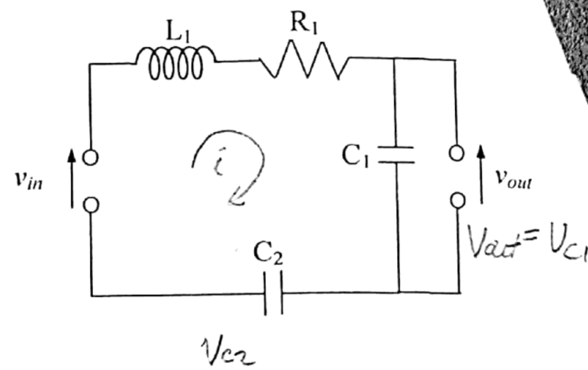
Midterm 3 2019 Dr Bone

Mechatronics (McMaster University)

5 1. For the electrical circuit shown, derive the Laplace transfer function $\frac{V_{out}(s)}{V_{in}(s)}$.



Mesh Analysis

$$V_{in} = L_1 \frac{di}{dt} + iR_1 + \frac{1}{C_1}\int i + \frac{1}{C_2}\int i$$

convert to Laplace

$$V_{in}(s) = L_1 i s + iR_1 + \frac{i}{C_1 s} + \frac{i}{C_2 s} \quad (1)$$

Also know:

$$V_{out} = V_{C1} = \frac{1}{C_1}\int i$$

Laplace

$$V_{out}(s) = \frac{i}{C_1 s} \Rightarrow i = V_{out}(s) C_1 s \quad (2)$$

Sub (2) into (1) + rearrange to get transfer function

$$V_{in}(s) = i\left(L_1 s + R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right)$$

$$V_{in}(s) = V_{out}(s)\, C_1 s\left(L_1 s + R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right)$$

$$V_{in}(s) = V_{out}(s)\left(C_1 L_1 s^2 + C_1 R_1 s + 1 + \frac{C_1 s}{C_2 s}\right)$$

$$V_{in}(s) = V_{out}(s)\left(\frac{C_1 C_2 L_1 s^3 + C_1 C_2 R_1 s^2 + C_2 s + C_1 s}{C_2 s}\right)$$

$$\boxed{\therefore \frac{V_{out}(s)}{V_{in}(s)} = \frac{C_2}{C_1 C_2 L_1 s^2 + C_1 C_2 R_1 s + (C_1 + C_2)}}$$

Please note: The sample answer to #2 given below was chosen since it presents the solution in a very clear and organized way. However, after taking the Laplace transform the variable names should have been changed to uppercase. For example $\mathscr{L}(q_{out}(t)) = Q_{out}(s)$. We did not deduct any marks for this error.

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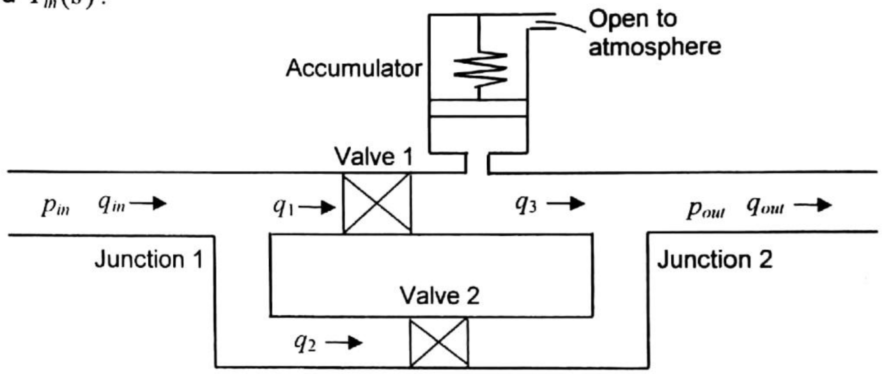
2. The circuit diagram for a hydraulic system is given below. It consists of two junctions, two valves and one accumulator. The valves are different sizes. The pressures p_{in} and p_{out} are larger than atmospheric pressure. All pipes are short.

(a) Write the governing equations for the junctions, valves and accumulator in terms of the variables shown on the diagram.

(b) Starting with your answer to (a), use Laplace transforms to solve for $P_{out}(s)$ as a function of $Q_{out}(s)$ and $P_{in}(s)$.



a) Junctions: $q_{in} = q_1 + q_2 \qquad q_{out} = q_2 + q_3$

Valves: $p_{in} - p_{out} = R_1 q_1 \qquad p_{in} - p_{out} = R_2 q_2$

Accumulator: $q_1 - q_3 = C\left(\dfrac{d(p_{out} - p_{atm})}{dt}\right) = C\dfrac{dp_{out}}{dt}$

b) $q_1 - q_3 = Cs\,p_{out}$, $\quad p_{out} = (q_1 - q_3)\dfrac{1}{Cs}$

$q_1 = (p_{in} - p_{out})\dfrac{1}{R_1}$

$q_3 = q_{out} - q_2$, $\quad q_2 = \dfrac{p_{in} - p_{out}}{R_2}$

$$q_3 = q_{out} - \frac{(p_{in} - p_{out})}{R_2}$$

$$= \frac{R_2 q_{out} - p_{in} + p_{out}}{R_2}$$

Answer to question 2 continued

$$p_{out} = \left[\frac{(p_{in} - p_{out})}{R_1} - \frac{(R_2 q_{out} - p_{in} + p_{out})}{R_2} \right] \frac{1}{C_s}$$

$$p_{out} = \left[\frac{R_2(p_{in} - p_{out}) - R_1(R_2 q_{out} - p_{in} + p_{out})}{R_1 R_2} \right] \frac{1}{C_s}$$

