Question 1: Answer the following questions (10 points):

1) After a fork(), the child process and the parent process have no shared address space. (T)

2) If scheduling of a task set is not feasible using RM algorithm, it cannot be scheduled by DM algorithm. (F)

3) Soft real-time tasks are those which do not have any time bounds associated with them. (F)

4) The cyclic executive to schedule n periodic tasks needs to pre-store the schedule for a certain amount of time. The minimum length of time is given by the period of the task with the lowest priority. (F)

5) Under the Priority Ceiling Protocol, a task can be blocked only before it starts executing, never once it has started. (F)

6) A task set consisting of three tasks, T1, T2, and T3, with identical period **and deadline**, is RM-schedulable if and only if the total processor utilisation is at most 1. (T)

7) Priority Inversion is a phenomenon that cannot occur if tasks are independent. (T)

8) For discrete approximation of continuous system, it is possible that discrete system is stable, but the corresponding continuous is unstable. (T)

9) The z-transform of a system response is given by $C(z) = \frac{1}{4}\frac{z^{-1}(1-z^{-4})}{(1-z^{-1})^2}$. It is final value is 1. (T)

10) The number of the root locus segments which do not terminate on **open-loop** zeroes is the difference between the number of **open-loop** poles and zeros. (T)

Student Name: ID:

Question 2: Multiple Choice Questions (10 points):

1) The impulse response of a system is given by $\mathrm{Y(z)} = \frac{z^3+2z^2+2}{z^3-25z^2+0.6z}$. Determine the values of y(nT) at the first four sampling instants.

a) y(0) = 1, y(T) = 27, y(2T) = 674.4, y(3T) = 16845.8

b) y(0) = 1, y(T) = 27, y(2T) = 647, y(3T) = 660.05

c) y(0) = 1, y(T) = 647, y(2T) = 47, y(3T) = 27

d) y(0) = 0, y(T) = 27, y(2T) = 47, y(3T) = 60.05

2) Which protocol can solve the priority inversion problem without introducing deadlock?

a) priority inheritance protocol

b) priority inversion protocol

c) **priority ceiling protocol**

d) none of the mentioned

3) The clock of the Raspberry Pi slows down at a rate of 40×10^{-6} seconds per second. Suppose that you can connect the device to a clock server that allows you to correct the time to within 5 seconds of the true time. You would like the displayed time to be accurate to within one minute of the true time. How often should the device be synchronized to the clock server?

a) 20 days

b) **382 hours**

c) 417 hours

d) 34.7 hours

4) If one thread opens a file with read privileges then

a) other threads in the another process can also read from that file

b) **other threads in the same process can also read from that file**

c) any other thread can not read from that file

d) all of the mentioned

5) The root locus is the trace of the roots of the characteristic equation in the s-plane?

a) as the input of the system is changed

b) as the output of the system is changed

c) **as a system parameter is changed**

d) as the sensitivity is changed

6) Which is the controller that will improve the transient response?

a) P b) I **c) D** d) P and I

7) If a set of pre-emptive and periodic tasks cannot be successfully scheduled by EDF, we can infer that:

a) they cannot be scheduled by rate monotonic scheduling algorithm

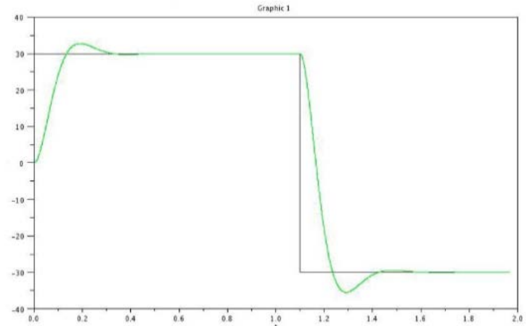
b) they cannot be scheduled by deadline monotonic scheduling algorithm

c) they cannot be scheduled by cyclic executive algorithm

d) All of the above

8) In order to simulate the given signal as below, you have to change the value of ( ) module:

a) Signal Generator b) Integrator **c) PID controller** d) Mux



9) Which of the following scheduling algorithm is used in real-time OS but not general purpose OS.

