Section 3 Actuators

3.1 Introduction

An actuator is a device that converts energy into a force or torque that produces motion. In this section we will cover common mechanisms, electric actuators, hydraulic actuators and pneumatic actuators.

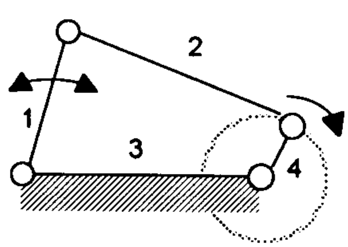
3.2 Mechanisms

Although mechanisms are not actuators themselves they are commonly used in mechatronic designs to transform motion or to provide a mechanical advantage. The actuator provides the input to the mechanism. A mechanism with mechanical advantage outputs a larger force or torque than that input by the actuator. Note that since the power output cannot be larger than the power input the increase in force is combined with a decrease in speed.

3.2.1 Four-bar Mechanisms and Cams

A rigid body used to connect two joints is termed a “link”. A common family of mechanisms uses four links and is termed “four-bar mechanisms”. These mechanisms employ either single axis rotary joints (or “pin joints”) or single axis linear translation joints (or “slider joints”). Changing the joints used, or the lengths of the links, produces different output motions for the same input motion. A link that is capable of making full rotation is termed a “crank”. Three important four-bar mechanisms are: crank-rocker, double-crank, and slider-crank.

The crank-rocker transforms rotary motion into oscillatory motion. An example is shown on the left of Figure 3.1. Continuous rotation of the crank (link 4) produces oscillatory motion of the output link (link 1, termed the “rocker”). A crank-rocker mechanism is used to produce the wiping motion of automotive windshield wipers. When the lengths of links 1 and 4 are equal, and the lengths of links 2 and 3 are equal, a “double-crank” mechanism is produced (see right side of Figure 3.1 for an example). With a double-crank mechanism the input and output links will both rotate fully and at the same speed. So one application of this mechanism is to transmit rotary motion to a different location. As the rotation occurs links 1 and 4, and 2 and 3, remain parallel to each other. This parallel motion is also useful in some applications. An example will be shown later in this section.



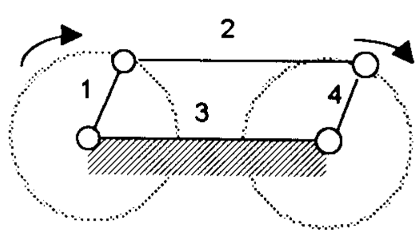
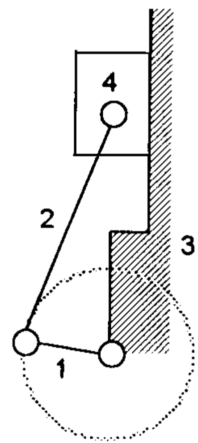


Figure 3.1 Kinematic diagrams for two types of four-bar mechanisms. Left: crank-rocker. Right: double-crank.

With a slider-crank mechanism one of the pin joints is replaced with a slider joint, as shown in Figure 3.2. A slider-crank mechanism is used to convert rotary motion to linear oscillatory motion or vice-versa. This mechanism has applications in pumps (input is link 1 and output is link 4) and in internal combustion engines (input is link 4 and output is link 1).



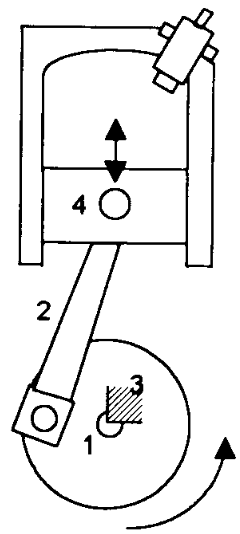


Figure 3.2 Left: Kinematic diagram for a slider-crank mechanism. Right: Application to an internal combustion engine.

A cam and cam follower is another mechanism that transforms rotary motion into oscillatory motion. An example is shown in Figure 3.3. The rotation of the cam is the input, and the output is the linear oscillatory motion of the follower. The shape of the cam can be changed to produce different output motion trajectories. Examples are shown in Figure 3.4. Cams and cam followers are used to open/close the valves for automotive engines.

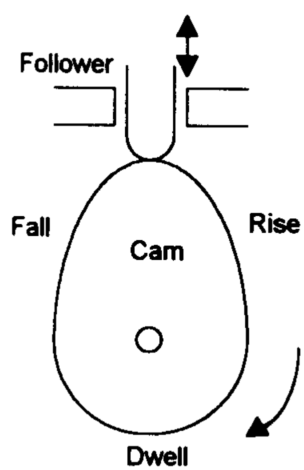


Figure 3.3 Cam and cam follower.

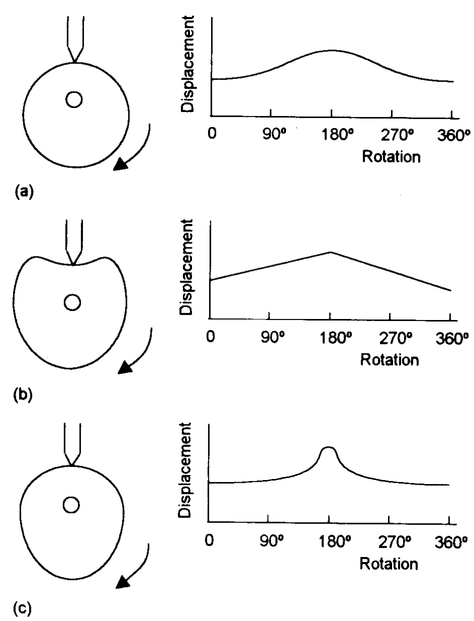


Figure 3.4 Cam examples.

Other types of mechanisms include gear trains, belt drives and chain drives. These may be used to transform rotary motion to rotary motion, or rotary motion to linear motion, or linear motion to rotary motion. More details will be provided later on.

3.2.2 Mechanisms for Conversion of Rotary to Linear and Linear to Rotary Motion

In mechatronic designs conversion from rotary to linear motion or vice-versa is often involved. One reason is that some actuators are limited to either rotary or linear motion. To provide linear motion, the best design might be a combination of a mechanism and a rotary actuator. A similar argument can be made when rotary motion is desired. Note that mechanisms may also be applied when designing a sensing system. There are four common mechanisms used to convert from rotary to linear motion (and vice-versa).

1) Lead screws and ball screws: A combination of a precisely made screw and nut. The nut is prevented from rotating. As a result the nut translates linearly along the screw when the screw is rotated. A ball screw employs ball bearings to convert the sliding contact between the screw and nut into rolling contacts. This greatly reduces both wear and the power lost to friction. A cutaway view of a ball screw and nut is shown in Figure 3.5.

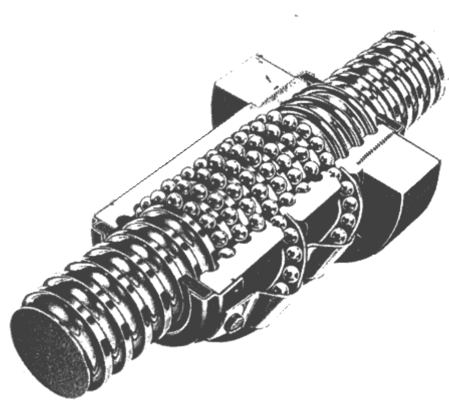


Figure 3.5 A cutaway view of a ball screw and nut.

Advantages: Ball screws have very good accuracy (a few microns) and a large motion range.

Disadvantages: High cost. Typically cannot be used to convert linear to rotary motion.

Example Applications: CNC machine tools and robots.

Governing Equations

The amount of linear motion per revolution is termed the "lead", l. Assuming the torque required to accelerate the screw is negligible then:

$$\tau = \frac{Fl}{(2\pi / rev)\eta_s} \tag{3.1}$$

where τ is the input torque, F is the load, l is the lead and η_s is the efficiency of the screw. If the friction of the screw is negligible then $\eta_s = 1$.

