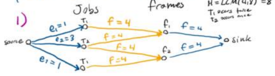
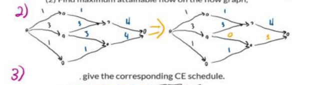
(1) For a frame size of 4, give the directed graph with edge capacity

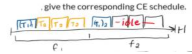
$f = 4$ $T_1(4,1)$ $T_2(8,3)$ $H = LCM(4,8) = 8$



(2) Find maximum attainable flow on the flow graph.



. give the corresponding CE schedule.

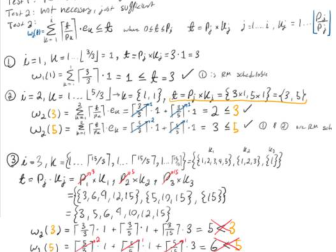


2 $T_1(3,1)$ $T_2(5,1)$ $T_3(15,3)$

1) RM

Test 1: Periods are not integer multiples of each other

Test 2: not necessary, just sufficient

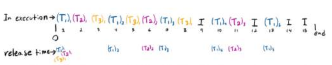


$\to$ 2) schedule

$T_1(3,1)$ $T_2(5,1)$ $T_3(15,3)$

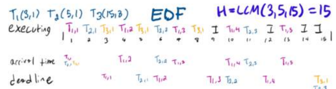
high prio → low prio

$H = LCM(3,5,15) = 15$



result → $T_1, T_2, T_3, T_1, T_3, T_2, T_1, T_3, I, T_1, T_2, I, T_1, I, I$

$T_1(3,1)$ $T_2(5,1)$ $T_3(15,3)$ EDF $H = LCM(3,5,15) = 15$



Schedule $T_1\ T_2\ T_3\ T_1\ T_3\ T_2\ T_1\ T_3\ I\ T_1\ T_2\ I\ T_1\ I\ I$

8. Given the task set: T1(4, 1); T2(6, 1); T3(12, 2). What is the largest suitable frame size f?

A. 3 B. 4 C. 6 D. 12

C1: $f \geq 2$

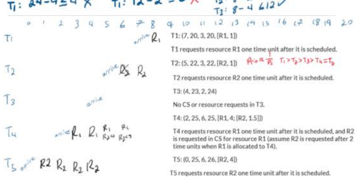
C2: $H = LCM(4,6,12) = 12$, $F = \{1,2,3,4,6,12\}$

C3: $2f - GCD(P_i, F) \leq D_i = P_i$

f = 12; T_1: $24 - 4 \leq 4$ ✗

f = 6; T_1: $12 - 2 \leq 6$ ✗

f = 4; T_1: $8 - 4 \leq 4$ ✓, T_2: $8 - 2 \leq 6$ ✓, T_3: $8 - 4 \leq 12$ ✓ ∴ 4



T1: (7, 20, 3, 20, [R1, 1])

T1 requests resource R1 one time unit after it is scheduled.

T2: (5, 22, 3, 22, [R2, 1])

T2 requests resource R2 one time unit after it is scheduled.

T3: (4, 23, 2, 24)

No CS or resource requests in T3.

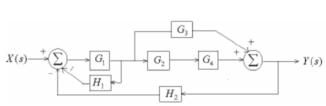
T4: (2, 25, 6, 25, [R1, 4; [R2, 1.5]])

T4 requests resource R1 one time unit after it is scheduled, and R2 is requested in CS for resource R1 (assume R2 is requested after 2 time units when R1 is allocated to T4).

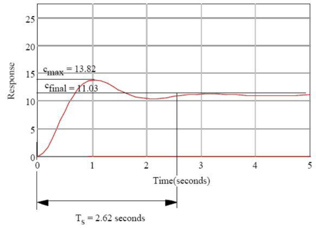
T5: (0, 25, 6, 26, [R2, 4])

T5 requests resource R2 one time unit after it is scheduled.

Quiz Solutions



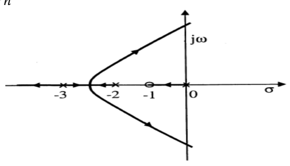
$T(s) = \dfrac{G_1(G_2G_4+G_3)}{1+G_1H_1+H_2G_1(G_2G_4+G_3)}$



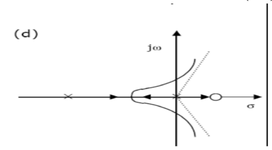
$T_s = 2.62$ seconds

Find transfer function: $\%OS = 25.3\%, T_s = 2.62$ $\zeta = \dfrac{-ln(\%OS/100)}{\sqrt{\pi^2+ln^2(\%OS/100)}} = 1.37/3.43 \approx 0.4$

