Prac exam 2020

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1 Bandpass Filter Construction

(20 points) This time we construct a bandpass filter by putting a low-pass and a high-pass filter into series. Placing a zero at $\frac{F_s}{2}$ and normalizing at DC results in a simple low-pass filter $H_1(\omega)$ Placing a zero at DC normalizing at $\frac{F_s}{2}$ results in a simple high-pass filter $H_2(\omega)$ We construct a new H putting H_1 and H_2 into series.

What is the gain for the new H at DC, $\frac{F_s}{4}$ and $\frac{F_s}{2}$ of this filter $H(\omega)$? Make a table so we easy find our answers !

$$\omega_0 = 2\pi f_0 \qquad * \; f_0 = f/f_s = \frac{f_s/2}{f_s} = \frac{1}{2}$$

$$= 2\pi/2$$

$$= \pi \leftarrow F_s/2$$

$$\omega_0 = 2\pi f_0 \qquad * \; f_0 = f/f_s = \frac{f_s/4}{f_s} = \frac{1}{4}$$

$$= 2\pi \frac{1}{4}$$

$$= \pi/2 \leftarrow F_s/4$$

$$H\left(\tfrac{F_s}{2}\right) = H(\pi) \qquad \tilde{H}_1(\omega) = (e^{-i\omega} - e^{i\pi})$$

$$H(\pi) = 0 \qquad = e^{-i\omega} + 1$$

$$\tilde{H}_1(0) = 2$$

$$H_1(0) = 1 \qquad H_1(\omega) = \frac{1}{2}(e^{-i\omega} + 1)$$

$$H_2(\pi) = 1 \qquad \tilde{H}_2(\omega) = (e^{-i\omega} - 1)$$

$$H_2(0) = 0 \qquad \tilde{H}_2(\pi) = -2$$

$$H_2(\omega) = -\frac{1}{2}(e^{-i\omega} - 1)$$

$$H_T(\omega) = H_1 H_2 = -\frac{1}{4}(e^{-i\omega} + 1)(e^{-i\omega} - 1)$$

$$= -\frac{1}{4}$$

$$= -\frac{1}{4}(e^{-2i\omega} - 1)$$

$$H(\tfrac{F_s}{4}) = H(\tfrac{\pi}{2}) = -\frac{1}{4}(e^{-i\pi} - 1) = \boxed{\frac{1}{2}}$$

$$H(\tfrac{F_s}{2}) = H(\pi) = -\frac{1}{4}(e^{-i2\pi} - 1) = \boxed{\emptyset}$$

2 Frequency Content

(10 points)

Compute the frequency content of a discrete square wave signal $x(n)$ with a period of 4. (so the signal goes $\cdots, 1, 1, -1, -1, 1, 1, -1, -1, \cdots$.

$$P = 4 \qquad x(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$$

$$X(\omega) = \sum x(n) e^{-i\omega n}$$

$$= \delta(\emptyset) e^{-i\omega\emptyset} + \delta(1-1) e^{-i\omega 1} - \delta(2-2) e^{-i\omega 2}$$

$$- \delta(3-3) e^{-i\omega 3}$$

$$= 1 + e^{-i\omega} - e^{-2i\omega} - e^{-3i\omega}$$

4 State Space Equation

(20 points) Give the state space equations (A, B, C, D) for the system that has the frequency response

$$H(\omega) = 1 - e^{-\omega} + e^{-2\omega}$$

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$$y(n) = x(n) - \underbrace{x(n-1)}_{S_2} - \underbrace{x(n-2)}_{S_1}$$

$$S_1 = x(n-2) \longrightarrow S_1(n+1) = x(n-1) = S_2$$
$$S_2 = x(n-1) \longrightarrow S_2(n+1) = x(n)$$

$$S(n+1) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} S(n) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x(n)$$

$$y(n) = \begin{pmatrix} -1 & -1 \end{pmatrix} S(n) + (1)\, x(n)$$

5 Filtering

(10 points, 2 each) Given the frequency response

$$H(\omega) = \sin(2\omega)$$

of a discrete system. Compute the output to the input (just giving numbers results in no points)

1. $x(n) = \cos(\frac{\pi}{4}n)$
2. $x(n) = 2$
3. $x(n) = \cos(\frac{3\pi}{4}n)$
4. $x(n) = \cos(\pi n)$
5. $x(n) = 0$

$$H(\omega) = \sin(2\omega) = \frac{1}{2i}\left(e^{2i\omega} - e^{-2i\omega}\right)$$