The equivalent inertia of a load connected to a ballscrew is:

$$J = M\left(\frac{l}{(2\pi / rev)}\right)^2 \tag{3.2}$$

where J is the equivalent rotational inertia of the mass M connected to the linear section of the screw.

Question: How will equation (3.1) change if the torque required to accelerate the screw is not negligible?

2) Rack and pinion:

This mechanism is illustrated in Figure 3.6. The governing equation for the motion is:

Linear motion per revolution = $(2\pi/rev)r_p$ (3.3)

where r_p is the radius of the pitch circle for the pinion. If the pinion is attached to the actuator then the force output is given by:

$$F_{out} = \frac{\tau_{in}}{r_p}\eta_{rp} \tag{3.4}$$

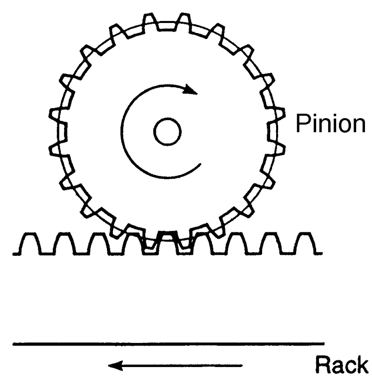


Figure 3.6 A rack and pinion mechanism.

where η_{rp} is the efficiency. If the friction of the rack and pinion is negligible then $\eta_{rp} = 1$. If the rack is attached to the actuator then the torque output is given by:

$$\tau_{out} = F_{in} r_p \eta_{rp} \tag{3.5}$$

The equivalent inertia of a load connected to the rack is:

$$J = Mr_p^2 \tag{3.6}$$

Advantages: Large motion range. Less expensive than ball screw. Can be used to convert rotary to linear motion and vice-versa.

Disadvantages: Less accurate than ball screw due to wear and backlash.

3) Timing belt

A timing belt is a belt with teeth similar to gear teeth that prevent the belt from slipping on its pulleys. This may be used with two toothed pulleys to convert rotary motion to linear motion (or vice-versa) in a manner similar to the rack and pinion. The payload is attached to the belt and the belt tension is usually maintained using a spring. Typically, one of pulleys is driven by a rotary actuator. An example is shown in Figure 3.7.

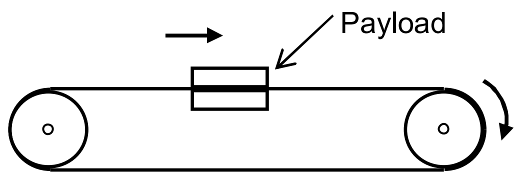


Figure 3.7 Use of a timing belt for motion conversion.

Advantages: Lowest cost solution.

Disadvantages: Only suitable for driving small loads due to the flexibility of the belt. The rubber belt teeth are sensitive to wear. Less accurate than ball screw and rack and pinion.

Example Applications: Photocopiers and desktop scanners.

4) Cam and cam follower

Advantages: Compact. Suitable for very high speed motion.
Disadvantage: Small motion range. Cannot be used to convert linear to rotary motion.

Example Application: Used in some robot grippers to open/close the jaws. See Figure 3.8 for an example.

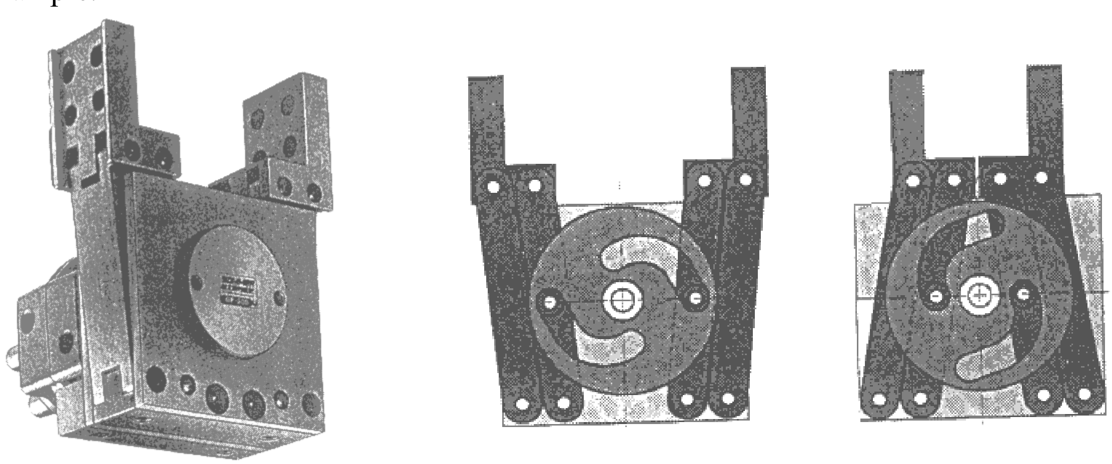


Figure 3.8 A robot gripper. The gripper is opened (middle image) and closed (right image) using a positive-return cam and two double-crank mechanisms. The double-crank mechanisms keep the jaws parallel to make the gripping action more effective.

3.2.3 Mechanisms Providing Mechanical Advantage

Electric motors are the most common actuators used in mechatronic designs. Electric motors have the disadvantage that they do not produce the high torques at low speeds needed for many applications. Therefore a mechanism providing mechanical advantage is necessary. The most common solution is to use a gearbox. Typically we want the gearbox to be compact, lightweight, and have low inertia. "Planetary" gearboxes and "harmonic drive" reducers are often used.

A planetary gearbox consists of an internal ring gear, planet gears, a planet carrier and a sun gear. The sun gear is the input and the planet carrier is the output. The internal ring gear is kept stationary. The reduction from a single stage is typically 5:1. See Figure 3.9 for an example. Several such stages can be combined to produce high reduction in a small package. The disadvantage of a planetary gearbox is the gear backlash introduces position errors.

A harmonic drive uses a completely different approach. It consists of a wave generator, a flexspline and a circular spline (see Figure 3.10). Normally the wave generator is the input, the flexspline is the output and the circular spline is kept fixed. As the elliptical shaped wave generator rotates it causes the flexspine to flex and rotate slowly relative to the fixed circular spline. A single stage harmonic drive can produce reductions of 300:1 and is lighter than a comparable planetary gearbox. Since the harmonic drive does not use gears it does not suffer from backlash, however the flexibility of the flexspline does lead to position errors. (If you're interested, more information on how harmonic drives work is available at: www.harmonic-drive.com).

To end this introduction, two examples showing the tremendous size range of gears that are produced are shown in Figure 3.11.

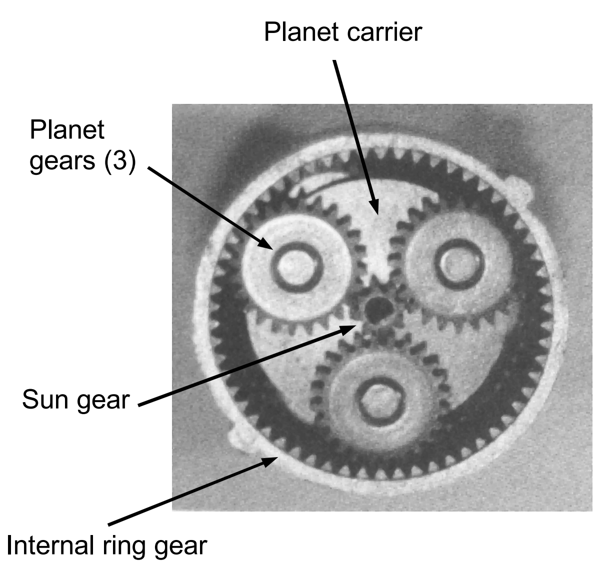


Figure 3.9 A typical planetary gearbox.