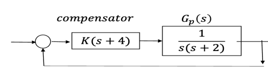
$T_s \approx \dfrac{4}{\zeta\omega_n} \to \omega_n = \dfrac{4}{0.4\times T_s} = 3.82$ $G(s) = \dfrac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$ $Y(s) = \dfrac{K}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ using FVT $c_{final} = \dfrac{sK}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \dfrac{K}{\omega_n^2} = 11.03$ $G(s) = \dfrac{160.95}{s^2+3.056s+14.59}$



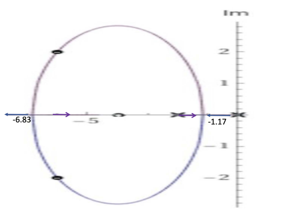
$G(s)H(s) = k\dfrac{(s+1)}{s(s+2)(s+3)}$



$G(s) = \dfrac{K(1-s)}{s(s+3)}$



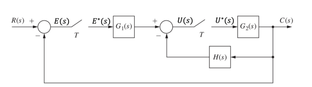
Root locus:



Z Transform of $x(n) = \frac{1}{2}^n [u(n) - u(n-10)]$ $Z[x(n)] = \sum_{n=0}^{9}\left(\frac{1}{2}\right)^n z^{-n} = \dfrac{1-\left(\frac{1}{2z}\right)^{10}}{1-\frac{1}{2z}} = \dfrac{2z-\left(\frac{1}{2}\right)^{10}z^{-9}}{2z-1}$

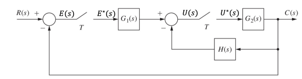
Inverse Z Transform of $F(z) = \dfrac{3+2z^{-1}}{2+3z^{-1}+z^{-2}}$ $\dfrac{3+2z^{-1}}{2+3z^{-1}+z^{-2}} = \dfrac{3z^2+2z}{2z^2+3z+1}$ $F(z)/z = \dfrac{3z+2}{2z^2+3z+1}$, partial fractions gives: $\dfrac{A}{2z+1} + \dfrac{B}{z+1}, A = B = 1$ $F(z)/z = \dfrac{1}{2z+1} + \dfrac{1}{z+1} = \dfrac{\frac{1}{2}}{z+\frac{1}{2}} + \dfrac{1}{z+1}$

$F(z) = \dfrac{\frac{1}{2}z}{z+\frac{1}{2}} + \dfrac{z}{z+1}$ $f(k) = \frac{1}{2}(-\frac{1}{2})^k + (-1)^k, k \geq 0$



$G_1(s)E*(s) - G_2(s)H(s)U*(s) = U(s)$ $R(s) - C(s) = E(s)$ $G_2(s)U*(s) = C(s)$ take z transforms: $G_1(z)E(z) - Z[G_2(s)H(s)]U(z) = U(z)$ (1) $R(Z) - C(z) = E(z)$(2) $G_2(z)U*(z) = C(z)$(3)

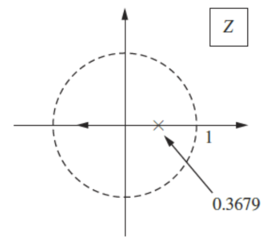
$G_1(z)E(z) = [\overline{G_2H}(z) + 1]U(z)$ (4) multiply (2) by $G_1(z)G_2(z)$ $G_1(z)G_2(z)[R(z) - C(z)] = G_1(z)G_2(z)E(z)$ (5) sub $G_1(z)E(z)$ in (5) with (4) $G_1(z)G_2(z)[R(z)-C(z)] = G_2(z)[\overline{G_2H}(z) + 1]U(z)$ then sub $G_2(z)U(z)$ as C(z) in (3) we get: $G_1(z)G_2(z)[R(z) - C(z)] = [\overline{G_2H}(z) + 1]C(z)$ $\therefore G_1(z)G_2(z)R(z) - G_1(z)G_2(z)C(z) = [\overline{G_2H}(z) + 1]C(z)$ $\therefore \dfrac{C(z)}{R(z)} = \dfrac{G_1(z)G_2(z)}{1+\overline{G_2H}(z)+G_1(z)G_2(z)}$ char equation is denominator set to 0



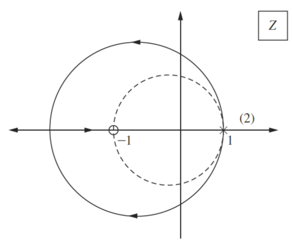
$T = 1$ (a) closed loop char equation of discrete system open loop transfer function $G(z) = (1-z^{-1})Z\left[\dfrac{1}{s(s+1)}\right]_{(T=1)}$

$G(z) = \dfrac{z-1}{z}\dfrac{(1-e^{-1})z}{(z-1)(z-e^{-1})} = \dfrac{0.6321}{z-0.3679}$

