## Example: Tree for a Query

Using the relations Bars(name, addr) and Sells(bar, beer, price), find the names of all the bars that are either on Maple St. or sell Bud for less than \$3.

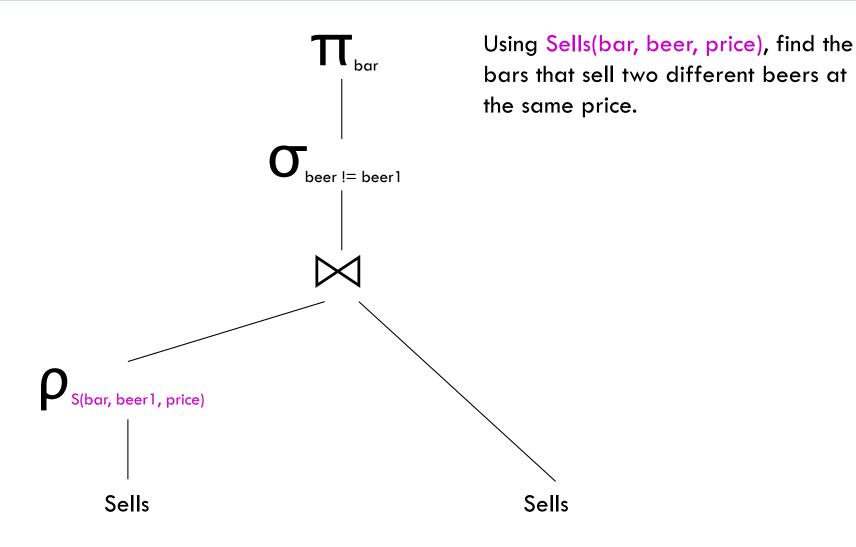
### As a Tree:

Using the relations Bars(name, addr) and Sells(bar, beer, price), find the names of all the bars that are either on Maple St. or sell Bud for less than \$3. Sells Bars

### **Example:** Self-Join

- Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.
- □ Strategy: by renaming, define a copy of Sells, called S(bar, beer1, price). The natural join of Sells and S consists of quadruples (bar, beer, beer1, price) such that the bar sells both beers at this price.

### The Tree



### Schemas for Results

- Union, intersection, and difference: the schemas of the two operands must be the same, so use that schema for the result.
- Selection: schema of the result is the same as the schema of the operand.
- Projection: list of attributes tells us the schema.

### Schemas for Results

- Product: schema is the attributes of both relations.
  - Distinguish two attributes with the same name.
- □ Theta-join: same as product.
- Natural join: union of the attributes of the two relations. Keep only one copy of the equated attributes.
- Renaming: the operator tells the schema.

**s** ⋈ T

39

R

A	В
1	1
2	1
3	3

$$\pi_{\scriptscriptstyle B}(R) \cap \rho_{\scriptscriptstyle T(B)}(\pi_{\scriptscriptstyle C}(S))\text{:}$$

1 3

$$R\bowtie (S\bowtie \rho_{T(B,C)}(R))$$

A	В	С	D
1	1	1	2
2	1	1	2
3	3	3	4
3	3	3	5

### The Extended Algebra

 $\delta$  = eliminate duplicates from bags.

T =sort tuples.

Y = grouping and aggregation.

Outerjoin: avoids "dangling tuples" = tuples that do not join with anything.

### **Duplicate Elimination**

$$\square$$
 R1 :=  $\delta$ (R2).

 □ R1 consists of one copy of each tuple that appears in R2 one or more times.

## **Example:** Duplicate Elimination

Α	В
1 3	2 4 2
	1

$$\delta_{(R)} = \begin{bmatrix} A & B \\ 1 & 2 \\ 3 & 4 \end{bmatrix}$$

### Sorting

- $\square$  R1 :=  $\mathsf{T}_L$  (R2).
  - $\square$  L is a list of some of the attributes of R2.
- R1 is the list of tuples of R2 sorted first on the value of the first attribute on L, then on the second attribute of L, and so on.
  - Break ties arbitrarily.

# **Example:** Sorting

R =	(	Α	В	
		1	2	
		3	4	
		5	2	

$$T_{B}(R) = \begin{pmatrix} A & B \\ 5 & 2 \\ 1 & 2 \\ 3 & 4 \end{pmatrix}$$

### Aggregation Operators

- Aggregation operators are not formally operators of relational algebra.
- Rather, they apply to entire columns of a table and produce a single result.
- The most important examples: SUM, AVG, COUNT, MIN, and MAX.

# **Example:** Aggregation

R = (	Α	В	)
	1	3	
	3	4	
	3	2	

$$SUM(A) = 7$$

$$COUNT(A) = 3$$

$$MAX(B) = 4$$

$$AVG(B) = 3$$

## Grouping Operator

- R1:=  $Y_L$  (R2). L is a list of elements that are either:
  - 1. Individual (grouping) attributes.
  - 2. AGG(A), where AGG is one of the aggregation operators and A is an attribute.
    - An arrow and a new attribute name renames the component.

