ENG PHYS 2A04 Tutorial 11

Electricity and Magnetism

Your TAs today

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Chapter 6

Problems

Problem 6.3

Problem 6.4

Problem 6.5

Problem 6.7

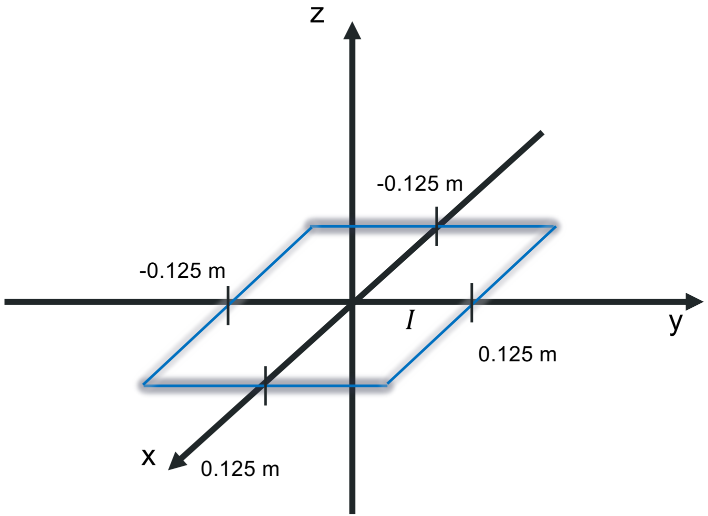
Problem 6.3 – Question

A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the x- or y-axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a) $\mathbf{B} = \hat{\mathbf{z}}\, 20e^{-3t}$ (T)

(b) $\mathbf{B} = \hat{\mathbf{z}}\, 20 \cos x \cos 10^3 t$ (T)

(c) $\mathbf{B} = \hat{\mathbf{z}}\, 20 \cos x \sin 2y \cos 10^3 t$ (T)



Problem 6.3 – Details

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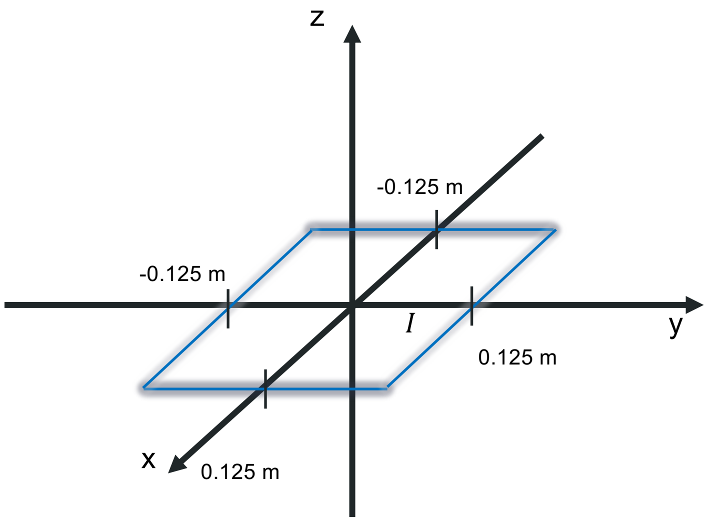
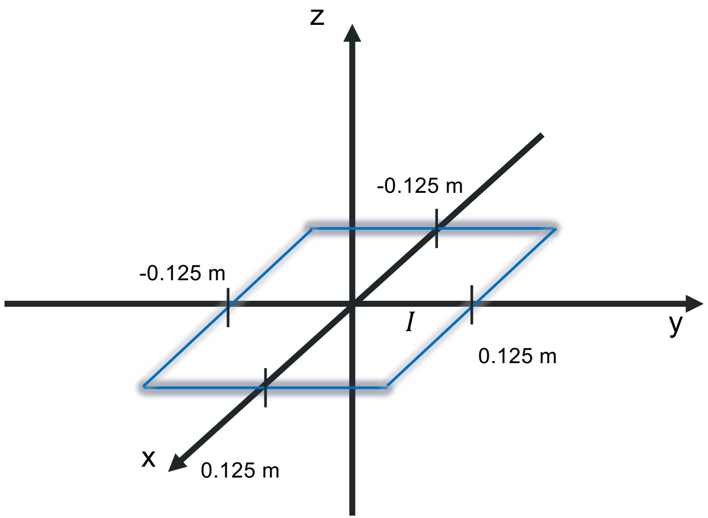


Solution? → Apply Faraday's Law!