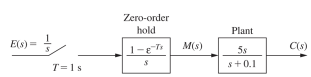
$T(z) = \dfrac{KG(z)}{1+KG(z)}$ char equation is $1 + KG(z) = z - 0.3679 + 0.6321K = 0$ (b) K value for Marginally stable when $z = -1$ or ($z = 1\angle 180°$ + $1 + KG(-1) = -1 - 0.3679 + 0.6321K = 0$ $K = 1.3679/0.6321 = 2.16$



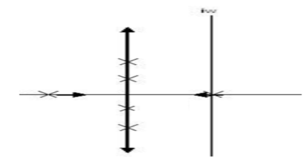
(c) K Value for which analog system with all sampling removed and $G_p(s) = \dfrac{K}{s+1}$ is stable: $1+G_p(s) = 1+\dfrac{K}{s+1} = 0$ $s = -K - 1$ $K \geq -1$ leads to stable system in analog system the system is always stable when $K \geq 0$ Adding sampling component destabilizes the system: $0 < K < 2.16$ $G(z) = \dfrac{0.05K(z+1)}{(z-1)^2}$ plot z-plane root locus



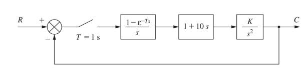
Two open loop poles, one finite open loop zero (and one infinite open loop zero) Breakaway points: $dG(z)/dz = 0$ $0.05K/(z-1)^2 + (-2)0.05K(z+1)/(z-1)^3 = 0$ hence $z = -3$ is break in point



(1) find $C(z)$ $G(z) = \dfrac{z-1}{z}Z[\dfrac{5s}{s(s+0.1)}] = \dfrac{z-1}{z} \times \dfrac{5z}{z-e^{-0.1}} = \dfrac{5(z-1)}{z-0.905}$ $C(z) = \dfrac{z}{z-1}G(z) = \dfrac{z}{z-1} \times \dfrac{5(z-1)}{z-0.905} = \dfrac{5z}{z-0.905}$ (2) find system response at sampling instances$c(nT)$ $c(nT) = 5 \times 0.905^n$ (3) find input of the plant $M(s)$ $M(s) = \dfrac{1}{s} \to C(s) = \dfrac{1}{s}\dfrac{5s}{s+0.1} = \dfrac{5}{s+0.1} \to$ $c(t) = 5\times e^{-0.1t} \to c(nT) = 5 \times 0.905^n$ **root locus of** $T(s) = \dfrac{9K}{10s^2+6s+9k}$ $10s^2 + 6s + 9K = 0$ $s = \dfrac{-6\pm\sqrt{36-360k}}{20}$

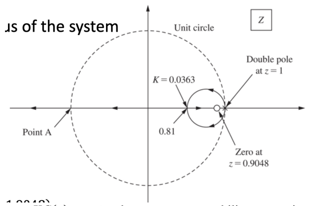


z transform of sequence $X(k) = z^{-2}(1 + 2z^{-1})(1 - 2z^{-1})(1 + z^{-1})$ what is sequence x(k)? $X(z) = z^{-2} + z^{-3} - 4z^{-4} - 4z^{-5}$ infers: x(k): x(0) = x(1) = 0 x(2) = x(3) = 1 x(4) = x(5) = -4 x(k) = 0 for k > 5



open loop: $KG(s) = \dfrac{1-e^{-sT}}{s}[\dfrac{K(1+10s)}{s^2}]$ z transform: $KG(z) = \dfrac{10.5K(z-0.9048)}{(z-1)^2}$ compute: $\dfrac{dG(z)}{z} = \dfrac{10.5K}{(z-1)^2} + \dfrac{(-2)\times 10.5K(z-0.9048)}{(z-1)^2} = 0, z = 0.81$ $KG(z) = \dfrac{z-1}{z}Z[\dfrac{K(1+10s)}{s^3}] = K\dfrac{z-1}{z}Z[\dfrac{1}{s^3} + \dfrac{10}{s^2}]$ $KG(z) = K\dfrac{z-1}{z}[\dfrac{1/2T^2z(z+1)}{(z-1)^3} + \dfrac{10Tz}{(z-1)^2}]$ since $T = 1$ $KG(z) = K\dfrac{z-1}{z}[\dfrac{1/2z(z+1)}{(z-1)^3} + \dfrac{10z}{(z-1)^2}] = K[\dfrac{1/2(z+1)}{(z-1)^2} + \dfrac{10}{z-1}]$ $KG(z) = K\dfrac{1/2(z+1)+10(z-1)}{(z-1)^2} = K\dfrac{10.5z-9.5}{(z-1)^2} = \dfrac{10.5K(z-0.9048)}{(z-1)^2}$

