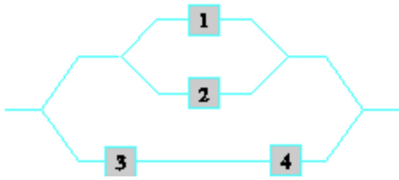
Assignment 2

Sunday, May 15, 2022 9:11 PM

Problem #1: Consider the system of components connected as in the figure below. Components 1 and 2 are connected in parallel, so that subsystem works if and only if either 1 or 2 works. Since 3 and 4 are connected in series, that subsystem works if and only if both 3 and 4 work. Suppose that $P(1 \text{ works}) = 0.75$, $P(2 \text{ works}) = 0.75$, $P(3 \text{ works}) = 0.8$, and $P(4 \text{ works}) = 0.85$. Find the probability that the system works.



$P(3 \text{ and } 4) = P(3) \times P(4) = 0.8 \times 0.85 = 0.68$

$P(1 \text{ or } 2) = P(1) + P(2) - P(1)P(2) = 0.75 + 0.75 - (0.75)(0.75)$

$= 0.9375$

$P(\text{works}) = P(1 \text{ or } 2) + P(3 \text{ and } 4) - P(1 \text{ or } 2) \cdot P(3 \text{ and } 4)$

$= 0.9375 + 0.68 - (0.9375)(0.68)$

$= 0.98$

Problem #2: Consider purchasing a system of audio components consisting of a receiver, a pair of speakers, and a CD player. Let A_1 be the event that the receiver functions properly throughout the warranty period. Let A_2 be the event that the speakers function properly throughout the warranty period. Let A_3 be the event that the CD player functions properly throughout the warranty period. Suppose that these events are (mutually) independent with $P(A_1) = 0.95$, $P(A_2) = 0.88$, and $P(A_3) = 0.71$.

(a) What is the probability that at least one component needs service during the warranty period?

(b) What is the probability that exactly one of the components needs service during the warranty period?

$P(A_1) = P(R) = 0.95 \Rightarrow P(R') = 0.05$

$P(A_2) = P(S) = 0.88 \Rightarrow P(S') = 0.12$

$P(A_3) = P(CD) = 0.71 \Rightarrow P(CD') = 0.29$

a.) $P(\geq 1 \text{ component needs repair}) = 1 - P(\text{all work})$

$= 1 - P(R) \cdot P(S) \cdot P(CD)$

$= 1 - (0.95)(0.88)(0.71)$

$= 0.40644$

b.) $P(\text{exactly one}) = P(R \text{ fails}) + P(S \text{ fails}) + P(CD \text{ fails})$

$= P(R') \cdot P(S) \cdot P(CD) + P(S') \cdot P(R) \cdot P(CD) + P(CD') \cdot P(R) \cdot P(S)$

$$= (0.05)(0.88)(0.71) + (0.12)(0.95)(0.71) + (0.29)(0.95)(0.88)$$
$$= 0.03124 + 0.08094 + 0.24244$$
$$= 0.35462$$

Problem #3: A box consists of 13 components, 7 of which are defective.

(a) Components are selected and tested one at a time, without replacement, until a non-defective component is found. Let X be the number of tests required. Find $P(X = 4)$.

(b) Components are selected and tested, one at a time without replacement, until two *consecutive* non defective components are obtained. Let X be the number of tests required. Find $P(X = 5)$.

7 defective (D) , 6 non-defective (N)

a.) order is DDDN

$$P(X=4) = \frac{7}{13} \times \frac{6}{12} \times \frac{5}{11} \times \frac{6}{10} = \frac{21}{286}$$

b.) possible orders are : DDDNN , DNDNN , NDDNN

$$P(DDDNN) = \frac{7}{13} \times \frac{6}{12} \times \frac{5}{11} \times \frac{6}{10} \times \frac{5}{9} = \frac{35}{858}$$

$$P(DNDNN) = \frac{7}{13} \times \frac{6}{12} \times \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} = \frac{14}{429}$$

$$P(NDDNN) = \frac{6}{13} \times \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} = \frac{14}{429}$$

$$P(X=5) = P(DDDNN) + P(DNDNN) + P(NDDNN)$$

$$= \frac{35}{858} + \frac{14}{429} + \frac{14}{429} = \frac{7}{66}$$

Problem #4: An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are 0.94 and 0.85, respectively. Let X be the number of components in the assembly that meet specifications.

