ENG PHYS 2A04 Tutorial 2

Electricity and Magnetism

Your TAs today

- Tommy Kean

  keant1@mcmaster.ca

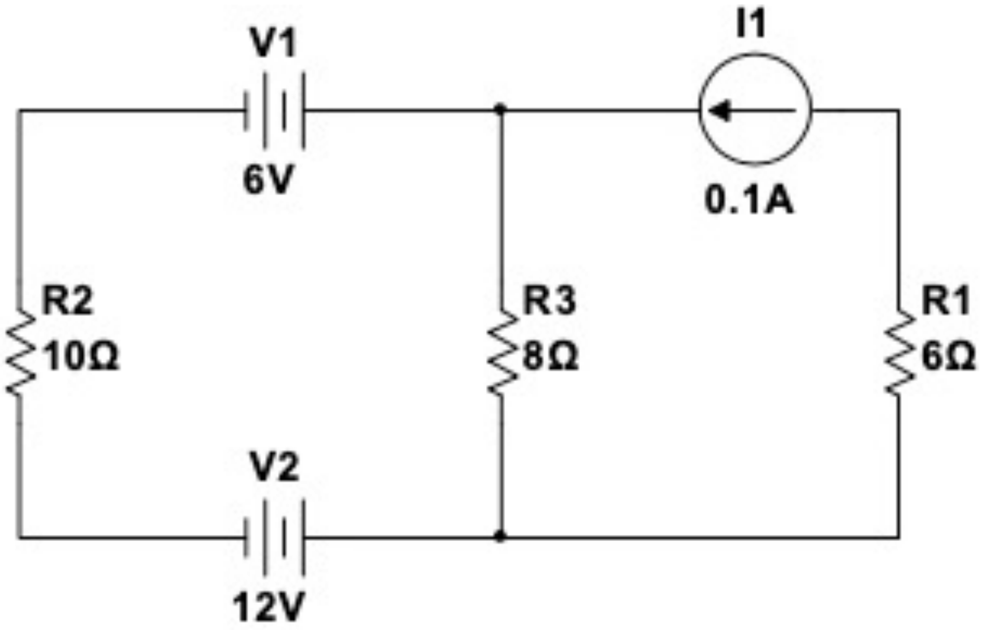
- Muhammad Munir

  munirm6@mcmaster.ca

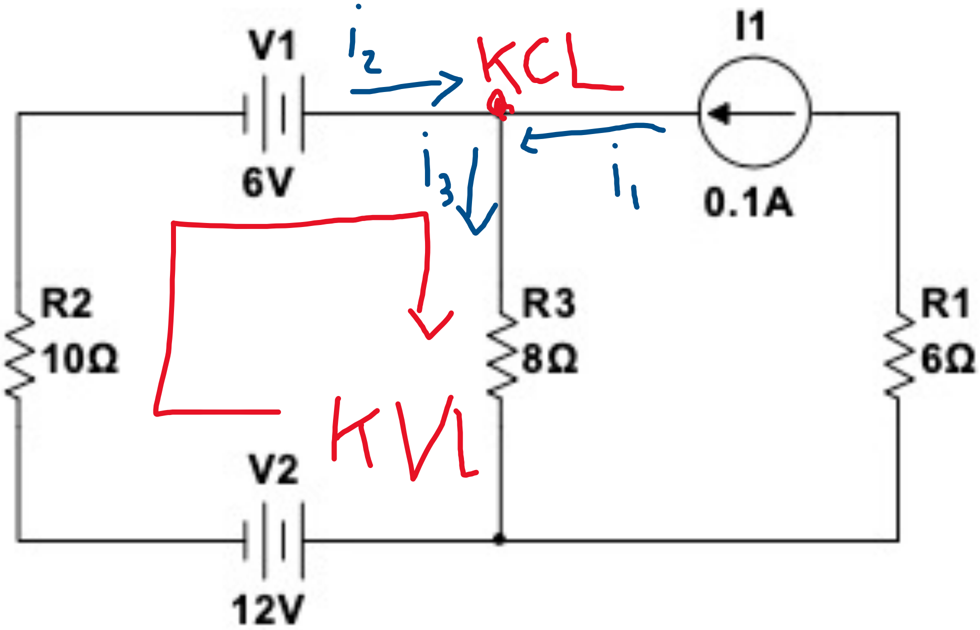
Chapter 28: DC Circuits (Serway, 9th edition)

