Generative Models

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Applications of Machine Learning (4AL3)

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McMaster University

ENGINEERING

Review

- Discriminative vs Generative Models
- Naïve Bayes Theorem

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

- Text to Vector Conversion: Bag Of Words
- Naïve Bayes Classification applied to test

Training Naïve Bayes: Review

function TRAIN NAIVE BAYES(D, C) **returns** V, $\log P(c)$, $\log P(w|c)$

for each class $c \in C$ # Calculate $P(c)$ terms
  N_{doc} = number of documents in D
  N_c = number of documents from D in class c
  $\boxed{logprior[c] \leftarrow \log \frac{N_c}{N_{doc}}}$
  $V \leftarrow$ vocabulary of D
  $bigdoc[c] \leftarrow$ **append**(d) **for** d $\in$ D **with** class c
  for each word w in V # Calculate $P(w|c)$ terms
    $count(w,c) \leftarrow$ # of occurrences of w in $bigdoc[c]$
    $loglikelihood[w,c] \leftarrow \log \frac{count(w,c) + 1}{\sum_{w' \text{ in } V} (count(w',c) + 1)}$
return *logprior*, *loglikelihood*, V

function TEST NAIVE BAYES(*testdoc*, *logprior*, *loglikelihood*, C, V) **returns** best c

for each class $c \in C$
  $sum[c] \leftarrow logprior[c]$
  for each position i in *testdoc*
    $word \leftarrow testdoc[i]$
    if $word \in V$
      $sum[c] \leftarrow sum[c] + loglikelihood[word,c]$
return $\text{argmax}_c\ sum[c]$

Training Naïve Bayes: Review

- Goal is to learn probabilities

	Label	documents
Training	-	just plain boring
Training	-	entirely predictable and lacks energy
Training	-	no surprises and very few laughs
Training	+	very powerful
Training	+	the most fun film of the summer
Test	?	predictable with no fun

$$P(-) = \frac{3}{5}$$

$$P(+) = \frac{2}{5}$$

$$\frac{\text{Number of d in class c}}{\text{Number of documents (d)}}$$

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Training Naïve Bayes : Review

- Goal is to learn probabilities:

$P(\text{predictable}|-)$ $\frac{1+1}{14+20}$

$P(\text{predictable}|+)$ $\frac{0+1}{9+20}$

$P(\text{no}|-)$ $\frac{1+1}{14+20}$

$P(\text{no}|+)$ $\frac{0+1}{9+20}$

$P(\text{fun}|-)$ $\frac{0+1}{14+20}$

$P(\text{fun}|+)$ $\frac{1+1}{9+20}$

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Using add -1 smoothening $\frac{\text{Count}(f_i, c)+1}{\sum_{f \in V}(\text{Count}(f, c)) + |V|}$

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```

$P(-)P(S|-)$

$$\frac{3}{5} \times \frac{2 \times 2 \times 1}{34 \times 34 \times 34}$$

$P(+)P(S|+)$

$$\frac{2}{5} \times \frac{1 \times 1 \times 2}{29 \times 29 \times 29}$$

Maximum of the two ?

Converting Text to Vectors: Review

- Techniques used:
  - Bag of Words

word	frequency
It	6
I	5
the	4
satirical	1
whimsical	1
would	1
adventure	1
and	3

"I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!"

Converting Text to Vectors : Review

with respect to document (**not** position within a sentence) : More accurate description is “**relative** frequency of occurrence in a document”

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the number of documents in which each term can be found

“It manages to be **whimsical** and romantic while laughing at the conventions of the fairy tale genre.”

“I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be **whimsical** .

Generative vs Discriminative Models

Generative models model the problem

$$\mathrm{P}(x, y) = X, Y \rightarrow [0,1]$$

Discriminative models model the problem

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- Can work without labels, so can compute $\mathrm{P}(x)$

