Real Time Systems and Control Applications

Contents

Review for Midterm 3

Midterm 3 Will Be Different From Previous Midterms

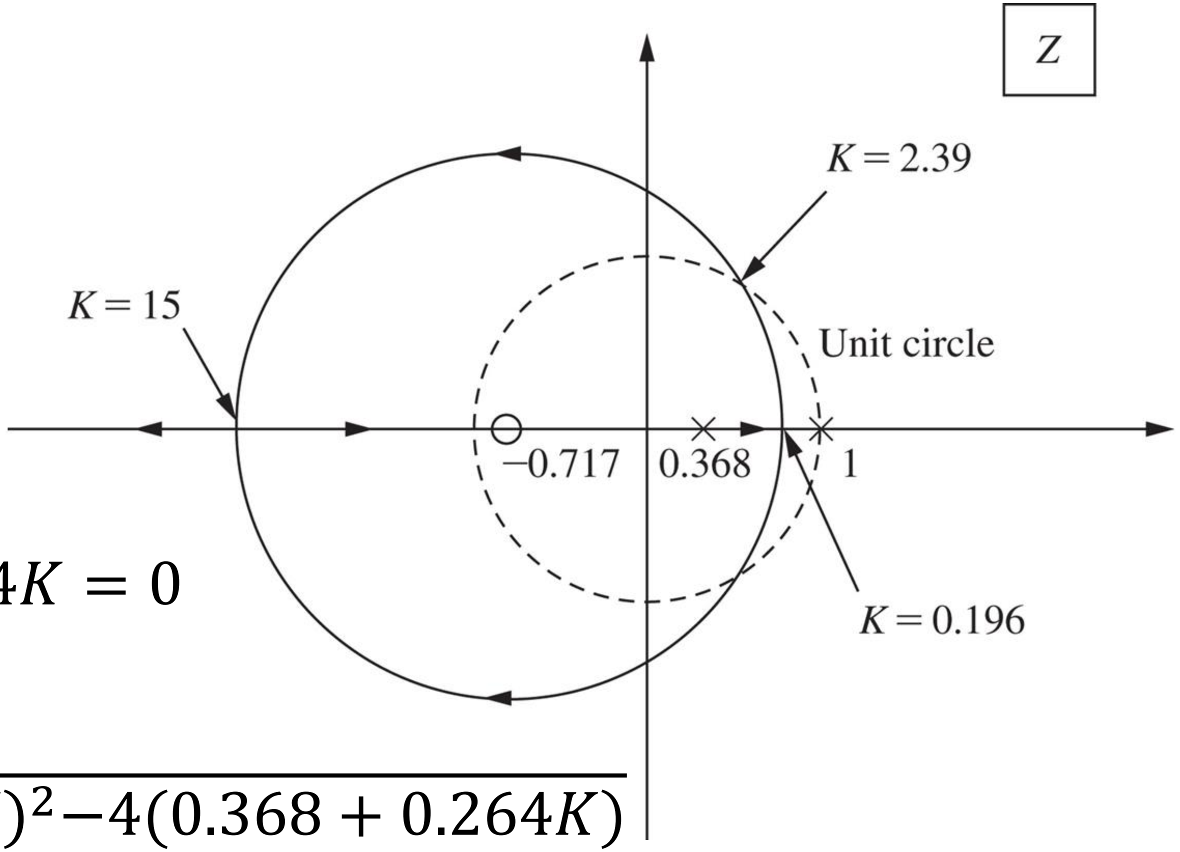
- No True and False Questions
- No multiple choices
- Short-answered questions only
- One-point bonus question will be given

Contents

- Laplace Transform and Inverse Laplace Transform
- Closed-loop Transfer Function
- Closed-loop poles and stability region
- Time Response of the First and Second Order Systems (time constant, peak time, overshoot percentage, time-domain response c(t))
- Z-Transform, inverse z-transform, and mapping from s-plane to z-plane
- Root locus in s-plane and z-plane
- Closed-loop Transfer Function for sampled-data systems

Revisit Previous Example

Open-loop TF: $G(z) = \frac{0.368K(z+0.718)}{(z-1)(z-0.368)}$



How to determine the value of K, which yields marginally stable?