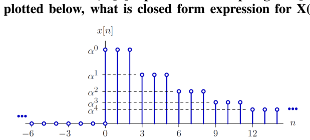
us of the system



$1 + KG(z) = 0$ when $z = -1$ stability range is $0 < K < 0.2$ **Z transform y(k), if y(k+2)-5y(k+1)+6(k)=0, y(0) = 0, y(1) = 2** Note: Time shifting property of z-transform $Z[x(n+k)] = z^kX(z) - z^k\sum_{i=0}^{k-1}x(i)z^{-i}$ $Z[x(n-k)] = z^{-k}X(z) + z^{-k}\sum_{i=0}^{k-1}x(-i)z^i$ $Z[y[k+2]] - 5Z[y[k+1]] + 6Z[y[k]] = 0$

$z^2Y(z) - zy(0) - zy(1) - 5zY(z) + 5zy(0) + 6Y(z) = 0$ $(z^2 - 5z + 6)Y(z) = 2z$ $Y(z) = \dfrac{2z}{z^2-5z+6}$

$Y(z) = \dfrac{2}{z-3} + \dfrac{2}{z-2}$ $y(k) = 2\times 3^k - 2\times 2^k$ **x[n] = [1,4,7] output [-1,-4], system difference equation**: $y(n) + 4y(n-1) + 7y(n-1) = -x(n) - 4x(n-1)$ **A discrete-time signal has z-transform X(z), if Y(z) = X(-z) is the z-transform of the signal, then the z-transform of a signal is given by, what is it's final value?:** 1 **Find Z transform of x[n] defined as x[n] = u[n], where is a positive real number, u[n] represents unit-step. signal x[n] is plotted below, what is closed form expression for X(z)?**

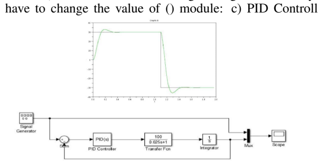


$X(z) = \sum_{n=-\infty}^{\infty}x[n]z^{-n}$ $X(z) = \sum_{n=0}^{\infty}a^nz^{-n}$ $\sum_{n=0}^{\infty}r^n = \dfrac{1}{1-r}$ for $|r| < 1$ $X(z) = \sum_{n=0}^{\infty}(\alpha z^{-1})^n$ $X(z) = \dfrac{1}{1-\alpha z^{-1}}$ for $|\alpha z^{-1}| < 1$ $X(z) = \dfrac{1}{1-\alpha/z}$ $X(z) = \dfrac{z}{z-a}$ with ROC $|z| > |\alpha|$ The characteristic equation of a sampled data system is given by $q(z) = z^2 + (2K - 1.75)z + 2.5 = 0$ where K > 0. What is the range of K for a stable system? Answer:

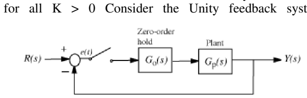
The roots of q(z) is $\dfrac{(1.75-2K)\pm\sqrt{(1.75-2K)^2-10}}{2}$ if $(1.75-2K)^2 - 10 \geq 0$ there are two real roots in this case, either $(1.75 - 2K) \geq \sqrt{10}$ or $(1.75 - 2K) \leq -\sqrt{10}$. The first scenario is not possible because K > 0, secondary scenario one of the root must be smaller than $\dfrac{(1.75-2K)}{2} \leq \dfrac{-\sqrt{10}}{2}$ which is smaller than -1. So the system with not be stable if $(1.75 - 2K)^2 - 10 \geq 0$ Consider the case where $(1.75 - 2K)^2 - 10 < 0$ there are two conjugate roots. By forcing the two conjugate roots $x \pm jy$ to have amplitude 1, we will have $x = \dfrac{1.75-2K}{2}$ and $y = \dfrac{\sqrt{(1.75-2K)^2-10}}{2}$. So we need $\dfrac{(1.75-2K)^2}{4} + \dfrac{10-(1.75-2K)^2}{4} = 1$ This is not possible for any K value. So system is unstable for all K values.

