ENGPHYS 2A04 Assignment 6 Solutions

1. **Charge and Current Distributions**

a)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}\sin^2\phi \, r \, dr \, d\phi$$

$$= \frac{\rho_{s0} r^2}{2}\bigg|_0^a \int_0^{2\pi}\left(\frac{1-\cos 2\phi}{2}\right) d\phi$$

$$= \frac{\rho_{s0} a^2}{4}\left(\phi - \frac{\sin 2\phi}{2}\right)\bigg|_0^{2\pi}$$

$$= \frac{\pi a^2}{2}\rho_{s0}$$

b)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0} e^{-r} r \, dr \, d\phi$$

$$= 2\pi\rho_{s0}\int_0^a r e^{-r} dr$$

$$= 2\pi\rho_{s0}[-re^{-r} - e^{-r}]_0^a$$

$$= 2\pi\rho_{s0}[1 - e^{-a}(1+a)]$$

c)

$$Q = \int \rho_s \, ds = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}\cos\phi \, r \, dr \, d\phi$$

$$= \frac{\rho_{s0} r^2}{2}\bigg|_0^a \sin\phi \bigg|_0^{2\pi}$$

$$= 0$$

d)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0} e^{-r}\sin^2\phi \, r \, dr \, d\phi$$

$$= \rho_{s0}\int_{r=0}^{a} r e^{-r} \, dr \int_{\phi=0}^{2\pi}\sin^2\phi \, d\phi$$

$$= \rho_{s0}[1 - e^{-a}(1+a)] \cdot \pi$$

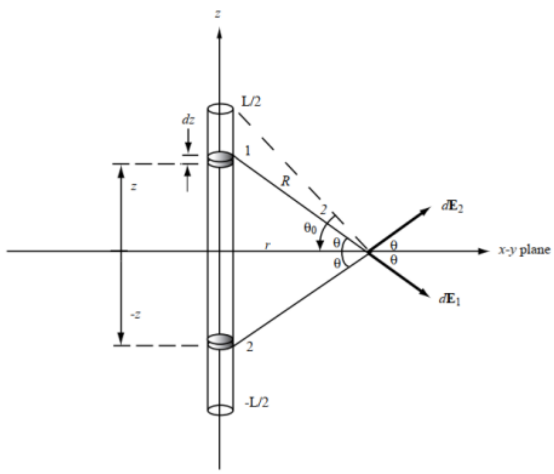
$$= \pi\rho_{s0}[1 - e^{-a}(1 + a)]$$

2. **Coulomb's Law**

$$d\mathbf{E} = d\mathbf{E_1} + d\mathbf{E_2} = \hat{\mathbf{r}}\frac{2\rho_l \cos\theta\, dz}{4\pi\varepsilon_0 R^2} = \hat{\mathbf{r}}\frac{\rho_l \cos\theta\, dz}{2\pi\varepsilon_0 R^2}$$

Our integration variable is z, but it will be easier to integrate over the variable θ from $\theta = 0$ to

$$\theta_0 = \sin^{-1}\frac{L/2}{\sqrt{r^2 + (L/2)^2}}$$



Here, with $R = r/\cos\theta$, and $z = r\tan\theta$ and $dz = r\sec^2\theta\ d\theta$, we have

$$\mathbf{E} = \int_{z=0}^{L/2} d\mathbf{E} = \int_{\theta=0}^{\theta_0} d\mathbf{E} = \int_0^{\theta_0} \hat{r}\frac{\rho_l}{2\pi\varepsilon_0}\frac{\cos^3\theta}{r^2} r\sec^2\theta\, d\theta$$

$$= \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\int_0^{\theta_0}\cos\theta\ d\theta$$

$$= \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\sin\theta_0$$

$$= \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\frac{L/2}{\sqrt{r^2 + (L/2)^2}}$$

For $L \gg r$,

$$\frac{L/2}{\sqrt{r^2 + (L/2)^2}} \approx 1$$

$$\mathbf{E} = \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\ \text{(infinite line of charge)}$$

3. **Gauss's Law**

Symmetry of the spherical shape indicates that D is radially oriented.

$$\mathbf{D} = \hat{\mathbf{R}} D_r$$

Gauss's Law at any radius or R.

$$\oint_S \mathbf{D} \cdot ds = Q$$

$$\oint_S \hat{\mathbf{R}} D_r \cdot \hat{\mathbf{R}} ds = Q$$

$$4\pi R^2 D_R = Q$$

$$D_R = \frac{Q}{4\pi R^2}$$

For R < a, there is no charge in the cavity (hollow). Therefore, Q =0.

$$D_R = 0, R \leq a$$

For $a \leq R \leq b$,

$$Q = \int_{R=a}^{R} \rho_v dV = \int_{R=a}^{R} -\frac{\rho_{v_0}}{R^2} \cdot 4\pi R^2 \, dR = -4\pi \rho_{v_0}(R-a)$$

Therefore,

$$D_R = \frac{-4\pi \rho_{v_0}(R-a)}{4\pi R^2}, \quad a \leq R \leq b$$

$$D_R = -\frac{\rho_{v_0}(R-a)}{R^2}, \quad a \leq R \leq b$$

For $R \geq b$,

$$Q = \int_{R=a}^{b} \rho_v dV = \int_{R=a}^{b} -\frac{\rho_{v_0}}{R^2} \cdot 4\pi R^2 \, dR = -4\pi \rho_{v_0}(b-a)$$

$$D_R = \frac{-4\pi \rho_{v_0}(b-a)}{4\pi R^2}, \qquad R \geq b$$

$$D_R = -\frac{\rho_{v_0}(b-a)}{R^2}, \quad a \leq R \leq b$$

4. **Electric Scalar Potential**

- Defined as the voltage difference between two points in a circuit
  - Represents the amount of work or potential energy required to move a unit of charge from <u>one point to another</u> (voltage difference)

    - More accurate representation of voltage
        - Voltage – amount of potential energy between two points in a circuit
        - Voltage represents the “difference” in potential and therefore is more accurate for electric scalar potential
- Is scalar because of:
    - Path independence in a conservative electric field
    - Work is only present when there is work being done against the electric field, if the work done is going with the field, there is no work done
    - Therefore, work is only dependent whether it is along or going against the electric field since field is conservative it is <u>not</u> dependent on path
    - Electric potential which is a calculation of work done will therefore also be a scalar quantity not dependent on direction