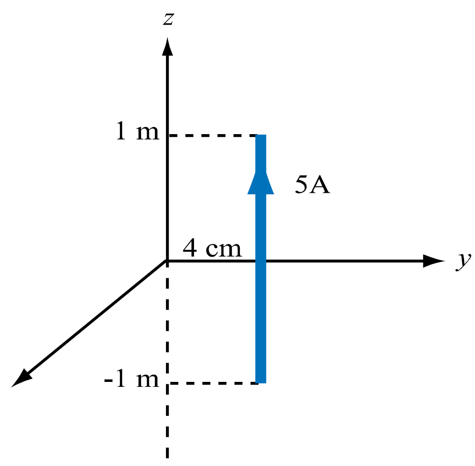

Figure P5.5: Problem 5.5.

Problem 5.5 In a cylindrical coordinate system, a 2-m-long straight wire carrying a current of 5 A in the positive z-direction is located at $r = 4$ cm, $\phi = \pi/2$, and $-1\text{ m} \le z \le 1$ m.

(a) If $\mathbf{B} = \hat{\mathbf{r}}\,0.2\cos\phi$ (T), what is the magnetic force acting on the wire?

(b) How much work is required to rotate the wire once about the z-axis in the negative ϕ-direction (while maintaining $r = 4$ cm)?

(c) At what angle ϕ is the force a maximum?

Solution:

(a)

$$\begin{aligned}\mathbf{F} &= I\boldsymbol{\ell}\times\mathbf{B}\\ &= 5\hat{\mathbf{z}}2\times[\hat{\mathbf{r}}\,0.2\cos\phi]\\ &= \hat{\boldsymbol{\phi}}\,2\cos\phi.\end{aligned}$$

At $\phi = \pi/2$, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$. Hence,

$$\mathbf{F} = -\hat{\mathbf{x}}\,2\cos(\pi/2) = 0.$$

(b)

$$\begin{aligned}W &= \int_{\phi=0}^{2\pi}\mathbf{F}\cdot d\mathbf{l} = \int_0^{2\pi}\hat{\boldsymbol{\phi}}[2\cos\phi]\cdot(-\hat{\boldsymbol{\phi}})r\,d\phi\bigg|_{r=4\text{ cm}}\\ &= -2r\int_0^{2\pi}\cos\phi\,d\phi\bigg|_{r=4\text{ cm}} = -8\times10^{-2}\,[\sin\phi]_0^{2\pi} = 0.\end{aligned}$$

The force is in the $+\hat{\boldsymbol{\phi}}$-direction, which means that rotating it in the $-\hat{\boldsymbol{\phi}}$-direction would require work. However, the force varies as $\cos\phi$, which means it is positive when $-\pi/2 \leq \phi \leq \pi/2$ and negative over the second half of the circle. Thus, work is provided by the force between $\phi = \pi/2$ and $\phi = -\pi/2$ (when rotated in the $-\hat{\boldsymbol{\phi}}$-direction), and work is supplied for the second half of the rotation, resulting in a net work of zero.

(c) The force **F** is maximum when $\cos\phi = 1$, or $\phi = 0$.