important

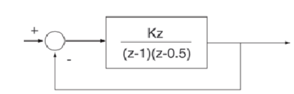
1) After a fork(), the child process and the parent process have no shared address (T) 2) If scheduling of a task set is not feasible using RM algorithm, it cannot be scheduled by DM algorithm. (F) 3) Soft real-time tasks are those which do not have any time bounds associated with them. (F) 4) The cyclic executive to schedule n periodic tasks needs to pre-store the schedule for a certain amount of time. The minimum length of time is given by the period of the task with the lowest priority. (F) 5) Under the Priority Ceiling Protocol, a task can be blocked only before it starts executing, never once it has started. (F) 6) A task set consisting of three tasks, T1, T2, and T3, with identical period and deadline, is RM-schedulable if and only if the total processor utilisation is at most 1. (T) 7) Priority Inversion is a phenomenon that cannot occur if tasks are independent. (T) 8) For discrete approximation of continuous system, it is possible that discrete system is stable, but the corresponding continuous is unstable. (T) 1 9) The z-transform of a system response is given by $C(z) = \frac{1}{4}\dfrac{z^{-1}(1-z^{-4})}{(1-z^{-1})^2}$ value is 1. (T) after simplifying: $\dfrac{z^4-1}{z^3(z-1)^2}$ $lim_{z\to 1}(z-1)\dfrac{z^4-1}{z^3(z-1)^2}\dfrac{1}{4} = \dfrac{1-1}{1(1-1)}\dfrac{1}{4}$ 10) The number of the root locus segments which do not terminate on open-loop zeroes is the difference between the number of open-loop poles and zeros. (T) 11) The Impulse response of a system given by $Y(z) = \dfrac{z^3+2z^2+2}{z^3-25z^2+0.6z}$ Determine the values of y(nT) at the first four sampling instants. a) y(0) = 1, y(T) = 27, y(2T) = 674.4, y(3T) = 16845.8 12) Which protocal can solve the priority inversion problem without introducing deadlock? c) priority ceiling protocol 3) The clock of the Raspberry Pi slows down at a rate of 40×10^{-6} seconds per second. Suppose that you can connect the device to a clock server that allows you to correct the time to within 5 seconds of the true time. You would like the displayed time to be accurate to within one minute of the true time. How often should the device be synchronized to the clock server? b) 382 hours $\dfrac{60-5}{40\times 10^{-6}}$ seconds 4) If one thread opens a file with read privileges then b) other threads in the same process can also read from that file 5) The root locus is the traces of the roots of the characteristic in the s-plane? c) as a system parameter is changed 6) Which is the controller that improves the transient response? c) D 7) If a set of pre-emptive and periodic tasks cannot be successfully scheduled by EDF, we can infer that: a) they cannot be scheduled by rate monotonic scheduling algorithm b) they cannot be scheduled by deadline monotonic scheduling algorithm c) they cannot be scheduled by cyclic executive algorithm **d) All of the above** 8) In order to simulate the given signal as below you have to change the value of () module: c) PID Controller



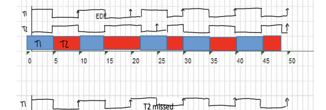
9) Which of the following scheduling algorithm is used in real-time OS but not general purpose OS b) rate monotonic scheduling 10) Consider s-domain function $Y(s) = \dfrac{10}{s(s+2)(s+6)}$ Let T be the sampling time. Then, in the z-domain the function Y(s) is: c) $Y(z) = \dfrac{5}{6}\dfrac{z}{z-1} - \dfrac{5}{4}\dfrac{z}{z-e^{-2T}} + \dfrac{5}{12}\dfrac{z}{z-e^{-6T}}$ **Determine stability range of K > 0** Consider sampled-data system with closed loop system transfer function $T(z) = K\dfrac{z^2+2z}{z^2+0.2z-0.5}$ For what K Value, the system stable? Answer: Ampltiude of roots must be smaller than 1, so the system is stable for all K > 0 Consider the Unity feedback system:



$G_p(s) = \dfrac{K}{s+1}$ sampling time T = 0.1 sec, what is max value of K for stable closed loop system Answer: open loop transfer function is $G(z) = (1-z^{-1})Z\left[\dfrac{K}{s(s+1)}\right] = K\dfrac{z-1}{z}\left[\dfrac{z}{z-1} - \dfrac{z}{z-e^{-T}}\right] = K(1 - \dfrac{z-1}{z-e^{-T}})$ T = 0.1 so, $G(z) = K(\dfrac{1-e^{-0.1}}{z-e^{-0.1}}) = \dfrac{0.1K}{z-0.9}$ The characteristic equation is z - 0.9 + 0.1K = 0, considering the root locus and force z= -1 we will have K = 19 so for K < 19, the system is stable also ok if the answer is 20 Consider the following discrete system, what value of K makes the system stable?



$G(z) = \dfrac{kz}{z^2-z=0.5} = kz(z^2 - z + 0.5)^{-1}$ $\dfrac{dG(z)}{dz} = k(z^2-z+0.5)^{-1} + Kz(-1)(z^2-z+0.5)^{-2}(2z-1)$ $= k\left[\dfrac{1}{z^2-z+0.5} - \dfrac{z(2z-1)}{(z^2-z+0.5)^2}\right] = k\dfrac{-z^2+0.5}{(z^2-z+0.5)^2} = 0$ $z = \pm\sqrt{0.5} = \pm 0.707$ Break-away point 0.707 and break-in point -0.707 char equation: $z^2 + (K - 1.5)z + 0.5 = 0$ sub z with -1 , we get k =3 , so system is stable when K < 3 **System consists of three periodic tasks T1 = (3,1), T2 = (5,2) and T3 = (8,3)** a) EDF schedulable? U = 1/3 + 2/5 + 3/8 = 1.108 > 1, not schedulable b) Suppose we want to reduce the execution time of T3 in order to make it EDF schedulable . What is the minimum amount of reduction necessary to make the system schedulable max ratio of e3/T3 is given by 1 - 1/3 -2/5 = 4/15 max time e/8 = 4/15 , e = 32/15 min reduction is 3 - 32/15 = 13/15 = 0.867 **Given T1(10;5), T2(25;12)** a) Graphically construct EDF schedule for 50 time units b) Use same task set for constructing a schedule based on RM algorithm c) Use suitable schedulability tests to veriy your answer and show if task set is schedulable under either EDF or RM