	$H(\omega)$	$\lvert H(\omega)\rvert$	$\measuredangle H(\omega)$	out
$\pi/4$	$= e^{i\pi/2} - e^{-i\pi/2}$ $= (i + i)/2i$ $= 1$	1	$\emptyset$	$y(n) = \cos(\frac{\pi}{4}n)$
$\emptyset$	$\emptyset$	$\emptyset$	—	$y(n) = \emptyset$

$\emptyset$	$\emptyset$	$\emptyset$	—	$y(n) = \emptyset$
$\frac{3\pi}{4}$	$= \frac{e^{i3\pi/2} - e^{-i3\pi/2}}{2i}$ $= (-i - i)/2i$	$\|-1\|$	$-\pi$	$y(n) = \cos(\frac{3\pi}{4} n - \pi)$
π	$= \frac{e^{i2\pi} - e^{-i2\pi}}{2i}$ $= \emptyset$	$\emptyset$	—	$y(n) = \emptyset$
$\emptyset$	$\emptyset$	$\emptyset$	—	$y(n) = \emptyset$

7 Short Questions

(10 points, 2 each) For each of the following frequency responses find the corresponding impulse response **or** system. (some are discrete some are continuous !)

1. $G(s) = s$
2. $G(z) = z$
3. $H(\omega) = \delta(\omega - \omega_c) + \delta(\omega + \omega_c)$
4. $G(s) = \frac{s}{s+1}$
5. $G(z) = 1 + \frac{1}{1+z^{-1}}$

1) $Y/X = s$

$Y = Xs$

$y = x'$ $\quad x = e^{i\omega t}$

$H(\omega)e^{i\omega t} = i\omega e^{i\omega t}$

$\underline{H(\omega) = i\omega}$

2) $Y/X = z$

$Y = Xz$

$y(n-1) = x(n)$

$H(\omega)e^{-i\omega} = 1$

$\underline{H(\omega) = 1/e^{-i\omega}}$

4) $G = \frac{s}{s+1}$

$Ys + Y = Xs$

$y' + y = x'$

5) $Y/X = 1 + \frac{1}{1+z^{-1}}$

$= \frac{1 + z^{-1} + 1}{1 + z^{-1}}$

$2 + z^{-1}$

$$Ys + Y = Xs$$
$$y' + y = x'$$
$$i\omega H e^{i\omega t} + H e^{i\omega t} = e^{i\omega t} i\omega$$
$$H(1 + i\omega) = i\omega$$
$$H = \frac{i\omega}{1 + i\omega}$$

$$= \frac{\qquad}{1 + z^{-1}}$$
$$= \frac{2 + z^{-1}}{1 + z^{-1}}$$

$$Y + Yz^{-1} = 2X + Xz^{-1}$$
$$y(n) = 2x(n) + x(n-1) + y(n-1)$$
$$H(\omega) = 2 + e^{-i\omega} + H(\omega)e^{-i\omega}$$
$$H(\omega) = \frac{2 + e^{-i\omega}}{1 + e^{-i\omega}}$$

3) $h(t) = \frac{1}{2\pi} \int H(\omega) e^{i\omega t} \, d\omega$

$$= \frac{1}{2\pi}\left(\int \delta(\omega - \omega_c) e^{i\omega t} + \int \delta(\omega + \omega_c) e^{i\omega t} \right)$$

$\bigstar$ $\int f(\omega)\delta(\omega - \alpha)$ identity

$$= \frac{1}{2\pi}\left(e^{-i\omega_c t} + e^{i\omega_c t} \right)$$
$$= \frac{1}{\pi} \cos(\omega_c t)$$

8 Stability

(10 points, 5 each) For the two systems below determine α, β so they are stable:

1. $y' + \alpha y + x' + x = 0$
2. $y(n+1) = x(n) - \beta y(n-1) + x(n-1)$

1) $Ys + \alpha Y = -Xs - X$

$$Y(s - \alpha) = X(-s - 1)$$
$$G = \frac{-s - 1}{s - \alpha} = \frac{s + 1}{\alpha - s}$$

$$\alpha - s = \emptyset$$
$$s = \alpha$$
$$\boxed{\alpha < \emptyset}$$

$$\delta = a$$

$$\boxed{r < 0}$$

2) $y(n+1) = x(n) - \beta y(n-1) + x(n-1)$

$$Yz = X - \beta Y z^{-1} + X z^{-1}$$

$$Y(z + \beta z^{-1}) = X(1 - z^{-1})$$

$$G = \frac{1 - z^{-1}}{z + \beta z^{-1}}$$

$$0 = z + \beta z^{-1}$$

$$z^2 = -\beta$$

$$z = \sqrt{-\beta}$$

$$1 > |z|$$

$$1 > |\sqrt{-\beta}|$$

$$1 > |-\beta|$$

$$-1 < \beta < 1$$