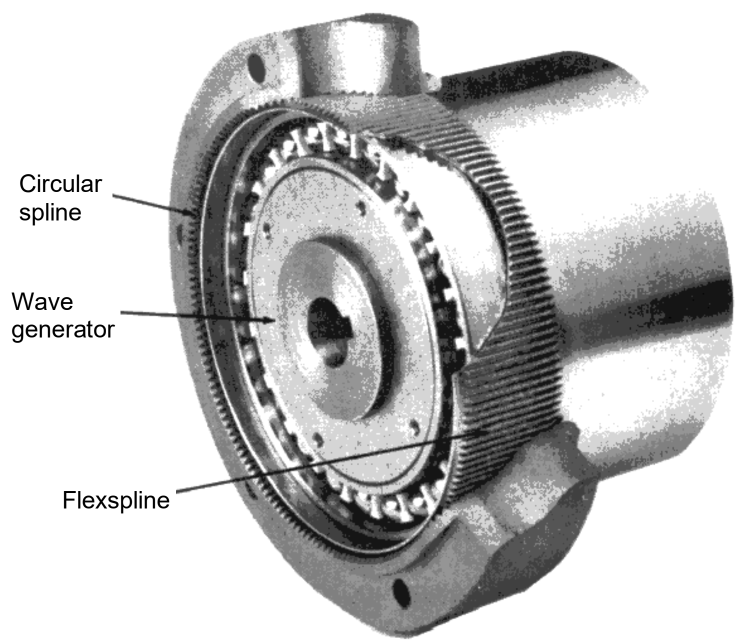
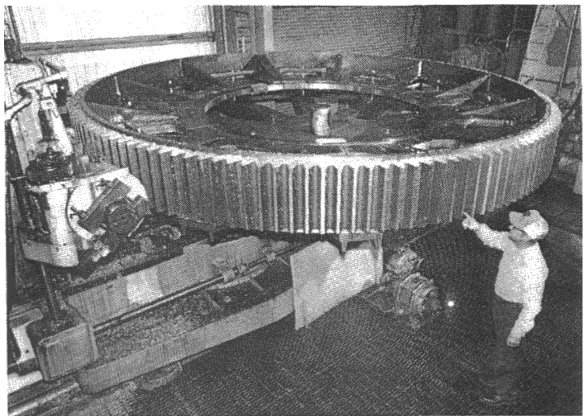


Figure 3.10 Cutaway view of a harmonic drive.



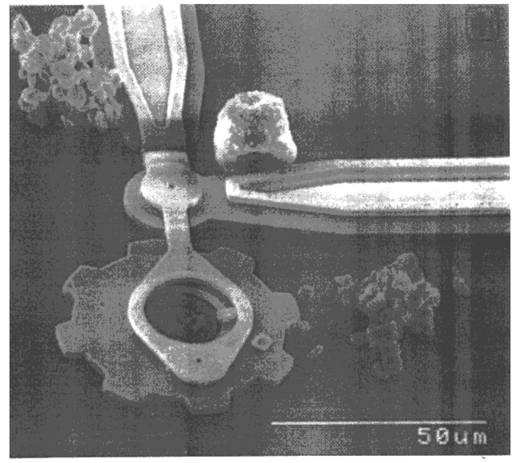


Figure 3.11 Gears come in many sizes. Left: a roughly 5 m diameter gear being machined after casting. Right: a roughly 5×10^{-5} m diameter gear that has been etched from silicon as part of a MEMS device (Note: the item at the centre of the picture is a grain of pollen).

Gearbox Governing Equations

Assuming friction losses are negligible, the output torque increases with a gear ratio greater than one, while the angular velocity and acceleration decrease. In equation form:

$$\omega_{out} = \frac{1}{N_r}\omega_{in} \quad (3.7)$$

$$\dot{\omega}_{out} = \frac{1}{N_r}\dot{\omega}_{in} \quad \text{and} \quad (3.8)$$

$$\tau_{out} = N_r \tau_{in} \eta_g \quad (3.9)$$

where N_r is the gear ratio, ω is the angular velocity, $\dot{\omega}$ is the angular acceleration, τ is the torque, and η_g is the efficiency. It should be noted that in many situations η_g is not a constant, e.g. it may vary with ω and/or due to changes in gear lubrication.

For the remainder of this section, we assume that the gearbox and coupling inertia is negligible relative to the load/motor, and that the gearbox is 100% efficient (i.e. $\eta_g = 1$). A motor is often connected to through a gearbox to a load as shown in Figure 3.12 (left). To properly design or select the motor it is necessary to find out what the required motor torque is. The required torque is the sum of the motor's own inertial load, and the external load scaled by the gearbox. The scaled external load is applied directly to the motor's shaft, and is known as the "reflected torque", see Figure 3.12 (right). The reflected torque is the combination of all inertial loads and non-inertial (e.g. friction or gravity) loads, as seen by the motor. In equation form it is:

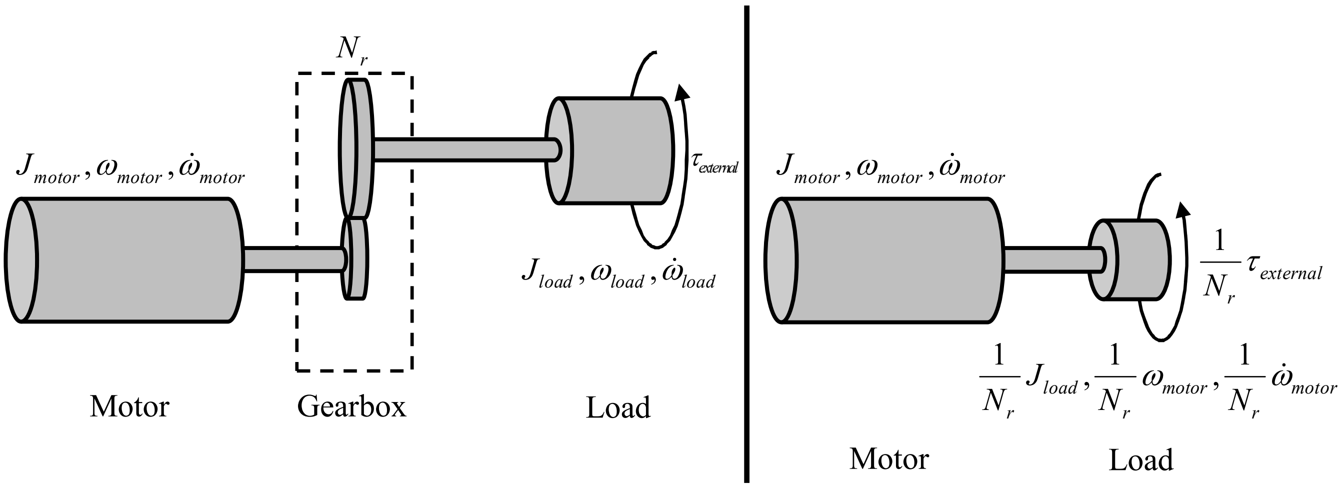


Figure 3.12 Left: Schematic of a motor connected to a load via a gearbox. Right: Load as reflected to the motor.

$$\begin{aligned}\tau_{reflected} &= \frac{1}{N_r}J_{load}\dot{\omega}_{load} + \frac{1}{N_r}\tau_{external} \\ &= \frac{1}{N_r}J_{load}\left(\frac{\dot{\omega}_{motor}}{N_r}\right) + \frac{1}{N_r}\tau_{external} \\ &= \frac{1}{N_r^{\,2}}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}\end{aligned} \tag{3.10}$$

In this case the reflected torque is comprised of the inertial torque of the load scaled by the gearbox, and the non-inertial torque of the load scaled by the gearbox. The load inertial torque includes a squared gear ratio, because both the inertia and the angular acceleration are written relative to the motor.

The total torque required of the motor is then the sum of the motor's inertial torque and the reflected torque as follows:

$$\begin{aligned}\tau_{motor} &= J_{motor}\dot{\omega}_{motor} + \tau_{reflected} \\ &= J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^{\,2}}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}\end{aligned} \tag{3.11}$$

3.3 Electrical Actuators

We will cover solenoids, voice coils, DC motors, and stepper motors. Examples will be shown in class.

1) Solenoids: These devices consist of a coil and a soft iron core. When current flows through the coil the core is pulled inwards.

   Advantages: Fast response, simple design, compact and do not wear.

   Disadvantages: The output force and range of motion are small. The output force is unidirectional (A spring or a 2nd actuator is needed for the other direction).