Utilization 0.5 + 12/25 = 0.98 < 1 , task set is EDF schedulable RM (necessary and sufficient condition): for i =1, t1 = 10, $w_1(t)$ = e1 < 10 for i =2, t1 = 10, t2 = 20, t3 = 25, w2(t1) = e1 + e2 = 17 > t1 = 10 w2(t2) = 2e1 + e2 = 10 + 12 = 22 > t2 = 20 w2(t3) = 3e1 + e2 = 15 + 12 = 27 > t3 = 25 task set is not RM schedulable **Discrete approximation of Continuous System** $D(s) = 25\dfrac{s+1}{s+15}$ a) Derive the corresponding Differential Equation $y' + 15y = 25x' + 25x$ or $u' + 15u = 25e' + 25e$ b) Substitute the expression for the frequency variable s corresponding to forward rectangular rule, in the transfer function and derive D(z) Forward Rectangular Rule: $s \to \dfrac{z-1}{T}$ hence $D(z) = 25\dfrac{z-1/T+1}{z-1/T+15} = \dfrac{25z-24.375}{z-0.625}$ c) For discrete transfer function given in (b) find a digital implementation with a sampling rate of 40 Hz (1/T = 40) $u(z)(1 - 0.625z^{-1}) = e(z)(25 - 24.375z^{-1})$ $u(k+1) = 0.625u(k) + 25e(k+1) - 24.375e(k)$ **Given**

T1(5; 10; 4; 10 [R1; 2])

T2(2; 15; 6; 15 [R3; 4[R2 ; 1]])

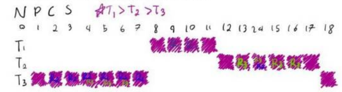
T3(0; 20; 8; 20 [R2; 6[R3; 3]])

The resource R1 is required by T1 after it has executed for 1 time unit.

The resource R2 is required by task T2 after it has executed for 2 time units and by Task T3 after it has executed 1 time unit.

The resource R3 is required by task T2 after it has executed 1 time unit and by task T3 after it has executed 3 time units.

N P C S $T_1 > T_2 > T_3$



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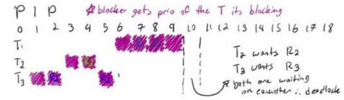
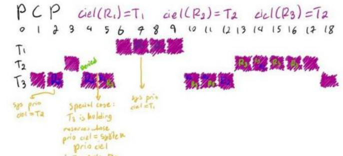
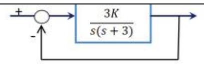


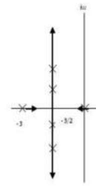
P C P $ceil(R_1) = T_1$ $ceil(R_3) = T_2$ $ceil(R_3) = T_2$



Q1: System given as



- Sketch the root locus for the closed loop feedback system. Must show the starting, branching-off and ending points of the root locus. (Consider only positive values for K)



Answer: First, find the closed-loop transfer function, $T(s) = \frac{3K}{s^2+3s+3K}$. Hence, the characteristic equation is

$$s^2 + 3s + 3K = 0$$

The root of the characteristic equation is $s = \frac{-3\pm\sqrt{9-12K}}{2}$, the intersection of the root locus and real axis is $9 - 12K = 0$, so $\boldsymbol{K = 0.75}$ makes $\boldsymbol{s = -1.5}$ is the break away point.

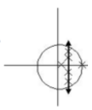
- (b) For what values of K, (if any) the system is not stable?

Answer: The system is stable for all values of K>0

- (c) An approximate closed loop z-domain transfer function of the above system was digitized assuming T = 0.3, as:

$$T(z) = \frac{3K}{11z^2 - 12z + 1 + 3K}$$

Sketch a z-domain root locus for the given discrete-time system.



