

Mechatronic Engineering: The synergistic integration of elements of mechanical, electrical, control and computer engineering.

through the design

and manufacturing

process

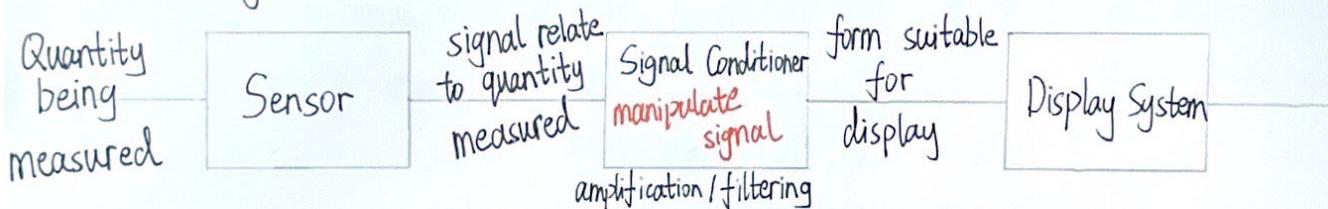
benefit: shorter development cycles, lower costs, increased quality, increased reliability, increased performance and increased benefits to the customer.

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Mechatronic Design: One that uses all the elements of mechatronic design engineering to achieve the design objectives and/or a design that is some way optimal.

Mechatronics involves "systems": one or more input and one or more output interested in relationship between input & output

Measurement System:



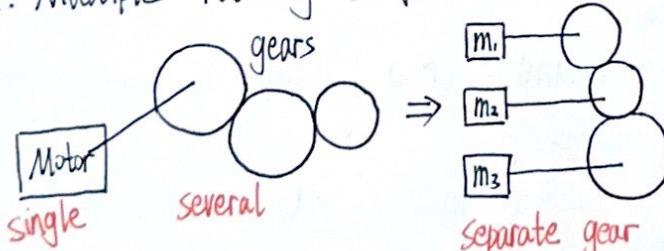
In the past, sequential control was implemented using cam-operated switches

Today, Programmable Logic Controller (PLC) is used.

Computer Simulation: based on mathematical models, allow preliminary mechatronic designs to be tried out quickly, inexpensively and safely

Mechatronic Systems:

1. Multiple rotating shafts



removing gears reduces cost

removing gears eliminates wear and backlash

failure of a single motor has lower impact

2. Integrated sensor-controller-amplifier-actuator



only power and network signal

one Master controller, other follow

no additional hardware

no extensive wiring

3. Hollow integrated motor-gearbox

PowerHub { DC motor
bearing
harmonic drive

twisting of
the cables

⇒ make centre hollow

4. Computer hard disk drive

read/write arm } reduce inertia, allowing acceleration and velocity of arm
motor } to be increased (reduce seek time)

5. Electronic bathroom scale

Weight → mechanical mechanism → electromechanical force sensor

6. Artificial hand for small child.

electric motor is used only when hand must be opened to grip an object
spring force is used to close the hand and perform gripping action.

{ Distributed Control: multiple microprocessors used in mechatronic systems
common controller greater reliability (less wiring, failure of one μP is less critical)
still required improved performance (computing done in parallel, less data
less expensive transmission delay)

Centralized Control: signal travel through relative long cables ⇒ easy for failure
by insulation wear and fracture and wire fatigue.

Control System: keep its output as close as possible to its reference input
(setpoint)
cable between controller and actuator fails ⇒ spring-loaded brake

cable between sensor and controller fails:

closed-loop { 1. A change in the process dynamics or a disturbance causes the
is better output to change
2. The changes are small enough so that closed-loop is stable.
3. The controller receives correct signal from the sensor

If wrong signal sent from sensor, open-loop controller will be unaffected

Sensor: A device that produces a signal related to the quantity measured.
input is the quantity; output is a voltage

① Static performance: steady-state ② Dynamic performance: varies over time

Range: The limits between which the quantity being measured can vary
e.g. thermometer range: -20°C to 100°C

Full Scale: Difference between maximum and minimum values that the sensor can measure. $\text{Quantity}_{\max} - \text{Quantity}_{\min}$

Resolution: Smallest change in the quantity being measured that is detectable by the sensor. Easy to measure but do not truly indicate quality.

Repeatability: The ability of a sensor to produce the same output for repeated applications of the same input, also called precision,
 $= \pm 3\sigma_y$, σ_y is standard deviation to 99.7% confidence interval
Random error normally distributed

Accuracy: The maximum extent the output of the sensor may be incorrect over its range. include both random and deterministic error

$$\text{Accuracy} = \pm (\max(\text{abs}(Y_{\text{true}} - Y_{\text{sensor}})) + 3\sigma_y)$$

large Y_{true} : True value measured, Y_{sensor} mean value of calibrated output

Sensitivity: Ratio of the magnitude of the sensor output to the magnitude of the quantity being measured.

Small Cross-sensitivity: With directional sensors, the sensitivity of the sensor output to inputs orthogonal to the measurement direction.

Deadband: The band of input values for which there is zero output.

Linearity: The maximum error between the mean values of the sensors outputs and the true value being measured.

$$\text{Linearity} = \pm \max(\text{abs}(Y_{\text{true}} - Y_{\text{sensor}}))$$

Hysteresis: The **maximum** difference between the calibrated sensor outputs for continuously increasing and decreasing inputs.



Response Time: The time the output takes to reach 95% of the input.

Rise Time: Time required for the output to increase from 10% to 90% of its steady-state value.

Settling Time: The time required for the output to settle within $\pm 1\%$ of its steady-state value.

Time Constant: The time required for the output to reach 63.2% of its steady-state

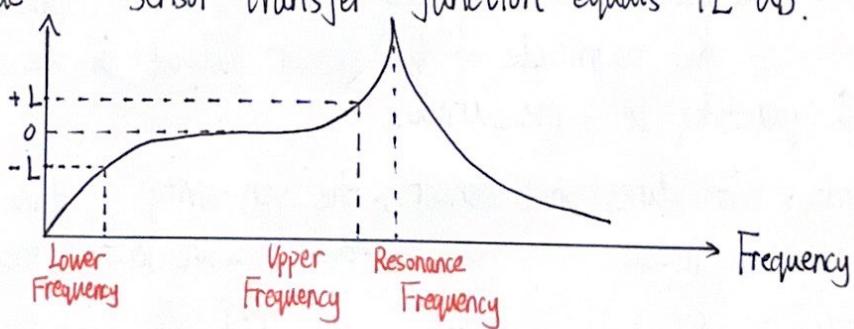
Stability: The ability of a sensor to produce the same output when measuring a constant input over an extended period of time.

drift: a change in output under these conditions.

Bandwidth: The **lowest** frequency at which the magnitude of the sensor transfer function drops by 3dB.

{ Lower frequency limit: The **lowest** frequency at which the magnitude of the sensor transfer function equals $-L$ dB.

Upper frequency limit: The **lowest** frequency at which the magnitude of the sensor transfer function equals $+L$ dB.



Sensor Calibration :

1. Applying a constant input at the start of the range of the sensor.
2. Measuring the sensor output multiple times (at least 100)
3. Calculating the mean value and standard deviation of the measured outputs
4. Increasing the input and hold it constant
5. Repeat 2 and 3. 6) Repeat 4 and 5 until input is at end of range
- 7) Fitting a calibration line to mean values of output (measured), using linear regression $\Rightarrow y = A x \rightarrow$ independent

$$A = \frac{\sum xy}{\sum x^2}$$

$$A = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

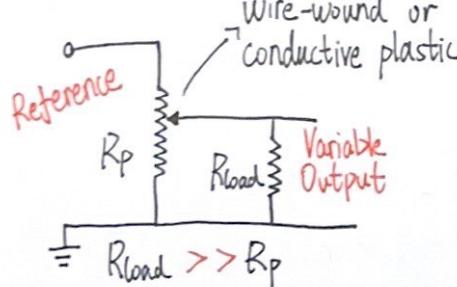
$$B = \frac{\sum y - A \sum x}{n}$$

$$\text{For } y = Ax \quad Y_{\text{sensor}} = \frac{Y_{\text{volt}}}{A}$$

$$\text{For } y = Ax + B \quad Y_{\text{sensor}} = \frac{Y_{\text{volt}} - B}{A}$$

Displacement Sensors

1. Potentiometer : a resistance element with sliding contact that can move

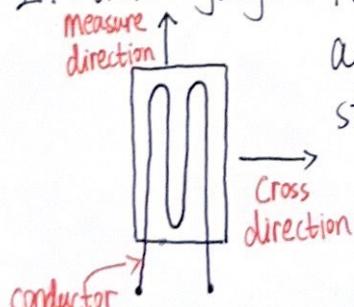


Wire-wound or conductive plastic
Advantage: not temperature sensitive
Disadvantage: limited resolution (wire diameter)
plastic resistor
Advantage: unlimited resolution
Disadvantage: temperature sensitive

Advantage: Low cost, Large range

Disadvantage: Output affected by load resistance, analog out, sensor is affected by dirt and wire

2. Strain gauge : A wire, metal foil or semiconductor strip that can be stuck onto a surface to measure its strain. The change in resistance due to strain can be converted into a voltage.



tiny change in resistance $\frac{\Delta R}{R} = G E$ - strain
voltage
strain factor
positive: tensile
negative: compressive
flexible elements

Small range (less than 1m) \Rightarrow Cantilever, Ring Shaped, V shaped

Temperature Compensation: very sensitive to changes in temperature.