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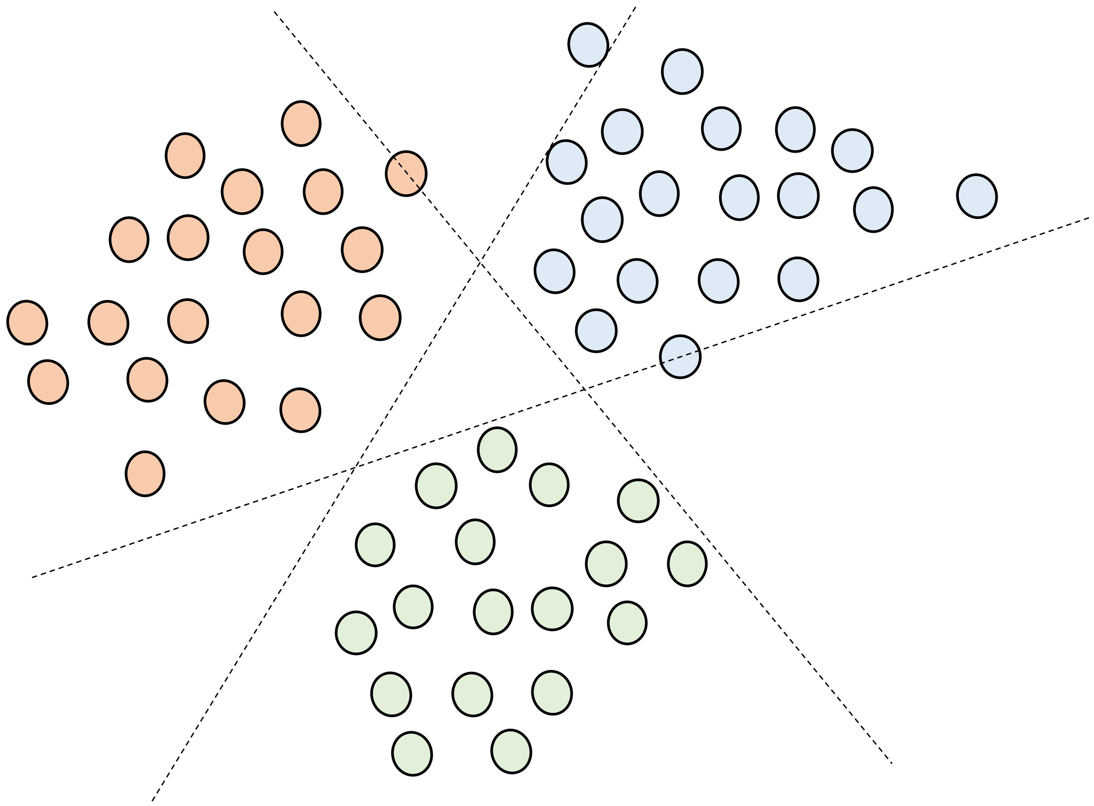
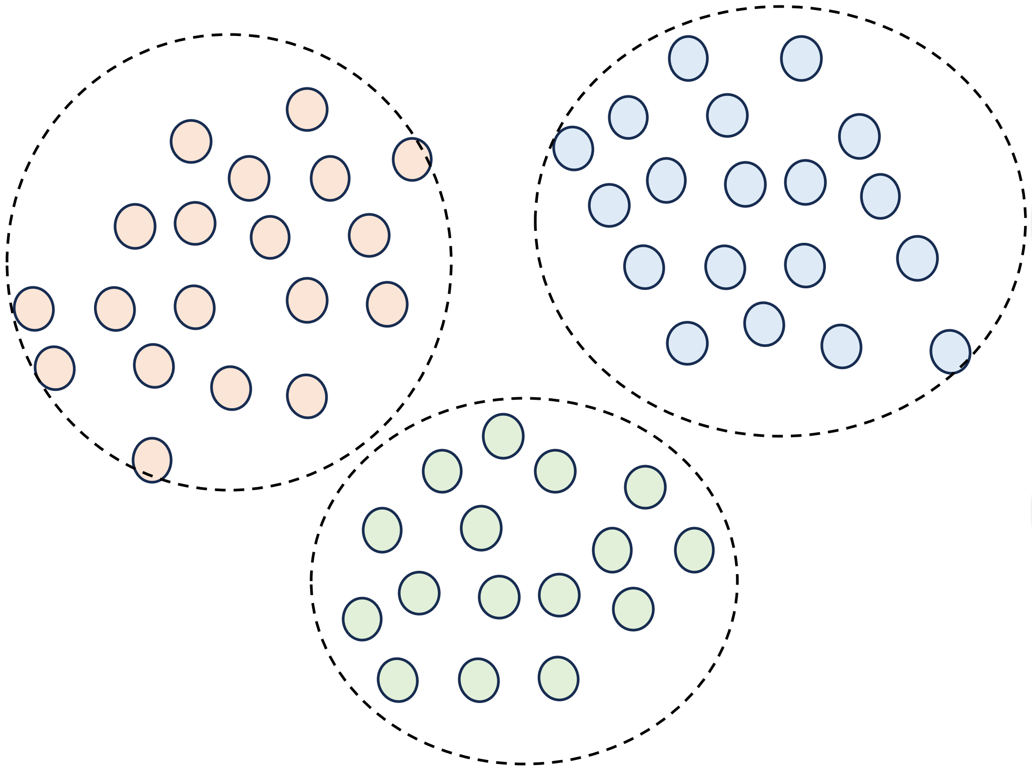
Clustering



Discriminative Model



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What other generative approaches exist?

