H3

Introduction

The circuit below was inspired by the provided exemplar, modelled to fit the specifications outlined in the deliverables. The circuit was solved analytically for the voltage across the capacitor as a function of time using voltage dividers. Digitally and physically, the time constant was found to match results from the analytical solution (using multisim and the Hantek).

Problem Framing:

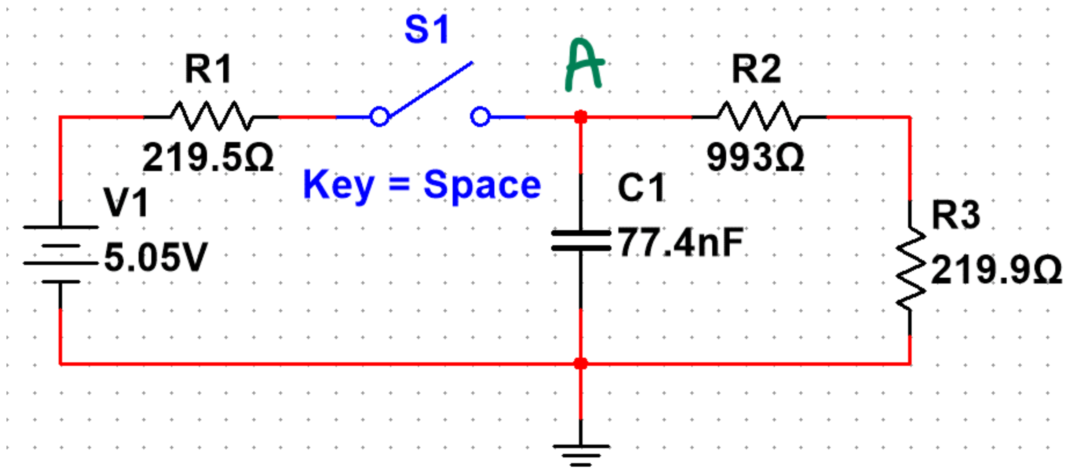


Figure 1: Circuit Diagram

The goal is to determine the voltage through the capacitor as a function of time, digitally with Multisim using the Tektronix cursors to measure voltage and time differences, and physically with the Hantek oscilloscope using the same strategy, and finally physically using the same strategy. Additionally, another goal was to determine the resistance of the Hantek via discharging the capacitor.

It is important to recognize the behavior of the capacitor in different scenarios that are seen throughout this lab. When fully discharged, the capacitor behaves like a wire, allowing current to pass through but carrying no voltage. When fully charged, the capacitor behaves as a voltage supply, meaning that it acts does not allow current to pass through it but does supply voltage to connected components (in this case, R2 and R3).

Analytical Solution:

Charging

To calculate the voltage in terms of time, the voltage of the capacitor at a time of infinity and the time constant "τ" are needed. The voltage of the capacitor at time = 0 was assumed to be zero. Next, to find the voltage when the capacitor is fully charged (time of infinity), a voltage divider was used.

$$V(inf) = V1\left(\frac{R2+R3}{R1+R2+R3}\right) = 5.05\left(\frac{993+219.9}{993+219.5+219.9}\right) = 4.2761 \text{ V}$$

Next, the time constant was determined. The resistance seen by the capacitor at time = infinity is:

R = R1||(R2+R3)

R = [(219.5)$^{-1}$ + (993 + 219.9)$^{-1}$]$^{-1}$ = 185.864 Ω

The time constant can be calculated with the formula $\tau = R * C$, where C = 77.4 nF, the measured capacitance of the capacitor.

$$\tau = R * C = (185.864)(77.4 * 10^{-9}) = 0.0000143858736 \approx 14.4\ \mu s$$

Having derived this, the values acquired were substituted into the formula for voltage of a capacitor as a function of time (below). Note that the first term is cancelled since V(0) is assumed to be 0.

$$V(t) = V(0) * e^{-\frac{t}{\tau}} + V(\infty) * \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$V_c(t) = (4.28\,V)\left[1 - e^{\left(\frac{-t}{14.4 * 10^{-6}\,s}\right)}\right]$$

Discharging

To derive the formula when discharging, the same values are needed. The capacitor is discharging from its fully charged state, so the voltage of the capacitor at time = 0 is the voltage when fully charged: 4.28 V (calculated above). The voltage at time = ∞ is 0, since the capacitor is fully discharged.

$$V(\infty) = 0, \qquad V(0) = 4.28\,V$$

Next, the time constant was determined once again. The resistance seen by the capacitor while discharging is R2 in series with R3, since the open switch caused R1 to be disconnected from the capacitor. Note that because of this, the time constant is larger.

R = (R2+R3)

R = (993 + 219.9) = 1212.9 Ω

Time constant: $\tau = R * C$, where C = 77.4 nF.

$$\tau = R * C = 1212.9 * (77.4 * 10^{-9}) = 0.00009387846 \approx 93.9\ \mu s$$

The values acquired were substituted into the formula for voltage of a capacitor as a function of time. Note that the second term is cancelled since $V(\infty) = 0$.

$$V(t) = V(0) * e^{-\frac{t}{\tau}} + V(\infty) * \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$V_c(t) = (4.28\,V) * e^{\left(\frac{-\mathrm{t}}{93.9 * 10^{-6}\,\mathrm{s}}\right)}$$

Digital Solution Via Multisim:

All measurements were taken using the following configuration:

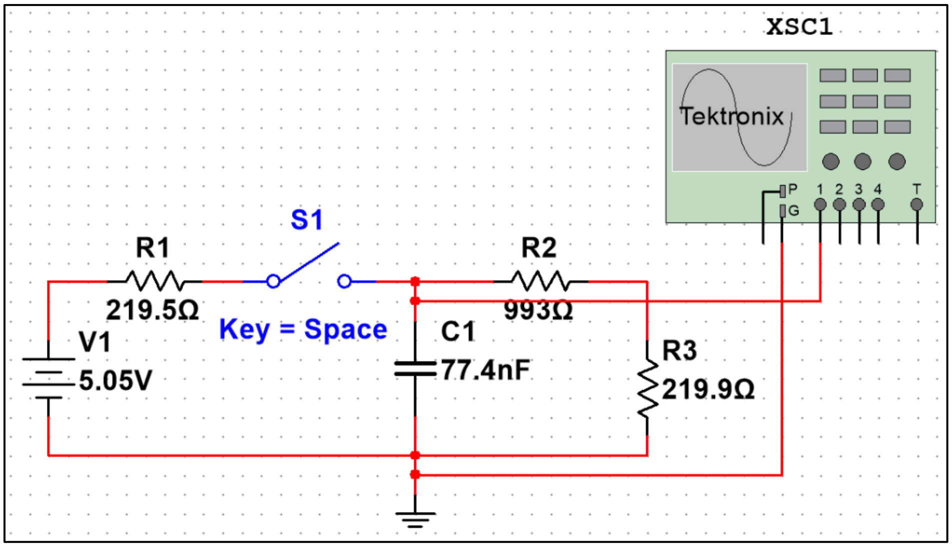
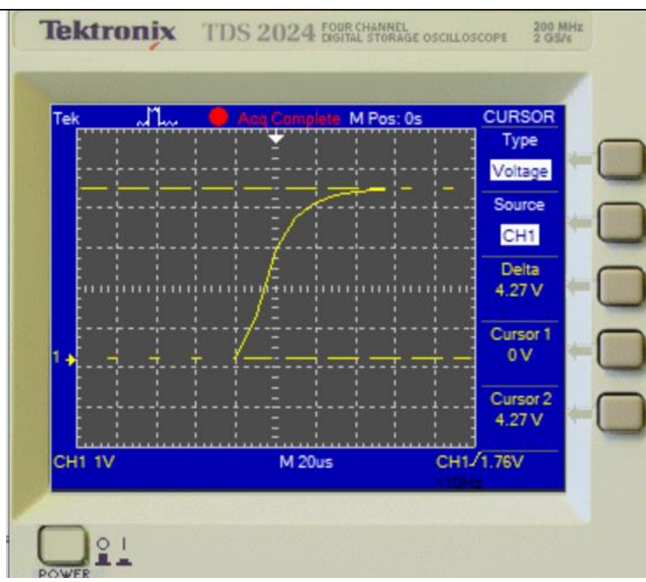
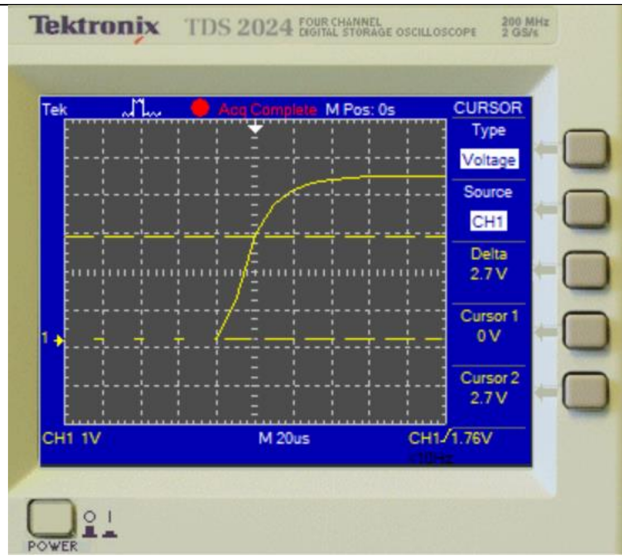
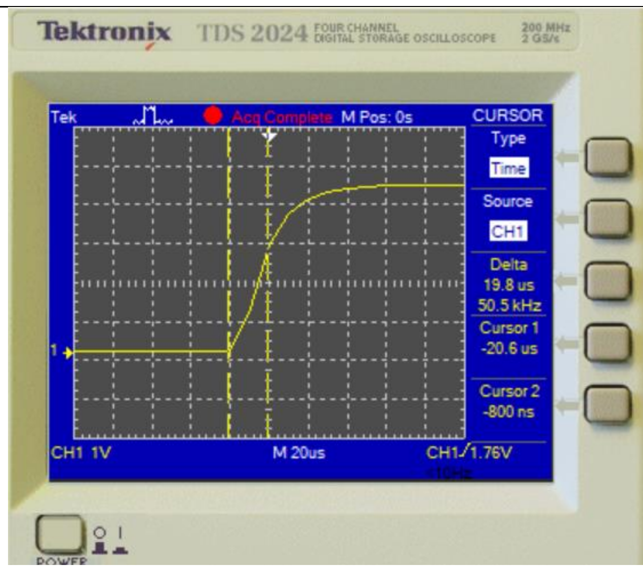
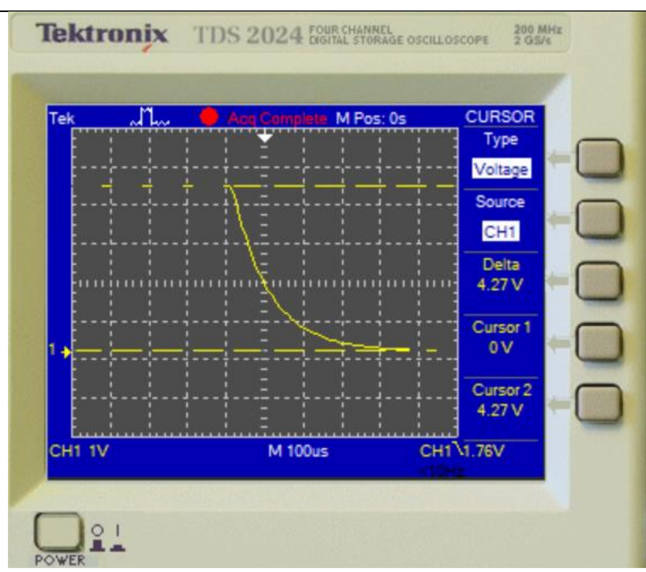


Figure 2: Circuit Diagram with Tektronix oscilloscope

The following are images of the measurements taken with the Tektronix:

Measurement type	**Image**	**Result (w/ uncertainty)**
Charging – Voltage @ $t = \infty$		$V_c(\infty) = (4.27 \pm 0.2)\ V$ $V_c(0) = 0\ V$

<table>
<tr>
<td>Charging –
Voltage @
$t = \tau$</td>
<td>
</td>
<td>Analytical Calculation for voltage at t=τ
$V_\tau = (4.27) * \left(1 - e^{\frac{-\tau}{\tau}}\right)$
$V_\tau = 4.27 * 0.632$
$V_\tau \approx 2.70\ V$
Position in Multisim:
$V_c(\tau) = (2.70 \pm 0.2)\ V$</td>
</tr>
<tr>
<td>Charging –
Solving for
time @ $V_c(\tau)$</td>
<td>
</td>
<td>$\tau = (19.8 \pm 4)\ \mu s$</td>
</tr>
<tr>
<td>Discharging –
Voltage drop
@ $t = \infty$</td>
<td>
</td>
<td>$V_c(0) = (4.27 \pm 0.2)\ V$
$V_c(\infty) = 0\ V$</td>
</tr>
</table>

