V}'$$



$$\vec{\boldsymbol{B}}(\vec{\boldsymbol{R}}) = \int_{s'} \frac{\mu \vec{\boldsymbol{J}}_s(\vec{\boldsymbol{R}}') \times (\vec{\boldsymbol{R}} - \vec{\boldsymbol{R}}')}{4\pi |\vec{\boldsymbol{R}} - \vec{\boldsymbol{R}}'|^3} ds' = \frac{\mu}{4\pi} \int_{\mathcal{V}'} \frac{\vec{\boldsymbol{J}}_s \times \hat{\boldsymbol{R}}}{R^2} ds'$$



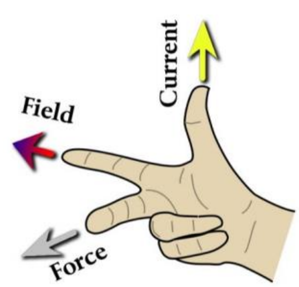
$$\vec{\boldsymbol{B}}(\vec{\boldsymbol{R}}) = \int_{l'} \frac{\mu I d\vec{\boldsymbol{l}}' \times (\vec{\boldsymbol{R}} - \vec{\boldsymbol{R}}')}{4\pi |\vec{\boldsymbol{R}} - \vec{\boldsymbol{R}}'|^3} = \frac{\mu I}{4\pi} \int_{l'} \frac{d\vec{\boldsymbol{l}}' \times \hat{\boldsymbol{R}}}{R^2}$$

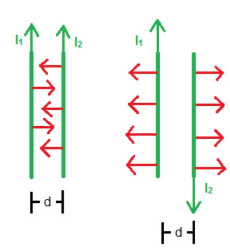
On a line

Magnetic field of a loop $B = \frac{\mu N I a^2}{2(a^2+z^2)^{1.5}} \hat{z}$

At z = 0 $B = \mu * \frac{NI}{2*a} \hat{z}$

At points far away $B = \frac{\mu N I a^2}{2z^3} \hat{z}$





$$F_{parallel\,wires} = \frac{\mu_0 l I_1 I_2}{2\pi d}$$

$\mu_0 = 1.26 \, x \, 10^{-6} \, T \, m/A$

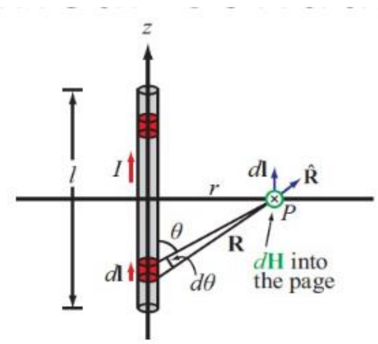
Same direction Current, attract force

Different direction current, repel force

$Magnetic\ field\ of\ a\ linear\ conductor$ $B = \frac{\mu N I a}{2\pi r\sqrt{4r^2+a^2}}\ \hat{\phi}$

R is the distance from center to point P, a is length of wire segment

For an infinity long wire $B = \frac{\mu N I}{2\pi r}\ \hat{\phi}$



Ampere's law (Gauss's Law for magnetism) (net magnetic flux through a closed Gaussian surface is 0)

$\oint_C\ \vec{H}\cdot\vec{dl} = I = \int J\cdot ds$ $\qquad\qquad \nabla\times\vec{H} = \vec{J}$

H field for long wire: $r1 \le a \to \vec{H} = \frac{r_1 * I}{2\pi a^2}\ \hat{\phi}$ $\qquad r2 \ge a \to \vec{H} = \frac{I}{2\pi r}\ \hat{\phi}$

H field for toroidal coil: $r < a \to \vec{H} = 0$ $\qquad r < a \to \vec{H} = 0$ $\qquad a < r < b \to\ \vec{H} = -\frac{NI}{2\pi r}\ \hat{\phi}$

