Q1

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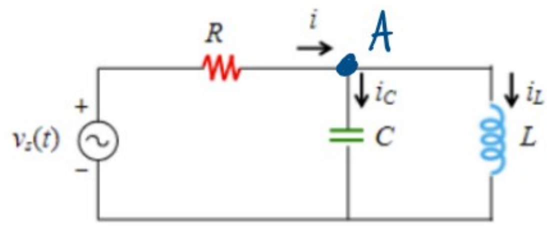
1. **AC Circuit Analysis**

The circuit shown below has the voltage source given by the equation:

$$V_s(t) = 7\cos(2 * 10^4 t - 60°)$$

If the circuit has the following values of $R = 50\ \Omega$, $C = 12\ \mu F$, and $L = 0.6$ mH, use phasor analysis to acquire an expression for the current flowing through the inductor $i_L(t)$.



$V_s = 7\cos(2\times10^4 t - 60°)$ [V]

$\tilde{V}_s = 7e^{-j60°}$ V $\quad R = 50\Omega$

$Z_C = 1/j\omega C = (j(2\times10^4)(12\times10^{-6}))^{-1}$

$Z_C = -4.17j$ [Ω]

$Z_L = j\omega L = j(2\times10^4)(0.6\times10^{-3})$

$Z_L = 12j$ [Ω]

V_A = Voltage divider @ A

$V_A = V_s \dfrac{(Z_L^{-1} + Z_C^{-1})^{-1}}{}$

$$V_A = V_S \frac{(Z_L + Z_C)}{R + (Z_L^{-1} + Z_C^{-1})^{-1}}$$

$$V_A = V_S \frac{((12j)^{-1} + (-4.17j)^{-1})^{-1}}{50 + ((12j)^{-1} + (-4.17j)^{-1})}$$

$$V_A = V_S \left(-6.38j / 50 - 6.38j\right)$$

$$V_A = (0.0161 - 0.126j) V_S$$

$$r = \sqrt{0.0161^2 + 0.126^2}$$

$$r = 0.127$$

$$\phi = \arctan\left(\frac{-0.126}{0.0161}\right)$$

$$\phi = -82.7^\circ$$

$$\tilde{V}_A = (0.127 e^{-j82.7^\circ})(7 e^{-j60^\circ})$$

$$\tilde{V}_A = 0.889 e^{-j142.7^\circ}$$

$$\alpha = 0.889 \cos(-142.7)$$

$$\alpha = -0.707$$

$$\beta = 0.889 \sin(-142.7)$$

$$\beta = -0.539$$

w/ Ohm's law

$$V_A = i_L \cdot Z_L$$

$$i_L = V_A / Z_L$$

$$i_L = \frac{-0.707 - 0.539j}{12j} \leftarrow V_A = -0.707 - 0.539j$$

$$i_L = \frac{-0.707 + -0.534j}{12j} \leftarrow V_A = -0.707 + -0.534j$$

$$i_L = -0.0449 + 0.0589j \rightarrow r = \sqrt{0.0449^2 + 0.0589^2}$$

$$r = 0.741$$

$$\phi = 180° + \tan^{-1}(-0.0589/0.0449)$$

$$\phi = 127.3°$$

$$\boxed{i_L = 0.741 e^{-j127.3°}}$$

Q2

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2. **Transmission Lines**

A transmission line of length l is connected to a sinusoidal voltage source with a certain frequency f. If we assume the velocity of the wave propagates at a certain velocity c, in which of the following conditions shown below is it reasonable to ignore the transmission line effects within the solution of the circuit? In addition, explain the difference between a dispersive and non-dispersive transmission line.

