

Prac exam 2020

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1 Bandpass Filter Construction

(20 points) This time we construct a bandpass filter by putting a low-pass and a high-pass filter into series. Placing a zero at $\frac{F_s}{2}$ and normalizing at DC results in a simple low-pass filter $H_1(\omega)$. Placing a zero at DC normalizing at $\frac{F_s}{2}$ results in a simple high-pass filter $H_2(\omega)$. We construct a new H putting H_1 and H_2 into series.

What is the gain for the new H at DC, $\frac{F_s}{4}$ and $\frac{F_s}{2}$ of this filter $H(\omega)$? Make a table so we easy find our answers !

$$\omega_s = 2\pi f_s \quad \text{if } f_s = f/f_s = \frac{f_s/2}{f_s} = \frac{1}{2}$$

$$= 2\pi/2$$

$$= \pi \leftarrow F_s/2$$

$$\omega_0 = 2\pi f_s \quad \text{if } f_0 = f/f_s = \frac{f_s/4}{f_s} = \frac{1}{4}$$

$$= 2\pi \frac{1}{4}$$

$$= \pi/2 \leftarrow F_s/4$$

$$H\left(\frac{F_s}{2}\right) = H(\pi) \quad \tilde{H}_1(\omega) = (e^{-i\omega} - e^{i\pi})$$

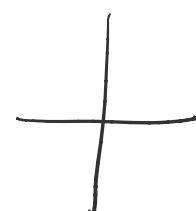
$$H(\pi) = 0 \quad \tilde{H}_1(\omega) = e^{-i\omega} + 1$$

$$H_1(0) = 1 \quad H_1(\omega) = \frac{1}{2}(e^{-i\omega} + 1)$$

$$\overline{H_2(\pi)} = 1 \quad \overline{\tilde{H}_2(\omega)} = \overline{(e^{-i\omega} - 1)}$$

$$H_2(0) = 0 \quad \tilde{H}_2(\pi) = -2$$

$$H_2(\omega) = -\frac{1}{2}(e^{-i\omega} - 1)$$



$$H_T(\omega) = H_1 H_2 = \frac{1}{4}(e^{-i\omega} + 1)(e^{-i\omega} - 1)$$

$$= -\frac{1}{4}$$

$$= -\frac{1}{4}(e^{-2i\omega} - 1)$$

$$\mathcal{H}\left(\frac{F_s}{4}\right) = \mathcal{H}\left(\frac{\pi}{2}\right) = -\frac{1}{4}(e^{-i\pi} - 1) = \boxed{\frac{1}{2}}$$

$$\mathcal{H}\left(\frac{F_s}{2}\right) = \mathcal{H}(\pi) = -\frac{1}{4}(e^{-i2\pi} - 1) = \boxed{\emptyset}$$

2 Frequency Content

(10 points)

Compute the frequency content of a discrete square wave signal $x(n)$ with a period of 4. (so the signal goes $\dots, 1, 1, -1, -1, 1, 1, -1, -1, \dots$.

$$P = 4 \quad x(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$$

$$\begin{aligned} \widehat{x}(\omega) &= \sum x(n) e^{-i\omega n} \\ &= \delta(0)e^{-i\omega 0} + \delta(1-1)e^{-i\omega 1} - \delta(2-2)e^{-i\omega 2} \\ &\quad - \delta(3-3)e^{-i\omega 3} \\ &= 1 + e^{-i\omega} - e^{-2i\omega} - e^{-3i\omega} \end{aligned}$$

4 State Space Equation

(20 points) Give the state space equations (A, B, C, D) for the system that has the frequency response

$$H(\omega) = 1 - e^{-\omega} + e^{-2\omega}$$

$$1/(1,\omega) = 1 - e^{-\omega} + e^{-2\omega}$$

$$H(\omega) = 1 - e^{-\omega} + e^{-2\omega}$$

$$y(n) = x(n) - \underbrace{x(n-1)}_{S_2} - \underbrace{x(n-2)}_{S_1}$$

$$\begin{aligned} S_1 &= x(n-2) \rightarrow S_1(n+1) = x(n-1) = S_2 \\ S_2 &= x(n-1) \rightarrow S_2(n+1) = x(n) \end{aligned}$$

$$S(n+1) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} S(n) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x(n)$$

$$y(n) = (-1 \quad -1) S(n) + (1) x(n)$$

5 Filtering

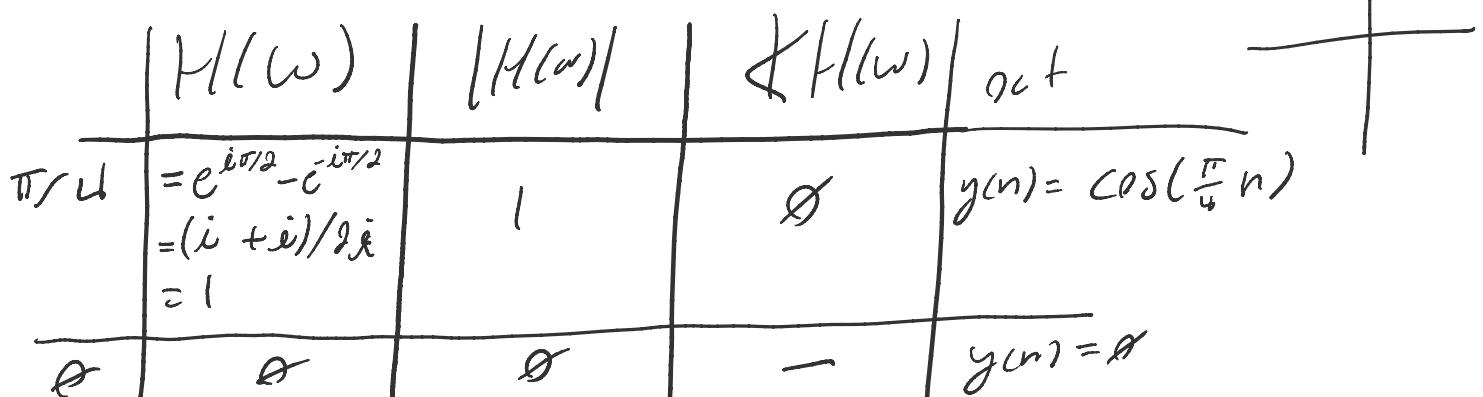
(10 points, 2 each) Given the frequency response

