Q1

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March 26, 2022 2:00 PM

1. **Magnetic Forces and Torques**
An electron with a speed of $4\text{x}10^6$ m/s is projected along the positive x direction into a medium containing uniform magnetic flux density $\mathbf{B} = (\hat{\mathbf{x}}7 - \hat{\mathbf{z}}4)$ T. Determine the initial acceleration and direction of the electron (at the moment it is projected into the medium).

$$v = 4\times10^6 \text{ m/s}$$

$$\vec{B} = \langle 7, \emptyset, -4 \rangle$$

$$\vec{F} = m\vec{a} = q\vec{v}\times\vec{B}$$

$$\vec{a} = \frac{e\vec{v}\times\vec{B}}{m_e}$$

$$\vec{a} = \frac{(-1.6\times10^{-19})\langle 4\times10^6, \emptyset, \emptyset\rangle \times \langle 7, \emptyset, -4\rangle}{9.1\times10^{-31}}$$

$$\vec{a} = \frac{(-1.6\times10^{-19})\langle \emptyset, 16\times10^6, \emptyset\rangle}{9.1\times10^{-31}}$$

$$\vec{a} = \frac{(-1.6\times10^{-19})(16\times10^6)\,\hat{y}}{9.1\times10^{-31}}$$

4.1×10^{-31}

$$\vec{a} = -\hat{y} \cdot 2.81 \times 10^{18} \text{ m/s}$$

Q2

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2. **Magnetic Forces and Torques**

In a cylindrical coordinate system, a 4 m long straight wire carrying a current of 8 A in the positive z-direction is located at r = 5 cm, $\phi = \frac{\pi}{2}$, and $-2\ m \le z \le 2\ m$.

a. If $\mathbf{B} = \hat{\mathbf{r}}0.3\cos\phi$ (T), what would be the magnetic force acting on the wire?

b. How much work is required to rotate the wire once about the z axis in the negative ϕ direction) while maintain r = 5 cm)?

c. At what angle ϕ is the force maximum?

a)

$$\vec{F} = I l \times B$$

$$\vec{F} = 8 \cdot 4\,\hat{z} \times \hat{r}\,0.3\cos\phi$$

$$\vec{F} = (9.6\cos\phi)\,\hat{\phi}$$

$$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$$

$$\star\ \phi = \pi/2$$

$$\hat{x} = \hat{r}\,\cancel{\cos(\pi/2)}^{\,0} - \hat{\phi}\,\cancel{\sin(\pi/2)}^{\,1}$$

$$\hat{x} = -\hat{\phi}$$

$$\hat{\phi} = -\hat{x}$$

$$\vec{F} = -9.6\cos\phi\ \hat{x}$$

$$\vec{F} = -9.6\,\cancel{\cos(\pi/2)}^{\,0}\ \hat{x}$$

$\vec{\ }$

$F = -9.6\cos(\pi/2)\,\hat{x}$

$\vec{F} = \emptyset$

b) $r = 5\,\text{cm}$

$W = \int_{\phi} \vec{F} \cdot d\vec{l}$

$\star\; dl = \hat{r}\,dr + \hat{\phi}\,r\,d\phi + \hat{z}\,dz$

$$\frac{dl}{d\phi} = \hat{\phi}\,r$$

$$dl = \hat{\phi}\,r\,d\phi$$

rotating in negative ϕ direction

$\therefore dl = -\hat{\phi}\,r\,d\phi$

$$W = \int_0^{2\pi} (9.6\cos\phi)\,\hat{\phi} \cdot (-r)\,\hat{\phi}\,d\phi$$

$$W = \int^{2\pi} -9.6\,r\cos\phi\,d\phi$$

$$W = \int_0^{2\pi} -9.6 r \cos\phi \, d\phi$$

★ $r = 5\text{cm}$

$$W = (-9.6)(0.04)\int_0^{2\pi} \cos\phi \, d\phi$$

$$W = -0.384\left(\cancelto{0}{\sin\phi}\right)\Big|_0^{2\pi}$$

$$W = 0$$

c) $F = 9.6\cos\phi$

$$\frac{dF}{d\phi} = \sin\phi$$

find max/min

$$0 = \sin\phi$$

$\phi = 0, \pi$ (for a full rotation)