Problem 1 Example in Lecture

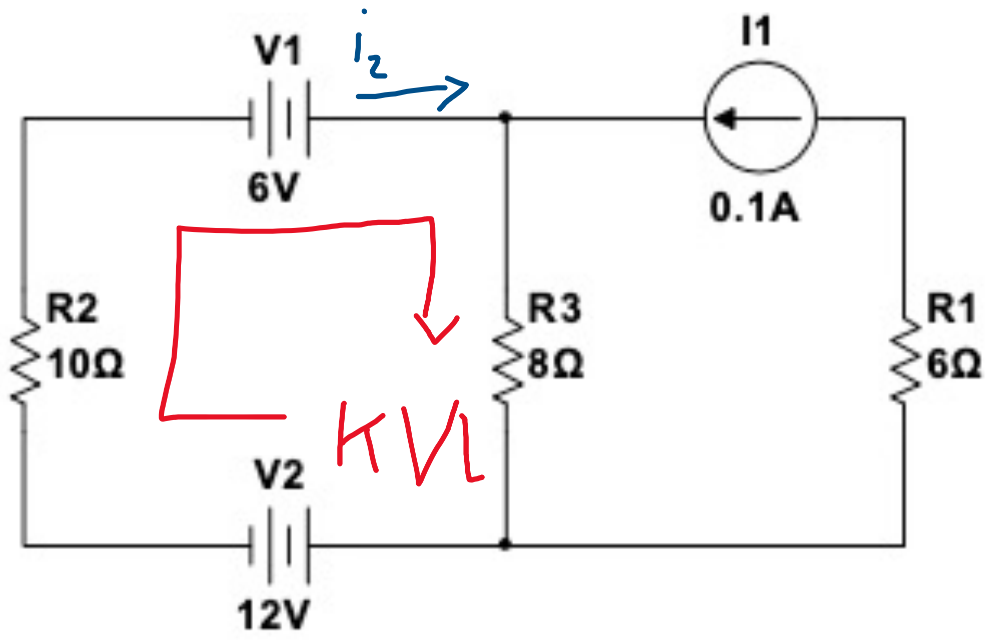
Find the current across R_1, R_2 and R_3



Example in Lecture



Example in Lecture



Step 1 - Loop Rule (KCL) :

$$\sum_{closed\ loop} \Delta V = 0$$

$$V_1 + V_{R_3} + V_2 + V_{R_2} = 0$$

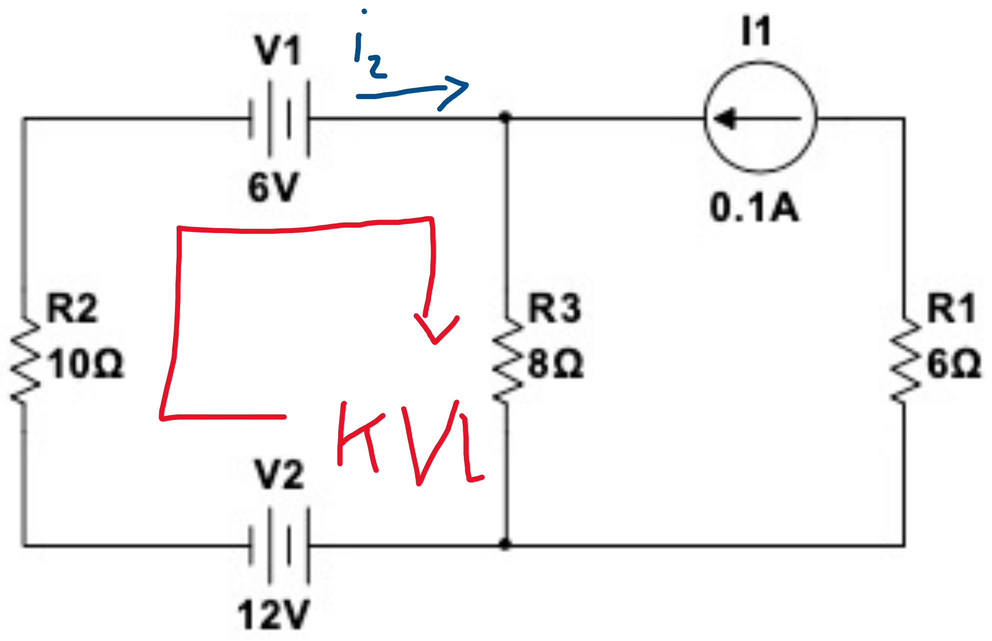
$$V_1 + (i_3 R_3) + V_2 + (i_2 R_2) = 0$$

$$V_1 + i_3 R_3 + V_2 + i_2 R_2 = 0$$

$$(-6) + (12) + \ i_3(8) + i_2(10) = 0$$

$$6 + \ 8i_3 + 10i_2 = 0$$

Example in Lecture



Step 2 - Junction Rule :

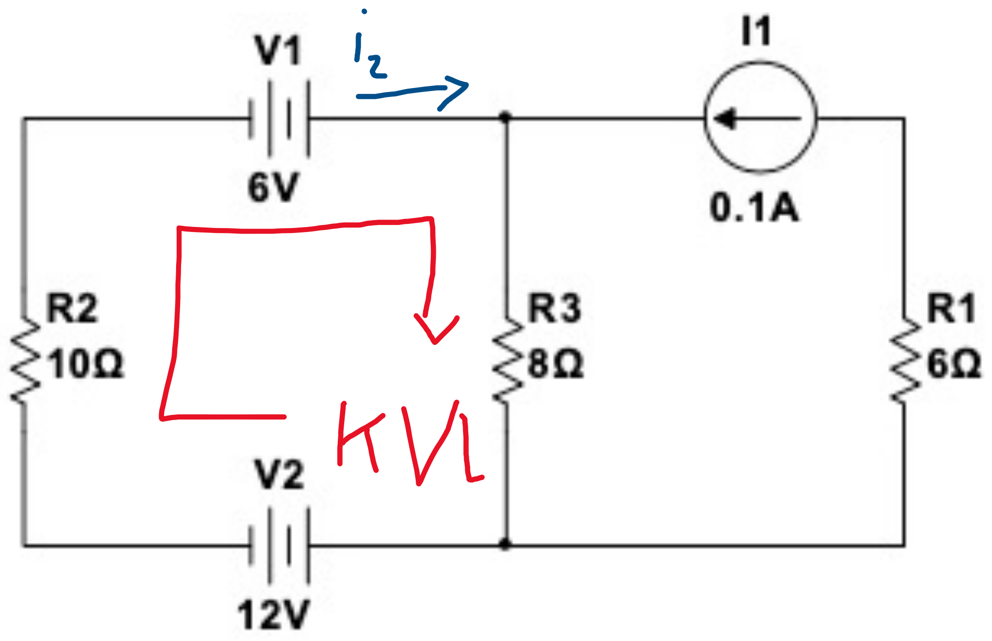
$$\sum_{junction} \mathrm{i} = 0$$

$$i_1 + i_2 - i_3 = 0$$

$$0.1 + i_2 - i_3 = 0$$

$$i_3 = 0.1 + i_2$$

Example in Lecture



Step 3 - Substitute:

$$6 + \ 8i_3 + 10i_2 = 0$$
$$6 + \ 8(0.1 + i_2) + 10i_2 = 0$$
$$6 + \ 0.8 + 8i_2 + 10i_2 = 0$$
$$6.8 + 18\, i_2 = 0$$
$$i_2 = \frac{-6.8}{18}$$
$$i_2 = -0.377$$

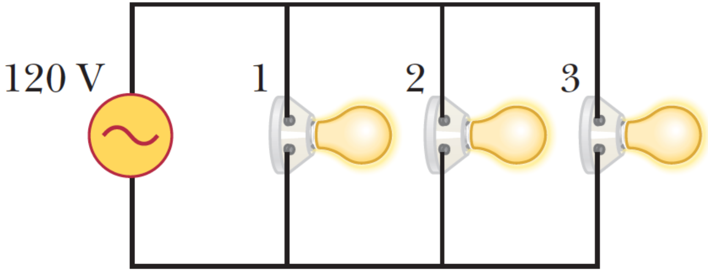
$$i_3 = \ 0.1 + i_2$$
$$i_3 = \ 0.1 + (-0.377)$$
$$i_3 = \ -0.277$$

$$i_1 = 0.1\,A,\ i_2 = \ -0.377\,A,\ i_3 = \ -0.277\,A$$

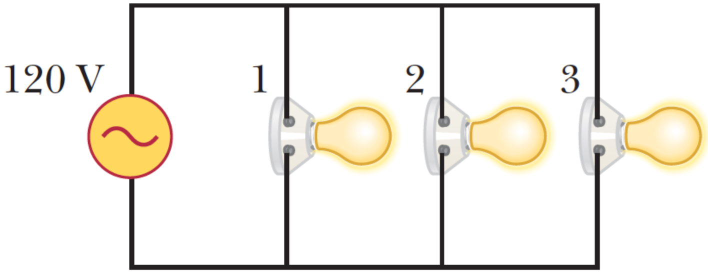
Chapter 33: AC Circuits (Serway, 9th edition)

Problem 2

The figure below shows three lightbulbs connected to a 120-V AC (rms) household supply voltage. Bulb 1 has a power rating of 40 W, bulb 2 has a 75 W rating and bulb 3 has a 60 W rating. Find (a) the rms current in each bulb and (b) the resistance of each bulb. (c) What is the total resistance of the combination of the three lightbulbs:



Problem 2



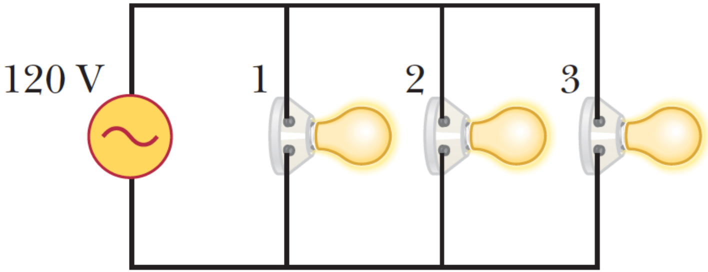
Find the RMS Current:

$$I_{1,rms} = \frac{P_1}{V} = \frac{40W}{120V} = 0.33A$$

$$I_{2,rms} = \frac{P_2}{V} = \frac{75W}{120V} = 0.625A$$

$$I_{3,rms} = \frac{P_3}{V} = \frac{60W}{120V} = 0.5A$$

Problem 2



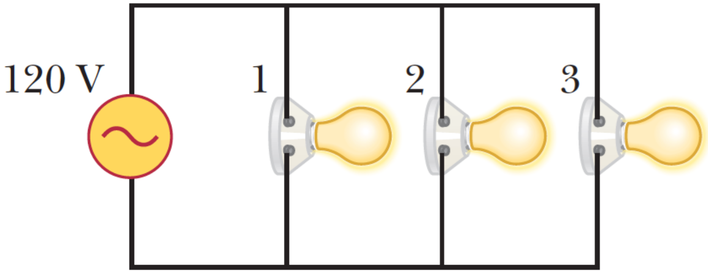
Find the resistance of each bulb :

$$R_1 = \frac{V^2}{P_1} = \frac{(120V)^2}{40W} = 360\ \Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{(120V)^2}{75W} = 192\ \Omega$$

$$R_3 = \frac{V^2}{P_3} = \frac{(120V)^2}{60W} = 240\ \Omega$$

Problem 2



the total resistance of the combination of the three lightbulbs :

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{total} = \left(\frac{1}{360} + \frac{1}{192} + \frac{1}{240}\right)^{-1} = 82.3\ \Omega$$

Problem 3

What maximum current is delivered by an AC source M with ΔV_{max} = 48.0 V and f=90.0 Hz when connected across a 3.70-μF capacitor?

