Name: ....
Student Number: ....

Software Engineering/Mechatronics 3DX4

DAY CLASS — Dr. Mark Lawford
DURATION OF EXAMINATION: 2.5 Hours
McMaster University Final Examination — April 2018

THIS EXAMINATION PAPER INCLUDES 5 PAGES AND 5 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special Instructions: The use of two double sided $8\frac{1}{2}'' \times 11''$ pages of notes is permitted during this exam. You may use the McMaster Standard (Casio FX-991 MS or MS+) calculator. Answer all questions in the provided answer booklets. Fill in your name and student number and sign each booklet you use. This paper must be returned with your answers, with your name and student number filled in.

Make sure you show all steps when answering questions. Just writing down the final answer will not get many marks. A table of some useful Laplace and Z transforms can be found on the last page.

The following figure (Fig. 1) is the basis of many of the questions.

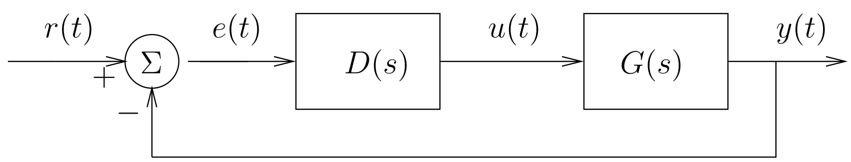
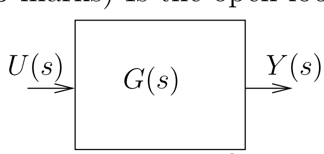


Figure 1: Closed loop system model

Here we can consider $G(s)$ as physical plant we want to control and $D(s)$ is the controller. In each question we will provide the transfer function for $G(S)$ and state or ask what type of control should be used for $D(s)$.

1. **Stability** (25 marks)

   a) (5 marks) Is the open loop plant

   where

   $$G(s) = \frac{s+3}{s^4 + 23s^3 + 182s^2 + 580s + 600}$$

   stable or unstable? Justify your answer.

   b) (5 marks) Compute the closed loop transfer function of the system in Fig. 1 in the case when

   $$D(s) = K \quad \text{and} \quad G(s) = \frac{s+3}{s^4 + 23s^3 + 182s^2 + 580s + 600}$$

c) (10 marks) For what values of K is the closed loop system stable?

d) (5 marks) Calculate the $j\omega$-axis crossings (frequency). From your answer to (1c) you know the value of K where this occurs.

2. **Root Locus, Time Response and Controller Design** (25 marks)

Consider Fig. 1 in the case when:

$$D(s) = K \text{ and } G(s) = \frac{s+3}{(s+2)(s+5)(s+6)(s+10)}$$

a) (5 marks) Sketch the root locus for the control system. You do not need to calculate the exact breakaway/breakin points.

b) (5 marks) For $K = 430$ the system is stable and the closed loop poles are approximately located at