H field inside long solenoid: $r > a \to\ \vec{H} \approx 0$ $\qquad r < a \to \vec{H} = N * \frac{I}{L}\ \hat{z}$

H field of current sheet: $z > 0 \to \vec{H} = -\frac{J}{2}\ \hat{y},\ J\ is\ current\ density$ $\qquad z < 0 \to\ \vec{H} = \frac{J}{2}\ \hat{y}$

Magnetic vector potential & Magnetic material

$\vec{B} = \nabla\times\vec{A}\ (Wb/m^2)\ \to A\ is\ magnetic\ vector\ potential$ $\qquad \overrightarrow{A(R)} = \int_{v'} \frac{\mu * \vec{J}\ (\vec{R'})}{4\pi|\vec{R} - \vec{R_l}|}\ dV'\ (Wb/m^2)$

Spin Magnetic Moments: $m = I * A = \frac{-e*v}{2\pi r} * \pi r^2 = -\frac{e*L}{2m}$ $\qquad$ Angular momentum: $L = m * v * r$

Magnetization: $\vec{B} = \mu_0\vec{H}\ \ in\ free\ space$ $\qquad \vec{B} = \mu_0\vec{H} + \mu_0\vec{M}\ in\ magnetic\ material$

$\vec{M} = X_m * \vec{H} \to\ X_m\ is\ magnetic\ susceptibility$ $\qquad$ M is sum of magnetic dipole moments in medium

$\vec{B} = \mu_0\vec{H} + \mu_0\ X_m * \vec{H} = \mu_0(1 + X_m)\vec{H} \to\ (1 + X_m) = \mu_r$

	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Primary magnetization mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis (see Fig. 5-22)
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminum, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m Typical value of μ_r	$\approx -10^{-5}$ ≈ 1	$\approx 10^{-5}$ ≈ 1	$\lvert\chi_m\rvert \gg 1$ and hysteretic $\lvert\mu_r\rvert \gg 1$ and hysteretic

$n = \frac{N_e}{N_{atoms}} \to \#\ of\ electrons\ per\ atom$

$B_m = \mu_0 * M = \mu_0 * N_e * m_s$

$N_e = \frac{B_m}{\mu_0 * m_s}$

$n = \frac{N_e}{N_{atoms}} = \frac{B_m}{\mu_0 * m_s * N_{atoms}}$

$\mu_1 * H_{1z} = \mu_2 * H_{2z}$

Inductor

$L = \frac{Magnetic\ Flux}{Current}$ $\qquad \phi_m = \iint_s\ \vec{B}\cdot\vec{ds}\ (Wb) \to total\ magntic\ flux$ $\qquad Self\ inductance = L = \frac{\phi m}{I} = \frac{\int_S \vec{B}\cdot\vec{ds}}{\oint_C\ \vec{H}\cdot\vec{dl}}\ \ (H\ or\ Wb/A)$

Self-inductance in a solenoid: $\phi_m = \frac{\mu * N * I}{L} * S \to S\ is\ area\ of\ one\ loop$ $\qquad L = \frac{\mu * N^2}{L} * S$ $\qquad I = \frac{BL}{\mu N}$

Energy stored in solenoid: $W = \frac{1}{2} * L * I^2 = \frac{1}{2} * \frac{B^2}{\mu}$ $\qquad$ Energy density $w = \frac{1}{2}\mu H^2$ $\qquad$ Total energy in any volume $W = \frac{1}{2}\int_V\ \mu H^2\ dV$

Self-inductance of toroid: $L = \frac{\mu * N^2}{2\pi r_m} * S \to\ r_m = \frac{a+b}{2}$ $\qquad Mutual\ Inductance\ L_{12} = \frac{N_2}{I_1}\int_{S2}\vec{B_1}\cdot\vec{ds}$

$Potential\ energy\ W = \int_0^I i * v * dt = L\int_0^I i * di = \frac{1}{2}LI^2(J)$