① The temperature of the sensor must be kept **constant**

② Mount a pair of sensors on **opposite** side → best

approach

Advantage: no wear

Disadvantage: **small** range unless mechanical element is added, **analog** output
temperature sensitive

3. **Encoders:** used in **motion control** systems of robots and machine tools.
produce **digital** output for angular (or linear) displacement.

	common	optical	magnetic	capacitive	inductive
Incremental		✓	✓	✓	✓
Absolute		✓	✓		

Incremental Encoder: Output is a measure of **relative** displacement

light → [Diagram] light sensor
rotating: blocked / unblocked ⇒ OFF / ON

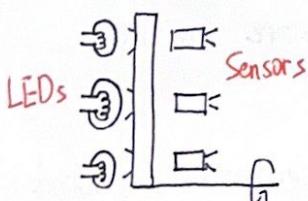
absolute displacement: Channel A and Channel B

[Diagram] half slot width

ON OFF determine the rotation direction.

improve resolution: **quadrature counting**

Absolute Encoder: each position gives a **unique** output (binary number)



problem: range is **limited** to one revolution.

use an incremental encoder with a counter

Advantage: no wear, digital output is relatively **insensitive** to noise and **easy** to interface

Disadvantage: **limited** resolution, susceptibility to **large** vibrations

Digital Sensor

Advantage: Noise Immunity

Analog Sensor

Advantage: potentially **infinite** resolution

Disadvantage: **Limited** resolution

Disadvantage: **sensitive** to noise

Few purely sensor exists

digitize to analog : often transmit digitally: **long** distance (increases noise)

4. Hall Effect Sensor : measure the **flux density** of a magnet

analog output: linear or employ a threshold → non-contact proximity switch

↳ automotive application

Advantage: non-contact (no wear), **immune** to the working environment

Velocity Sensors

1. By differentiating displacement or position

Advantage: **low** cost

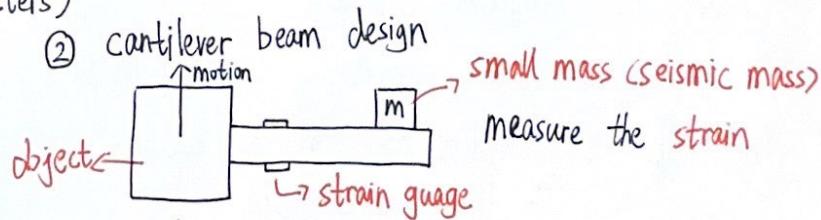
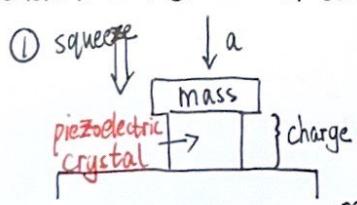
Disadvantage: Amplifies **high** frequency noise, numerical differentiation adds **time delay**

2. Tachogenerator or Tachometer : produces an **analog** voltage that is **proportional** to the velocity

motor as generator, brush / brushless, not used so much

Disadvantage: brushes create **noise**, are subject to wear.

Acceleration Sensors (Accelerometers)



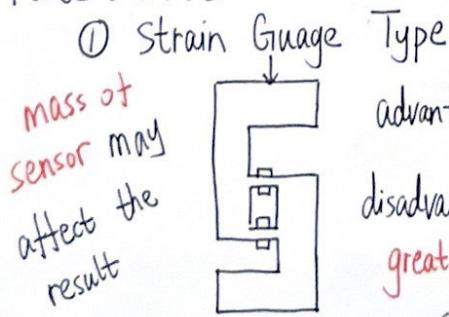
$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Larger $\frac{m}{k}$, Larger sensitivity of the sensor.
will limit the bandwidth.

Advantage: simpler to use than displacement and velocity sensors, no wear

Multiple axis (X-Y-Z) and angular acceleration sensors are available

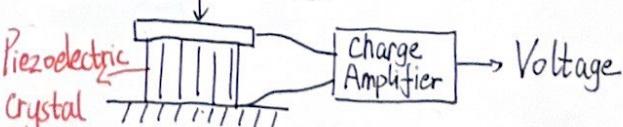
Force Sensors



advantage: Low cost

disadvantage: Worse linearity
greater temperature sensitivity.

② Piezoelectric Type



Advantage: Better linearity and temp sensitivity
Disadvantage: Higher cost, amplifier must be designed carefully to prevent stability problems

Design issue: piezoelectric avoids problem with cross-sensitivity
flexible elements must be designed to avoid cross-sensitivity
Both need to be as stiff as possible to maximize natural sensitivity

Pressure Sensors

Employing a tiny diaphragm incorporates strain gauges to measure deflection when pressure is applied.

Temperature Sensors

1. Bimetallic Strips: two metal strip bonded back to back having different coefficient of thermal expansion. The different expansions cause strip to bend. Used as temperature controlled switch



Issue: fatigue of the metals.

2. Resistance Temperature Detectors: consists coil of wire, resistance changes when temperature changes

Advantage: simplicity, linearity

Disadvantage: small sensitivity to temperature.

Small temperature sensor

- 1. measure very locally \Rightarrow temp distribution
- 2. heat up faster \Rightarrow fast response time
- 3. fit in tight location

3. Thermistors: mixture of metal oxides, the resistance is very sensitive to the temperature.

Advantage: can be easily formed into different shapes, large sensitivity, can be very small

Disadvantage: Very non-linear response so special calibration required.

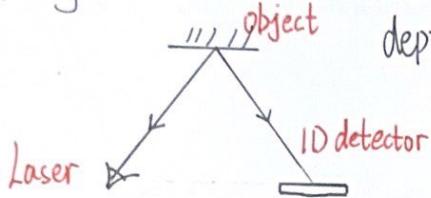
4. Thermocouples: traditional temperature sensors but less popular. Small voltage produced when two metals joined, function of metals and temperature

Advantage: Large range, Excellent linearity

Disadvantage: Very small sensitivity, small output makes it very susceptible to noise, Special electronics required

Depth Sensors (range-sensing)

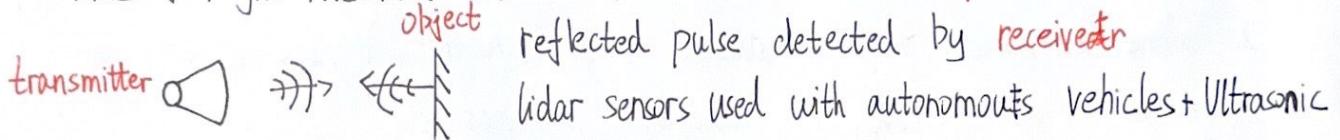
1. **Triangulation Method:** Laser and 1D detector form a triangle



depth determined by geometry

one of ways humans and animals detect the depth

2. **Time of Flight Method:** based on $\text{distance travelled} = \text{speed} \times \text{time}$



Advantage: Greater range, not affected by optical properties of the object, transmitter can be located close to receiver without affecting performance, not affected by dirt, lower cost.

Disadvantage: Worse accuracy, susceptible to indirect reflections.

Proximity Sensors

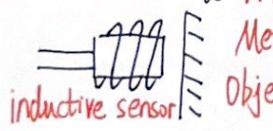
used to detect when or whether a location is within a specified proximity

1. **Microswitch:** spring-wedged mechanical switch.



Disadvantage: Object sensed must come into contact and apply small force. Dynamics of spring and mass can cause false outputs. Subject to wear.

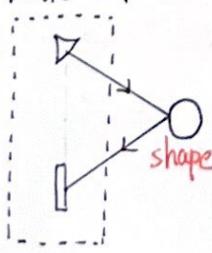
2. **Inductive:** used change in magnetic circuit formed by coil, detect presence of a metal object.



Disadvantage: Can detect presence of metal objects only. Small range, require power.

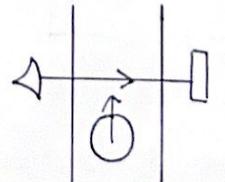
3. **Photoelectric:** consists of light and light detector (phototransistor)

Retro-reflective



Contained within one sensor
not require separate mounting
and alignment
smaller range, greater sensitivity
to optical properties

Thru-beam



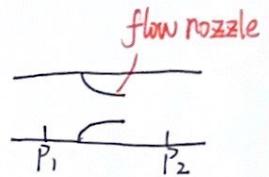
Two parts aligned

Advantage: Both types have a much **larger** range

Disadvantage: Suitable for **opaque** objects only, sensitive to **dirt**, sensitive to **ambient light**, requires power.