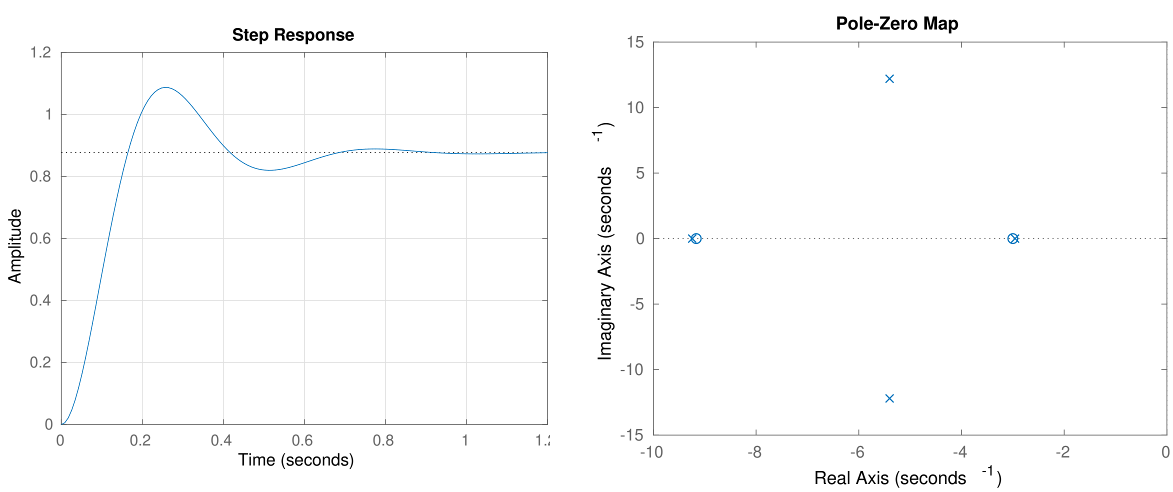
$$s = -2.7 \pm j6.1, -2.9, -14.7$$

Assuming that the closed loop pole at $s = -2.9$ cancels the system zero at $s = -3$, what is the damping ratio of the dominant poles? Assuming ideal 2nd order behaviour, what do you expect the resulting percent overshoot to be for the system?

c) (5 marks) Your manager said that she is happy with % overshoot of your controller design for $K = 430$, but she would like you to design a PD compensator to cut the settling time in half while keeping the same % overshoot. Assuming ideal 2nd order behaviour, what would be the new location for the dominant poles?

d) (5 marks) Design a PD compensator to place the dominant poles at the location you computed in part (2c).

e) (5 marks) The actual step response of the compensated closed loop system (i.e. $D(s) = K(s + z_c)$) is shown below on the left and the closed loop pole zero map is shown on the right.



Does the percent overshoot and settling time appear to correspond to the ideal second order behaviour your manager wanted for the compensated system? Does the closed loop pole zero map indicate that this should be the case?

3. **Steady State Error** (15 marks)

   Assume that

$$G(s) = \frac{s+3}{s^4 + 23s^3 + 182s^2 + 580s + 600} = \frac{s+3}{(s+2)(s+5)(s+6)(s+10)}$$

   in Fig.1.

   a) (5 marks) Assuming the feedback configuration shown in Fig. 1 and $D(s) = K$ is chosen so that the closed loop system is stable, what is the type of the system? What is its static error constant for a given gain K? What is the steady state error of the closed loop system in response to a unit step input?

   b) (5 marks) The system requires the elimination of the steady state error and robustness with respect to step disturbances. You colleague (who graduated from McMaster) suggests changing the controller from $D(s) = K$ to an integral controller $D(s) = \frac{K}{s}$. Is this a good choice? Does it eliminate the steady state error and still have the same % overshoot for an appropriate choice of K? What might have changed? Justify your answer.

   c) (5 marks) For the control $D(s) = \frac{K}{s}$ with the plant $G(s)$ as above, what type of input (step, ramp, parabola) now results in a steady state error? What is the static error constant of the system and control in this case?

4. **State Space Control** (20 marks)

   Assume that the plant is given by:

$$G(s) = \frac{s+3}{s^4 + 23s^3 + 182s^2 + 580s + 600}$$

   a) (5 marks) Given $G(s)$ what is the controller canonical (phase variable) state space representation of the plant $G(s)$?

   b) (5 marks) A state feedback control $u = -\mathbf{Kx}$ is to be used to:

      - place one of the closed loop poles is to be placed at $s = -3$ to exactly cancel the system zero,
      - place the dominant poles at $s = -5.4 \pm j12.2$ to get the desired % overshoot and settling time, and
      - place the final pole at $s = -54$ so it won't interfere.

      What would the desired characteristic equation of the designed closed loop system be?

   c) (5 marks) Compute a state feedback control matrix K to place the poles at the desired locations.