# Applying $Y_L(R)$

- Group R according to all the grouping attributes on list L.
  - That is: form one group for each distinct list of values for those attributes in R.
- Within each group, compute AGG(A) for each aggregation on list L.
- Result has one tuple for each group:
  - The grouping attributes and
  - 2. The group's aggregations.

# Example: Grouping/Aggregation

R =

A	В	O
1	2	3
4	5	6
1	2	5

$$Y_{A,B,AVG(C)\rightarrow X}$$
 (R) =  $??$ 

First, group R by A and B:

A	В	С
,	C	9
	2	3
1	2	5
4	5	6

Then, average C within groups:

Α	В	Х
1	2	4
4	5	6

### Recall: Outerjoin

- $\square$  Suppose we join  $R \bowtie_C S$ .
- □ A tuple of R that has no tuple of S with which it joins is said to be dangling.
  - Similarly for a tuple of S.
- Outerjoin preserves dangling tuples by padding them NULL.

## Example: Outerjoin

(1,2) joins with (2,3), but the other two tuples are dangling.

R FULL OUTERJOIN S =

Α	В	С
1	2	3
4	5	NULL
NULL	6	7

## Outer Join – Example

#### instructor

ID	name	dept_name
10101	Srinivasan	Comp. Sci.
12121	Wυ	Finance
15151	Mozart	Music

#### teaches

ID	course_id
10101	CS-101
12121	FIN-201
76766	BIO-101

### instructor teaches

■ Left Outer Join	
instructor 🖂	<sub>FT</sub> teaches

ID	name	dept_name	course_id
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

ID	name	dept_name	course_id
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	NULL

### Outer Join - Example

#### instructor

ID	name	dept_name
10101	Srinivasan	Comp. Sci.
12121	Wυ	Finance
15151	Mozart	Music

#### teaches

ID	course_id
10101	CS-101
12121	FIN-201
76766	BIO-101

■ instructor teaches

ID	name	dept_name	course_id
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
76766	null	null	BIO-101

instructor teaches

ID	name	dept_name	course_id
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wυ	Finance	FIN-201
15151	Mozart	Music	null
76766	null	null	BIO-101

### Operations on Bags

A **bag** = a set with repeated elements

All operations need to be defined carefully on bags

- $\sigma_{C}(R)$ : preserve the number of occurrences
- $\Pi_A(R)$ : no duplicate elimination
- Cartesian product, join: no duplicate elimination
   Important! Relational Engines work on bags, not sets!

## Why Bags?

- SQL, the most important query language for relational databases, is actually a bag language.
- Some operations, like projection, are more efficient on bags than sets.

### Operations on Bags

- Selection applies to each tuple, so its effect on bags is like its effect on sets.
- Projection also applies to each tuple, we do not eliminate duplicates.
- Products and joins are done on each pair of tuples, so duplicates in bags have no effect on how we operate.

# **Example:** Bag Selection

R

Α	В
1	2
5	6
1	2

 $\mathbf{O}_{A+B<5}$  (R) =

Α	В
1	2
1	2

# **Example:** Bag Projection

R

Α	В
1 5	2 6
1	2

$$\prod_{A} (R) =$$

Α
1
5
1

# Example: Bag Product

R

Α	В
1	2
5	6
1	2

S

В	С
3 7	4 8

$$RXS =$$

Α	R.B	S.B	U
1	2	3	4
1	2	7	8
5	2 2 6 6 2 2	3	4
5	6	7	8
1	2	3	4
1	2	7	8

# Example: Bag Theta-Join

R(	Α,	В )
	1	2
	5	6
	1	2

$$_{R}$$
  $\bowtie$   $_{R.B < S.B}$   $S =$ 

Α	R.B	S.B	U
1	2	3	4
1	2	7	8
5	6	7	8
1	2	3	4
1	2	7	8

### Bag Union

- An element appears in the union of two bags the sum of the number of times it appears in each bag.
- □ Example:  $\{1,2,1\}$   $\cup$   $\{1,1,2,3,1\}$  =  $\{1,1,1,1,1,2,2,3\}$

### Bag Intersection

- An element appears in the intersection of two bags the minimum of the number of times it appears in either bag
- $\square$  Example:  $\{1,2,1,1\} \cap \{1,2,1,3\} = \{1,1,2\}.$

### Bag Difference

- □ An element appears in the difference A B of bags as many times as it appears in A, minus the number of times it appears in B.
- □ Example:  $\{1,2,1,1\} \{1,2,3\} = \{1,1\}$ .

### Beware: Bag Laws != Set Laws

- Some, but not all algebraic laws that hold for sets also hold for bags.
- □ Example: the commutative law for union  $(R \cup S = S \cup R)$  does hold for bags.
  - Since addition is commutative, adding the number of times x appears in R and S doesn't depend on the order of R and S.

### **Example:** A Law That Fails

- $\square$  Set union is idempotent, meaning that  $S \cup S = S$ .
- $\square$  However, for bags, if x appears n times in S, then it appears 2n times in  $S \cup S$ .
- $\square$  Thus  $S \cup S != S$  in general.
  - $\blacksquare$  e.g.,  $\{1\} \cup \{1\} = \{1,1\} != \{1\}.$

What about Intersection?

Intersection is idempotent for sets and bags.