Flow Rate Sensors

1. Flow Nozzle: restricts the flow causing a **pressure drop** ($P_1 - P_2$)



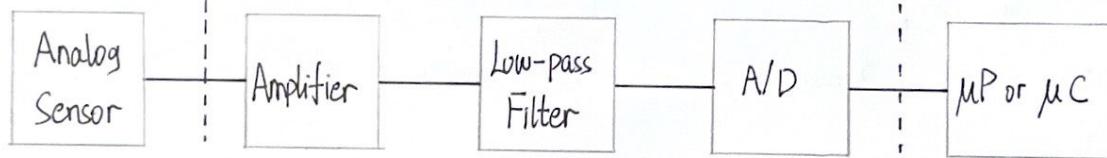
2. Turbine Flow Meter: propeller attached to a tachogenerator, less pressure drop

Advantage: **Less** expensive, **Less** susceptible to wear.

Disadvantage: **Less** accurate

Sensor Interfacing

Signal Conditioning System



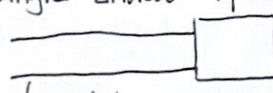
increase voltage
to suitable value

reduce amplitude of
high frequency noise

analog → digital

long cable
more negative effect

Single-Ended Inputs

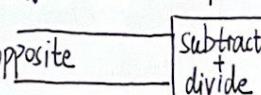


$$\frac{0.3V}{\downarrow} \Rightarrow \text{noise} \Rightarrow \frac{0.5V}{\uparrow} \Rightarrow \text{Input } 0.8V.$$

relative common ground noise added

send **three signals**, need four lines

Differential Inputs

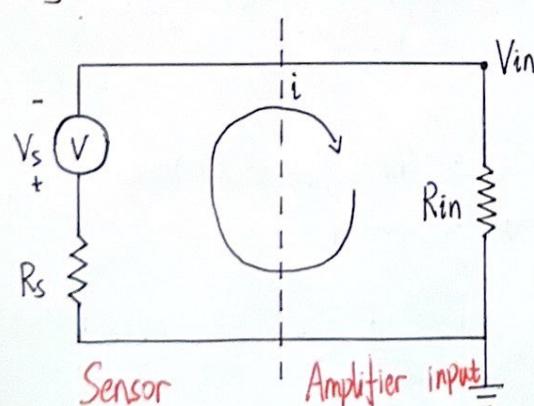


$$\frac{+0.3V}{\uparrow} \Rightarrow \text{noise} \Rightarrow \frac{-0.3V}{\downarrow} \Rightarrow \frac{0.35 - (-0.25)}{2} = 0.3V$$

greatly reduce common noise **Better**
used analog and digital

Differential disadvantage: added **cost** (more complex input circuitry & extra wire needed)

Loading Effect



Sensor gives **output voltage**, received by Amplifier

$$V_{in} = V_s \left(\frac{R_{in}}{R_{in} + R_s} \right) \quad \text{ideally want } \approx 1$$

Each hardware element must have a **small output impedance** and **large input impedance** for loading effects to be **minimized**

happen between any **connections** of input & output

Error of resolution of an ADC

$$\text{ADC accuracy in volts} \leftarrow A_{\text{ADC}} = \pm \frac{V_{\text{FS}}}{2^{\text{ENOB}}} \rightarrow \begin{array}{l} \text{ADC's full scale Voltage} \\ \text{ADC's Effective number of bits} \end{array}$$

Propagation of Error

When uncertain quantities are multiplied or divided, the total uncertainty can be calculated by summing relative uncertainties.

if $R = \frac{XYZ}{P}$ $\Rightarrow \Delta R = |R| \left(\left| \frac{\Delta X}{X} \right| + \left| \frac{\Delta Y}{Y} \right| + \left| \frac{\Delta Z}{Z} \right| + \left| \frac{\Delta P}{P} \right| \right)$

uncertainty of R Relative uncertainties of each element

When uncertain quantities are added or subtracted, the total uncertainty can be calculated by summing absolute uncertainties

if $R = X + Y - Z \Rightarrow \Delta R = |\Delta X| + |\Delta Y| + |\Delta Z|$

Errors Due to Time Varying Inputs

approximate the measurement system's transfer function first order

$$\frac{Y_{\text{out}}(s)}{Y_{\text{true}}(s)} = \frac{K_s}{\tau_s s + 1} \quad \begin{array}{l} \text{steady state gain} \\ \text{usually 1} \end{array} \quad \tau_s = 0.455 t_r \rightarrow \text{rise time}$$

dominant time constant

$$= \frac{1}{W_b} = \frac{1}{2\pi f_b} \rightarrow \text{bandwidth}$$

A step input applied at $t=0$

$$y_{\text{out}}(t) = y_{\text{out}(0)} e^{-\frac{t}{\tau_s}} + K_s (1 - e^{-\frac{t}{\tau_s}}) y_{\text{true}}$$

How long should

system wait to read output $\leftarrow t \geq -\tau_s \ln \left(\frac{0.1 |a_y|}{\max(y_{\text{max}} - y_{\text{out}(0)}, y_{\text{out}(0)} - y_{\text{min}})} \right)$ Rule of thumb: Wait until the effect of the transient on y_{out} drops to $\frac{1}{10}$ of accuracy

Worst case error $\Delta y_{\text{out}}(t) = |a_y| + \max(y_{\text{max}} - y_{\text{out}(0)}, y_{\text{out}(0)} - y_{\text{min}}) e^{-\frac{t}{\tau_s}}$

An input consisting one or more sinusoids

$$y_{\text{true}}(t) = A_{\text{true}} \sin(\omega t) \Rightarrow y_{\text{out}}(t) = A_{\text{out}}(t) \sin(\omega t + \phi(\omega))$$

Input

$$= K_s M(\omega) A_{\text{out}} \sin(\omega t + \phi(\omega))$$

$$M(\omega) = \frac{1}{\sqrt{1+\omega^2 z_s^2}} \text{ magnitude} \quad \phi(\omega) = -\tan^{-1}(\omega z_s) \text{ phase shift in radians} = \frac{\text{error in phase}}{\text{input & output}}$$

$$t_d = -\frac{\phi}{\omega} \text{ time delay due to shift} \quad \Delta A_{out}(\omega) = |A_y| + (\sqrt{1+\omega^2 z_s^2} - 1) A_{out}(\omega) \text{ Worst case error}$$

Error Caused by Numerical Differentiation

$$\text{estimate velocity } V_{est}(kT) = \frac{p(kT) - p(k-1)T}{T} = \frac{p_{true}(kT) - p_{true}(k-1)T}{T} + \frac{e_p(kT) - e_p(k-1)T}{T} \xrightarrow{\text{measurement error form of amplified noise}}$$

Worst case velocity error $\Delta V_{est} = \frac{T}{2} \max(|\dot{a}_{true}|) + \frac{2\Delta P}{T}$ Smaller T \Rightarrow smaller truncation + larger amplification of position measurement noise
 phase lag \leftarrow truncation backward differencing

$$T_{opt} = \sqrt{\frac{4\Delta P}{\max(|\dot{a}_{true}|)}} \xrightarrow{\text{position}}$$

$$\Delta V_{est} = \frac{T}{2} \max(|\dot{a}_{true}|) + \frac{2\Delta P}{T} \xrightarrow{\substack{\text{sensed by an} \\ \text{incremental encoder}}}$$

$$= \frac{T}{2} \max(|\dot{a}_{true}|) + \frac{\text{encoder position resolution}}{T}$$

Optimal sampling period value

need to consider the frequency of noise

Sensed by a position sensor with analog output

$$99.7\% \text{ conf } \Delta V_{est} = \frac{T}{2} \max(|\dot{a}_{true}|) + \frac{6\Delta P}{T} \xrightarrow{\substack{\text{standard deviation}}}$$

Actuator: a device that converts energy into a force or torque that produces motion.

e.g. motor applies torque to robot, furnace heats the air

Mechanism: commonly used in mechatronic designs to transform motion or to provide a mechanical advantage

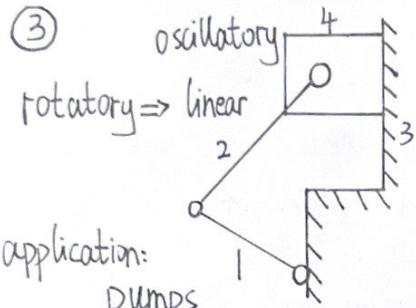
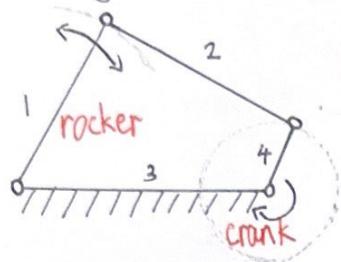
actuator $\xrightarrow{\text{input}}$ mechanism $\xrightarrow{\text{larger force or torque}}$ increase in force \Rightarrow decrease in speed

link: a rigid body used to connect two joints

crank: a link that is capable of making full rotation

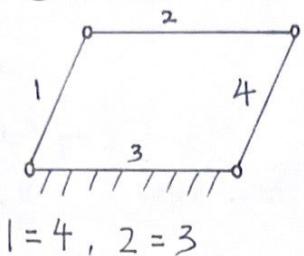
four-bar mechanisms ① crank-rocker ② double-crank ③ slider-crank