   Control: On/off.

   Applications: Commonly used to actuate pneumatic and hydraulic valves.

2) Voice coils: These consist of a permanent magnet (PM), a stationary iron core and a movable coil, as shown in Figure 3.13. The iron core intensifies the magnetic field produced by the PM.

   Advantages: Bidirectional force output, fast response, simple design, compact and do not wear.

   Disadvantages: Small motion range. Higher cost due to PM.
   Control: The output force is proportional to the current through the coil so open-loop force control is possible. Continuous control of the velocity or position is also possible but requires sensor feedback (*i.e.* closed-loop control).

   Applications: Positioning of read/write head of hard disk drive, control of the spool position with hydraulic or pneumatic servo valves.

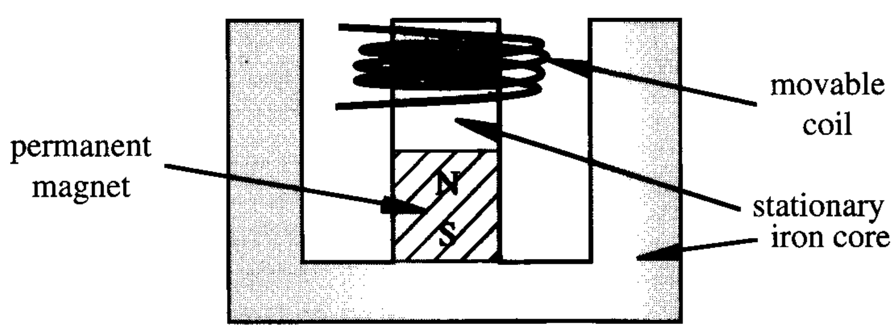


Figure 3.13: Voice Coil

3) DC Motors: In mechatronic designs, PM DC brush motors and PM DC brushless motors are commonly used. The structure of a PM DC brush motor is shown in Figure 3.14. In the past a set of windings was required to generate the magnetic field strength needed to produce a large torque. Recent improvements in magnetic materials have allowed these "field windings" to be replaced with PMs. This eliminates the heat generated by the field windings. A PM DC brush motor includes a stationary "stator" consisting of PMs, a rotating armature or "rotor" consisting of several coils, and a "commutator". The commutator uses a split ring and metal or graphite brushes to switch the direction of current flow through the armature coils. This creates a magnetic attraction that drives the rotation.

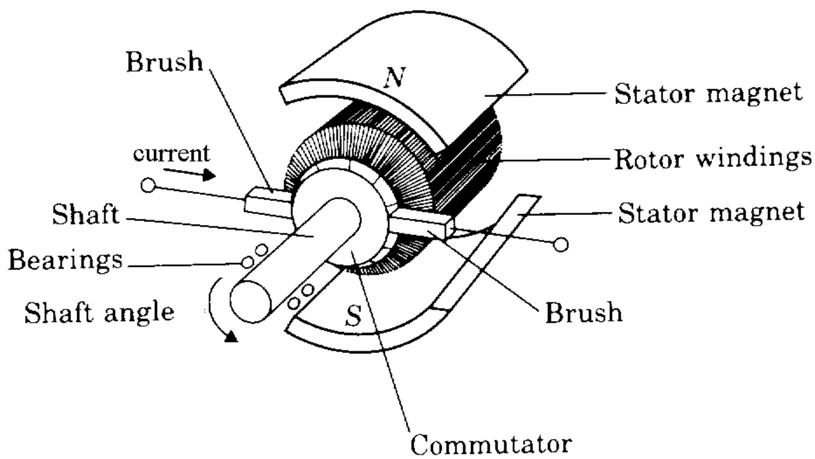


Figure 3.14 The components of a PM DC brush motor.

Advantages of PM DC brush motors: Relatively easy to control and interface.

Disadvantages of PM DC brush motors: Relatively low torque output. The brushes wear, cause friction losses, and can cause arcing. The arcing creates electrical noise and can be a safety problem in some applications (*e.g.* painting robots).

A newer option is the PM DC brushless motor. With a brushless motor the rotor consists of permanent magnets and the stator consists of several coils that are switched using external electronics to produce the rotation. The basic structure is shown in Figure 3.15. To ensure proper switching the rotor position must be sensed using an absolute encoder or several hall effect sensors.

Advantages of PM DC brushless motors: Relatively easy to control and interface. No problems due to brushes, and greater reliability than brush motors. Better commutation allows higher speeds (up to 100 KPRM vs. 15 KRPM for brush type). They can produce higher continuous torque than brush motors since the windings are attached to the motor case, producing much better heat dissipation.

Disadvantages of PM DC brushless motors:

- More expensive than brush type.
- More difficult to produce a constant torque output than brush type. This torque variation is known as "torque ripple" and can produce oscillations in the motor speed. This is mainly a problem for low speed motion. At high speeds the increased momentum of the rotor makes it less sensitive to torque ripple. The torque ripple can also be reduced by using a more advanced (and more expensive) amplifier and/or controller.

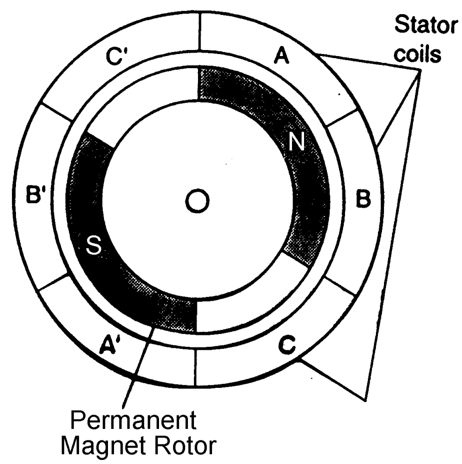


Figure 3.15 Basic structure of a PM DC brushless motor.

Control: DC motors are relatively easy to control because:
(i) The output torque is roughly proportional to the input current and
(ii) For a constant load torque, the speed is roughly proportional to the input voltage.
Continuous control of torque, velocity and position is possible.

Applications: DC motors are used for positioning in numerous mechatronic devices, such as robots, machine tools, and video cameras.

Mathematical Modelling of PM DC Motors

A mathematical model is useful at this stage for helping to design mechatronic systems that use PM DC motors. We will cover the derivation of this model later in the course. The electrical elements are modelled by the equation:

$$V_a = K_b \omega + L_a \frac{di_a}{dt} + R_a i_a \qquad (3.13)$$

where: $V_a \equiv$ armature voltage, $K_b \equiv$ back emf constant, $\omega \equiv$ angular velocity $L_a \equiv$ armature inductance, $R_a \equiv$ armature resistance and $i_a \equiv$ armature current. The mechanical elements are modelled by the equation:

$$J\frac{d\omega}{dt} = K_t i_a - K_d\omega - \tau_{load} \tag{3.14}$$

where: $J \equiv$ moment of inertia of the motor, $\tau_{load} \equiv$ torque due to the load, $K_t \equiv$ torque constant and $K_d \equiv$ damping constant (due to viscous friction).

Example 3.1

Say we need to accelerate a 50 kg mass at 20 m/s^2 using a linear actuator consisting of a DC motor and a ball screw with a 15 mm lead.

(a) Assuming that friction and the moment of inertia of the ball screw are negligible, what motor torque is required?

(b) If the maximum velocity is 2 m/s, what is the maximum motor rpm?

Solution

(a) From Newton's second law:

$$F = ma = (50kg)(20m/s^2) = 1000N$$

From equation (3.1), where η_s=1 (based on the given assumptions):

$$\tau = \frac{Fl}{(2\pi/rev)\eta_s} = \frac{(1000\ N)(15\ mm/rev)(1\ m/1000\ mm)}{(2\pi/rev)1} = 2.4\ Nm$$

(b) For a ball screw, the linear velocity equals the product of the angular velocity and the lead:

$$v = \dot{\theta}\, l$$

So we have:

$$\dot{\theta} = \frac{v}{l} = \frac{(2\ m/s)(60\ s/min)}{0.015\ m/rev} = 8000\ rev/min \text{ or } rpm$$

Example 3.2

We are analysing an actuator consisting of a PM DC motor driving a ballscrew. The ballscrew has a lead of 2 mm. Assume the friction and moment of inertia of the ballscrew may be neglected. We wish to drive a load of 500 N at a speed of 0.25 m/s. The motor parameters are (Maxon motor, model 148877):
J=135 gcm^2, K_b=0.007 V/rpm, K_t=63.7 mNm/A, L_a=0.33 mH, R_a = 1.16 Ohm and K_d=3.5 × 10^{-5} Nm/rpm.