Characteristic equation:

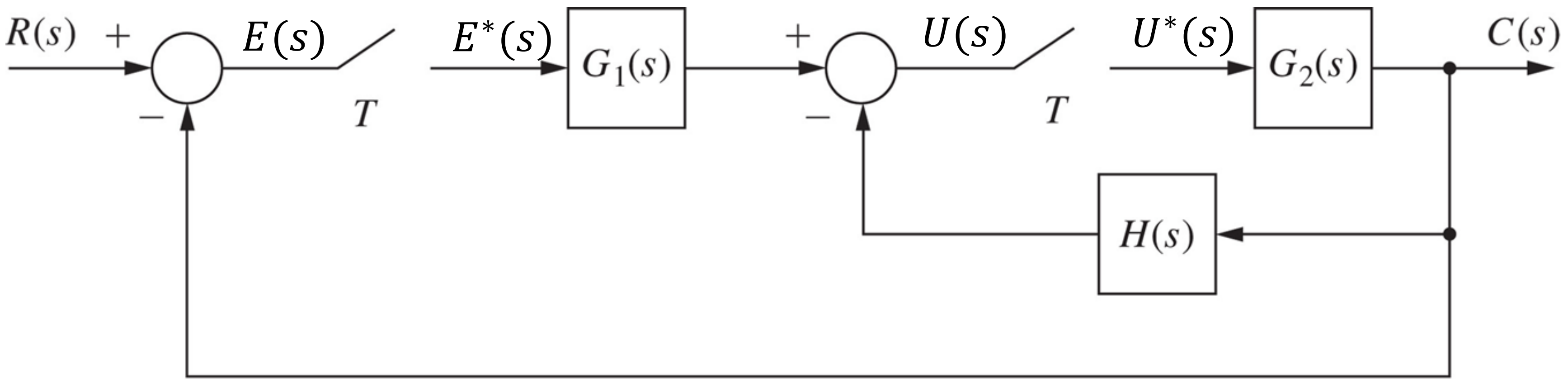
$$z^2 + (0.368K - 1.368)z + 0.368 + 0.264K = 0$$

$$z = \frac{(1.368 - 0.368K) \pm \sqrt{(1.368 - 0.368K)^2 - 4(0.368 + 0.264K)}}{2}$$

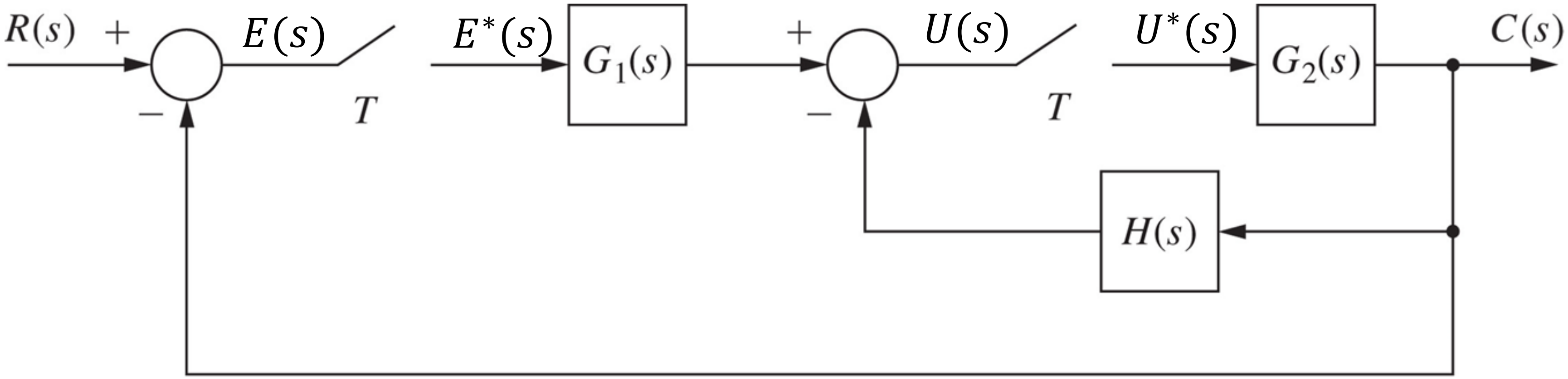
By forcing the real and imaginary part of z to be zero , $K = 2.39$

Question 1

- The system has two samplers, what is the system characteristic equation?