$$H(\omega) = \sin(2\omega)$$

of a discrete system. Compute the output to the input (just giving numbers results in no points)

1. $x(n) = \cos(\frac{\pi}{4}n)$
2. $x(n) = 2$
3. $x(n) = \cos(\frac{3\pi}{4}n)$
4. $x(n) = \cos(\pi n)$
5. $x(n) = 0$

$$H(\omega) = \sin(2\omega) = \frac{1}{2j}(e^{2i\omega} - e^{-2i\omega})$$



θ	θ	θ	-	$y(n) = \theta$
$\frac{3\pi}{4}$	$= e^{\frac{i3\pi}{2}} - e^{-i3\pi/2}$ $= (-i - i)/2i$	$ -1 $	$-\pi$	$y(n) = \cos\left(\frac{3\pi}{4}n - \pi\right)$
π	$= e^{\frac{i2\pi}{2}} - e^{-i2\pi/2}$ $= 0$	0	-	$y(n) = 0$
θ	θ	θ	-	$y(n) = \theta$

7 Short Questions

(10 points, 2 each) For each of the following frequency responses find the corresponding impulse response or system.
(some are discrete some are continuous !)

1. $G(s) = s$
2. $G(z) = z$
3. $H(\omega) = \delta(\omega - \omega_c) + \delta(\omega + \omega_c)$
4. $G(s) = \frac{s}{s+1}$
5. $G(z) = 1 + \frac{1}{1+z^{-1}}$

$$1) \frac{Y}{X} = s$$

$$Y = Xs$$

$$y = x' \quad \cancel{x = e^{j\omega t}}$$

$$H(\omega)e^{j\omega t} = j\omega e^{j\omega t}$$

$$\underline{H(\omega) = j\omega}$$

$$2) \frac{Y}{X} = z$$

$$Y = Xz$$

$$y(n-1) = x(n)$$

$$H(\omega)e^{-j\omega} = 1$$

$$\underline{H(\omega) = 1/e^{-j\omega}}$$

$$4) G = \frac{s}{s+1}$$

$$YS + Y = Xs$$

$$y' + y = x'$$

$$5) \frac{Y}{X} = 1 + \frac{1}{1+z^{-1}}$$

$$= \frac{1+z^{-1}+1}{1+z^{-1}}$$

$$2+z^{-1}$$

$$Y_S + Y = X_S$$

$$y' + y = x'$$

$$iwH e^{j\omega t} + H e^{-j\omega t} = e^{j\omega t} \sin w$$

$$\begin{aligned} H(1+iw) &= iw \\ H &= \frac{iw}{1+iw} \end{aligned}$$

$$\begin{aligned} &= \frac{1 - z^{-1}}{1 + z^{-1}} \\ &= \frac{2 + z^{-1}}{1 + z^{-1}} \end{aligned}$$

$$Y + Yz^{-1} = 2X + Xz^{-1}$$

$$y(n) = 2x(n) + x(n-1) + y(n-1)$$

$$H(\omega) = 2 + e^{-j\omega} + H(\omega)e^{-j\omega}$$

$$H(\omega) = \frac{2 + e^{-j\omega}}{1 + e^{-j\omega}}$$

$$3) h(t) = \frac{1}{2\pi} \int H(\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \left(\int \delta(\omega - \omega_c) e^{j\omega t} + \int \delta(\omega + \omega_c) e^{j\omega t} \right)$$

~~if~~ $\int f(\omega) \delta(\omega - \alpha) d\omega$

$$= \frac{1}{2\pi} (e^{-j\omega_c t} + e^{j\omega_c t})$$

$$= \frac{1}{\pi} \cos(\omega_c t)$$

8 Stability

(10 points, 5 each) For the two systems below determine α, β so they are stable:

$$1. y' + \alpha y + x' + x = 0$$

$$2. y(n+1) = x(n) - \beta y(n-1) + x(n-1)$$

$$1) Y_S + \alpha Y = -X_S - X$$

$$Y(s-\alpha) = X(-s-1)$$

$$G = \frac{-s-1}{s-\alpha} = \frac{s+1}{\alpha-s}$$

$$\alpha - s = 0$$

$$\overbrace{s}^{\approx 0} = \alpha$$

$$\begin{array}{c} \text{unstable} \\ S = a \\ \boxed{|\alpha| < 0} \end{array}$$

$$2) y(n+1) = x(n) - \beta y(n-1) + x(n-1)$$

$$Yz = X - \beta Yz^{-1} + Xz^{-1}$$

$$Y(z + \beta z^{-1}) = X(1 - z^{-1})$$

$$G = \frac{1 - z^{-1}}{z + \beta z^{-1}}$$

$$\begin{aligned} \phi &= z + \beta z^{-1} & | > |z| \\ z^2 &= -\beta & | > |\sqrt{-\beta}| \\ z &= \sqrt{-\beta} & | > |\beta| \\ && -1 < \beta < 1 \end{aligned}$$