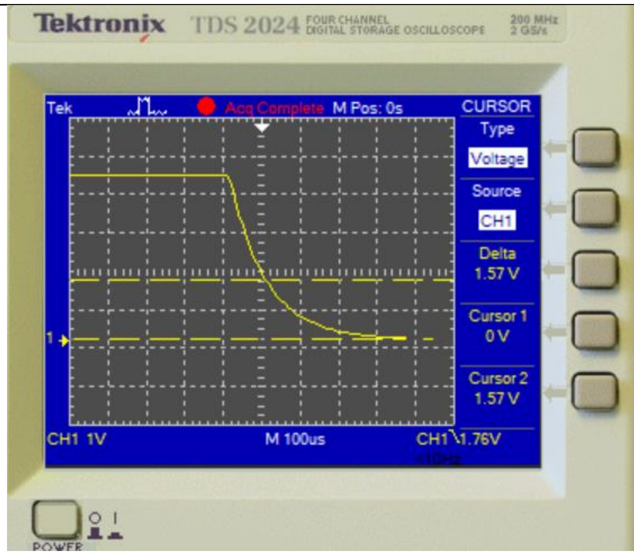
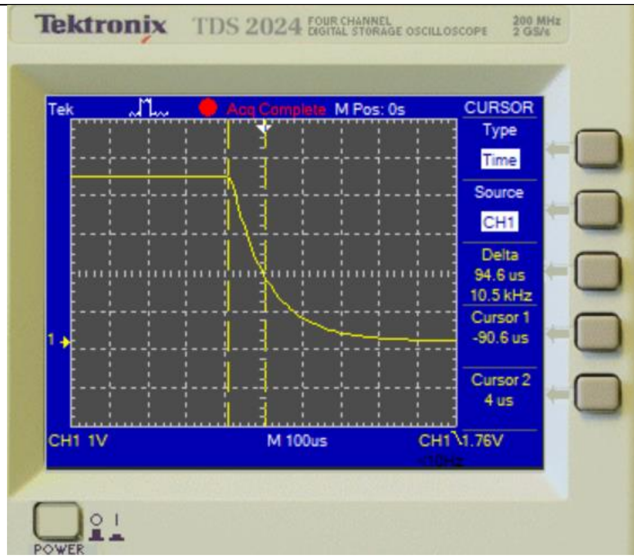
<table>
<tr>
<td>Discharging –
Voltage @
$t = \tau$</td>
<td>
</td>
<td>Analytical Calculation for voltage at t=τ
$V_\tau = (4.27) * \left(e^{\frac{-\tau}{\tau}}\right)$
$V_\tau = 4.27 * 0.368$
$V_\tau \approx 1.57\ V$
Position in Multisim:
$V_c(\tau) = (1.57 \pm 0.2)\ V$</td>
</tr>
<tr>
<td>Discharging –
Solving for
time @ $V_c(\tau)$</td>
<td>
</td>
<td>$\tau = (94.6 \pm 20)\ \mu s$</td>
</tr>
</table>

Table 1: Measured values with Tektronix oscilloscope

With the values acquired, the equations of voltage in terms of time were derived:

$$\textit{Charging:}\quad V_c(t) = [(4.27 \pm 0.2)\ V] * [1 - e^{\frac{-t}{(19.8 \pm 4)*10^{-6}\,s}}]$$

$$\textit{Discharging:}\quad V_c(t) = [(4.27 \pm 0.2)\ V] * [e^{\frac{-t}{(94.6 \pm 20)*10^{-6}\,s}}]$$

Analysis

The time constants in both instances are relatively close to the time constants yielded from the analytical calculations. However, I noticed that the discharging time constant is within error while the charging time constant is not. This is mainly due to the method of measurement used. The method of lining up the time cursors with the voltage cursors is inaccurate, as it relies on “eyeballing it” and estimation when measuring time. Next time, I should use the method described by Dr. Minnick, where he measured for time = 2τ, and solved analytically for τ. This method was more accurate 80% of the voltage (4.27 V) results in a number that's easier to work with. This may have reduced the aspect of estimation in measuring for the time constant, as the change in time between the initial voltage and V_C*80% would line up closely with the gridlines on the Tektronix, reducing "eye-balling".

Physical Solution Via Hantek and Breadboard:

The physical and digital solutions are identical. The oscilloscope's cursor functions were used to measure differences in time as well as differences in voltage to determine the time constant & initial/final voltages.

All measurements were taken using the following breadboard configuration:

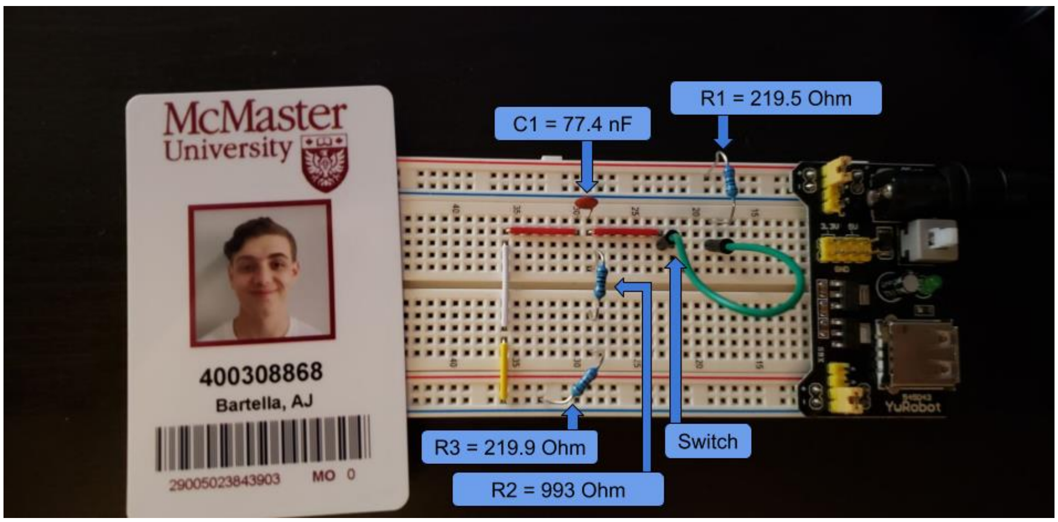


Figure 3: Physical breadboard configuration

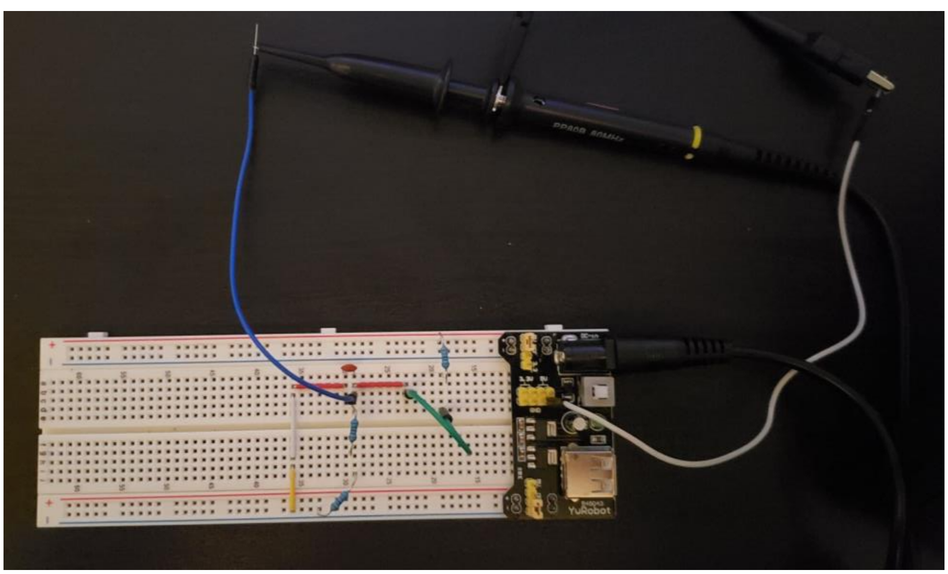
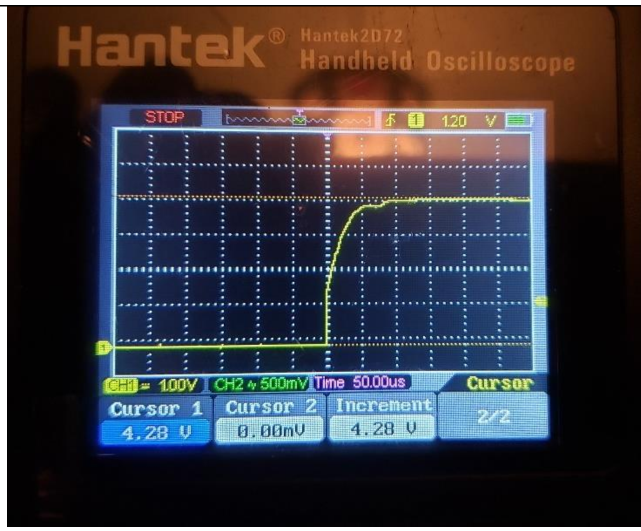
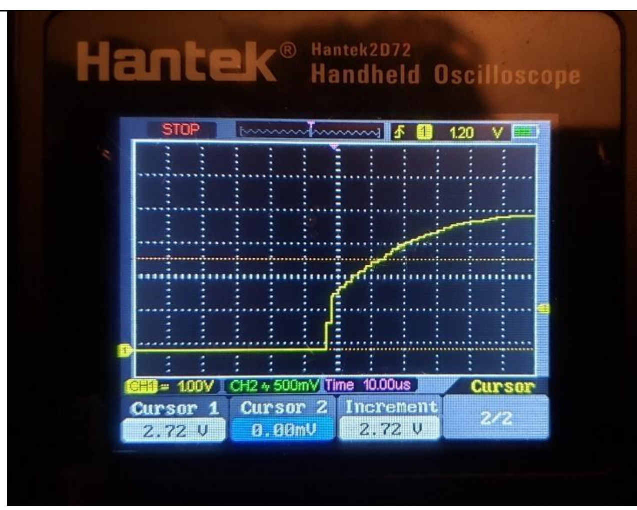
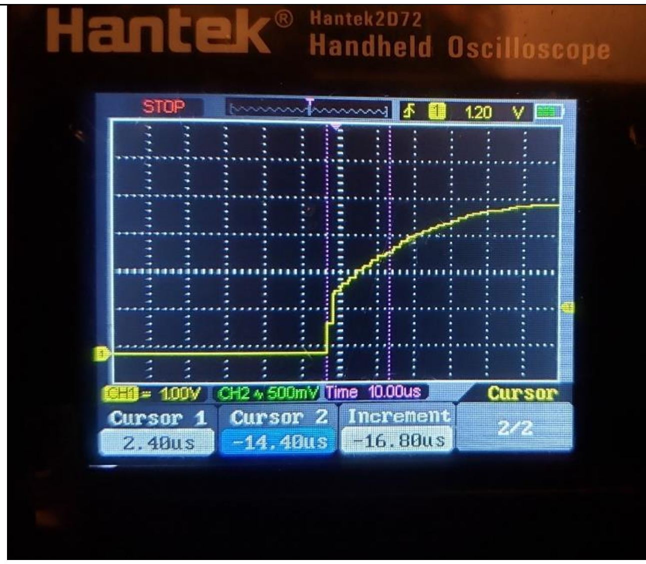
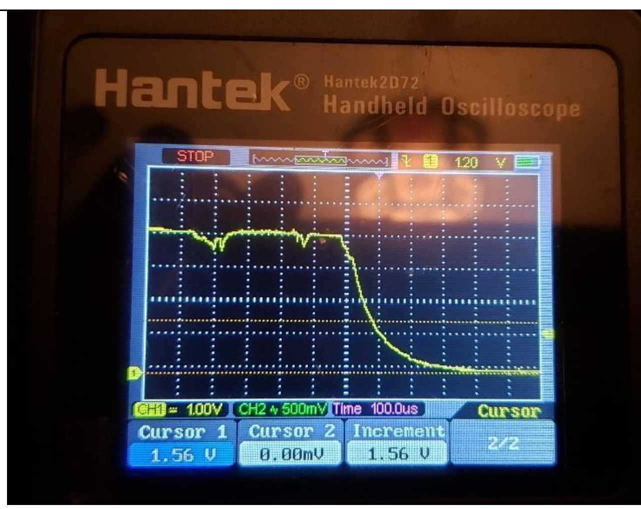


Figure 4: Hantek Measurement Configuration

The above picture demonstrates the configuration of the Hantek when measuring the voltage through capacitor. Note that the blue wire is connected to the oscilloscope probe and the white wire is connected to ground.

