McMaster University Final Exam

Name ____________________
Student Number ____________________

MECH ENG 4K03/6K03 ROBOTICS

INSTRUCTOR NAME: Fengjun Yan

DURATION OF EXAMINATION: 2.5 HRS

December 2020

THIS EXAMINATION PAPER INCLUDES 5 PAGES (3 PAGES FOR QUESTIONS AND 2 PAGES FOR FORMULAS) AND 5 QUESTIONS.

Use of Casio FX-991 MS or MS Plus calculator.

Questions:

1. (8 points) Give the definitions of the following terms in robotics.

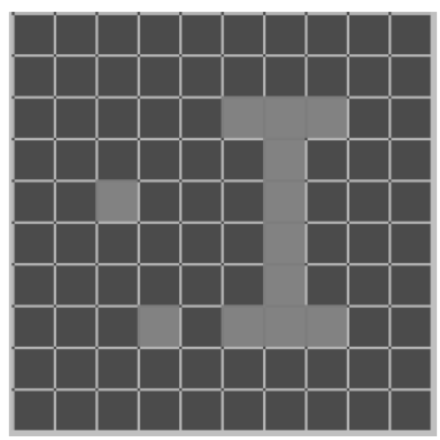
1) Hard automation
2) Flexible automation
3) Planar robot
4) Dextrous workspace

2. Short Answer Questions

1) (5 points) If the transformation matrices, ${}^{C}T_{D}$, ${}^{A}T_{B}$, ${}^{A}T_{E}$ and ${}^{E}T_{D}$, are known, derive the transformation equation for ${}^{B}T_{C}$ in terms of these matrices.

2) (5 points) Given the grayscale input image of the letter "I" from an optical character recognition application:

$$A = \begin{bmatrix} 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 130 & 130 & 130 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 130 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 130 & 75 & 130 & 130 & 130 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \end{bmatrix}$$

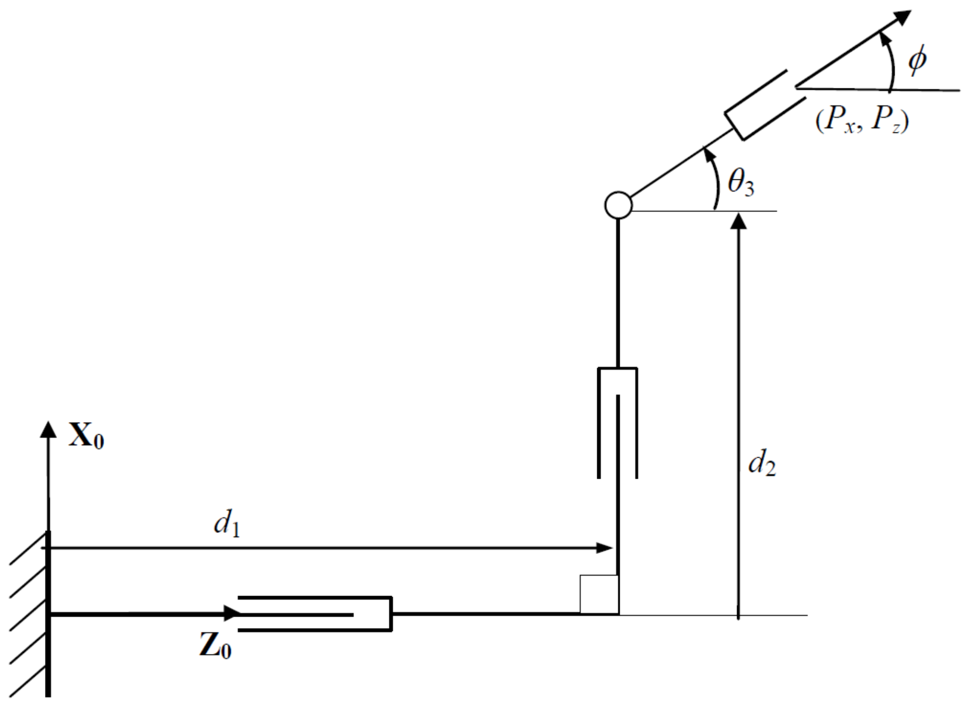


Show a sample calculation for the row=8, col=5 pixel, if we apply a Laplacian 1 filter to the input image.

