Real Time Systems and Control Applications (Fall 2019)
MECHTRON 4AA4/SFWRENG 4AA4/SFWRENG 6GA3 Final Exam

Duration: 150 Minutes McMaster University (CAS) Instructor: Dr. Wenbo He

Student Name: ID:

This test paper includes 12 pages and 6 questions. You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions:
1. Put all answers on the test paper.
2. Try to keep your answers brief. Use point form if need be.
3. Other than two-page double-sided crib sheets, no memory aids or text books of any kind are allowed during the test.
4. Electronic calculator may be utilized.
5. The mark weighting for each question is given within the square brackets.
6. The burden of communication is upon you. Solutions not properly explained will not be considered correct. If we cannot decode "what you wrote, we cannot grade it as a correct answer.
7. The necessary Laplace and Z transforms are given on the last page of the test paper.

Grading

Section	Grade	
Question #1 – True or False	10 points	_____
Question #2 – Multiple Choice	10 points	_____
Question #3 – Determine the Stability Range	10 points	_____
Question #4 – Scheduling Algorithms	11 points	_____
Question #5 – Discrete Approx. of CCS	6 points	_____
Question #6 – Z Transform and Discrete Signal	3 points	_____
Total:	50points	_____

Question 1: Answer the following questions (10 points):

1) If scheduling of a task set is not feasible using RM algorithm, it cannot be scheduled by DM algorithm. (T/F)

2) Judging if a system is soft real-time, firm real-time and hard real-time systems depends on if a user can tolerate the consequence of deadline missing. (T/F)

3) Soft real-time tasks are those which do not have any time bounds associated with them. (T/F)

4) The cyclic executive to schedule n periodic tasks needs to pre-store the schedule for a certain amount of time. The minimum length of time is given by the period of the task with the lowest priority. (T/F)

5) Under the Priority Ceiling Protocol, a task can be blocked only before it starts executing, never once it has started. (T/F)

6) A task set consisting of three tasks, T1, T2, and T3, with identical period P and identical deadline D, is RM-schedulable if and only if the total processor utilisation is at most 1. (T/F)

7) Priority Inversion is a phenomenon that cannot occur if tasks are independent. (T/F)

8) For discrete approximation of continuous system, it is possible that discrete system is stable, but the corresponding continuous is unstable. (T/F)

9) The z-transform of a system response is given by $C(z) = \frac{1}{4}\frac{z^{-1}(1-z^{-4})}{(1-z^{-1})^2}$. It is final value is 1. (T/F)

10) The number of the root locus segments which do not terminate on open-loop zeroes is the difference between the number of open-loop poles and zeros. (T/F)

Question 2: Multiple Choice Questions (10 points):

1) The impulse response of a system is given by $Y(z) = \frac{z^3+2z^2+2}{z^3-25z^2+0.6z}$. Determine the values of y(nT) at the first four sampling instants.

a) y(0) = 1, y(T) = 27, y(2T) = 674.4, y(3T) = 16845.8

b) y(0) = 1, y(T) = 27, y(2T) = 647, y(3T) = 660.05

c) y(0) = 1, y(T) = 647, y(2T) = 47, y(3T) = 27

d) y(0) = 0, y(T) = 27, y(2T) = 47, y(3T) = 60.05

2) Which protocol can solve the priority inversion problem without introducing deadlock?

a) priority inheritance protocol

b) priority inversion protocol

c) priority ceiling protocol

d) none of the mentioned

~~3) The clock of the Raspberry Pi slows down at a rate of 40×10^{-6} seconds per second. Suppose that you can connect the device to a clock server that allows you to correct the time to within 5 seconds of the true time. You would like the displayed time to be accurate to within one minute of the true time. How often should the device be synchronized to the clock server?~~

~~a) 20 days~~

~~b) 382 hours~~

~~c) 417 hours~~

~~d) 34.7 hours~~

4) If one thread opens a file with read privileges then

a) other threads in the another process can also read from that file

b) other threads in the same process can also read from that file

c) any other thread can not read from that file

d) all of the mentioned

5) The root locus is the trace of the roots of the characteristic equation in the s-plane?

a) as the input of the system is changed

b) as the output of the system is changed

c) as a system parameter is changed

d) as the sensitivity is changed

6) Which is the controller that will improve the transient response?

a) P b) I c) D d) P and I

7) If a set of pre-emptive and periodic tasks cannot be successfully scheduled by EDF, we can infer that:

a) they cannot be scheduled by rate monotonic scheduling algorithm

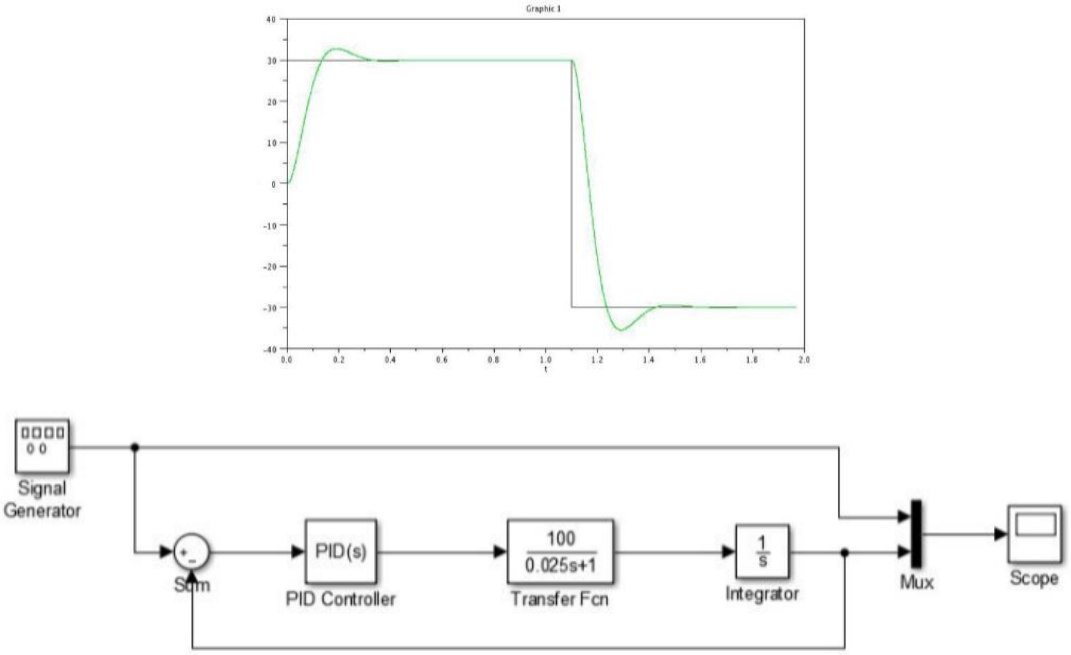
b) they cannot be scheduled by deadline monotonic scheduling algorithm

c) they cannot be scheduled by cyclic executive algorithm

d) All of the above

8) In order to simulate the given signal as below, you have to change the value of ( ) module:

a) Signal Generator b) Integrator c) PID controller d) Mux



9) Which of the following scheduling algorithm is used in real-time OS but not general purpose OS.

a) round robin

b) rate monotonic scheduling

c) first come first served

d) priority-based scheduling

10) Consider s-domain function $\mathrm{Y}(s) = \frac{10}{s(s+2)(s+6)}$. Let T be the sampling time. Then, in the z-domain the function Y(z) is:

a) $\mathrm{Y}(z) = \frac{1}{6}\frac{z}{z-1} - \frac{z}{z-e^{-2T}} + \frac{5}{6}\frac{z}{z-e^{-6T}}$

b) $\mathrm{Y}(z) = \frac{1}{6}\frac{z}{z-1} - \frac{5}{4}\frac{z}{z-e^{-2T}} + \frac{5}{12}\frac{z}{z-e^{-6T}}$

c) $\mathrm{Y}(z) = \frac{5}{6}\frac{z}{z-1} - \frac{5}{4}\frac{z}{z-e^{-2T}} + \frac{5}{12}\frac{z}{z-e^{-6T}}$

d) $\mathrm{Y}(z) = \frac{5}{6}\frac{z}{z-1} - \frac{z}{z-e^{-2T}} + \frac{5}{6}\frac{z}{z-e^{-6T}}$

Question 3: Determine the Stability Range of K>0

1) Consider a sampled-data system with the closed-loop system transfer function

$$\mathrm{T}(z) = K\frac{z^2 + 2z}{z^2 + 0.2z - 0.5}$$

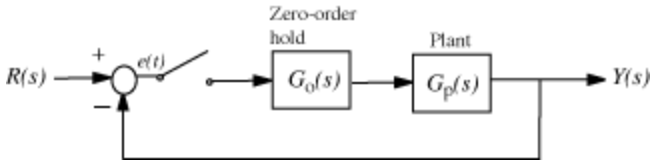
For what K value, the system stable? [2 points]

2) The characteristic equation of a sampled data system is given by

$$q(z) = z^2 + (2K - 1.75)z + 2.5 = 0,$$

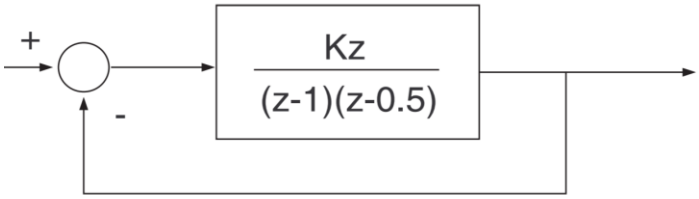
where K > 0. What is the range of K for a stable system? [3 points]

3) Consider the unity feedback system,



where $G_p(s) = \frac{K}{s+1}$, with the sampling time T = 0.1 sec. What is the maximum value for K for a stable closed-loop system? (3 points)

4) Consider the following discrete system, what value of K makes the system stable? [2 points]



Question 4: Scheduling Algorithms

1） A system consists of three periodic tasks: T1 = (3, 1), T2 = (5, 2), and T3 = (8, 3).