The following are images of the measurements taken with the Hantek. Note that the uncertainties are half of the smallest increment on the scope (multiplied by 2 for two cursors), plus 5% for the error of the Hantek:

Measurement type	**Image**	**Result (w/ uncertainty)**
Charging – Voltage @ $t = \infty$		$V_c(\infty) = \big(4.28 \pm (0.2 + 0.214)\big)V$ $V_c(\infty) = (4.28 \pm 0.414)\ V$ $V_c(0) = 0\ V$
Charging – Voltage @ $t = \tau$		Analytical Calculation for voltage at t=τ $V_\tau = (4.28) * \left(1 - e^{\frac{-\tau}{\tau}}\right)$ $V_\tau = 4.27 * 0.632$ $V_\tau \approx 2.70\ V$ Position on Hantek: $V_c(\tau) = \big(2.72 \pm (0.2 + 0.136)\big)V$ $V_c(\tau) = (2.72 \pm 0.336)\ V$

Charging – Solving for time @ $V_c(\tau)$	 	$\tau = (16.80 \pm (2 + 0.84))\ \mu s$ $\tau = (16.80 \pm 2.84)\ \mu s$
Discharging – Voltage drop @ $t = \infty$	 	$V_c(0) = (4.28 \pm 0.414)\ V$ $V_c(\infty) = 0\ V$
Discharging – Voltage @ $t = \tau$	 	Analytical Calculation for voltage at t=τ $V_\tau = (4.27) * \left(e^{\frac{-\tau}{\tau}}\right)$ $V_\tau = 4.27 * 0.368$ $V_\tau \approx 1.57\ V$ Position on Hantek: $V_c(\tau) = \left(1.56 \pm (0.2 + 0.078)\right)V$ $V_c(\tau) = (1.56 \pm 0.278)\ V$

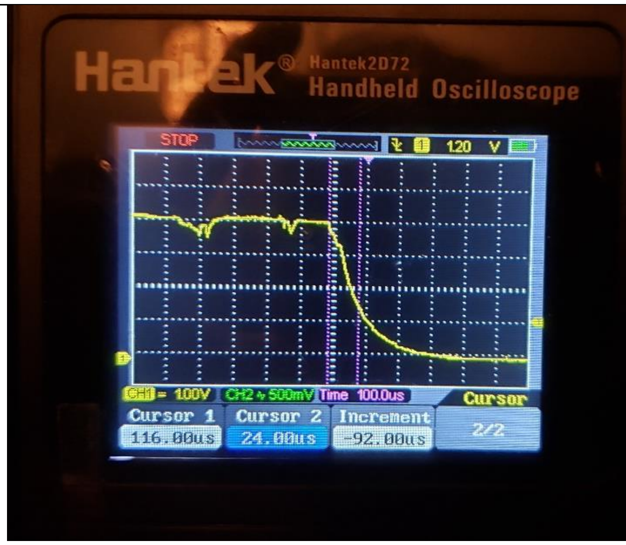
Discharging – Solving for time @ $V_c(\tau)$		$\tau = \left(92.00 \pm (20 + 4.6)\right)\mu s$ $\tau = (92.00 \pm 24.6)\mu s$

Table 2: Measured values with Hantek oscilloscope

From the results above, the formula for voltage in terms of time can be derived:

$$\textit{Charging:}\quad V_c(t) = [(4.28 \pm 0.414)\ V] * [1 - e^{\frac{-t}{(16.80 \pm 2.84) * 10^{-6}\ s}}]$$

$$\textit{Discharging:}\quad V_c(t) = [(4.28 \pm 0.414)\ V] * [e^{\frac{-t}{(92.00 \pm 24.6) * 10^{-6}\ s}}]$$

Analysis

Observing the results derived from the physical analysis, we can see that all values match the results from the analytical solution and the multisim solution (when accounting for error of course).

Probe Impedance

Finally, the impedance of the Hantek's probe was measured by discharging the capacitor onto it. After having measured the time constant using the same methods outlined previously, the resistance of the probe can be found using the formula $\tau = R * C$.

The following configuration was used. Note that the ground of the probe is connected between the capacitor and ground, so that when the yellow wire is disconnected from the breadboard, the voltage output by the capacitor will flow only into the probe's ground wire, thus discharging the capacitor into a circuit composed of only itself and the Hantek's probe.

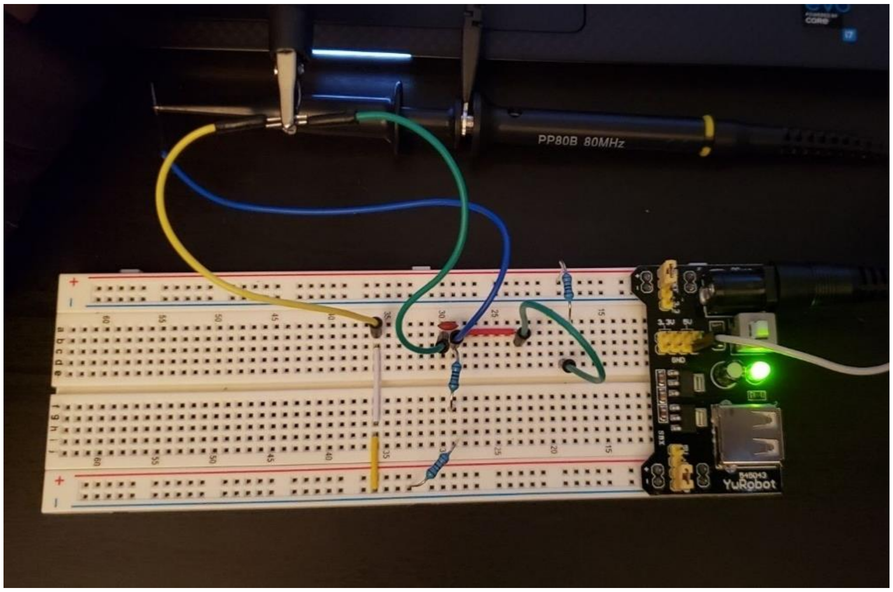
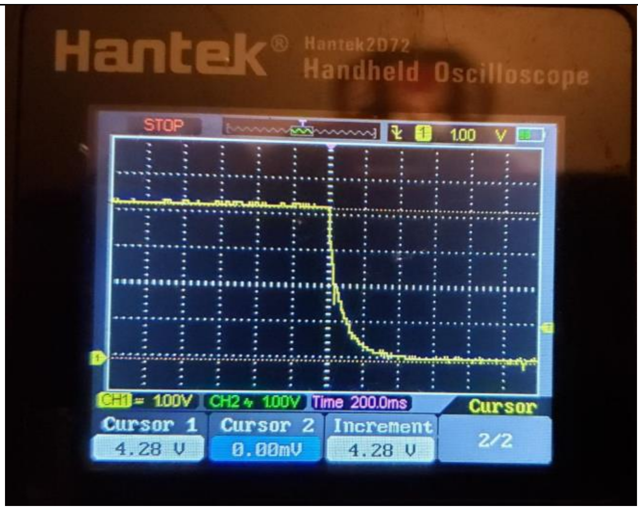


Figure 4: Discharging into probe configuration

Measurement type	**Image**	**Result (w/ uncertainty)**
Discharging – Voltage drop @ $t = \infty$		$V_c(0) = (4.28 \pm 0.414)\ V$ $V_c(\infty) = 0\ V$

