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Q1

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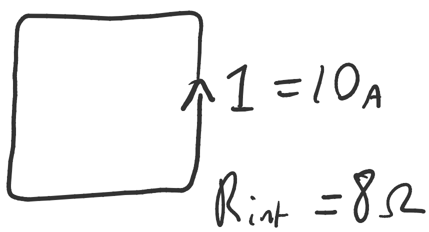
1. A stationary conducting loop with an internal resistance of 8 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 10 A flows through it. What will the current be if the loop is opened to create a small gap and a 5 Ω resistor is connected across its open ends?

time varying field → V_{emf}

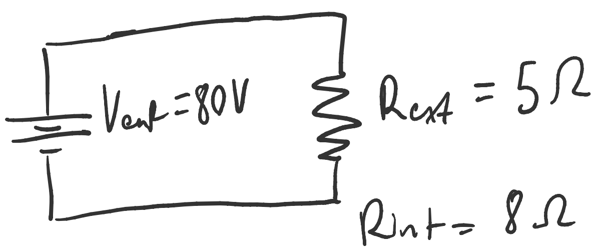
(1)



$V_{emf} = IR = 10 \cdot 8$

$V_{emf} = 80\,V$

(2)



$V = IR \rightarrow I = V/R$

$I = V_{emf}/(R_{ext} + R_{int})$

$I = 80/13$

$I = 6.154\,A$

Q2

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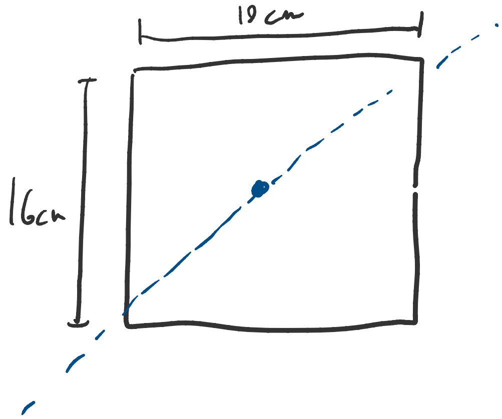
2. A rectangular conducting loop 10 cm × 16 cm with a small air gap in one of its sides is spinning at 7200 revolutions per minute. If the field $\boldsymbol{B}$ is normal to the loop axis and its magnitude is 3.5×10^{-6} T, what is the peak voltage induced across the air gap?



$$\omega = \frac{2\pi \cdot 7200}{60} = 753.982 \text{ rad/s}$$

$\vec{B} \perp$ loop axis

$$A = (0.1\,m)(0.16\,m)$$
$$A = 0.016\ m^2$$

$$V_{emf} = A\omega B_0 \sin(\omega t)$$

Max V_{emf} occurs @ $\sin(\omega t) = 1$

$$\left(\begin{aligned} \omega t &= \frac{\pi}{2} \\ t &= \frac{\pi}{2\omega} \\ t &= \pi/2(753.982) \\ t &= 2.083\text{ ms} \end{aligned} \right)$$

sub $\sin(\omega t) = 1$ (or $t = 2.083$ ms)

$$(V_{emf})_{max} = (0.016)(753.982)(3.5\times10^{-6})(1)$$

$$\boxed{(V_{emf})_{max} = 4.22\times10^{-5}\ V}$$

Q3

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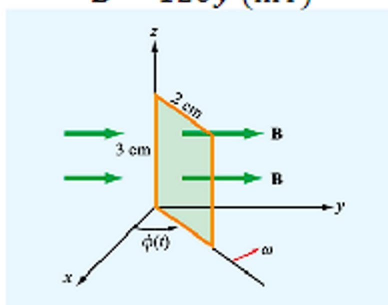
3. The rectangular conducting loop shown in the figure below rotates at 1,200 revolutions per minute in a uniform magnetic flux density given by:

$$\boldsymbol{B} = 120\hat{\boldsymbol{y}}\ (\mathrm{mT})$$



Determine the current induced in the loop if its internal resistance is 250 mΩ.