① rotatory \Rightarrow oscillatory



① crank-rocker ② double-crank ③ slider-crank

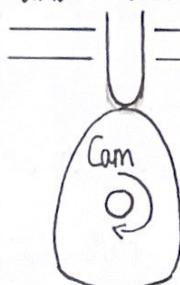
② double-crank



all in parallel

inputs and outputs links will both rotate fully and at the same speed

Cam and Cam follower



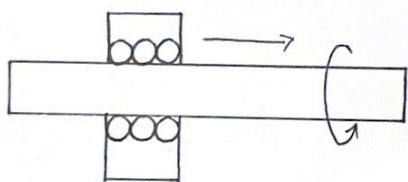
rotatory \Rightarrow oscillatory motion

shape of cam can be used to produce different output motion

Mechanisms for Conversion of Rotatory to linear and linear to Rotatory

1. Linear screws and ball screws

a combination of a precisely made screw and nut (prevented from rotating)



ball bearing: convert sliding contact between screw and nut into rolling contact

reduce wear and power loss to friction

Advantage: Ball screws have very good accuracy and large motion range

Disadvantage: High cost, typically cannot be used to convert linear to rotatory

Example Application: CNC machine tools and robots

$$\tau = \frac{FL}{(2\pi/\text{rev})\eta s}$$

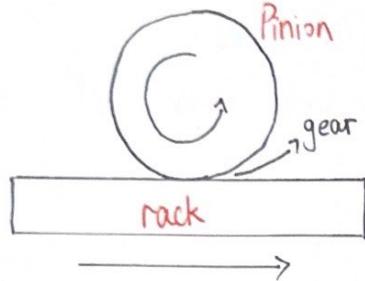
load
input torque $= 1$ if no friction
amount of linear motion per revolution (lead)
efficiency of the screw

Assume: Torque to accelerate the screw is negligible

$$J = M \left(\frac{l}{(2\pi/\text{rev})} \right)^2$$

mass
equivalent rotational inertia
lead

2. Rack and Pinion



if **rack** attached to actuator

$$\tau_{\text{out}} = F_{\text{in}} r_p \eta_{rp}$$

linear motion per revolution = $(2\pi/\text{rev}) r_p$ → radius of the pitch circle

$$F = \frac{\tau_{\text{in}}}{r_p} \eta_{rp} \rightarrow \begin{matrix} \text{force} & \leftarrow \\ \text{output} & \end{matrix} \quad \begin{matrix} \text{torque} \\ \downarrow \end{matrix} \quad \begin{matrix} \text{efficiency}, = 1 \text{ if no friction} \\ \downarrow \end{matrix}$$

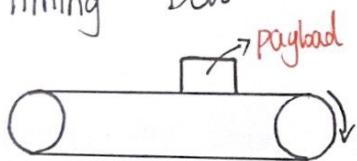
$$J = M r_p^2$$

inertia ↓ mass

Advantage: Large motion range, less expensive than ball screw. Can be used to convert rotatory to linear motion and vice-versa.

Disadvantage: Less accurate than ball screw due to wear and backlash.

3. Timing Belt



belt with teeth, prevent from slipping

typically one of pulleys is driven by a rotatory actuator

Advantage: Low cost solution

Disadvantage: Only suitable for driving small loads due to the flexibility of the belt. The rubber belt teeth are sensitive to wear.
Less accurate than ball screw and pinion

Example Application: Photo copiers and ~~desktop~~ desktop scanners

4. Cam and Cam follower

Advantage: Compact. Suitable for very high speed motion

Disadvantage: Small motion range. Cannot be used to convert linear to rotatory motion

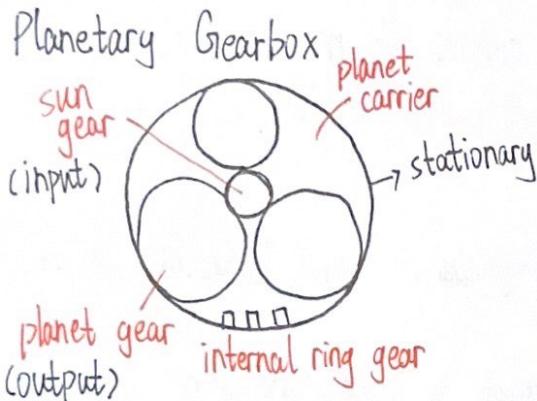
Example Application: Used in some robot grippers to open / close the jaws

Electric motor: the **most common** actuators used in mechatronic designs.

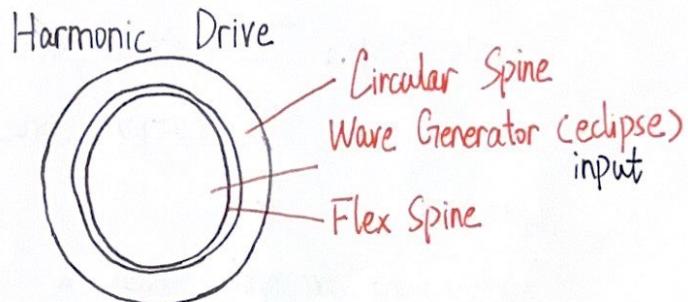
Do not produce high torques at low speeds

compact, low weight

gearbox: most common solution ① Planetary gearbox ② harmonic drive reducers
low inertia



Disadvantage: gear backlash
introduces position errors



Does not suffer from backlash

But the **flexibility of the flex spine** does lead to position error.

Gearbox Equation

$$\omega_{out} = \frac{1}{N_r} \omega_{in}$$

angular velocity

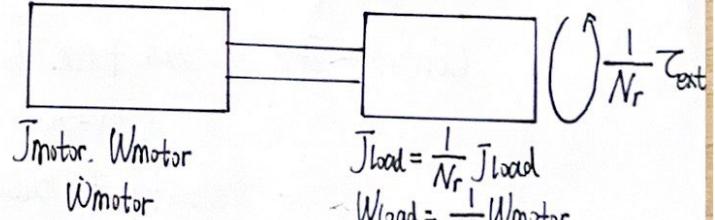
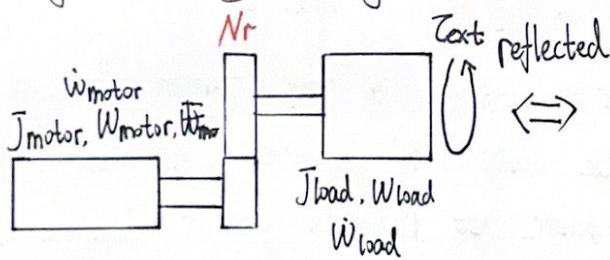
$$\dot{\omega}_{out} = \frac{1}{N_r} \dot{\omega}_{in}$$

angular acceleration

$$T_{out} = N_r T_{in} \eta_g$$

torque

N_r : gear ratio
 η : efficiency



$$T_{reflected} = \frac{1}{N_r} J_{load} \dot{\omega}_{load} + \frac{1}{N_r} T_{ext} = \frac{1}{N_r} J_{load} \frac{1}{N_r} \dot{\omega}_{motor} + \frac{1}{N_r} T_{external}$$

$$= \frac{1}{N_r^2} J_{load} \dot{\omega}_{motor} + \frac{1}{N_r} T_{ext}$$

$$J_{motor} = J_{motor} \dot{\omega}_{motor} + T_{reflected}$$

$$= J_{motor} \dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load} \dot{\omega}_{motor} + \frac{1}{N_r} T_{ext}$$

Electrical Actuators

1. Solenoids: consists of a coil and a soft iron core. When current flows through the coil the core is pulled inwards.

Advantage: Fast response, simple design and do not wear

Disadvantage: The output force and range of motion are small.
The output force is ~~unidirectional~~ undirectional

Control: on / off

Application: Commonly used to actuate pneumatic and hydraulic valves.

2. Voice Coils: consist of a permanent magnet, a stationary iron core and a movable coil. The iron core intensifies the magnetic field produced by the PM.

Advantage: Bidirectional force output, fast response, simple design, compact and do not wear

Disadvantage: Small motion range, Higher cost due to PM

Control: The output force is proportional to the current through the coil so open-loop force control is possible. Continuous control of velocity is also possible but requires sensor feedback.

Application: Positioning of read/write head of hard disk, control of the spool position with hydraulic or pneumatic servo valves.