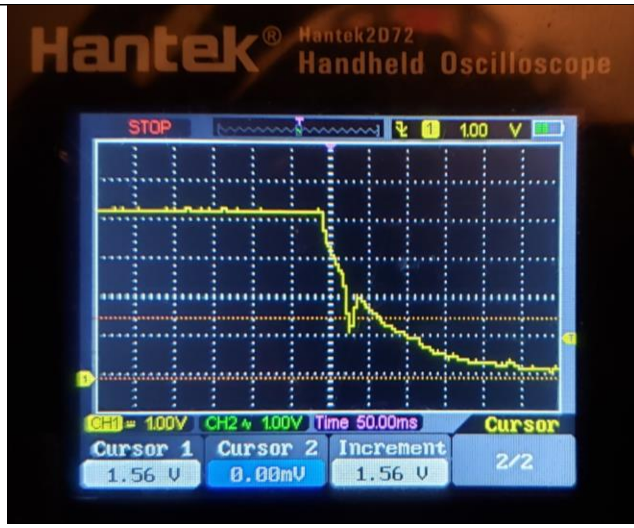
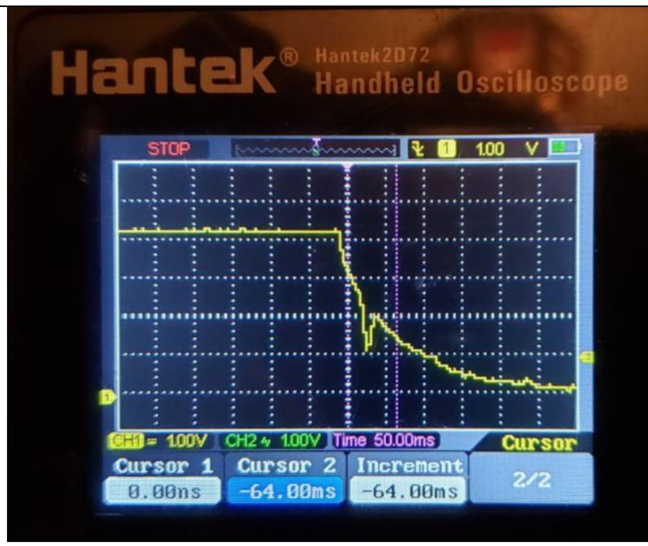
Discharging – Voltage @ $t = \tau$		Analytical Calculation for voltage at t=τ $V_\tau = (4.28) * \left(e^{\frac{-\tau}{\tau}}\right)$ $V_\tau = 4.28 * 0.368$ $V_\tau \approx 1.58\, V$ Position on Hantek: $V_c(\tau) = (1.56 \pm 0.278)\, V$
Discharging – Solving for time @ $V_c(\tau)$		$\tau = (64.00 \pm 13.2)\, ms$

Table 3: Measured values with Hantek oscilloscope

Having acquired the time constant, the resistance of the probe can be calculated mathematically.

$$\tau = (64.00 \pm 13.2) * 10^{-3}\, s, \qquad C = 77.4 * 10^{-9}\, F$$

$$\tau = R * C \rightarrow R = \tau / C$$

$$R = \left((64.00 \pm 13.2) * 10^{-3}\right) / (77.4 * 10^{-9})$$

$$R = 826{,}873.3850129\, \Omega$$

$$R_{Probe} = (0.827 \pm 0.170)\, M\Omega$$

Analysis

The impedance of the Hantek is stated to be 1.0 MΩ with an uncertainty of 0.02 MΩ. While my results are similar to the provided values, the measured resistance just barely outside of error (if we consider the lower bound of the given impedance and the upper bound of the measured impedance). A possible cause for the discrepancy in the results is the estimation factor in this lab. As previously mentioned, lining up the cursors is not a very accurate way to position the time cursor and gauge the time constant. Another cause is evidently the physical setup, as poor contact between probe and jumpers, or poor contact between elements and the breadboard can cause for error in the measurements. All in all, despite the error I think this analysis was a success, as the yielded results were similar to the actual values.

Answers to Deliverable Questions

The initial voltage across the capacitor is 4.28 V. This is because at the time that the switch is thrown (time = 0), the capacitor has been fully charged, since V(0) of a discharging process is equal to v(∞) of a charging process. The energy stored in a capacitor can be expressed with the equation below:

$$U = \frac{Q^2}{2C}$$

Where Q is the charge of the capacitor, and C is the capacitance. We can rearrange the definition of capacitance $C = \frac{Q}{V}$ to yield $Q = V * C$. Substituting this gives:

$$U = \frac{C * V^2}{2}$$

So at a voltage of 4.28 V (the initial voltage), we can find the energy stored in the capacitor:

$$U = \frac{(77.4 * 10^{-9}\ F)(4.28\ V)^2}{2}$$

$$U = 7.08922 * 10^{-7} J, \qquad U = 709\ nJ$$

Therefore, the energy stored in the capacitor at the initial voltage is approximately 709 nJ.

The capacitor forms an RC circuit with the Hantek. When the switch is thrown, the voltage supply is disconnected, and the capacitor is connected only to the Hantek in series. The capacitor is being discharged by the Hantek probe because they form an RC circuit. Since capacitors resist change in voltage, the capacitor begins discharging voltage to minimize change in voltage over time (to avoid a sudden drop to 0V and rather gradually drop to 0V).

Considering the calculations in the previous section, the impedance is estimated to be $(0.827 \pm 0.170)\ M\Omega$. This result is relatively accurate but just outside of uncertainty, since the Hantek manual states the impedance to be approximately 1 MΩ.

Results

Below is a table comparing the determined results for each method (not including resistance of probe).

	V(t) - Charging Formula	V(t) - Discharging formula
Analytical	$V_c(t) = (4.28\,V)\left[1 - e^{\left(\frac{-t}{14.4\,*10^{-6}\,s}\right)}\right]$	$V_c(t) = (4.28\,V) * e^{\left(\frac{-\text{t}}{93.9*10^{-6}\,\text{s}}\right)}$
Multisim (Tektronix)	$V_c(t) = [(4.27 \pm 0.2)\,V] * [1 - e^{\frac{-t}{(19.8\pm 4)*10^{-6}\,s}}]$	$V_c(t) = [(4.27 \pm 0.2)\,V] * [e^{\frac{-t}{(94.6\pm 20)*10^{-6}\,s}}]$
Physical (Hantek)	$V_c(t) = [(4.28 \pm 0.414)\,V] * [1 - e^{\frac{-t}{(16.80\pm 2.84)*10^{-6}\,s}}]$	$V_c(t) = [(4.28 \pm 0.414)\,V] * e^{\frac{-t}{(92.00\pm 24.6)*10^{-6}\,s}}$

Table 4: Summary of Results

Analysis

The results of all 3 methods of analysis are quite similar, as all results are within uncertainty of one-another. The formula of the curves from the multisim and physical solutions are relatively consistent with the analytical solution's curves, though there is still clearly error present. What can be observed from the time constants of the results is that discharging generally took between 4.7 and 6.5 times longer than charging. This is due to the reduced resistance in the circuit after the switch was closed. Since there was less resistance to reduce voltage, it took longer for all the voltage to be dissipated from the capacitor.

