Q1

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a) $m_e = 3850\ kg$

$\mathcal{E} = 0.78$

$m_c = 1600\ kg$

$a = uniform = a_c$

$V_\emptyset = 0.35\ m/s$

$V = 3.6\ m/s$

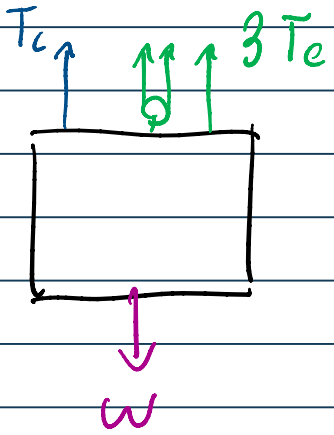
$t = 7\ s$

$V = V_\emptyset + a\ t$

$3.6 = 0.35 + a\ (7)$

$a = 0.464\ m/s^2$

b)



$T_c = ?$

$\uparrow \Sigma F_y = m\ a$

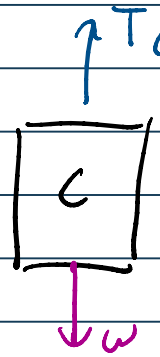
$w = 9.81 \cdot 3850$

$\uparrow \Sigma F_y = ma$ $\qquad w = 9.81 \cdot 3850$

$(3850)(0.464) = T_c + 3T_e - w$

$1786.4 = T_c + 3T_e - 37768.5$

(1) $39554.9 = T_c + 3T_e$



$w = 1600 \cdot 9.81$

$\uparrow \Sigma F_y = ma$

$(1600)(0.464) = T_c - 1600 \cdot 9.81$

$T_c = 16438.4\ N$

$\underline{T_c = 16438\ N}$

C) $T_e = ?$

(1) $39554.9 = T_c + 3T_e$

$39554.9 = 16438.4 + 3T_e$

$$T_c = 7705.5\ N$$

$$\underline{T_c \doteq 7706\ N}$$

d) $P_{AVG} = \vec{F} \cdot \vec{V}_{AVG}$

$$P_{AVG} = 3(7706) \cdot \left(\frac{0.35 + 3.6}{2}\right)$$

$$\underline{P_{AVG} \doteq 45658\ W}$$

$$P_{max} = \vec{F} \cdot \vec{V}_{max}$$

$$P_{max} = 3(7706) \cdot (3.6\ m/s)$$

$$\underline{P_{max} \doteq 83225\ W}$$

e) $P_{electrical} = P_{in}$

$$\mathcal{E} = \frac{P_{out}}{P_{in}} = 0.78$$

$$0.98 = \frac{45658}{(P_{in})_{avg}}$$

$(P_{in})_{avg} \doteq 58536\ W$

$(P_{in})_{max} = \frac{83225}{0.78}$

$(P_{in})_{max} = 106\ 699\ W$

f) $V = \omega r \rightarrow r = 0.18\ m$

$\omega = \frac{V_{max}}{r} = \frac{3.6}{0.18} = 20\ rad/s$

$\omega_{rpm} = \frac{20 \times 60}{2\pi} = 191\ rev/min$

Q2

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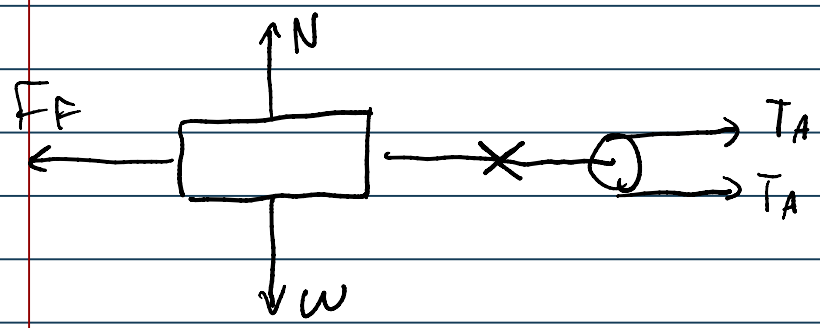
$m = 15\ kg \qquad \bar{T} = 25\ t^{1.5}$