Answer



$$\begin{cases} G_1(s)E^*(s) - G_2(s)H(s)U^*(s) = U(s) \\ R(s) - C(s) = E(s) \\ G_2(s)U^*(s) = C(s) \end{cases}$$

Taking z-transform, we have

$$\begin{cases} G_1(z)E(z) - Z[G_2(s)H(s)]U(z) = U(z) & (1) \\ R(z) - C(z) = E(z) & (2) \\ G_2(z)U^*(z) = C(z) & (3) \end{cases}$$

Continued…

$$
\begin{cases}
G_1(z)E(z) - \overline{G_2H(z)}U(z) = U(z) & (1)\\
R(z) - C(z) = E(z) & (2)\\
G_2(z)U(z) = C(z) & (3)
\end{cases}
$$

Then from (1), we have:

$$G_1(z)E(z) = \left[\overline{G_2H(z)} + 1\right]U(z) \qquad (4)$$

Multiply (2) by $G_1(z)G_2(z)$, we get:

$$G_1(z)G_2(z)[R(z) - C(z)] = G_1(z)G_2(z)E(z) \qquad (5)$$

Substitute $G_1(z)E(z)$ in (5) with (4),

$$G_1(z)G_2(z)[R(z) - C(z)] = G_2(z)[\overline{G_2H(z)} + 1]U(z)$$

Continued…

Then substituting $G_2(z)U(z)\ as\ C(z)$ as in (3), we will get

$$G_1(z)G_2(z)[R(z) - C(z)] = [\overline{G_2H(z)} + 1]C(z)$$

$$\therefore G_1(z)G_2(z)R(z) - G_1(z)G_2(z)C(z) = [\overline{G_2H(z)} + 1]C(z)$$

Therefore, $\frac{C(z)}{R(z)} = \frac{G_1(z)G_2(z)}{1+\overline{G_2H(z)}+G_1(z)G_2(z)}$

- The characteristic equation is $1 + \overline{G_2H(z)} + G_1(z)G_2(z) = 0$

Question 2

Consider the system with open loop transfer function as

$$G(z) = \frac{0.05K(z+1)}{(z-1)^2}$$

Plot the z-plane root locus.

Answer

$$G(z) = \frac{0.05K(z+1)}{(z-1)^2}$$

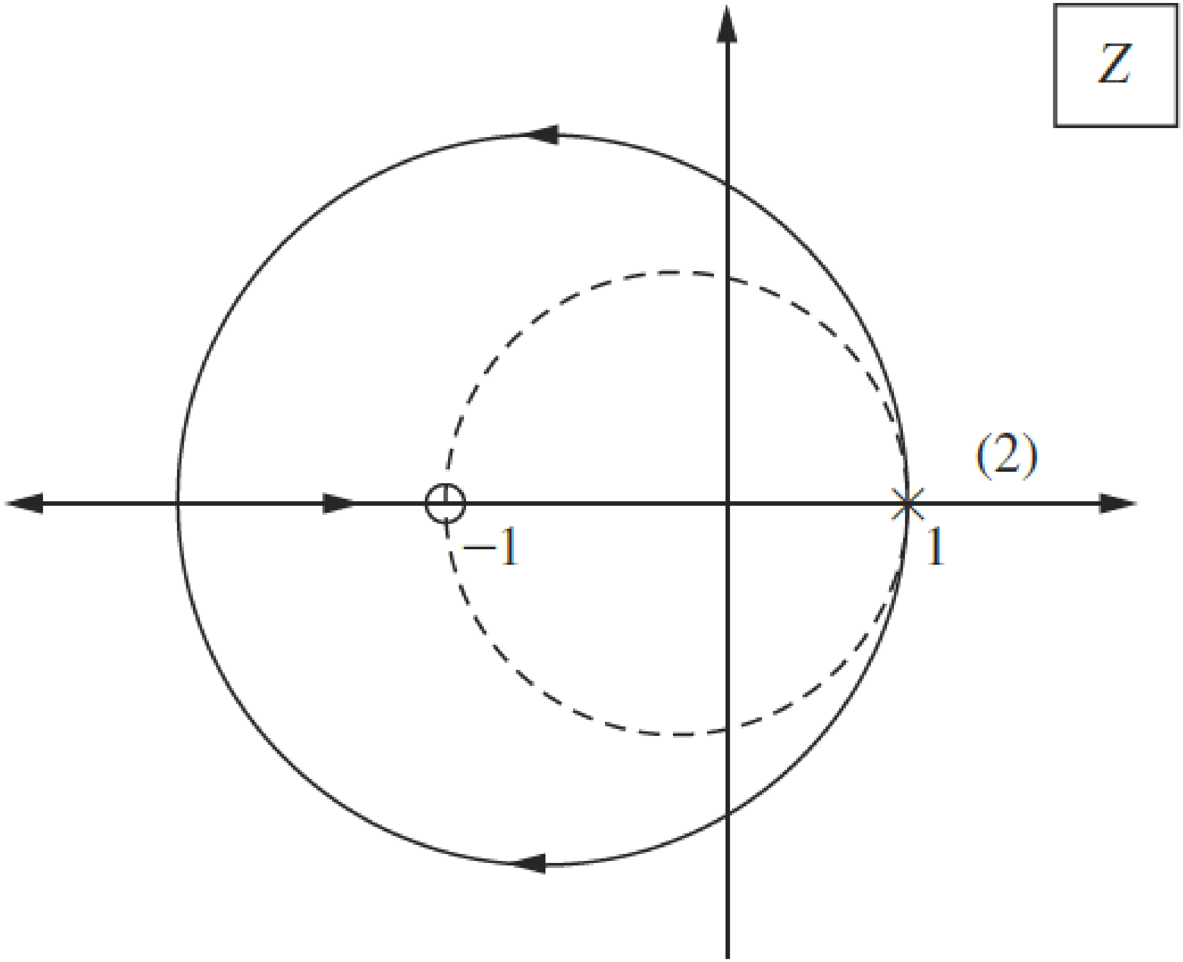
- Two open loop poles, and one finite open loop zero (and one infinite open loop zero).