Faraday's Law

- A time varying magnetic field creates transformer emf V
- A moving loop with time-varying surface area in static field B create motional emf
- A moving loop in time-varying field B is transformer emf + motional emf $\quad V = AwB_0 \sin(wt)$

$$V_{emf} = -N * \frac{d\phi_m}{dt} = -N\frac{d}{dt}\int_S \vec{B} \cdot \overrightarrow{ds}\ (V) \qquad B_0 = \frac{V}{A*w}$$

$$\oint_C \vec{E} \cdot \overrightarrow{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{ds} \qquad \nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \qquad V = -A\frac{d}{dt}(B_0 \cos(wt + \alpha_0))$$

Lenz's Law

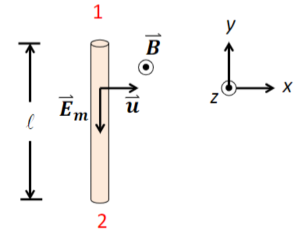
- The current in the loop is always in a direction that opposes the change of magnetic flux that produced I

Moving Conductor in a static magnetic field

$$Motional\ EMF = V_{12} = \int_2^1 (\vec{u} \times \vec{B}) \cdot \overrightarrow{dl}$$

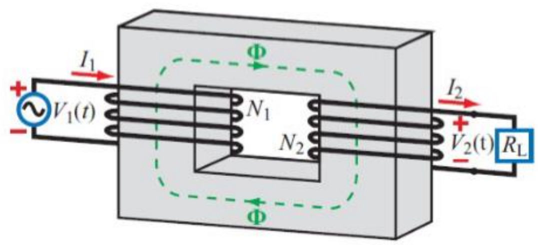
$$\vec{u} \times \vec{B} = u\hat{x} \times B_0\hat{z} = -uB_0\hat{y} \qquad V_{emf} = -uB_0L$$

$$V_{emf} = -\int_S \frac{d\vec{B}}{dt} \cdot \overrightarrow{ds} + \int_2^1 (\vec{u} \times \vec{B}) \cdot \overrightarrow{dl}$$



Transformers

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \rightarrow turns\ ratio \rightarrow\ N_2 > N_1 \text{ Step up traernsform}; N_2 < N_1\ \text{"Step down traernsform"} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$



$$P_1 = I_1V_1 = P_2 = I_2V_2$$

$$R_{in} = \frac{V_1}{I_1} = (\frac{N_1}{N_2})^2 * R_L$$

$$Z_{in} = (\frac{N_1}{N_2})^2 * Z_L$$

$$V_{emf} = -N * \frac{d\phi_m}{dt} = A * w * B_0 * \sin(wt + C_0)$$

DC Generators: Same components as AC generator, main difference is contacts to the rotating loop are made using a split ring called commutator.

Motor: Electrical to mechanical energy $\qquad$ Generators works opposite

Motors are devices into which energy is transferred by electrical transmission while energy is transferred out by work

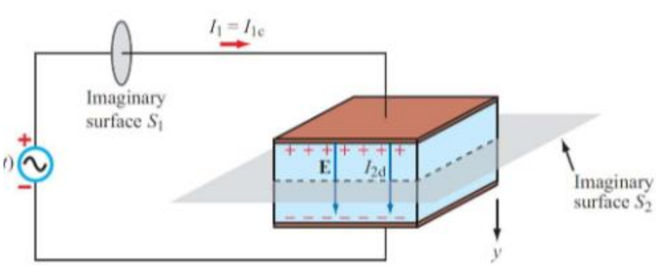
EM Motor: A current is supplied to the coil by a battery and the torque acting on the current carrying coil causes it to rotate

- Induced back emf, acts to reduce the current in the coil
- The back emf increases in magnitude as the rotational speed of coil increases
- $I = \frac{V_{app} - V_{emf}}{R}$

