ENGPHYS 2A04 Tutorial 3

Electricity and Magnetism

Your TAs Today

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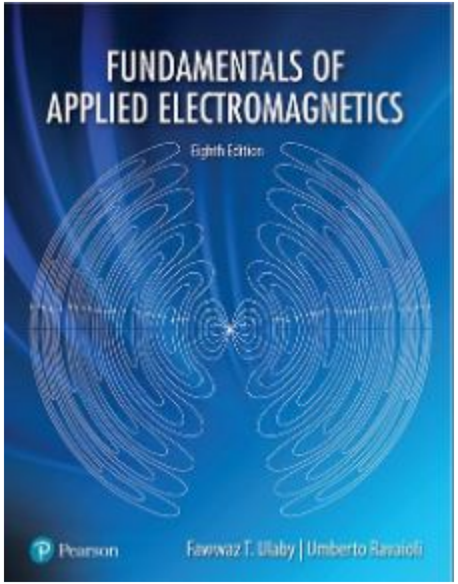
Your Textbook

Fundamentals of Applied Electromagnetics Eighth Edition.

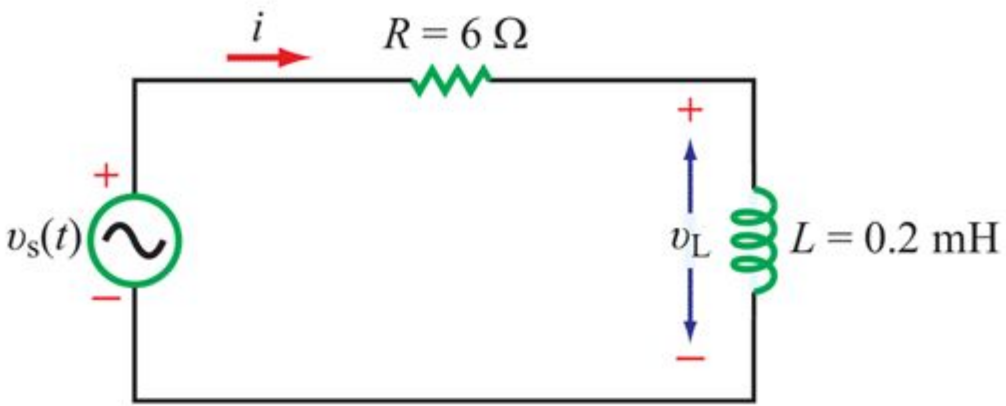
Ulaby & Ravaioli

Seventh edition also acceptable, with some inconsistencies



Lecture Problem (Example 1-4 Page 40)

The voltage source of the RL circuit shown below is given by:
V_s (t) =5 sin(4*10^4t-30 °). Find an expression for the voltage across the inductor.



Lecture Problem Solution

Before converting to the phasor domain, we express the initial equation in terms of cosine.

$$\begin{aligned} v_s(t) &= 5\sin(4\times 10^4 t - 30^\circ) \\ &= 5\cos(4\times 10^4 t - 120^\circ) \qquad \text{(V)}. \end{aligned}$$

The corresponding voltage phasor equation is

$$\widetilde{V}_s = 5e^{-j120^\circ} \qquad \text{(V)},$$

Lecture Problem Solution

The voltage loop equation of the RL circuit is $$Ri + L\frac{di}{dt} = v_s(t).$$

The corresponding phasor equation is $$R\widetilde{I} + j\omega L\widetilde{I} = \widetilde{V}_s.$$

Solving for current phasor ***I*** the equation becomes:

$$I = \frac{V_s}{R + j\omega L}$$

$$I = \frac{5e^{-j120°}}{6 + j4 * 10^4 * 2 * 10^{-4}}$$

$$I = \frac{5e^{-j120°}}{6 + j8}$$

$$I = \frac{5e^{-j120°}}{10e^{j53.1°}}$$

$$I = 0.5e^{-j173.1°}$$

Lecture Problem Solution

The voltage phasor across the inductor is related to the current *I* by

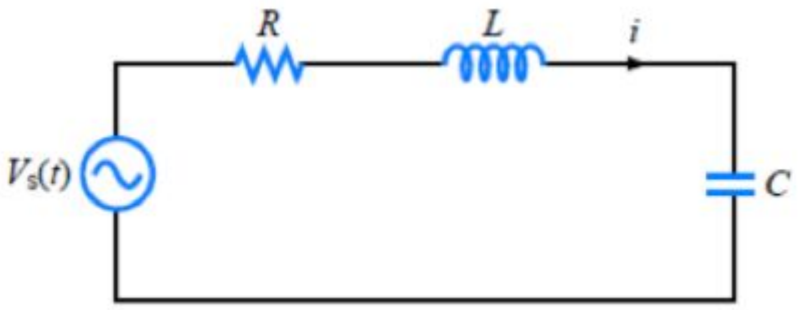
$$
\begin{aligned}
\widetilde{V}_{\mathrm{L}} &= j\omega L\widetilde{I} \\
&= j4 \times 10^4 \times 2 \times 10^{-4} \times 0.5e^{-j173.1^\circ} \\
&= 4e^{j(90^\circ - 173.1^\circ)} = 4e^{-j83.1^\circ} \qquad \text{(V)},
\end{aligned}
$$

Corresponding Instantaneous Voltage $V_L(t)$ is:

$$
\begin{aligned}
v_{\mathrm{L}}(t) &= \mathfrak{Re}\left[\widetilde{V}_{\mathrm{L}} e^{j\omega t}\right] \\
&= \mathfrak{Re}\left[4e^{-j83.1^\circ} e^{j4\times 10^4 t}\right] \\
&= 4\cos(4 \times 10^4 t - 83.1^\circ)
\end{aligned}
$$

Problem 33.4 - Phasor Method

A series RLC AC circuit has: $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu F$, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.



a) Determine the inductive reactance and the capacitive reactance of the circuit.

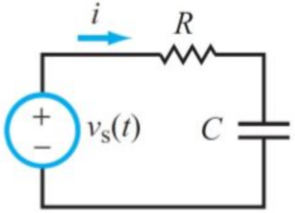
b) Find the impedance and phase angle between the current and voltage.

c) Find the maximum current in the circuit.

d) Find both the maximum voltage and the instantaneous voltage across each element.

AC Phasor Analysis

Step 1
Adopt Cosine Reference
(Time Domain)



$v_s(t) = 12 \sin(\omega t - 45^\circ)$ (V)

Step 2
Transfer to Phasor Domain

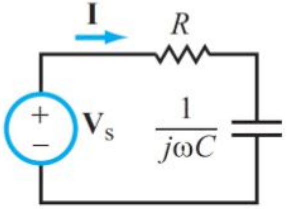
$i \rightarrow \mathbf{I}$

$v \rightarrow \mathbf{V}$

$R \rightarrow \mathbf{Z}_R = R$

$L \rightarrow \mathbf{Z}_L = j\omega L$

$C \rightarrow \mathbf{Z}_C = 1/j\omega C$



$\mathbf{V}_s = 12 e^{-j135^\circ}$ (V)

Step 3
Cast Equations in
Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

(apply Ohm's and Kirchoff's laws)

Step 4
Solve for Unknown Variable
(Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$$

Step 5
Transform Solution
Back to Time Domain

$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}]$$
$$= I_0 \cos(\omega t - \phi_i) \text{ (A)}$$

Example 33.4 Initial Steps

First create the time domain cosine equation for the given circuit using the information given.

$$V_s(t) = \Delta V_{max} \cos(\omega t + \phi)$$

$$V_s(t) = 150 \cos(377t + \phi)$$

Then transfer the equation into the phasor domain.

$$i \rightarrow \tilde{I}$$

$$V_s(t) = \tilde{V} = \Delta V_{max} e^{j\phi}$$

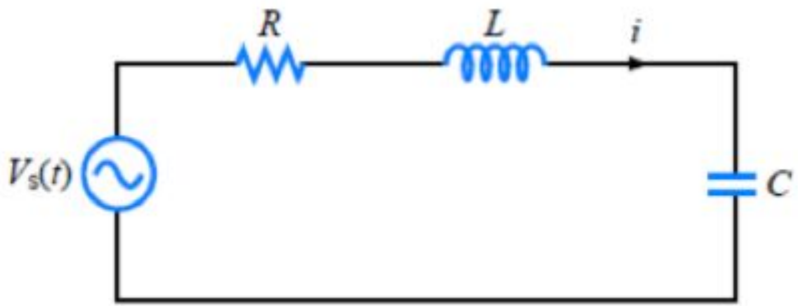
$$R \rightarrow Z_R = R = 425\ \Omega$$

$$L \rightarrow Z_L = j\omega L = jX_L$$

$$C \rightarrow Z_C = \frac{1}{j\omega C} = -jX_C$$

Example 33.4 Solution to A)