- Breakin point satisfies

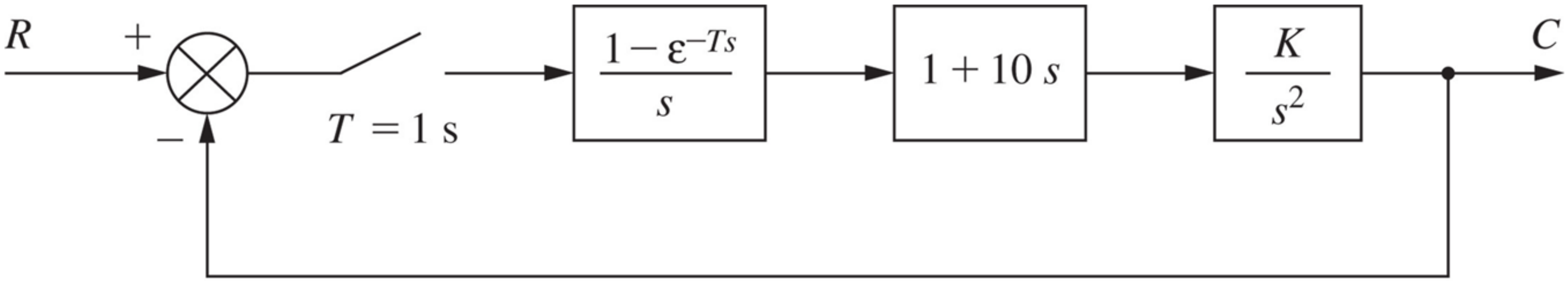
$$\frac{dG(z)}{dz} = 0$$

$$\frac{0.05K}{(z-1)^2} + (-2)\frac{0.05K(z+1)}{(z-1)^3} = 0$$

Hence, $z = -3$ is the breakin point



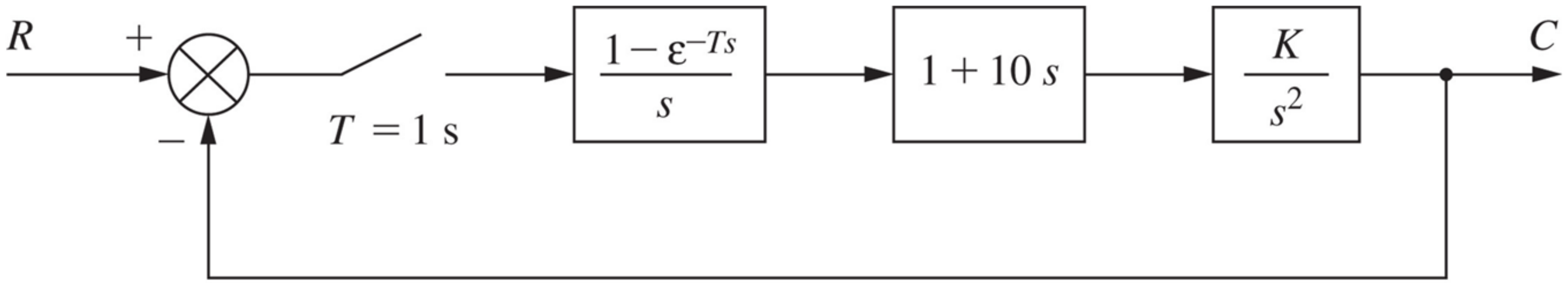
Question 3



- Plot the root locus of the system in z-plane.

- Find the range of K, which makes the system stable.