a) round robin

b) rate monotonic scheduling

c) first come first served

d) priority-based scheduling

10) Consider s-domain function $\mathrm{Y}(s)=\frac{10}{s(s+2)(s+6)}$. Let T be the sampling time. Then, in the z-domain the function Y(z) is:

a) $\mathrm{Y(z)}=\frac{1}{6}\frac{z}{z-1}-\frac{z}{z-e^{-2T}}+\frac{5}{6}\frac{z}{z-e^{-6T}}$

b) $\mathrm{Y(z)}=\frac{1}{6}\frac{z}{z-1}-\frac{5}{4}\frac{z}{z-e^{-2T}}+\frac{5}{12}\frac{z}{z-e^{-6T}}$

c) $\mathbf{Y(z)}=\boldsymbol{\frac{5}{6}\frac{z}{z-1}-\frac{5}{4}\frac{z}{z-e^{-2T}}+\frac{5}{12}\frac{z}{z-e^{-6T}}}$

d) $\mathrm{Y(z)}=\frac{5}{6}\frac{z}{z-1}-\frac{z}{z-e^{-2T}}+\frac{5}{6}\frac{z}{z-e^{-6T}}$

Question 3: Determine the Stability Range of K>0

1) Consider a sampled-data system with the closed-loop system transfer function

$$\mathrm{T(z)}=K\frac{z^2+2z}{z^2+0.2z-0.5}$$

For what K value, the system stable? [2 points]

Answer: Amplitude of roots must be smaller than 1, So the system is stable for all K>0.

2) The characteristic equation of a sampled data system is given by

$$q(z)=z^2+(2K-1.75)z+2.5=0,$$

where K > 0. What is the range of K for a stable system? [3 points]

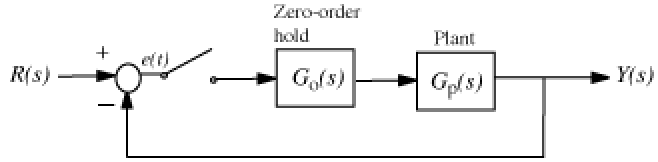
Answer: The roots of q(z) is $\frac{(1.75-2K)\pm\sqrt{(1.75-2K)^2-10}}{2}$

If $(1.75-2K)^2-10\geq 0$, there are two real roots. In this case, either $(1.75-2K)\geq\sqrt{10}$ or $(1.75-2K)\leq-\sqrt{10}$. The first scenario, $(1.75-2K)\geq\sqrt{10}$ is not possible because $K>0$. The second scenario $(1.75-2K)\leq-\sqrt{10}$, one of the root must be smaller than $\frac{(1.75-2K)}{2}\leq\frac{-\sqrt{10}}{2}$, which is smaller than -1. So the system will not be stable, if $(1.75-2K)^2-10\geq 0$.

Consider the case where $(1.75-2K)^2-10<0$, there are two conjugate roots. By forcing the two conjugate roots $x\pm jy$ have amplitude 1. We will have:

$x=\frac{(1.75-2K)}{2}$ and $y=\frac{\sqrt{(1.75-2K)^2-10}}{2}$, So we need $\frac{(1.75-2K)^2}{4}+\frac{10-(1.75-2K)^2}{4}=1$.
This is not possible for any K value. So the system is unstable for all K values.

3) Consider the unity feedback system,



where $G_p(s) = \frac{K}{s+1}$, with the sampling time T = 0.1 sec. What is the maximum value for K for a stable closed-loop system? (3 points)

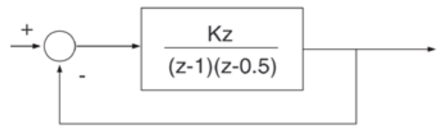
Answer: Open-loop transfer function is G(z)=$(1-z^{-1})Z\left[\frac{K}{s(s+1)}\right] = K\frac{z-1}{z}\left[\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right] = K(1 - \frac{z-1}{z-e^{-T}})$

