H4

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ENGPHYS 2E04

Introduction

The circuit below was inspired by the sample lab, modelled to fit the specifications outlined in the deliverables. The circuit was solved analytically and digitally (using Multisim) to find the Bode plot, transfer function, centre frequency, and -3dB frequency. Physically (using the Hantek and Excel), an estimation of the bode plot was found, as well as the centre and -3dB frequencies.

Problem Framing:

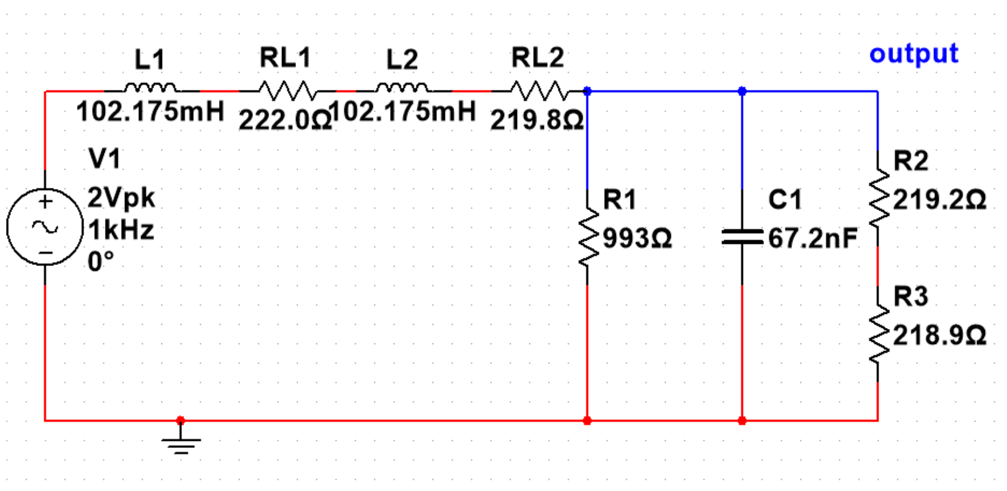


Figure 1: Circuit Diagram

The goal is to determine the bode plot, -3dB frequency, centre frequency, and the gain at those frequencies at the output node (blue). This was done analytically with Maple, digitally with the Multisim single frequency AC sweep, and physically by taking the gain and phase across the output node at a variety of frequencies and plotting these values.

The phenomena observed during this lab are primarily due to the behavior of inductors and capacitors at different frequencies. When at low frequency, the capacitor behaves like an open since it limits to infinite impedance and acts as a short at high frequencies. The opposite is true for inductors: they act as shorts at low frequency, and limit to infinite impedance at high frequencies.

Analytical Solution:

To determine the transfer function of this circuit at the output node, voltage dividers were used. The transfer function is defined as the change in gain with respect to frequency:

$$H \equiv \frac{V_{out}}{V_{in}}$$

Where Vout and Vin are functions of frequency.

The impedance of leftmost elements in series (L1, L2, RL1, RL2) are represented by Zleft, while the impedance of the elements on the right (R1, R2, R3, C1) are represented by Zpar. The total impedance is equal to these two groups in series. Note that RL1 and RL2 are representations of the internal resistance of the inductors.

The impedances of each group were derived:

$$Z_{par} = ZC1||R1||(R2+R3) \quad \rightarrow \quad Z_{par} = \left(\frac{1}{ZC1} + \frac{1}{R1} + \frac{1}{R2+R3}\right)^{-1}$$

$$Z_{left} = ZL1 + ZL2 + RL1 + RL2$$

With these formulas, the voltage divider at the output node can be derived.

$$V_{out} = V_1 * \frac{Z_{par}}{Z_{par} + Z_{left}}$$

In the case of this circuit, the input voltage is the voltage from the AC current source, V1. These values were substituted into the transfer function and simplified with maple.

```
restart :
L1 := 102.175e−3 : L2 := 102.175e−3 :
RL1 := 222 : RL2 := 219.8 :
C1 := 67.2e−9 :
R1 := 993 : R2 := 219.2 : R3 := 218.9 :
V1 := 2 : Vin := V1 :
```

$$ZC1 := \frac{1}{I \cdot 2 \cdot \mathrm{Pi} \cdot f \cdot C1} : ZL1 := I \cdot 2 \cdot \mathrm{Pi} \cdot f \cdot L1 : ZL2 := I \cdot 2 \cdot \mathrm{Pi} \cdot f \cdot L2 :$$

$$Zpar := \frac{1}{\frac{1}{ZC1} + \frac{1}{R1} + \frac{1}{R2 + R3}} : Zleft := ZL1 + ZL2 + RL1 + RL2 :$$

$$Vo := \frac{Zpar}{Zpar + Zleft} V1 :$$

$$H := \frac{Vo}{Vin} : H := simplify(\%);$$

$$\boxed{H := \frac{1.000000 \times 10^{6}}{2.453359961 \times 10^{6} - 0.5421302639 f^{2} + 4410.328056 \mathrm{I} f}}$$

Figure 2: Maple code for transfer function

Type of circuit

We can qualitatively determine the type of the circuit by observing its structure. At low frequencies inductors, L1 and L2, act as a short and the capacitor, C1, acts as open (since it would have infinite impedance). In our case, at low frequency voltage would be allowed to reach the output node.

At high frequencies, the inductors act as opens and the capacitor acts as short, thereby stopping all voltage from reaching the output node. Therefore, by observation, this circuit is a low-pass filter.

We can confirm this quantitatively, by observing the behavior of the transfer function and taking the limit as frequency approaches 0 and infinity. The maximum gain occurs when frequency = 0, since it can be observed from the transfer function that *H* is inversely proportional to frequency. Also, the minimum gain will occur when frequency approaches infinity. In this case,

$$H_{max} = \frac{10^6}{2.453359961*10^6 - 0 + 0} = 0.4076042716 = -7.795225466 \text{ dB}$$

$$H_{min} = \lim_{f \to \infty} H = \frac{10^6}{(\infty)} = 0 \ \ (= -\infty \ dB)$$

The centre frequency for a low pass filter is the frequency that results in the highest gain. As proven above, the highest gain occurs at a frequency of 0 Hz. Therefore, the centre frequency is also 0 Hz.

-3db Frequency

The -3dB (or cutoff) frequency of a low-pass filter is the frequency at which half of the input voltage is being passed through to the output.

$$|H_{cutoff}| = \frac{|H_{max}|}{\sqrt{2}}$$

This frequency was solved for by using the *fsolve* function in maple (see appendix for maple code). Results are rounded to 4 significant digits:

$$H_{cutoff} = 0.2882197446 \approx 0.2882 \approx -10.80 dB$$

$$\omega_{cutoff} = 3750.197559 \approx 3750 \, rad/s$$

$$f_{cutoff} = 596.8624790 \, Hz \approx 596.9 \, Hz$$

The -3dB frequency is also the frequency when the gain is 3dB less than the gain at the centre frequency. If we subtract 3dB from -7.795, we get -10.795. This value rounds to -10.80 when accounting for significant digits, approximately matching our analytical result.

Bode plot

Using the *plots* package in maple, the real and imaginary parts of the transfer function were plotted to create a bode plot, representing frequency and phase as functions of time. Note that the gain at the cutoff frequency is also represented by the green line, and that the curves are in terms of frequency (not ω) for easier comparison to multisim and physical solutions.

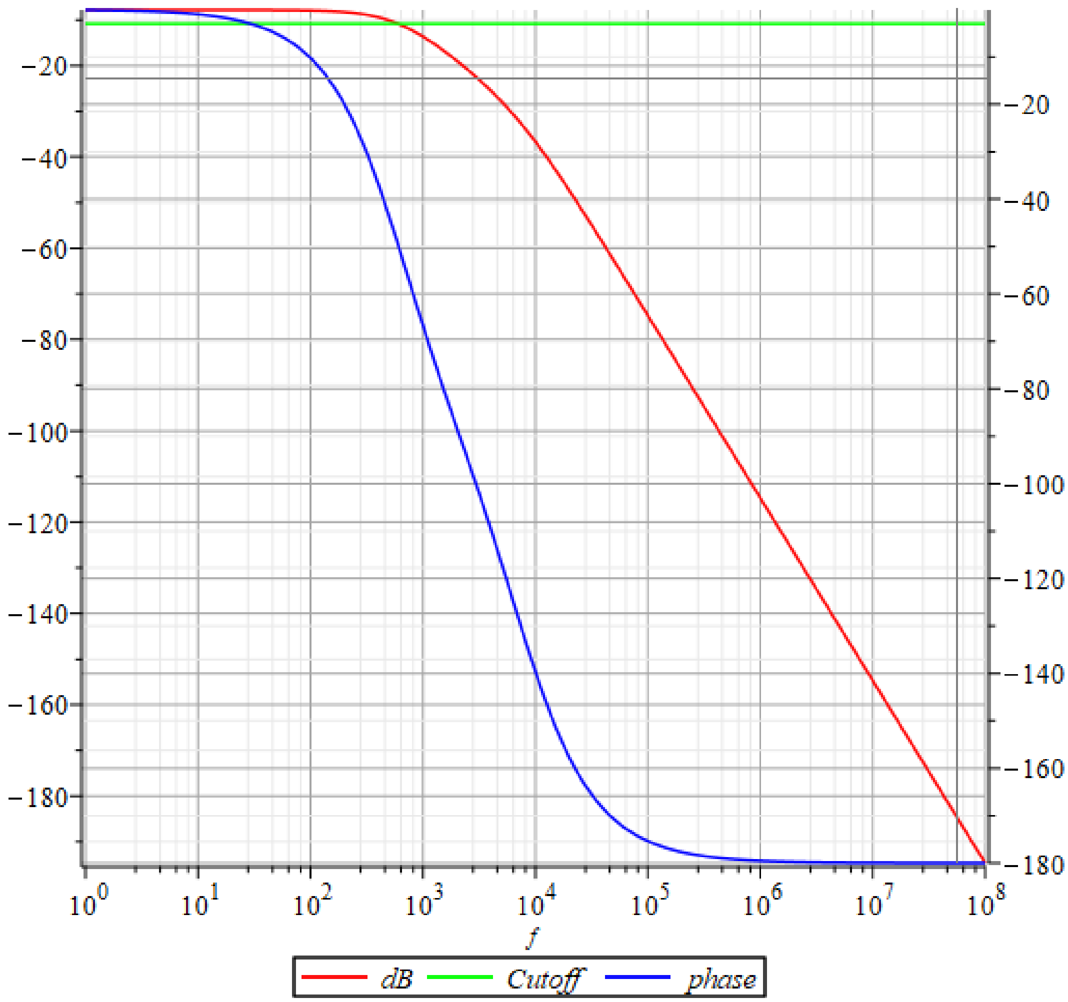
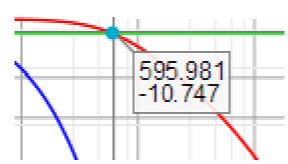