As discussed previously, there are many possible causes for error in this lab. The method of lining up the cursors for time and voltage to determine the time constant is prone to error, as it involves estimation via the position of the cursor on the screen ("eyeballing it"). Another cause of error affecting the physical and digital solution is the inaccuracy of measuring with the cursors. Finally, poor contact with the breadboard and using a jumper wire as a switch instead of a proper switch could have caused error in physical measurements.

Conclusion

The goal was to find the voltage of the capacitor as a function of time analytically, using the Tektronix oscilloscope in Multisim, and physically using the Hantek and breadboard. Observing the results, the physical analysis matched the analytical solution within uncertainty, while the multisim solution did not (though the multisim solution was similar).

In addition to this, physically finding the resistance of the Hantek probe using the time constant formula. In this case, the result was not within error of the given value in the manual but was very similar.

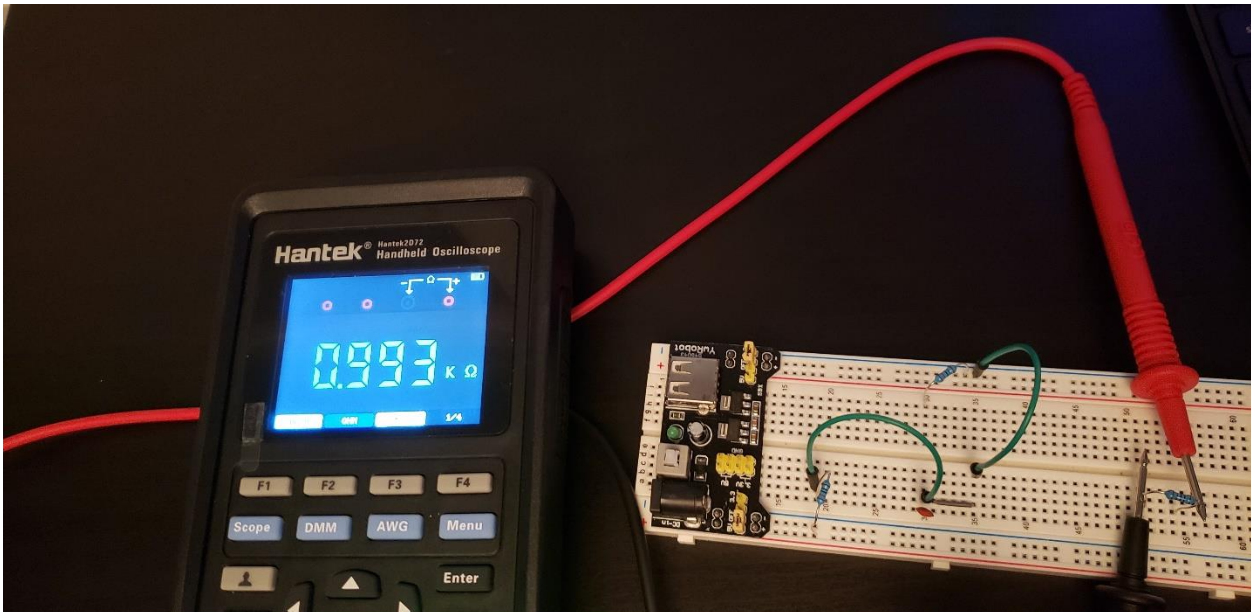
My experience with this lab was much more positive than the previous. Because of my prior knowledge from last year, I had a better understanding of the content, allowing me to apply the theory effectively into the physical and digital analysis. During this lab I learned that the voltages throughout a circuit can vary even in a DC circuit because of capacitors, and this showed me how an oscilloscope can be used in a DC circuit (I thought they only had applications in an AC environment). Also, I learned how to adjust the trigger settings on an oscilloscope to capture only data that is wanted.

All in all, I had a much better time with this lab. My prior knowledge on the topics being explored guided me through the different stages of the lab. Despite this, I still learned a lot from this lab, mainly including skills with the Hantek.

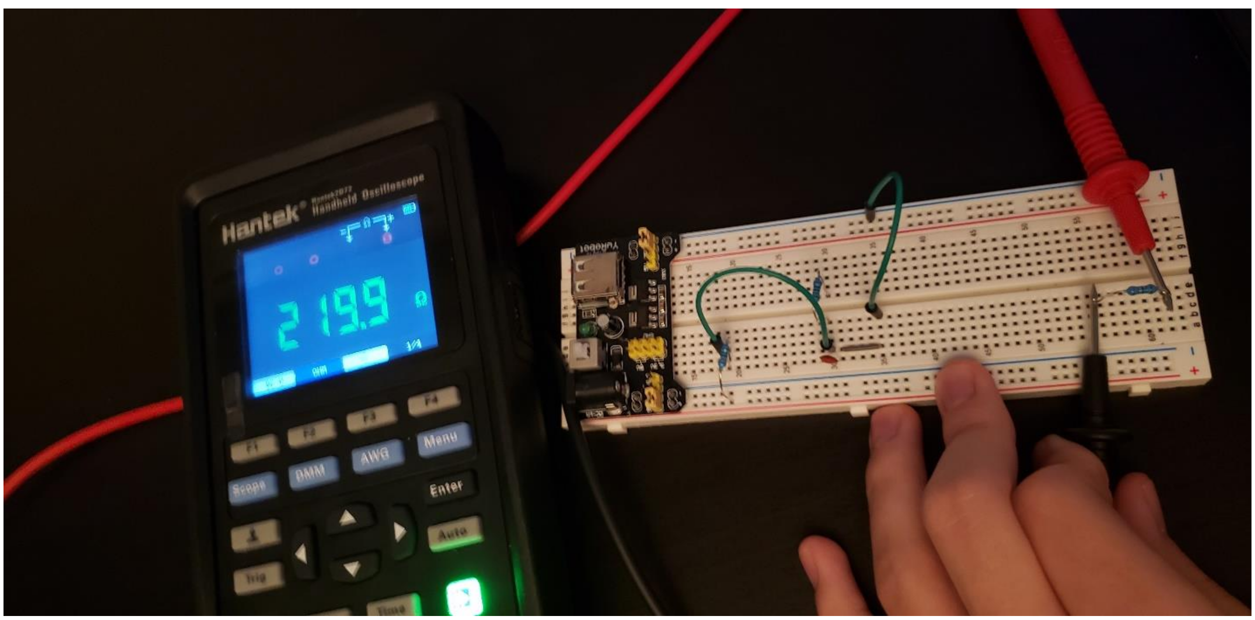
Appendix 1: Measurement pictures

Note all measurements were taken in the same manner as in the picture below.