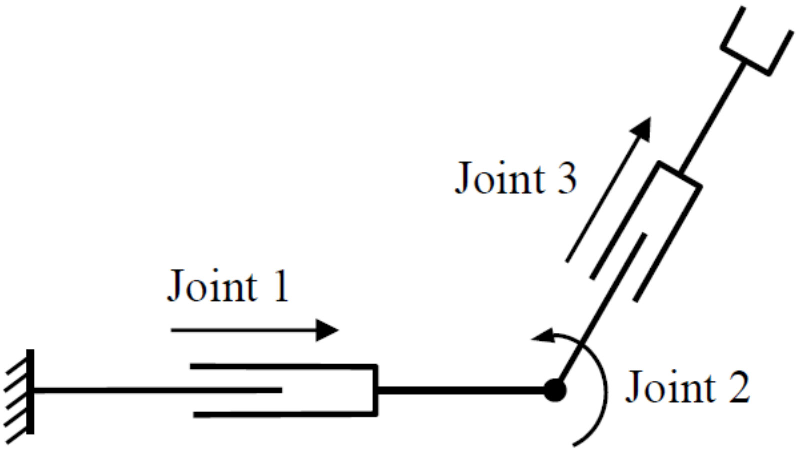
3) (5 points) Which of the following matrices is a valid representation for a frame? Explain your answer.

$$A = \begin{bmatrix} 0.77 & -0.64 & 0 & 3 \\ 0.64 & 0.77 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ B = \begin{bmatrix} 0 & \sqrt{3}/2 & 0 & 3 \\ -1 & 0 & 0 & 0.5 \\ 0 & 1/2 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ C = \begin{bmatrix} 0 & \sqrt{3}/2 & -1/2 & 3 \\ -1 & 0 & 0 & 0.5 \\ 0 & 1/2 & -\sqrt{3}/2 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

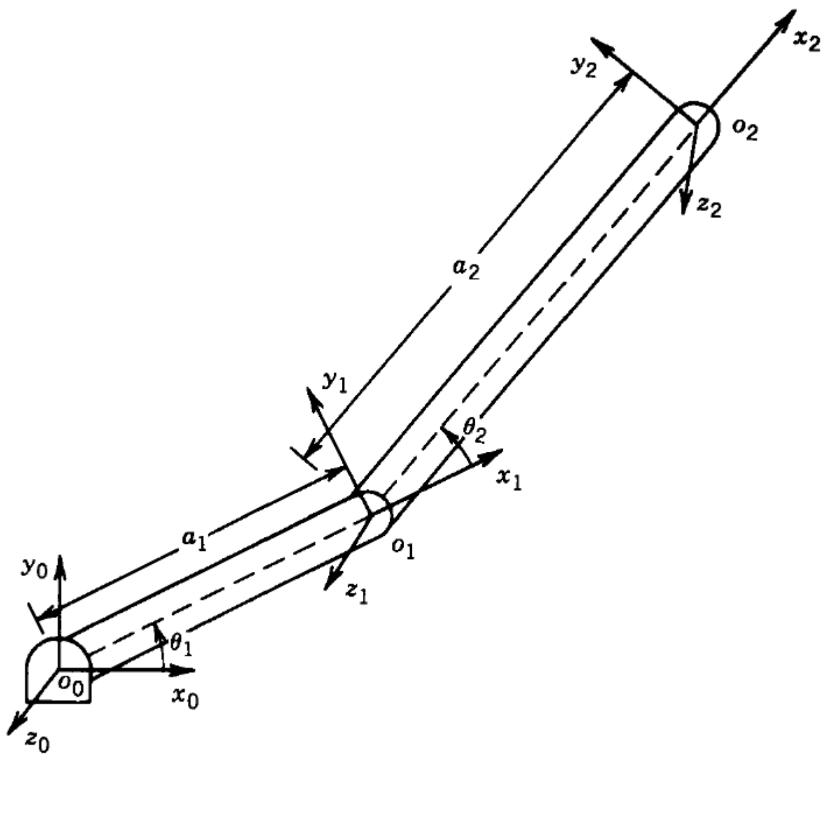
3. (25 points) For the PPR planar robot shown in the following Figure:
a) Assign the frames using the D-H method.
b) Determine the D-H parameters and put them in the standard table form. Identify the joint variables.
c) Draw a diagram of the robot that properly shows the D-H frames, the joint variables, and any *d* or *a* parameters that are non-zero.
d) Calculate the *A* matrices and ${}^0T_3$
e) Its joint variables are d_1, d_2, and θ_3. Its end-effector position and orientation are given by P_x, P_z and $\emptyset$. Derive its inverse kinematics.