Problem 6.3 – Solution (a)



Faraday's Law states: $V_{emf} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s}$

$N = 100 \text{ turns}, d\mathbf{s} = \hat{\mathbf{z}}\, dxdy$

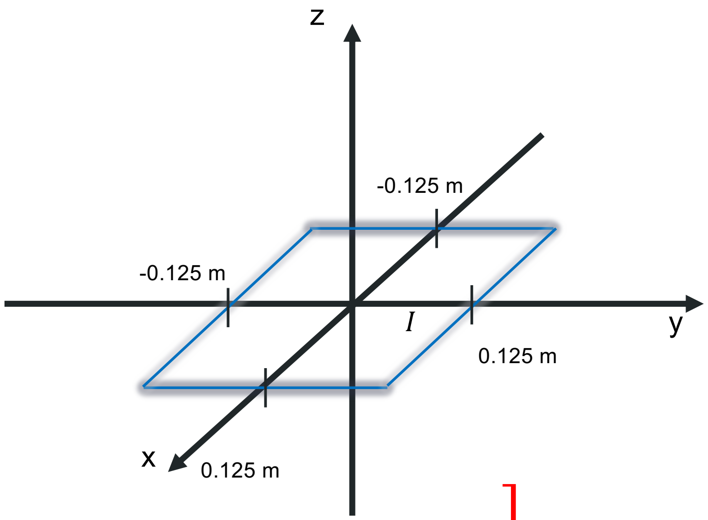
$\mathbf{B} = \hat{\mathbf{z}}\, 20e^{-3t} \text{ (T)}$

$$\rightarrow V_{emf} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = -N\frac{d}{dt}\left[\int_{-0.125}^{0.125}\int_{-0.125}^{0.125} \hat{\mathbf{z}}\, 20e^{-3t}\cdot\hat{\mathbf{z}}\, dxdy\right]$$

$$= -100\frac{d}{dt}\left[(20e^{-3t})(0.25)^2\right] = -100(0.0625)(20)\frac{d}{dt}\left[e^{-3t}\right] = -125(-3)e^{-3t}$$

$$\rightarrow\therefore \boldsymbol{V_{emf} = 375e^{-3t}} \textbf{ (V)}$$

Problem 6.3 – Solution (b)



$$V_{emf} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s},\ N = 100 \text{ turns},\ d\mathbf{s} = \hat{\mathbf{z}}\,dxdy$$

$$\mathbf{B} = \hat{\mathbf{z}}\,20\cos x\cos 10^3 t\ (\mathrm{T})$$

$$\rightarrow V_{emf} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = -N\frac{d}{dt}\left[\int_{-0.125}^{0.125}\int_{-0.125}^{0.125}\hat{\mathbf{z}}\,20\cos x\cos 10^3 t\cdot\hat{\mathbf{z}}\,dxdy\right]$$

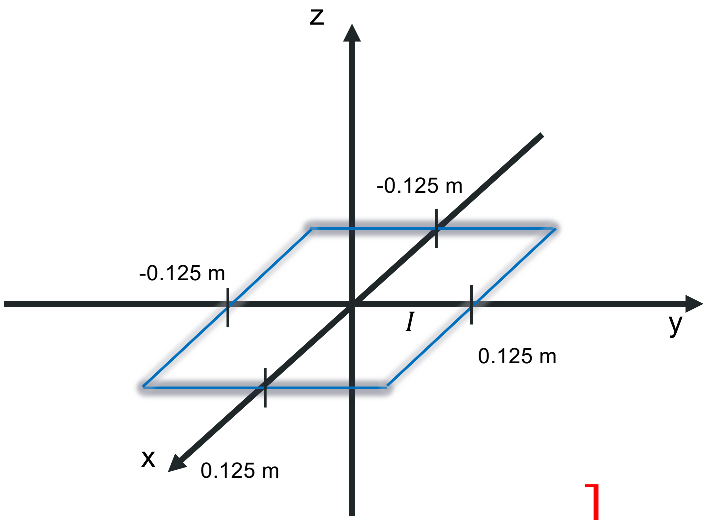
$$= -100\frac{d}{dt}[20\cos 10^3 t\,(\sin 0.125 - \sin -0.125)(0.25)]$$