(a) What armature current and armature voltage will be required?
(b) The maximum continuous voltage rating is 48 V and the maximum continuous current rating is 3.6 A for this motor. Assuming the load must be driven for an extended period of time, are we within these ratings?

Solution

From the given information:

$$\tau_{load} = \frac{Fl}{(2\pi / rev)\eta_s} = \frac{(500N)(2mm/rev)(1m/1000mm)}{2\pi / rev} = 0.159Nm$$

$$\omega = \frac{v}{l} = \frac{(0.25\ m/s)(60\ s/min)}{0.002\ m/rev} = 7500 \text{ rpm}$$

Since the speed and load are constant, from equations (3.13) and (3.14), V_a and i_a will be constant. Since the speed is constant $\frac{d\omega}{dt} = 0$ and equation (3.14) gives:

$$0 = K_t i_a - K_d \omega - \tau_{load}$$

$$K_t i_a = K_d \omega + \tau_{load}$$

$$i_a = \left(K_d \omega + \tau_{load}\right) / K_t$$

= ((3.5 × 10^{-5} Nm/rpm)(7500 rpm) + 0.159 Nm) / (63.7 × 10^{-3} Nm/A)
= 6.62 A

Since i_a is constant $\frac{di_a}{dt} = 0$ and equation (3.13) is reduced to:

$$V_a = K_b \omega + R_a i_a$$

= (0.007 V/rpm)(7500 rpm) + (1.16 Ohm)(6.62 A)
= 60.2 V

(b) Based on our answer to (a), we will exceed the continuous current and voltage ratings. Therefore we must either reduce the load and/or speed, or select a more powerful motor.

More Information about the Position Control of PM DC Motors

Position control of a PM DC motor is only possible using closed-loop control. An incremental encoder is the most common source of position feedback. The control algorithm is typically executed on a μC, and the output from the μC is usually a Pulse-Width-Modulated (PWM) signal. With PWM the width of a high pulse is varied relative to a fixed period. This is illustrated in Figure 3.16a.

We will term the high voltage V_{high} and the low voltage V_{low}. If T_{high}=Period (the pulse width equals the period) then the output voltage simply equals V_{high}. If T_{high}=0 the output voltage equals V_{low}. But if $0<T_{high}<Period$ we have:

$$\begin{aligned} V_{out} &= V_{average} + High\ frequency\ signal \\ &= \frac{T_{high}V_{high} + (Period - T_{high})V_{low}}{Period} + High\ frequency\ signal \end{aligned} \tag{3.15}$$

In other words we can adjust the average level of the output voltage by adjusting T_{high}. This is a very convenient form of digital to analog conversion since no additional hardware is required. Equation (3.15) is illustrated for the $T_{high} = \frac{1}{2}$ Period case in Figure 3.16b. Note that the frequency response of the motor will filter out the high frequency component of V_{out}.

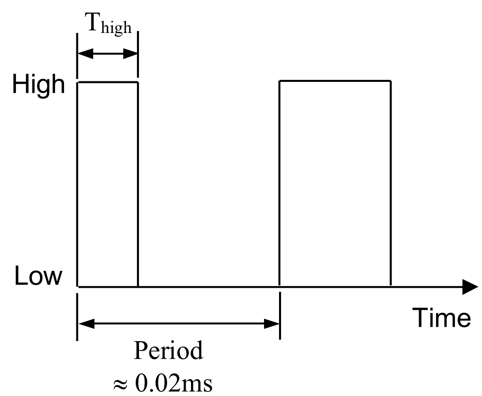


Figure 3.16a Pulse-width-modulation (PWM).

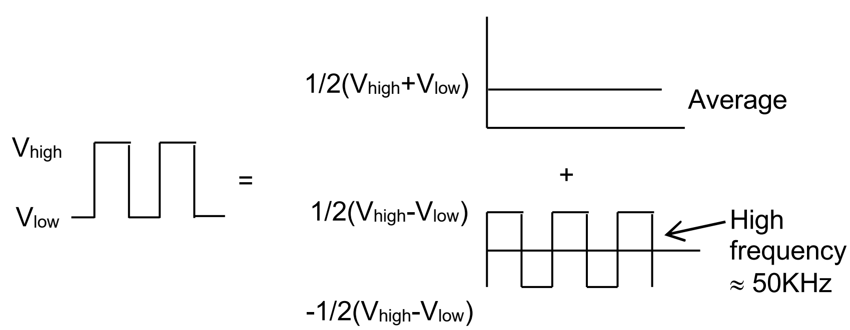


Figure 3.16b PWM example.

Advantages of PWM: Inexpensive. May be used to switch the transistors in a "switching amplifier". Switching transistors on/off is more energy efficient that operating them in their linear region. (The same idea is used with the switching power supplies used with computers).

Disadvantages of PWM: The high frequency component is actually high frequency noise. This can be picked up by other elements of the control system (such as sensors) and affect the control performance.

4) Linear Motors

A linear motor is a DC, AC, or stepper motor whose stator has been "unwrapped" into a linear form. This idea is shown in Figure 3.17.

Advantages:
-Capable of achieving very high acceleration and range of speeds (0.02mm/s - 6m/s).
-Can be made non-contact if brushless or stepper design is used with an air bearing (no friction or wear).
-Can be 2-D (planar).
-No backlash.
-Smaller size for the same range of motion (see Figure 3.18).
-Better reliability.

Disadvantages:
-High cost.
-Sensitive to dirt due to open design (need bellow type covers or forced air).
-To increase the range of motion the size of the motor must be increased leading to increased cost (and not just length of ball screw or other mechanism for rotary to linear motion conversion).
-No mechanical advantage from ball screw (or other mechanism) to help provide force.

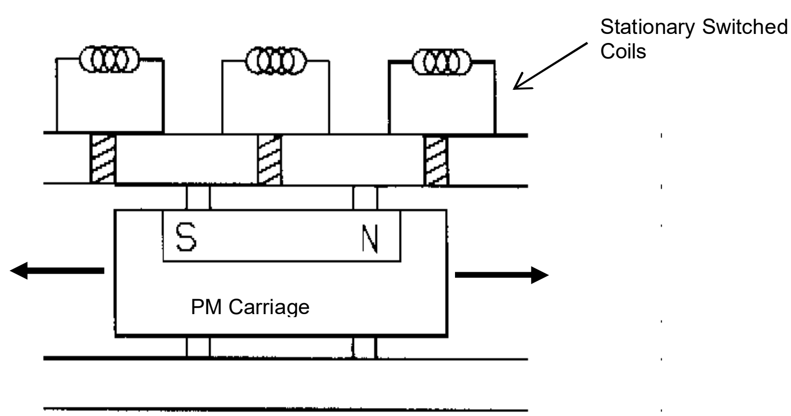
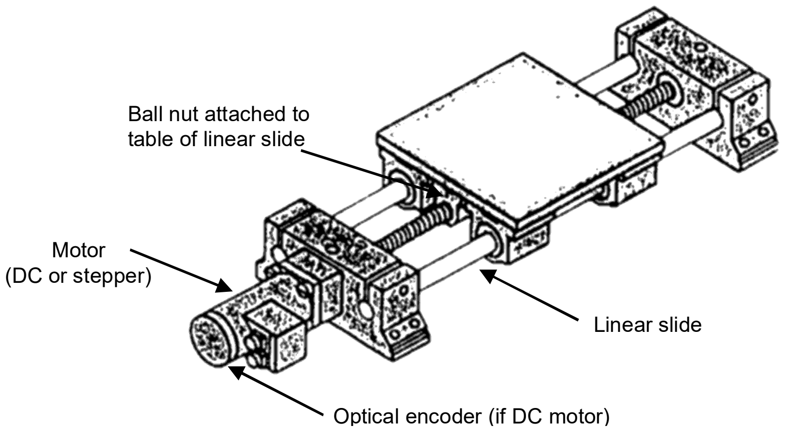


Figure 3.17 One form of linear motor.