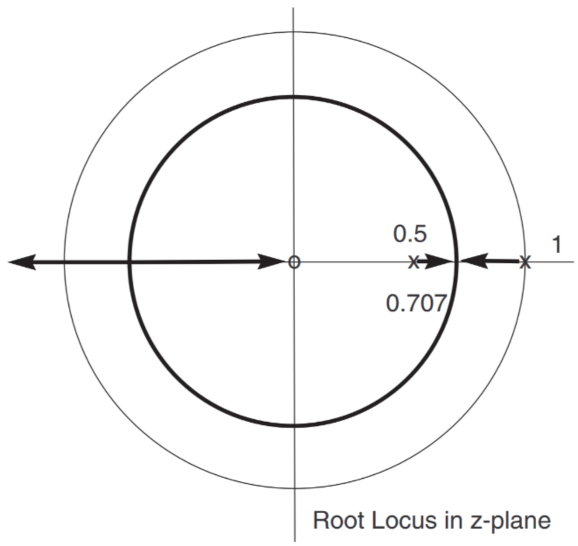
T=0.1, so G(z)= $K\left(\frac{1-e^{-T}}{z-e^{-T}}\right) = K\left(\frac{1-e^{-0.1}}{z-e^{-0.1}}\right) = \frac{0.1\,K}{z-0.9}$

The characteristic equation is $\mathrm{z} - 0.9 + 0.1\mathrm{K} = 0$, considering the root locus and force z=-1, we will have K=19. So for K<19, the system is stable.

Note that it also OK if the answer is 20.

4) Consider the following discrete system, what value of K makes the system stable? [3 points]





Break-away point 0.707 and break-in point -0.707

Characteristic equation:

$$z^2 + (K - 1.5)z + 0.5 = 0$$

Substitute z with -1, we get K=3. So system is stable when K<3.

Question 4: Scheduling Algorithms

1) A system consists of three periodic tasks: T1 = (3, 1), T2 = (5, 2), and T3 = (8, 3).

(a) Is the above set of tasks schedulable using EDF algorithm? Justify your answer. [1]

(b) Suppose we want to reduce the execution time of T3 in order to make the task system schedule-able according to the EDF algorithm. What is the minimum amount of reduction necessary for the system to be schedulable (tasks may execute for a fraction of a time unit)? [2]

Answer:

(a) U=1/3+2/5+3/8=1.108>1, So it is not schedulable.

(b) Maximum ratio of e3/T3 is given by $1-\frac{1}{3}-\frac{2}{5}=\frac{4}{15}$.

So maximum time $\frac{e}{8}=\frac{4}{15} \rightarrow e=\frac{32}{15}$

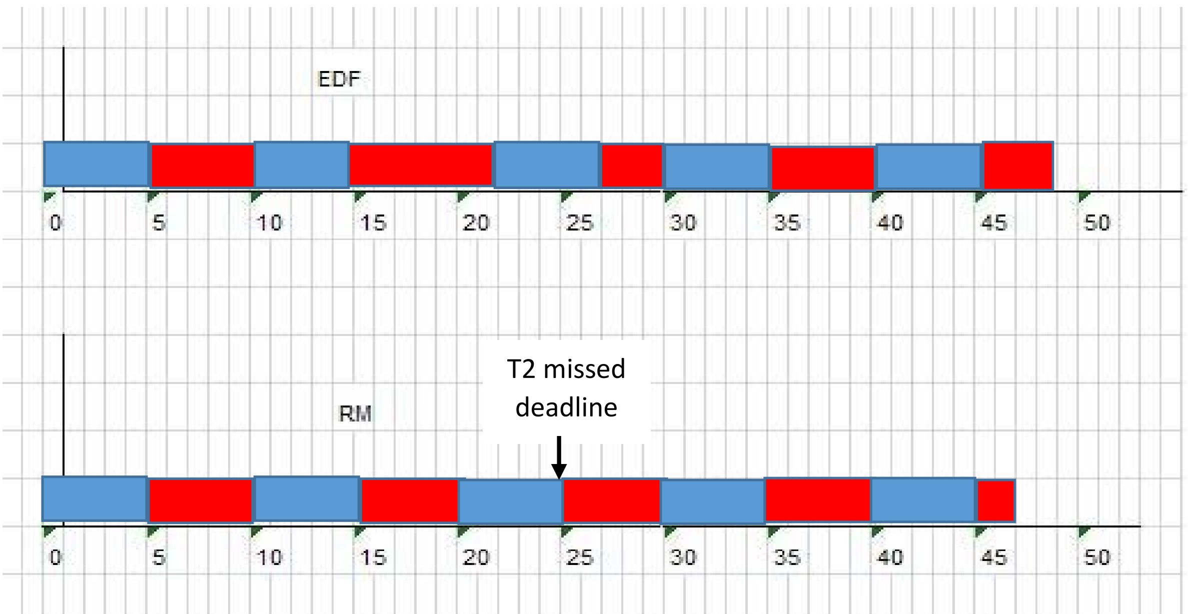
So minimum reduction of execution time is $3-\frac{32}{15}=\frac{13}{15}=0.867$

