



Exam 2020, questions and answers

Mechatronics (McMaster University)

- 8 1. A straight calibration line was fit to temperature sensor data. The slope of the calibration line was found to be $0.05 \text{ V}/^\circ\text{C}$. The standard deviation of the sensor output was not affected by the input, and equalled 0.025 V . The input and the mean values of the calibrated sensor output for a series of tests are listed in the table below. Using the given information, determine values for the following performance specifications:

a) sensitivity, b) repeatability, c) hysteresis, d) linearity, and e) accuracy.

Test number	Temperature Input ($^\circ\text{C}$)	Mean Calibrated Output ($^\circ\text{C}$)
1	0	0
2	10	8
3	25	23
4	50	48
5	75	73
6	100	98
7	75	77
8	50	53
9	25	27
10	10	11
11	0	0

a) Sensitivity = slope = $0.05 \text{ V}/^\circ\text{C}$

b) Repeatability = $\pm 3 \sigma_y$
 $= \pm 3 \frac{\sigma_{\text{volts}}}{\text{slope}}$
 $= \pm \frac{3(0.025)}{0.05} = \pm 1.5^\circ\text{C}$

c) hysteresis =
 max | input increasing - input decreasing |
 $= |48 - 53| = 5^\circ\text{C}$

d) linearity = $\pm \max | Y_{\text{sensor}} - Y_{\text{actual}} |$
 $= \pm |53 - 50| = \pm 3^\circ\text{C}$

e) accuracy = $\pm (\max | Y_{\text{actual}} - Y_{\text{sensor}} | + 3 \sigma_y)$
 $= \pm \left(|53 - 50| + 3 \frac{\sigma_{\text{volts}}}{\text{slope}} \right)$
 $= \pm \left(3 + \frac{3(0.025)}{0.05} \right)$
 $= \pm 4.5^\circ\text{C}$

- 8 2. A force sensor has a sensitivity of 50 mV/N, a range of ± 150 N and an accuracy of $\pm 0.2\%$ of full scale.

- a) Assuming other sources of error in the measurement system are insignificant, if the input is 60 N, what is the worst case measurement error?
 b) The sensor's output impedance is 1 k Ω . It is connected to an ADC with a 50 k Ω input impedance. Repeat part (a) including the effect of these impedances.
 c) The ADC has a 12-bit resolution with 10.8 effective bits. Its input range is ± 10 V. Repeat part (b) including this source of error.

a)

$$\Delta F = a_{\text{sensor}} = \pm 0.2\% \text{ of full scale} = \pm 0.2\% (150 - (-150)) \\ = \pm 0.2\% (300) = \pm 0.6 \text{ N}$$

b) $V_s = (50 \frac{\text{mV}}{\text{N}}) (60 \text{ N}) = 3000 \text{ mV} = 3 \text{ V}$

$$V_{in} = V_s \left(\frac{R_{in}}{R_{in} + R_s} \right) = (3000) \left(\frac{50}{50 + 1} \right) = 2941 \text{ mV}$$

$$\Delta V = 3000 - 2941 = 59 \text{ mV}$$

$$\Delta F = |F_{out}^*| \left(\left| \frac{a_{\text{sensor}}}{F_{in}} \right| + \left| \frac{\Delta V}{V_s} \right| \right) \\ = (60 \text{ N}) \left(\left| \frac{\pm 0.6 \text{ N}}{60 \text{ N}} \right| + \left| \frac{59}{3000} \right| \right) \\ = \pm 1.78 \text{ N}$$

c) $a_{\text{ADC}} = \pm \frac{V_{FS}}{2^{\text{ENOB}}} = \pm \frac{(10 - (-10))}{2^{10.8}} = \pm 0.0112 \text{ V}$

$$\Delta F = |F_{out}^*| \left(\left| \frac{a_{\text{sensor}}}{F_{in}} \right| + \left| \frac{\Delta V}{V_s} \right| + \left| \frac{a_{\text{ADC}}}{V_{in}} \right| \right) \\ = (60 \text{ N}) \left(\left| \frac{\pm 0.6 \text{ N}}{60 \text{ N}} \right| + \left| \frac{59 \text{ mV}}{3000 \text{ mV}} \right| + \left| \frac{\pm 0.0112}{3} \right| \right) \\ = \pm 2.00 \text{ N}$$

- 6 3. A measurement system consisting of an accelerometer and signal conditioner has a range of $\pm 500 \text{ m/s}^2$, an accuracy of $\pm 0.15\%$ of full scale and a bandwidth of $3 \times 10^4 \text{ rad/s}$.