3. DC Motors: PM DC brush & PM DC brushless motors

in the past rotor windings needed to generate magnetic field strength nowadays replace windings with PMs, eliminates heat generated by rotor windings

stationary **stator** (PMs), rotating **rotor** (several coils)

commutator: uses **split** ring and metal or graphite brushes to switch direction.

Brush:

Advantage: Relatively easy to control and interface

Disadvantage: Relatively low torque output, the brushes **wear**, cause **friction losses**, and can ~~cause~~ cause **arcing** \Rightarrow create electric noise and can be a safety problem in some application.

Brushless: rotor (permanent magnets) stator (~~several~~ several coils)

Advantage: ~~Relatively~~ Relatively easy to control and interface, No problems due to brushes, greater reliability, better communication, can produce higher continuous torque, better heat dissipation

Disadvantage: More expensive, more difficult to produce constant torque output at high speeds increased momentum less sensitive to torque ripple

Control: easy to control because output torque is proportional to input current for a constant load torque, speed is roughly proportional to input voltage
Continuous control of torque, velocity and position is possible.

Application: positioning in numerous mechatronic devices, such as robots, machine tools, and video cameras

Mathematical Modelling

$$V_a = K_b W + L_a \frac{di}{dt} + R_a i_a$$

V_a : armature voltage R_a : armature resistance

K_b : back emf constant i_a : armature current

L_a : armature inductance

$$J \frac{d\omega}{dt} = K_t i_a - K_d W - T_{load}$$

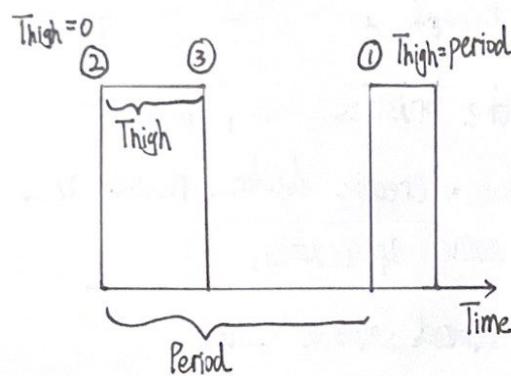
J : moment of inertia of the motor

T_{load} : torque due to load

K_d : damping constant

K_t : torque constant

Position control of PM DC motor is only possible using **Closed-loop control**.
incremental encoder is the most common source of position feedback
control algorithm executed on μ C $\xrightarrow{\text{output}}$ Pulse - Width - Modulated signal



- ① output voltage = V_{high}
- ② output voltage = V_{low}
- ③ $V_{\text{out}} = V_{\text{average}} + \text{High frequency signal}$
 $= \frac{\text{Thigh}V_{\text{high}} + (\text{Period} - \text{Thigh})V_{\text{low}}}{\text{Period}} + \text{High frequency signal}$
filter out **high frequency component**

Advantage: **Inexpensive**. May be used to switch the transistors in a "switching amplifier"
Switching transistors on/off is **more energy efficient** than in linear region

Disadvantage: high frequency component is actually **high frequency noise**. can be picked up by other elements and affect control performance

4. Linear Motors: DC, AC or stepper motor whose stator has been **unwrapped** into a linear form.

Advantage: Capable of achieving **very high** acceleration and range of speeds
Can be made **no contact**, can be **2-D**, **no backlash**, **smaller size**,
Better reliability

Disadvantage: **High cost**, **Sensitive** to dirt, to increase range of motion the **size should be increased** \Rightarrow increased cost, **no mechanical advantage** for ball screw (or other mechanism) to help provide force

5. Stepper Motor: moves in small angular increments

① PM Stepper Motor
accomplished by electronically switching the **polarity** of the stator windings.

Full stepping mode: reverse the polarity of a pair of poles. step angle = 90°

Half stepping mode: one of pair is shutting off. step angle = 45°

disadvantage: average torque output is reduced.

Microstepping: position is incremented by decreasing the current in one pair.

disadvantage: more complex and expensive controller required.

② VR Stepper Motor

rotor and stator are cut with small grooves to form teeth
rotor moves the position of lowest reluctance.

Step angle = $360^\circ / \text{rotor teeth} * \text{num of pair of poles (phase)}$

③ Hybrid stepper Motor

combines the best features of the two above

Advantage: less expensive, less complex, no potential for closed-loop instability

Disadvantage: less torque than DC motor of small size, care must be taken so that controller and motor remain synchronised, small acceleration and payloads, no useful in presence of large disturbance, less smooth & low efficiency

Control: performed open-loop, control position, velocity and acceleration

Application: Any positioning application that does not involve large loads or accelerations

Under steady state, $\frac{dw}{dt} = 0, \frac{dia}{dt} = 0$, no friction

$$V_a = K_b W + R_a i_a \quad \begin{matrix} \nearrow \text{input} \\ \text{load} = K_t i_a - K_d W \end{matrix}$$

$$\eta_{\text{motor}} = \frac{\text{mechanical output}}{\text{electrical input}} = \frac{T_{\text{load}} W}{V_a i_a} = \frac{K_t i_a W}{K_b W i_a + R_a i_a^2}$$

$$\eta_{\text{motor+gear}} = \eta_{\text{motor}} \eta_g \quad \begin{matrix} \nearrow \text{max speed} \\ = \frac{K_t i_a W}{K_b W i_a + R_a i_a^2} \eta_g \end{matrix}$$

$$N_{r,\text{opt}} = \sqrt{\frac{J_{\text{load}}}{J_{\text{motor}}}}$$

closest smaller available gear ratio should selected

J_r will be maximized when gear ratio is above

$$\text{Ratio}_g = \frac{T_{\text{load}} / N_r^2}{J_{\text{motor}}} \quad \begin{matrix} \parallel \\ \text{I will maximize power} \end{matrix}$$

inertia matching: keep inertia ratio as close to 1 as possible, with 1 to 10. Larger than 10 will be unstable.

$$T_{\text{motor,RMS}} = \sqrt{\frac{\sum_{i=1}^n T_{\text{motor},i}^2 t_i}{\sum_{i=1}^n t_i}}$$

$T_{\text{motor},i}$: the torque for the i^{th} period

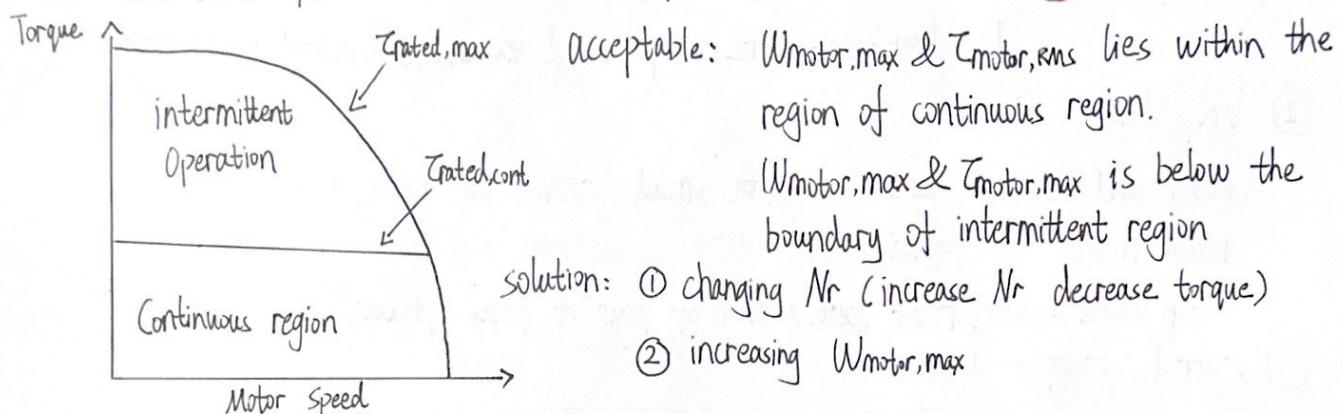
t_i : duration of the i^{th} period

n : number of periods

thermally damaged if work too long

a motor can operate at its maximum rated torque $T_{\text{rated,max}}$ **intermittently**

motor can also produce a smaller torque $T_{\text{rated,cont}}$ **continuously**



Selection procedure for a DC motor and gearbox

1. Calculate desired motion profiles if not provided
2. Based on the load, motor data and ~~mechanisms~~ mechanisms used, select **gear ratio** maximizes acceleration, or let inertia ratio closest to 1.
3. Based on velocity profile, find W_{max} . Compare $W_{\text{motor,max}} = N_r W_{\text{max}}$ to $W_{\text{rated,max}}$. Until a suitable one is found.
4. Based on the material provided, calculate $T_{\text{motor,max}}$.
5. Check if $T_{\text{motor,max}} < T_{\text{rated,cont}}$
6. Compute $T_{\text{motor,RMS}}$. Check if $T_{\text{motor,RMS}} < T_{\text{rated,cont}}$
7. Select a higher gear ratio if possible, or select a more powerful motor.
8. Check if temp rise is acceptable. If not then try increase gear ratio. If no sufficient then either try additional cooling, or select a more powerful motor.