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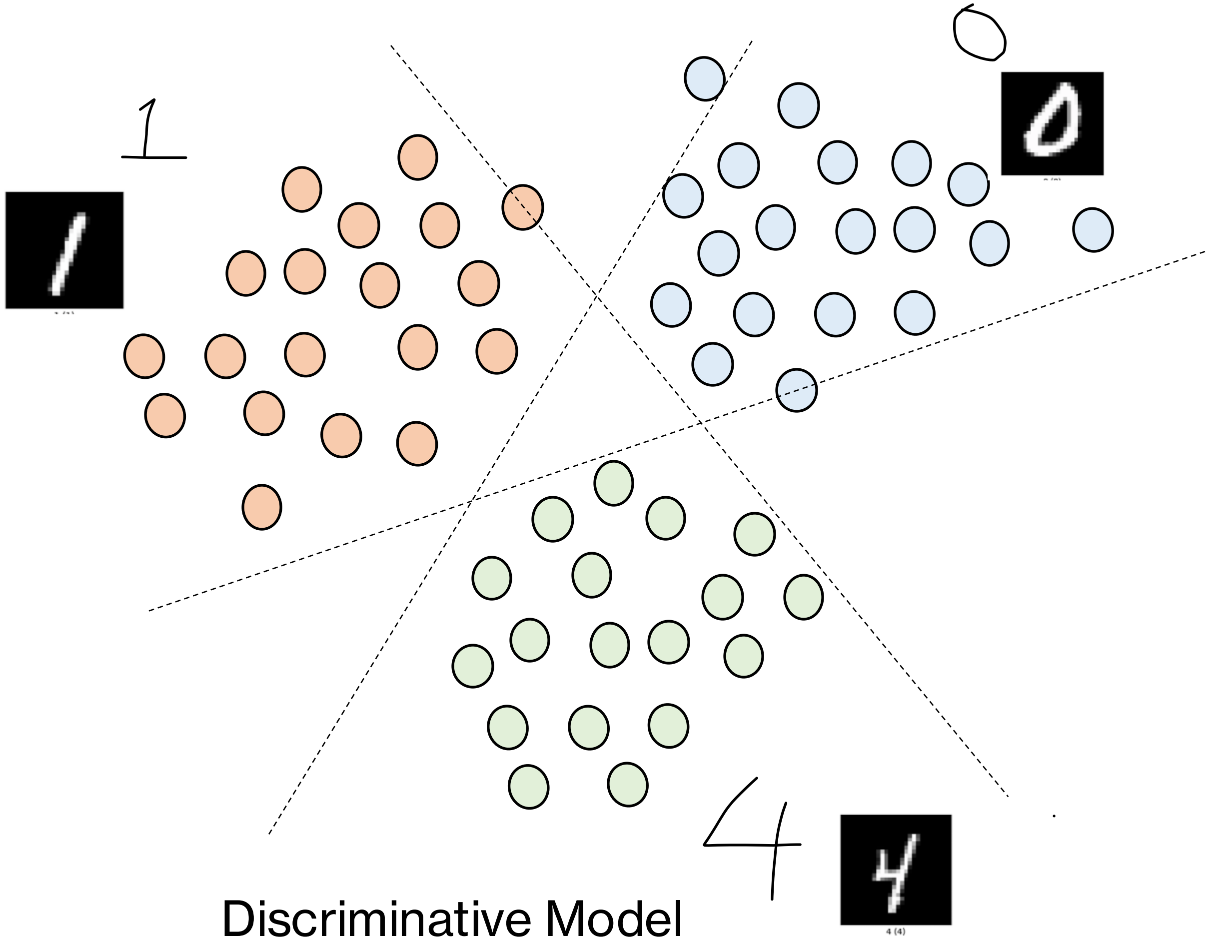