$$= -100(20)(0.25)(0.125-(-0.125))\frac{d}{dt}[\cos 10^3 t] = -125(-1000\sin 10^3 t)$$

$$\rightarrow \therefore \boldsymbol{V_{emf} = 125\sin 10^3 t\ \ (\mathrm{kV})}$$

Problem 6.3 – Solution (c)

$$V_{emf} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s},\ N = 100 \text{ turns},\ d\mathbf{s} = \hat{\mathbf{z}}\,dxdy$$



$$\mathbf{B} = \hat{\mathbf{z}}\,20\cos x \sin 2y \cos 10^3 t \ \ (\mathrm{T})$$

$$\rightarrow V_{emf} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = -N\frac{d}{dt}\left[\int_{-0.125}^{0.125}\int_{-0.125}^{0.125} \hat{\mathbf{z}}\,20\cos x\sin 2y\cos 10^3 t\cdot\hat{\mathbf{z}}\,dxdy\right]$$

$$\boxed{\underline{\underline{=\mathbf{0}}}} \qquad \boxed{\cos -2y = \cos 2y}$$

$$= -100\frac{d}{dt}\left[20\cos 10^3 t\left(\frac{-\cos 2y + \cos(-2y)}{2}\right)(\sin 0.125 - \sin -0.125)\right]$$

$$\rightarrow \therefore \boldsymbol{V_{emf} = 0\ (\mathrm{V})}$$

Problem 6.4 – Question

A stationary conducting loop with internal resistance of 0.5 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 5 A flows through it. What will the current be if the loop is opened to create a small gap and a 2 Ω resistor is connected across its open ends?

Problem 6.4 – Details

A stationary conducting loop with internal resistance of 0.5 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 5 A flows through it. What will the current be if the loop is opened to create a small gap and a 2 Ω resistor is connected across its open ends?

Goal → find current in closed loop

Let:

- V_{emf} = induced emf
- $I = 5\ \mathrm{A}$ = current of stationary conducting loop
- $R = 0.5\ \Omega$ = internal resistance of stationary conducting loop
- $I' = ?$ = current of new loop with $2\ \Omega$ resistor inserted
- $R' = 2\ \Omega$ = equivalent resistance

Problem 6.4 – Solution

A stationary conducting loop with internal resistance of 0.5 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 5 A flows through it. What will the current be if the loop is opened to create a small gap and a 2 Ω resistor is connected across its open ends?

Does V_{emf} change with the resistance? → NO → $V_{emf} = I \cdot R = I' \cdot R'$

$$\rightarrow I' = \frac{I \cdot R}{R'} = \frac{5\ \mathrm{A} \cdot 0.5\ \Omega}{2\ \Omega + 0.5\ \Omega} \rightarrow \therefore \boldsymbol{I' = 1\ \mathrm{A}}$$

Let:

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Problem 6.5

A circular-loop TV antenna with 0.02 m^2 area is in the presence of a uniform-amplitude 300-MHz signal. When oriented for maximum response, the loop develops an emf with a peak value of 30 (mV). What is the peak magnitude of B of the incident wave?

Problem 6.5

Solution:

TV loop antennas have one turn.

