ME4K03 ROBOTICS

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Solutions for Assignment #5

1. Planar RRR robot operating in the vertical plane.

Given :

$$\begin{cases} a_1 = 0.5m \\ a_2 = 0.5m \\ a_3 = 0.1m \end{cases} \text{ and } \begin{cases} m_1 = 10kg \\ m_2 = 10kg \\ m_3 = 2kg \end{cases} \Rightarrow \begin{cases} (\frac{1}{2}m_1 + m_2 + m_3)ga_1 = 83.385Nm \\ (\frac{1}{2}m_2 + m_3)ga_2 = 34.335Nm \\ \frac{1}{2}m_3 ga_3 = 0.981Nm \end{cases}$$

$$J(q) = \begin{bmatrix} -a_1S\theta_1 - a_2S\theta_{12} - a_3S\theta_{123} & -a_2S\theta_{12} - a_3S\theta_{123} & -a_3S\theta_{123} \\ a_1C\theta_1 + a_2C\theta_{12} + a_3C\theta_{123} & a_2C\theta_{12} + a_3C\theta_{123} & a_3C\theta_{123} \\ 1 & 1 & 1 \end{bmatrix} \quad (1)$$

$$G(q) = \begin{bmatrix} (\frac{1}{2}m_1 + m_2 + m_3)ga_1C\theta_1 + (\frac{1}{2}m_2 + m_3)ga_2C\theta_{12} + \frac{1}{2}m_3ga_3C\theta_{123} \\ (\frac{1}{2}m_2 + m_3)ga_2C\theta_{12} + \frac{1}{2}m_3ga_3C\theta_{123} \\ \frac{1}{2}m_3ga_3C\theta_{123} \end{bmatrix} \quad (2)$$

a) For the given configuration:

$$\begin{cases} \theta_1 = 45^\circ \\ \theta_2 = -75^\circ \\ \theta_3 = 30^\circ \end{cases} \Rightarrow \begin{cases} S\theta_1 = 0.707 \;,\; C\theta_1 = 0.707 \\ S\theta_{12} = -0.5 \;,\; C\theta_{12} = 0.866 \\ S\theta_{123} = 0 \;,\; C\theta_{123} = 1 \end{cases} \text{ and } F = \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} 0 \\ 5\times 9.81 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 49.05 \\ 0 \end{bmatrix} N$$

$$(1) \Rightarrow J(q) = \begin{bmatrix} -0.104 & 0.25 & 0 \\ 0.887 & 0.533 & 0.1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow J(q)^T = \begin{bmatrix} -0.104 & 0.887 & 1 \\ 0.25 & 0.533 & 1 \\ 0 & 0.1 & 1 \end{bmatrix}$$

$$(2) \Rightarrow G(q) = \begin{bmatrix} 89.668 \\ 30.715 \\ 0.981 \end{bmatrix} Nm$$

$$\tau = J(q)^T F + G(q) \Rightarrow \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} -0.104 & 0.887 & 1 \\ 0.25 & 0.533 & 1 \\ 0 & 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 49.05 \\ 0 \end{bmatrix} + \begin{bmatrix} 89.668 \\ 30.715 \\ 0.981 \end{bmatrix} = \begin{bmatrix} 133.175 \\ 56.859 \\ 5.886 \end{bmatrix} Nm$$

b) We need the Jacobian matrix for the new configuration B and also the inverse matrices for both A and B:

$$A: \begin{cases} \theta_1 = 45^\circ \\ \theta_2 = -75^\circ \\ \theta_3 = 30^\circ \end{cases} \Rightarrow J(q) = \begin{bmatrix} -0.104 & 0.25 & 0 \\ 0.887 & 0.533 & 0.1 \\ 1 & 1 & 1 \end{bmatrix} \text{and } J(q)^{-1} = \begin{bmatrix} -1.793 & 1.035 & -0.104 \\ 3.257 & 0.429 & -0.043 \\ -1.464 & -1.464 & 1.146 \end{bmatrix}$$

$$B: \begin{cases} \theta_1 = 45^\circ \\ \theta_2 = -5^\circ \\ \theta_3 = -40^\circ \end{cases} \Rightarrow J(q) = \begin{bmatrix} -0.675 & -0.321 & 0 \\ 0.837 & 0.483 & 0.1 \\ 1 & 1 & 1 \end{bmatrix} \text{and } J(q)^{-1} = \begin{bmatrix} -17.579 & -14.750 & 1.475 \\ 33.805 & 30.977 & -3.098 \\ 16.226 & 16.226 & 2.623 \end{bmatrix}$$

Since we want the robot to apply a precise force in the X_0 direction, we need to know the relationship of F_x and joint torques. The general statics equation is:

$$F = (J(q)^{-1})^T(\tau - G(q))$$

Since the question concerns the effect of the torque resolution we only need to include τ and not $G(q)$.

$$\text{For A: } (J(q)^{-1})^T(\tau) = \begin{bmatrix} -1.793 & 3.257 & -1.464 \\ 1.035 & 0.429 & -1.464 \\ -0.104 & -0.043 & 1.146 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$\text{For B: } (J(q)^{-1})^T(\tau) = \begin{bmatrix} 17.579 & 33.805 & 16.226 \\ -14.750 & 30.977 & -16.226 \\ 1.475 & -3.098 & 2.623 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

Change in F_x because of a differential torque of 0.1Nm in each joint

$$\text{for A: } \begin{cases} \Delta F_x \text{ due to } \tau_1 = (-1.793)(0.1) = -0.1793N \\ \Delta F_x \text{ due to } \tau_2 = (3.257)(0.1) = 0.3257N \\ \Delta F_x \text{ due to } \tau_3 = (-1.464)(0.1) = -0.1464N \end{cases}$$

$$\text{for B: } \begin{cases} \Delta F_x \text{ due to } \tau_1 = (17.579)(0.1) = 1.758N \\ \Delta F_x \text{ due to } \tau_2 = (33.805)(0.1) = 3.381N \\ \Delta F_x \text{ due to } \tau_3 = (-16.226)(0.1) = -1.623N \end{cases}$$

From the above results, it is evident that the robot can apply a much more precise force in the X0 direction when in configuration A.

<u>Alternate Calculation Method:</u> Rather than comparing the contributions of the individual joints, one can compare the sum of the absolute values. This covers the worst situation (*i.e.* the largest possible force variation). The calculations are:

for A: $\Delta F_x = |(-1.793)(0.1)| + |(3.257)(0.1)| + |(-1.464)(0.1)| = 0.65N$

for B: $\Delta F_x = |(-17.579)(0.1)| + |(33.805)(0.1)| + |(-16.226)(0.1)| = 6.8N$