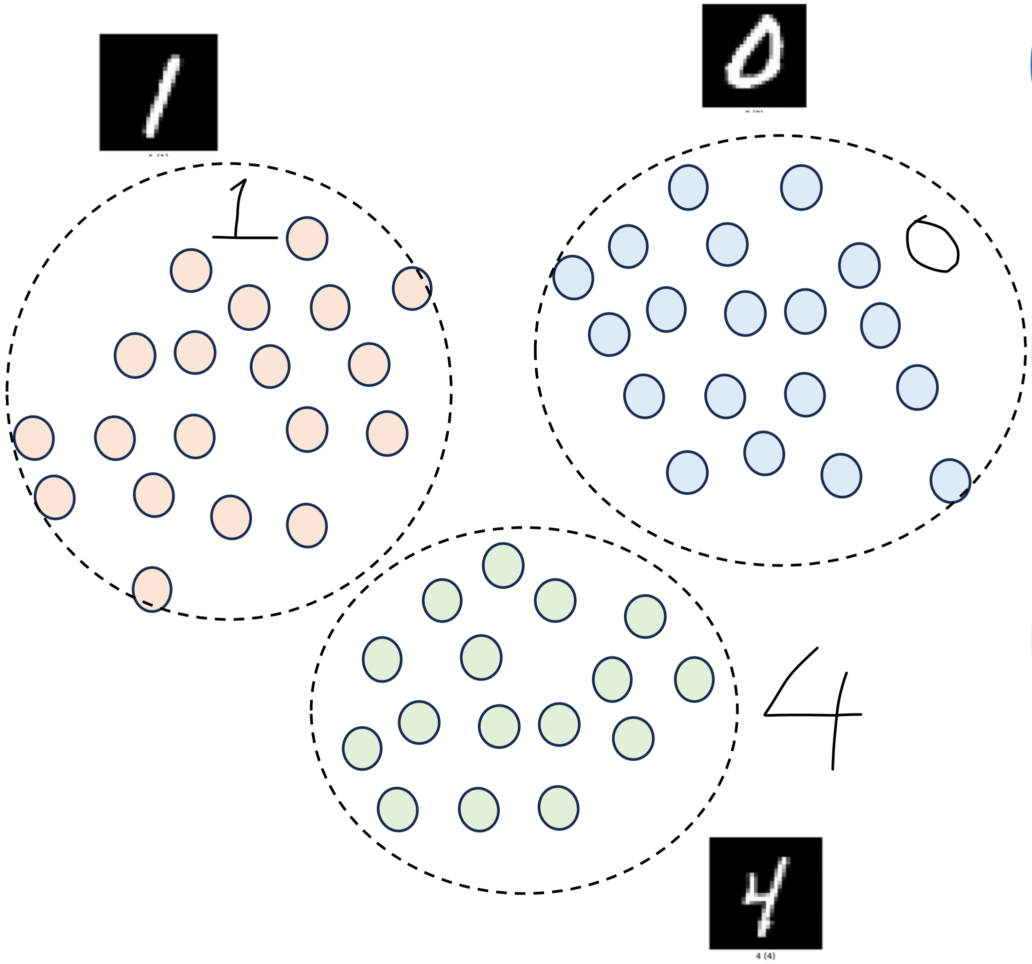
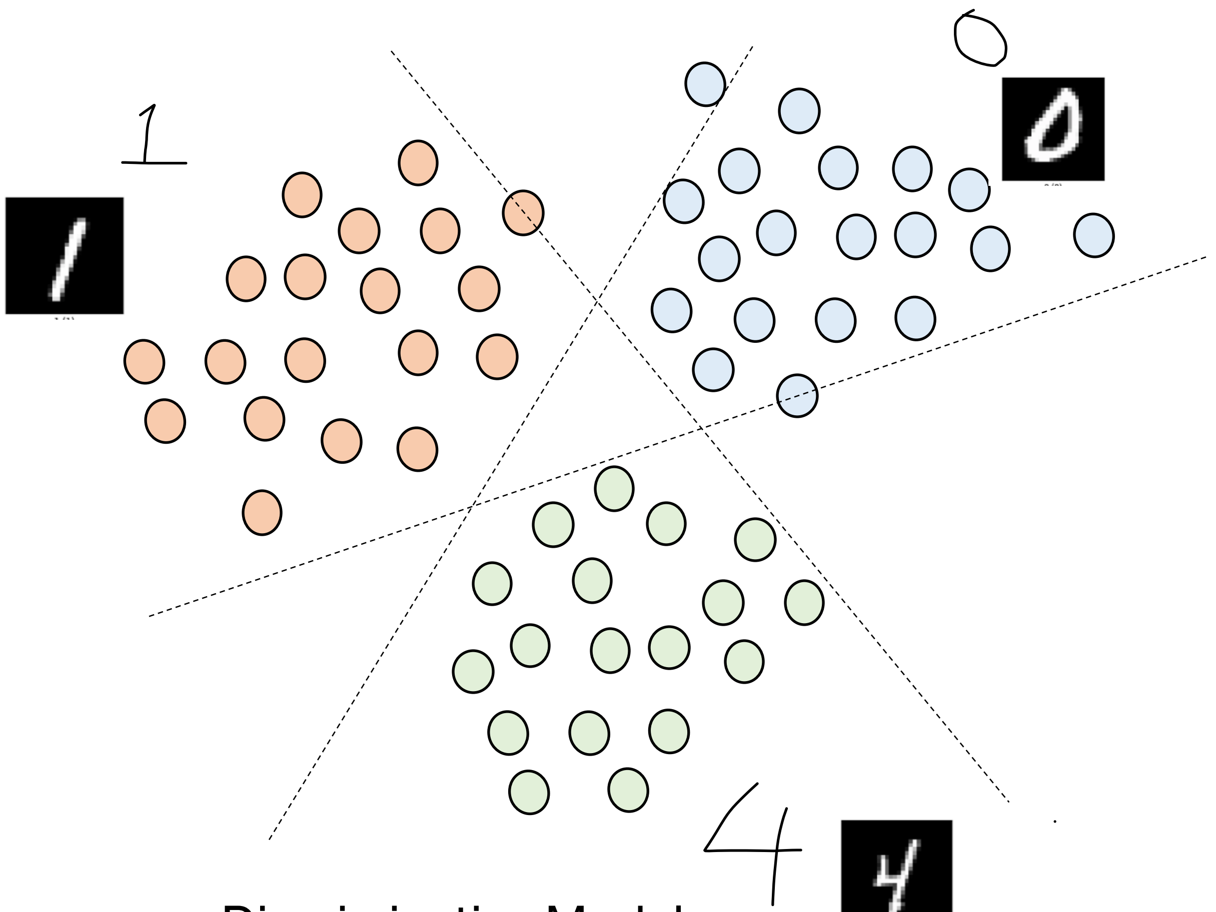


