Problem 1 (10 points) Suppose you enter two numbers x and y from the keyboard on your computer, store them in double precision variables, and compute `x*y-(x-y)`. Assuming that this expression is evaluated in double precision (with rounding to the nearest), calculate a bound for the error in the computed result. It is sufficient to give a good approximation for this bound.

Problem 2 (5 points) Describe an approach for computing $\sin(x)/(x - \sqrt{x^2 - 1})$ such that loss of significance is avoided.

Problem 3 (5 points) Calculate the first two intervals generated by the bisection method when applied to $f(x) = x^3 - x^2 + 1$ with an initial interval $[-1, 1]$.

Problem 4 (15 points) Suppose that r is a double root of $f(x)$, $f \in R \to R$. That is $f(r) = f'(r) = 0$. Suppose f, f', f'' are continuous in a neighbourhood of r.

Assume that you apply Newton' s method to find this root of f. Denoting $e_n = r - x_n$, show that

$$e_{n+1} \approx \frac{1}{2} e_n$$

near r. Therefore, Newton's method is linear convergent near a double root.

Problem 5 (10 points) Solve the system

$$\begin{bmatrix} 2 & 3 & 0 \\ -1 & 2 & -1 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 9 \end{bmatrix} \tag{1}$$

using Gaussian elimination with scaled partial pivoting. In your calculations, you can carry four digits after the decimal point, and a nonzero digit before the decimal point.

Problem 6 (10 points) You are given a nonsingular $n \times n$ matrix A. Describe a $O(n^3)$ method for computing A^{-1}.

You do not have to write a program. Giving the steps in terms of formulas and explanations is sufficient.

Problem 7 (10 points) What straight line best fits the data

x	1	2	3	4
y	0	1	1	2

in the least squares sense.

THE END