$$P_j = I^2 R_{\text{Hot}} \quad \begin{matrix} \text{motor thermal resistance} \\ \text{increases as the temperature increases} \end{matrix}$$

Power loss current typically gives at 25°C

Copper winding resistance:

$$R_{Hot} = R_{25} (1 + 0.00392(T_{Hot} - 25))$$

desired operating temperature

Thermal resistance $R_{th} = R_{th1} + R_{th2}$ — indicator of heat transfer from housing to ambient

↳ characterizes heat transfer from windings to housing

Winding Temperature:

$$T_w(t) = T_{initial} + (P_j R_{th} + T_a - T_{initial})(1 - e^{-\frac{t}{T_w}})$$

\downarrow amount of time to reach 63.2% of steady state temp
 \downarrow ambient temp

if t is greater than T_{tw} , can be simplified: $T_w = T_a + P_j R_{th}$

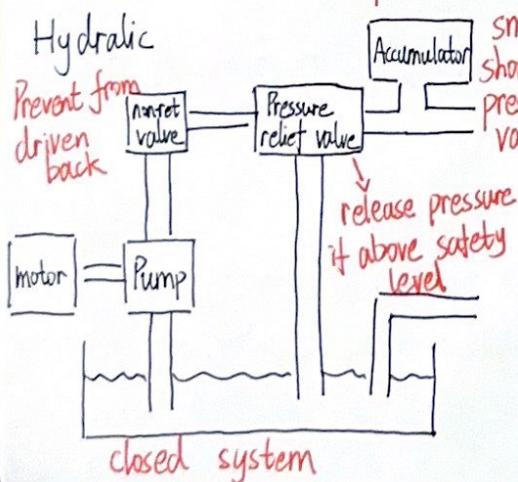
if motor is turned off when $T_w > T_a$, the temperature falls:

$$T_w(t) = T_{initial} + (T_a - T_{initial})(1 - e^{-\frac{t}{T_w}})$$

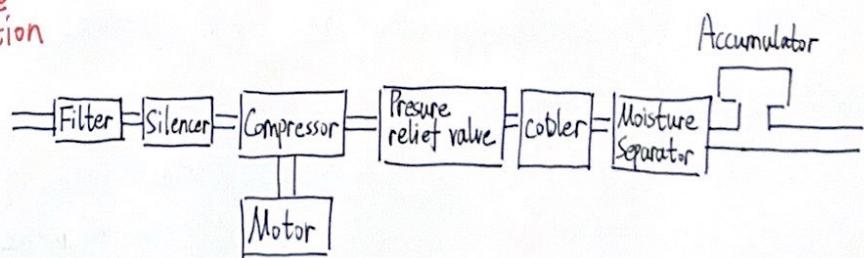
Pneumatic and Hydraulic Actuators

Advantage: provide much higher ratios of (force or torque) to (mass and size) than electromechanical actuators. Less expensive when accurate control not required. Mechanically stiffer than electrical actuators. Many companies have a factory wide high pressure air supply.

Disadvantage: The compressibility of air makes continuous position control very difficult. Hydraulic actuators are more expensive than pneumatic actuators. Hydraulic oil is toxic and flammable. Hydraulic actuators operate at high pressures. Pneumatic and hydraulic are much larger and more complex than electrical ones.



Pneumatic: operate at lower pressure



return air are exhausted into the atmosphere

Pneumatic and Hydraulic Cylinders: Two types. Double acting & Single acting with spring return
 Double acting: has two bidirectional ports, possible to continuously control
 Single acting: a single hose and simpler valve. Disadvantage of only supply unidirectional force, not suitable for position or force control applications.

Output force in the extend direction

$$F_{\text{extend}} = P_{\text{gauge}} A_{\text{extend}} - P_{\text{gauge}} A_{\text{retract}}$$

Output velocity $V = \frac{Q}{A}$ \rightarrow volume flow rate

$$A_{\text{extend}} = \frac{\pi D_{\text{bore}}^2}{4}, A_{\text{retract}} = \frac{\pi (D_{\text{bore}}^2 - D_{\text{rod}}^2)}{4} \text{ for single acting cylinder}$$

Control Valves: used to direct the flow, and/or to change the flow rate, and/or to change the pressure finite position if valves can either be open or closed.

continuous (infinite) valves: proportional valve allows orifice area to be adjusted in proportional to an input voltage.

higher cost servo valve has built in closed-loop control system give faster and more accurate response.

Valve sizing equation

$$C_v = 422 \times 10^4 Q \sqrt{\frac{P}{\Delta P}} \quad \text{or} \quad Q = 2.37 \times 10^5 C_v \sqrt{\frac{\Delta P}{P}}$$

Q : required volume flow rate
 ΔP : pressure drop
 P : density

P for air:

$$P = \frac{P_2}{R_g T} = \frac{P_1 - \Delta P}{R_g T}$$

P_2 : absolute outlet pressure T : air temperature
 R_g : gas constant
 P_1 : absolute inlet pressure

Piezoelectric Actuators

Applied electric charge parallel to the direction of polarization of the crystal causes expansion and force output.

made of several layers (each < 1mm)

Advantage: Better energy efficiency, suitable for miniaturization, capable of nanometre resolution, capable of response times < 1ms, capable of large output force, large stiffness when pushing, no wear

Disadvantage: very **small range** of motion, **low** polling force unless preloading is used, large displacements require **large voltages**. **Sensitive** to temperature changes. Requires **complex** form of closed-loop control to achieve **low frequency** displacements. Subject to **fatigue failure**

Application: Inkjet printer heads, piezoelectric valves, vibratory feeders, optical alignment, precision assembly and precision ~~machining~~ machining.

Shape Memory Alloy Actuators

at **low** temperature, pushed and the shape is changed; at **high** temperature, SMA spring recovers its original shape and push back the shaft.

Advantage: **Simple** one piece design, material can be shaped for the application. **Large** force per unit area, no wear.

Disadvantage: **Slow** response speed, **small** range of motion, operating temperature is approximately 90°C , subject to fatigue failure, **large** hysteresis, **unidirectional**, **low** energy efficiency.

Control: Typically by controlling the electric current flowing through the actuator, this allows the temperature and resulting displacement to be altered.

Applications: valves, electronic locks, safety devices.

Ultrasonic Motors

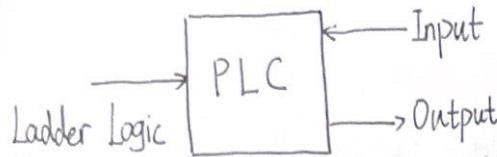
a special type of **piezoelectric** actuator, can be linear or rotary. **electric driven**

Advantage: provide **high torque** at low speed. **Small and lighter**, can produce **large acceleration** due to small rotor inertia. Rotor is **locked** when power is off. Suitable for **miniaturization**.

Disadvantage: Relies on **friction** to provide torque. Subject to **fatigue and wear**.

Driven by **large voltages** relative to DC motors. Torque is **not controllable**

Application: lens motors in autofocus cameras.



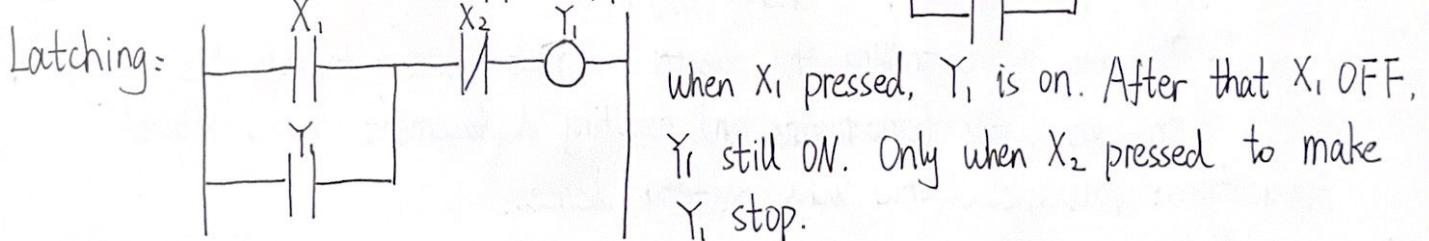
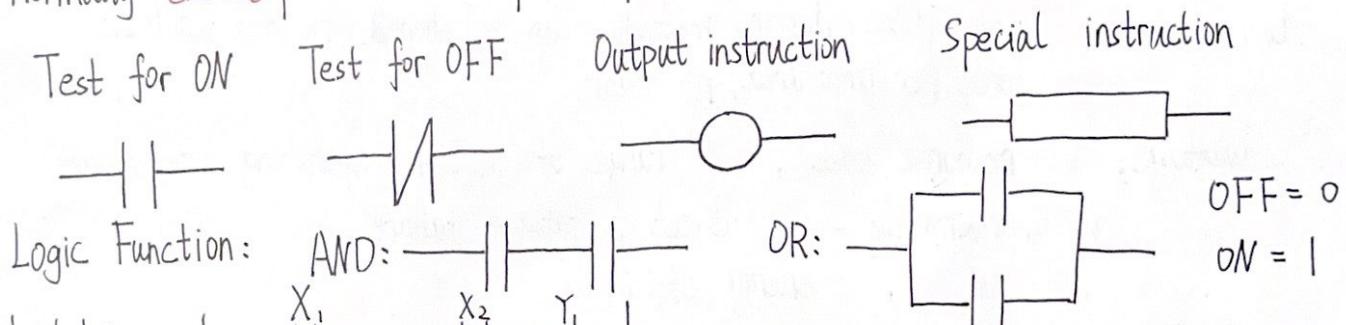
Ladder Logic: most common programming language
consists of a series of groups of **instructions / rungs**