Discriminative Model

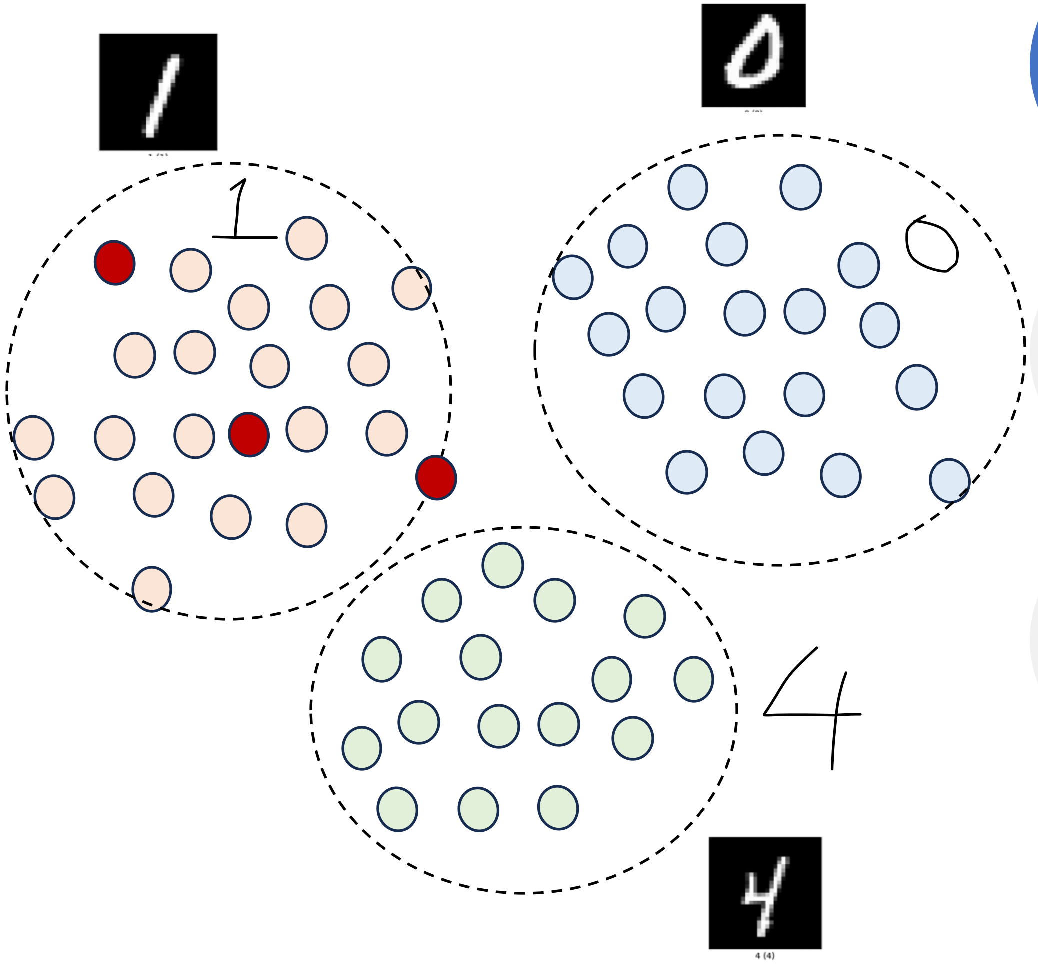


Generative Model

Clustering



Discriminative Model



Generative Model

Clustering

- We may cluster the data into subgroups and partition the entire data distribution based on similarity.
- The goal of clustering algorithms is to find homogenous subgroups among the observations.
- When a new data instance arrives, then we find the subgroup it might belong to, or is closest to.
- Relies on data distribution to build a classification model.
- They are most challenging, and tackle more difficult tasks

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$$C_1 \cup C_2 \cup \cdots \cup C_k = \{1,2,\ldots,n\}$$

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    - Let us assume that if i^{th} observation is in the k^{th} cluster, then $i \in C_k$
    - Let us assume that within a cluster the variation (how much different each of the samples are from each other) is defined by $W(C_k)$

Clustering

- Types of Clustering:
  - K-Means
    - Good clustering is one for which the within-cluster variation is as small as possible.
      - Optimization Problem: $\underset{C_1, C_2, \ldots, C_k}{\textit{minimize}} \left\{ \sum_{k=1}^{K} W(C_k) \right\}$

$$W(C_k) = \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2$$

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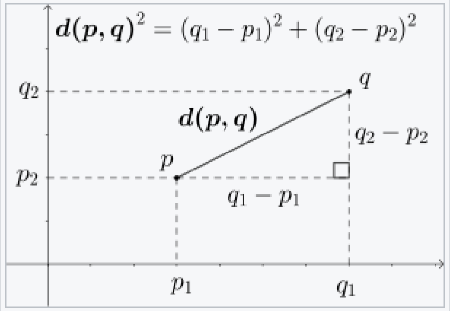

Using the Pythagorean theorem to compute two-dimensional Euclidean distance

Distance between two points $d(p, q) = |p, q|$

Picture Source: Wikipedia

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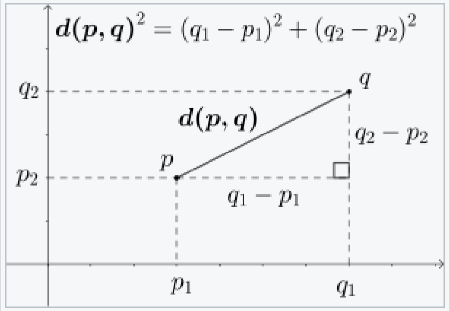

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Distance between two points $d(p,q) = |p,q|$

In high dimensionality : $d(p,q) = \sqrt{(p-q)^2}$

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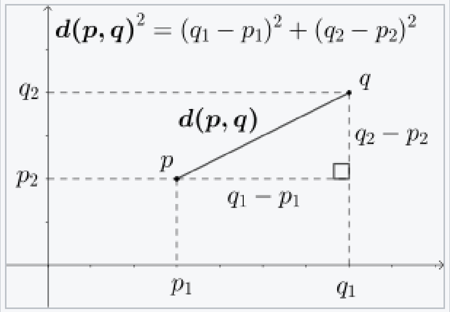

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Where $|C_k|$ = denotes the number of observations in k^{th} cluster

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Clustering : Training

- Types of Clustering:
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1. Randomly assign a number, from 1 to K, to each of the observations. These serve as initial cluster assignments for the observations.

