ENGPHYS 2A04 Winter 2022 – Assignment 5
DUE MONDAY Feb 28, 8AM

1. **Vector Algebra**
   a. Determine whether the vector $\boldsymbol{C}$ is perpendicular to both $\boldsymbol{A}$ and $\boldsymbol{B}$ given:

$$\boldsymbol{A} = \boldsymbol{4\hat{x} + 5\hat{y}}, \qquad \boldsymbol{B} = \boldsymbol{7\hat{x} + 6\hat{y} + 8\,\hat{z}}, \qquad \boldsymbol{C} = \boldsymbol{\hat{x} + 5\hat{z}}$$

   Show your work (2) and clearly state whether $\boldsymbol{C}$ is perpendicular to $\boldsymbol{A}$ and $\boldsymbol{B}$

$$\boldsymbol{A} \cdot \boldsymbol{C} = 4(1) + 5(0) + 0\,(5) = 4$$
$$\boldsymbol{B} \cdot \boldsymbol{C} = 7(1) + 6(0) + 8\,(5) = 47$$

   $\boldsymbol{A}$ and $\boldsymbol{B}$ and are not perpendicular since the dot product $\boldsymbol{A} \cdot \boldsymbol{B} \neq \boldsymbol{0}$
   $\boldsymbol{A}$ and $\boldsymbol{C}$ and are not perpendicular since the dot product $\boldsymbol{A} \cdot \boldsymbol{C} \neq \boldsymbol{0}$

   b. Find a vector $\boldsymbol{P}$ whose magnitude is 12 and whose direction is perpendicular to both vectors $\boldsymbol{Q}$ and $\boldsymbol{S}$, given: (3)

$$\boldsymbol{Q} = 5\hat{\boldsymbol{x}} \;+ 3\hat{\boldsymbol{y}}, \qquad \boldsymbol{S} = 20\hat{\boldsymbol{y}} - \hat{\boldsymbol{z}}.$$

   The vector $\boldsymbol{P}$ is represented by:

$$\boldsymbol{P} = \frac{\boldsymbol{12}\,(\boldsymbol{Q} \times \boldsymbol{S})}{|\boldsymbol{Q} \times \boldsymbol{S}|}$$

   First find the vector orthogonal to $\boldsymbol{Q}$ and $\boldsymbol{S}$.

$$\boldsymbol{Q} \times \boldsymbol{S} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{z} \\ 5 & 3 & 0 \\ 0 & 20 & -1 \end{vmatrix}$$
$$= [(3)(-1) + (20)(0)]\hat{\boldsymbol{x}} - [(5)(-1) + (0)(0)]\hat{\boldsymbol{y}} + [(5)(20) + (0)(3)]\hat{\boldsymbol{z}}$$
$$= -3\hat{\boldsymbol{x}} + 5\hat{\boldsymbol{y}} + 100\hat{\boldsymbol{z}}$$

   Find the magnitude of the perpendicular vector

$$|\boldsymbol{Q} \times \boldsymbol{S}| = \sqrt{(-3)^2 + (5)^2 + (100)^2} = \sqrt{10034} \approx 100.17$$

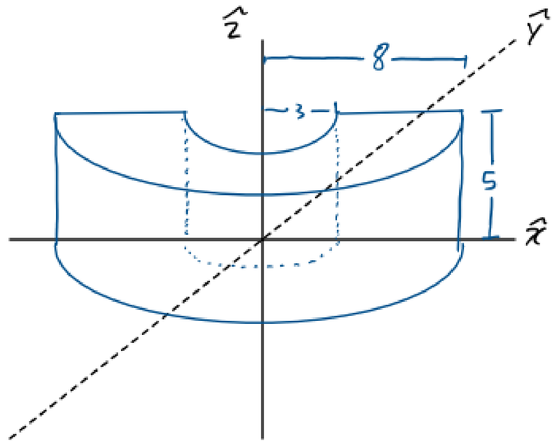
   Substitute to find $\boldsymbol{P}$.

