Name: ....
Student Number: ....

Mechatronics 3DX4

DAY CLASS — Dr. Onaizah Onaizah
DURATION OF EXAMINATION: 2.5 Hours
McMaster University Final Examination — April 19, 2023

THIS EXAMINATION PAPER INCLUDES 5 PAGES AND 5 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

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Make sure you show all steps when answering questions. Just writing down the final answer will not get many marks. A table of some useful Laplace transforms can be found on the last page.

1. **State-space representation - 10 marks**

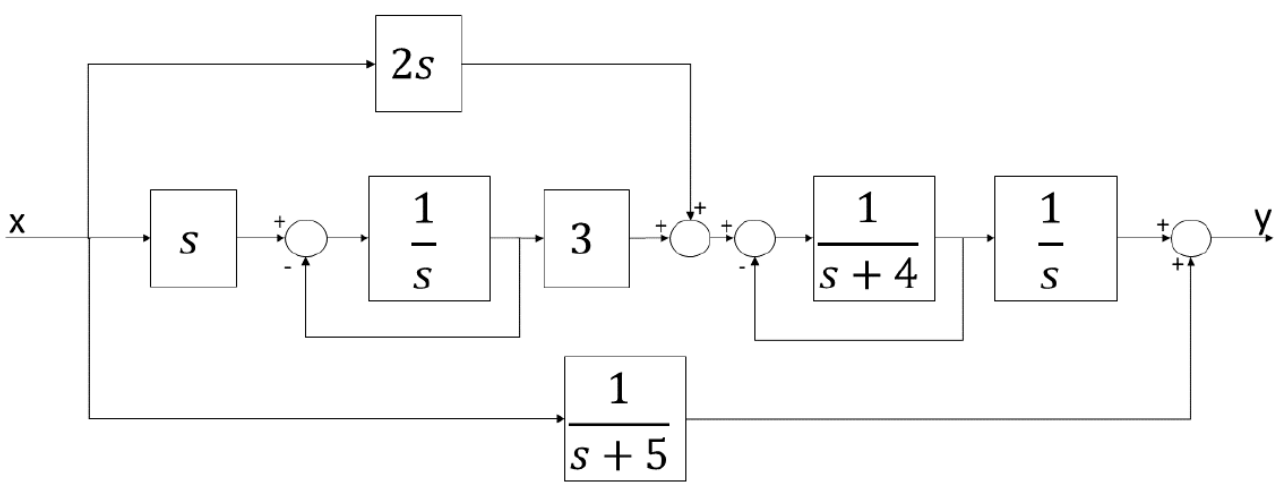
   Find the transfer function $G(s) = \frac{Y(s)}{U(s)}$ for the following system represented in state-space. Is this system stable?

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & -3 & -8 \\ 0 & 5 & 3 \\ -3 & -5 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 3 & 6 \end{bmatrix} \mathbf{x}$$

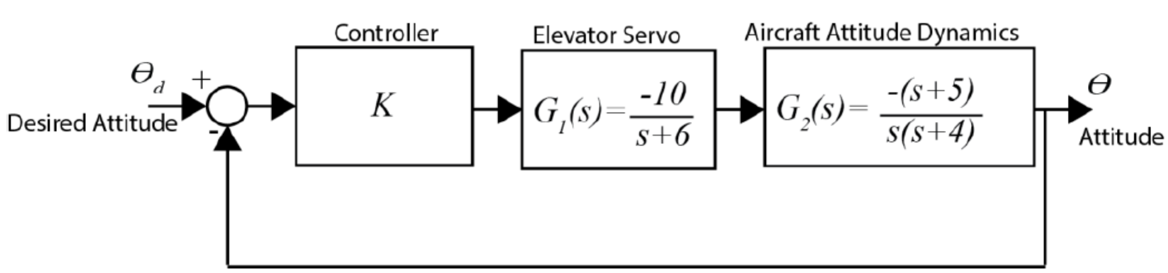
2. **Block Diagram - 10 marks**

   What is the closed loop transfer function $\frac{Y(s)}{X(s)}$?



3. **Stability and Steady State Error - 25 marks**

   Consider the following aircraft altitude control system. Here, the altitude is controlled using a proportional controller.



For this system, please conduct the following analysis:

(a) (10 marks) Determine the range of the controller gain K which results in the closed loop system being stable.