$$F(0) = 9.6\cancelto{1}{\cos(0)} > 0$$

$$F(\pi) = 9.6\cancelto{-1}{\cos(\pi)} = \ldots$$

$$F(\pi) = 9.6\,\overbrace{\cos(\pi)}^{-1} = -9.6 < 0 \leftarrow \text{negative direction}$$

$\therefore$ force is maximum when $\phi = 0$

Q3

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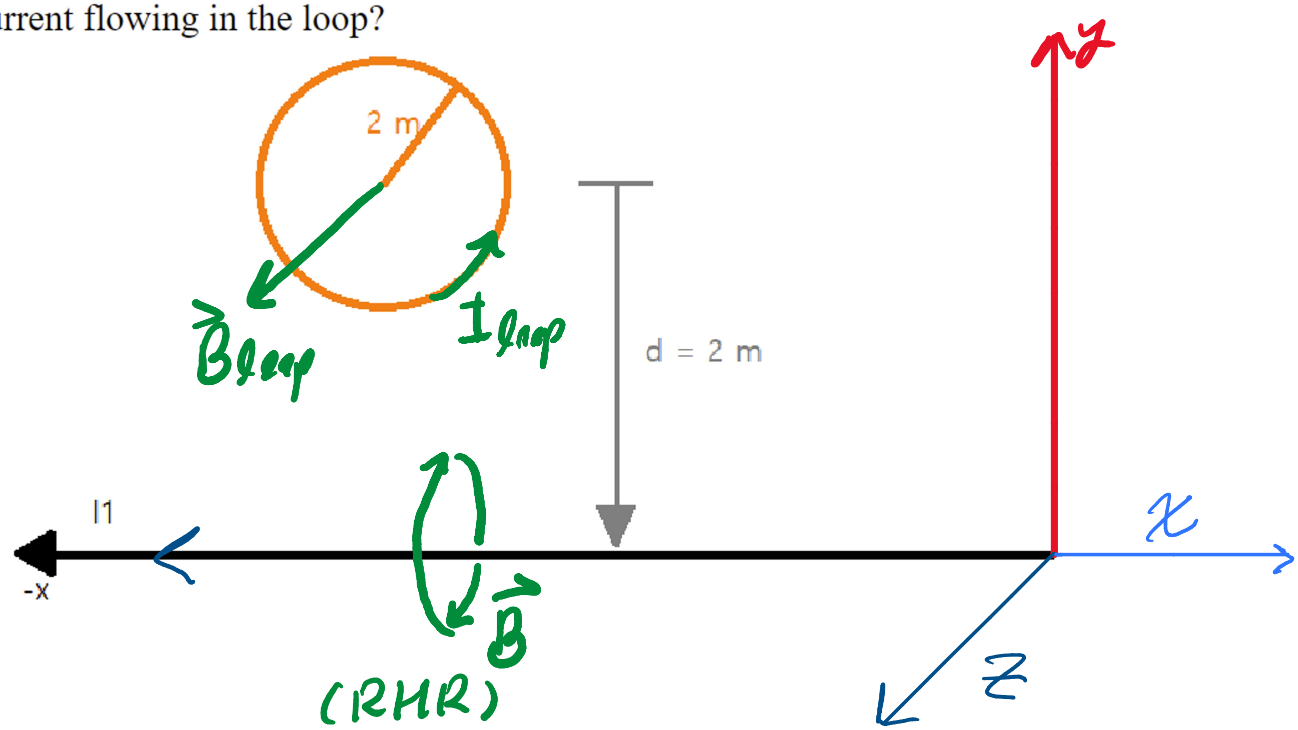
3. **Biot-Savart Law**