(a) l = 5.3 cm, f = 37 kHz,
(b) l = 8.2 km, f = 475 Hz,
(c) l = 7.7 cm, f = 770 MHz,
(d) l = 2.3 mm, f =98 GHz

find l/λ for each, compare to 0.01

$$\frac{l}{\lambda} = \frac{fl}{c}$$

a) $$\frac{(37\times10^{3}\,Hz)(5.3\times10^{-2}\,m)}{3\times10^{8}\,m/s}$$

$$= 6.53667\times10^{-6} < 0.01$$

$\therefore$ it is reasonable to ignore transmission line effects

b) $\frac{(475\ Hz)(8.2 \times 10^3 m)}{3 \times 10^8\ m/s}$

$= 0.01298 > 0.01$

$\therefore$ it is <u>not</u> reasonable to ignore transmission line effects

c) $\frac{(770 \times 10^6\ Hz)(7.7 \times 10^{-2} m)}{3 \times 10^8\ m/s}$

$= 0.197633 > 0.01$

$\therefore$ it is <u>not</u> reasonable to ignore transmission line effects

d) $\frac{(98 \times 10^9\ Hz)(2.3 \times 10^{-3} m)}{3 \times 10^8\ m/s}$

$= 0.7513 > 0.01$

$\therefore$ <u>definitely</u> <u>cannot</u>

ignore transmission line effects

Dispersive transmission line

→ signal distortion due to phase delay & reflections

Non-Dispersive

→ no signal distortion

Q3

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3. **Transmission Lines**

A two-wire gold transmission line is embedded in an unknown dielectric material that has the following parameters: $\varepsilon_r = 2.7$ and $\sigma = 3.2 * 10^{-6}$ S/m. The wires are separate by a width of 1 cm and their radii are 1 mm each. Calculate the line parameters *R'*, *L'*, *G'*, and *C'* at 1 GHz. (Refer to Appendix B for μ_c and σ_c of gold) Assume that $\mu_c = \mu_0$ for the dielectric material.

$\mu_c = 1.0$ H/m $\quad \sigma_c = 4.1 \times 10^7$ S/m

$f = 1\times10^9$ Hz

two-wire

$D = 1$cm $\quad r = 1$mm

$$R' = \frac{2R_s}{\pi d}$$

$d = 2$ mm

$$L' = \frac{\mu}{\pi}\ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$$

$$G' = \pi\sigma / \ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$$

$$C' = \pi\varepsilon / \ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$$

$$\ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$$

$$= \ln\left(\frac{1\times10^{-2}}{2\times10^{-3}} + \sqrt{\left(\frac{1\times10^{-2}}{2\times10^{-3}}\right)^2 - 1}\right)$$

$$= \ln\left(5 + \sqrt{5^2 - 1}\right)$$

$$= \ln\left(5 + \sqrt{24}\right)$$

$$= 2.292$$

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

$$R_s = \sqrt{\pi (10^9 Hz)(1.0\ H/m)/(4.1 \times 10^7\ S/m)}$$

$$R_s \doteq 8.754$$

$$R' = \frac{2R_s}{\pi d}$$

$$= \frac{2(8.754)}{\pi(2 \times 10^{-3})}$$

$$= 2786.48\ \Omega/m$$

$$R' \doteq 2.8\ k\Omega/m$$

$$L' = \frac{\mu}{\pi} \ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$$

$$= \frac{4\cancel{\pi} \times 10^{-7}}{\cancel{\pi}}(2.292)$$

$$L' = 9.168 \times 10^{-7}\ H/m$$

$$L' \doteq 9.2 \times 10^{-7}\ H/m$$

$$G' = \pi\sigma / \ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$$

$$= \pi\, 3.2 \times 10^{-6} / 2.292$$

$$= 4.3862 \times 10^{-6}\ S/m$$

$$G' \doteq 4.39 \times 10^{-6}\ S/m$$

$G' \doteq 4.39 \times 10^{-6}\ S/m$

$C' = \pi \varepsilon / \ln\left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right)$

$= \pi 2.7 \cdot 8.854 \times 10^{-12} / 2.292$

$= 3.2767 \times 10^{-11}\ F/m$

$C' \doteq 3.3 \times 10^{-11}\ F/m$