The displacement current

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow \frac{\partial \vec{D}}{\partial t} = \vec{J_d} \rightarrow displacement\ current\ density \qquad I = \frac{\varepsilon A}{d} * \frac{dV}{dt} \qquad R = \frac{d}{\sigma A}$$

$$\oint_C \vec{H} \cdot \overrightarrow{dl} = \int_S \vec{J} \cdot ds' + \int_S \frac{\partial \vec{D}}{\partial t} \cdot ds' = I_C(conduction\ current) + I_D(Displacement\ current) = I$$



$$In\ perfect\ conducting\ wire: I_1 = I_{1c} + I_{1d} = -CV_0 w \sin(wt)$$

$$In\ perfect\ conducting\ capacitor:$$

$$I_2 = I_{2c} + I_{2d} = -\frac{\varepsilon A}{d} V_0 w \sin(wt) = I_1$$

Continuity of current flow through the circuit

- The displacement current behaves like a real current
- The displacement current accounts for polarization in the medium
- The perfect wire has infinite conductivity
    - If it has finite conductivity, then D in the wire would be non-zero and I_1 would consist of both conduction and displacement currents
- A magnetic field can be produced either by currents or by changing electric fields

$\nabla\cdot(\nabla\times \mathrm{H}) = \nabla\cdot J + \frac{\partial \nabla D}{\partial t} = 0$ *Continuity equation*: $\nabla\cdot\vec{J} = \frac{-\partial\rho}{\partial t}$

$\oint_S \vec{J}\cdot ds' = 0$ *for steady currents* $= \sum_i I_i = 0$

- An electric field can be produced either by charges or changing magnetic fields

EM waves

Maxwell's equations in free space → $\rho_V = 0$ & $J = 0$

1) $\nabla\cdot\vec{\boldsymbol{D}} = \rho_V(x,y,z) \longrightarrow \nabla\cdot\vec{\boldsymbol{D}} = 0; \boxed{\nabla\cdot\vec{\boldsymbol{E}} = 0}$

2) $\boxed{\nabla\times\vec{\boldsymbol{E}} = -\frac{\partial\vec{\boldsymbol{B}}}{\partial t}}$

3) $\boxed{\nabla\cdot\vec{\boldsymbol{B}} = 0}$

4) $\nabla\times\vec{\boldsymbol{H}} = \vec{\boldsymbol{J}} + \frac{\partial\vec{\boldsymbol{D}}}{\partial t} \longrightarrow \nabla\times\vec{\boldsymbol{H}} = \frac{\partial\vec{\boldsymbol{D}}}{\partial t}; \boxed{\nabla\times\vec{\boldsymbol{B}} = \varepsilon_0\mu_0\frac{\partial\vec{\boldsymbol{E}}}{\partial t}}$

Proposed wave solution: $\vec{\boldsymbol{E}} = E_0 e^{i(\omega t - \beta z)}$

Plug wave solution into left side $\quad \boxed{\nabla^2\vec{\boldsymbol{E}} = \varepsilon_0\mu_0\frac{\partial^2\vec{\boldsymbol{E}}}{\partial t^2}} \quad$ Plug wave solution into right side

$$\frac{\partial^2\mathbf{E}}{\partial z^2} = -\beta^2\mathbf{E}_0 e^{i(\omega t-\beta z)} = -\beta^2\mathbf{E} \qquad \mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} = -\mu_0\varepsilon_0\omega^2\mathbf{E}_0 e^{i(\omega t-\beta z)} = -\mu_0\varepsilon_0\omega^2\mathbf{E}$$

Result: $-\beta^2\mathbf{E} = -\mu_0\varepsilon_0\omega^2\mathbf{E}$

Phase velocity: $\boxed{u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\times10^8\ \mathrm{m/s}}$ ⬅ Unification of *electricity*, *magnetism* and *light*!

