14 **1.** A <u>rodless pneumatic cylinder</u> will be used to move a 15 kg mass vertically upwards. The mass is subjected to a 70 N friction force. The mass should be moved 0.4 m in 0.5 s using a period of constant acceleration, followed by an equal period of constant deceleration. The motion should start and end at rest. The supply pressure is 6×10^5 Pa gauge and the air temperature is 30 °C. Assume that the pressure drop across the valve is the same for the return flow as for the intake flow and that the air is returned to the atmosphere. If a pressure drop of 8×10^4 Pa across the valve is desired, determine:

(a) The maximum force required from the cylinder.
(b) The maximum velocity of the mass.
(c) The minimum bore diameter required.
(d) The minimum valve flow coefficient required.

a) $\frac{1}{4} a t_{move}^2 = x \rightarrow a = \frac{4(0.4\,m)}{(0.5\,s)^2} = 6.4\,m/s^2$

<u>accelerating</u>

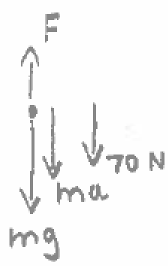
$ma = F - mg - 70\,N$

$F = m(a+g) + 70\,N$

$= (15\,kg)(6.4\,m/s^2 + 9.81\,m/s^2) + 70\,N$

$= 313.15\,N$

<u>decelerating</u>



$ma = -F + mg + 70\,N$

$F = -m(a-g) + 70\,N$

$= -(15\,kg)(6.4\,m/s^2 - 9.81\,m/s^2) + 70\,N$

$= 121.15\,N$

The maximum force required from the cylinder is 313.15N, during the acceleration period

b) $V_{max} = \frac{1}{2} a t_{move}$

$= \frac{1}{2}(6.4\, m/s^2)(0.5\, s) = \boxed{1.6\, m/s}$

c) $F = (P_{supply} - \Delta P)A - (\underbrace{P_{sump}}_{0\ \text{gauge}} + \Delta P)A$

$= (P_{supply} - 2\Delta P)A$

$A = \frac{F}{P_{supply} - 2\Delta P} = \frac{313.15 N}{6\times10^5 Pa - 2(8\times10^4 Pa)}$

$= 7.12\times10^{-4}\, m^2$

$A = \frac{\pi}{4}D^2 \rightarrow D = \sqrt{\frac{4}{\pi}A} = \sqrt{\frac{4}{\pi}(7.12\times10^{-4} m^2)}$

$\boxed{= 0.03\ m} = 30\ mm$

d) $\rho = \frac{(6\times10^5\, Pa + 101\times10^3\, Pa) - (8\times10^4\, Pa)}{(287\, \frac{m^2}{s^2 K})(303\, K)} = 7.14\, kg/m^3$

$C_v = (4.22\times10^4 m^{-2})\, v A \sqrt{\frac{\rho}{\Delta P}}$

$= (4.22\times10^4 m^{-2})(1.6\, m/s)(7.12\times10^{-4} m^2)\sqrt{\frac{7.14\, kg/m^3}{8\times10^4\, Pa}}$

$\boxed{C_v = 0.454}$

20 **2.** A disk is to be driven by a DC motor and gear box. The disk should be accelerated at 200 rad/s^2 for 0.45 s, and decelerated at 100 rad/s^2 for 0.9 s. After the deceleration, the load should be kept stationary for 0.6 s before the operating cycle restarts. The disk is subjected to a 3.2 Nm friction torque and has a moment of inertia of 1.25×10^{-2} kgm^2. The motor parameters are: moment of inertia = 4.0 $\times 10^{-5}$ kgm^2, max. torque = 2.5 Nm, max. continuous torque = 1.0 Nm and max. speed = 800 rad/s. You may assume the torque ratings are independent of the speed. **The available gear ratios are: 2, 4, 6, etc.**

a) Determine the best gear ratio for this application using the method of section 3.4.3.

b) Using the gear ratio you calculated in part (a), determine the temperature the motor will reach after this operating cycle has been repeated for several hours. The ambient temperature is 25 °C, motor's torque constant is 0.1 Nm/A, its armature resistance at its maximum temperature is 3.0 ohm, and its total thermal resistance is 1.5 °C/W.

a)

$$N_r = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{1.25\times10^{-2}}{4\times10^{-5}}} = 17.67 \simeq 17.7 \rightarrow \text{Round down to nearest even gear ratio } N = 16$$