2) Given the task set: $T_1(10; 5)$; $T_2(25; 12)$

(a) Graphically construct an EDF schedule for 50 time units in the space provided in Figure 1. [2]

(b) Use the same task set for constructing a schedule based on RM algorithm in the space provided in Figure 1. [2]

(c) Use suitable schedulability tests to verify your answer and show if the task set is schedulable under either EDF or RM algorithm. [4]



(c) Utilization 0.5+12/25=0.98<1, so the task set is EDF schedulable.

RM (necessary and sufficient condition):
For i=1, t1=10, w_1(t)=e1<10
For i=2, t1=10, t2=20 and t3=25,
w_2(t1) = e1+e2=17>t1=10,
w_2(t2)=2e1+e2=10+12=22>t2=20
w+2(t3)=3e1+e2=15+12=27>t3=25
So the task set is not RM schedulable.

Question 5: Discrete Approximation of Continuous System

The transfer function of a controller is given by:

$$\mathrm{D}(\mathrm{s}) = 25\frac{s+1}{s+15}$$

(a) Derive the corresponding differential equation [2].

(b) Substitute the expression for the frequency variable s corresponding to forward rectangular rule, in the transfer function and derive D(z) [2].

(c) For the discrete transfer function given in (b), find a digital implementation with a sampling rate of 40 Hz [2].

Answer:

(a) $y' + 15y = 25x' + 25x$ or $u' + 15u = 25e' + 25e$, etc.

(b) Forward rectangular rule: $s \leftarrow \frac{z-1}{T}$. Hence, $\mathrm{D}(\mathrm{z}) = 25\frac{\frac{z-1}{T}+1}{\frac{z-1}{T}+15} = \frac{25z-24.375}{z-0.625}$

(c) $u(z)(1-0.625z^{-1}) = e(z)(25-24.375z^{-1})$ →

$u(k+1) = 0.625u(k) + 25e(k+1) - 24.375e(k)$

Question 6: Given the difference equation

$$y(k+2) - \frac{3}{4}y(k+1) + \frac{1}{8}y(k) = e(k)$$

where $y(0) = y(1) = 0$, $e(0) = 0$, and $e(k) = 1, k = 1,2, \ldots$

(a) Solve difference equation direction for $y(k), 2 \le k \le 4$. [1 point]
(b) Solve for $y(k)$ as a function of k using z-transform. [2 points]

Solution to (a)

$$y(k+2) = e(k) + \frac{3}{4}y(k+1) - \frac{1}{8}y(k)$$

$$y(2) = 0 + \frac{3}{4}(0) - \frac{1}{8}(0) = 0$$

$$y(3) = 1 + \frac{3}{4}(0) - \frac{1}{8}(0) = 1$$

$$y(4) = 1 + \frac{3}{4}(1) - \frac{1}{8}(0) = 7/4$$

Solution to (b)

$$E(z) = \mathfrak{z}[u(k-1)] = z^{-1}\left[\frac{z}{z-1}\right] = \frac{1}{z-1}$$

$$\left[z^2 - \frac{3}{4}z + \frac{1}{8}\right]Y(z) = E(z)$$

$$\frac{Y(z)}{z} = \frac{1}{z\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} \cdot \frac{1}{z-1} = \frac{-8}{z} + \frac{8/3}{z-1} + \frac{-16}{z-\frac{1}{2}} + \frac{64/3}{z-1/4}$$

$$\therefore y(k) = -8\delta(0) + \frac{8}{3} - 16\left(\frac{1}{2}\right)^k + \frac{64}{3}\left(\frac{1}{4}\right)^k$$

$$\therefore y(0) = 0;\ y(1) = 0;\ y(2) = 0;\ y(3) = 1;\ y(4) = \frac{7}{4}$$