$\mu_s = 0.68 \qquad V(0) = 0$

$\mu_a = 0.32$

a) $\bar{T}_A = ?$



$\Sigma F = 0$

$\Sigma F_y = 0 \rightarrow N = w = (9.81)(15)$

$N = 147.15\ N$

$\Sigma F_x = 0 = 2T_A - F_F \qquad \star F_F = \mu_s N$

$0 = 2T_A - 0.68(147.15)$

$T_A = 50.0\ N$

b) $\emptyset = 2T_A - F_F$

$\emptyset = 50 t_b^{3/2} - 0.68(147.15)$

$t_b = \left(\frac{0.68(147.15)}{50}\right)^{2/3}$

$\underline{t_b = 1.59\ s}$

c) $t = 2.1(1.59) = 3.34\ s$

$\Sigma F_x = ma = 2T_A - F_F \rightarrow F_F = \mu_k N$

$25 \cdot a = 50 t^{3/2} - 0.32(147.15)$

$a = (2t^{3/2} - 1.883)\ m/s^2$

$\frac{d}{dt} a = v \rightarrow V = \int a\, dt$

$V = \int (2t^{3/2} - 1.883)\, dt$

$V = 2\left(\frac{2}{5}\right) t^{5/2} - 1.883t$

$V(3.34) = \frac{4}{5}(3.34)^{5/2} - 1.883(3.34)$

$V(3.34) = 10.02 \text{ m/s}$

d) $V = \frac{ds}{dt}$

$\int V\,dt = s$

$s(t) = \int \left(\frac{4}{5} t^{5/2} - 1.883\, t\right) dt$

$s(t) = \frac{4}{5}\int t^{5/2} - 1.883 \int t\,dt$

$s(t) = \frac{4}{5}\left(\frac{2}{7}\right) t^{7/2} - \frac{1.883}{2} t^2$

$s(t) = \frac{8}{35} t^{7/2} - 0.9415\, t^2$

$s(3.34) = \frac{8}{35}(3.34)^{7/2} - 0.9415(3.34)^2$

$s(3.34) = 5.06 \text{ m}$

Q3

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$m = 125\ kg$
$k = 5250\ N/m$
$c = 1400\ Ns/m$

a) $\omega_d = ?$ & $f_d = ?$

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

$$\omega_d = \sqrt{\frac{2 \cdot 5250}{125} - \left(\frac{1400}{2 \cdot 125}\right)^2}$$

$$\omega_d = 7.26\ rad/s$$

$$f_d = \frac{\omega_d}{2\pi} = \frac{7.26}{2\pi}$$

$$f_d = 1.15\ Hz$$

b) motion in terms of time

position $\rightarrow y = D\left[e^{-(c/2m)t} \sin(\omega_d t + \phi)\right]$

velocity $\rightarrow \dot{y} = D\left[e^{-(c/2m)t} \omega_d \cos(\omega_d t + \phi) - \frac{c e^{-(c/2m)t} \sin(\omega_d t + \phi)}{2m}\right]$

accel $\rightarrow \ddot{y} = D\left[c^2 e^{-(c/2m)t} \sin(\omega_d t + \phi) - 4c\omega_d m e^{-(c/2m)t} \cos(\omega_d t + \phi) - 4m^2 \omega_d^2 e^{-(c/2m)t} \sin(\omega t + \phi)\right] / (4m^2)$

Initial conditions: $\dot{y}(0) = 0$, $y(0) = 0.08$

$$y(\emptyset) = 0.08 = D\left[e^{-(1400/(2\cdot 125))\emptyset} \sin(7.26(\emptyset) + \phi)\right]$$

(1) $0.08 = D\sin(\phi)$

$$\dot{y}(\emptyset) = \dot{\emptyset} = D\left[e^{-(1400/(2\cdot 125))\emptyset}(7.26)\cos(7.26\cdot\emptyset + \phi) - \frac{(1400)\, e^{-(1400/(2\cdot 125))\emptyset} \sin(7.26\cdot\emptyset + \phi)}{2\cdot 125}\right]$$

$$\frac{\dot{\emptyset}}{D} = \frac{D(7.26\cos(\phi) - 5.6\sin(\phi))}{D} \quad (2)$$