(a) Find the mean of X.

(b) Find the variance of X.

a.) $E(x) = p_{1st} + p_{2nd} = 0.94 + 0.85 = 1.79$

b.) $Var(x) = (0.94^2 + 0.85^2) - 1.79$

$= -0.1839$

$Var(X) = 0.1839$

Problem #5: A manufacturing process has 46 customer orders to fill. Each order requires one component part that is purchased from a supplier. However, typically 6% of the components are identified as defective, and the components can be assumed to be independent.

☆ **(a)** If the manufacturer stocks 48 components, what is the probability that the 46 orders can be filled without reordering components?

(b) Let X be the number of good (i.e., non-defective) components among the 48 in stock. Find the mean of X.

(c) Find the variance of X [from part (b)].

6% defective, so 94% non-defective

$n = 48$, $p = 94\%$

a.) $P(X=46) = \binom{n}{x} p^x (1-p)^{n-x}$

$= \binom{48}{46} (0.94)^{46} (1-0.94)^{48-46}$

$= \binom{48}{46} (0.94)^{46} (0.06)^2 = 0.235772$

b.) X = # of good items among the 48

$\mu = E(X) = n \cdot p = 48(0.94) = 45.12$

c.) $Var(X) = n \cdot p \cdot (1-p) = (48)(0.94)(1-0.94) = 2.7072$

Problem #6: The probability that a randomly selected box of a certain type of cereal has a particular prize is 0.20. Suppose that you purchase box after box until you have obtained 3 of these prizes.

(a) What is the probability that you purchase exactly 8 boxes?

(b) What is the probability that you purchase at least 9 boxes?

(c) How many boxes would you expect to purchase, on average?

probability of prize is 0.20 $\Rightarrow$ $p = 0.2$

a.) Negative Binomial Distribution so $X = 8$, $r = 3$

$$P(X=8) = \binom{x-1}{r-1}(1-p)^{x-r}\cdot p^r = \binom{8-1}{3-1}(1-0.2)^{8-3}(0.2)^3$$

$$= \binom{7}{2}(0.8)^5(0.2)^3 = 0.0550502$$

b.) $P(X \geq 9) = 1 - P(X < 9)$ $\swarrow$ $P(X=8) + P(X=7) \ldots$

$$= 1 - 0.20308224$$

$$= 0.796918$$

https://keisan.casio.com/exec/system/1180573210

number of failures before k successes x	6	x=0,1,2,..
number of successes k	3	k=1,2,..
probability of success p	0.2	0≦p≦1

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Negative Binomial distribution	value
probability mass f	0.058720256
lower cumulative P	0.261802496
upper cumulative Q	0.79691776
mean	12

c.) Average $\rightarrow$ $E(X)$

$$E(x) = \frac{r}{p} = \frac{3}{0.2} = 15$$

Problem #7: A geologist has collected 25 specimens of basaltic rock and and 17 specimens of granite. The geologist instructs a laboratory assistant to randomly select 7 of the specimens for analysis.

(a) What is the probability that at least 4 of the selected specimens are granite?

(b) What is the expected number of granite specimens in the sample?

(c) If this same process is repeated every day, how many days (on average) will it take before getting a sample consisting entirely of granite?