Problem 3

What maximum current is delivered by an AC source with ΔV_{max} = 48.0 V and f=90.0 Hz when connected across a 3.70-μF capacitor?

1. Find the V_{RMS}

$$\Delta V_{RMS} = \frac{\Delta V_{max}}{\sqrt{(2)}}$$

$$\Delta V_{RMS} = 33.94\ V$$

Problem 3

What maximum current is delivered by an AC source M with ΔV_{max} = 48.0 V and f=90.0 Hz when connected across a 3.70-μF capacitor?

2. Find the reactance

$$x_c = \frac{1}{2\pi f C}$$

$$x_c = \frac{1}{2\pi(90\ Hz)(3.7\times10^{-6})}$$

$$x_c = 477.94\Omega$$

Problem 3

What maximum current is delivered by an AC source M with ΔV_{max} = 48.0 V and f=90.0 Hz when connected across a 3.70-µF capacitor?

3. Find the I_{rms}

$$I_{rms} = \frac{\Delta V_{RMS}}{x_c}$$

$$I_{rms} = \frac{33.94\ V}{477.94\Omega}$$

$$I_{rms} = 0.071\ A$$

Problem 3

What maximum current is delivered by an AC source M with ΔV_{max} = 48.0 V and f=90.0 Hz when connected across a 3.70-µF capacitor?

3. Find the I_{rms}

$$I_{peak} = I_{rms}\sqrt{2}$$

$$I_{peak} = (0.071\ A)\sqrt{2}$$

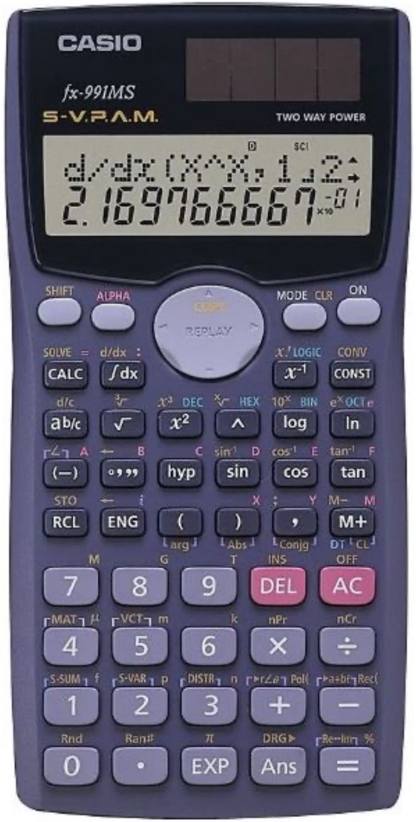
$$I_{peak} = 0.100\ A$$

Section 1-7: Review of Phasors
(Ulaby, 8th ed.)

McMaster’s Standard Calculator

- Casio FX-991MS
- 2nd Edition (left) available at McMaster Bookstore: $21.99 (link below)
- Will need to solve phasor equations

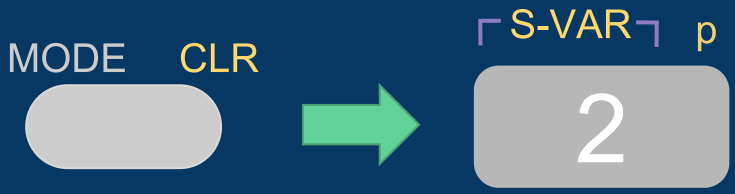
https://campusstore.mcmaster.ca/cgi-mcm/ws/gmdetail.pl?pwsPRODIDG1=2207018&sType=gm&proddesc=Casio%20fx%2D991%20Calculator&pwsGROUP=



Phasor Conversion – Calculator Setup

Enable complex numbers:

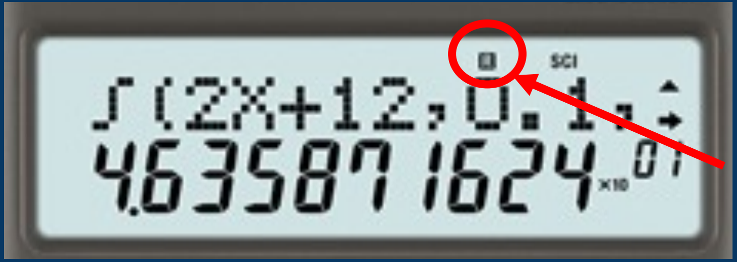
- “CMPLX” appears at top of display



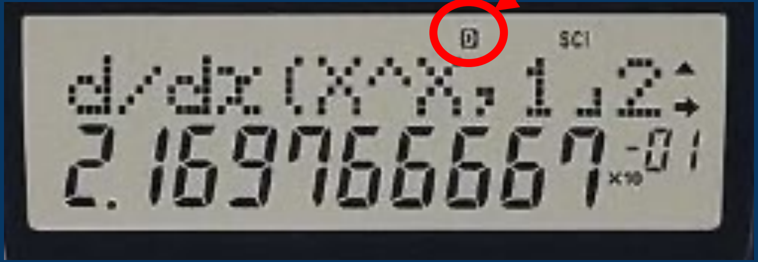
Ensure calculator is in degree mode

- For consistent units in polar form

GOOD!

