Quiz_1 L02

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MECHTRON 3DX4 Tutorial Quiz 1 L02: Laplace Transforms

1. **Laplace Transforms** (10 marks)
Laplace transform tables are on Page 2!

a) (5 marks) Assume that you are given a system with $G(s) = \frac{s+5}{s^2+3s-4}$. What is the time domain output, $c(t)$, in response to a step input (i.e. $r(t) = u(t)$)?

M -4
A 3
N 4, -1

$r(t) = u(t)$

$R(s) = 1/s$

$C(s) = R(s)G(s)$

$C(s) = \left(\frac{1}{s}\right)\left(\frac{s+5}{s^2+3s-4}\right)$

$C(s) = \frac{s+5}{s(s+4)(s-1)}$

$C(s) = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s-1}$

$\frac{0+5}{(0+4)(0-1)} = -\frac{5}{4} = A$

$\frac{-4+5}{(-4)(-4-1)} = \frac{1}{20} = B$

$\frac{1+5}{(1)(1+4)} = \frac{6}{5} = C$

$C(s) = -\frac{5}{4s} - \frac{1}{20(s+4)} + \frac{6}{5(s-1)}$

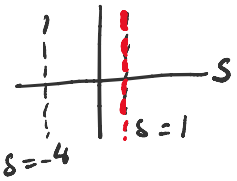
$c(t) = \mathcal{L}^{-1}(C(s))$

$= -\frac{5}{4}\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{1}{20}\mathcal{L}^{-1}\left(\frac{1}{s+4}\right) + \frac{6}{5}\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)$

$= -\frac{5}{4} - \frac{1}{20}e^{-4t} + \frac{6}{5}e^{t}$

b) (5 marks) Could you have applied the Final Value Theorem to get the value of $\lim_{t\to\infty} c(t)$ (i.e. the steady state response) for question (1a) without doing the inverse Laplace transform? Justify your answer.

The poles occur at $s = -4$ & $s = 1$. Since one of the poles is not in the left side of the s-plane, then the system is not BIBO stable. Therefore, FVT cannot be applied.



Justification

using final value theorem yields:

$\lim_{t\to\infty} c(t) = \lim_{s\to 0} C(s) \cdot s$ ✱ $C(s) = \frac{s+5}{s^3+3s^2-4s}$

$= \lim_{s\to 0} s \cdot \frac{s+5}{s^3+3s^2-4s}$

$= \lim_{s\to 0} \frac{s+5}{s^2+3s-4}$

$= \frac{-5}{4}$

From 1a)

$C(\infty) = \lim_{t\to\infty}\left[-\frac{5}{4} - \frac{1}{20}e^{-4t} + \frac{6}{5}e^{t}\right]$ ($e^{-4t} \to 0$, $e^{t} \to \infty$)

$= -\frac{5}{4} + \infty$

$= \infty$

$-\frac{5}{4} \neq \infty$

this result confirms that FVT cannot be applied and that inverse laplace transform is required