A series RLC AC circuit has: R = 425 Ω, L = 1.25 H, C = 3.50 μF, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.



a) Determine the inductive reactance and capacitive reactance of circuit.

$$X_L = \omega L = (377s^{-1})(1.25\ H)$$

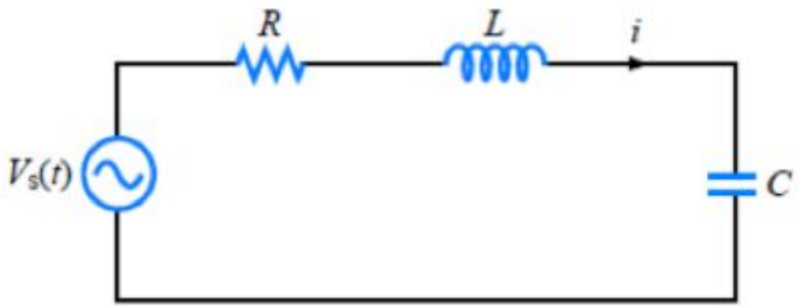
$$X_L = 471\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377s^{-1})(3.50 * 10^{-6}\ F)}$$

$$X_C = 758\ \Omega$$

Example 33.4 Solution to B)

A series RLC AC circuit has: $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu F$, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.



b) Find the impedance and phase angle between current and voltage

$$Z = Z_R + Z_L + Z_C = R + jX_L - jX_C$$

$$Z = 425 + 471j - 758j$$

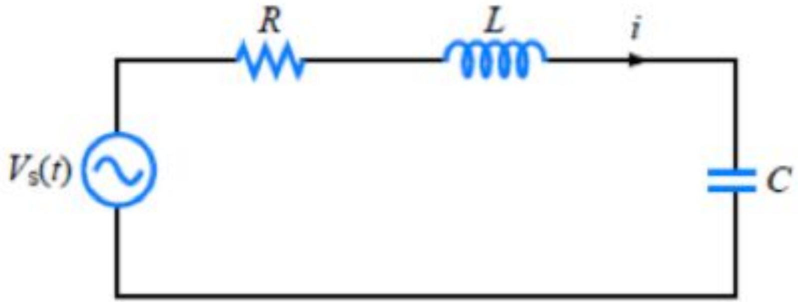
$$Z = 425 - 287j\ \Omega$$

Convert to phasor using method from before.

$$Z \cong 513\ \Omega \angle -34.0°$$

Example 33.4 Solution to C)

A series RLC AC circuit has: R = 425 Ω, L = 1.25 H, C = 3.50 μF, ω = 377 s^{-1}, and ΔV_{max} = 150 V.



c) Find the maximum current by applying Kirchoff's Rules in phasor form and solve for unknowns.

$$\tilde{V} = V_R + V_L + V_C$$

$$\tilde{I} = (Z_R + Z_L + Z_C) = \tilde{I}Z$$

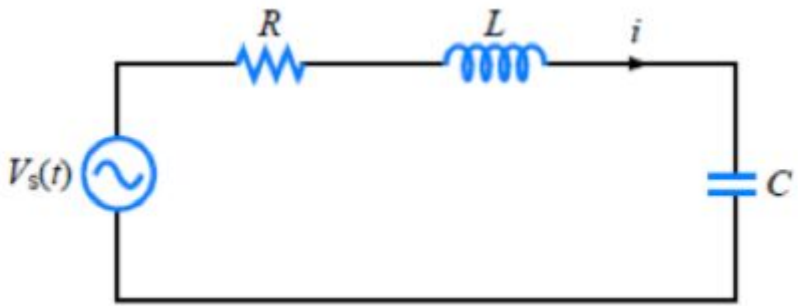
$$\tilde{I} = \frac{\tilde{V}}{Z}$$

The max current occurs when: $\tilde{V} = \Delta V_{max}$

$$\tilde{I}_{max} = \frac{\Delta V_{max}}{|Z|} = \frac{150\ V}{513\ \Omega} = 0.292\ A$$

Example 33.4 Solution to D) Max Voltage

A series RLC AC circuit has: R = 425 Ω, L = 1.25 H, C = 3.50 μF, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.



c) Find both the maximum voltage and the instantaneous voltage across each element.