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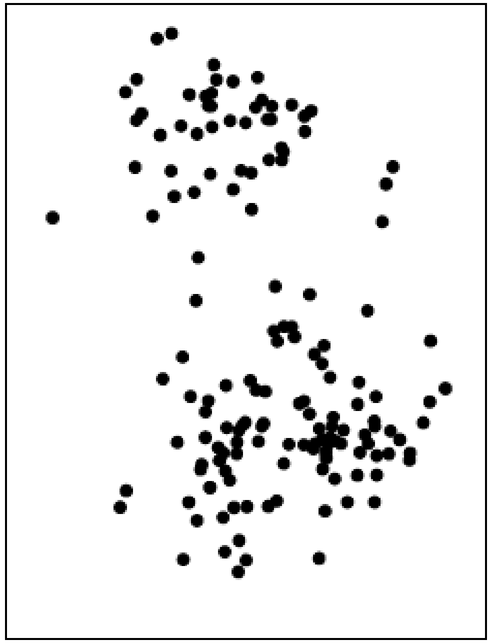

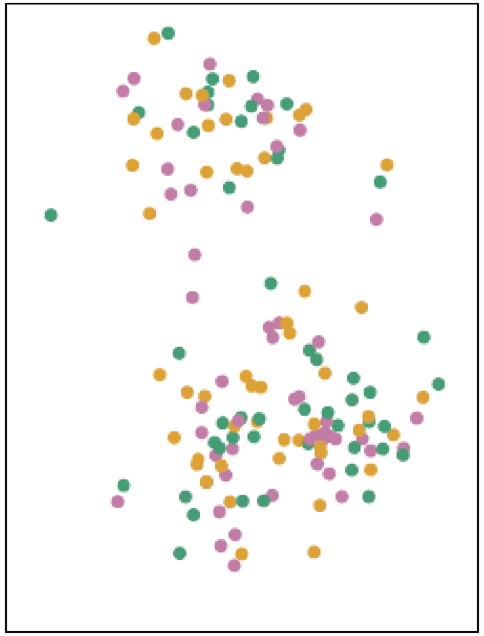

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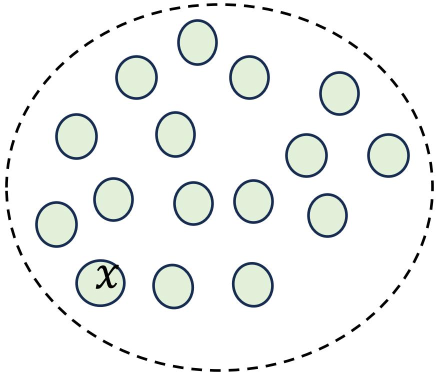
1. Randomly assign a number, from 1 to K, to each of the observations. These serve as initial cluster assignments for the observations.
2. Iterate until the cluster assignments stop changing:
   (a) For each of the K clusters, compute the cluster *centroid*. The kth cluster centroid is the vector of the p feature means for the observations in the kth cluster.

$$\mu_i = \frac{1}{|S_i|} \sum_{x \in S_i} x$$



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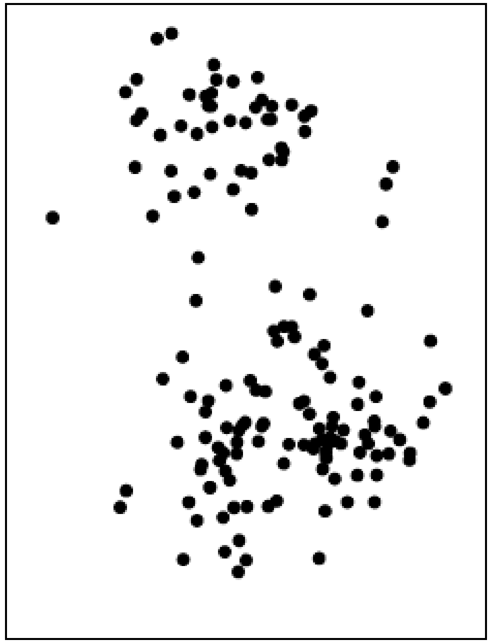

