In-Class Test (#2)

Name ____________________
Student Number ____________________

ROBOTICS 4K03

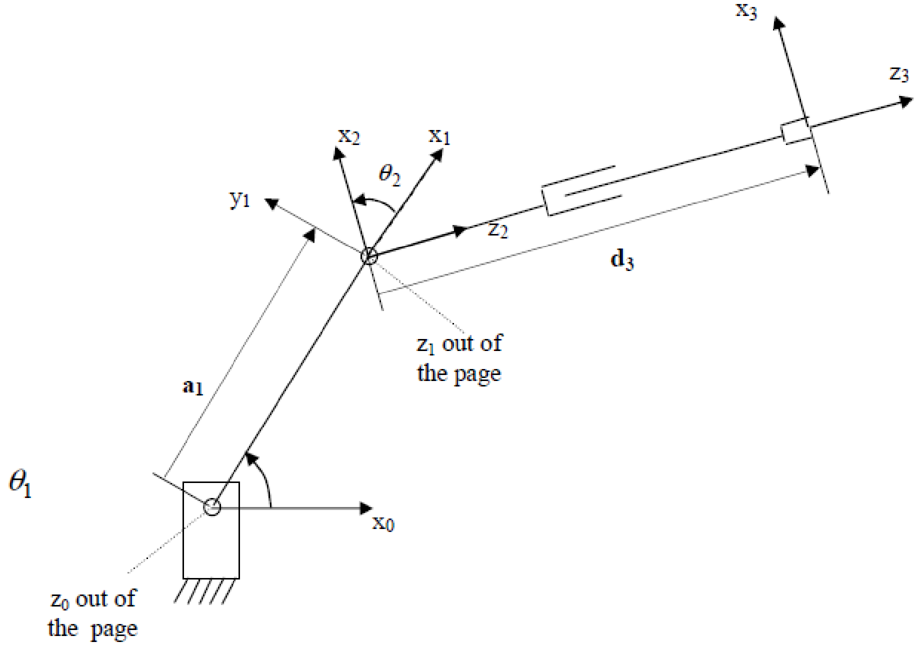
Questions:
1. (40 points) For the RRP planar robot shown in the following figure:

1) Assign the frames using the D-H method.
2) Determine the D-H parameters and put them in a table. Identify joint variables.
3) Draw a diagram of the robot that properly shows the D-H frames, the joint variables, and any *d* or *a* parameters that are non-zero.
4) Compute the *A* matrices and ${}^{0}T_{3}$.

Solution:

Parts (a) and (c) are done together in following figure below:

<u>View of plane normal to joints 1 and 2</u>



Part (b)

The D-H parameters are listed in the table below. Those with * are the joint variables.

n+1	θ	d	a	α
1	$*\theta_1$	0	a_1	0
2	$*\theta_2$	0	0	$90°$
3	0	d_3^*	0	0

Part (d)

The A matrices can be obtained using the formula

$$^{n}T_{n+1} = A_{n+1} = Rot(Z,\theta_{n+1}) * Trans(0,0,d_{n+1}) * Trans(a_{n+1},0,0) * Rot(X,\alpha_{n+1})$$

Substituting the parameters from the table, we have results as follows.

$$A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ s\theta_2 & 0 & -c\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} {}^{0}T_3 &= A_1 \cdot A_2 \cdot A_3 \\ &= \begin{bmatrix} c\theta_{12} & 0 & s\theta_{12} & a_1 c\theta_1 + d_3 s\theta_{12} \\ s\theta_{12} & 0 & -c\theta_{12} & a_1 s\theta_1 - d_3 c\theta_{12} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

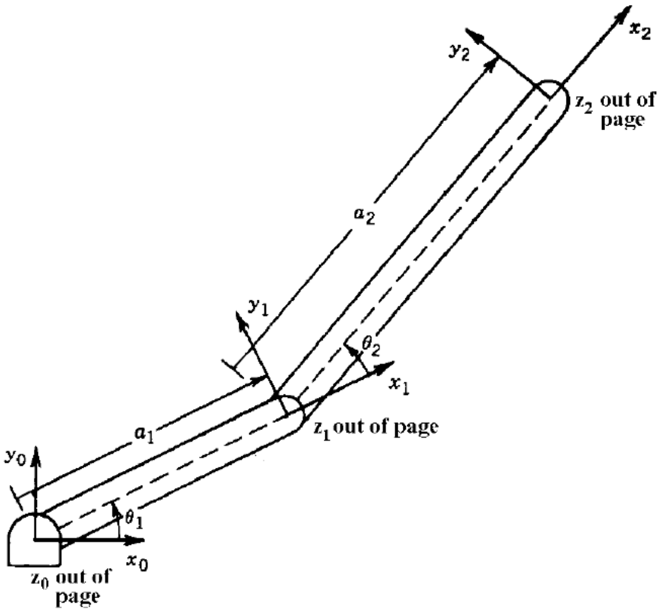
2. (60 points) For the planar RR robot shown in the following figure, if $a_1 = 0.4m$ and $a_2 = 0.3m$:

1) Compute the A matrices and $^{0}T_2$.

2) Compute the Jacobian matrix.

3) Calculate v_x and v_y when $\theta_1 = 35^\circ$, $\theta_2 = 75^\circ$, $\dot{\theta}_1 = 100^\circ / s$, $\dot{\theta}_2 = -50^\circ / s$

Solution:



The parameters are shown in table below:

n+1	θ	d	a	α
1	θ_1	0	a_1	0
2	θ_2	0	a_2	0

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward kinematics solution is

$${}^0T_2 = A_1 * A_2 = \begin{bmatrix} C\theta_{12} & -S\theta_{12} & 0 & a_1 C\theta_1 + a_2 C\theta_{12} \\ S\theta_{12} & C\theta_{12} & 0 & a_1 S\theta_1 + a_2 S\theta_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J(\theta)=\begin{bmatrix} -a_1S\theta_1-a_2S\theta_{12} & -a_2S\theta_{12} \\ a_1C\theta_1+a_2C\theta_{12} & a_2C\theta_{12} \end{bmatrix}=\begin{bmatrix} -0.4\sin(35^0)-0.3\sin(35^\circ-75^\circ) & -0.3\sin(35^\circ-75^\circ) \\ 0.4\cos(35^0)+0.3\cos(35^\circ-75^\circ) & 0.3\cos(35^\circ-75^\circ) \end{bmatrix}$$

$$=\begin{bmatrix} -0.0366 & 0.1928 \\ 0.5575 & 0.2298 \end{bmatrix}m$$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}=J(\theta)\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}=\begin{bmatrix} -a_1S\theta_1-a_2S\theta_{12} & -a_2S\theta_{12} \\ a_1C\theta_1+a_2C\theta_{12} & a_2C\theta_{12} \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}=\begin{bmatrix} -0.0366 & 0.1928 \\ 0.5575 & 0.2298 \end{bmatrix}\begin{bmatrix} 1.7453 \\ -0.8727 \end{bmatrix}=\begin{bmatrix} -0.23 \\ 0.77 \end{bmatrix}m/s$$

So the tool velocities in the world frame are $v_x=-0.23\,m/s$, $v_y=0.77\,m/s$