Q1

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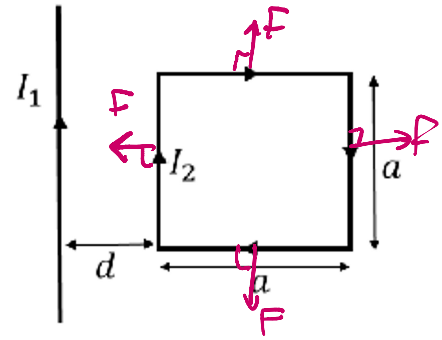
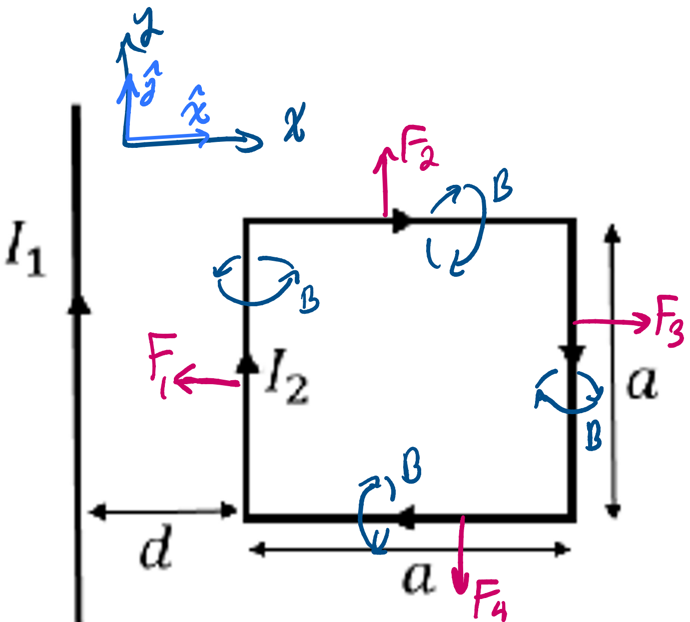
1. The wire on the left carries a current I_1, while the loop on the right carries a current of I_2. The loop is positioned a distance d from the wire, and has dimensions $a \times a$. Find a simplified expression for the magnetic force acting on the loop – and remember, this is a vector quantity.





$$\vec{F} = I\vec{l} \times \vec{B}$$

$$F = I l B \sin\theta$$

$$F = I_2 L_2 \left(\frac{\mu_0 I_1}{2\pi r}\right)$$

$$\vec{F}_{net} = \vec{F}_1 + \cancel{\vec{F}_2} + \vec{F}_3 + \cancel{\vec{F}_4}$$ * by symmetry, $F_2 = -F_4$

$$\vec{F}_{net} = -I_2 a \left(\frac{\mu_0 I_1}{2\pi d}\right)\hat{x} + I_2 a \left(\frac{\mu_0 I_1}{2\pi (d+a)}\right)\hat{x}$$

$$\vec{F}_{net} = \left(\frac{1}{d+a} - \frac{1}{d}\right)\frac{I_2 I_1 \mu_0 a}{2\pi}\,\hat{x}$$

$$\vec{F}_{net} = \left(\frac{d}{d(d+a)} - \frac{d+a}{d(d+a)}\right)\left(\frac{I_2 I_1 \mu_0 a}{2\pi}\right)\hat{x}$$

$$\vec{F}_{net} = \frac{-a}{d(d+a)}\left(\frac{I_2 I_1 \mu_0 a}{2\pi}\right)\hat{x}$$

$$\boxed{\vec{F}_{net} = -\frac{I_2 I_1 \mu_0 a^2}{2\pi d(d+a)}\hat{x} \quad [N]}$$

