EP2A04 Tutorial 1

TUTORIAL TAS:
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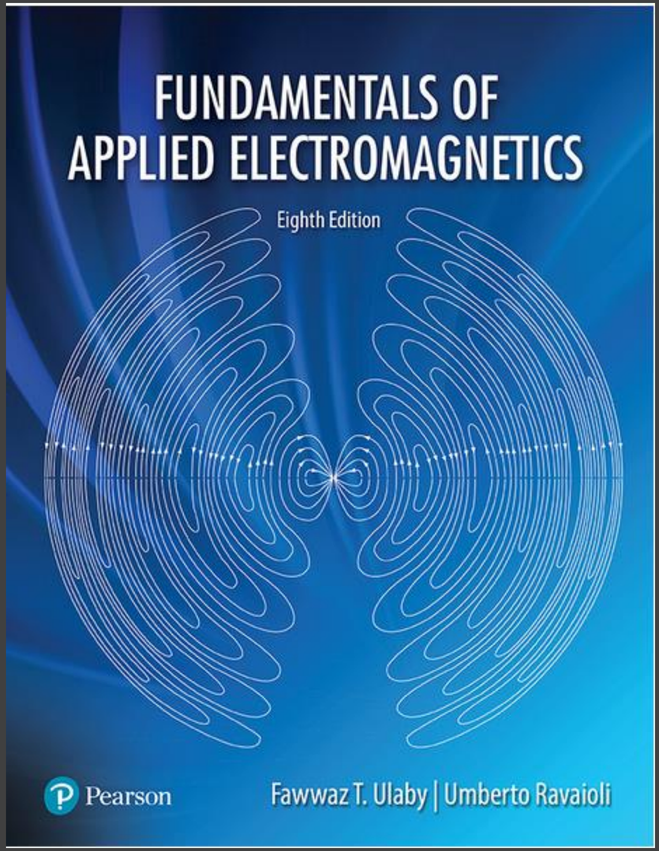
Your Textbook

Fundamentals of Applied Electromagnetics, Eighth Edition

Ulaby & Ravaioli

Seventh edition also acceptable, with some inconsistencies



Tutorial Problem 1 (1.1, or 1.3 in 7th ed.)

A harmonic wave traveling along a string is generated by an oscillator that completes 180 vibrations per minute. If it is observed that a given crest, or maximum, travels 300 cm in 10 s, what is the wavelength?

Goal: **Wavelength**

Givens: **Frequency, Phase Velocity**

$$u_P = f\lambda \rightarrow \lambda = \frac{u_P}{f}$$

Convert to SI: $f = 180\frac{1}{60s} = 3\frac{1}{s} = 3Hz$ $\quad u_P = \frac{300cm}{10s} = \frac{3m}{10s} = 0.3\frac{m}{s}$

Solution: $\lambda = \frac{u_P}{f} = \frac{0.3}{3} = 0.1m$

Tutorial Problem 2 (1.2)

For the pressure wave described in Example 1-1, plot the following:

A) $p(x, t)$ vs x at $t = 0$.

B) $p(x, t)$ vs t at $x = 0$.

Be sure to use appropriate scales for and so that each of your plots covers at least two cycles.

The equation: $$\boldsymbol{p(x, t) = 10\cos\left(2\pi \times 10^3 t - \frac{4\pi}{3}x + \frac{\pi}{3}\right)}$$

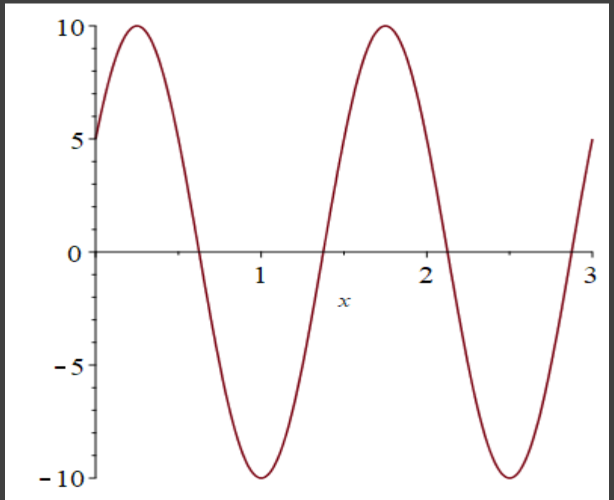
Tutorial Problem 2 (1.2) *continued…*

The equation: $\boldsymbol{p(x,t) = 10\cos\left(2\pi \times 10^3 t - \frac{4\pi}{3}x + \frac{\pi}{3}\right)}$

A) Substituting $t = 0$:

$$p(x,0) = 10\cos\left(-\frac{4\pi}{3}x + \frac{\pi}{3}\right)$$

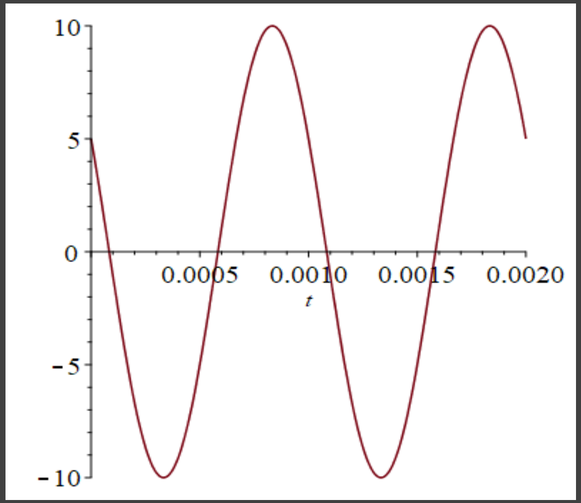
Plotting this for 2 wavelengths ($\lambda = \frac{3}{2}$):



B) Substituting $x = 0$:

$$p(0,t) = 10\cos\left(2\pi \times 10^3 t + \frac{\pi}{3}\right)$$

Plotting this for 2 periods ($T = \frac{1}{f} = 0.001s$):



Tutorial Problem 3 (1.8)

Two waves on a string are given by the functions:

$$y_1(x,t) = 4\cos(20t - 30x) \quad (cm)$$

$$y_2(x,t) = -4\cos(20t + 30x) \quad (cm)$$

Where x is in centimeters. The waves are said to interfere constructively when their superposition $|y_S| = |y_1 + y_2|$ is a maximum, and they interfere destructively when $|y_S|$ is a minimum.