Max voltage occurs when: $\tilde{I} = \tilde{I}_{max}$

$$\Delta V_R = \tilde{I}_{max} Z_R = (0.292\ A)(425\ \Omega)$$

$$\Delta V_R = 124\ V$$

$$\Delta V_L = \tilde{I}_{max} Z_L = (0.292\ A)(471j\ \Omega)$$

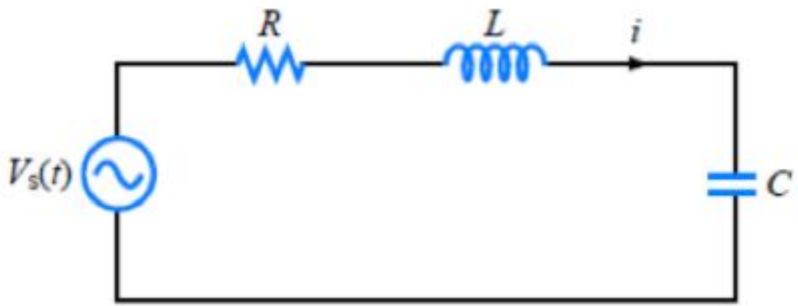
$$\Delta V_L = 138j\ V$$

$$\Delta V_C = \tilde{I}_{max} Z_C = (0.292\ A)(-758j\ \Omega)$$

$$\Delta V_C = -222j\ V$$

Example 33.4 Solution to D) Instantaneous Voltage

A series RLC AC circuit has: R = 425 Ω, L = 1.25 H, C = 3.50 μF, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.



c) Find both the maximum voltage and the instantaneous voltage across each element.

Transform all solutions back to time domain.

$$v(t) = \Re\mathrm{e}\left[V_0 e^{j\phi} e^{j\omega t}\right] = V_0 \cos(\omega t + \phi)$$

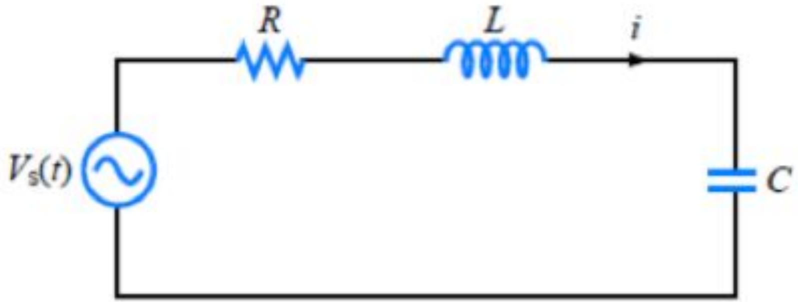
$$\Delta V_R = V_0 e^{j\phi} = 124\ V$$

$$\Delta V_L = V_0 e^{j\phi} = 138j = 138 e^{\frac{j\pi}{2}}\ V$$

$$\Delta V_C = V_0 e^{j\phi} = -222j = 222 e^{-\frac{j\pi}{2}}\ V$$

Example 33.4 Solution to D) Instantaneous Voltage

A series RLC AC circuit has: R = 425 Ω, L = 1.25 H, C = 3.50 μF, ω = 377 s^{-1}, and ΔV_{max} = 150 V.



c) Find both the maximum voltage and the instantaneous voltage across each element.

$$v(t) = \Re e\left[V_0 e^{j\phi} e^{j\omega t}\right] = V_0 \cos(\omega t + \phi)$$

$$v_R(t) = \Re e\left[\Delta V_R e^{j\omega t}\right] = \Re e\left[124 e^{j377t}\right]$$

$$v_R(t) = 124 \cos 377t$$

$$v_L(t) = \Re e\left[\Delta V_L e^{j\omega t}\right] = \Re e\left[138 e^{\frac{j\pi}{2}} e^{j377t}\right]$$

$$v_L(t) = 138 \sin 377t$$

$$v_C(t) = \Re e\left[\Delta V_C e^{j\omega t}\right] = \Re e\left[222 e^{\frac{-j\pi}{2}} e^{j377t}\right]$$

$$v_C(t) = -222 \sin 377t$$

Example 2.1

A transmission line of length l connects to a sinusoidal voltage source with an oscillation frequency f. Assuming that the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit.

a) $l = 20$ cm, $f = 20$ kHz,
b) $l = 50$ km, $f = 60$ Hz
c) $l = 20$ cm, $f = 600$ MHz,
d) $l = 1$ mm, $f = 100$ GHz,

Example 2.1 Solution

Transmission line effects are negligible when $\frac{l}{\lambda} < 0.01$

It may be necessary to account for both the phase shift due to time delay and the presence of reflected signals that may have bounced back.