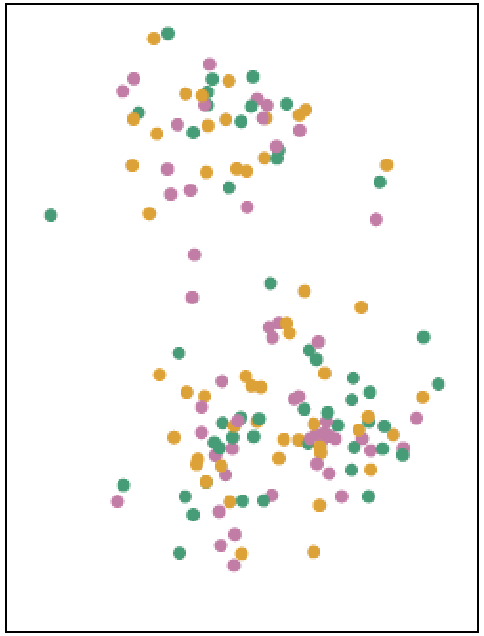

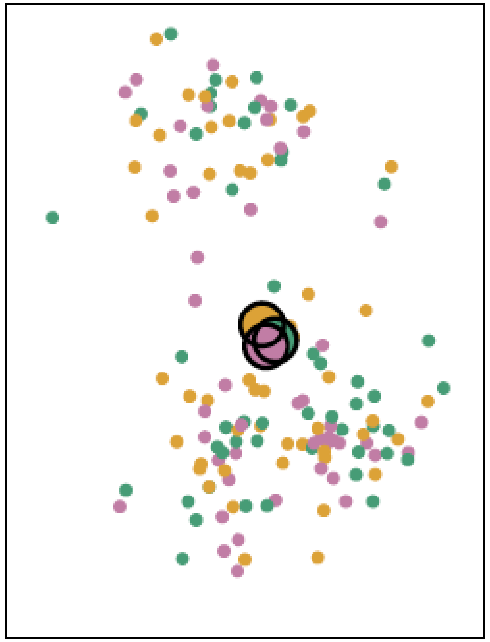

Clustering : Training

- Types of Clustering:
  - K-Means algorithm calculating centroid

Observation	X_1	X_2
A	7	9
B	3	3
C	4	1
D	3	8

Centroid

Cluster	X_1	X_2
A,B	$(7+3)/2 = 5$	$(9+3)/2 = 6$
C,D	$(4+3)/2 = 3.5$	$(1+8)/2 = 4.5$

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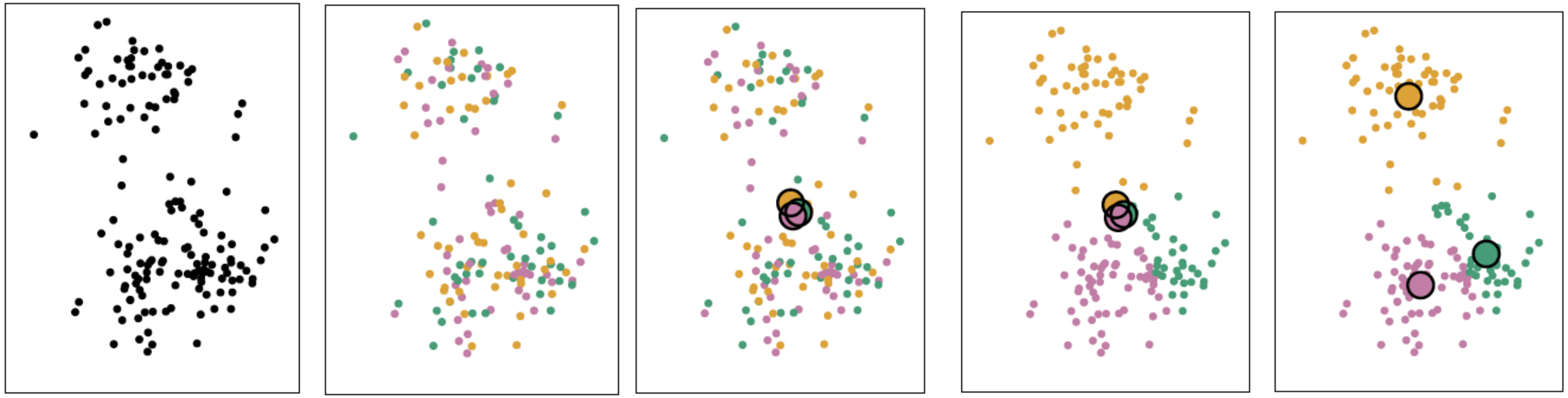

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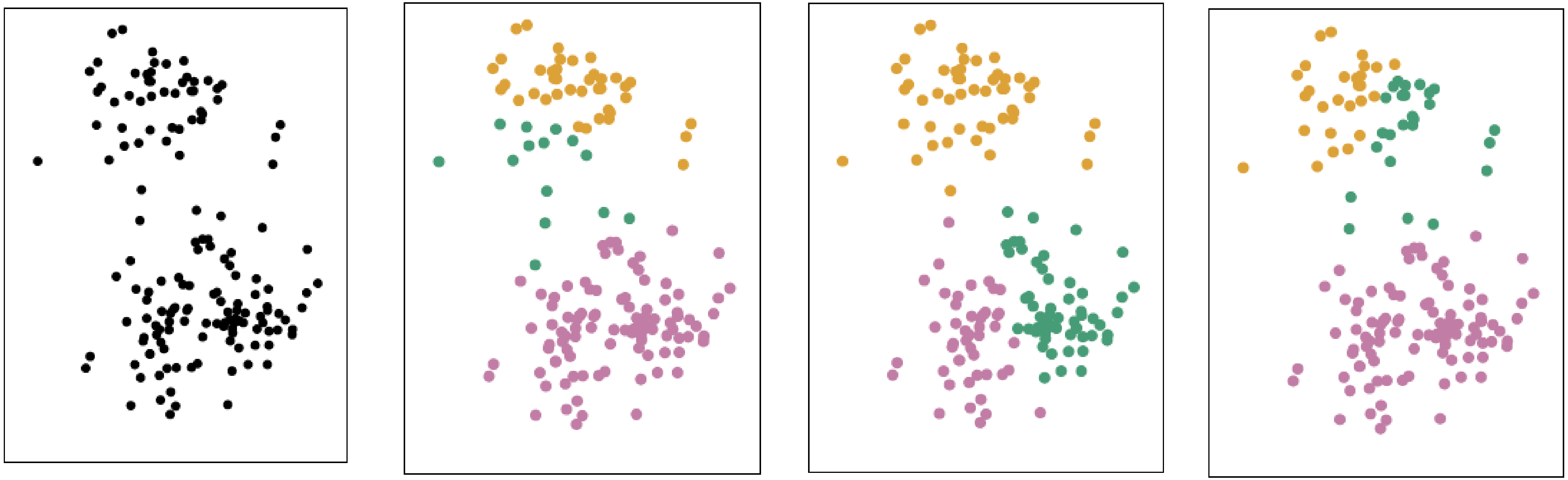