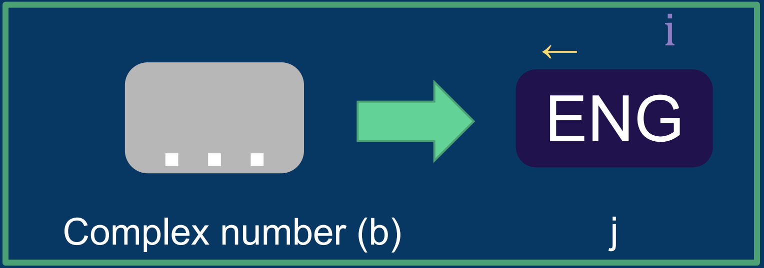


BAD!

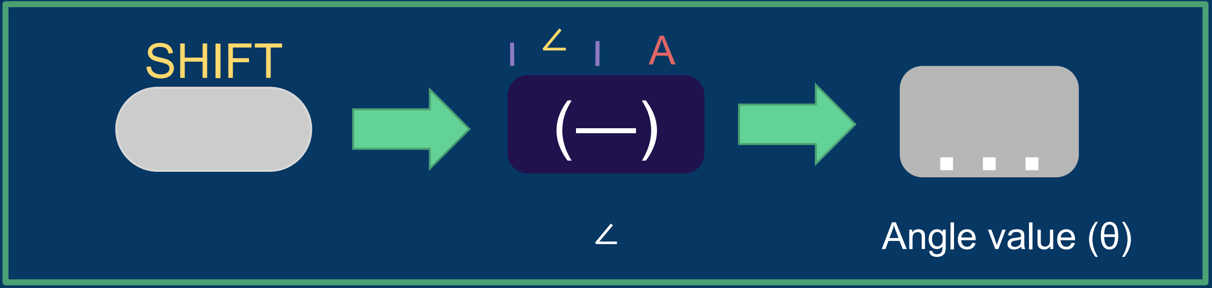


Phasor Conversion – Inputting Complex Values

To enter complex number in rectangular form:

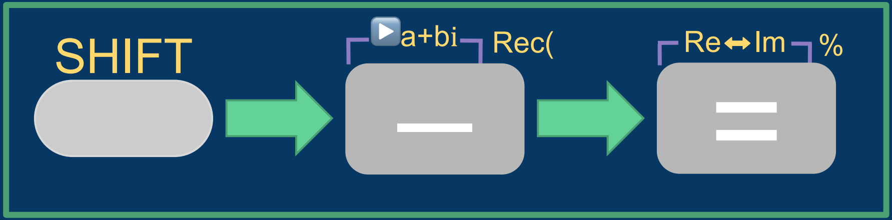


To enter angle value in polar form:

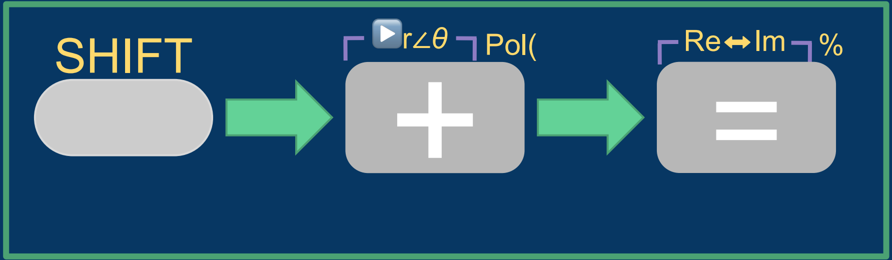


Phasor Conversion – Output Viewing

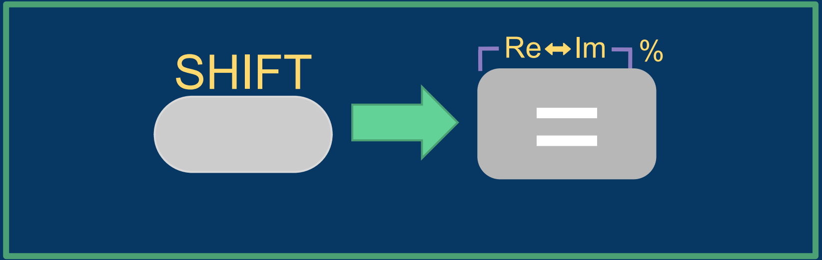
To view answer in rectangular form:



To view answer in polar form:



To change between real and imaginary (rectangular) or radius and angle (polar):



Phasor Conversion - Question

Convert the following phasors to rectangular form (a + bj):

a) $9 \angle -60°$

b) $12 \angle -45°$

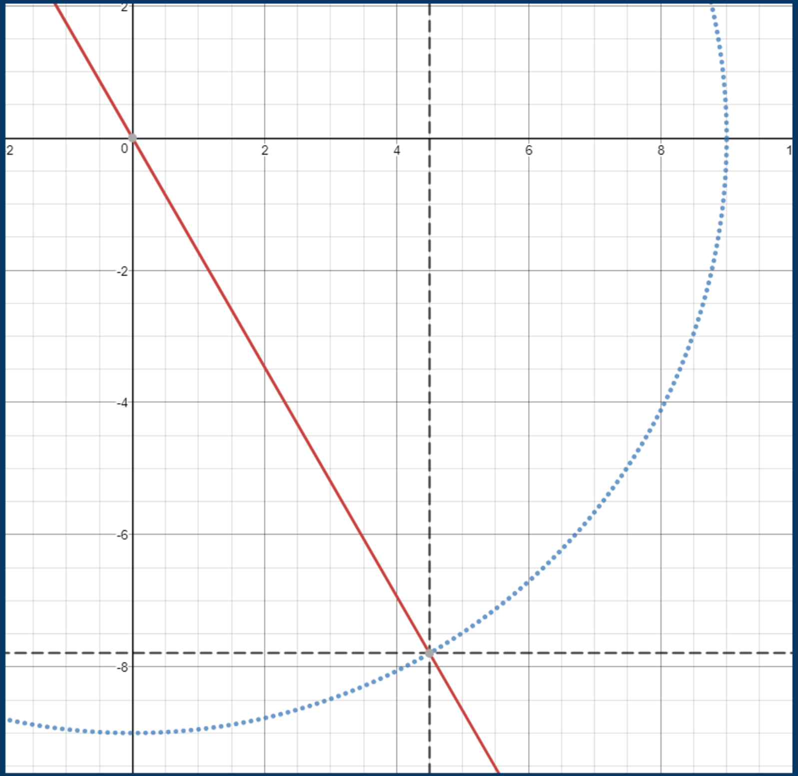
Convert the following phasors to polar form ($r \angle \theta$):