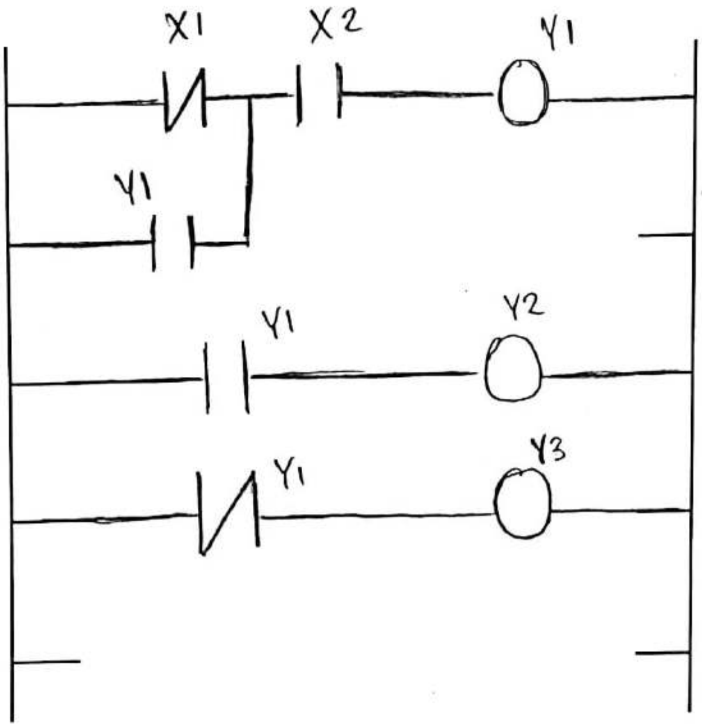
$$p_{out} = \frac{R_2 p_{in} - R_2 p_{out} - R_1 R_2 q_{out} + R_1 p_{in} - R_1 p_{out}}{R_1 R_2 C_s}$$

$$R_1 R_2 C_s p_{out} = R_2 p_{in} - R_2 p_{out} - R_1 R_2 q_{out} + R_1 p_{in} - R_1 p_{out}$$

$$R_1 R_2 C_s p_{out} + R_2 p_{out} + R_1 p_{out} = R_2 p_{in} - R_1 R_2 q_{out} + R_1 p_{in}$$

$$\boxed{p_{out} = \frac{(R_1 + R_2) p_{in} - R_1 R_2 q_{out}}{(R_1 R_2 C_s + R_1 + R_2)}}$$

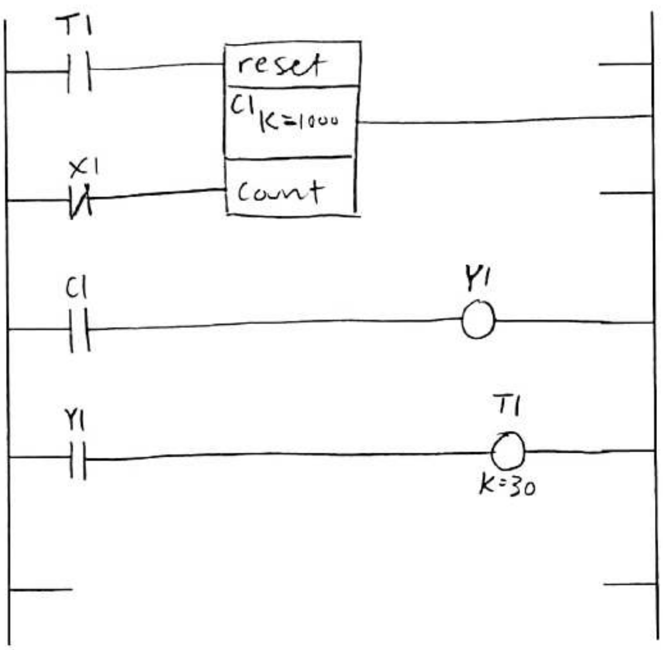
3. A motor and two lights are to be controlled using a PLC. Pressing the start button should turn on the motor. The motor should remain on after the button is no longer being pressed. The motor should be turned off (or not turn on initially) whenever the stop button is pressed. Both switches are normally closed momentary pushbutton switches. The green light should be on when the motor is running. The red light should be on when the motor is stopped. **Using the signals defined in the table below, and the instructions taught in this course, write the required ladder logic program in the space provided below.**



Signal	Description
X1	Normally closed start switch
X2	Normally closed stop switch
Y1	Motor
Y2	Green light
Y3	Red light

The answer given below is the most common correct answer to #4 (other correct answers exist):

9 **4.** A machine produces parts. Where the parts come out, it is equipped with a thru-beam proximity sensor. When a part is in front of the sensor it outputs an off signal, otherwise it outputs an on signal. For quality control, every 1000th part should be ejected from the machine using a solenoid actuator (i.e., eject 1000th part, 2000th part, 3000th part, etc.). The solenoid actuator is at the same location as the proximity sensor. The solenoid actuator requires a 0.3 s on signal to eject a part. Assume that the time gap between parts is greater than 0.3 s. The PLC timer resolution equals 0.01 s. **Using the signals defined in the table below, and the instructions taught in this course, write the required ladder logic program in the space provided below.**



Signal	Description
X1	Proximity sensor
Y1	Solenoid
T1	Timer for solenoid
C1	Counter