At maximum orientation with the loop area of A and Uniform magnetic field of B=$|B|$,

$\Phi = \int B.ds = \pm BA$

F=300 MHz, $\omega = 2\pi \times 300 \times 10^6 = 6\pi \times 10^8 rad/s$

$$V_{emf} = -N\frac{d\Phi}{dt} = -A\frac{d}{dt}[B_0 \cos(\omega t + \alpha_0)] = AB_0\omega \sin(\omega t + \alpha_0)$$

$$-1 \leq \sin(\omega t + \alpha_0) \leq 1$$

$$V_{emf} = AB_0\omega$$

$$B_0 = \frac{V_{emf}}{A\omega} = \frac{0.03V}{0.02m^2(6\pi \times 10^8 rad/s)}$$

$$B_0 = 0.8nA/m$$

Problem 6. 7

The rectangular conducting loop shown in Fig. P6.7 rotates at 6,000 revolutions per minute in a uniform magnetic flux density given by

$$\mathbf{B} = \hat{\mathbf{y}}50 \text{ (mT)}.$$

Determine the current induced in the loop if its internal resistance is 0.5 Ω.

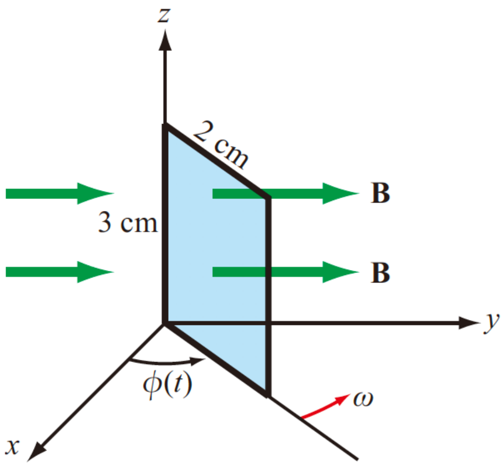

Figure P6.7: Rotating loop in a magnetic field (Problem 6.7).

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Analysis:

A moving loop with a time-varying in a static field B

$$I_{ind} = \frac{V_{emf}}{R_i} \Rightarrow \boxed{V_{emf} = V_{emf}^{m}} = \boxed{-}\frac{d\phi}{dt}$$

Lenz's Law

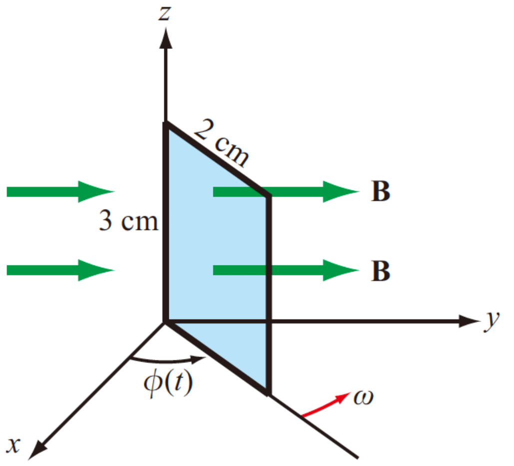

Figure P6.7: Rotating loop in a magnetic field (Problem 6.7).

Solution:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S} = \hat{\mathbf{y}}\, 50 \times 10^{-3} \cdot \hat{\mathbf{y}}(2 \times 3 \times 10^{-4}) \cos\phi(t) = 3 \times 10^{-5} \cos\phi(t),$$

$$\phi(t) = \omega t = \frac{2\pi \times 6 \times 10^3}{60} t = 200\pi t \quad \text{(rad/s)},$$

$$\Phi = 3 \times 10^{-5} \cos(200\pi t) \quad \text{(Wb)},$$

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = 3 \times 10^{-5} \times 200\pi \sin(200\pi t) = 18.85 \times 10^{-3} \sin(200\pi t) \quad \text{(V)},$$

$$I_{\text{ind}} = \frac{V_{\text{emf}}}{0.5} = 37.7 \sin(200\pi t) \quad \text{(mA)}.$$

Don’t forget the direction of current, because current is a vector.

The direction of the current is CW (if looking at it along $-\hat{\mathbf{x}}$-direction) when the loop is in the first quadrant ($0 \le \phi \le \pi/2$). The current reverses direction in the second quadrant, and reverses again every quadrant.