An infinitely long wire carrying a 30 A current in the negative x direction is place along the x axis near a 40 turn circular loop located in the x-y plane as shown below. If the magnetic field at the center of the loop is zero, what is the direction and magnitude of the current flowing in the loop?



$$B = \frac{\mu_0 I}{2\pi R}$$

$$\underset{\text{@ center of loop}}{\vec{B}_{wire}} = -\frac{\mu_0 I_1}{2\pi d}\hat{z} \quad \text{(into the page)}$$

$$\vec{B}_{wire} = -\frac{30\mu_0}{2\pi 2}\hat{z} = -\frac{15\mu_0}{2\pi}\hat{z}$$

$$\vec{B}_{loop} = \frac{\mu_0 I N}{2a} \hat{z}$$

$\therefore$ net field $= \varnothing$, direction of $\vec{B}_{wire}$ = opposite direction of $\vec{B}_{loop}$

$$\vec{B}_{loop} = \frac{\mu_0 40 I_{loop}}{2(2)} \hat{z} = 10 \mu_0 I_{loop} \hat{z}$$

Net field @ center $= \varnothing$

$$\varnothing = \vec{B}_{wire} + \vec{B}_{loop}$$

$$\vec{B}_{wire} = -\vec{B}_{loop}$$

$$\frac{15 \cancel{\mu_0}}{2\pi} = 10 \cancel{\mu_0} I_{loop}$$

$$\frac{15}{20\pi} = I_{loop}$$

$$I_{loop} = 0.2387324 \text{ A}$$

$I_{loop} \doteq 0.239\ A$

By RHR: $\because \vec{B} = \hat{z}$, $\therefore \vec{I} = CCW$

$\vec{I}_{loop} \doteq 0.239\ A$ CCW

Q4

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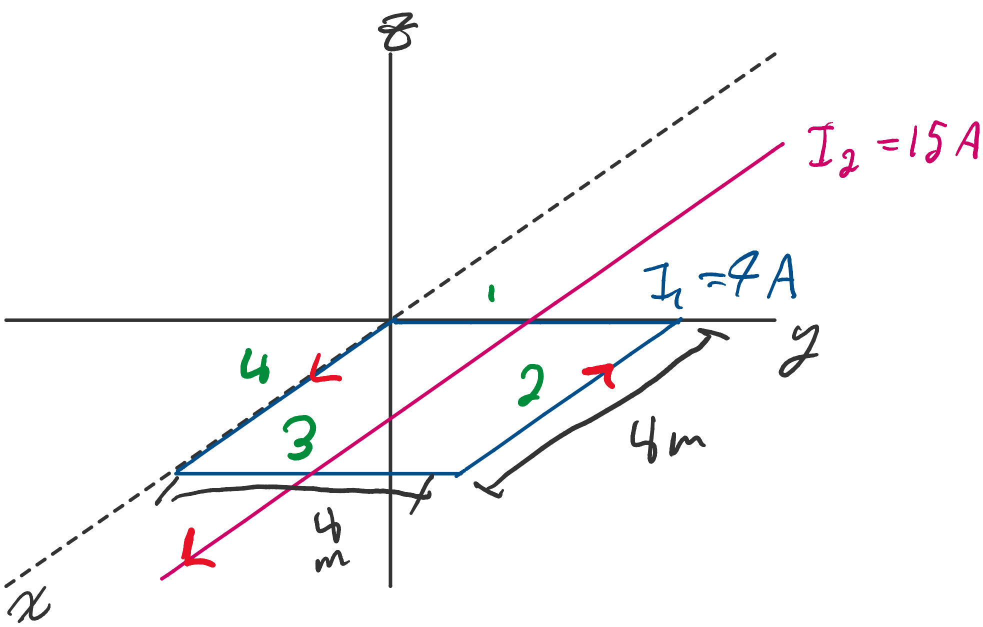
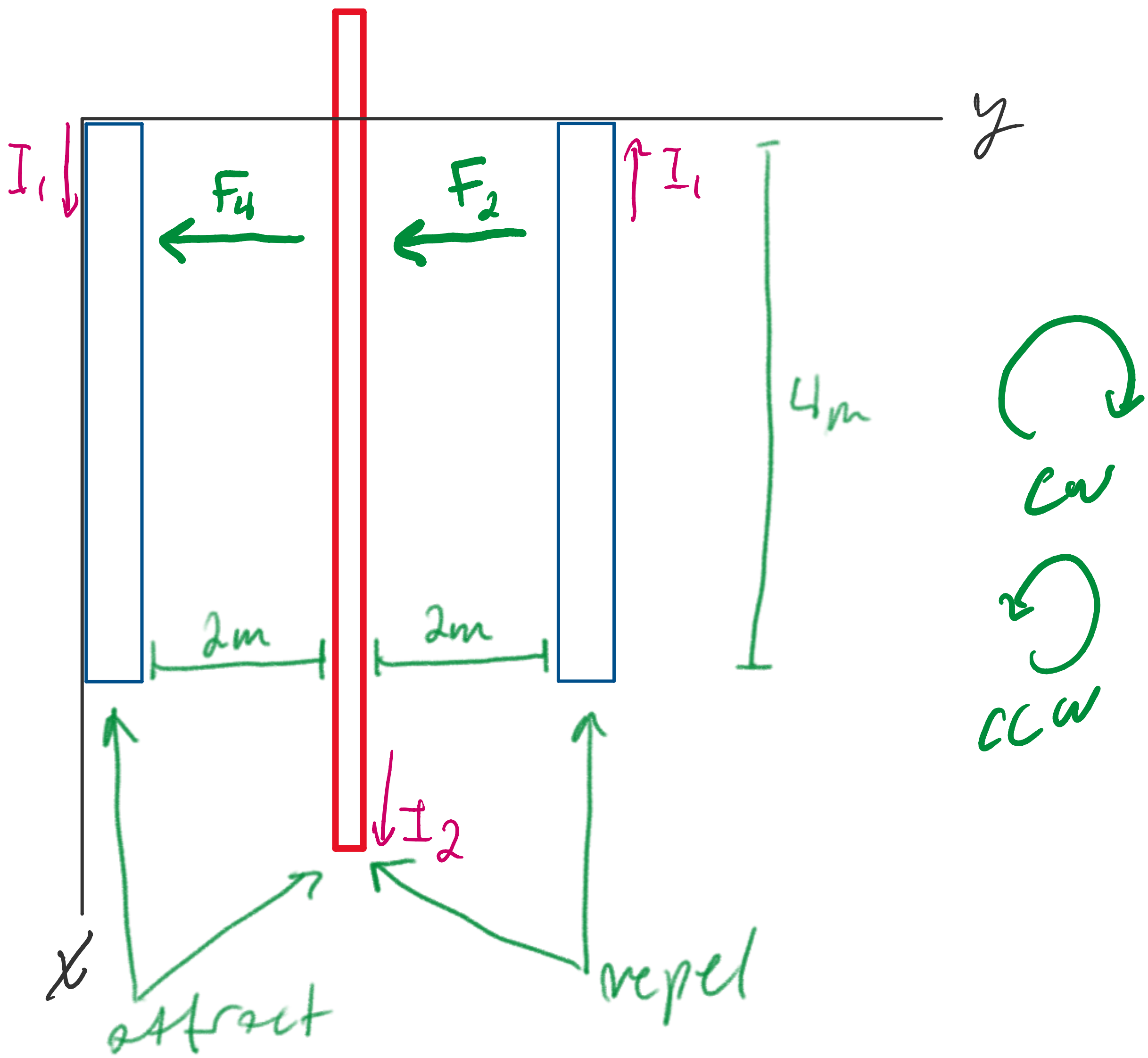
4. **Biot-Savart Law**
A square loop is placed onto the x-y plane. The square loop has 4 m sides and carries a current of $I_1 = 7$ A. If a straight, long conductor carrying a current $I_2 = 15$ A is introduced and placed just above the midpoints of the two loop's sides, determine the net force acting on the loop.