(3) $\frac{0.08}{D} = \sin(\phi)$ ★ $D \neq \emptyset$

(4) $\dot{\emptyset} = 7.26\cos(\phi) - 5.6\sin(\phi)$

$\dot{\emptyset} = 7.26\sin(\phi + \frac{\pi}{2}) - 5.6\sin\phi$

Solve (4) for ϕ

$\phi = 0.91377$

$\phi = 0.914$ radians

$D = \frac{0.08}{\sin(0.914)} = 0.101$

$$y = 0.101\left[e^{-5.6t}\sin(7.26t + 0.914)\right]$$

$$\dot{y} = 0.101\left[e^{-5.6t}\cdot 7.26\cdot\cos(7.26t + 0.914)\right.$$

$$\left. - 1400\, e^{-5.6t}\sin(7.26t + 0.914)\right]$$

$$\ddot{y} = 0.101\left[(1.96\times10^{6})e^{-5.6t}\sin(7.26t + 0.914)\right.$$

$$-(5.98\times10^{6})e^{-5.6t}\cos(7.26t + 0.914)$$

$$\left.-(3.29\times10^{6})e^{-5.6}\sin(7.26t + 0.914)\right]/(6.25\times10^{4})$$

c) no vibration = overdamped or critically damped

$$c \geq c_c$$

$$c \geq 2m\omega_n$$

$$c \geq 2(125)\sqrt{2.5250/125}$$

$$\underline{c \geq 2291.3\ \text{Ns/m}}$$

Q4

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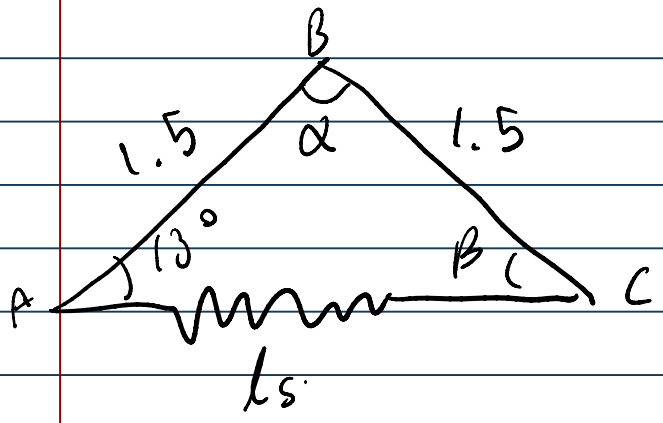


$$\frac{\sin(13)}{1.5} = \frac{\sin(\beta)}{1.5}$$

$$\beta = 13°$$

$$\angle\alpha = 180 - \beta - \theta$$

$$\angle\alpha = 154°$$

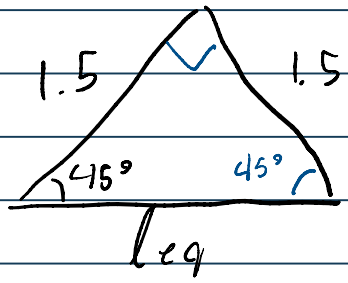
$$\frac{\sin(154°)}{l_s} = \frac{\sin(14°)}{1.5}$$

$$l_s = \frac{1.5\sin(154°)}{\sin(14°)}$$

$$l_s = 2.72\ m$$

$l_s = 2.72\ m$

b) $V_e = \frac{1}{2} k s^2$



$s = l_s - l_{equilibrium}$

$s = 2.72 - \sqrt{1.5^2 + 1.5^2}$

$s = 0.599\ m$

$V_e = \frac{1}{2}\left(5 \frac{N}{m}\right)(0.599)^2$

$V_e = 0.897\ J$

Q5

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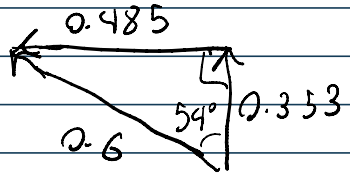
$\ell_D = 1.1\,m \quad a_C = 8.0\,m/s^2 \uparrow$