a) $3 + 9j$

b) $-0.4 + 0.3j$

Phasor Conversion - Solution

a) Convert the following to rectangular form (a + bj):



$9 \angle -60°$

9 | SHIFT | (-) | (-) | 6 | 0

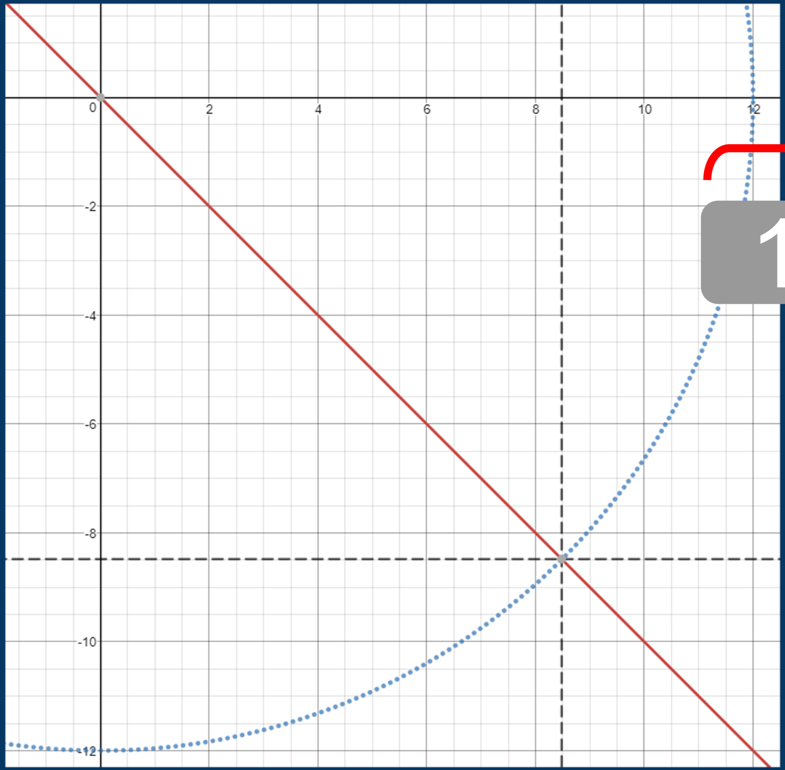
= a + bj

= | SHIFT | =

Answer: 4.5 - 7.79j

Phasor Conversion - Solution

b) Convert the following to rectangular form (a + bj):