31

Maxwell's equations in a dielectric

In dielectric: $\rho_V \approx 0$ and $\vec{\boldsymbol{J}} \approx 0$, $\vec{\boldsymbol{D}} = \varepsilon\vec{\boldsymbol{E}}$, $\vec{\boldsymbol{B}} = \mu\vec{\boldsymbol{H}}$ Same solution as free space but with:

1) $\nabla\cdot\vec{\boldsymbol{E}} = 0$

2) $\nabla\times\vec{\boldsymbol{E}} = -\frac{\partial\vec{\boldsymbol{B}}}{\partial t}$

3) $\nabla\cdot\vec{\boldsymbol{B}} = 0$

4) $\nabla\times\vec{\boldsymbol{B}} = \varepsilon\mu\frac{\partial\vec{\boldsymbol{E}}}{\partial t}$

$$\boxed{u_p = \frac{1}{\sqrt{\mu\varepsilon}}}$$

$\mu\approx\mu_0,\ \varepsilon>\varepsilon_0 \Rightarrow \boxed{u_p < c \text{ inside matter}}$

$u_p = c/n$

$\boxed{\text{where n = refractive index} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}}$

Maxwell's equations in a conductor

In conductor: $\vec{\boldsymbol{J}} = \sigma\vec{\boldsymbol{E}}$

1) $\nabla\cdot\vec{\boldsymbol{E}} = 0$ Note: excess charge dissipates very fast so $\rho_V \approx 0$

2) $\nabla\times\vec{\boldsymbol{E}} = -\frac{\partial\vec{\boldsymbol{B}}}{\partial t}$

3) $\nabla\cdot\vec{\boldsymbol{B}} = 0$

4) $\nabla\times\vec{\boldsymbol{B}} = \mu\vec{\boldsymbol{J}} + \varepsilon\mu\frac{\partial\vec{\boldsymbol{E}}}{\partial t} \longrightarrow \nabla\times\vec{\boldsymbol{B}} = \mu\sigma\vec{\boldsymbol{E}} + \varepsilon\mu\frac{\partial\vec{\boldsymbol{E}}}{\partial t}$

$$\boxed{\begin{aligned}\frac{\partial^2\mathbf{E}}{\partial z^2} &= \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} + \mu\sigma\frac{\partial\mathbf{E}}{\partial t}\\ \frac{\partial^2\mathbf{B}}{\partial z^2} &= \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2} + \mu\sigma\frac{\partial\mathbf{B}}{\partial t}\end{aligned}}$$

Solutions: $\vec{\boldsymbol{E}} = E_0 e^{i(\omega t - \gamma z)}$

$\vec{\boldsymbol{B}} = B_0 e^{i(\omega t - \gamma z)}$

Where γ is the complex propagation constant:

$$\gamma^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega$$

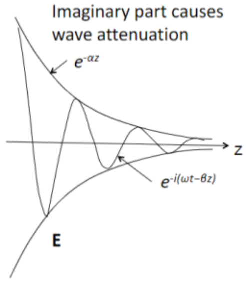
Skin Depth

$\vec{E} = E_0 e^{i(\omega t - \gamma z)}$
$= E_0 e^{-\alpha z} e^{i(\omega t - \beta z)}$

The skin depth is that **distance below the surface of a conductor where the current density has diminished to 1/e of its value at the surface.**

Skin depth = $1/\alpha$

Imaginary part causes wave attenuation



For a good conductor, $\sigma >> \varepsilon\omega$

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$$

$$\approx \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{\frac{\sigma}{\varepsilon\omega}}$$