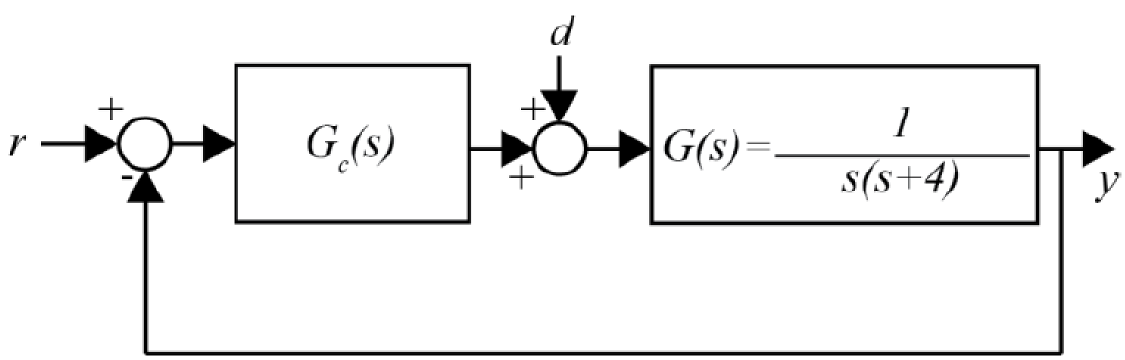
(b) (5 marks) For a choice of K that stabilizes the system, what would be the steady state error to step input of $\theta_d(t) = 5u(t)$, $t >= 0$.

(c) (5 marks) For a choice of K that stabilizes the system, what would be the steady state error to ramp input of $\theta_d(t) = 3tu(t)$, $t >= 0$.

(d) (5 marks) For a choice of K that stabilizes the system, what would be the steady state error to parabolic input of $\theta_d(t) = t^2u(t)$, $t >= 0$.

4. **Controller Design - 30 marks**

We are trying to develop a position controller for a wheeled robot driven by a DC motor. We are measuring the speed using a tachometer and are using feedback to speed up the response and limit system overshoot. The dynamics of the system are shown in the figure below.



(a) (7 marks) First, assuming the disturbance (labelled above as d) is zero, design a proportional controller ($G_c(s) = K_p$) to speed up the response of the system and have a peak time of less than 1 second for a unit step input for the reference.

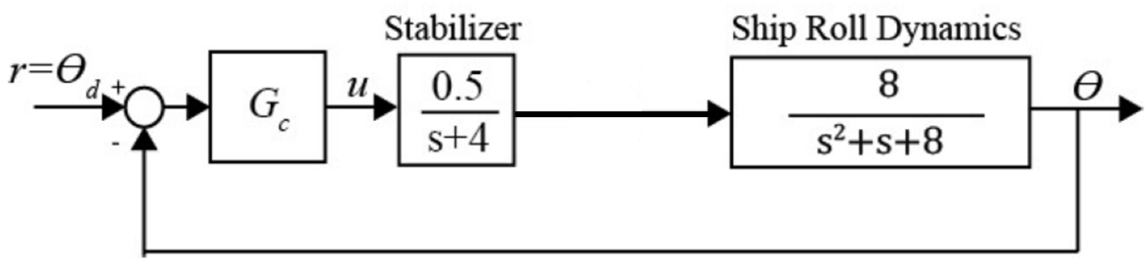
(b) (4 marks) Also assuming the disturbance (labelled above as d) is zero, design another proportional controller to ensure that the overshoot for a unit step input to the reference remains less than 5%.

(c) (9 marks) Also assuming the disturbance (labelled above as d) is zero, can you meet both a peak time of less than 1 second and overshoot of less than 5% for a step input to the reference with the same proportional controller? If not, choose another control approach that would speed up the response and limit overshoot. Determine the control gains to meet the requirement.

(d) (3 marks) For the controller designed in part c), what would the steady state error be to a unit step input to the reference.

(e) (7 marks) For the controller designed in part c), what would the steady-state error be due to a unit step input in the disturbance. If non-zero, suggest what you might add to your controller to get zero steady-state error (no need to design the controller).

5. **Routh-Hurwitz Criterion and Root Locus - 25 marks**



(a) (7 marks) Assume $G_c(s) = K$. Determine the transfer function from r to θ. For what range of K is the closed-loop system stable.

(b) (8 marks) Sketch the root locus for the system. Be sure to calculate any relevant information such as asymptotes, break-in or breakaway points, departure or arrival angles, and any imaginary-axis crossing points.

(c) (10 marks) Design a controller that results in the closed-loop system having an overshoot of 4% or smaller and the system will have a settling time less than 8 seconds.

Item no.	*f(t)*	*F(s)*
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s)-sf(0-)-\dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s)-\sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

The End