Program **never wait** at a rung, starts again when running at the end of system

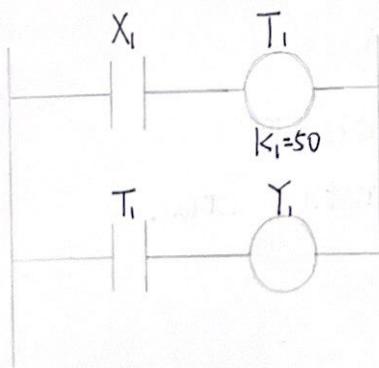
1. Each rung must **begin** with one or more "Test for ON" or "Test for OFF" and **end** with an output instruction or a special instruction.
2. Each output instruction should **occur once** in a program.

normally **open** pushbutton: press provides ON, not press provides OFF

normally **closed** pushbutton: press provides OFF, not press provides ON



Timer: Delay-On Timer: The logic input must be held for ON for greater than delay period for the timer output to turn ON.
Timer resets and turned OFF if OFF signal is passed.



When X_1 is **ON** and stays **ON**, timer T_1 starts counting until $50 \times a_s$ has passed. The T_1 will be ON and put output Y_1 work.

timer resolution : a_s

Marker: internal signals that are not connected to an external input or output device. OFF/ON state is **not lost** when PLC **temporarily loses power**.

Counter:

Reset
C_i
$K_i = k$
Count

 will not count when the Reset is ON, and will reset the accumulator to zero.
Count until **k** rising edge to make C_i ON.

Data Handling Instructions: operate data (bits word word)

Move:

MOV	S	D
-----	---	---

 Compare:

?	S	D
---	---	---

 Add:

+	D ₀	D _i
---	----------------	----------------

 Can also sub
move S to D storage ON if $S \oplus D$ $D_i = D_0 + D_i$

Retentive Timer:

Start/Stop
RT _i
K = ?
Reset

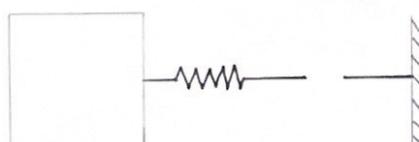
works only when Start/Stop is ON. Stops if the signal is OFF. Set RTA_i to zero if Reset is ON.

Up Down Counter (Reversible Counter)

UDCI	K = ?
UP	
DOWN	
RESET	

Let UDCA_i to increment or decrement. Make UDCA_i zero if the RESET is ON.

Mechanical System



$$\text{Spring Force: } F = Kx \quad \text{deflection}$$

$$\text{Damping Force: } F = Cv = C\dot{x} \quad \text{velocity}$$

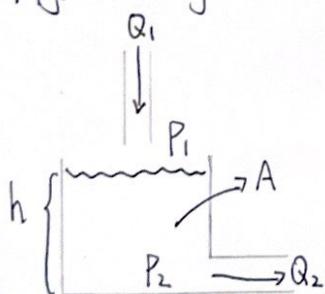
$$\text{Newton's Second Law: } \sum F = ma = m\ddot{x}$$

Electrical System

$$\text{Inductor: } V = L \frac{di}{dt} \quad \text{Capacitor: } i = C \frac{dv}{dt} \quad \text{Resistor: } V = iR$$

Kirchhoff's Law $\left\{ \begin{array}{l} 1. \text{ The sum of currents at a junction must equal to zero} \\ 2. \text{ The sum of voltage drops in a loop must equal to Voltage supply} \end{array} \right.$

Hydraulic System



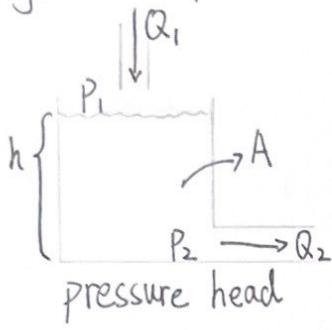
Hydraulic resistance: a liquid flow through a **valve** or a **decrease in pipe diameter**

$$P_1 - P_2 = RQ \rightarrow \text{Volume flow rate}$$

Pressure Drop ↓
Hydraulic Resistance

dissipates energy

Hydraulic Capacitance



$$Q_1 - Q_2 = \frac{dV}{dt} \quad \text{change in volume} \quad V = Ah \quad \text{cross-section area}$$

$$Q_1 - Q_2 = A \frac{dh}{dt} \quad P_2 - P_1 = \rho gh \quad \text{change in pressure}$$

$$C = \frac{A}{\rho g} \Rightarrow Q_1 - Q_2 = C \frac{d(P_2 - P_1)}{dt} \quad P_2 - P_1 = \frac{1}{C} \int (Q_1 - Q_2) dt$$

pressure head
Capacitance

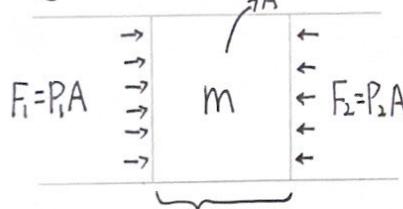
$P_1 = 0$ if the tank is open at top and gauge pressure is used.

$$Q_1 - Q_2 = A \frac{dx}{dt} \quad x = \frac{F}{k} = \frac{(P_2 - P_1)A}{k} \quad \text{displacement of the piston}$$

Diagram of a cylinder with a piston. The top surface has pressure P_1 . The bottom surface has pressure P_2 and flow Q_2 to the right. The area is A . The displacement is x . An arrow labeled Q_1 points to the left from the piston.

$P_1 = 0$ if open at top and gauge pressure is used

Hydraulic Inertia



$$\sum F = (P_1 - P_2)A = ma = ALP \frac{dV}{dt} \quad V = \frac{Q}{A}$$

$$I = \frac{LP}{A} \quad P = \text{density of the block}$$

Inertia

$$P_1 - P_2 = I \frac{dQ}{dt}$$

Kirchhoff's law $\left\{ \begin{array}{l} 1. \text{ Sum of flow rates at a junction} = 0 \\ 2. \text{ Sum of pressure drops around loop} = \text{supply pressure} \end{array} \right.$

Pneumatic Systems

mass flow rate is used $P_1 - P_2 = R \dot{m}$ \rightarrow Pneumatic Resistance

Pneumatic Capacitance : the same as hydraulic accumulator

$$\dot{m}_1 - \dot{m}_2 = V \frac{dP}{dt} \quad PV = mRgT$$

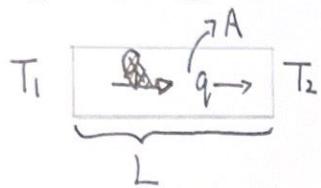
$\dot{m}_1 \rightarrow$

$$C = \frac{V}{RgT} \Rightarrow \dot{m}_1 - \dot{m}_2 = C \frac{dP}{dt} \quad \text{change of pressure}$$

Pneumatic Inertia : $I = \frac{L}{A}$ $P_1 - P_2 = I \frac{d\dot{m}}{dt}$

Thermal System

thermal resistance: $T_1 - T_2 = Rq \rightarrow$ rate of heat flow



unidirectional conduction through solid: $R = \frac{L}{A K} \rightarrow$ thermal conductivity

liquid and gas: $R = \frac{1}{A h} \rightarrow$ coefficient of heat transfer

if $T_1 > T_2$, flow right

thermal capacitance: $q_1 - q_2 = mc \frac{dT}{dt} = C \frac{dT}{dt}$

C: specific heat capacity

m: mass

T: temperature

Non-linear Elements

approximate non-linear to linear

- elements that having an operating region over which linearity may be assumed
- elements that may be approximated by a linear system at particular condition

if $y = f(x) \Rightarrow \Delta y = \left(\frac{df}{dx} \Big|_{x=x_0} \right) \Delta x$. for multiple input, just add all

Dead Time (Time Delay)

$$out = \begin{cases} 0 & t < \tau_d \\ in(t-\tau_d) & t \geq \tau_d \end{cases} \quad \tau_d: \text{period of dead time}$$

$\frac{Out(s)}{In(s)} = e^{-\tau_d s}$ in Laplace transform

minimize dead time:

- make actuator and sensor are close together \Rightarrow transport delay
- send high-level command and let it perform locally.