$$\boldsymbol{P} = \frac{12}{\sqrt{10034}}(-3\hat{\boldsymbol{x}} + 5\hat{\boldsymbol{y}} + 100\hat{\boldsymbol{z}}) \approx 0.12(-3\hat{\boldsymbol{x}} - 5\hat{\boldsymbol{y}} + 100\hat{\boldsymbol{z}})$$

2. **Coordinate Systems**
   a. Provide a sketch (1) and find the volume described by (2)

$$3 \le r \le 8;\ \pi \le \varphi \le 2\pi;\ 0 \le z \le 5$$

$$Ans: \frac{275\pi}{2}\ units^3$$



$$V = dV$$
$$= \iiint r\,dr\,d\theta\,dz$$
$$= \int_{z=0}^{5} \int_{\phi=\pi}^{2\pi} \int_{r=3}^{8} r\,dr\,d\phi\,dz$$
$$= \int_0^5 \int_\pi^{2\pi} \left[\tfrac{1}{2}r^2\right]_3^8 d\theta\,dz$$
$$= \int_0^5 \int_\pi^{2\pi} \left[\tfrac{1}{2}(8^2 - 3^2)\right) d\theta\,dz$$
$$= \frac{55}{2} \int_0^5 \int_\pi^{2\pi} d\theta\,dz$$
$$= \frac{55}{2} \int_0^5 [2\pi - \pi]\,dz$$
$$= \frac{55}{2} \int_0^5 (\pi)\,dz$$
$$= \frac{55\pi}{2} \int_0^5 dz$$
$$= \frac{55\pi}{2}(5-0)$$
$$= \frac{275}{2}\pi$$

b. The surface area described by: (2)

$$0 \leq R \leq 2;\ 180 \leq \theta \leq 270^\circ;\ 45^\circ \leq \varphi \leq 90^\circ$$

$$Ans: \pi\ units^2$$

Convert Degrees to radians

$180^\circ = \pi$ rad

$270^\circ = \frac{3\pi}{2}$ rad

$45^\circ = \frac{\pi}{4}$ rad

$90^\circ = \frac{\pi}{2}$ rad

$$S = \int_{\phi=\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\theta=\pi}^{\frac{3\pi}{2}} R^2 \sin\theta \, d\theta \, d\phi \Big|_{R=2}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\pi}^{\frac{3\pi}{2}} (2)^2 \sin\theta \, d\theta \, d\phi$$

$$= 4 \int_{\frac{\pi}{4}}^{\pi/2} \int_{\pi}^{\frac{3\pi}{2}} \sin\theta \, d\theta \, d\phi$$

$$= 4 \int_{\frac{\pi}{4}}^{\pi/2} \left[-\cos\theta\right]_{\theta=\pi}^{\frac{3\pi}{2}} \, d\phi$$

$$= 4 \int_{\frac{\pi}{4}}^{\pi/2} \left[-\cos\left(\tfrac{3\pi}{2}\right) - (-\cos\pi)\right] d\theta$$

$$= 4 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[0 + \cos\pi\right] d\theta$$

$$= 4 \left[\cos\pi\right]_{\pi/4}^{\pi/2}$$

$$= 4 \left[(-1)\right]_{\pi/4}^{\pi/2}$$

$$= 4 \left[-\frac{\pi}{2} + \frac{\pi}{4}\right]$$

$$= 4 \left[-\frac{\pi}{4}\right]$$

$$= -\pi$$

Correction *. The bounds should of θ are $0 \leq \theta \leq 180^\circ$. So instead the above coordinates should have been:

$$0 \leq R \leq 2; 90 \leq \theta \leq 180^\circ;\ 45^\circ \leq \varphi \leq 90^\circ$$

$$= 4\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[-\cos(\pi) - \left(-\cos\left(\frac{\pi}{2}\right)\right)\right] d\theta$$

$$= 4\int_{\frac{\pi}{4}}^{\pi/2} \left[-\cos(\pi) + 0\right] d\theta$$

$$= 4\left[-\cos\pi\right]_{\pi/4}^{\pi/2}$$

$$= 4\left[(1)\right]_{\pi/4}^{\pi/2}$$

$$= 4\left[\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{4}\right)\right]$$

$$= 4\left[\frac{\pi}{4}\right]$$

$$= \pi$$

3. **Gradient** Find the gradient of the following scalar functions.