$$= \sqrt{\frac{\sigma\mu\omega}{2}}$$

$$\text{skin depth} = \frac{1}{\alpha} = \sqrt{\frac{2}{\sigma\mu\omega}}$$

<u>Example: typical metal</u>

$\sigma \sim 10^7\ \Omega^{-1}\mathrm{m}^{-1}$

$\mu = \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$

<u>At visible frequencies:</u>

$\omega \sim 10^{15}$ Hz Skin depth ~ 13 nm

<u>In the RF:</u>

$\omega \sim 10^{9}$ Hz Skin depth ~ 13 μm

57

σ *is area charge density* ; J *is current density*; ρ *is charge density*

Line charge

$E = \frac{\rho}{2\pi\varepsilon r}$ $V = \frac{\rho}{4\pi\varepsilon}\ln\left(\frac{\sqrt{l^2+4r^2}-l}{\sqrt{l^2+4r^2}-l}\right)$, *r is the distance* $B = \frac{\mu NI}{2\pi r}$

Ring charge

$E = \frac{\lambda * 2\pi r * z}{4\pi\varepsilon(x^2+r^2)^{1.5}}$ $B = \frac{\mu N I r^2}{2(r^2+z^2)^{1.5}}$

Disk charge

$E = \frac{1}{2\varepsilon} * \sigma * (1 - \frac{z}{\sqrt{z^2+r^2}})$

Infinite plane charge

$E = \frac{\sigma}{2\varepsilon}$ $B = \frac{\mu J}{2}$

Point charge

$E = \frac{Q}{4\pi\varepsilon r^2}$

Toroidal coil

$r < a \to \vec{H} = 0$ $r < a \to \vec{H} = 0$ $a < r < b \to \vec{H} = -\frac{NI}{2\pi r}\hat{\phi}$

Long Solenoid

$r > a \to \vec{H} \approx 0$ $r < a \to \vec{H} = N * \frac{I}{L}\hat{z}$

Long Wire

$r1 \leq a \to \vec{H} = \frac{r_1 * I}{2\pi a^2}\hat{\phi}$ $r2 \geq a \to \vec{H} = \frac{I}{2\pi r}\hat{\phi}$

Sphere

Constants

Charge of an electron $e = 1.602 * 10^{-19}C$ Electrical permittivity $\varepsilon = 8.854 * 10^{-12}F/m$

Magnetic permittivity $\mu = 4\pi * 10^{-7}\ H/m$ Gravitational constant $G = 6.67 * 10^{-11}\ N * m^2/kg^2$

Electron mass $m = 9.11 * 10^{-31}\ kg$ Proton mass $m = 1.67 * 10^{-27}kg$

Light speed $c = 3 * 10^8\ m/s$

PREFIX	SYMBOL	MAGNITUDE	PREFIX	SYMBOL	MAGNITUDE
exa	E	10^{18}	milli	m	10^{-3}
peta	P	10^{15}	micro	μ	10^{-6}
tera	T	10^{12}	nano	n	10^{-9}
giga	G	10^{9}	pico	p	10^{-12}
mega	M	10^{6}	femto	f	10^{-15}
kilo	k	10^{3}	atto	a	10^{-18}

Theory

Electrical Properties of particles

- SI unit of charge: coulomb <named for a French physicist, Charles-Augustin de Coulomb>
- Represents the charge on $6.241 * 10^8$ electrons Charge of an electron $e = 1.602 * 10^{-19} C$
- Charge conservation: cannot create or destroy charge, only transfer
- Superposition: vector sum of forces due to each point charge to get total force or field
- Coulomb's experiments show <like charge repel> <force acts along the line joining the charges> <Force proportional to charges>

Magnetic field

- Individual electric charges can be isolated but magnetic poles always exist in pairs
- Magnetic fields are induced by moving charges
- Static condition: charges are stationary or moving with constant velocity
- Under static condition, electric and magnetic fields are independent. In dynamic conditions, they become coupled

Waves

- Types of waves <Circular waves> <Plane and cylindrical waves> <Spherical wave>
- A medium is said to be lossless if it does not attenuate the amplitude of the wave traveling within it or on its surface
- The phase velocity is the velocity of the wave pattern as it moves through the medium