State Space Method

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad \begin{aligned} x(t) &= [x_1(t), x_2(t), \dots, x_n(t)]^T \\ \dot{x}(t) &= [\dot{x}_1(t), \dot{x}_2(t), \dots, \dot{x}_n(t)]^T \\ y(t) &= [y_1(t), y_2(t), \dots, y_p(t)]^T \end{aligned} \quad \begin{aligned} \text{states} & \quad u(t) : \text{input} \\ \downarrow & \quad [u_1(t), u_2(t), \dots, u_m(t)]^T \end{aligned}$$

usually zero

- advantage:
- Matrix form of state space makes very well suited for computer simulations
 - Much simpler to use with multiple input & output systems
 - The conversion of a transfer function into state space is straightforward

approximate $\dot{x}(t)$: $x(t+\Delta t) = x(t) + \Delta t * \dot{x}(t)$ at small time change

Discrete Time Model:

signals are measured at each sampling period/interval T

Objective of control system:

1. Regulating: Keeping the process output close to a constant value of the reference input in the presence of external disturbance.

2. Servoing: Making the process output respond to changes in the reference input in a specific way.

Control Algorithms

ON-OFF Control: output is binary (either ON or OFF)

rapid switching causes rapid actuator wear and fatigue

Advantage: Extreme simple to implement. Cannot go unstable

Disadvantage: Output oscillations are unavoidable.

PID Control: most popular. $\frac{U(s)}{E(s)} = K_p(1 + \frac{1}{s}K_I + sK_D) \Rightarrow \frac{U(z)}{E(z)} = K_p(1 + K_I \frac{T_z}{z-1} + K_D \frac{z-1}{T_z})$

tuning: selection of controller gains or other parameters to achieve a desired closed-loop performance.

K_p improves speed of response

K_I eliminates steady state error but also add phase lag to the system

K_D adds damping and phase lead to the ~~overshoot~~ response

{ Set all gains equal to zero

Increase K_p until step response has excessive overshoot.

Increase K_D until the overshoot of step response has the desired value

Increase K_I until steady state error is eliminated.

Model-Based Digital Control

Digital control system **stable** if all poles are all inside the **unit circle**

Ragazzini's Method: if $Q(z) = \frac{D(z)G(z)}{1+D(z)G(z)}$ is a digital transfer function

$$H(z) \text{ is the desired closed-loop function} \Rightarrow D(z) = \frac{U(z)}{E(z)} = \frac{1}{G(z)} \frac{H(z)}{1-H(z)}$$

1. Causality Rule: the dead time of $H(z)$ must be **greater than or equal to** the dead time of $G(z)$. Normally pick $H(z) = G(z)$ dead time
dead time = (highest power z denominator - lowest power z numerator) * T
in sampling period of the digital system.

2. Stability Rule: 1) Poles of $H(z)$ must not lie outside unit circle
2) $H(z)$ must contain as zeros all of the **zeros of $G(z)$** that lie out of the unit circle.

$\downarrow 3)$ ~~$H(z)$~~ must contain as zeros all of the **poles of $G(z)$** that lie outside the unit circle

3. Steady-State Error: $E(z) = R(z)(1-H(z))$ final value when $z=1$

$H(1)=1$ to have **zero** error for step input and **finite** error to ramp input.

$$\text{error in ramp input: } e(\infty) = \sum_{i=1}^n \frac{T}{1-p_i} - \sum_{j=1}^m \frac{T}{1-q_j} \Rightarrow \text{zeros}$$

poles \Leftarrow

4. Transient Response: $z = e^{Ts} = e^{aT+jbT} = e^{aT} (\cos(bT) + j \sin(bT))$

5. Sensitivity to Modeling Errors: closer of poles to origin, faster response.

greater difference between poles of $H(z)$ and $G(z)$, more sensitive
the controller will be to modeling errors.

inverse z transform: multiplying $z^{-n} \Rightarrow x(k-n)$

$$D(z) = \frac{U(z)}{E(z)} \Rightarrow u(k) = ? \quad e(k) \& u(k-n)$$

output at kT error previous output at $(k-1)T$

Designing for Safety

Risk Assessment and Risk Reduction : identifying all of the risks faced by anyone who may harm by the mechatronic system, and changing the design until all risks are reduced to an acceptable level.

- 1) Determination of the limits of the mechatronics system. ☹ must
- 2) An analysis to identify potentially hazardous phenomena.
- 3) Evaluation of the degree of risk

System Faults and Fault Management

{ Random hardware component failures, analyzed statistically
Systematic faults due to the design of the system. not random
Errors in the design specifications

Fault Avoidance: prevent faults due to the design of the system or errors in the design specifications.

Fault Removal: finding and correcting faults before system enters service

Fault Detection: detect faults during service, allow appropriate actions

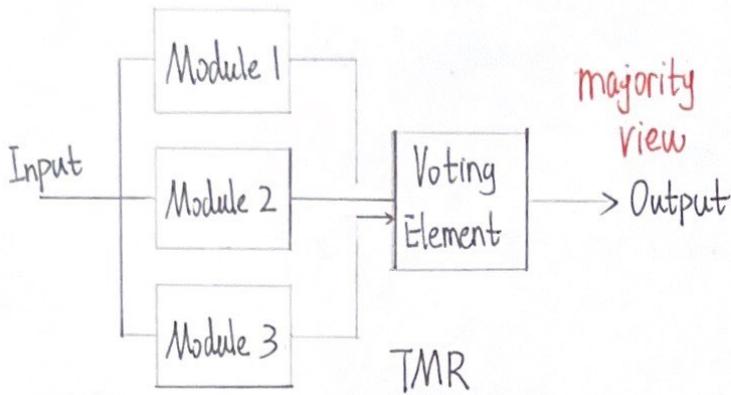
Fault Tolerance: allow system to operate properly with faults.

Fault management categories:

System architecture: main approach, provides protection against
random component failure and some types of systematic faults

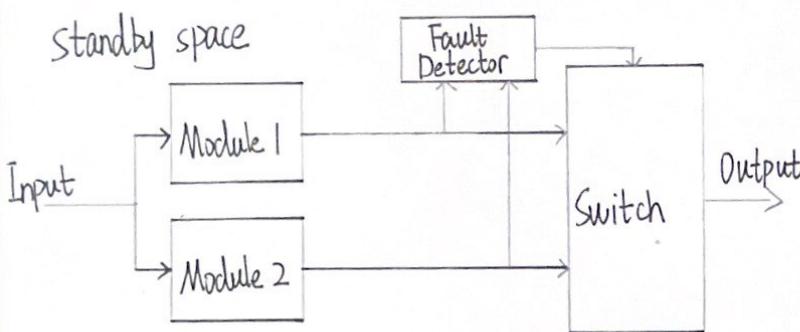
Reliability engineering: techniques for predicting the probability of failure and altering design until meets required reliability.

Quality management: maintaining high quality standards through design, manufacturing and testing processes.

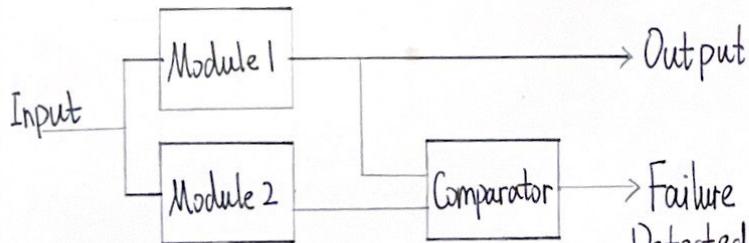


failure of single module → correct output
 assume **independent** module faults
 common-mode faults ⇒ design diversity
 redundant module have same functionality
 but design independently
 ↓
 no improve reliability

Watchdog timer: common fault detection technique. A hardware timer will ~~not~~ turn on an output if not reset before timing period finishes.



Fault Detector detect failure
 ↓
 Switch choose good module
 disadvantage: **reliance** on reliable
 fault detection, and disruption that
 occurs during detection & reconfiguration



Outputs of two modules are
compared using hardware / software
 failure detected if not same
 Module 2 ⇒ digital ~~twin~~ twin

Self-checking pair

partitioning a system ⇒ desired design approach

aids the comprehension of the system, provides **isolation** between modules such that the effects of faults are **localized** as much as possible, reliability of modules is easier to test and verify

cares must be taken for hierarchy partitioning to prevent unsafe design

High level → command: operate if safe → lowest level: check state of safety switch and control actuators.
not practical for all systems

Self-testing: done when system is in service

Safety critical elements of a system should use components that have an extensive record of reliable operation.

Proper signage, manuals, and training procedures is needed. also safety systems.
Systems should be designed for ease of maintenance