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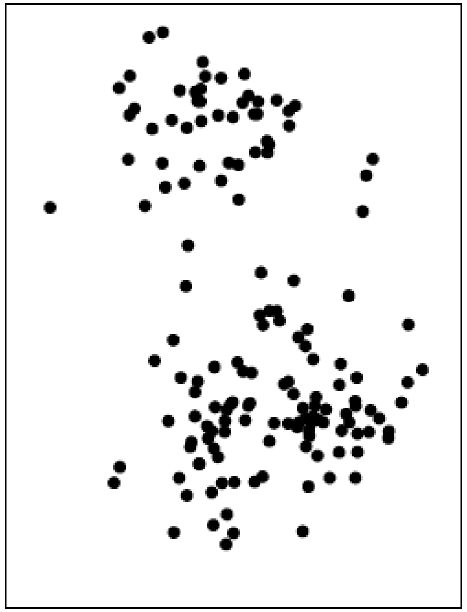

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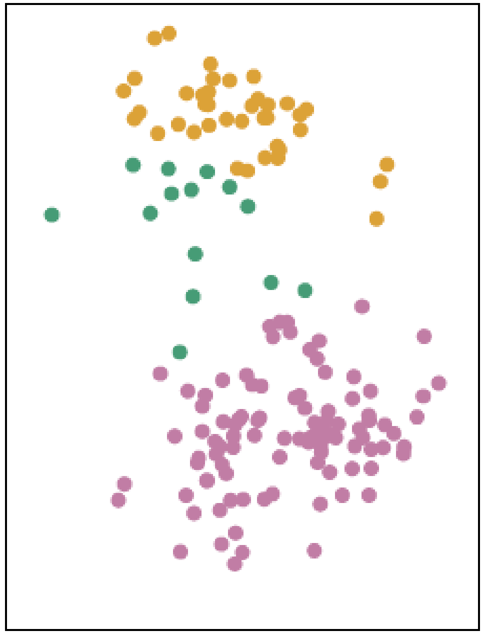

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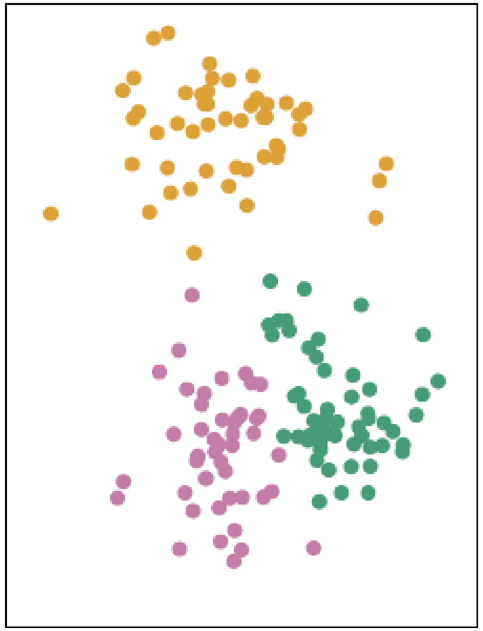

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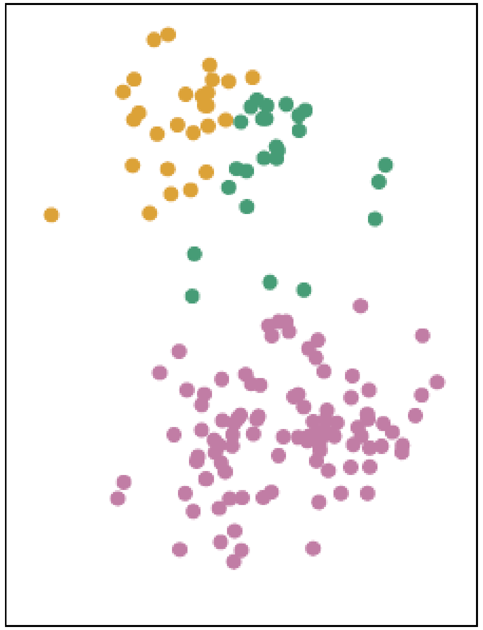

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2. Iterate until the cluster assignments stop changing:

   (a) For each of the K clusters, compute the cluster *centroid*. The kth cluster centroid is the vector of the p feature means for the observations in the kth cluster.

   (b) Assign each observation to the cluster whose centroid is closest (where *closest* is defined using Euclidean distance).

Readings

Required Readings:

Introduction to Statistical Learning

- Chapter 4 – Section 4.4 page 158 – 164
- Chapter 12 – Section 12.4 page 521 - 525

Thank You

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