Recall the gradient is given by $\nabla \mathrm{f(x,y,z)} = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$. Find the partial derivative with respect to each component.

a. $\boldsymbol{M} = 10/(x^2+z^2)$ (1)

$$\nabla \boldsymbol{M} = -\frac{20x}{(x^2 + z^2)^2}\hat{\boldsymbol{x}} - \frac{20z}{(x^2 + z^2)^2}\hat{\boldsymbol{z}}$$

b. $\boldsymbol{A} = xy^3z^2$(1)

$$\nabla \boldsymbol{A} = y^3z^2\hat{\boldsymbol{x}} + 3xy^2z^2\hat{\boldsymbol{y}} + 2xy^3z\hat{\boldsymbol{z}}$$

c. $\boldsymbol{T} = e^R \sin\theta$ (1)

$$\nabla \boldsymbol{T} = \frac{\partial \mathrm{T}}{\partial \mathrm{R}}\widehat{\boldsymbol{R}} + \frac{1}{R}\frac{\partial \mathrm{T}}{\partial \theta}\widehat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial \mathrm{T}}{\partial \phi}\widehat{\boldsymbol{\phi}}$$

$$\nabla \boldsymbol{T} = e^R \sin(\theta)\,\widehat{\boldsymbol{R}} + \frac{e^R\cos\theta}{R}\widehat{\boldsymbol{\theta}}$$

d. $\boldsymbol{H} = R^3\cos^2\theta$ (1)

$$\nabla \boldsymbol{H} = \frac{\partial \mathrm{H}}{\partial \mathrm{R}}\widehat{\boldsymbol{R}} + \frac{1}{R}\frac{\partial \mathrm{H}}{\partial \theta}\widehat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial \mathrm{H}}{\partial \phi}\widehat{\boldsymbol{\phi}}$$

$$\nabla \boldsymbol{H} = 3R^2 \cdot cos^2(\theta)\widehat{\boldsymbol{R}} - 2R^2 \cdot cos(\theta) \cdot sin(\theta)\widehat{\boldsymbol{\theta}}$$

e. $\boldsymbol{S} = xy^2 - z^2$(1)

$$\nabla \boldsymbol{S} = y^2\hat{\boldsymbol{x}} + 2xy\hat{\boldsymbol{y}} - 2z\hat{\boldsymbol{z}}$$

4. **Divergence.** The Divergence theorem states the surface integral of a vector field over a closed surface, the flux through the surface, is equal to the volume integral of the divergence over the region within the surface. Given the Divergence theorem states:

$$\int_v \nabla \cdot \boldsymbol{E}\, d\boldsymbol{v} = \oint_S \boldsymbol{E} \cdot d\boldsymbol{s}$$

The vector field V is given by:

$$\boldsymbol{V} = x^2\hat{\boldsymbol{x}} + y^3\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$$

Verify the divergence theorem by computing

a. The total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each, parallel to the cartesian axes (2)

The closed surface has 6 sides:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{s} = F_{top} + F_{bottom} + F_{right} + F_{left} + F_{front} + F_{back}$$

We must find the flux through each side. The calculation of the flux through each side is shown on the next page.

$$\begin{aligned}\oint \boldsymbol{E} \cdot d\boldsymbol{s} &= F_{top} + F_{bottom} + F_{right} + F_{left} + F_{front} + F_{back} \\ &= (4) + (4) + (4) + (4) + (4) + (-4) \\ &= 16\end{aligned}$$

b. The integral of $\nabla \cdot \boldsymbol{V}$ over the cubes volume. (2)

$$\iiint \nabla \cdot \vec{E}\, d\vec{v} = \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} \nabla \cdot (x^2\hat{x} + y^3\hat{y} + z\hat{z})\, dz\, dy\, dx$$

$$= \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} (3y^2 + 2x + 1)\, dz\, dy\, dx$$