Figure 3: Bode plot made using Maple



Observing the graph, we can confirm that we do in fact have a low pass filter, since the gain is high at low frequencies and drops sharply as frequency increases. We can also confirm that the maximum gain does occur at a frequency of zero, confirming our calculated centre frequency. In terms of the cutoff frequency, there is in fact only one -3dB frequency (this is correct since our filter is lowpass), and the values do line up with the calculated frequency (see the approximate point of intersection of the green cutoff line and red transfer function below). Therefore, the graph matches our mathematically calculated properties of the circuit.

Observing the phase curve, we can see that when the impedance of capacitor is dominating and the inductor is acting as a short (low frequency), the phase is small (close to zero). The opposite is true at high frequency: when the inductor is cutting voltage to the output node, the magnitude of the phase is very high (180 degrees), which means the very (low voltage) at the output node is lagging the input voltage by a half cycle. This makes sense, since we know that the voltage across an inductor lags the input voltage by $\frac{\pi}{2}$ (equal to 180 degrees), so this curve behaviour demonstrates the dominance of the inductors at high frequency determining the lag of the voltage at the output node.

Multisim Solution

The following configuration was used in multisim to acquire graphs and make all measurements:

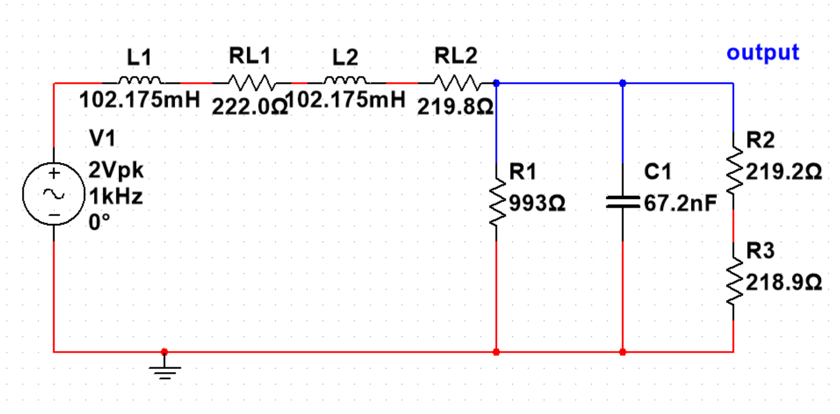


Figure 4: Multisim Circuit

Rather than the bode plotter, Single Frequency AC sweep was used to create the bode plot below.

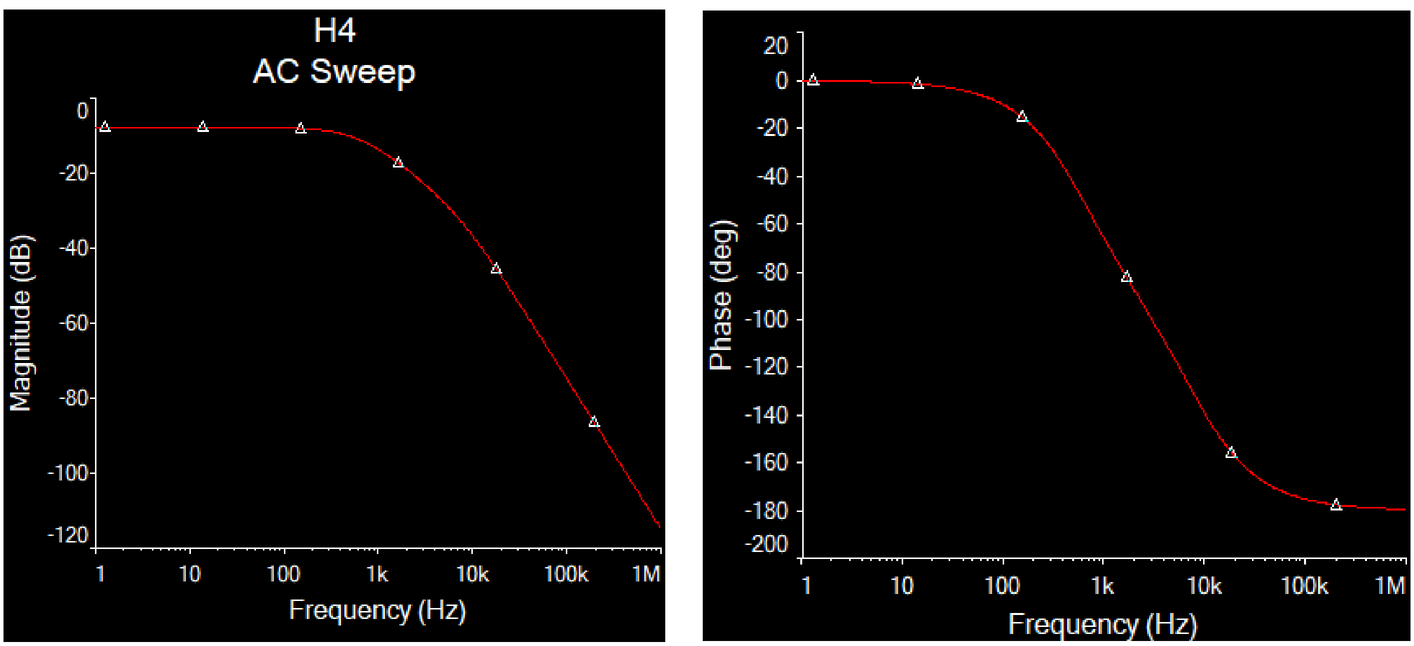


Figure 5: Multisim Bode plot

At first glance, the shape of both curves matches the graphs acquired in the analytical solutions exactly. The gain plot approaches just above -10 dB as frequency approaches 0, and $-\infty\ dB$ as frequency approaches infinity. The same can be said about the phase graph: the phase at low frequency is near 0, and the phase at high frequency approaches -180 (or a half cycle). To further confirm the properties of the virtual bode plots, we can employ cursors to measure the amplitude at the centre frequency and cutoff frequency. This would validate the digital results matching the analytical results.

Centre frequency

We defined previously the centre frequency to be the frequency at an amplitude of -7.795 dB. We can set the y-value of the cursor to -7.795 to find the associated frequency to see if it matches our previous answer:

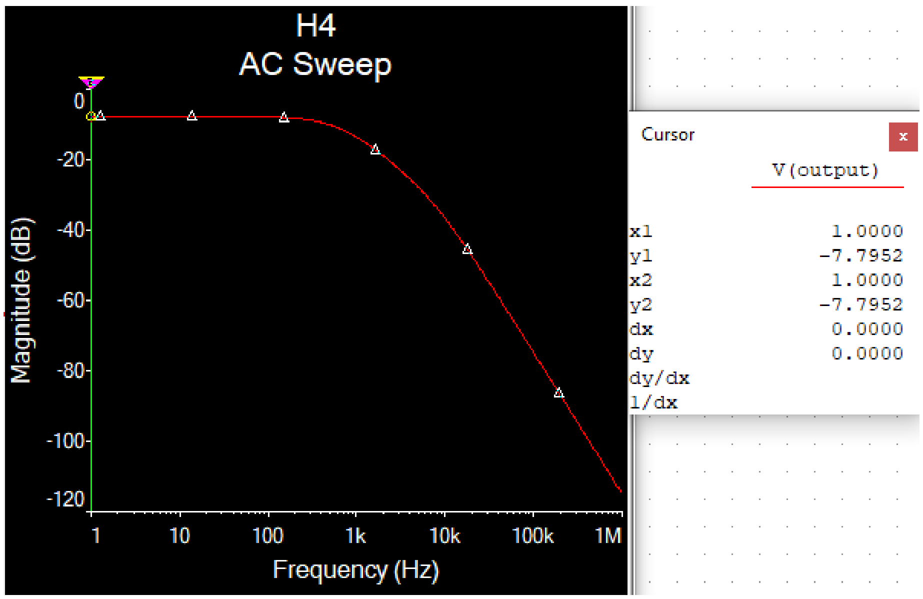


Figure 6: Centre frequency measurement in Multisim

Just like in the analytical solution, the maximum gain on this graph occurs at the minimum frequency value of the graph, 1 Hz. Overall, we can observe that gain increases as the graph approaches 0.