Speed check: $\omega_{motor,rated} = 800$ rad/s

$$\omega_{Load,max} = \dot{\omega}_{Load} \times t = 200 \times 0.45 = 90 \text{ rad/s}$$

$$\omega_{motor,max} = N_r \cdot \omega_{Load,max} = 16 \times 90 = 1440 \text{ rad/s}$$

$\omega_{motor,max} > \omega_{motor,rated} \rightarrow$ Find a new gear ratio

$$N_{r\,new} = \frac{\omega_{motor,rated}}{\omega_{Load,max}} = \frac{800}{90} = 8.88 \rightarrow \text{Round down to nearest even gear ratio } N = 8 \checkmark$$

$$\text{Ratio } j = \frac{J_{load}}{N_r^2 \cdot J_{motor}} = \frac{1.25\times10^{-2}}{8^2\times4\times10^{-5}} = 4.88$$

$1 < 4.88 < 10 \checkmark$

$$\omega_{motor,max} = N_{r\,new} \cdot \omega_{Load,max} = 8 \times 90 = 720 \text{ rad/s}$$

$$\dot{\omega}_{Load} = 200 \text{ rad/s}$$

$$\dot{\omega}_{motor\,acc.} = N_{r,new} \cdot \dot{\omega}_{Load} = 8 \times 200 = 1600 \text{ rad/s}^2$$

$$T_{motor,acc} = J_{motor} \cdot \dot{\omega}_{motor,acc} + \left(\frac{1}{N_r^2}\right) J_{Load} \cdot \dot{\omega}_{motor} + \left(\frac{1}{N_r}\right) T_{ext}$$

$$T_{motor} = 4\times10^{-5}\times1600 + \frac{1}{8^2}\times1.25\times10^{-2}\times1600 + \frac{1}{8}\times3.2 = 0.7765$$

$$T_{motor,max} = 0.7765 \text{ N.m} < T_{count\ torqe,max} = 1 \text{ N.m}$$

b)

$$\dot{\omega}_{motor,dec.} = N_{r\,new} \cdot \dot{\omega}_{load,dec} = 8 \times (-100) = -800 \ rad/s^2$$

$$\tau_{motor,dec} = J_{motor} \cdot \dot{\omega}_{\substack{motor\\dec}} + \left(\frac{1}{N_r^2}\right) J_{load} \cdot \dot{\omega}_{\substack{motor\\dec}} + \left(\frac{1}{N_r}\right) \tau_{ext}$$

$$\tau_{motor,acc} = 4 \times 10^{-5} \times (-800) + \frac{1}{8^2} \times 1.25 \times 10^{-2} \times (-800) + \frac{1}{8} \times 3.2$$

$$= 0.2118 \approx 0.21 \ N.m$$

$$\tau_{motor,\ stationary} = 0 \ N.m$$

$$\tau_{motor,\ rms} = \sqrt{\frac{\sum_{i=1}^{n} \tau_{motor,i}^2 \, t_i}{\sum_{i=1}^{n} t_i}} = \sqrt{\frac{0.78^2 \times 0.45 \times 0.21^2 \times 0.9}{0.45 + 0.9 + 0.6}}$$

$$= 0.3998 \approx 0.4 \ N.m$$

$$I_{rms} = \frac{\tau_{rms}}{k_t} = \frac{0.3998}{0.1} = 3.998 \approx 4 \ A$$

$$T_W = T_A + P.R_t = T_A + I_{rms}^2 R_{hot} R_{th} = 25 + 4^2 \times 3 \times 1.5 = 96.9$$

$$\approx 97 ^\circ C$$

6 **3. Based on the material covered in this course**, answer the following questions in the spaces provided:

a) List one advantage of a cam and cam follower mechanism:

Answer: high speed

b) Other than cost, list one advantage of pneumatic actuators compared to electromechanical actuators:

Answer: higher force & torque ratios (compared to mass)

c) Other than "no wear", list one advantage and one disadvantage of a piezoelectric actuator:

Advantage: high resolution - nanometres

Disadvantage: small range

d) List one advantage and one disadvantage of an ultrasonic motor:

Advantage: higher torque at low speeds

Disadvantage: high wear due to using friction

e) A motor is connected to a rack and pinion mechanism to produce linear motion. You know the values of the pitch radius, efficiency and desired force. What is the equation for the required motor torque?

Answer: $T_{in} = \frac{F_{out} r_p}{\eta_{rp}}$