Note: integral bounds are omitted for simplicity

$$= \iiint 3y^2\, dz\, dy\, dx + \iiint 2x\, dz\, dy\, dx + \iiint dz\, dy\, dx$$

$$= (x\, y^3 z + x^2 yz + xyz)\Big|_{z=-1}^{1}\Big|_{y=-1}^{1}\Big|_{x=-1}^{1}$$

$$= xyz\,(y^2 + x + 1)\Big|_{z=-1}^{1}\Big|_{y=-1}^{1}\Big|_{x=-1}^{1}$$

$$= 16$$

c. Does the theorem hold true? (1)

Yes, the theorem holds since the surface integral of a vector field equals the volume integral of the divergence.

$$F_{top} = \int_{x=-1}^{1} \int_{y=-1}^{1} (x^2 \hat{x} + y^3 \hat{y} + z\hat{z})\Big|_{z=1} \cdot (\hat{z}\, dy\, dx)$$

$$= \int_{x=-1}^{1} \int_{y=-1}^{1} (z|_{z=1})\, dy\, dx$$

$$= \int_{x=-1}^{1} \int_{y=-1}^{1} dy dx$$

$$= [x]_{-1}^{1} \int_{y=-1}^{1} dy$$

$$= (1-(-1))[y]_{-1}^{1}$$

$$= 2(1-(-1))$$

$$= 4$$

$$F_{bottom} = \int_{x=-1}^{1} \int_{y=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{z=-1} \cdot (-\hat{z}\, dy\, dx)$$

$$= \int_{x=-1}^{1} \int_{y=-1}^{1} -z|_{z=-1}\, dy\, dx$$

$$= \int_{x=-1}^{1} \int_{y=-1}^{1} -(-1)\, dy\, dx$$

$$= [x]_{-1}^{1} [y]_{-1}^{1}$$

$$= 4$$

$$F_{Right} = \int_{x=-1}^{1} \int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{y=1} \cdot (\hat{y}\, dz\, dx)$$

$$= \int_{x=-1}^{1} \int_{z=-1}^{1} y^3|_{y=1}\, dz\, dx$$

$$= \int_{x=-1}^{1} \int_{z=-1}^{1} (1)^3\, dz\, dx$$

$$= 4$$

$$F_{left} = \int_{x=-1}^{1} \int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{y=-1} \cdot -\hat{y}\, dz\, dx$$

$$= \int_{x=-1}^{1} \int_{z=-1}^{1} -y^3|_{y=-1}\, dz\, dx$$

$$= \int_{x=-1}^{1} \int_{z=-1}^{1} -(-1)^3\, dx dz$$

$$= 4$$

$$F_{front} = \int_{y=-1}^{1} \int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{x=1} \cdot \hat{x}\, dz\, dy$$

$$= \int_{y=-1}^{1} \int_{z=-1}^{1} x^2 \Big|_{x=1} \, dz \, dy$$

$$= \int_{y=-1}^{1} \int_{z=-1}^{1} (1)^2 \, dz \, dy$$

$$= 4$$

$$F_{back} = \int_{y=-1}^{1} \int_{z=-1}^{1} \left(x^2 \hat{x} + y^3 \hat{y} + z \hat{z}\right)\Big|_{x=-1} \cdot -\hat{x} \, dz \, dy$$

$$= \int_{y=-1}^{1} \int_{z=-1}^{1} -x^2 \Big|_{x=-1} \, dz \, dy$$

$$= \int_{y=-1}^{1} \int_{z=-1}^{1} -(-1)^2 \, dz \, dy$$

$$= -4$$

5. **Curl.** Stokes's theorem is a powerful equation that allows the conversion of a surface integral of the curl of a vector over an open surface S into a line integral, such as in the calculation of current through a closed magnetic field loop. Given that Stokes's theorem states:

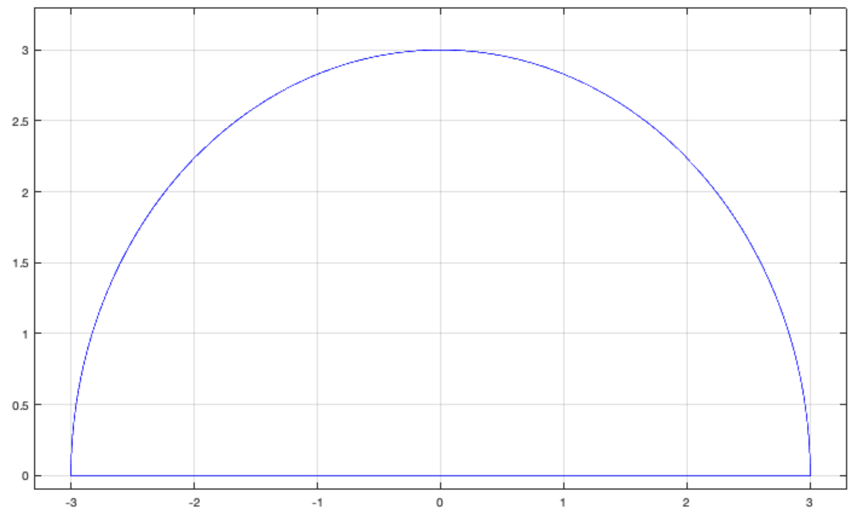
$$\int_S (\nabla \times \boldsymbol{B}) \cdot d\boldsymbol{s} = \oint_C \boldsymbol{B} \cdot d\boldsymbol{l}$$

Verify Stokes's theorem for the vector field:

$$\boldsymbol{B} = r\cos\phi\,\hat{\boldsymbol{r}} + \sin\phi\ \hat{\boldsymbol{\phi}}$$

a. By evaluating $\oint_C \boldsymbol{B} \cdot d\boldsymbol{l}$ over the semicircular contour shown below (2)

b. By evaluating $\int_S (\nabla \times \mathbf{B}) \cdot d\boldsymbol{s}$; over the semicircular contour shown below (3)



<u>Solution</u>:

Part a)

$$\oint_C \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{L_1} \boldsymbol{B} \cdot d\boldsymbol{l} + \int_{L_2} \boldsymbol{B} \cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{B} \cdot d\boldsymbol{l}$$

$$\boldsymbol{B} \cdot d\boldsymbol{l} = \left(r\cos\phi\,\hat{\boldsymbol{r}} + \sin\phi\ \hat{\boldsymbol{\phi}}\right) \cdot \left(dr\,\hat{\boldsymbol{r}} + r d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\boldsymbol{z}}\right) = r\cos\phi\,dr + r\sin\phi\,d\phi$$

$$\int_{L_1} \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{(r=0)}^{3} r\cos\phi\,dr\Big|_{\phi=0,\ z=0} + \int_{(\phi=0)}^{0} r\sin\phi\ d\phi\Big|_{z=0}$$

$$= \frac{1}{2}r^2\Big|_{r=0}^{3} + 0$$

$$= \frac{9}{2}$$

$$\int_{L_2} \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{(r=3)}^{3} r\cos\phi\,dr\Big|_{z=0} + \int_{(\phi=0)}^{\pi} r\sin\phi\ d\phi\Big|_{z=0}$$

$$= 0 + (-3\cos\phi)|_{\phi=0}^{\pi}$$

$$= 6$$

$$\int_{L_3} \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{(r=3)}^{2} r\cos\phi\, dr \bigg|_{\phi=\pi,\ z=0} + \int_{(\phi=\pi)}^{\pi} r\sin\phi\ d\phi \bigg|_{z=0}$$
$$= -\frac{1}{2}r^2\bigg|_{r=3}^{0} + 0$$
$$= \frac{9}{2}$$
$$\oint_C \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{L_1} \boldsymbol{B} \cdot d\boldsymbol{l} + \int_{L_2} \boldsymbol{B} \cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{B} \cdot d\boldsymbol{l}$$
$$\oint_C \boldsymbol{B} \cdot d\boldsymbol{l} = \frac{9}{2} + 6 + \frac{9}{2}$$
$$= 15$$