a) If the current output is 100 m/s^2 and the true acceleration suddenly changes to a new value, how long should the mechatronic system wait before taking its next reading?

b) If the measured acceleration is a $9 \times 10^3 \text{ rad/s}$ sinusoid with a 200 m/s^2 amplitude, what is the worst case error in the measured amplitude?

$$a) \tau_s = \frac{1}{\omega_b} = \frac{1}{3 \times 10^4 \text{ rad/s}} = 3.33 \times 10^{-5} \text{ s}$$

$$|a_y| = |(\pm 0.15\%)(500 - (-500))| = 1.5 \text{ m/s}^2$$

$$y_{\max} = 500 \text{ m/s}^2 \text{ and } y_{\min} = -500 \text{ m/s}^2$$

$$y_{\text{out}}(0) = 100 \text{ m/s}^2$$

$$t \geq -\tau_s \ln \left[\frac{0.1 |a_y|}{\max(y_{\max} - y_{\text{out}}(0), y_{\text{out}}(0) - y_{\min})} \right]$$

$$t \geq -3.33 \times 10^{-5} \ln \left[\frac{(0.1)(1.50)}{\max(500 - 100, 100 - (-500))} \right]$$

$$\therefore t \geq 2.76 \times 10^{-4} \text{ s}$$

$$b) \Delta A_{\text{out}}(\omega) = |a_y| + (\sqrt{1 + \omega^2 \tau_s^2} - 1) A_{\text{out}}(\omega)$$

$$= 1.5 + (\sqrt{1 + (9 \times 10^3)^2 (3.33 \times 10^{-5})^2} - 1) 200$$

$$= \pm 10.3 \text{ m/s}^2$$

- 6 4. A force control system will estimate the derivative by computing the backward difference of the measured force. The force measurement has a repeatability of ± 4 N. The maximum second derivative equals 600 N/s^2 .

The worst case error of the estimated derivative must be less than or equal to 180 N/s . Find the smallest and largest values of sampling period that can be used.

Hint: The answer does not involve the optimal sampling period.

Define $P = \text{measured force}$
 $V = 1^{\text{st}} \text{ derivative of force}$
 $a = 2^{\text{nd}} \text{ derivative of force}$

$$\pm 3\sigma_P = \pm 4 \text{ N}$$

$$\sigma_P = \frac{4}{3} = 1.33 \text{ N}$$

$$\Delta V_{\text{est}} = \frac{T}{2} \max(|a_{\text{true}}|) + \frac{6\sigma_P}{T}$$

$$180 = \frac{T}{2} (600) + \frac{6(1.33)}{T}$$

$$\therefore \left(\frac{600}{2}\right)T^2 - 180T + 8 = 0$$

Solution is $T = 0.048 \text{ s}$ and $T = 0.552 \text{ s}$

- 12 **5. Based on the material covered in this course**, answer the following questions in the spaces provided:

a) A flow nozzle is a simple, inexpensive sensor for measuring flow rate.

b) Excluding Hall effect sensors, a inductive proximity sensor is the least sensitive to dirt.

c) An ultrasonic sensor measures depth using the time-of-flight method.

d) List one advantage and one disadvantage of a thermistor:

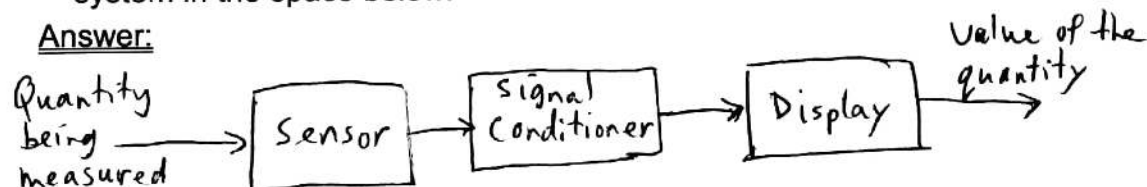
Advantage: large sensitivity (OR very small OR easily formed into different shapes)

Disadvantage: non-linear response

e) A seismic mass is used with a acceleration sensor. Decreasing the seismic mass decreases the sensor's sensitivity specification and increases the sensor's bandwidth (or upper frequency limit) specification.

f) The concurrent engineering approach (that came before mechatronics engineering) integrated design and manufacturing

g) Draw a block diagram showing the three elements of a typical measurement system in the space below.



h) Label the three frequencies identified by the dashed lines on the plot below.