DC & AC

- In DC, we assume <The current to be steady state> <Wires have negligible resistance>
- In AC, we assume <The current has varying magnitude and direction> <negligible resistance> <Kirchhoff's law applies>
- Capacitor is a device that stores charge separation
- Inductor, creates strong magnetic flux for a given current and opposes changes in current flow
- RMS current, average value of current
- Phasor is a complex number in polar form, simplifies the process of solving the equation
- Impedance: accounts for the AC resistance and the phase change

Transmission lines

- Encompass all structure and media that serve to transfer energy or information between two points
- A system of conductors that transfer electromagnetic signals form one place to another
- Transverse electromagnetic: electric and magnetic fields are orthogonal to one another, and bother are orthogonal to direction
- Characteristic impedance: the ratio of forward voltage wave to forward current wave or the negative ratio of the backward traveling voltage wave term to the backward current wave term
- Lossless transmission line: R' and G' are negligible
- Voltage standing wave ratio, S, varies from 1 to infinity, describes the strength of the voltage amplitude standing wave pattern
- Wave impedance, ratio of the total voltage to the total current
- Dispersive: wave velocity is not constant as a function of frequency. Has distortion of the wave shape

Vector analysis

- Divergence: If the field has positive divergence then the net flux out of surface S is positive, which can be viewed as if volume contains a source of field lines
- If E has negative divergence then V may be viewed as containing a sink of field lines
- If E is uniform the same amount of flux enters V as it leaves it, the divergence is 0
- Curl: describes the rotational property, curl B is the circulation of B per unit area, with the area s of the contour C being oriented such that the circulation is maximum

Electrostatics and Magnetostatics

- Electric field/flux and electric charge are connected by Coulomb's law
- Magnetic field/flux and electric current are connected by Biot-Savart law
- Under general case (time varying conditions): charges move around and are accelerated and there's coupling between fluxes
- Static condition: Charges are permanently fixed in space, move at a steady rate, electric and magnetic fields de coupled
- Coulomb's law: show the E field is constant, find electric field given charge
- Gauss's law: Find charge given a field, determine the electric flux density (D) when charge distribution possesses symmetry
- Electric scaler potential: Amount of work or potential energy required to move a unit of charge from one point to another
    - Path independence in a conservative electric field, work is only presented when there's work being done against the electric field, work is dependent whether it is along or going against the electric field
- Electric dipoles: consists of two-point charges of equal magnitude but opposite polarity. Applications: dielectrics, antennas
- Types of dipoles: Permanent <One atom in molecule attracts electrons more than the other, a polar molecule> Instantaneous<electrons happen to concentrate in one place, a temporary dipole> Induced <A permanent dipole or applied electric field near another atom/molecule induces a dipole, repels the electrons of the other molecule>
- Induced electric field in the material (Polarization field) is weaker than and opposite in direction to the applied electric field
- Conductors <materials in which some of the electrons are free electrons>

