Solutions

1. (8 points) Give the definitions of the following terms in robotics.

1) Hard automation
2) Flexible automation
3) Planar robot
4) Dextrous workspace

Hard automation: specalized machines for high volume manufacturing in the form. Whenever the product is changed the machines must be shutdown and the hardware changed or "retooled".

Flexible automation: use of robots in place of the specalized machines used in hard automation. The robots may be reprogrammed when the product changes so minimal retooling is required

Planar robot: a robot whose end-effector motion is limited to a single plane

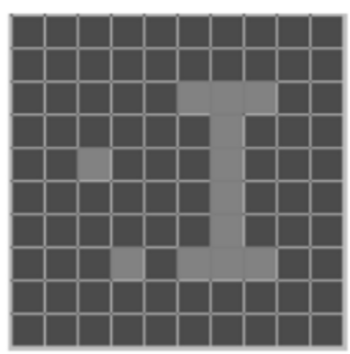
Dextrous workspace: the volume of space the end-effector can reach with any desired orientation

2. Short Answer Questions

1) (5 points) If the transformation matrices, ${}^{C}T_D$, ${}^{A}T_B$, ${}^{A}T_E$ and ${}^{E}T_D$, are known, derive the transformation equation for ${}^{B}T_C$ in terms of these matrices.

2) (5 points) Given the grayscale input image of the letter "I" from an optical character recognition application:

$$A = \begin{bmatrix} 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 130 & 130 & 130 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 130 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 130 & 75 & 130 & 130 & 130 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \end{bmatrix}$$



Show a sample calculation for the row=8, col=5 pixel, if we apply a Laplacian 1 filter to the input image.

3) (5 points) Which of the following matrices is a valid representation for a frame? Explain your answer.

$$A = \begin{bmatrix} 0.77 & -0.64 & 0 & 3 \\ 0.64 & 0.77 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; B = \begin{bmatrix} 0 & \sqrt{3}/2 & 0 & 3 \\ -1 & 0 & 0 & 0.5 \\ 0 & 1/2 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; C = \begin{bmatrix} 0 & \sqrt{3}/2 & -1/2 & 3 \\ -1 & 0 & 0 & 0.5 \\ 0 & 1/2 & -\sqrt{3}/2 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

a) ${}^{C}T_{D}$, ${}^{A}T_{B}$, ${}^{A}T_{E}$, ${}^{E}T_{D}$ $\rightsquigarrow$ get ${}^{B}T_{C}$

$${}^{B}T_{C} = ({}^{A}T_{B})^{-1} ({}^{A}T_{E})({}^{E}T_{D}) ({}^{C}T_{D})^{-1}$$

b) $\begin{bmatrix} 75 & 75 & 75 \\ 130 & 75 & 130 \\ 75 & 75 & 75 \end{bmatrix}$ laplacian filter: $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$$S = 0 - 1 + 0 - 1 + 5 - 1 + 0 - 1 + 0 = 1$$

$$f_{85} = \frac{1}{1} \left| 75(-1) + 130(-1) + 75(5) + 130(-1) + 75(-1) \right|$$
$$= 35$$

check max pixel value:

$$f_{84} = \frac{1}{1} \left| (-1)75 + (-1)75 + 5(130) + (-1)75 + (-1)75 \right| = 350$$

$$f_{84}, \text{scaled} = 255$$

$$f_{85}, \text{scaled} = \left(\frac{255}{350}\right) 35 \simeq 26$$

c) $\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 0 \text{ or } 1 \end{bmatrix}$ $\vec{n} \cdot \vec{o} = 0$, $\vec{n} \cdot \vec{a} = 0$, $\vec{o} \cdot \vec{a} = 0$

A: $\vec{n} \cdot \vec{o} = 0$, $\vec{n} \cdot \vec{a} = 0$, $\vec{o} \cdot \vec{a} = 0$

B: $\vec{n} \cdot \vec{o} = 0$, $\vec{n} \cdot \vec{a} = 0$, $\vec{o} \cdot \vec{a} \neq 0$