Despite this observation, due to the logarithmic nature of the plot, we can't measure the gain at exactly zero. We can approximate the gain at 1 Hz and 0 Hz to be the same, but we can adjust the plot to see the frequency at exactly 0 Hz.

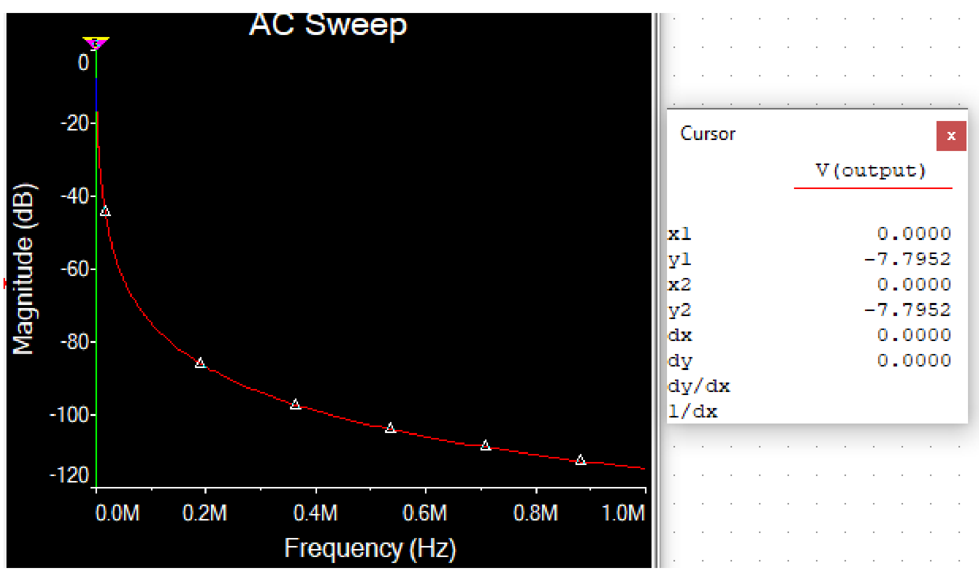


Figure 7: Centre frequency measurement with linear scale

If we represent the frequency linearly (see picture above), we see the same gain (at a precision of 4 decimal places) occurs at 0.0 Hz, confirming the estimation that centre frequency is 0 Hz.

Note that there is no uncertainty applied to the yielded value. This is because the **exact** value was entered as the position of the cursor, so there is no error due to the increment the cursor moves at.

-3db Frequency

From the analytical solution the gain at -3dB frequency was calculated to be approximately -10.80 dB.

To find the -3dB frequency, we can set the cursor's y-value to the -3db gain, and find the associated x-value:

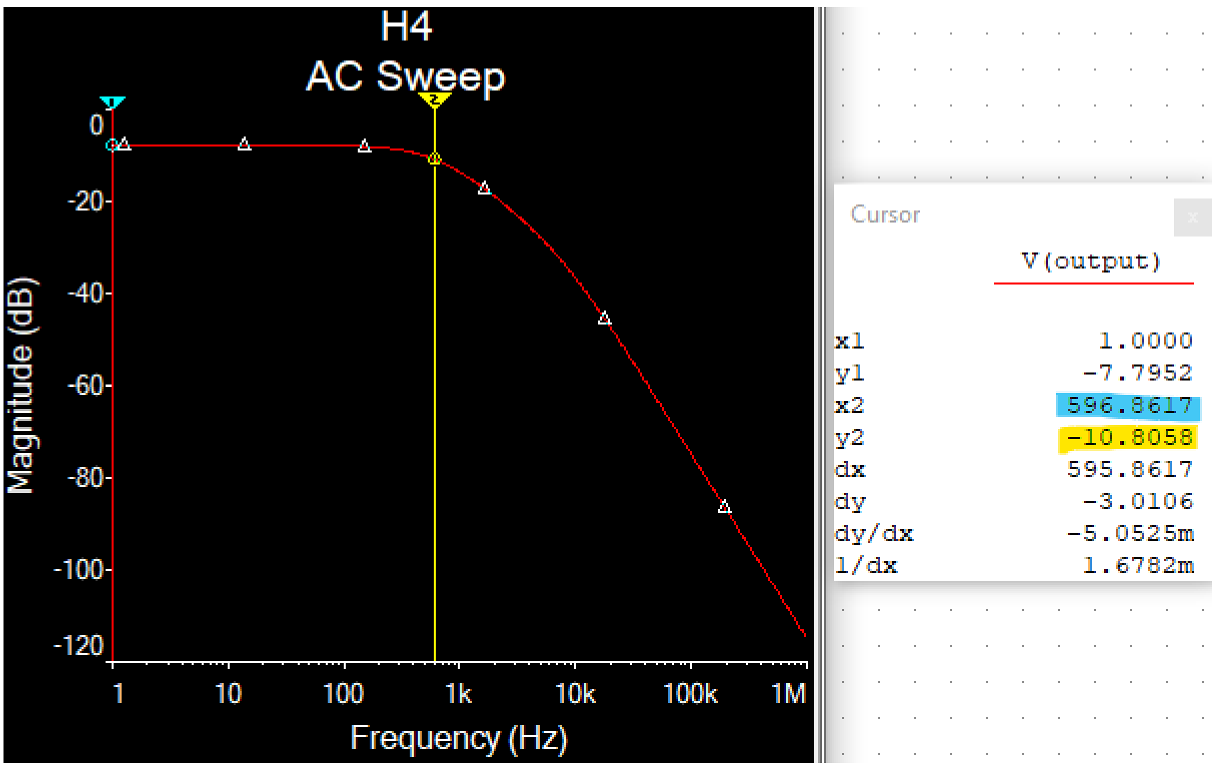


Figure 8: -3dB frequency in Multisim

The yielded -3dB frequency is approximately 596.9 Hz, which also approximately matches the analytical results.

Both the -3dB frequency and the centre frequency match in the analytical and multisim solutions. Also, as previously mentioned, the analytical and digital plots have the same behaviour at frequencies of zero and infinity, thus confirming that the bode plot in multisim is the exact same as the analytical bode plot.

Below is a table summarizing the results from the digital solution and comparing to analytical results (to four significant digits). Notice that they match exactly.

	Centre Frequency	*-3dB Frequency*
Digital	0 Hz	596.9 Hz
Analytical	0 Hz	596.9 Hz

Table 1: Summary of digital and Analytical results

Physical Solution

All measurement were made using the following configuration. The blue wire is at the output node and is connected to the Hantek probe (channel 1), the green wire is at the input node and is connected to the Hantek probe (channel 2).

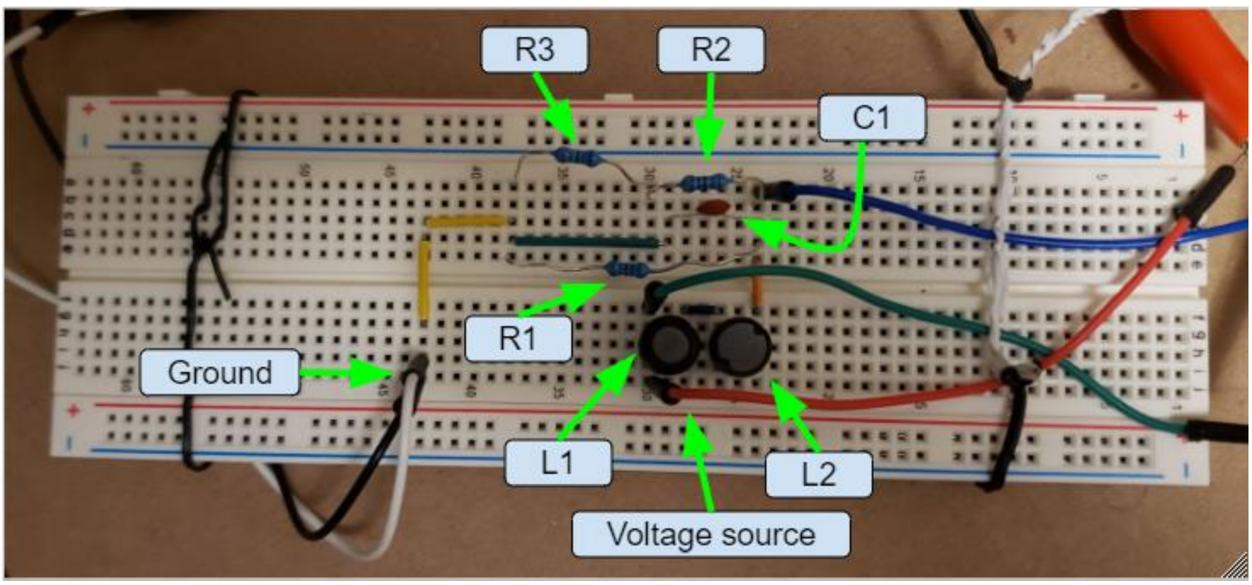


Figure 9: Physical circuit configuration

For the physical solution, the amplitude and phase of the output voltage relative to the input voltage were measured at various frequencies. Through Excel, these points were plotted to create a rough bode plot. For measurement on the Hantek, the peak-to-peak measurement function was used to find the amplitude of the waves, and the time cursors were used to find the phase difference. Due to the methods of measurement, there is the cursor error in addition to the standard 5% on the phase difference, while there is only 5% error on the amplitude since the Hantek is measuring it directly.