$\ell_B = 0.6\,m \quad \theta = 54°$

$V_C = 4.5\,m/s \downarrow \quad \phi = 27°$

a)

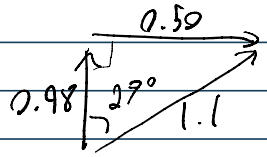


general plane motion

$V_B = \omega_{AB} \times r_{AB}$

$V_B = \omega_{AB}\hat{k} \times (-0.485\hat{i} + 0.353\hat{j})$

(1) $V_B = -0.485\,\omega_{AB}\hat{j} - 0.353\,\omega_{AB}\hat{i}$



$V_{B/C} = \omega_{BC} \times r_{BC}$

$V_{B/C} = \omega_{BC}\hat{k} \times (0.5\hat{i} + 0.98\hat{j})$

$V_{B/C} = 0.50\,\omega_{BC}\hat{j} - 0.98\,\omega_{BC}\hat{i}$

$V_B = V_C + V_{B/C}$

(2) $V_B = -4.5\hat{j} + 0.50\,\omega_{BC}\hat{j} - 0.98\,\omega_{BC}\hat{i}$

solve (1) & (2) by components

(3) $y: -4.5 + 0.50\omega_{BC} = -0.485\omega_{AB}$

$x: -0.98\omega_{BC} = -0.853\omega_{AB}$

(4) $\omega_{BC} = 0.360\omega_{AB}$

sub (4) into (3) & solve

$-4.5 + 0.50(0.36\omega_{AB}) = -0.485\omega_{AB}$

$-4.5 = -0.665\omega_{AB}$

$\underline{\omega_{AB} = (6.77\hat{k})\ rad/s}$

sub ω_{AB} into (4)

$\omega_{BC} = 0.364(6.77)$

$\underline{\omega_{BC} = (2.46\hat{k})\ rad/s}$

b) $a_B = a_C + \alpha_{BC} \times r_{BC} - \omega_{BC}^2 \cdot r_{BC}$

$a_B = \alpha_{AB} \times r_{B/A} - \omega_{AB}^2 r_{B/A}$

$a_B = \alpha_{AB} \times (-0.485\hat{i} + 0.353\hat{j}) - (6.77)^2 (-0.485\hat{i} + 0.353\hat{j})$

(1) $a_B = -0.485\hat{j}\,\alpha_{AB} - 0.353\hat{i}\,\alpha_{AB} - 22.2\hat{i} + 16.18\hat{j}$

$a_B = 8.0\hat{j} + \alpha_{BC}(0.5\hat{i} + 0.98\hat{j}) + (2.46)^2(0.5\hat{i} + 0.98\hat{j})$

$$a_B = 8.0\hat{j} + \alpha_{BC}(0.5\hat{i} + 0.98\hat{j}) + (2.46)^2(0.5\hat{i} + 0.98\hat{j})$$

(2) $$a_B = 8.0\hat{j} + 0.5\hat{j}\alpha_{BC} - 0.98\hat{i}\alpha_{BC} + 3.03\hat{i} + 5.93\hat{j}$$

$$x: -0.353\alpha_{AB} - 22.2 = -0.98\alpha_{BC} + 3.03$$

$$\alpha_{AB} = \frac{25.23 - 0.98\alpha_{BC}}{-0.353}$$

(3) $$\alpha_{AB} = -71.47 + 2.78\alpha_{BC}$$

$$j: 8.0 + 0.5\alpha_{BC} + 5.93 = -0.485\alpha_{AB} + 16.18$$

$$0.5\alpha_{BC} = 2.25 - 0.485(-71.47 + 2.78\alpha_{BC})$$

$$0.5\alpha_{BC} = 2.25 + 34.66 - 1.35\alpha_{BC}$$

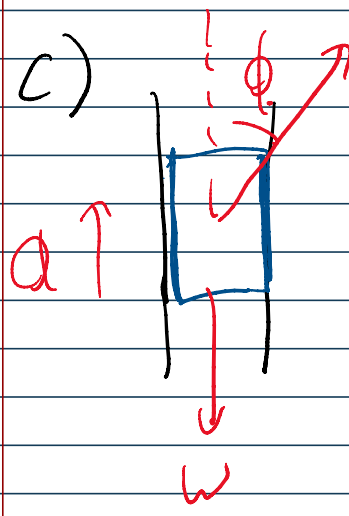
$$1.85\alpha_{BC} = 36.91$$

$$\alpha_{BC} = 19.95 \text{ rad/s}^2$$

$$\alpha_{AB} = -71.7 + 2.78(19.95)$$

$$\alpha_{AB} = -16.23 \text{ rad/s}^2$$

c)