Answer: The closed loop posed are given as $z = \frac{12\pm\sqrt{144-44(3K+1)}}{22}$, so the break away point is $z = \frac{12}{22} = 0.55$, and Starting points are $(\frac{2}{22}, 0)$ and (1,0)

- (d) For what values of K, the system is stable?
- Answer: We have $z = \frac{12\pm\sqrt{100-132K}}{22}$. If z has two conjugate roots, then $z = \frac{12\pm j\sqrt{132K-100}}{22}$. On the unit circle, $(\frac{12}{22})^2 + \frac{132K-100}{22^2} = 1$ ➔ $K = \frac{22^2}{132}$ ... $= \frac{440}{132} = 3.333$. The stable range is (0, 3.333).

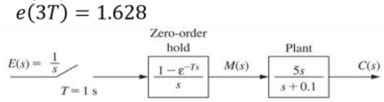
Find the z-transform of the following function $E(s) = \frac{15}{(s+2)(s+5)}$. If the sampling time T=0.1s, please find the sampled time sequence $e(nT)$ using power series method for $1 \le n \le 3$.

- Answer: $E(s) = \frac{15}{(s+2)(s+5)} = \frac{5}{s+2} - \frac{5}{s+5}$

$$E(z) = \frac{5z}{z - e^{-2T}} - \frac{5z}{z - e^{-5T}}\bigg|_{T=0.1} = \frac{1.061z}{(z - 0.8187)(z - 0.6065)}$$

$$e(T) = 1.061,$$
$$e(2T) = 1.512,$$
$$e(3T) = 1.628$$

Q3



- Find the transfer function of $C(z)$.

Answer: $G(z) = \frac{z-1}{z}Z\left[\frac{5s}{s(s+0.1)}\right] = \frac{z-1}{z} \times \frac{5z}{z-e^{-0.1}} = \frac{5(z-1)}{z-0.905}$

$$C(z) = \frac{z}{z-1}G(z) = \frac{z}{z-1} \times \frac{5(z-1)}{z-0.905} = \frac{5z}{z-0.905}$$

- Find the system response at the sampling instances $c(nT)$.

Answer: $c(nT) = 5 \times 0.905^{nT}$

- Determine the input of the plant $M(s)$, then calculate $c(t)$.

Answer:

$M(s) = \frac{1}{s}$ ➔ $C(s) = \frac{1}{s}\frac{5s}{s+0.1} = \frac{5}{s+0.1}$ ➔ $c(t) = 5 \times e^{-0.1t}$ ➔ $c(nT) = 5 \times 0.905^{nT}$

- Given the following frequency domain function: $F(s) = \frac{zs+a}{s(s-a)}$

Justify whether the Final Value Theorem can or cannot be used to find the steady state value of $f(t)$.

Answer: Not applicable, since the steady state tends to infinity.

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty}(3e^{at} - 1) = \infty$$

Q5: Solve $x(k)$, if the difference equation is given:

$$x(k) - 3x(k-1) + 2x(k-2) = e(k), where$$

$$e(k) = \begin{cases} 1, & k = 0,1 \\ 0, & k \ge 2 \end{cases}$$

$$x(-2) = x(-1) = 0$$

$$E(z) = \sum_{k=0}^{\infty} e(k)z^{-k} = 1 + z^{-1}$$

Answer:

$$X(z)(1 - 3z^{-1} + 2z^{-2}) = E(z) = 1 + z^{-1}$$

$$X(z) = \frac{z^2}{(z-1)(z-2)} \times \frac{z+1}{z} = \frac{z(z+1)}{(z-1)(z-2)} = \left(\frac{-2z}{z-1} + \frac{3z}{z-2}\right)$$

$$x(k) = -2 + 3 \times 2^k$$

- Consider a function in z-domain $F(z) = \frac{2z^2+z}{z^2-1.5z+0.5}$, What is the sampled value $f(k)$.

$$\frac{F(z)}{z} = \frac{2z+1}{z^2-1.5z+0.5} = \frac{2z+1}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5} = \frac{6}{z-1} + \frac{-4}{z-0.5}$$

$$F(z) = 6\frac{z}{z-1} - 4\frac{z}{z-0.5}$$

$$f[k] = 6u[k] - 4 \cdot 0.5^k$$

$$f = \{2, 4, 5, 5.5, \cdots\}$$

Use Z transform to solve y(k), if y(k+2)-5y(k+1)+6(k)=0, where y(0)=0 and y(1)=2.