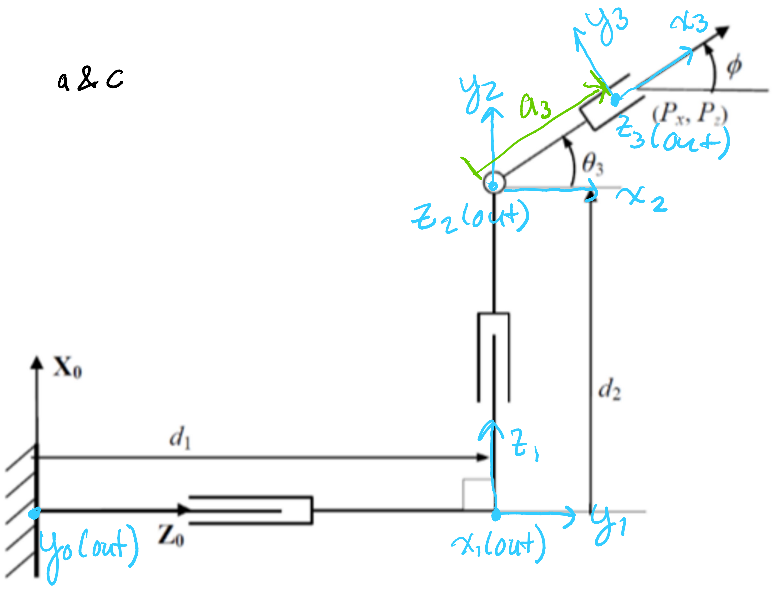
C: $\vec{n} \cdot \vec{o} = 0$, $\vec{n} \cdot \vec{a} = 0$, $\vec{o} \cdot \vec{a} \neq 0$

$\therefore$ A is valid representation

$\therefore$ H is valid representation

3. (25 points) For the PPR planar robot shown in the following Figure:
a) Assign the frames using the D-H method.
b) Determine the D-H parameters and put them in the standard table form. Identify the joint variables.
c) Draw a diagram of the robot that properly shows the D-H frames, the joint variables, and any d or a parameters that are non-zero.
d) Calculate the A matrices and ${}^0T_3$
e) Its joint variables are d_1, d_2, and θ_3. Its end-effector position and orientation are given by P_x, P_z and $\emptyset$. Derive its inverse kinematics.

a & c



b)

n+1	θ	d	a	α
1	90	d_1*	0	90
2	90	d_2*	0	90
3	θ_3*	0	a_3	0

* joint variables

d) $A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$${}^0T_3 = \begin{bmatrix} S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 + d_2 \\ 0 & 0 & 1 & 0 \\ C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e) $P_x = d_2 + a_3 S\theta_3$ $\quad P_z = d_1 + a_3 C\theta_3$ $\quad \boxed{\emptyset = \theta_3}$

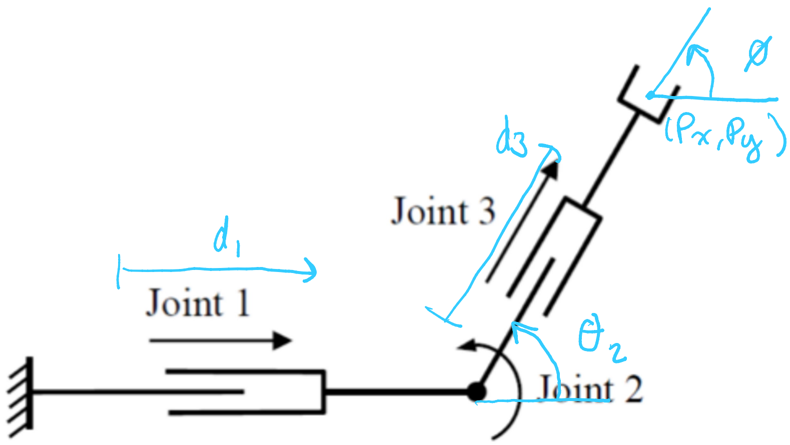
$\boxed{d_2 = P_x - a_3 S\theta_3}$ $\quad \boxed{d_1 = P_z - a_3 C\theta_3}$

$$\boxed{d_2 = P_x - a_3 S\theta_3} \qquad \boxed{d_1 = P_z - a_3 C\theta_3}$$

$$\sin\theta_3 = \frac{P_x - d_2}{a_3} \qquad \cos\theta_3 = \frac{P_z - d_1}{a_3}$$

$$\boxed{\theta_3 = \operatorname{atan2}\left(\frac{P_x - d_2}{a_3}, \frac{P_z - d_1}{a_3}\right)}$$

4. (25 points) For the planar PRP robot shown in the following Figure:
a) Derive the 3x3 manipulator Jacobian matrix. (The form used for calculating the linear velocity and angular velocity of the tool).
b) Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.
c) Draw the robot in a singular configuration and indicate which degree (s) of freedom have been lost.



joint variables:
d_1, θ_2, d_3

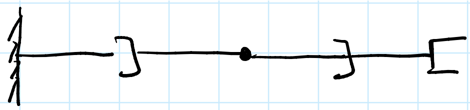
$$P_x = d_1 + d_3 C\theta_2$$
$$P_y = d_3 S\theta_2$$

$$J(q) = \begin{bmatrix} \partial P_x/\partial d_1 & \partial P_x/\partial\theta_2 & \partial P_x/\partial d_3 \\ \partial P_y/\partial d_1 & \partial P_y/\partial\theta_2 & \partial P_y/\partial d_3 \\ 0 & 1 & 0 \end{bmatrix}$$

a) $$J = \begin{bmatrix} 1 & -d_3 S\theta_2 & C\theta_2 \\ 0 & d_3 C\theta_2 & S\theta_2 \\ 0 & 1 & 0 \end{bmatrix}$$