Line length

Frequency of the oscillating wave

$$\boxed{\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{lf}{c} = \frac{lf}{3\times10^8 \ m/s}}$$

Wavelength

Velocity of wave propagation

Speed of light

$$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{lf}{c} = \frac{lf}{3 * 10^8\ m/s}$$

Example 2.1 Solution

a) $l = 20\ cm,\ f = 20\ kHz$ $= \frac{lf}{3 * 10^8\ m/s} = \frac{20 * 10^{-3}\ m * 20 * 10^3\ Hz}{3 * 10^8\ m/s} = 1.33 * 10^{-5}$ ← negligible

b) $l = 50\ km,\ f = 60\ Hz$ $= \frac{lf}{3 * 10^8\ m/s} = \frac{50 * 10^3\ m * 60\ Hz}{3 * 10^8\ m/s} = 0.01$ ← included

c) $l = 20\ cm,\ f = 600\ MHz,$ $= \frac{lf}{3 * 10^8\ m/s} = \frac{20 * 10^{-2}\ m * 600 * 10^6\ Hz}{3 * 10^8\ m/s} = 0.4$ ← included

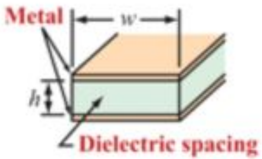
d) $l = 1\ mm,\ f = 100\ GHz,$ $= \frac{lf}{3 * 10^8\ m/s} = \frac{1 * 10^{-3}\ m * 20 * 10^9\ Hz}{3 * 10^8\ m/s} = 0.33$ ← included

Example 2.4

A 1 GHz parallel-plate transmission line consists of 1.2 cm wide copper strips separated by a 0.15 cm thick layer of polystyrene.

Appendix B gives $\mu_c = \mu_0 = 4\pi * 10^{-7}\, H/m$ and $\sigma_c = 5.8 * 10^{7} S/m$ for copper, and $\varepsilon_r = 2.6$

for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume that $\mu_c = \mu_0$ and $\sigma \approx 0$ for polystyrene.



Example 2.4 Solution

From table 2-1, the following parameter formulas can be determined for a parallel-plate:

$$R' = \frac{2R_S}{w}$$

$$L' = \frac{\mu h}{w} \qquad R_S = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

$$G' = \frac{\sigma w}{h}$$

$$C' = \frac{\epsilon w}{h}$$

For copper:

$$w = 1.2\ cm = 0.012\ m$$
$$\mu_c = \mu_0 = 4\pi * 10^{-7}\ H/m$$
$$\sigma_c = 5.8 * 10^{8}\ S/m$$

Other values:

$$f = 1\ GHz = 1 * 10^{9}\ Hz$$
$$\varepsilon = 8.854 * 10^{-12}\ F/m$$

For Polystyrene:

$$h = 0.15\ cm = 0.0015\ m$$
$$\mu = \mu_0 = 4\pi * 10^{-7}\ H/m$$
$$\sigma = 0$$
$$\varepsilon_r = 2.6$$

Example 2.4 Solution

From table 2-1, the following parameter formulas can be determined for a parallel-plate:

$$R' = \frac{2R_S}{w} \qquad R' = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{1.2 * 10^{-2}}\sqrt{\frac{\pi * 1 * 10^{9} * 4\pi * 10^{-7}}{5.8 * 10^{7}}} = 1.38\ \Omega/m$$

$$L' = \frac{\mu h}{w} \qquad L' = \frac{\mu h}{w} = \frac{4\pi * 10^{-7} * 0.0015}{0.012} = 1.57 * 10^{-7}\ H/m$$

$$G' = \frac{\sigma w}{h} \qquad G' = \frac{\sigma w}{h} = 0\ S/m$$

$$C' = \frac{\epsilon w}{h} \qquad C' = \frac{\epsilon w}{h} = \frac{\varepsilon_0 \varepsilon_r w}{h} = \frac{8.854 * 10^{-12} * 2.6 * 0.012}{0.0015} = 1.84 * 10^{-10}\ F/m$$