$12 \angle -45°$

1 2 SHIFT (-) (-) 4 5

= a + bj

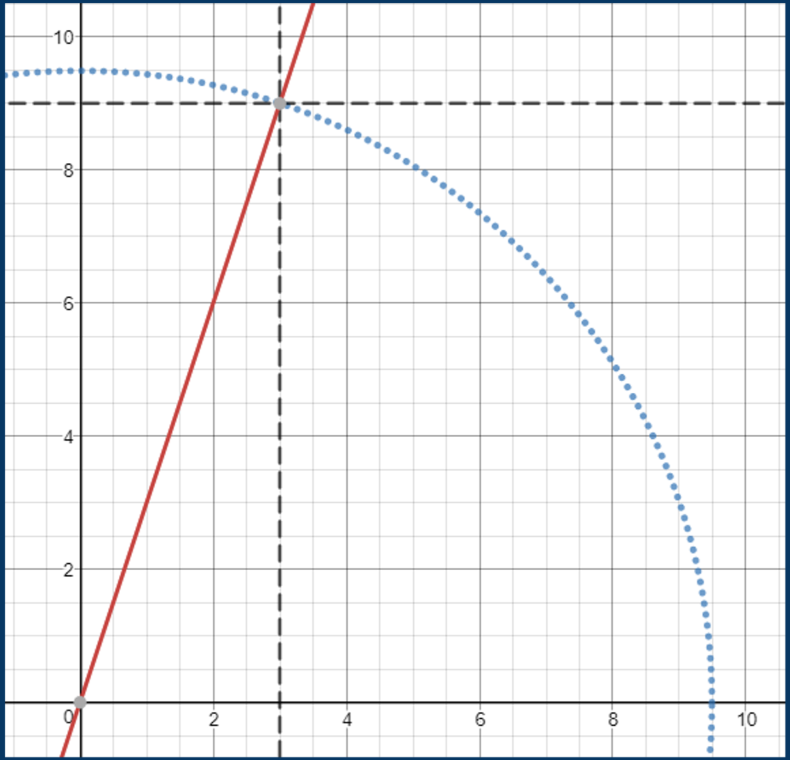
=

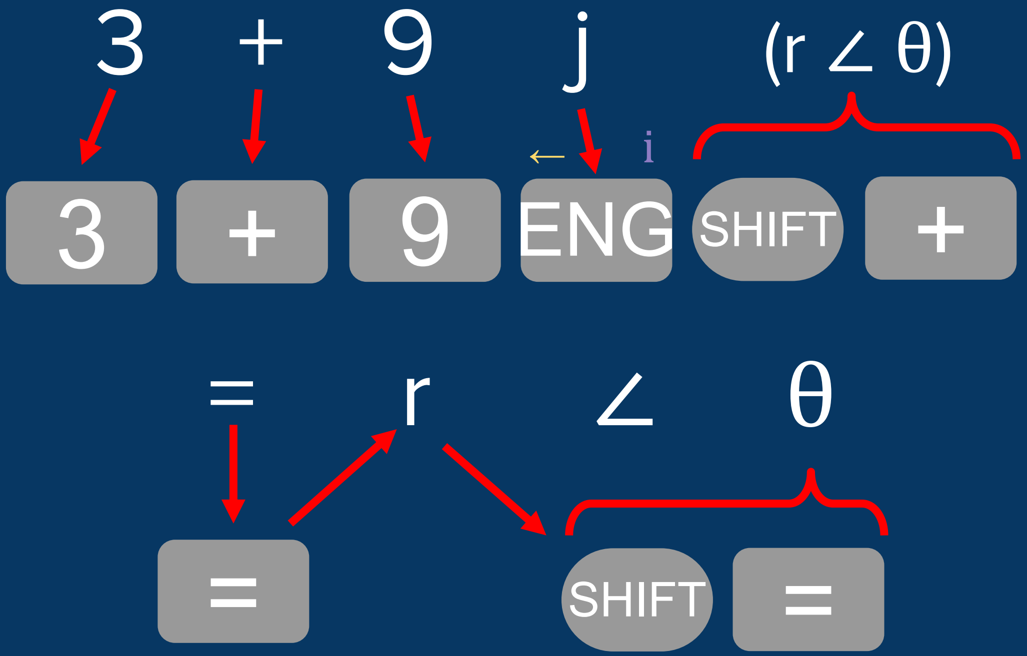
SHIFT =

Answer: 8.48 – 8.48j

Phasor Conversion - Solution

a) Convert the following to polar form (r ∠ θ):

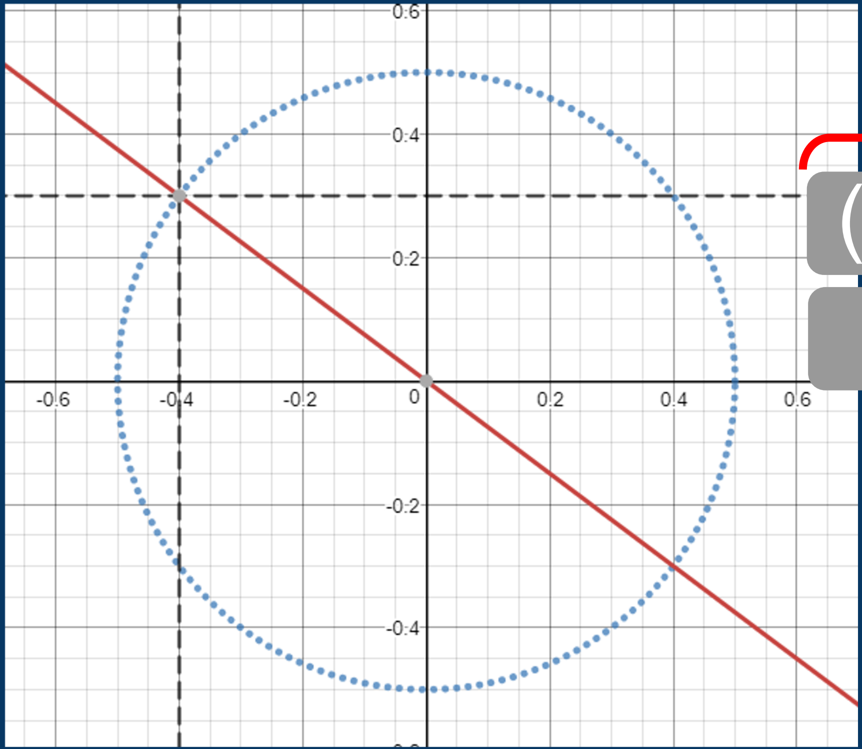




Answer: 9.49 ∠ 71.56°

Phasor Conversion - Solution

a) Convert the following to polar form (r ∠ θ):



$-0.4 \quad + \quad 0.3 \quad j$

(-) 0 . 4 + 0 . 3 ENG

← i

(r ∠ θ) = r ∠ θ

SHIFT + = SHIFT =

Answer: 0.5 ∠ 143.13°

Problem 1.28 – Question

Find the phasors of the following time functions:

a) $v(t) = 9\cos\left(\omega t - \frac{\pi}{3}\right)$ (V)

b) $v(t) = 12\sin\left(\omega t + \frac{\pi}{4}\right)$ (V)

c) $i(x,t) = 5e^{-3x}\sin\left(\omega t + \frac{\pi}{6}\right)$ (A)

d) $i(t) = -2\cos\left(\omega t + \frac{3\pi}{4}\right)$ (A)

e) $i(t) = 4\sin\left(\omega t + \frac{\pi}{3}\right) + 3\cos\left(\omega t - \frac{\pi}{6}\right)$ (A)

Problem 1.28 – Solution (a)

a) $v(t) = 9\cos\left(\omega t - \frac{\pi}{3}\right)$ (V)

Adopt cosine reference ($v(t) = V_0 \cos(\omega t + \phi)$). Is this already done?