Time Shifting Property of Z-Transform

$$x(n+k) \xleftrightarrow{ZT} z^k X(z)$$
$$x(n-k) \xleftrightarrow{ZT} z^{-k} X(z)$$

If the initial conditions are not neglected, then

$$Z[x(n+k)] = z^k X(z) - z^k \sum_{i=0}^{k-1} x(i)z^{-i}$$
$$Z[x(n-k)] = z^{-k} X(z) + z^{-k} \sum_{i=0}^{k-1} x(-i)z^{i}$$

Solve $y[k+2] - 5y[k+1] + 6y[k] = 0$, where $y[0] = 0$, $y[1] = 2$.

$$\mathcal{Z}\{y[k+2]\} - 5\mathcal{Z}\{y[k+1]\} + 6\mathcal{Z}\{y[k]\} = 0$$

Taking z transforms:

$$z^2Y(z) - zy[0] - zy[1] - 5zY(z) + 5zy[0] + 6Y(z) = 0$$

Rearranging and using initial conditions:

$$(z^2 - 5z + 6)Y(z) = 2z$$
$$Y(z) = \frac{2z}{z^2 - 5z + 6}$$

Using partial fractions:

$$Y(z) = \frac{2}{z-3} + \frac{2}{z-2}$$

Using inverse transforms straight from the table to get the solution:

$$y[k] = 2 \times 3^k - 2 \times 2^k$$

Find $f(k)$, for the $F(z) = \frac{z(z+2)(z+5)}{(z-0.4)(z-0.6)(z-0.8)}$

$$\frac{(z+2)(z+5)}{(z-0.4)(z-0.6)(z-0.8)} = \frac{A}{(z-0.4)} + \frac{B}{(z-0.6)} + \frac{C}{(z-0.8)}$$

➔

A(z^2-1.4z+0.48)+B(z^2-1.2z+0.32)+c(z^2-z+0.24)=z^2+7z+10

A+B+C=1

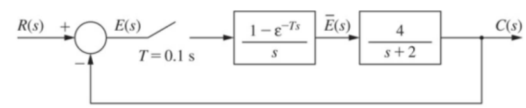
1.4A+1.2B+C=-7

0.48A+0.32B+0.24C=10

$$F(z) = 162 \times \frac{z}{(z-0.4)} - 364 \times \frac{z}{(z-0.6)} + 203 \times \frac{z}{(z-0.8)}$$

Hence,

$$f(k) = 162 \times (0.4)^k - 364 \times (0.6)^k + 203 \times (0.8)^k$$



- We can express the system output as

$$T(z) = \frac{G(z)}{1 + G(z)}$$

where $G(z) = Z\left[\frac{1-e^{-Ts}}{s}\frac{4}{s+2}\right] = \frac{z-1}{z}Z\left[\frac{4}{s(s+2)}\right] = \frac{z-1}{z}\frac{2(1-e^{-2T})z}{(z-1)(z-e^{-2T})}$

If $T = 0.1s$, $G(z) = \frac{0.3625}{z-0.8187}$.

Hence,

$$T(z) = \frac{G(z)}{1 + G(z)} = \frac{0.3625}{z - 0.4562}$$

- Since $R(z) = \frac{z}{z-1}$, we have $C(z) = \frac{G(z)}{1+G(z)}R(z)$.
- Hence, $C(z) = \frac{0.3625}{z-0.4562}\frac{z}{z-1} = \frac{0.667z}{z-1} - \frac{0.667z}{z-0.4562}$
- $c(kT) = 0.667[1 - 0.4562^k]$
- Given $u(k+1) = (1 - bT)u(k) + K_0(aT - 1)e(k) + K_0e(k+1)$, what is the corresponding z-transform?

$$\frac{U(z)}{E(z)} = \frac{K_0(aT-1)z^{-1} + K_0}{1 + (bT-1)z^{-1}} = \frac{K_0z + K_0(aT-1)}{z + (bT-1)}$$

$$u(k+1) - (1 - bT)u(k) = K_0(aT - 1)e(k) + K_0e(k+1)$$
$$zU(z) - (1 - bT)U(z) = K_0(aT - 1)E(z) + zK_0E(z)$$
$$U(z)[z - (1 - bT)] = E(z)[K_0(aT - 1) + zK_0]$$
$$U(z)/E(z) = [K_0(aT - 1) + zK_0]/[z + (bT - 1)]$$

- $D(s) = \frac{U(s)}{E(s)} = \frac{a}{s+a}$ u(kT)=u(k)

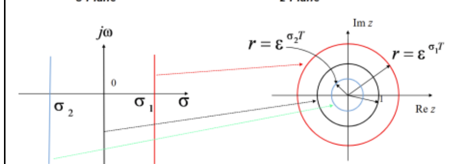
$U(s)(s+a) = aE(s)$ ➔ Laplace Transform ➔ $\frac{u(k+1)-u(k)}{T} + a\,u(k) = a\,e(k)$

- The difference equation is:

$$u(k+1) = (1 - aT)u(k) + aTe(k)$$

- The corresponding z-transform is:

$$\frac{U(z)}{E(z)} = \frac{aTz^{-1}}{1 + (aT-1)z^{-1}} = \frac{aT}{z + (aT-1)}$$



Vertical lines on the s-plane map to circles with constant radius on the z-plane.