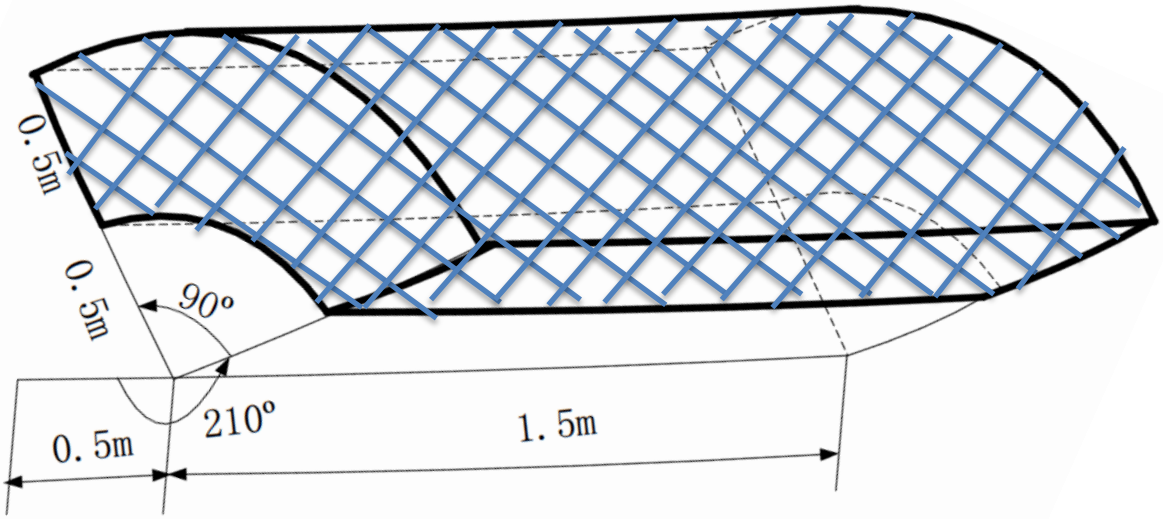
Questions:

1. (20 points) Give the definitions of the following terms in robotics.

1) Accuracy: The maximum error between the desired location of the end-effector (normally commanded by the user program) and its actual location.
2) Repeatability: A measure of the robot's ability to position the end-effector in the same place repeatedly.
3) Major axes: The first three joints of the robot.
4) Minor axes: The joints of the robot after the major axes.

2. (35 points) Draw the workspace for a planar PRP robot with the following properties: range of first joint is 0.5m-2m, range of second joint is $210^\circ - 300^\circ$, range of third joint is 0.5m-1m.



3. (45 points)

a) Calculate the transformation matrix representing a series of transformations:
Step 1. a rotation of -45° about the Y-axis of reference frame,
Step 2. a translation of [2, -4, 5] along the X, Y, and Z axes of the current frame,
Step 3. a rotation of 60° about X-axis of reference frame.

Solution for a):

We can start by using an identity matrix as a place keeper. So we have:

$$T = I$$

After step 1 the result is:

$$T = Rot(Y, -45^\circ) * I$$

After step 2 the result is:

$$T = Rot(Y, -45^\circ) * I * Trans(2, -4, 5)$$

After step 3 the equation for the desired combined transformation is:

$$T = Rot(X, 60^\circ) * Rot(Y, -45^\circ) * I * Trans(2, -4, 5)$$

The matrix resulting from equation about equals:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{4} & 0.5 & -\frac{\sqrt{6}}{4} & 0 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} & 0.5 & -\frac{\sqrt{6}}{4} & -(2+\frac{7\sqrt{6}}{4}) \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & \frac{7\sqrt{2}}{4} - 2\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.7071 & 0 & -0.7071 & -2.121 \\ -0.615 & 0.5 & -0.615 & -6.287 \\ 0.3535 & 0.866 & 0.3535 & -0.9895 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) In Step 3, to achieve the same transformation (same physical movement), what is the equivalent transformation matrix relative to the current frame in step 3? (Note: For question (b), symbolic solution is ok.)

Solution for (b):

After step 3 the equation for the desired combined transformation is:

$$T = Rot(Y,-45^\circ) * I * Trans(2,-4,5) * A$$

A can be calculated by

$$A = (Rot(Y,-45^\circ) * Trans(2,-4,5))^{-1} * T$$

The matrix resulting from equation about equals:

$$A = \left(\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)^{-1} * \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} & 0.5 & -\frac{\sqrt{6}}{4} & -(2+\frac{7\sqrt{6}}{4}) \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & \frac{7\sqrt{2}}{4}-2\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ 0 & 1 & 0 & 4 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & -\frac{7\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} & 0.5 & -\frac{\sqrt{6}}{4} & -(2+\frac{7\sqrt{6}}{4}) \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & \frac{7\sqrt{2}}{4}-2\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{6}}{4} & -\frac{3}{4} & -\frac{13}{4}+\sqrt{6}+\frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} & 0.5 & -\frac{\sqrt{6}}{4} & 2-\frac{7\sqrt{6}}{4} \\ \frac{3}{4} & \frac{\sqrt{6}}{4} & -\frac{1}{4} & \frac{1}{4}-\sqrt{6}-\frac{7\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Note: Just symbolic solution is OK to get full mark for (b).)