- Yes! $V_0 = 9$, $\phi = -\frac{\pi}{3}$

From sinusoidal time domain to phasor domain:

- $v(t) = V_0 \cos(\omega t + \phi) \rightarrow \mathbf{V} = \mathrm{V}_0 e^{j\phi}$

$$\therefore v(t) = 9\cos\left(\omega t - \frac{\pi}{3}\right) \text{ (V)} \rightarrow \boldsymbol{\tilde{V} = 9e^{-j\frac{\pi}{3}}\ (V)}$$

Problem 1.28 – Solution (b)

b) $v(t) = 12\sin\left(\omega t + \frac{\pi}{4}\right)$ (V)

Adopt cosine reference ($v(t) = V_0\cos(\omega t + \phi)$). Is this already done?

- No! → $v(t) = 12\sin\left(\omega t + \frac{\pi}{4}\right) = 12\cos(\omega t + \frac{\pi}{4} - \frac{\pi}{2}) = 12\cos(\omega t - \frac{\pi}{4})$
- $V_0 = 12, \phi = -\frac{\pi}{4}$

Sinusoidal time domain → phasor domain: $v(t) = V_0\cos(\omega t + \phi) \rightarrow \mathbf{V} = \mathrm{V}_0 e^{j\phi}$

$$\therefore v(t) = 12\sin\left(\omega t + \frac{\pi}{4}\right)(\mathrm{V}) = 12\cos\left(\omega t - \frac{\pi}{4}\right)(V) \rightarrow \boldsymbol{V}^{\sim} = \mathbf{12}\boldsymbol{e}^{-j\frac{\boldsymbol{\pi}}{\mathbf{4}}}\ (\boldsymbol{V})$$

Problem 1.28 – Solution (c)

c) $i(x,t) = 5e^{-3x}\sin\left(\omega t + \frac{\pi}{6}\right)$ (A)

Adopt cosine reference ($i(t) = V_0\cos(\omega t + \phi)$). Is this already done?

- No! →$i(x,t) = 5e^{-3x}\sin\left(\omega t + \frac{\pi}{6}\right) = 5e^{-3x}\cos(\omega t + \frac{\pi}{6} - \frac{\pi}{2}) = 5e^{-3x}\cos(\omega t - \frac{\pi}{3})$
- $V_0 = 5e^{-3x}$, $\phi = -\frac{\pi}{3}$

Sinusoidal time domain → phasor domain: $v(t) = V_0\cos(\omega t + \phi) \rightarrow \mathbf{V} = \mathrm{V}_0 e^{j\phi}$

$\therefore i(x,t) = 5e^{-3x}\sin\left(\omega t + \frac{\pi}{6}\right)$ (A) $= 5e^{-3x}\cos(\omega t - \frac{\pi}{3})\,(A) \rightarrow \boldsymbol{V^{\sim} = 5e^{-3x}e^{-j\frac{\pi}{3}}\,(V)}$

Problem 1.28 – Solution (d)

d) $i(t) = -2\cos\left(\omega t + \frac{3\pi}{4}\right)$ (A)

Adopt cosine reference ($i(t) = I_0 \cos(\omega t + \phi)$). Is this already done?

- Yes! $I_0 = -2,\ \phi = \frac{3\pi}{4}$
- One missing step: $-1 = e^{-j\pi} \rightarrow I_0 = -2 = 2(-1) = 2e^{-j\pi}$

Sinusoidal time domain → phasor domain $i(t) = I_0 \cos(\omega t + \phi) \rightarrow \mathbf{I} = \mathrm{I}_0 e^{j\phi}$

$$\therefore\ i(t) = -2\cos\left(\omega t + \frac{3\pi}{4}\right) \text{ (A)} \rightarrow I = 2e^{-j\pi}e^{j\frac{3\pi}{4}}\ (A) \rightarrow \boldsymbol{I = 2e^{-j\frac{\pi}{4}}\ (A)}$$

Problem 1.28 – Solution (e)

e) $i(t) = 4\sin\left(\omega t + \frac{\pi}{3}\right) + 3\cos\left(\omega t - \frac{\pi}{6}\right)\ (\mathrm{A})$

Adopt cosine reference ($i(t) = I_0 \cos(\omega t + \phi)$). Is this already done?

- Kinda… →$4\sin\left(\omega t + \frac{\pi}{3}\right) = 4\cos\left(\omega t + \frac{\pi}{3} - \frac{\pi}{2}\right) = 4\cos\left(\omega t - \frac{\pi}{6}\right)$
- $I_0 = 4, \phi = -\frac{\pi}{6}$ and $I_0 = 3, \phi = -\frac{\pi}{6}$

Sinusoidal time domain → phasor domain $i(t) = I_0 \cos(\omega t + \phi) \ \rightarrow \mathbf{I} = \mathrm{I}_0 e^{j\phi}$

$\therefore i(t) = 4\sin\left(\omega t + \frac{\pi}{3}\right) + 3\cos\left(\omega t - \frac{\pi}{6}\right)(\mathrm{A}) \rightarrow I = 4e^{-j\frac{\pi}{6}} + 3e^{-j\frac{\pi}{6}} \rightarrow \boldsymbol{I = 7e^{-j\frac{\pi}{6}}(A)}$