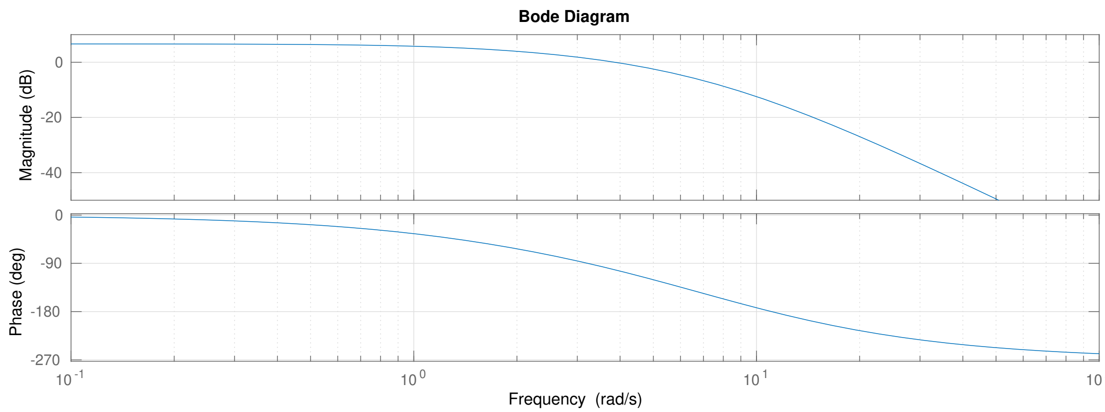
   d) (5 marks) Is the state space representation from part (4a) observable so that we can implement our state feedback controller given our current sensor outputs? Justify your answer.

5. **Bode Plot Margins, Bandwidth & Digital Control** (15 marks)

The open loop bode plot of $D(s)G(s)$ for:

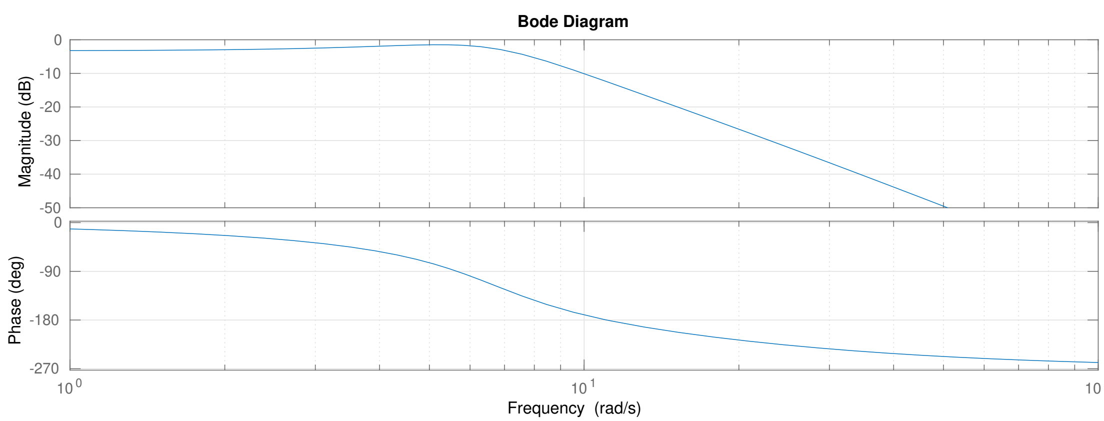
$$D(s) = K = 430, G(s) = \frac{s+3}{s^4 + 23s^3 + 182s^2 + 580s + 600}$$

is shown below:



a) (5 marks) What is the gain margin for the system? What is the phase margin? How much could the gain change by before the systems would be unstable? (Note your answer to (1c) might help with this question.)

b) (5 marks) The Bode plot for the closed loop system with $D(s) = K = 430$ is shown below.



What is the bandwidth of the closed loop control system? What is the minimum sampling frequency that could be used to implement a digital version of the system?

c) (5 marks) What is the formula to compute the Zero Order Hold Equivalent of the plant $G(s)$? For a sampling rate of 10 Hz, where would the pole corresponding to $s = -6$ be located in the z-plane? (Note: You do not have to solve for the complete Zero Order Hold equivalent of $G(s)$, just give the general formula and compute the z-plane pole location for $s = -6$).

TABLE 13.1 Partial table of z- and s-transforms

	$f(t)$	$F(s)$	$F(z)$	$f(kT)$
1.	$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(KT)$
2.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
3.	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a \to 0} (-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
4.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
6.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$	$\sin \omega kT$
7.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z - \cos \omega T)}{z^2 - 2z\cos \omega T + 1}$	$\cos \omega kT$
8.	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$	$e^{-akT}\sin \omega kT$
9.	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2 - ze^{-aT}\cos \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$	$e^{-akT}\cos \omega kT$

The End