Problem 6.4 A stationary conducting loop with an internal resistance of 0.5 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 5 A flows through it. What will the current be if the loop is opened to create a small gap and a 2-Ω resistor is connected across its open ends?

Solution: V_{emf} is independent of the resistance which is in the loop. Therefore, when the loop is intact and the internal resistance is only 0.5 Ω,

$$V_{\text{emf}} = 5\text{ A} \times 0.5\ \Omega = 2.5\text{ V}.$$

When the small gap is created, the total resistance in the loop is infinite and the current flow is zero. With a 2-Ω resistor in the gap,

$$I = V_{\text{emf}}/(2\ \Omega + 0.5\ \Omega) = 2.5\text{ V}/2.5\ \Omega = 1 \quad \text{(A)}.$$

Problem 6.8 A rectangular conducting loop 5 cm $\times$10 cm with a small air gap in one of its sides is spinning at 7200 revolutions per minute. If the field **B** is normal to the loop axis and its magnitude is 5×10^{-6} T, what is the peak voltage induced across the air gap?

Solution:

$$\omega = \frac{2\pi \text{ rad/cycle} \times 7200 \text{ cycles/min}}{60 \text{ s/min}} = 240\pi \text{ rad/s},$$

$$A = 5 \text{ cm} \times 10 \text{ cm}/(100 \text{ cm/m})^2 = 5.0 \times 10^{-3} \text{ m}^2.$$

From Eqs. (6.36) or (6.38), $V_{\text{emf}} = A\omega B_0 \sin\omega t$; it can be seen that the peak voltage is

$$V_{\text{emf}}^{\text{peak}} = A\omega B_0 = 5.0 \times 10^{-3} \times 240\pi \times 5 \times 10^{-6} = 18.85 \quad (\mu\text{V}).$$

Problem 6.7 The rectangular conducting loop shown in Fig. 6-20 (P6.7) rotates at 6,000 revolutions per minute in a uniform magnetic flux density given by

$$\mathbf{B} = \hat{\mathbf{y}}\,50 \quad \text{(mT)}.$$

Determine the current induced in the loop if its internal resistance is 0.5 Ω.

Solution:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S} = \hat{\mathbf{y}}\,50 \times 10^{-3} \cdot \hat{\mathbf{y}}(2 \times 3 \times 10^{-4}) \cos\phi(t) = 3 \times 10^{-5} \cos\phi(t),$$

$$\phi(t) = \omega t = \frac{2\pi \times 6 \times 10^3}{60}\, t = 200\pi t \quad \text{(rad/s)},$$

$$\Phi = 3 \times 10^{-5} \cos(200\pi t) \quad \text{(Wb)},$$

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = 3 \times 10^{-5} \times 200\pi \sin(200\pi t) = 18.85 \times 10^{-3} \sin(200\pi t) \quad \text{(V)},$$

$$I_{\text{ind}} = \frac{V_{\text{emf}}}{0.5} = 37.7 \sin(200\pi t) \quad \text{(mA)}.$$

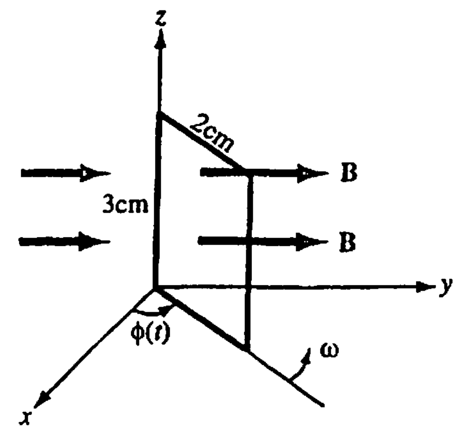

Figure P6.7: Rotating loop in a magnetic field (Problem 6.7).

The direction of the current is CW (if looking at it along $-\hat{\mathbf{x}}$-direction) when the loop is in the first quadrant ($0 \leq \phi \leq \pi/2$). The current reverses direction in the second quadrant, and reverses again every quadrant.

Problem 6.8 The transformer shown in Fig. P6.8 consists of a long wire coincident with the z-axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x–y plane and centered at the origin. The toroidal core uses iron material with relative permeability μ_r, around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure.

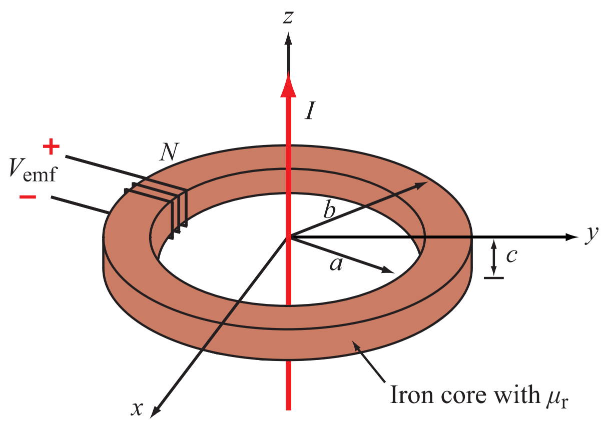

Figure P6.8: Problem 6.8.

(a) Develop an expression for V_{emf}.

(b) Calculate V_{emf} for $f = 60$ Hz, $\mu_r = 4000$, $a = 5$ cm, $b = 6$ cm, $c = 2$ cm, and $I_0 = 50$ A.

Solution:

(a) We start by calculating the magnetic flux through the coil, noting that r, the distance from the wire varies from a to b

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_a^b \hat{\mathbf{x}} \, \frac{\mu I}{2\pi r} \cdot \hat{\mathbf{x}} \, c \, dr = \frac{\mu c I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$V_{emf} = -N \frac{d\Phi}{dt} = -\frac{\mu c N}{2\pi} \ln\left(\frac{b}{a}\right) \frac{dI}{dt}$$

$$= \frac{\mu c N \omega I_0}{2\pi} \ln\left(\frac{b}{a}\right) \sin \omega t \quad \text{(V)}.$$

(b)

$$V_{emf} = \frac{4000 \times 4\pi \times 10^{-7} \times 2 \times 10^{-2} \times 100 \times 2\pi \times 60 \times 50 \ln(6/5)}{2\pi} \sin 377t$$

$$= 5.5 \sin 377t \quad \text{(V)}.$$