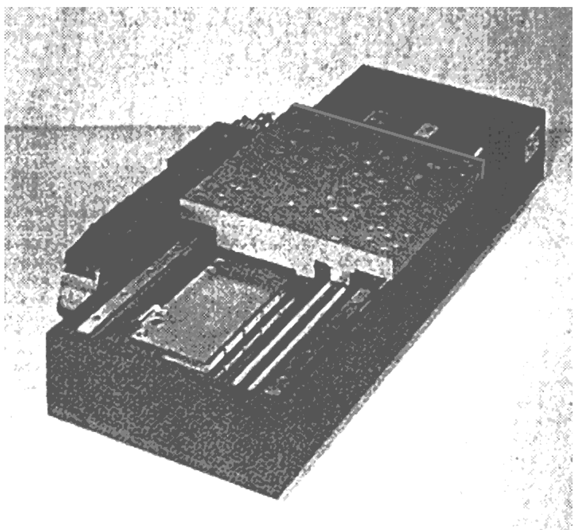


Figure 3. 18 Comparison of linear actuators. Top: A linear actuator consisting a motor, encoder, ball screw and linear slide. Bottom: When a linear motor is used, the motor, encoder and linear slide are closely integrated, resulting in a more compact actuator.

5) Stepper Motors (also known as Stepping Motors)

A stepper motor moves in series of small angular increments or angular "steps". A step by step movement is created by energising the stator windings in a particular sequence. There are three types of stepper motors: PM stepper motor, variable reluctance (VR) stepper motors, and hybrid stepper motors.

PM Stepper Motors

With this type of motor the rotor is a PM. The rotation is accomplished by electronically switching the polarity of the stator windings. There are three possible operating modes:

i) Full stepping mode

In this mode the rotor moves by one full step at a time. The energising sequence is:

1) reverse the polarity of a pair of poles.
2) rotor moves to its equilibrium position (midway between the poles).
3) reverse the polarity of the next pair of poles.

…

This mode is illustrated in Figure 3.19 for motor with a resolution of 4 full steps/rev. This motor has two pairs of poles, (A_1, A_2), and (B_1, B_2). The resulting step angle is 90°.

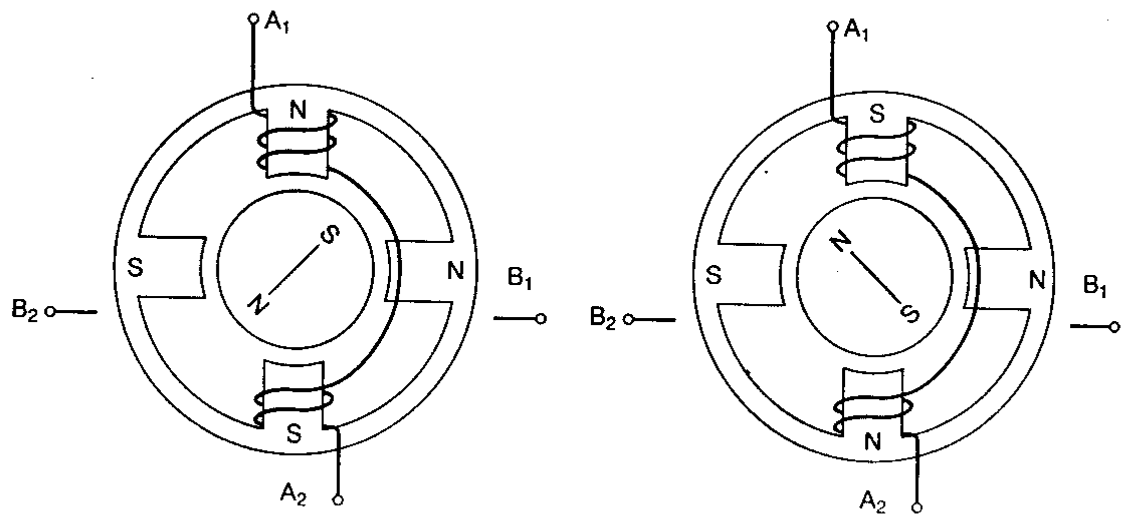


Figure 3.19 Example of full stepping mode. Left: Starting position. Right: Reversing the polarity of one pair of poles causes the rotor to rotate clockwise by a full step.

ii) Half stepping mode

If one pair of windings is shut off before the next full step the equilibrium position for the rotor will be ½ way between the full step positions. If this is action is added to the energising sequence the step angle will be halved. This is illustrated for the 4 full steps/rev motor in Figure 3.20. Note that the resulting step angle is 45°. The disadvantage of half stepping is that the average torque output is reduced since the windings are shut off part of the time.

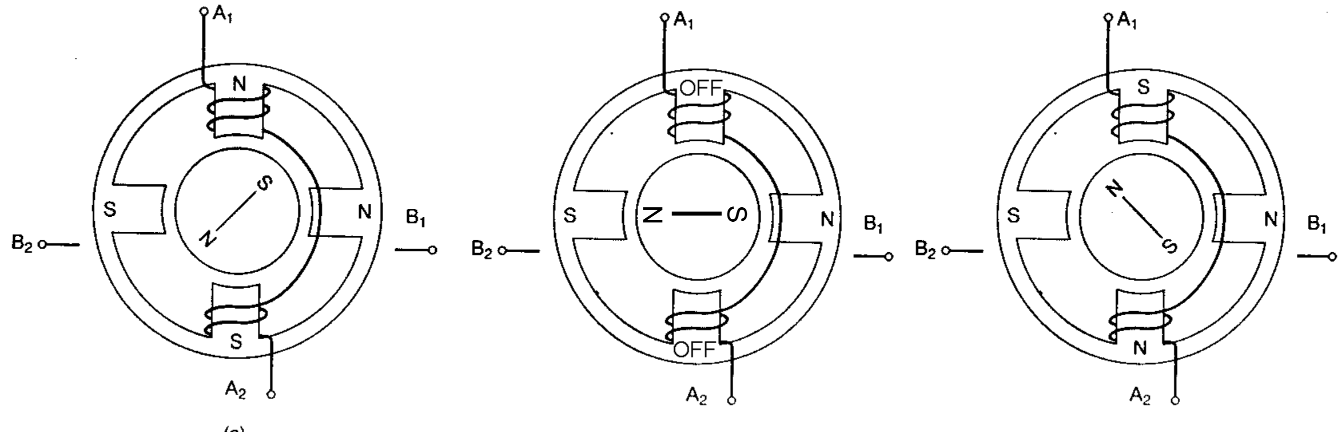


Figure 3.20 Example of half stepping mode. Left: Starting position. Centre: Shutting off a pair of poles causes the rotor to move 1/2 step clockwise. Right: Re-energising the poles with reversed polarity cause the rotor to rotate clockwise another 1/2 step.

ii) Microstepping

In this mode the equilibrium position of the rotor is changed by smaller increments by decreasing the current in one pair of energised poles. This allows the step angle to be decreased up to 250 times less than a full step. Like half stepping this mode produces lower torque than full stepping. It has the further disadvantage that a more complex and expensive controller is required.

VR Stepper Motors

The rotor of a VR stepping motor is made from soft iron. The rotor and the stator poles are cut with small grooves to form "teeth". See the example shown in Figure 3.21. When the poles are energized the rotor moves to the position of lowest reluctance (reluctance is analogous to the resistance of a magnetic circuit). This is a function of the alignment of the teeth on the rotor and on the poles. The step angle equals 360° divided by the product of the number of rotor teeth and the number of pairs of poles (also called the number of phases). So if the two phase motor we saw earlier was used with a 50 toothed iron rotor then the full step angle would be 360°/((2 phases)(50 teeth)) = 3.6°.