Below is a table of values measured to find gain at different frequencies (to four significant digits):

Freq (Hz)	Amplitude in	Amplitude out	Amp. Difference	Gain (decibels)
10	1.94	0.8	0.412	-7.694
300	1.96	0.68	0.347	-9.195
596.9	2	0.52	0.260	-11.70
1000	2	0.36	0.180	-14.90
1300	2	0.296	0.148	-16.59
3000	2	0.13	0.065	-23.74
8000	2.02	0.0384	0.01901	-34.42
40 000	2	0.003	0.0015	-56.48

Table 2: Hantek gain Measurements

In addition to the same data for the phase including uncertainty (see appendix D for uncertainty calculations in excel):

Freq (Hz)	Time difference	Time uncertainty	Phase in degrees	Phase uncertainty
10	0.00	4.00E-03	0.00	0.00
300	-2.80E-04	2.60E-05	-30.24	2.81
596.9	-2.40E-04	2.80E-05	-51.57	6.02
1000	-2.00E-04	1.00E-05	-72.00	3.60
1300	-1.60E-04	1.20E-05	-74.88	5.62
3000	-1.00E-04	5.00E-06	-108.00	5.40
8000	-4.44E-05	-2.20E-07	-127.87	0.63
40 000	-1.21E-05	-2.04E-07	-173.95	2.94

Table 3: Hantek phase and phase error Measurements

The data above can be summarized into a table and used to create a bode plot, and to find the -3dB frequency. Note the 5% error bars on the frequency of both graphs since frequency is also being measured directly by the Hantek (see pictures of measurements in appendix).

Freq	Gain (dB)	Phase (deg)	Gain uncertainty	Phase uncertainty
10	-7.6942349	0	0.384711743	0
300	-9.1949432	-30.24	0.459747159	2.808
596.9	-11.700533	-51.57216	0.585026652	6.016752
1000	-14.89455	-72	0.744727495	3.6
1300	-16.594766	-74.88	0.829738285	5.616
3000	-23.741733	-108	1.187086643	5.4
8000	-34.420403	-127.872	1.721020145	0.6336
400	-56.478175	-173.952	2.823908741	2.9376

Table 4: Summary of Hantek Gain and Phase measurements

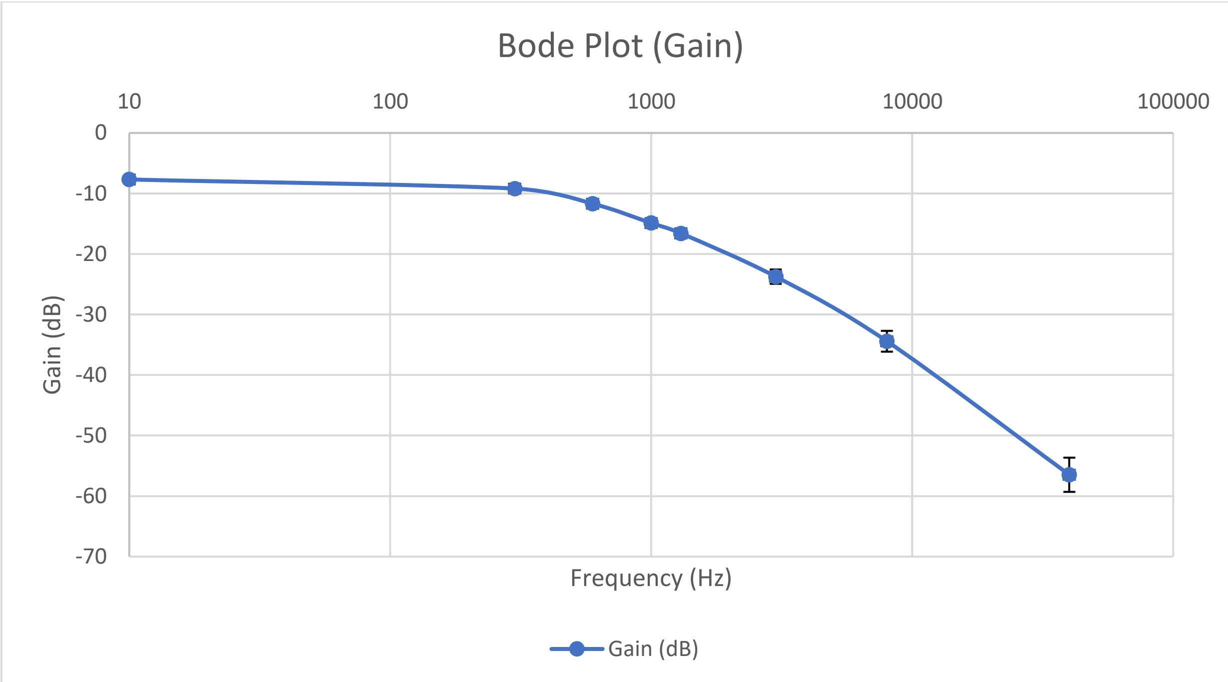


Figure 10: Hantek Bode plot (Gain) via Excel. Note: error bars are present but difficult to see.

Observing the bode plot, it almost exactly matches the shape of the bode plot in the analytical and digital solutions. It can be observed that the gain increases as frequency approaches 0, and gain decreases as frequency approaches infinity (again, matching previous solutions).

Though we cannot make measurements at 0 Hz, we instead made measurements 10 Hz, which will be approximated to 0 Hz since we know that gain increases as the frequency limits to zero. In addition to this, we know from previous solutions that there is almost no change in gain from 0 Hz to 20 Hz (all gain in this interval is approximately -7.9 dB), thereby allowing the gain at 0 Hz to be approximated by the gain at 10 Hz.

So, the centre frequency is approximately 0 Hz since that is demonstrated to be the point of highest gain on the physical plot. Specifically, the gain at a frequency of (approximately) 0 on this graph yields a gain of around -7.7 dB, matching the centre frequency from previous solutions (≈ -7.8 dB) when accounting absolute error of 0.38 Hz.

To verify the -3dB frequency, a measurement was taken on the Hantek at the frequency determined to be the -3dB frequency in the previous solutions: approximately 596.9 Hz. The analytical and Multisim solution yielded a gain of around -10.80 dB at this frequency, while the physical solution yielded a gain of around -11.70 dB.

Though these values are very close, they do not match within the error margins of the Hantek. This could be due to many factors unaccounted for, such as poor contact with the breadboard, or the Hantek’s inability to measure inductance of the physical inductors.

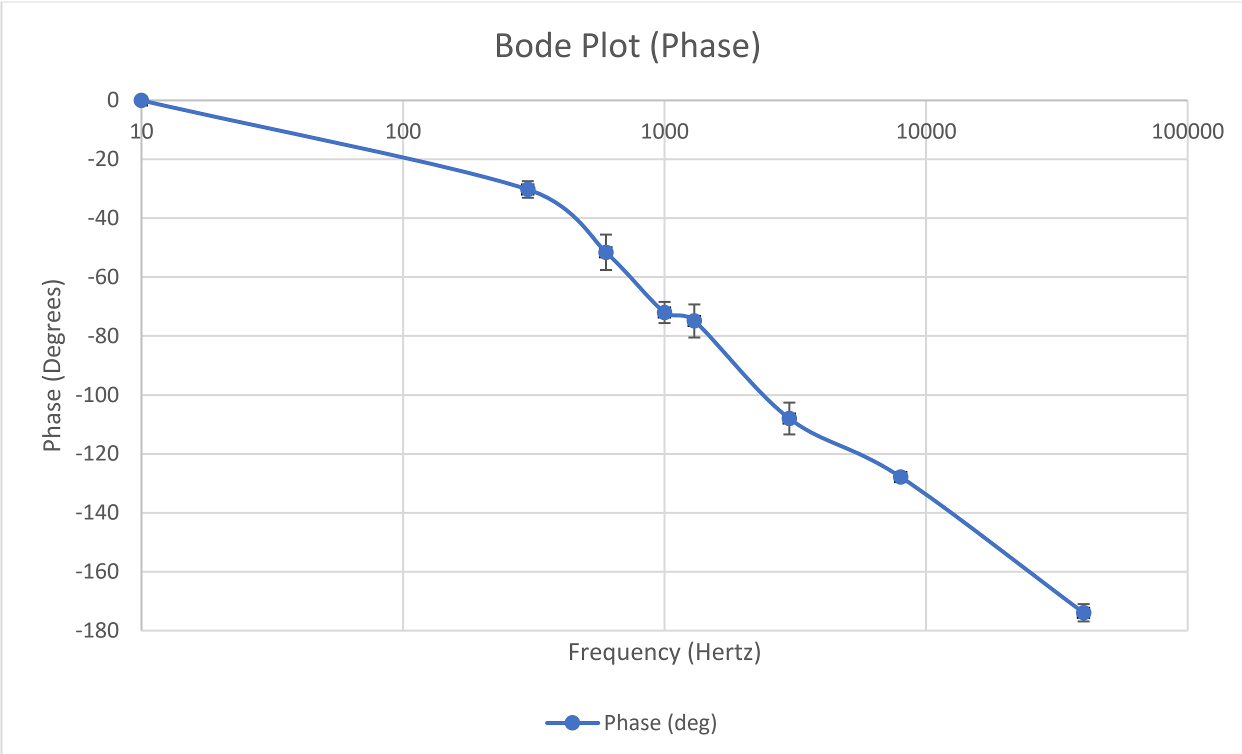


Figure 11: Hantek Bode plot (phase) via Excel

The phase plot (while much rougher) is also similar in shape and properties to the previously acquired phase plots. The phase clearly approaches 0 as frequency approaches zero, and the phase approaches 180 as frequency approaches infinity.