assuming current in loop is CCW
& current in conductor is in $+\hat{x}$ direction

compare sections w/ conductor
as if comparing 2 straight wires
(forces on parallel conductors)



→ parallel wires attract if current is the same direction, repel if opposite directions

→ note that segments 1 & 3 do not have force exerted on them by the conductor because they are perpendicular

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$$\frac{\quad}{2\pi r}$$

$$\vec{F}_1 = \emptyset, \quad \vec{F}_2 = \emptyset$$

Segment 2

$$\vec{F}_2 = \hat{y}\,\frac{\mu_0 (7A)(15A)(4m)}{2\pi(2m)}$$

$$\vec{F}_2 = 4.20\times10^{-5}\,\hat{y}\ N$$

Segment 4

$$\vec{F}_4 = \hat{y}\,\frac{\mu_0 (7A)(15A)(4m)}{2\pi(2m)}$$

$$\vec{F}_4 = 4.20\times10^{-5}\,\hat{y}\ N$$

$$\vec{F}_{net} = \cancel{\vec{F}_1}^{\,\emptyset} + \vec{F}_2 + \cancel{\vec{F}_3}^{\,\emptyset} + \vec{F}_4 = \vec{F}_2 + \vec{F}_4$$

$$\vec{F}_{net} = \cancel{\vec{F}_1} + \vec{F}_2 + \cancel{\vec{F}_3} + \vec{F}_4 = \vec{F}_2 + \vec{F}_4$$

$$\vec{F}_{net} = 8.40 \times 10^{-5}\,\hat{y}\ \text{N}$$

∴ Direction of current is not given:

if $\vec{I}_2$ was in $+\hat{x}$ & $\vec{I}_1$ was CCW

$$\vec{F}_{net} = 8.40 \times 10^{-5}\,\hat{y}\ \text{N}$$

if $\vec{I}_2$ was in $-\hat{x}$ & $\vec{I}_1$ was CCW,

$$\vec{F}_{net} = -8.40 \times 10^{-5}\,\hat{y}\ \text{N}$$

if $\vec{I}_2$ was in $-\hat{x}$ & $\vec{I}_1$ was CW,

$$\vec{F}_{net} = 8.40 \times 10^{-5}\,\hat{y}\ \text{N}$$

if $\vec{I}_2$ was in $+\hat{x}$ & $\vec{I}_1$ was CW,

$$\vec{F}_{net} = -8.40 \times 10^{-5}\,\hat{y}\ \text{N}$$

Q5

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5. **Bonus Question: Reading/Research Question**
   Ampere's Law can be applied to solve for the magnetic field. Compare Ampere's Law to Gauss's Law and the differences that exist between them.

Ampere's law describes magnetic field in terms of the flow of current. Ampere's law focuses on current's relationship with a magnetic field by examining the relationship of the magnetic field along a closed path and the current passing through the area enclosed by said path. Ampere's law can also be described as the effect on a magnetic field of a changing electric field.

$$\int \mathrm{B} \cdot \mathrm{dl} = \mu_o I$$

In contrast, the Gauss' law approach observes the net magnetic flux of a field over a closed surface and its proportionality to the charge enclosed in the surface. Rather than observing the integral of a field along a **path** like Ampere's law, Gauss' law focuses on the magnetic flux through a **surface.**

$$\oint \vec{B} \cdot \overrightarrow{dA} = 0$$

Sources:
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