Part b)

$$\nabla \times \mathbf{B} = \nabla \times (r\cos\phi\ \hat{\mathbf{r}} + \sin\phi\, \hat{\boldsymbol{\phi}})$$
$$= \left(\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{\phi}}0 - \frac{\partial}{\partial \mathrm{z}}(\sin\phi)\right)\hat{\mathbf{r}} + \left(\frac{\partial}{\partial z}(r\cos\phi) - \frac{\partial}{\partial r}0\right)\hat{\boldsymbol{\phi}}$$
$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}(r\sin\phi) - \frac{\partial}{\partial \phi}(r\cos\phi)\right)\hat{\boldsymbol{z}}$$
$$= (0)\hat{\mathbf{r}} + (0)\hat{\boldsymbol{\phi}} + \frac{1}{r}(\sin\phi + r\sin\phi)\hat{\boldsymbol{z}}$$
$$= \frac{1}{r}(\sin\phi + r\sin\phi)\hat{\boldsymbol{z}}$$
$$= \sin\phi\left(1 + \frac{1}{\mathrm{r}}\right)\hat{\boldsymbol{z}}$$
$$\int_S (\nabla \times \mathbf{B}) \cdot d\boldsymbol{s} = \int_{\phi=0}^{\pi}\int_{r=0}^{3} \sin\phi\left(1 + \frac{1}{\mathrm{r}}\right)\hat{\boldsymbol{z}} \cdot r dr d\phi \hat{\boldsymbol{z}}$$
$$= \int_{\phi=0}^{\pi}\int_{r=0}^{3} \sin\phi\left(1 + \frac{1}{\mathrm{r}}\right)\hat{\boldsymbol{z}} \cdot r dr d\phi \hat{\boldsymbol{z}}$$
$$= \int_{\phi=0}^{\pi}\int_{r=0}^{3} \sin\phi\ (\mathrm{r} + 1) dr d\phi$$
$$= \int_{\phi=0}^{\pi} \sin\phi\left[\frac{1}{2}\mathrm{r}^2 + r\right]_{r=0}^{3} d\phi$$
$$= \int_{\phi=0}^{\pi} \sin\phi\left[\left(\frac{9}{2}\right) + 3\right] d\phi$$
$$= 7.5\ [-\cos\phi]_{\phi=0}^{\pi}$$
$$= 7.5\ [(-\cos\pi - (-\cos 0)]$$
$$= 7.5\ [2]$$
$$= 15$$

Therefore, the theorem holds true.

6. **Bonus Question**: Answer one of the following questions. Clearly state whether a, b, or c is being answered.

   a. Find the values for $\boldsymbol{V} = ax^2\hat{\boldsymbol{x}} + by^3\hat{\boldsymbol{y}} + c\hat{\boldsymbol{z}}$ where the divergence at $P = (6{,}4{,}7)$ is equal to $\nabla \cdot \mathbf{V} = 10$ (2)

$$\nabla \cdot \mathbf{V} = \frac{\partial \mathbf{V}}{\partial \mathrm{x}} + \frac{\partial \mathbf{V}}{\partial \mathrm{y}} + \frac{\partial \mathbf{V}}{\partial \mathrm{z}}$$
$$10 = 2ax + 3by^2$$