Answer



- Open-loop function is

$$KG(s) = \frac{1-e^{-sT}}{s}\left[\frac{K(1+10s)}{s^2}\right]$$

- Applying the z-transform, we obtain

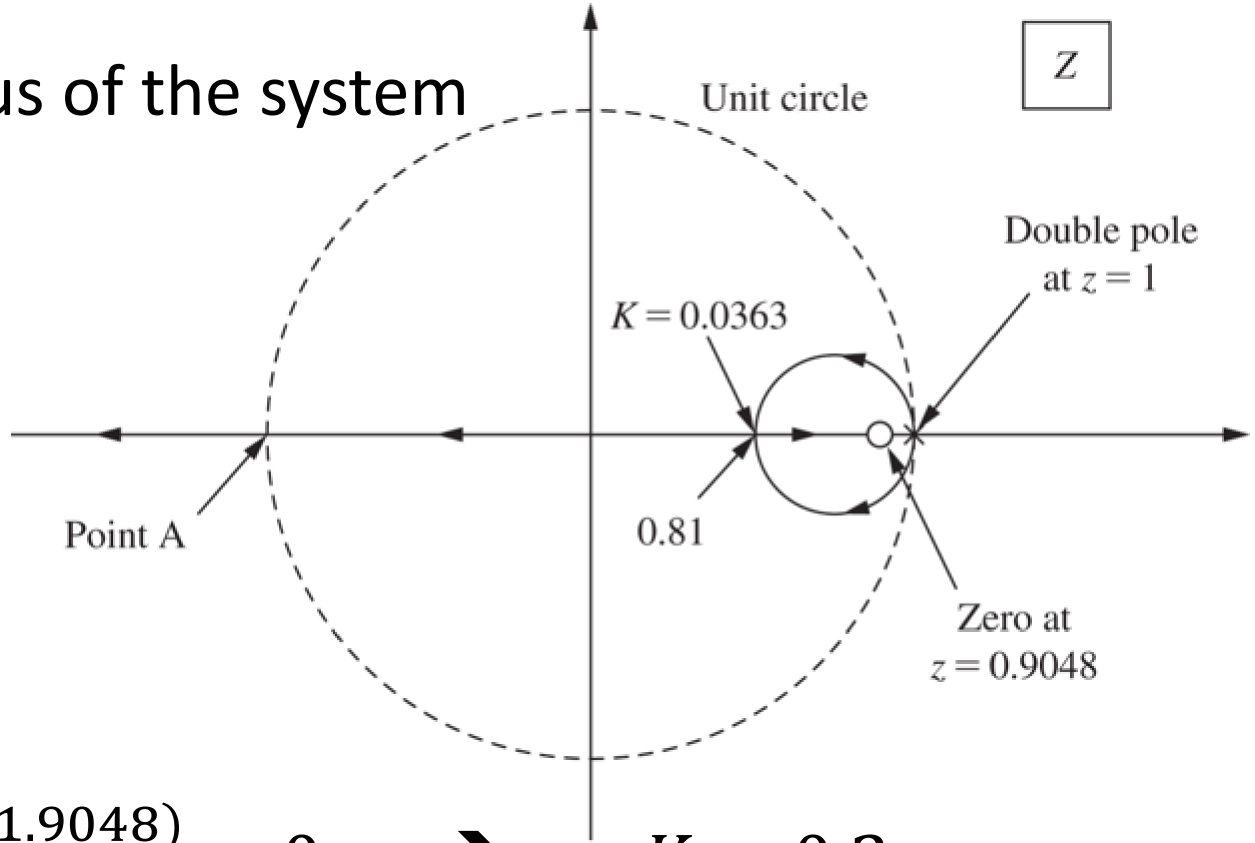
$$KG(z) = \frac{10.5K(z-0.9048)}{(z-1)^2}$$

- Compute the break in point:

$$\frac{dG(z)}{dz} = \frac{10.5K}{(z-1)^2} + \frac{(-2)\times 10.5K(z-0.9048)}{(z-1)^3} = 0$$

$$z = 0.81$$

This is the root locus of the system



At the point A, the system is marginally stable. To get the corresponding K, we need to have $1 + KG(z) = 0$ when $z = -1$.

$$\left.\frac{10.5K(z-0.9048)}{(z-1)^2}\right|_{z=-1} = \frac{10.5K(-1.9048)}{4} = 0 \quad \Rightarrow \quad K = 0.2$$

Hence, the stability range is $0 < K < 0.2$.

Root Locus from Closed-loop TF

- If the closed loop transfer function is given as $T(s) = \frac{9K}{10s^2+6s+9K}$

What does the root locus of the closed-loop system look like?

Answer: Note that the root is given as

$$s = \frac{-6 \pm \sqrt{36 - 360K}}{20}$$

This infers the root locus.