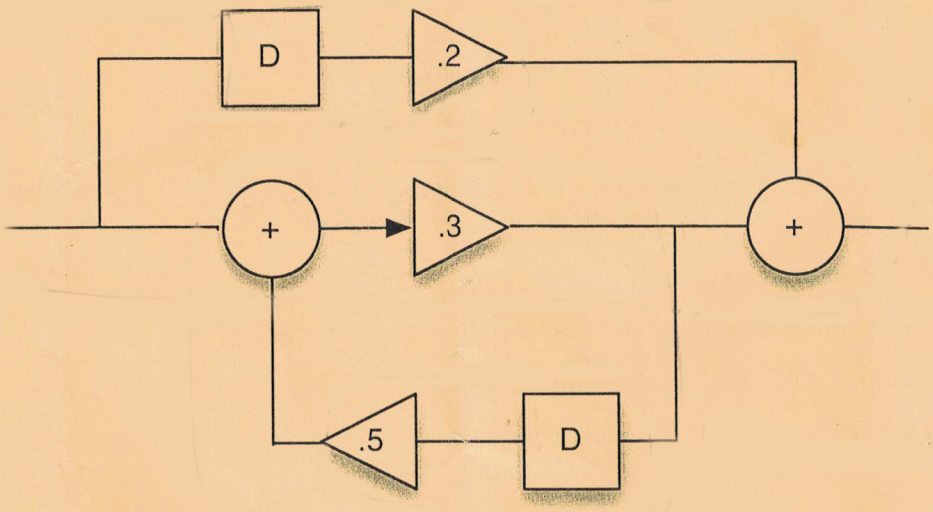
1 Block Diagrams

(*20 points*) Given the following block diagram of a discrete system, what is the Frequency response? (Nicely show your computation).



End of Question 1. Continued

2 Discrete Fourier Transform

(20 points) Given the signal $x(n) = 1 + \cos(\frac{\pi}{2}n)$, compute the DFT for $N = 4$. (You first will have to find ω_0.) To obtain full credits you have to give the four **numbers** X_0, X_1, X_2, X_3 **and** the way you compute them.

End of Question 2. Continued

Name:	ID:

3 Frequency Response

- *(10 points)* Compute the frequency response of

$$\ddot{y} + 2\omega_0\dot{y} + \omega_0^2 y = x$$

- (*10 points*) Given the system $y(n) = x(n) + x(n-1) - y(n-1)$ compute the output $y(n)$ to the input $x(n) = 1 + \cos(\frac{\pi}{2}n + \frac{\pi}{4}) + \sin(\pi n + \frac{3\pi}{2})$

End of Question 3. Continued

4 Impulse Response

(10 points) We designed a system that has the following Frequency response, compute the impulse response:

$$H(\omega) = \begin{cases} 1 & -\frac{\pi}{4} + k2\pi \leq \omega \leq \frac{\pi}{4} + k2\pi \\ 0 & \text{else} \end{cases}$$

Note, this H is 2π periodic (so it belongs to a discrete signal).

End of Question 4. Continued

Name:	ID:

5 DC Detection

(10 points)

We like to know the DC component (remember, DC is the constant offset) of the following signal:

$$x(n) = \delta(n+1) + \delta(n+2)$$

Compute the Fourier Transform and give the value, the result is a number.

End of Question 5, Continued

6 Design a Filter

(*20 points*) Give the difference equation of a causal, minimum order, discrete, real valued system that removes any signal with the frequency of 8'000Hz and passes 4'000 Hz with a gain of 1. The system samples at 16'000 Hz. You have to give the difference equation to get full credit.

Some Formulas

$$X(\omega) = \int_{-\infty}^{\infty} x(\omega) e^{-i\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega$$

$$X_k = \sum_{n=0}^{N-1} x(n) e^{-i\omega_0 nk}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i\omega_0 nk}$$

$$X_k = \frac{1}{p} \sum_{n=0}^{p-1} x(n) e^{-i\omega_0 nk}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{in\omega} d\omega$$

$$X(\omega) = \sum_{n=\infty}^{\infty} x(n) e^{-i\omega n}$$

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{i\omega_0 kt}$$

$$X_k = \frac{1}{p} \int_{0}^{p} x(t) e^{-i\omega_0 kt} dt$$

$$\int_{-\infty}^{\infty} e^{iat} dt = \begin{cases} 2\pi & a = 0 \\ 0 & else \end{cases}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-a) dt = f(a)$$

$$H(\omega) = \frac{H_1(\omega)}{1 - H_1(\omega) H_2(\omega)}$$