Hypergeometric distribution

a.) $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

$$P(X=4) = \frac{{}_{17}C_4 \times {}_{25}C_3}{} = 0.202904$$

a.) $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

$$P(X=4) = \frac{{}_{17}C_4 \times {}_{25}C_3}{{}_{42}C_7} = 0.202904$$

$$P(X=5) = \frac{{}_{17}C_5 \times {}_{25}C_2}{{}_{42}C_7} = 0.068811$$

$$P(X=6) = \frac{{}_{17}C_6 \times {}_{25}C_1}{{}_{42}C_7} = 0.011468$$

$$P(X=7) = \frac{{}_{17}C_7 \times {}_{25}C_0}{{}_{42}C_7} = 0.000721$$

$$P(X \geq 4) = 0.202904 + 0.068811 + 0.011468 + 0.000721$$
$$= 0.283904$$

b.) Expected number $\Rightarrow$ $E(X)$

$n = 7$; $K = 17$; $N = 42$

$$E(x) = n \times \frac{K}{N} = 7 \times \frac{17}{42} = \frac{17}{6}$$

c.) $$P(\text{all granite}) = \frac{{}_{17}C_7}{{}_{42}C_7} = \frac{187}{259407}$$

$$\#\text{ of days} = \frac{1}{P(\text{all granite})} = \frac{259407}{187} = 1387$$

Problem #8: Data from the Central Hudson Laboratory determined that the mean number of insect fragments in 225-gram chocolate bars was 14.4 (http://www.centralhudsonlab.com/chocolates.shtml). In a 41-gram bar the mean number of insect fragments would then be 2.62. Assume that the number of insect fragments follows a Poisson distribution.

(a) If you eat a 41-gram chocolate bar, find the probability that you will have eaten at least 2 insect fragments.

(b) If you eat a 41-gram chocolate bar every week for 9 weeks, find the probability that you will have eaten no insect fragments in exactly 6 of those weeks.

Poisson Distribution $\Rightarrow$ $\lambda = 2.62$ for 41g

$$P(X=x) = \frac{\lambda^x}{x!} e^{-\lambda} \Rightarrow P(X=x) = \frac{2.62^x}{x!} e^{-2.62}$$

a.) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X=1) - P(X=0)$

$= 1 - \frac{2.62^1}{1!} e^{-2.62} - \frac{2.62^0}{0!} e^{-2.62}$

$= 1 - 0.19074 - 0.0728$

$= 0.73646$

b.) $P(X=0) = \frac{2.62^0}{0!} e^{-2.62} = e^{-2.62}$

$P(X>0) = 1 - P(X=0) = 1 - e^{-2.62}$

$P(6 \text{ weeks}) = \binom{9}{6} (e^{-2.62})^6 (1 - e^{-2.62})^3$

$= \binom{9}{6} (0.0728)^6 (1 - 0.0728)^3$

$= 9.9675 \times 10^{-6}$

Problem #9: Let X denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. Suppose that X has the following *Rayleigh* pdf.

$$f(x) = \begin{cases} (x/\theta^2)\, e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) If $\theta = 95$, find the probability that the vibratory stress is between 85 and 280.

(b) If $\theta = 95$, then 78% of the time the vibratory stress is greater than what value?

a.) $P(a \leq x \leq b) = \int_a^b f(x)\,dx$

$P(85 \leq x \leq 280) = \int_{85}^{280} \frac{x}{\theta^2} \cdot e^{-\frac{x^2}{2\theta^2}}\,dx \qquad \theta = 95$

$= \int_{85}^{280} \frac{1}{\theta^2} \cdot x \cdot e^{-\frac{1}{2\theta^2} x^2}\,dx$

$= -e^{-\frac{x^2}{2\theta^2}} \Big|_{85}^{280}$

$= -e^{-\frac{280^2}{2 \cdot 95^2}} - \left(-e^{-\frac{85^2}{2 \cdot 95^2}} \right)$

$$= -e^{-\frac{280^2}{2\cdot 95^2}} - \left(-e^{-\frac{85^2}{2\cdot 95^2}}\right)$$

$$= 0.657143$$

b.) $P(X=x) = 78\% = 0.78$

$$0.78 = e^{-\frac{x^2}{2\cdot\theta^2}}$$

$$\ln 0.78 = \frac{-x^2}{2\cdot 95^2}$$

$$-4484.7275 = -x^2$$

$$x = 66.9681$$