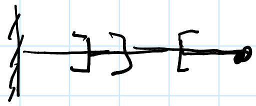
b) $$\det J = (1)\left[-1(S\theta_2)\right] + 0 + 0$$
$$= -S\theta_2$$
$$0 = -S\theta_2 \Rightarrow \theta_2 = 0^\circ, 180^\circ$$

c) $@\theta_2 = 0^\circ$



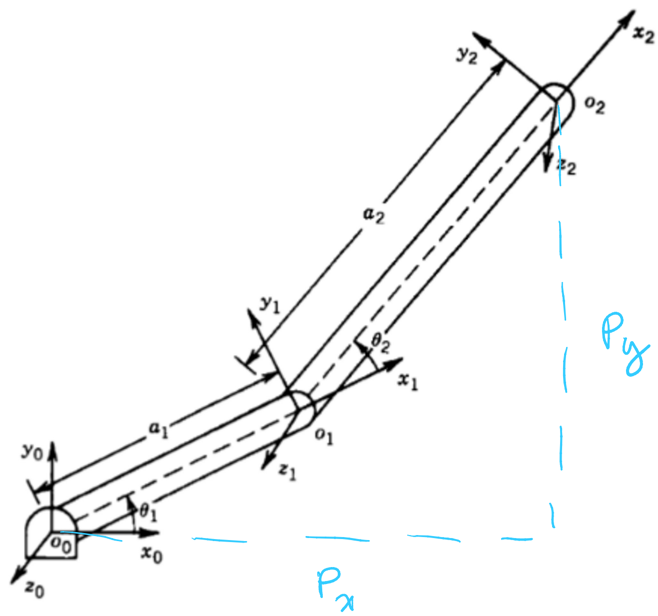
DOF in y lost

@ $\theta_2 = 180°$



DOF in y lost

5. (27 points) For the RR planar robot in the following figure, if $a_1 = 0.4$m and $a_2 = 0.3$m;
(a) Assuming the robot operates in the horizontal plane, calculate the joint torques such that the static force at the end-effector is $F_x = 20$N and $F_y = -15$N for the configuration $\theta_1 = 35^\circ$ and $\theta_2 = -75^\circ$
(b)Assuming the robot operates in the horizontal plane, calculate the static force applied by the end effector when $\tau_1 = 10$Nm, $\tau_2 = 5$Nm, $\theta_1 = 35^\circ$ and $\theta_2 = -75^\circ$.



assuming motion in a horizontal plane : $G = 0$

$P_x = a_1 C\theta_1 + a_2 C\theta_{12}$ joint variables:
$P_y = a_1 S\theta_1 + a_2 S\theta_{12}$ θ_1 & θ_2

$$J = \begin{bmatrix} \partial P_x/\partial\theta_1 & \partial P_x/\partial\theta_2 \\ \partial P_y/\partial\theta_1 & \partial P_y/\partial\theta_2 \end{bmatrix} = \begin{bmatrix} -a_1 S\theta_1 - a_2 S\theta_{12} & -a_2 S\theta_{12} \\ a_1 C\theta_1 + a_2 C\theta_{12} & a_2 C\theta_{12} \end{bmatrix}$$

a) $\theta_1 = 35°$, $\theta_2 = -75°$ $\Rightarrow$ $\theta_{12} = -40°$

$a_1 = 0.4\,m, \quad a_2 = 0.3\,m$

$$J = \begin{bmatrix} -0.03659 & 0.14284 \\ 0.55747 & 0.22981 \end{bmatrix}$$

$$\tau = J^T F = \begin{bmatrix} -0.03659 & 0.55747 \\ 0.14284 & 0.22981 \end{bmatrix} \begin{bmatrix} 20 \\ -15 \end{bmatrix}$$

$$= \begin{bmatrix} -9.09385 \\ 0.40965 \end{bmatrix} Nm$$

b) $\tau_1 = 10\,Nm, \quad \tau_2 = 5\,Nm, \quad \theta_1 = 35°, \quad \theta_2 = -75°$

$$\begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} -0.03659 & 0.55747 \\ 0.14284 & 0.22981 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{1}{\det(J)} \begin{bmatrix} 0.22981 & -0.55747 \\ -0.14284 & -0.03659 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\frac{1}{\det(J)} = \frac{1}{(-0.03659)(0.22981) - (0.14284)(0.55747)}$$

$$= -8.62728$$

$$= \begin{bmatrix} 4.221 \\ 18.215 \end{bmatrix} N$$

$F_x = 4.221\,N, \quad F_y = 18.215\,N$