A) What are the directions of propagation of waves $y_1(x,t)$ and $y_2(x,t)$?

B) At $t = (\pi/50)\ s$, what location x do the two waves interfere constructively, and what is the corresponding value of $|y_S|$?

C) At $t = (\pi/50)\ s$, what location x do the two waves interfere destructively, and what is the corresponding value of $|y_S|$?

Tutorial Problem 3 (1.8) *continued...*

A) What are the directions of propagation of waves $y_1(x,t)$ and $y_2(x,t)$?

$$y_1(x,t) = 4\cos(20t - 30x) \quad (cm)$$

The t and x terms have opposite signs, so the wave is propagating in the positive x-direction.

$$y_2(x,t) = -4\cos(20t + 30x) \quad (cm)$$

The t and x terms have the same sign, so the wave is propagating in the negative x-direction.

Tutorial Problem 3 (1.8) *continued…*

B) At $t = (\pi/50)\ s$, what location x do the two waves interfere constructively, and what is the corresponding value of $|y_S|$?

First, substitute in $t = (\pi/50)\ s$:

$$y_1(x, \pi/50) = 4\cos\left(20\left(\frac{\pi}{50}\right) - 30x\right) = 4\cos\left(\frac{2\pi}{5} - 30x\right)$$

$$y_2(x, \pi/50) = -4\cos\left(20\left(\frac{\pi}{50}\right) + 30x\right) = -4\cos\left(\frac{2\pi}{5} + 30x\right)$$

$$\left|y_S(x, \frac{\pi}{50})\right| = \left|4\cos\left(\frac{2\pi}{5} - 30x\right) - 4\cos\left(\frac{2\pi}{5} + 30x\right)\right| \quad \rightarrow \quad 2\sin(\alpha)\sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$\left|y_S\left(x, \frac{\pi}{50}\right)\right| = \left|4\left(2\sin\left(\frac{2\pi}{5}\right)\sin(30x)\right)\right| = \left|8\sin\left(\frac{2\pi}{5}\right)\sin(30x)\right|$$

Tutorial Problem 3 (1.8) *continued…*

B) At $t = (\pi/50)\ s$, what location x do the two waves interfere constructively, and what is the corresponding value of $|y_S|$?

$$\left|y_S\left(x, \frac{\pi}{50}\right)\right| = |7.61\sin(30x)|$$

To find where they constructively interfere, we find the peaks of the superposition by differentiating and finding the zeros:

$$\frac{dy_S}{dx} = \frac{d}{dx}(7.61\sin(30x)) = 30 * 7.61\cos(30x) = 228.3\cos(30x)$$

Find where $\frac{dy_S}{dx} = 0$. Cosine is 0 at $\theta = \left(n + \frac{1}{2}\right)\pi$, so:

$$x = \frac{\left(n + \frac{1}{2}\right)\pi}{30} = \frac{\pi}{60}, \frac{\pi}{20}, \frac{\pi}{12} \ldots (cm)$$

$$x = 0.052cm, 0.157cm, 0.262cm\ \ldots$$

The amplitude of the superposition at these values is:

$$\left|y_S\left(\frac{\pi}{60}, \frac{\pi}{50}\right)\right| = \left|7.61\sin\left(30\frac{\pi}{60}\right)\right| = 7.61\text{ cm}$$

Tutorial Problem 3 (1.8) *continued…*

C) At $t = (\pi/50)\ s$, what location x do the two waves interfere destructively, and what is the corresponding value of $|y_S|$?

$$\left|y_S\left(x, \frac{\pi}{50}\right)\right| = |7.61\sin(30x)|$$

By a similar logic, we can find the points of destructive interference by finding the zeros of the superposition. sin is zero at $\theta = n\pi$, so they interfere destructively at:

$$x = \frac{n\pi}{30} = 0, \frac{\pi}{30}, \frac{\pi}{15}\ \ldots (cm)$$
$$x = 0cm, 0.105cm, 0.209cm\ \ldots$$

The amplitude of the superposition at these values is:

$$\left|y_S\left(0, \frac{\pi}{50}\right)\right| = |7.61\sin(30(0))| = 0\ \text{cm}$$

Tutorial Problem 4 (1.12)

Given two waves characterized by

$$y_1(t) = 3\cos(\omega t)$$

$$y_2(t) = 3\sin(\omega t + 60°),$$

Does $y_2(t)$ lead or lag $y_1(t)$ and by what phase angle?

Sin(x) lag cos(x) by 90°. Adding in a phase shift of +60° will create a lead of 60°. The total effect is $y_2(t)$ lagging $y_1(t)$ by $30°$.

Good luck!

Assignment 1 is due 8AM on January 24.

No late submissions will be accepted – if your work is incomplete, submit whatever you have before the deadline for part marks!

Question 5 is bonus, to help us know where the class stands with circuits content.

Show all of your work for full marks.