4. (25 points) For the planar PRP robot shown in the following Figure:
a) Derive the 3x3 manipulator Jacobian matrix. (The form used for calculating the linear velocity and angular velocity of the tool).
b) Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.
c) Draw the robot in a singular configuration and indicate which degree (s) of freedom have been lost.



5. (27 points) For the RR planar robot in the following figure, if $a_1 = 0.4\text{m}$ and $a_2 = 0.3\text{m}$;
(a) Assuming the robot operates in the horizontal plane, calculate the joint torques such that the static force at the end-effector is $F_x = 20\text{N}$ and $F_y = -15\text{N}$ for the configuration $\theta_1 = 35^\circ$ and $\theta_2 = -75^\circ$
(b)Assuming the robot operates in the horizontal plane, calculate the static force applied by the end effector when $\tau_1 = 10\text{Nm}$, $\tau_2 = 5\text{Nm}$, $\theta_1 = 35^\circ$ and $\theta_2 = -75^\circ$.



Formulas

$$\mathrm{Rot}(X,\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \mathrm{C}\theta & -\mathrm{S}\theta & 0\\ 0 & \mathrm{S}\theta & \mathrm{C}\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

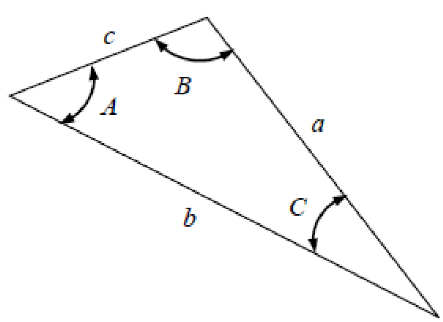
$$\mathrm{Rot}(Y,\theta)=\begin{bmatrix}\mathrm{C}\theta & 0 & \mathrm{S}\theta & 0\\ 0 & 1 & 0 & 0\\ -\mathrm{S}\theta & 0 & \mathrm{C}\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\mathrm{Rot}(Z,\theta)=\begin{bmatrix}\mathrm{C}\theta & -\mathrm{S}\theta & 0 & 0\\ \mathrm{S}\theta & \mathrm{C}\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\mathrm{Trans}(a,b,c)=\begin{bmatrix}1 & 0 & 0 & a\\ 0 & 1 & 0 & b\\ 0 & 0 & 1 & c\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$T^{-1}=\begin{bmatrix}n_x & n_y & n_z & -\vec{P}\bullet\vec{n}\\ o_x & o_y & o_z & -\vec{P}\bullet\vec{o}\\ a_x & a_y & a_z & -\vec{P}\bullet\vec{a}\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A_{n+1}={}^{n}T_{n+1}=\begin{bmatrix}\mathrm{C}\theta_{n+1} & -\mathrm{S}\theta_{n+1}\mathrm{C}\alpha_{n+1} & \mathrm{S}\theta_{n+1}\mathrm{S}\alpha_{n+1} & a_{n+1}\mathrm{C}\theta_{n+1}\\ \mathrm{S}\theta_{n+1} & \mathrm{C}\theta_{n+1}\mathrm{C}\alpha_{n+1} & -\mathrm{C}\theta_{n+1}\mathrm{S}\alpha_{n+1} & a_{n+1}\mathrm{S}\theta_{n+1}\\ 0 & \mathrm{S}\alpha_{n+1} & \mathrm{C}\alpha_{n+1} & d_{n+1}\\ 0 & 0 & 0 & 1\end{bmatrix}$$



$$\cos A=\left(b^2+c^2-a^2\right)/\left(2bc\right)$$

$$S\theta_1 C\theta_2 + C\theta_1 S\theta_2 = S(\theta_1+\theta_2) = S\theta_{12}$$

$$C\theta_1 C\theta_2 - S\theta_1 S\theta_2 = C(\theta_1+\theta_2) = C\theta_{12}$$

$$\text{if } a=\sin\theta \text{ and } b=\cos\theta \text{ then } \theta=\mathrm{atan2}(a,b)$$