Discussion and Analysis

Below is a table summarizing results from the 3 solutions.

	Centre Frequency	Max Gain	-3dB Frequency	Plot Similar to Analytical
Analytical	0 Hz	-7.795 dB	596.9 Hz	N/A
Digital	0 Hz	-7.795 dB	596.9 Hz	Yes
Physical	≈ 0 Hz	≈ (-7.694 ± 0.385) dB	≠ 596.9 Hz (gain was -11.70 dB instead of -10.80 dB)	Gain = Yes Phase = Roughly

Table 5: Summary of results from all measurement methods

As demonstrated previously, the analytical and digital solutions were the same in all aspects. The -3dB frequencies are both 596.9 Hz (4 significant digits), and the centre frequency/the frequency of highest gain were also precisely the same: a frequency of 0 Hz and a max gain of -7.795 dB. The plots of phase and gain (by visual observation) were also very similar to one another.

As for the physical solution, getting a smooth graph is difficult when only measuring at 9 frequencies, and it is not possible to measure the gain/phase at 0 Hz or greater than 100 kHz. For this reason, the shapes of the physical plots do not look exactly like the previous solutions but does generally follow the same trends and behaviour. On the physical gain and phase plot, behaviour when frequency limits to 0 Hz and infinity is the same as previous plots.

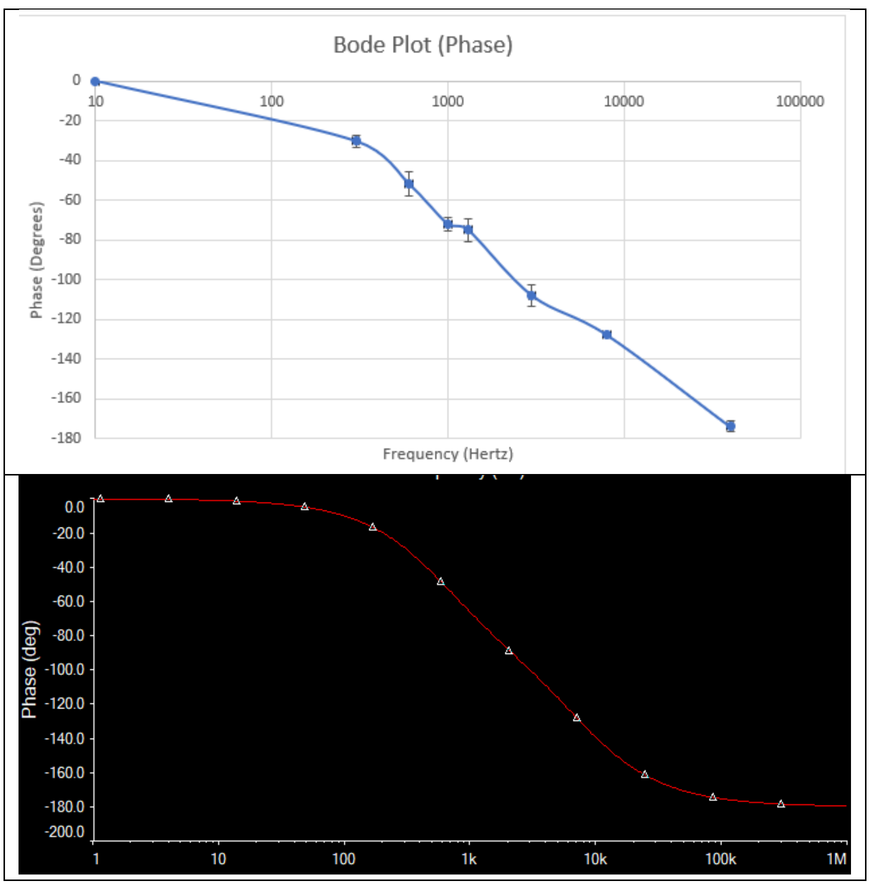


Table 6: Side-by-side of Hantek and Multisim Bode plot (Phase)

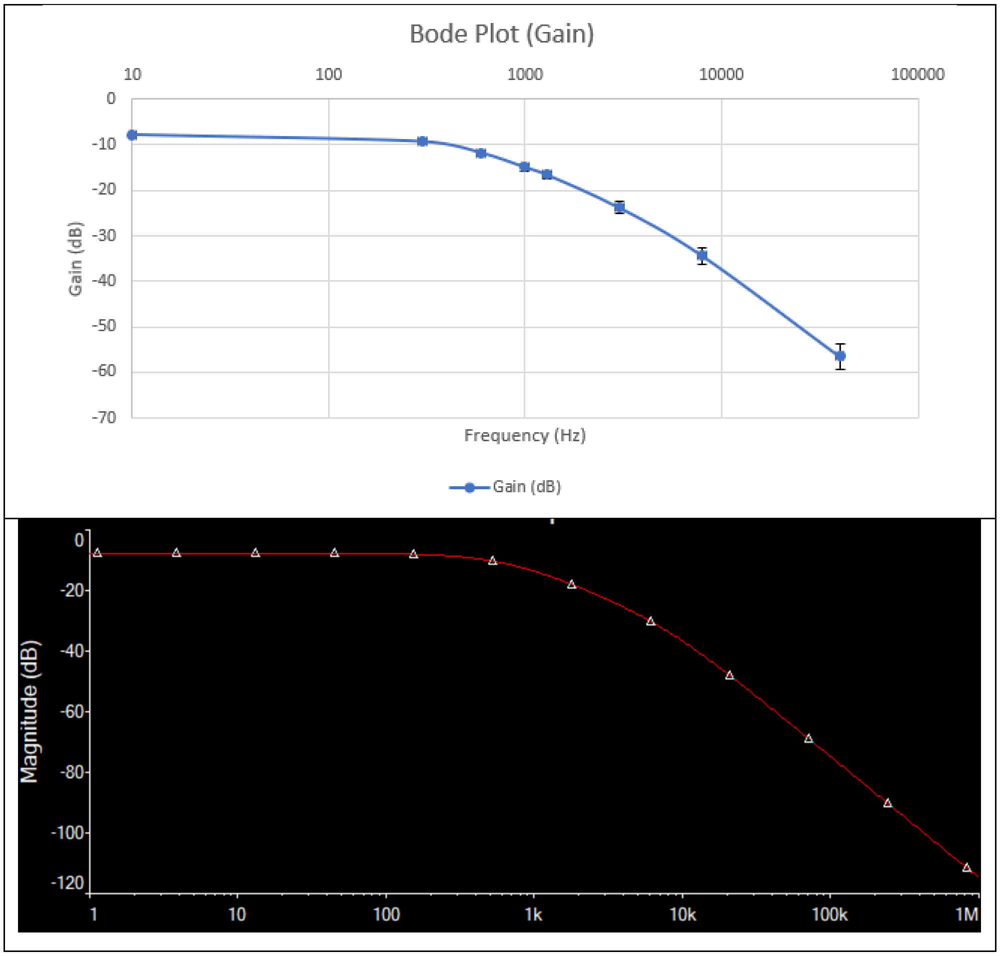


Table 7: Side-by-side of Hantek and Multisim Bode plot (Gain)

The max gain of the physical solution is approximated to be (-7.694 ± 0.385) dB @ 0 Hz (measurement was taken at 10 Hz), which is also the centre frequency. This matches the previous results within error margins. As for the -3dB frequency, when finding the gain at the previously derived -3dB frequency, the gain was (-11.70 ± 0.585) dB, which is similar to previous results, but not within error margins.

Error

There are many causes for error in this lab that could have caused discrepancies in the physical results (since in theory there should be no error in the analytical and digital results. Firstly, the Hantek is unable to measure inductance, so the inductance of the inductors in the physical circuit were assumed in the other solutions to be an estimation provided by Mr. Johnasson. This means that for all we know the physical results could exactly match the other results if we were able to measure and substitute the actual inductances into the analytical and multisim solutions. Another source for error is poor contact with the breadboard. Factors like dust are not accounted for in our calculations and could potentially cause poor contact between elements in the circuit. In addition, the nature of the capacitor could be a source of error, since Mr. Johnasson mentioned previously that they are very sensitive, and the measured capacitance can change depending on the position of the capacitor's poles. So, it is possible the capacitance was not measured correctly. Finally, the Hantek's error. Though it was accounted for it is still worth mentioning that at very low and high frequencies it is difficult to measure accurately with the cursors because of the clarity of the signal. When the signal becomes "fuzzy" it becomes difficult to measure phase.