$$R_{int} = 0.25\ \Omega$$

$$\vec{B} = 0.120\hat{y}\ (T)$$

$$A = 0.02 \cdot 0.03 = 6 \times 10^{-4}\ m^2$$

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

$$\Phi = 0.120\hat{y} \cdot (6 \times 10^{-4})\hat{y} \cos(\phi)$$

$$\phi = \omega t$$

$$= \left(\frac{2\pi}{60} \cdot 1200\right) t$$

$$= 125.664\, t$$

$$\Phi = (0.120)(6 \times 10^{-4})(\cos(125.644 t))$$

$$\Phi = (7.2 \times 10^{-5}) \cos(125.644 t)$$

$$\Phi = (7.2 \times 10^{-5})\cos(125.644t)$$

$$I = V/R = V_{emf} / R_{int}$$

$$* \; V_{emf} = -\frac{d}{dt}\Phi$$

$$I = \frac{-1}{0.25}\left(\frac{d}{dt}(7.2 \times 10^{-5})\cos(125.644t)\right)$$

$$I = -4(7.2 \times 10^{-5})\frac{d}{dt}(\cos(125.644t))$$

$$I = -2.88 \times 10^{-4}(-125.644\sin(125.644t))$$

$$\boxed{I = 0.0362\sin(125.644t)\ A}$$

Q4

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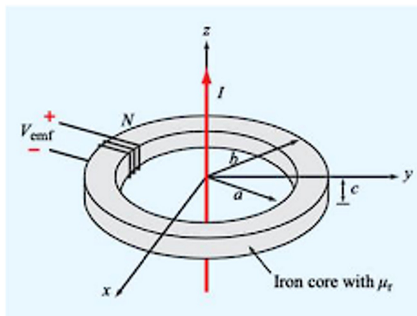
4. The transformer shown below consists of a long wire coincident with the z axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x–y plane and centered at the origin. The toroidal core uses iron material with relative permeability μr, around which 500 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure.

   a. Develop an expression for V_{emf}
   b. Calculate V_{emf} for $f = 60$ Hz, $\mu_r = 4500$, $a = 10\ cm$, $b = 12\ cm$, $c = 3\ cm$, and $I_0 = 60\ A$



$N = 500$, $I = I_0 \cos(\omega t)$, $\mu = \mu_r \mu_0$

a)

$$\Phi = \int_S \vec{B} \cdot d\vec{S} \qquad * B = \frac{\mu I}{2\pi r}\hat{x} \qquad * d\vec{S} = (c\,dr)\hat{x}$$

$$\Phi = \int_a^b \frac{\mu I}{2\pi r}\hat{x} \cdot c\,dr\,\hat{x}$$

$$\Phi = \frac{\mu I c}{2\pi}\int_a^b \frac{dr}{r}$$

$$\Phi = \frac{\mu I c}{2\pi}\ln(b/a)$$

$$V_{emf} = -N\frac{d\Phi}{dt}$$

$$= -\frac{N\mu\,(dI)\,c\,\ln(b/a)}{2\pi(dt)}$$

$$= -\frac{N\mu c \ln(b/a)}{2\pi} \frac{d}{dt}\left(I_0 \cos(\omega t)\right)$$

$$= -\frac{N\mu c I_0 \omega \ln(b/a)}{2\pi}\left(-\sin(\omega t)\right)$$

$$\boxed{V_{emf} = \frac{N\mu c I_0 \omega \ln(b/a)}{2\pi} \sin(\omega t) \quad [V]}$$

b) b. Calculate V_{emf} for $f = 60$ Hz, $\mu_r = 4500$, $a = 10\ cm$, $b = 12\ cm$, $c = 3\ cm$, and $I_0 = 60\ A$

$\mu_r = 4500$, $b = 12$ cm, $a = 10$ cm, $C = 0.03$m, $I_0 = 60$A

$\omega = 2\pi f = 120\pi$ [rad/s]

$N = 500 \qquad \mu = 4500 \times \mu_0 = 5.65 \times 10^{-3}$

$\ln(b/a) = \ln(1.2) = 0.18232$

$$V_{emf} = \frac{(0.03)(500)(5.655\times10^{-3})(60)(0.18232)(120\pi)}{2\pi} \cdot \sin(120\pi t)$$

$$\boxed{V_{emf} \doteq 55.67 \sin(120\pi t) \ [V]}$$