$$J(q)=\begin{bmatrix}\dfrac{\partial p_x(q)}{\partial q_1} & \dfrac{\partial p_x(q)}{\partial q_2}\\ \dfrac{\partial p_y(q)}{\partial q_1} & \dfrac{\partial p_y(q)}{\partial q_2}\end{bmatrix}$$

$$J(q)=\begin{bmatrix}\dfrac{\partial p_x(q)}{\partial q_1} & \dfrac{\partial p_x(q)}{\partial q_2} & \dfrac{\partial p_x(q)}{\partial q_3}\\ \dfrac{\partial p_y(q)}{\partial q_1} & \dfrac{\partial p_y(q)}{\partial q_2} & \dfrac{\partial p_y(q)}{\partial q_3}\\ \zeta_1 t_1 & \zeta_2 t_2 & \zeta_3 t_3\end{bmatrix}$$

$$J(q)=\begin{bmatrix}\dfrac{\partial p_x(q)}{\partial q_1} & \dfrac{\partial p_x(q)}{\partial q_2} & \dfrac{\partial p_x(q)}{\partial q_3}\\ \dfrac{\partial p_y(q)}{\partial q_1} & \dfrac{\partial p_y(q)}{\partial q_2} & \dfrac{\partial p_y(q)}{\partial q_3}\\ \dfrac{\partial p_z(q)}{\partial q_1} & \dfrac{\partial p_z(q)}{\partial q_2} & \dfrac{\partial p_z(q)}{\partial q_3}\end{bmatrix}$$

$$J(q)=\left[\begin{array}{cccc}\dfrac{\partial p_x(q)}{\partial q_1} & \dfrac{\partial p_x(q)}{\partial q_2} & \cdots & \dfrac{\partial p_x(q)}{\partial q_n}\\ \dfrac{\partial p_y(q)}{\partial q_1} & \dfrac{\partial p_y(q)}{\partial q_2} & \cdots & \dfrac{\partial p_y(q)}{\partial q_n}\\ \dfrac{\partial p_z(q)}{\partial q_1} & \dfrac{\partial p_z(q)}{\partial q_2} & \cdots & \dfrac{\partial p_z(q)}{\partial q_n}\\ \hdashline \zeta_1 \mathbf{z}_0(q) & \zeta_2 \mathbf{z}_1(q) & \cdots & \zeta_n \mathbf{z}_{n-1}(q)\end{array}\right]$$

$$z_i = {}^{0}R_i\begin{bmatrix}0\\0\\1\end{bmatrix} \quad \text{where } {}^{0}R_i=\prod_{k=1}^{i}{}^{k-1}R_k$$

if $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\det(J) = j_{11}(j_{33}j_{22} - j_{32}j_{23}) - j_{21}(j_{33}j_{12} - j_{32}j_{13}) + j_{31}(j_{23}j_{12} - j_{22}j_{13})$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J(q) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\tau = J(q)^T F$$

$$F_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i}$$

$$\tau_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i}$$

$$K_j = \tfrac{1}{2} m_j v_{cj}{}^2 + \tfrac{1}{2} I_j \omega_j{}^2$$

$$P_j = -m_j G^T p_{cj}$$

$$\dot{\theta}_{\max} = \frac{\theta_h - \theta_b}{t_h - t_b} = \ddot{\theta}_d t_b$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}_d{}^2 t_f{}^2 - 4\ddot{\theta}_d(\theta_f - \theta_i)}}{2\left|\ddot{\theta}_d\right|}$$

$\theta(t) = \theta_i + \frac{1}{2}\ddot{\theta}_d t^2$, $\dot{\theta}(t) = \ddot{\theta}_d t$,
and $\ddot{\theta}(t) = \ddot{\theta}_d$

$\theta(t) = \theta_i + \frac{1}{2}\ddot{\theta}_d t_b{}^2 + \ddot{\theta}_d t_b (t - t_b)$, $\dot{\theta}(t) = \ddot{\theta}_d t_b$,
and $\ddot{\theta}(t) = 0$

$\theta(t) = \theta_f - \frac{1}{2}\ddot{\theta}_d (t_f - t)^2$, $\dot{\theta}(t) = \ddot{\theta}_d (t_f - t)$,
and $\ddot{\theta}(t) = -\ddot{\theta}_d$

Gaussian $M = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, Mean $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Lap1 $M = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$, Lap2 $M = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

Sobel $M_h = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ and $M_v = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Prewitt $M_h = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $M_v = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$$F = A + c\left(A - F_{smooth}\right)$$

The End