Q2

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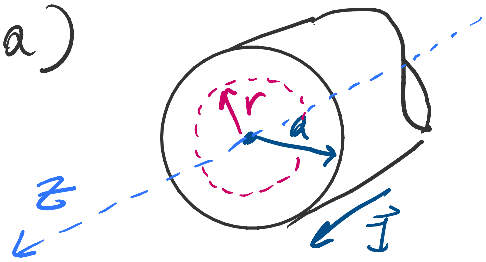
2. A cylindrical conductor, oriented along the z-axis, with a radius of a carries a current density of $\hat{\mathbf{z}}J_0e^{-kr}$. Calculate the magnetic field as a function of radial distances for distances
   a. Inside the conductor
   b. Outside the conductor

a) inside conductor $(\emptyset \leq r \leq a)$



$$\int \vec{H} \cdot d\vec{l} = \mu_\emptyset I$$

$$\star\ I = JA$$

$$I = \int J\,dA$$

enclosed area

$$I = \int_0^r J\, 2\pi r \cdot dr$$

$$\int \vec{H} \cdot d\vec{l} = \mu_\emptyset \int_0^r \vec{J} \cdot d\vec{r}$$

$$\vec{H}(\cancel{2\pi} r) = \mu_\emptyset \cancel{2\pi} \int_0^r J_\emptyset r e^{-kr} \hat{z}\, d\vec{r}$$

$$\vec{H} r = \hat{z} \mu_\emptyset J_\emptyset \int_0^r r e^{-kr} d\vec{r}$$

$$\vec{H} r = \hat{z} \mu_\emptyset J_\emptyset (1 - kre^{-kr} - e^{-kr}) \frac{1}{k^2}$$

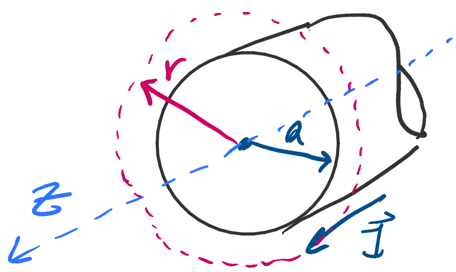
$$\boxed{\vec{H} = \frac{\hat{z} \mu_\emptyset J_\emptyset (1 - e^{-kr}(kr+1))}{rk^2} \quad [A/m]}$$

$$\vec{B} = \mu_\emptyset \vec{H}$$

$$\vec{B} = \hat{z} \mu_\emptyset^2 J_\emptyset (1 - e^{-kr}(kr+1))$$

$$\boxed{\vec{B} = \frac{\hat{z}\mu_0^2 J_0 (1 - e^{-kr}(kr+1))}{rk^2} \quad [T]}$$

b) outside conductor $(r > a)$



$$\oint \vec{H} \cdot d\vec{l} = \mu_0 \vec{I} \quad * \; I = JA$$

$$\oint \vec{H} \cdot d\vec{l} = \mu_0 \int_A \vec{J} \cdot d\vec{A}$$

$$\vec{H}(2\pi r) = \mu_0 \int_0^a \vec{J}\, 2\pi r \cdot dr$$

$$\cancel{2\pi} \vec{H} r = \mu_0 \cancel{2\pi} \int_0^a J_0 e^{-kr} r \,\hat{z}\, dr$$

$$\vec{H} r = \hat{z}\mu_0 J_0 \int_0^a r e^{-kr} dr$$

$$\vec{H} r = \hat{z}\mu_0 J_0 \frac{1}{k^2}\left[-kre^{-kr} - e^{-kr}\right]_0^a$$

$$\vec{H} = \frac{\hat{z}\mu_0 J_0}{r}\left(\frac{1 - ake^{-ak} - e^{-ak}}{k^2}\right)$$

$$\vec{H} = \frac{\hat{z}\mu_0 J_0}{rk^2}(1 - e^{-ak}(ak+1))$$

$$\boxed{\vec{H} = \frac{\hat{z}\mu_0 J_0 (1 - ke^{-ak}(a+1))}{rk^2} \quad [A/m]}$$

$$\boxed{\vec{H} = \frac{\mu_0 J_0 (1 - ke^{-ak}(a+1))}{rk^2} \ [A/m]}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\boxed{\vec{B} = \frac{\mu_0^2 J_0 (1 - ke^{-ak}(a+1))}{rk^2} \hat{z} \ [T]}$$

Q3

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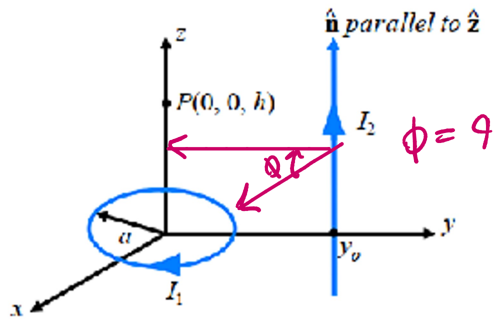
3. The loop centered at the origin in the figure below has a radius of 5cm, lies in the *x*-*y* plane, and carries a current of $I_1 = 8A$. A straight wire parallel to *z* intersects the point $y = 12cm$, and carries a current of $I_2 = 6A$. Calculate the magnetic field at the point P(0, 0, 10cm).