Conclusion

The goal was to find the transfer function, bode plot, -3dB frequency, and centre frequency at the output node of the circuit using AC sweep, mathematical calculations, and using the Hantek and breadboard. While the analytical and digital solutions were exactly alike, all methods showed similar results. The shapes of the plots were the same (roughly in the case of the physical solution), and the centre frequencies were all the same as well (within uncertainty). However, the -3dB frequency of the physical solution did not match the other solutions since the gain at the previously calculated cutoff frequency (596.9 Hz) differed between the physical and the multisim/analytical solutions. This could be due to a multitude of error causes, discussed in depth in the "Discussion and analysis" section.

I found this lab incredibly difficult to grasp. I read over the notes many times but couldn't put the pieces together in my head. I only truly understood the theory after doing the analytical component with the help of my classmates and tutorial sessions. After finishing that step, I was able to better visualize what was happening at the output node and how the theory applied to this.

During this lab I learned how capacitors and inductors can be used in real life. I can envision this type of system being used in musical applications. Specifically, audio engineers isolating sounds in a track, or speaker manufacturers restricting what frequency of sound can be played on each speaker in a speaker system. Overall, I enjoyed exploring the applications of filters and I hope to use this knowledge in a real-world setting in the future.

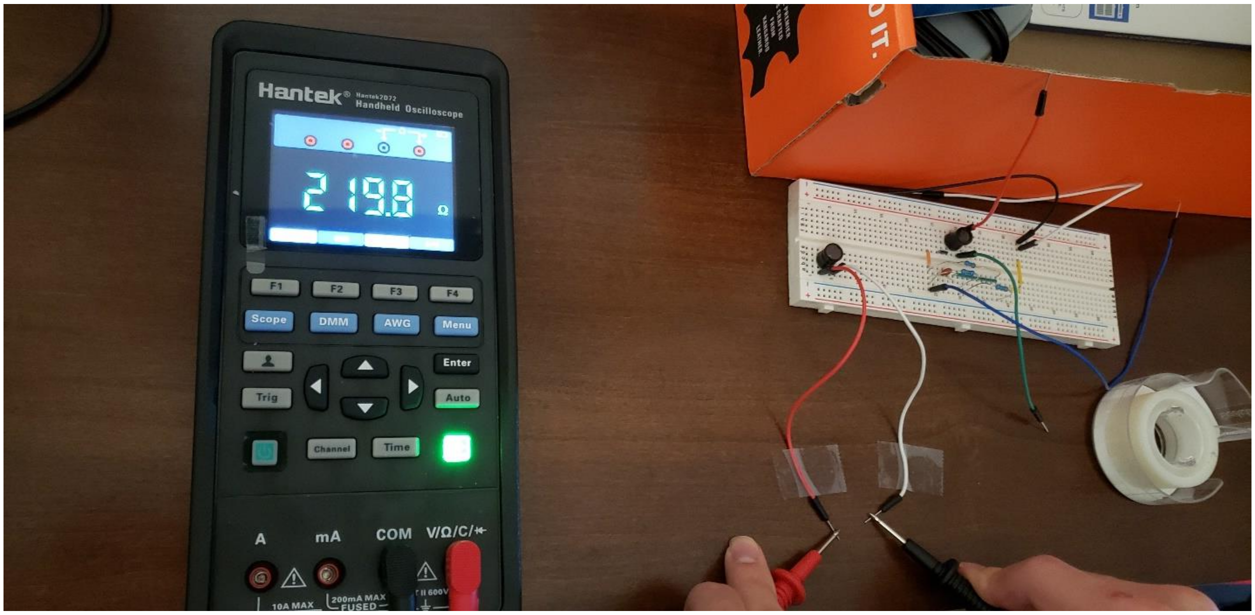
All in all, while I struggled at the beginning of this lab, I was able to gain a better grasp over the content and increase my level of understanding by completing the analytical section and observing what I had done and what the results look like.

Appendix A: Measurement pictures

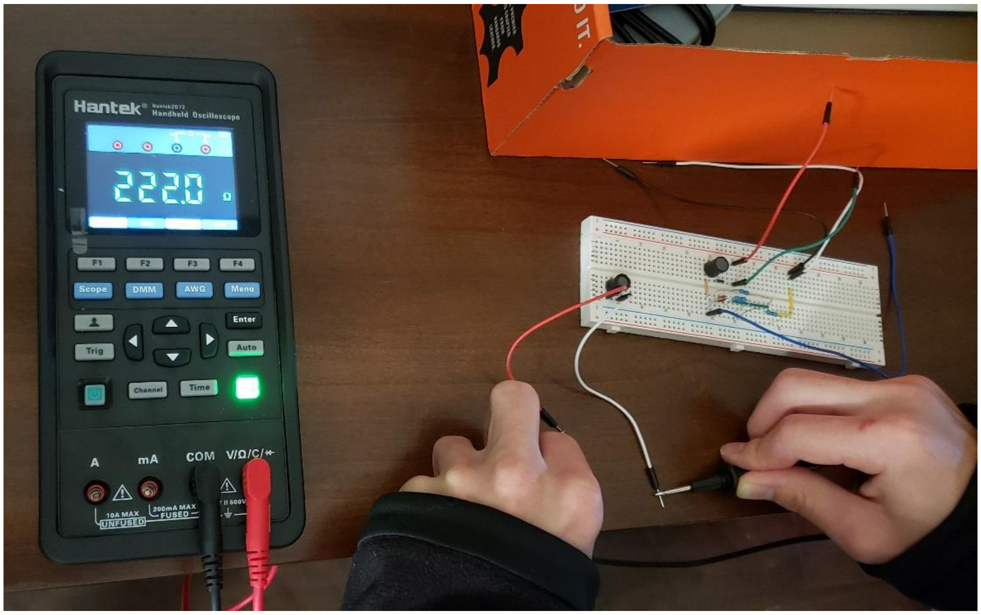
Note all measurements were taken in the same manner as in the picture below (in parallel with the component).

C1:

RL2:



RL1:


Hantek
Handheld Oscilloscope
222.0 Ω
F1
F2
F3
F4
Scope
DMM
AWG
Menu
Enter
Trig
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10A MAX
UNFUSED
200mA MAX
FUSED

R3:


Hantek
Handheld Oscilloscope
218.9 Ω
F1
F2
F3
F4
Scope
DMM
AWG
Menu
Enter
Trig
Auto
Channel
Time
A
mA
COM
V/Ω/C/
10A MAX
UNFUSED
200mA MAX
FUSED

R2:


Hantek
Handheld Oscilloscope
219.2
F1
F2
F3
F4
Scope
DMM
AWG
Menu
Enter
Trig
Auto
Channel
Time
A
mA
COM
V/Ω/C/
10A MAX
UNFUSED
200mA MAX
FUSED

R1:


Hantek
Handheld Oscilloscope
0.993
F1
F2
F3
F4
Scope
DMM
AWG
Menu
Enter
Trig
Auto
Channel
Time
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COM
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UNFUSED
200mA MAX
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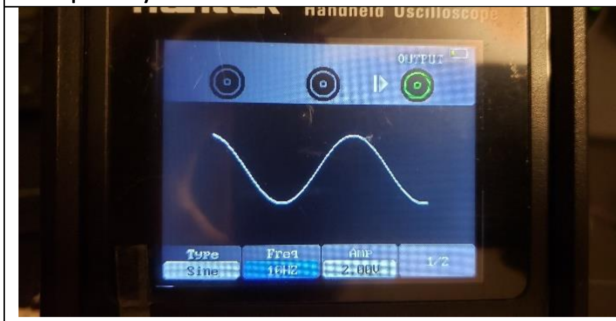
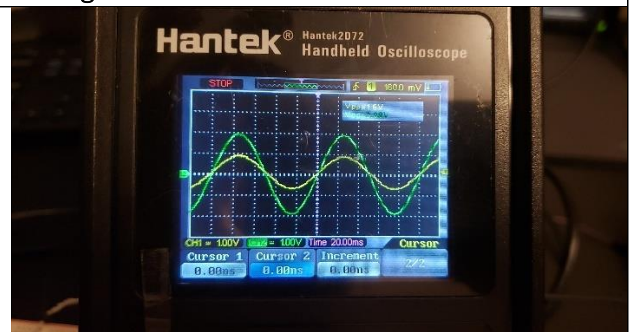
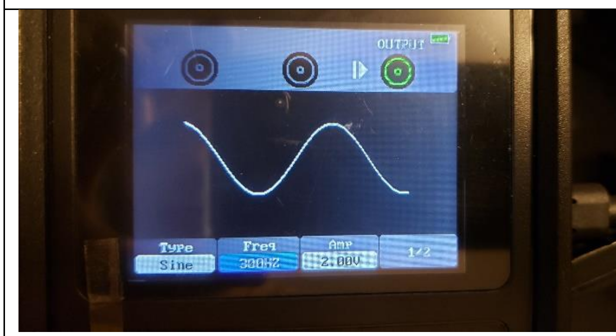
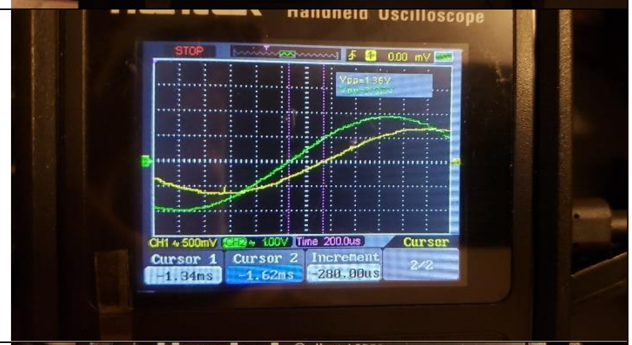
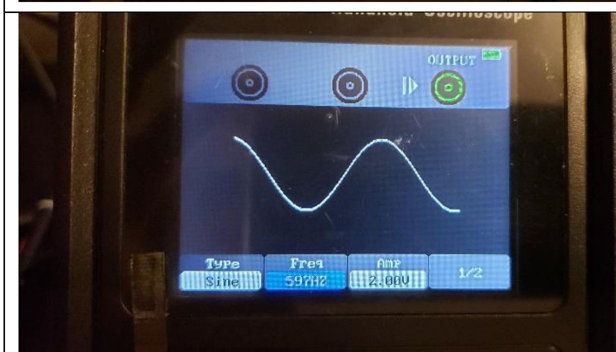
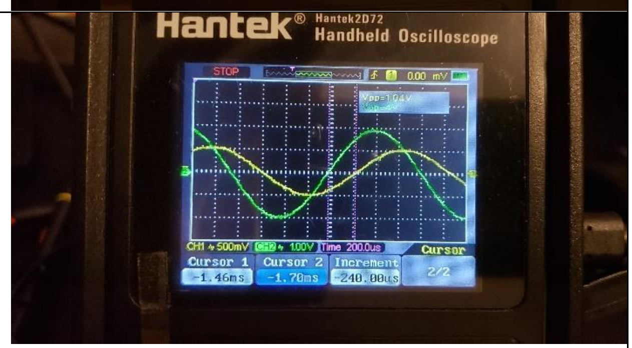
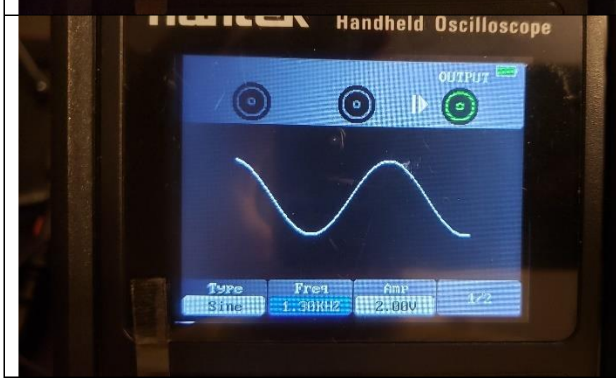
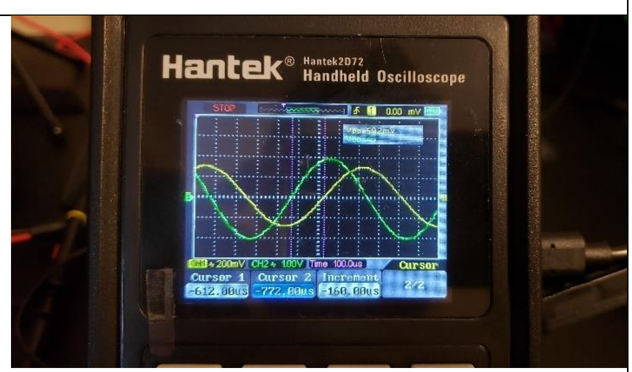
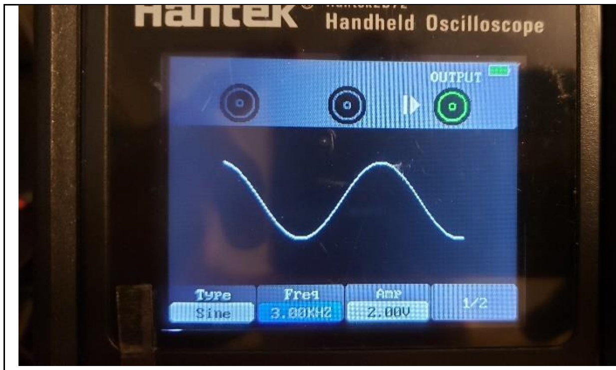
Appendix B: Physical solution measurements

Frequency	Voltage and Phase
	
	
	
	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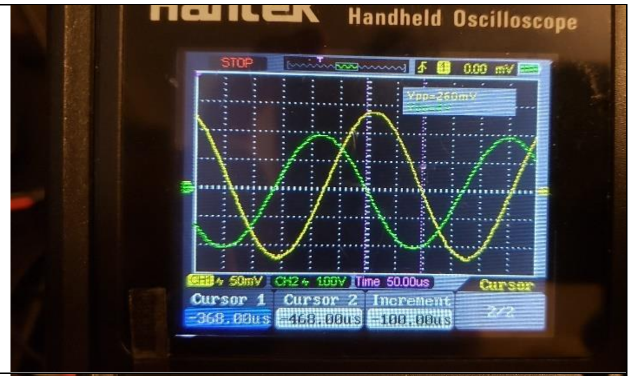
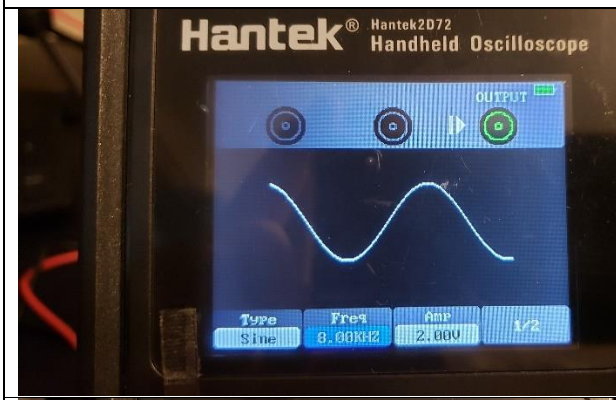
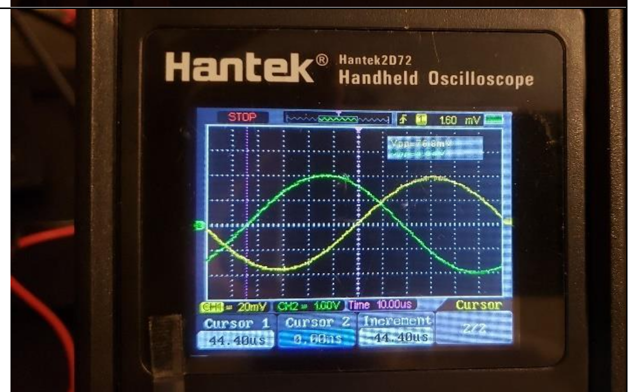
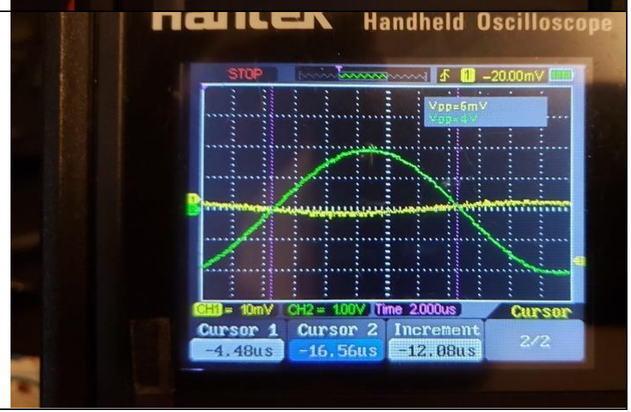
OUTPUT
Type
Sine
Freq
3.00KHZ
Amp
2.00V
1/2


Handheld Oscilloscope
STOP
0.00 mV
CH1 50mV
CH2 1.00V
Time 50.00us
Cursor
Cursor 1
-368.00us
Cursor 2
468.00us
Increment
100.00us
2/2


Hantek
Hantek2D72
Handheld Oscilloscope
OUTPUT
Type
Sine
Freq
8.00KHZ
Amp
2.00V
1/2


Hantek
Hantek2D72
Handheld Oscilloscope
STOP
160 mV
CH1 20mV
CH2 1.00V
Time 10.00us
Cursor
Cursor 1
44.40us
Cursor 2
0.00us
Increment
-44.40us
2/2


Hantek
Hantek2D72
Handheld Oscilloscope
OUTPUT
Type
Sine
Freq
40.00KHZ
Amp
2.00V
1/2


Handheld Oscilloscope
STOP
-20.00mV
Vpp=6mV
Vpp=4V
CH1 10mV
CH2 1.00V
Time 2.000us
Cursor
Cursor 1
-4.48us
Cursor 2
-16.56us
Increment
-12.08us
2/2

