Name:	ID:

McMaster University
Final Examination 2020, Term I
SFWRENG/ MECHTRON 3MX3
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Date: December 10, 2020

Duration: 120 Minutes **(There is no extra time to scan or submit!)**

This examination includes 3 pages and 8 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

WATCH YOUR TIME, you need to scan and submit in time !!!

- Make sure you have the time to scan your answers and submit them to Avenue to learn.
- Please keep answers brief and clean.
- Try to allocate your time sensibly, and don't "get stuck" on any one problem.
- By signing and submitting your submission you confirm that you understand what constitutes academic dishonesty and that you solved this exam by yourself.
- For example chatting with others during the exam about the exam is academic dishonesty. And if you chat you will most likely run out of time.

1	20	
2	10	
3	10	
4	20	
5	10	
6	10	
7	10	
8	10	
Mark	100	

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Signature

You are required to sign your submission to confirm you understand what constitutes academic dishonesty and that this submission is your own work.

1 Bandpass Filter Construction

(20 points) This time we construct a bandpass filter by putting a low-pass and a high-pass filter into series. Placing a zero at $\frac{F_s}{2}$ and normalizing at DC results in a simple low-pass filter $H_1(\omega)$ Placing a zero at DC normalizing at $\frac{F_s}{2}$ results in a simple high-pass filter $H_2(\omega)$ We construct a new H putting H_1 and H_2 into series.

What is the gain for the new H at DC, $\frac{F_s}{4}$ and $\frac{F_s}{2}$ of this filter $H(\omega)$? Make a table so we easy find our answers !

2 Frequency Content

(10 points)

Compute the frequency content of a discrete square wave signal $x(n)$ with a period of 4. (so the signal goes $\cdots, 1, 1, -1, -1, 1, 1, -1, -1, \cdots$.

3 Impulse Response/Frequency Response/Stability

(10 points, 5 points each)

1. Show, for a FIR system the frequency response is always defined (meaning it is finite).
2. We know that $|h(n)| < \alpha$. Does this mean the system is stable? Justify your answer by either showing it is stable or by giving bound input $x(n)$ that leads to an unstable response.

4 State Space Equation

(20 points) Give the state space equations (A, B, C, D) for the system that has the frequency response

$$H(\omega) = 1 - e^{-\omega} + e^{-2\omega}$$

5 Filtering

(10 points, 2 each) Given the frequency response

$$H(\omega) = \sin(2\omega)$$

of a discrete system. Compute the output to the input (just giving numbers results in no points)

1. $x(n) = \cos(\frac{\pi}{4}n)$
2. $x(n) = 2$
3. $x(n) = \cos(\frac{3\pi}{4}n)$
4. $x(n) = \cos(\pi n)$
5. $x(n) = 0$

6 Fourier Transform Based Filter

(10 points) In a digital system we can filter signals by first computing the FT then multiplying with the filter gains that correspond to the frequency bin and then transforming back. For simplicity we pick $N = 4$ and the signal is $x(0) = 1, x(1) = 0, x(2) = 1, x(3) = 0$, the filter is given by $H(\omega) = 1 - e^{-i2\omega}$. Note, this is all numbers, a forward FT, multiply, backward FT resulting in $y(0), y(1), y(2), y(3)$!

7 Short Questions

(10 points, 2 each) For each of the following frequency responses find the corresponding impulse response **or** system. (some are discrete some are continuous !)

1. $G(s) = s$
2. $G(z) = z$
3. $H(\omega) = \delta(\omega - \omega_c) + \delta(\omega + \omega_c)$
4. $G(s) = \frac{s}{s+1}$
5. $G(z) = 1 + \frac{1}{1+z^{-1}}$

8 Stability

(10 points, 5 each) For the two systems below determine α, β so they are stable:

1. $y' + \alpha y + x' + x = 0$
2. $y(n+1) = x(n) - \beta y(n-1) + x(n-1)$

THE END