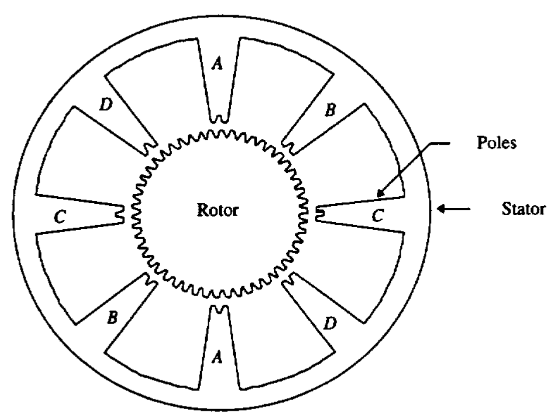


Figure 3.21 A VR stepper

Hybrid Stepper Motors

While VR stepper motors offer better resolution than PM stepper motors they generally also produce a lower output torque. Hybrid stepper motors combine the best features of the two designs and are typically the preferred choice. A resolution of 200 full steps/rev is common.

Note: Full stepping, half stepping and microstepping may be used with all three types of stepper motors.

Control of Stepper Motors

Control of the position, velocity and acceleration is accomplished by timed switching of the poles. Note that this control is performed open-loop.

Advantages of Stepper Motors

- Less expensive and less complex than DC motors since the control is open-loop (no sensing or complex controller is required).
- No potential for closed-loop instability.

Disadvantages of Stepper Motors

- The torque is less than a DC motor of the same size.
- Because the control is open-loop care must be taken so that the controller and motor remain synchronised. For example if the motor is moving and the controller tries to decelerate it too quickly the motor will continue to move and will move farther than the controller expects.
- Smaller acceleration and payloads must be used as a result of the open-loop control and limited torque.
- Not useful in the presence of large disturbances.
- Motion is less smooth than DC motors.
- Lower efficiency than DC motors

Applications: Any positioning application that does not involve large loads or large accelerations, *e.g.* 3D printers and desktop scanners.

3.4 Motor/Gearbox Selection and Optimization

Actuators are often combined with one or more mechanisms to provide the desired motion and/or mechanical advantage. For example, with many mechatronic systems an electric motor is combined with a gearbox (also called a "gearhead"). In the following subsections we will look at methods for effectively selecting the motor and gearbox.

3.4.1 Maximizing Energy Efficiency During Steady State Operation

Maximizing energy efficiency is desirable for reducing cost, and for improving sustainability. In battery powered mechatronic systems using PM DC motors, such as mobile robots, it is particularly important. When the motor armature voltage and the load torque are constant, the motor's speed will be constant, and the motor is operating at steady state. Under these steady state operating conditions, $\frac{d\omega}{dt}=0$ and $\frac{di_a}{dt}=0$, so (3.13) and (3.14) simplify to:

$$V_a = K_b\omega + R_a i_a \text{ and} \tag{3.16}$$

$$\tau_{load} = K_t i_a - K_d\omega \tag{3.17}$$

If we assume that the motor's friction is negligible (i.e. $K_d \approx 0$), then (3.17) can be further simplified to:

$$\tau_{load} = K_t i_a \tag{3.18}$$

The motor efficiency is given by:

$$\begin{aligned}\eta_{motor} &= \frac{\text{mechanical power output}}{\text{electrical power input}}\\ &= \frac{\tau_{load}\omega}{V_a i_a}\\ &= \frac{(K_t i_a)\omega}{(K_b\omega + i_a R_a) i_a}\\ &= \frac{K_t i_a \omega}{K_b i_a \omega + i_a^2 R_a}\end{aligned} \tag{3.19}$$

Examining (3.19), it is clear that operating the motor at its maximum speed will provide the maximum efficiency. However, if K_d cannot be neglected then the power lost to friction will increase as ω increases so the maximum efficiency will occur at a lower speed.

When the motor is connected is to a gearbox (and $K_d \approx 0$) the efficiency becomes:

$$\begin{aligned}\eta_{motor+gearbox} &= \eta_{motor}\eta_g\\ &= \frac{K_t i_a \omega}{K_b i_a \omega + i_a^2 R_a}\eta_g\end{aligned} \tag{3.20}$$

If we assume that a gearbox has already been selected, and its efficiency is constant, then once again the efficiency is maximized at maximum speed. However, this situation is unlikely to occur in practice. The efficiency of a gearbox is not usually constant (as previously discussed), plus a gearbox and motor are usually selected at the same time. The motor/gearbox combination must satisfy the speed and torque requirements of the application before efficiency can be considered. Since motors only come in certain sizes, and only a finite set of gear ratios are available, only a limited number of motor/gearbox combinations will satisfy those requirements. In general, the larger the gear ratio the lower is the gearbox efficiency. Unfortunately the relationship between N_r and η_g cannot be effectively modelled. It is still true that operating a motor at a higher speed will make it more efficient, but this will require a gearbox with a larger N_r and therefore a lower η_g, to meet the requirements of the application. So the best efficiency ($\eta_{motor+gearbox} = \eta_{motor}\eta_g$) will occur when the motor runs at an intermediate speed. If sufficient information is available from the motor and gearbox suppliers then this optimal combination of components and steady state motor speed can still be determined by numerically analyzing all of the combinations, but it cannot be solved using equations.

3.4.2 Optimal Gear Ratio for Maximum Load Acceleration and Maximum Power Transfer

Many mechatronic systems require moving a mass rapidly from one stationary position to another stationary position, e.g. robots, CNC machines, and DVD players. This requires an actuator capable of large acceleration (and deceleration). In this section we will look at maximizing the acceleration of a motor plus gearbox.

We will consider is a motor connected to a load through a gearbox, with the following assumptions:

- The load is inertial only,
- The gears have negligible inertia, and
- Friction is negligible.

As we saw in section 3.2.3, the torque required of the motor is then given by the sum of the motor's inertial torque, and the inertial torque of the load scaled by the gearbox, as follows:

$$\tau_{motor} = \dot{\omega}_{motor} J_{motor} + \left(\frac{1}{N_r}\dot{\omega}_{motor}\right)\left(\frac{1}{N_r}J_{load}\right) \tag{3.21}$$

Since we are maximizing the load acceleration, this value is written with respect to the load angular acceleration:

$$\tau_{motor} = N_r \dot{\omega}_{load} J_{motor} + \dot{\omega}_{load}\left(\frac{1}{N_r}J_{load}\right) \tag{3.22}$$

And finally, rearranging for acceleration

$$\dot{\omega}_{load} = \frac{\tau_{motor}}{N_r J_{motor} + \left(\frac{1}{N_r}J_{load}\right)} \tag{3.23}$$

It can be seen that by choosing N_r to minimize the denominator of (3.23), the load acceleration will be maximized. Taking the first derivative of the denominator, and setting it equal to zero gives:

$$\frac{\partial}{\partial N_r}\left(N_r J_{motor} + \left(\frac{1}{N_r} J_{load}\right)\right) = \left(J_{motor} + \left(\frac{-1}{N_r^{\,2}} J_{load}\right)\right) = 0 \qquad (3.24)$$

Which has the solution:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} \qquad (3.25)$$

Note that it is necessary to check the second derivative at this point to determine if we have found a maximum or a minimum. The result is:

$$\left.\frac{\partial^2}{\partial N_r^{\,2}}\left(N_r J_{motor} + \left(\frac{1}{N_r} J_{load}\right)\right)\right|_{N_r = N_{r,opt}} = \frac{2J_{load}}{\left(\frac{J_{load}}{J_{motor}}\right)^{3/2}} \qquad (3.26)$$

Since the inertias must be positive, (3.26) tells us that the second derivative will always be positive at this point, and thus $N_{r,opt}$ has minimized the denominator of (3.23). Therefore the angular acceleration is maximized and we conclude that the gear ratio at which the angular acceleration will be maximized is indeed given by (3.25).