Appendix C: Maple code

$>$

$$\mathit{restart}:$$
$$L1 := 102.175\mathrm{e}{-3} : L2 := 102.175\mathrm{e}{-3} :$$
$$RL1 := 222 : RL2 := 219.8 :$$
$$C1 := 67.2\mathrm{e}{-9} :$$
$$R1 := 993 : R2 := 219.2 : R3 := 218.9 :$$
$$V1 := 2 : Vin := V1 :$$
$$\mathrm{omega} = 2\cdot \mathrm{Pi}\cdot f :$$

$$ZC1 := \frac{1}{I\cdot \mathrm{omega}\cdot C1} : ZL1 := I\cdot \mathrm{omega}\cdot L1 : ZL2 := I\cdot \mathrm{omega}\cdot L2 :$$

$$Zpar := \frac{1}{\frac{1}{ZC1} + \frac{1}{R1} + \frac{1}{R2 + R3}} : Zleft := ZL1 + ZL2 + RL1 + RL2 :$$

$$Vo := \frac{Zpar}{Zpar + Zleft} V1 :$$
$$H := \frac{Vo}{Vin} : H := \mathit{simplify}(\%);$$

$$H := \frac{1.0000000 \times 10^{7}}{2.453359961 \times 10^{7} - 0.1373232\,\omega^{2} + 7019.255106\,\mathrm{I}\,\omega}$$

$$> Hmax := \mathit{limit}(H, \mathrm{omega} = 0);\ Hmin := \mathit{limit}(H, \mathrm{omega} = \mathrm{infinity});\ Hmax\_db := 20\cdot \log10(Hmax);\ 20\cdot \log10\left(\frac{\mathrm{abs}(Hmax)}{\mathrm{sqrt}(2.0)}\right);$$

$$Hmax := 0.4076042716$$
$$Hmin := 0.$$
$$Hmax\_db := -7.795225466$$
$$-10.80552542$$

$$> Hco := \frac{\mathrm{abs}(Hmax)}{\mathrm{sqrt}(2.0)};\ wco := \mathit{fsolve}(\mathrm{abs}(H) = Hco, \mathrm{omega} = 1\mathrm{e}3\,..1\mathrm{e}5);\ fco := \frac{wco}{2\,\mathrm{Pi}};\ Hco\_db := 20\cdot \log10(Hco);$$

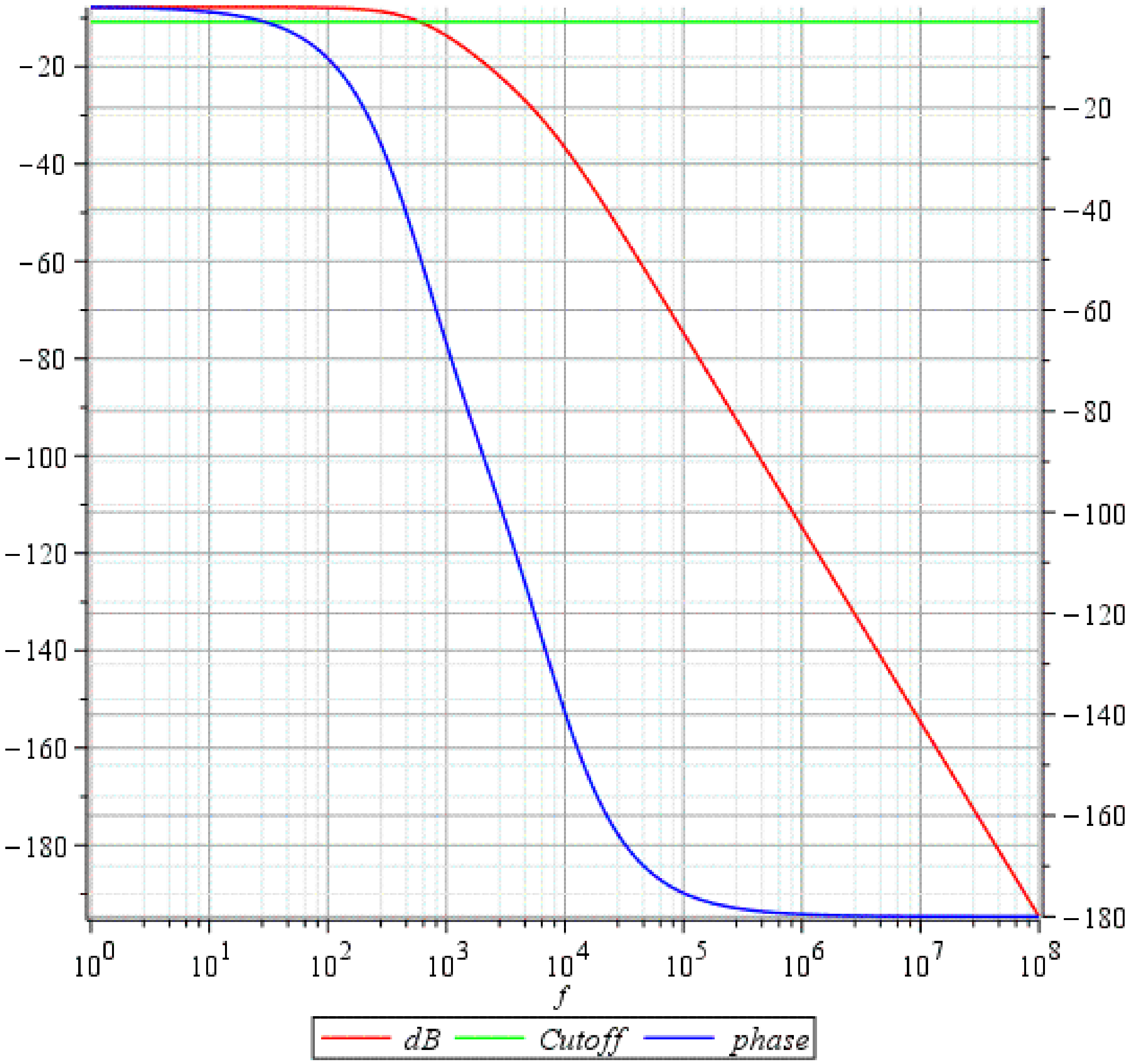
$$Hco := 0.2882197446$$
$$wco := 3750.197559$$
$$fco := 596.8624790$$
$$Hco\_db := -10.80552542$$

$>$

$>$

```
> "Plots in terms of frequency" :
omega := 2·Pi·f : Hfreq := Vo/Vin : Hfreq := simplify(%) : Hco := abs(Hmax)/sqrt(2.0) :
with(plots) :
semilogplot([20·log10(abs(H)), 20·log10(Hco)], f = 1 ..1e8, legend = ['Decibels', 'cutoff'],
    numpoints = 1000, gridlines = true, caption ="Decibels vs Frequency (Hz)") :
semilogplot(argument(H) * 180/Pi, f = 1 ..1e8, color ='blue', legend ='Phase', numpoints = 1000,
    gridlines = true, caption ="Phase vs Frequency (Hz)") :
```

```
> omega := 2·Pi·f : Hfreq := Vo/Vin : Hfreq := simplify(%) :
with(plots) :
dualaxisplot(
semilogplot([20·log10(abs(H)), 20·log10(Hco)], f = 1 ..1e8, color = ['red', 'green'], legend = [
    'dB', Cutoff]),
semilogplot(argument(H) * 180/Pi, f = 1 ..1e8, color ='blue', legend ='phase'), numpoints = 1000,
    gridlines = true);
```


−20
−40
−60
−80
−100
−120
−140
−160
−180
−20
−40
−60
−80
−100
−120
−140
−160
−180
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
f
dB
Cutoff
phase

>

>

$restart:$
$L1 := 102.175e-3 : L2 := 102.175e-3 :$
$RL1 := 222 : RL2 := 219.8 :$
$C1 := 67.2e-9 :$
$R1 := 993 : R2 := 219.2 : R3 := 218.9 :$
$V1 := 2 : Vin := V1 :$

$$ZC1 := \frac{1}{I \cdot 2 \cdot \mathrm{Pi} \cdot f \cdot C1} : ZL1 := I \cdot 2 \cdot \mathrm{Pi} \cdot f \cdot L1 : ZL2 := I \cdot 2 \cdot \mathrm{Pi} \cdot f \cdot L2 :$$

$$Zpar := \frac{1}{\frac{1}{ZC1} + \frac{1}{R1} + \frac{1}{R2 + R3}} : Zleft := ZL1 + ZL2 + RL1 + RL2 :$$

$$Vo := \frac{Zpar}{Zpar + Zleft} V1 :$$

$$H := \frac{Vo}{Vin} : H := simplify(\%);$$

$$H := \frac{1.000000 \times 10^{6}}{2.453359961 \times 10^{6} - 0.5421302639 f^{2} + 4410.328056 \mathrm{I} f}$$

>

Appendix D: Excel Table Sample Calculations

Gain

Formula:

Freq	p-p of in	amplitude of in	p-p of out	amplitude of out	amp difference	decibels	%uncertainty	Abs uncert.
f	Vin_{pk-pk}	$Vi = Vin_{pk-pk}/2$	$Vout_{pk-pk}$	$Vo = Vout_{pk-pk}/2$	A = Vo/Vi	B=20log(A)	5%	0.05*B

Calculations with sample values Values:

Freq	p-p of in	amplitude of in	p-p of out	amplitude of out	amp difference	decibels	%uncertainty	Abs uncert.
300	3.92	1.96	1.36	0.68	0.347	-9.195	5%	0.460

Phase

Formula

Freq	time diff	Increment	time uncertainty	Phase in degrees	Phase uncertainty
f	t	S = hantek time scale	S/5	P = F*t*360	0.05*P + S/5

Calculations with sample values Values:

Freq	time diff	Increment	time uncertainty	Phase in degrees	Phase uncertainty
f	-2.80E-04	2.00E-04	2.60E-05	-30.24	2.81