(a) Is the above set of tasks schedulable using EDF algorithm? Justify your answer. [1]

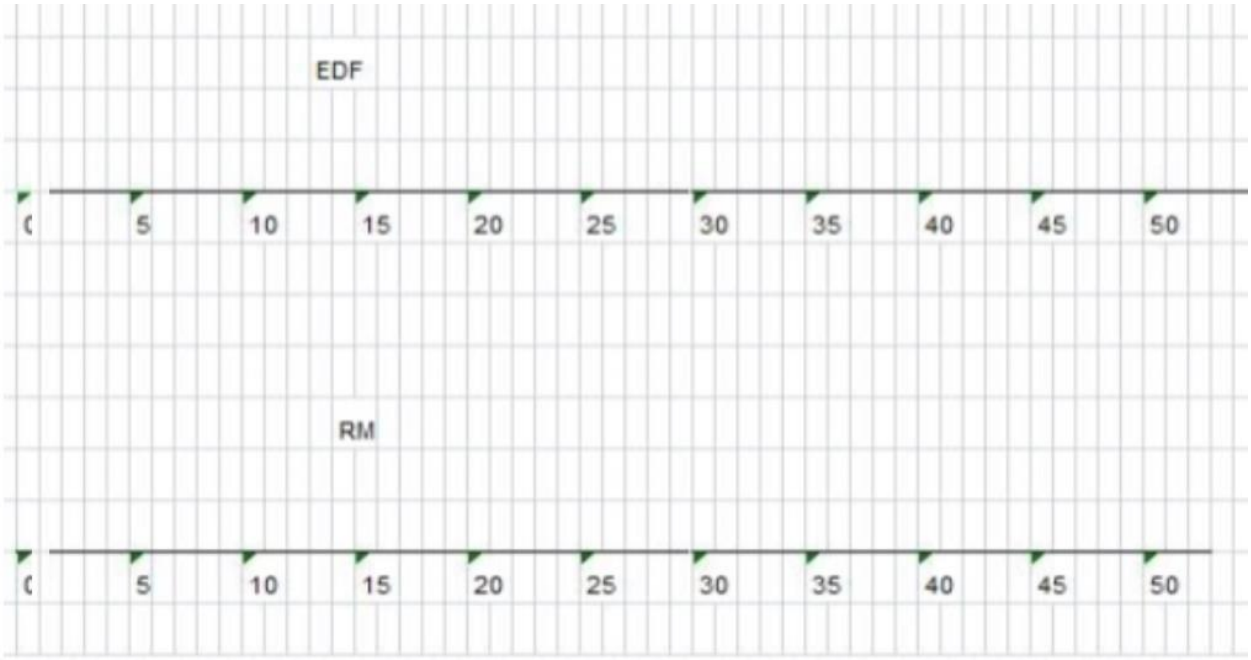
(b) Suppose we want to reduce the execution time of T3 in order to make the task system schedule-able according to the EDF algorithm. What is the minimum amount of reduction necessary for the system to be schedulable (tasks may execute for a fraction of a time unit)? [2]

2) Given the task set: $T_1(10; 5)$; $T_2(25; 12)$

(a) Graphically construct an EDF schedule for 50 time units in the space provided in Figure 1. [2]

(b) Use the same task set for constructing a schedule based on RM algorithm in the space provided in Figure 1. [2]

(c) Use suitable schedulability tests to verify your answer and show if the task set is schedulable under EDF and RM algorithm. [4]



Question 5: Discrete Approximation of Continuous System

The transfer function of a controller is given by:

$$\mathrm{D(s)} = 25\frac{s+1}{s+15}$$

(a) Derive the corresponding differential equation [2].

(b) Substitute the expression for the frequency variable s corresponding to forward rectangular rule, in the transfer function and derive D(z) [2].

(c) For the discrete transfer function given in (b), find a digital implementation with a sampling rate of 40 Hz [2].

Question 6: Given the difference equation

$$y(k+2) - \frac{3}{4}y(k+1) + \frac{1}{8}y(k) = e(k)$$

where $y(0) = y(1) = 0$, $e(0) = 0$, and $e(k) = 1, k = 1,2, \ldots$

(a) Solve difference equation direction for $y(k), 2 \leq k \leq 4$. [1 point]
(b) Solve for $y(k)$ as a function of k using z-transform. [2 points]

Table 1. Laplace and z- transforms that you may want to use

Laplace Domain	Time Domain (note)	Z Domain (t=kT)
1	$\delta(t)$ unit impulse	1
$\Gamma(s) = \frac{1}{s}$	$\gamma(t)$ (note)	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$T\frac{z}{(z-1)^2}$
$\frac{2}{s^3}$	t^2	$T^2\frac{z(z+1)}{(z-1)^3}$
$\frac{n!}{s^{(n+1)}}$	t^n	
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
	$b^k \quad (b = e^{-aT})$	$\frac{z}{z-b}$
$\frac{1}{(s+a)^2}$	te^{-at}	$T\frac{ze^{-aT}}{(z-e^{-aT})^2}$
$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{(b-a)}$	$\frac{z(e^{-aT}-e^{-bT})}{(b-a)(z-e^{-aT})(z-e^{-bT})}$

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