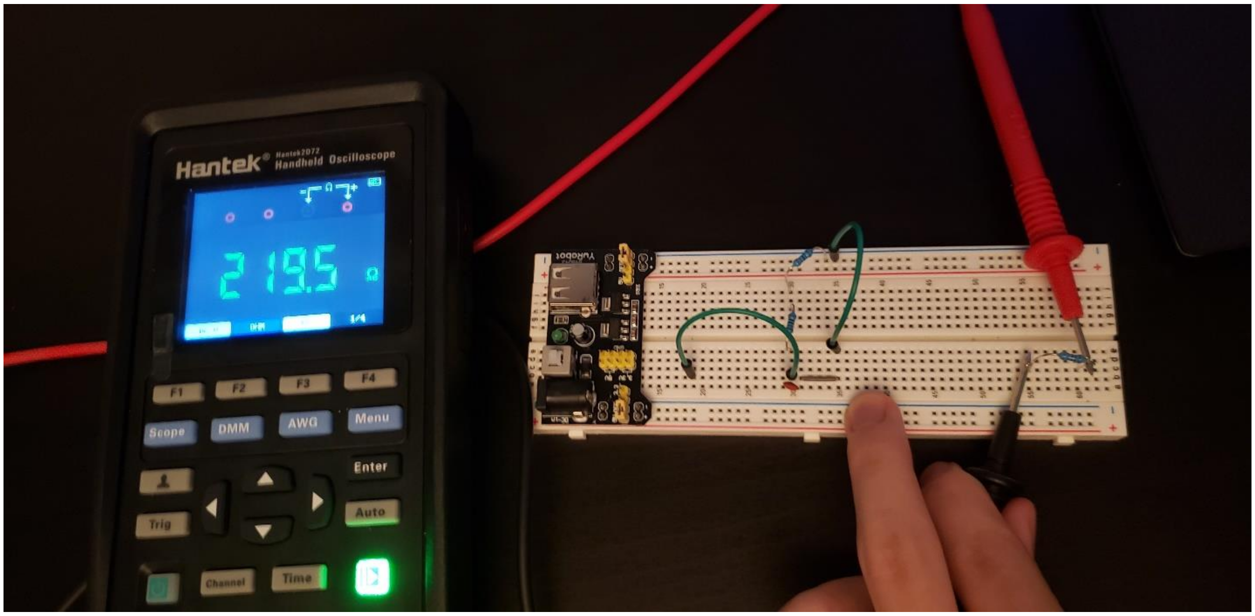
R2:



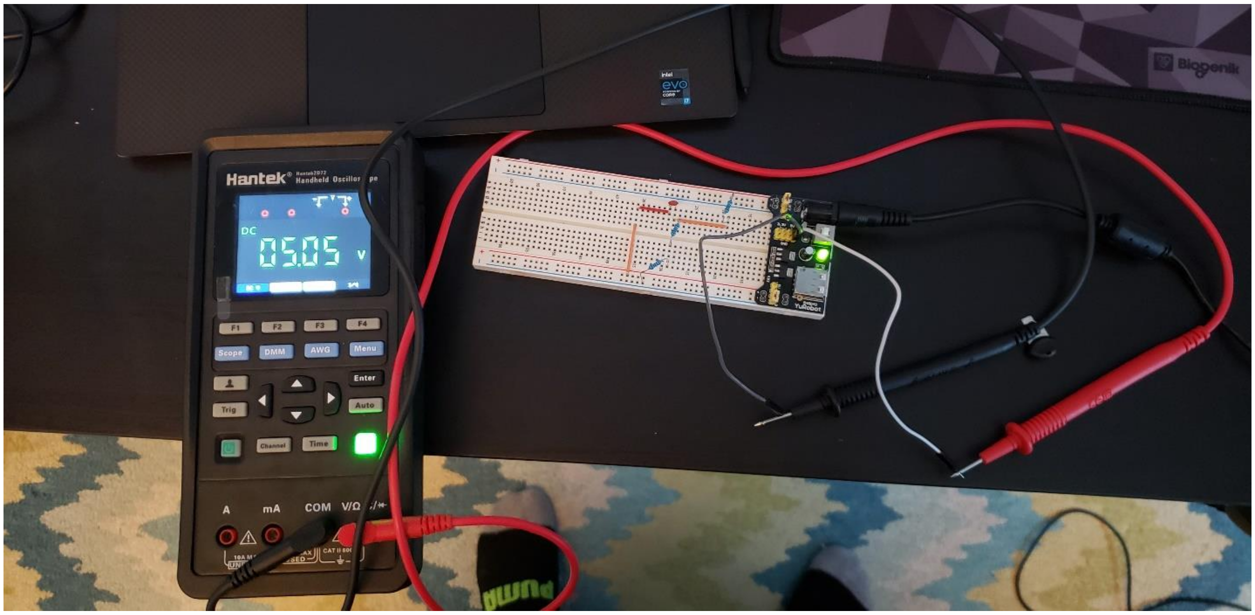
R3:



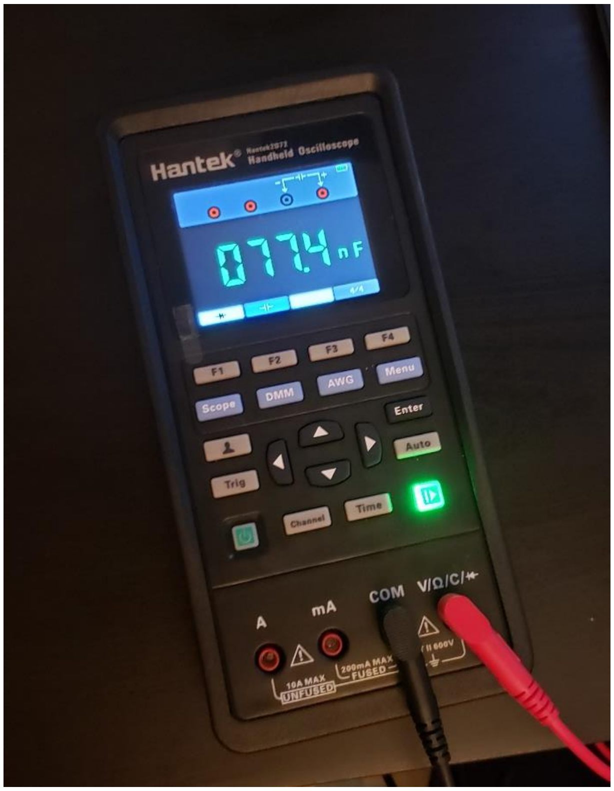
R1:



Voltage Supply:



C1:



(Please excuse the capacitor’s absence from the picture. It was measured in the same way as the resistors: in parallel with one probe on each end)