- Charge carriers <particle carry charge that is free to move, in a conducting medium, an electric field can exert force on these free particles, causing a net motion of the particles through the medium → current>
- Drift velocity: Steady state average velocity of the electrons.
- Mobility: Accounts for the effective mass of a charged particle and the average distance over which the applied electric field can accelerate it before it is stopped by colliding with an atom and then starts accelerating again
- Conductivity: A measure of how easily electrons can travel through the material under the influence of an externally applied electric field. For perfect dielectric, conductivity is 0. For perfect conductor, conductivity is infinity
- Electrostatic boundary conditions: at conductor boundary, E field direction is always perpendicular to conductor surface. At dielectric boundaries, electric field changes angle (it undergoes refraction)
- Capacitor: Types <Ceramic disk, film, electrolytic>
- Electrostatic potential energy: Work done in piling up charge onto plates of capacitor, energy stored in the electric field of the capacitor
- Current loops in magnetostatics: Must have closed path, otherwise charge would pile up. Total magnetic force on any closed current loop in a uniform magnetic field is zero, but there can be a force acting on a segment of the wire loop.
- Torque: acts on a closed current loop that causes the loop to rotate around its axis. It is maximum when the magnetic field and the surface normal of the loop are perpendicular
- Biot Savart law: Allows us to calculate the magnetic field due to a current carrying wire, surface current density or volume current density. Current loops create magnetic dipoles. Nearby current carry wires attract or repel due to forces.
- Ampere's law states that the line integral of H around a closed path is equal to the current I traversing the surface bounded by the path. Like Gauss's electrostatic law, can easily solve this involve symmetry
- Macroscopic magnetic properties: Diamagnetic, paramagnetic or ferromagnetic
- Magnetic vector potential: magnetic analogue of the electric scalar potential. Magnetization is the analogue of the electric polarization, and the magnetic permeability is a key material property
- Self-inductance: The ratio of the total magnetic flux through the cross-sectional area of the structure to the current flowing through it
- Mutual inductance: Magnetic coupling between two conducting structures. It is defined as the total flux through the second structure divided by the current in the first. And energy is stored in the magnetic field.
- Motional EMF: Only those segments of the circuit that cross magnetic field lines contribute to emf.
- EM motors: A current is supplied to the coil by a battery and the torque acting on the current carrying coil causes it to rotate
- Displacement current: does not carry free charges, it behaves like a real current, accounts for polarization in the medium
- Maxwell equations: Electric field can be produced either by charges or changing magnetic field
    - A magnetic field can be produced either by currents or changing electric field
    - Most of the E and H waves propagate in the dielectric
    - Current propagates near surface of conductor

For electricity

$$\boxed{\nabla\cdot\vec{\boldsymbol{D}} = \rho_V(x,y,z) \longleftrightarrow \oint_S \vec{\boldsymbol{D}}\cdot ds' = Q}$$

$$\boxed{\nabla\times\vec{\boldsymbol{E}} = 0 \longleftrightarrow \oint_C \vec{\boldsymbol{E}}\cdot dl' = 0}$$

For magnetism:

$$\boxed{\nabla\cdot\vec{\boldsymbol{B}} = 0 \longleftrightarrow \oint_S \vec{\boldsymbol{B}}\cdot ds' = 0}$$

$$\boxed{\nabla\times\vec{\boldsymbol{H}} = \vec{\boldsymbol{J}} \longleftrightarrow \oint_C \vec{\boldsymbol{H}}\cdot dl' = I}$$

Differential form — Integral form

Maxwell's Equations

	Differential Form	Integral Form	
1)	$\nabla\cdot\vec{\boldsymbol{D}} = \rho_V(x,y,z)$	$\oint_S \vec{\boldsymbol{D}}\cdot d\vec{s}' = Q$	Gauss's law
2)	$\nabla\times\vec{\boldsymbol{E}} = -\frac{\partial\vec{\boldsymbol{B}}}{\partial t}$	$\oint_C \vec{\boldsymbol{E}}\cdot d\vec{l}' = -\int_S \frac{\partial\vec{\boldsymbol{B}}}{\partial t}\cdot d\vec{s}'$	Faraday's law (stationary surface *S*)
3)	$\nabla\cdot\vec{\boldsymbol{B}} = 0$	$\oint_S \vec{\boldsymbol{B}}\cdot d\vec{s}' = 0$	Gauss's law, magnetism (no magnetic charges)
4)	$\nabla\times\vec{\boldsymbol{H}} = \vec{\boldsymbol{J}} + \frac{\partial\vec{\boldsymbol{D}}}{\partial t}$	$\oint_C \vec{\boldsymbol{H}}\cdot d\vec{l}' = \int_S \vec{\boldsymbol{J}}\cdot d\vec{s}' + \int_S \frac{\partial\vec{\boldsymbol{D}}}{\partial t}\cdot d\vec{s}'$	Ampere's law