$\phi = 90° \, CW = -90°$

$$\vec{H}_{total} = \vec{H}_1 + \vec{H}_2$$

Field due to loop: x & y components cancel due to symmetry

By RHR

$$\vec{H}_1 = (\vec{H}_1)_z = \frac{I_1 a^2}{2(a^2+h^2)^{3/2}} (-\hat{z})$$

$$\vec{H}_2 = \frac{I_2}{2\pi r} \hat{\phi}$$

✱ cartesian/cylindrical

$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$\phi = -90°$

$\phi = -90°$

$$\hat{\phi} = -\hat{x}\sin\cancel{(-90°)}^{-1} + \hat{y}\cos\cancel{(-90°)}^{0}$$

$$\hat{\phi} = \hat{x}$$

$$\vec{H}_2 = I_2\,\hat{x}/2\pi r$$

$$\vec{H}_{total} = \frac{-I_1 a^2\,\hat{z}}{2(a^2+h^2)^{3/2}} + \frac{I_2}{2\pi r}\hat{x}$$

sub $I_1 = 8A$, $I_2 = 6A$, $r = 0.12m$, $a = 0.05m$

$$\vec{H} = -\frac{(8)(0.05)^2\,\hat{z}}{2(0.05^2 + 0.10^2)^{3/2}} + \frac{6\hat{x}}{2\pi(0.12)}$$

$$\boxed{\vec{H} = 7.96\hat{x} - 7.16\hat{z} \quad A/m}$$

$$\vec{B} = \mu\vec{H} = \mu(7.96\hat{x} - 7.16\hat{z})$$

$$\boxed{\vec{B} = (10\hat{x} - 8.99\hat{z})\,\mu T}$$

Q4

April 4, 2022 12:05 AM

4. Consider a 5-meter long section of a coaxial transmission line, with an inner conductor radius of 3cm and an outer conductor inner radius of 8 cm. If the insulator is air and the line is carrying a DC current of 12A, how much magnetic energy is stored in the insulating medium?

$$L' = \frac{\mu_0}{2\pi} \ln(b/a)$$

$$* \; L = l \cdot L'$$

$$L = \frac{5 \cdot \mu_0 \ln(8/3)}{2\pi}$$

$$L = 9.81 \times 10^{-7} \text{ H}$$

$$E_m = \frac{1}{2} L I^2$$

$$E_m = \frac{1}{2}(9.81 \times 10^{-7})(12)^2$$

$$\boxed{E_m = 7.06 \times 10^{-5} \text{ J}}$$

Q5

April 4, 2022 12:36 AM

Article Title: "Recent challenges and advances in the sensorless commutation of brushless dc motors"

When implementing brushless DC motors there are many challenges surrounding the sensing of rotor position and achieving a fast startup.
Relating the ideas discussed to class material, the main cause of these issues are rooted in the implementation of a hall-sensor due to the presence of electromagnetic field interference, as observed in class through the interaction of multiple magnetic fields. As for a fast startup, it is necessary to detect Back EMF throughout the motor's operation, and sensorless methods (while better then Hall Sensor methods) are cause for startup delay.
Potential solutions include direct and indirect terminal voltage to detect Back EMF, while a strategy involving open/close loop reduced current startup process was used to reduce reverse-torque, thereby reducing startup delay. Despite these results, the journal concludes that a more robust method is needed to improve upon the design.

References:
U. K. Soni and R. K. Tripathi, “Recent challenges and advances in the sensorless commutation of brushless DC Motors,” *Recent Advances in Electrical & Electronic Engineering (Formerly Recent Patents on Electrical & Electronic Engineering)*, vol. 14, no. 1, pp. 90–113, 2021.