   Substitute $P = (6{,}4{,}7)$

$$10 = 2(6)a + 3(4)^2b + 0(7)$$
$$10 = 12a + 48b$$

   Provide any three values for (a, b, c) to satisfy the above equation. For example

$$a = 1, b = -\frac{2}{48}, c = any\ value$$

   b. Describe $\boldsymbol{A}$ in cylindrical coordinates and evaluate it at

$$P = (2, \pi, \pi/4)$$

   $\boldsymbol{A} = \sin^2\theta\ cos\varphi\widehat{\boldsymbol{R}} + \cos\theta\,\widehat{\boldsymbol{\theta}}\ - sin\varphi\ \widehat{\boldsymbol{\varphi}}$ (2)

Bonus Q

$\vec{A} = \sin^2\theta \cos\varphi \hat{R} + \cos\theta \hat{\theta} - \sin\varphi \hat{\varphi}$

Describe A in cylindrical coordinates

Evaluate at $P = (2, \pi, \pi/4)$

$\varphi = \phi$

Relations

Spherical to Cylindrical Coordinates	Vector Conversion
$r = R\sin\theta$	$A_r = A_R \sin\theta + A_\theta \cos\theta$
$\phi = \phi$	$A_\phi = A_\phi$
$z = R\cos\theta$	$A_z = A_R \cos\theta - A_\theta \sin\theta$

$$\vec{A}_{spherical} = \sin^2\theta \cos\phi \hat{R} - \sin\phi \hat{\phi} + \cos\theta \hat{\theta}$$

$$\vec{A}_{cylindrical} = A_r \hat{r} + A_\phi \hat{\phi} + A_z \hat{z}$$

$$= [A_R \sin\theta + A_\theta \cos\theta]\hat{r} + A_\phi \hat{\phi} + [A_R \cos\theta - A_\theta \sin\theta]\hat{z}$$

$$= [(\sin^2\theta \cos\phi)\sin\theta + (-\sin\phi)(\cos\theta)]\hat{r}$$
$$+ [-\sin\phi]\phi$$
$$+ [\sin^2\theta \cos\phi \cos\theta - \cos\theta \sin\theta]\hat{z}$$

$$= [\sin^3\theta \cos\phi - \sin\phi \cos\theta]\hat{r} - \sin\phi \hat{\phi}$$
$$+ [\sin^2\theta \cos\phi \cos\theta - \cos\theta \sin\theta]\hat{z}$$

Substitute $P_{spherical} = (2, \pi, \frac{\pi}{4})$

$$P = 2\hat{R} + \pi\hat{\theta} + \frac{\pi}{4}\hat{\phi}$$

$$A_{cylindrical} = [\sin^3(\pi)\cos(\tfrac{\pi}{4}) - \sin(\tfrac{\pi}{4})\cos(\pi)]\hat{r}$$
$$-(\sin\tfrac{\pi}{4})\hat{\phi}$$
$$+[\sin^2(\pi)\cos(\tfrac{\pi}{4})\cos(\pi) - \cos(\pi)\sin(\pi)]\hat{z}$$

$$= \hat{r} - \frac{1}{\sqrt{2}}\hat{\phi}$$

c. Convert the following coordinates

i. From cartesian to cylindrical and spherical coordinates: $P_1 = (5,10,15)$ (2)

Cylindrical:

$$r = \sqrt{125}$$
$$\phi = 63.43^{\circ} = 1.1\, rads$$
$$z = 15$$

Spherical:

$$R = \sqrt{350}$$
$$\theta = 36.69^{\circ} = 0.64\, rads$$
$$\phi = 63.43^{\circ} = 1.1\, rads$$

ii. From cylindrical to spherical and cartesian: $P_2 = \left(1, \frac{\pi}{2}, -1\right)$ (2)

Spherical:

$$R = \sqrt{2}$$
$$\phi = 90^{\circ} = \frac{\pi}{2}$$
$$\theta = -45^{\circ} = \frac{\pi}{2}\, rads$$

Cartesian:

$$x = 0$$
$$y = 1$$
$$z = -1$$

iii. From spherical to cylindrical: $P_3 = (4, \pi, \pi)$ (1)

Cylindrical:

$$r = 0$$
$$\phi = 180^{\circ} = \pi$$
$$z = -4$$

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal marks (except bonus questions).
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.