$\Sigma F = ma$

$+\uparrow \Sigma F_y = (9kg)(8 m/s)$

$\Sigma F_y = 72\ N$

$72 = -w + F_{BC}\cos\phi$

$72 = (-9.81)(9) + F_{BC}\cos\phi$

$160.29 = F_{BC}\cos(27°)$

$|F_{BC}| = 180.0\ N$

$F_{BC} = 180.0N\ [63° up\ from\ +x]$

$\vec{F}_{BC} = \langle 180\sin(27)\hat{i} + 180\cos(27)\hat{j} \rangle$

$\vec{F}_{BC} = \langle 81.72\hat{i} + 160.4\hat{j} \rangle$

Q6

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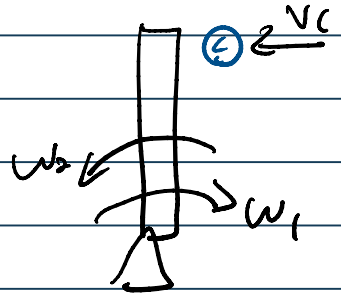

$L = 2.5\,m$

$m = 15\,kg$

$(V_C)_1 = 4.6\,m/s \leftarrow$

$\omega_1 = 1.4\,rad/s$ ↻

a) elastic, no energy lost

$e = 1$

$$e = \frac{(V_C)_2 - (V_{AB})_2}{(V_{AB})_1 - (V_C)_1} = 1$$

$(V_{AB})_1 = \omega_1 L$

$(V_C)_2 - (V_{AB})_2 = (2.5)(1.4) + 4.6\,m/s$

(1) $(V_C)_2 - (V_{AB})_2 = 8.1 \rightarrow (V_C)_2 = 8.1 + (V_{AB})_2$

$m_{AB}(V_{AB})_1 + m_C(V_C)_1 = m_{AB}(V_{AB})_2 + m_C(V_C)_2$

$(15)(2.5 \cdot 1.4) + (30)(4.6) = (15)(V_{AB})_2 + 30(V_C)_2$

$190.5 = 15(V_{AB})_2 + 30(8.1 + (V_{AB})_2)$

$190.5 = 15(V_{AB})_2 + 243 + 30(V_{AB})_2$

$$-52.5 = 45(V_{AB})_2$$

$$(V_{AB})_2 = -1.167 \text{ m/s}$$

$$(\omega_{AB})_2 = \frac{V}{r} = \frac{-1.167}{2.5} = -0.4668$$

$$\underline{(\omega_{AB})_2 = 0.467 \text{ rad/s CCW}}$$

$$(V_C)_2 = 8.1 + (V_{AB})_2$$

$$\underline{(V_C)_2 = 6.93 \text{ m/s} \rightarrow}$$

energy Eqt

$$E = \left[\frac{1}{2} m_{AB} (V_{AB})_1^2 + \frac{1}{2} m_C (V_C)_1^2\right]$$

$$- \left[\frac{1}{2} m_{AB} (V_{AB})_2^2 + \frac{1}{2} m_C (V_C)_2^2\right]$$

$$\Delta E = \frac{1}{2}(15)(1.167)^2 + \frac{1}{2}(10)(1.1)^2$$

$$\Delta E = \frac{1}{2}(15)(1.4 \cdot 2.5)^2 + \frac{1}{2}(30)(4.6)^2$$

$$-\left(\frac{1}{2}(15)(0.467 \cdot 2.5)^2 + \frac{1}{2}(30)(6.93)^2\right)$$

$$\Delta E = -321.3$$

$$E_{lost} = 321.3 \text{ J}$$