ENGPHYS 2A04 Winter 2022 – Assignment 1 Solutions

1. The electric field of a wave traveling in space is proportional to the following equation:

$$E(x,t) = \cos(3\pi \times 10^{14} t - \pi \times 10^{6} x)$$

   a) Find the direction of propagation, and the phase velocity of this wave. What is the significance of this value?

   b) Find the wavelength of this wave. What part of the electromagnetic spectrum does this fall under?

Solution:

a) The t and x terms have opposite signs, so the wave is travelling in the positive x direction. The phase velocity can be found according to $u_P = \frac{\omega}{\beta}$:

$$u_P = \frac{\omega}{\beta} = \frac{3\pi \times 10^{14}}{\pi \times 10^{6}} = 3 \times 10^{8}\,\frac{m}{s}$$

$$\boldsymbol{u_P = 3 \times 10^{8}\,\frac{m}{s}}$$

This is significant, because this value is approximately c, the speed of light in a vacuum.

b) The wavelength can be found according to $\beta = \frac{2\pi}{\lambda}$:

$$\beta = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{\pi \times 10^{6}} = 2 \times 10^{-6}\,m$$

$$\boldsymbol{\lambda = 2\,\mu m}$$

This wavelength, $2\mu m$, falls within the infrared part of the electromagnetic spectrum.

2. An electromagnetic wave is propagating in the z direction in a lossy medium with attenuation constant $\alpha = 0.4\,Np/m$. If the wave's electric-field amplitude is $63\,V/m$ at $z = 2m$, how far can the wave travel before its amplitude is reduced to $10\,V/m$?

Solution:

First, realize that the amplitude of the wave at a given z-value is represented by $A(z) = A_0 e^{-\alpha z}$. Since $A(2) = 63\,V/m$ is known, A_0 can be found:

$$A_0 = \frac{A(z)}{e^{-\alpha z}} = \frac{A(2)}{e^{-(0.4)(2)}} = \frac{63}{e^{-(0.4)(2)}} = 140.21\,V/m$$

With this knowledge, the amplitude function can be rearranged as follows:

$$A(z) = A_0 e^{-\alpha z}$$

$$\frac{A(z)}{A_0} = e^{-\alpha z}$$

$$ln\left(\frac{A(z)}{A_0}\right) = -\alpha z$$

$$z = \frac{-\,ln\left(\frac{A(z)}{A_0}\right)}{\alpha}$$

Substituting in the known values to find where $A(z) = 10\ V/m$:

$$z = \frac{-\,ln\left(\frac{10}{140.21}\right)}{0.4}$$

$$\therefore \boldsymbol{z = 6.6\ m}$$

3. On a windy day, the height of a wave on the lake (in meters) is described by:

$$h(x,t) = 1.1\sin(0.6t - 0.9x)$$

Determine the wavelength. Plot the height of the wave at $t = 2\ s$, over a distance of 2 wavelengths starting at $x = 0$.

Solution:

The wavelength can be found according to:

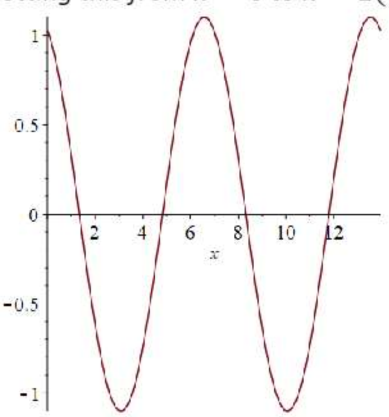
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.9}$$

$$\boldsymbol{\lambda = 6.98\ m}$$

At $t = 2\ s$, the expression simplifies to:

$$h(x,t) = 1.1\,sin(0.6(2) - 0.9x) = 1.1\,sin(1.2 - 0.9x)$$

Plotting this from $x = 0$ to $x = 2(6.98) = 13.96\ m$ yields:



4. The electric current through a point in a circuit is given by

$$i(t) = 0.06\cos(120\pi t)$$

Elsewhere in the circuit, the voltage across some component is given by

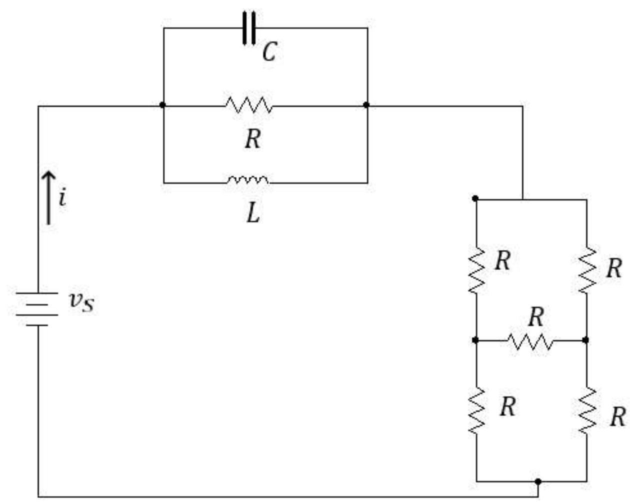
$$v(t) = 1.2\sin(120\pi t + 30°)$$

Is this voltage lagging or leading the current? By what phase angle?

Solution:

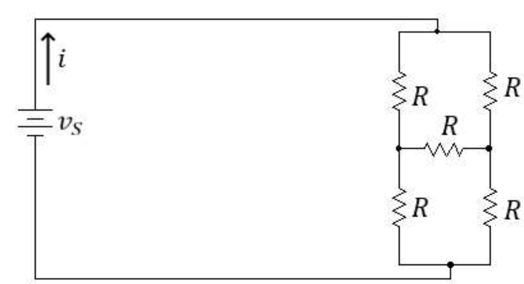
On its own, sin(x) will lag cos(x) by 90°. Adding in the 30° phase shift means that overall, ***the voltage lags the current by 60°.***

5. Find an expression for the current i in the circuit below, in terms of the source voltage v_S, the capacitance C, the inductance L, and the resistance R. Assume the voltage source is steady-state DC.

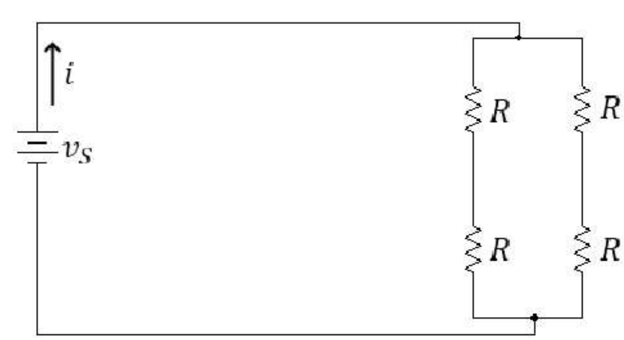


Solution:

In steady state DC, the inductor will short. This means the circuit can be simplified as follows:



By symmetry, the resistor in the center of the block will have no voltage difference across it – it can be treated as open, because no current will flow.



Finding the current is then trivial:

$$R_{equivalent} = \frac{1}{{}^{1}/_{2R} + {}^{1}/_{2R}} = \frac{1}{{}^{2}/_{2R}} = \frac{1}{{}^{1}/_{R}} = R$$

$$i = \frac{v_S}{R_{equivalent}} = \frac{v_S}{R}$$

$$\therefore \boldsymbol{i = \frac{v_S}{R}}$$