It can also be shown that in the case of combined inertial and constant torque loads, given by:

$$\dot{\omega}_{load} = \frac{\tau_{motor} - \frac{1}{N_r}\tau_{external}}{N_r J_{motor} + \left(\frac{1}{N_r} J_{load}\right)} \qquad (3.27)$$

That the result of the first and second derivatives (of the entire fraction) yield the same equation for optimal N_r, namely:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}}$$

This relation will give an exact value for the gear ratio, though in reality only a finite number of specific gear ratios are available. The closest smaller available gear ratio should be selected. A general rule of thumb is to keep the inertia ratio, given by (3.28), as close to 1 as possible, staying within the range 1 to 10. This is known as “inertia matching”. An inertia ratio larger than 10 can produce an unstable system due to the interaction of the small mechanical flexibility between the motor and load, their inertias and the position control loop.

$$Ratio_J = \frac{J_{load} / N_r^{\,2}}{J_{motor}} \qquad (3.28)$$

Lastly, it can be shown that $Ratio_J = 1$ will also maximize the mechanical power transmitted to the load which is relevant for steady state operation (i.e. constant speed).

3.4.3 Motor/Gearbox Selection for Dynamic Loads

As previously mentioned, many mechatronic applications involve dynamic operation in which velocities, accelerations and torques are required to change magnitude, direction and duration. The position, velocity and acceleration profiles for a simple operating cycle are shown in Figure 3.24. This cycle has four periods: constant acceleration ($0 \le t < 0.25$ s), constant velocity ($0.25 \le t < 0.75$ s), constant deceleration ($0.75 \le t < 1$ s) and idle ($1 \le t < 1.5$ s).

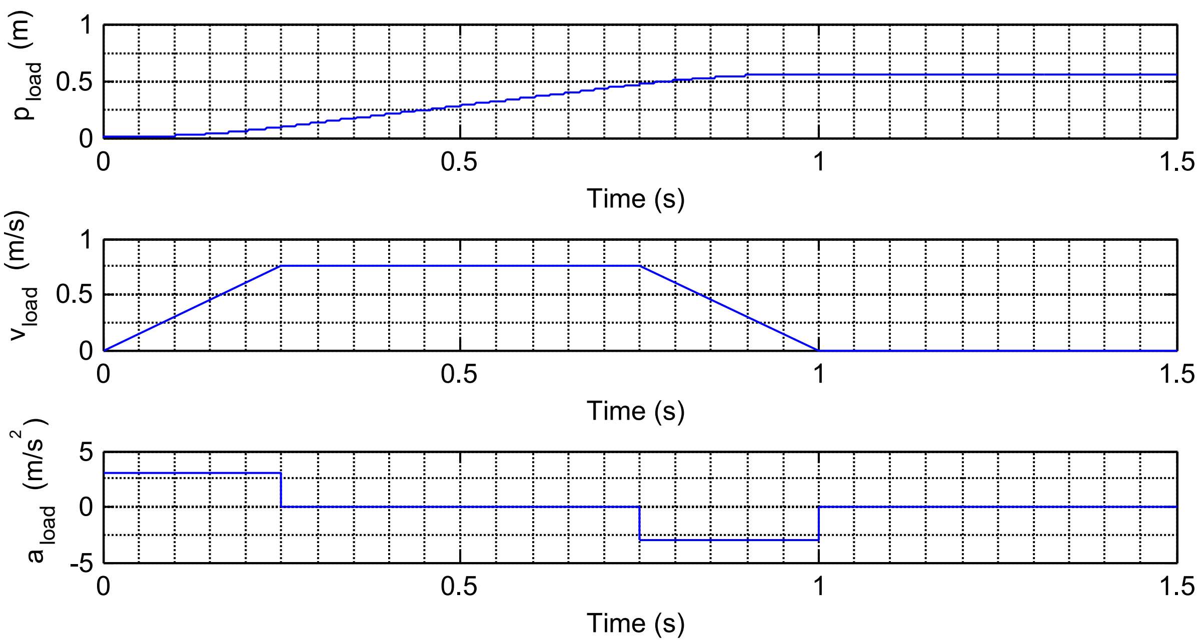


Figure 3.24 Example of position, velocity and acceleration profiles for an operating cycle with four periods.

The next step is to calculate the required motor torques and speeds from the operating cycle using standard physics equations, and the relevant equations from subsections 3.2.2 and 3.2.3. The results will be the motor velocity and motor torque for each period. To check that the motor is suitable for the application we need to calculate the maximum required motor torque, $\tau_{motor,\,max}$, and the maximum required motor speed, $\omega_{motor,\,max}$. It also may be necessary to calculate the RMS motor torque, $\tau_{motor,\,RMS}$. For a motor torque profile that is a piecewise constant function[1], the RMS value may be calculated using:

[1] The acceleration profile shown in Figure 3.24 is an example of a piecewise constant function. Finding the RMS value of more complex motor torque profiles is beyond the scope of this course.

$$\tau_{motor,RMS} = \sqrt{\sum_{i=1}^{n} \tau_{motor,i}^{2} t_i \Big/ \sum_{i=1}^{n} t_i} \tag{3.33}$$

where $\tau_{motor,\,i}$ is the torque for the i^{th} period, t_i is the duration of the i^{th} period and n is the number of periods.

A motor can operate at its maximum rated torque, $\tau_{rated,\,max}$, intermittently (i.e. for short periods of time followed by periods operating at a lower torque). Motors can also produce a smaller torque, $\tau_{rated,\,cont}$, continuously. If the motor is operated above its continuous torque rating ($\tau_{rated,\,cont}$) for too long it will be thermally damaged.

A motor is capable of producing different intermittent and continuous torques based upon its operating speed. These are characterized by performance curves, such as the one shown in Figure 3.25a for a DC motor.

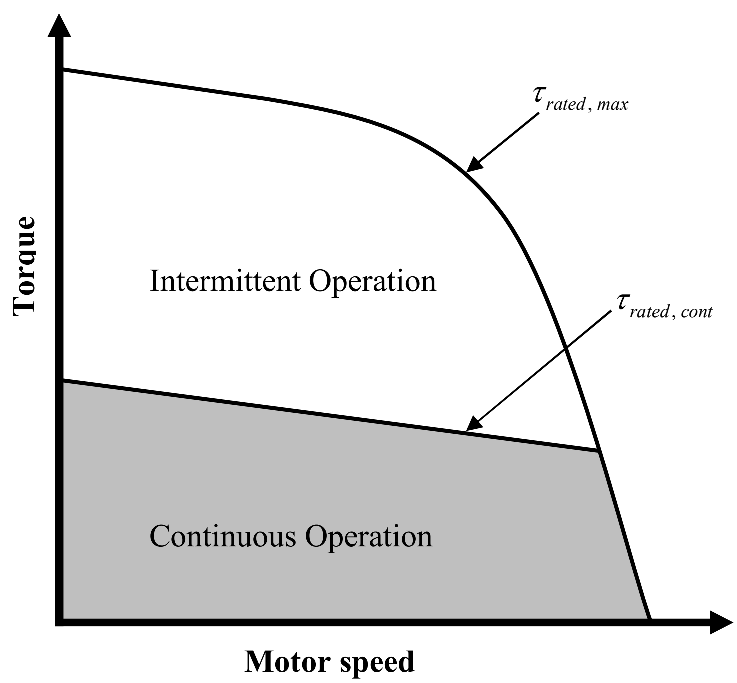


Figure 3.25a Typical DC motor performance curve

For a DC motor, if the point $\left(\omega_{motor,\,max}, \tau_{motor,\,RMS}\right)$ lies within the region of continuous operation, and the point $\left(\omega_{motor,\,max}, \tau_{motor,\,max}\right)$ is below the boundary of intermittent operation, then the motor/gearbox combination is acceptable. Note that the torques are checked at $\omega_{motor,\,max}$ to be conservative. If one of the points lies outside its acceptable region then the design is unacceptable. It may be possible to shift unacceptable point(s) into the acceptable region(s) by changing the gear ratio (or other mechanism parameter(s)) used. An example is shown in Figure 3.25b. In this example, $\tau_{motor,\,RMS}$ is too large (on the left). Increasing N_r decreases the required torques, while increasing $\omega_{motor,\,max}$, resulting in the acceptable design shown on the right.

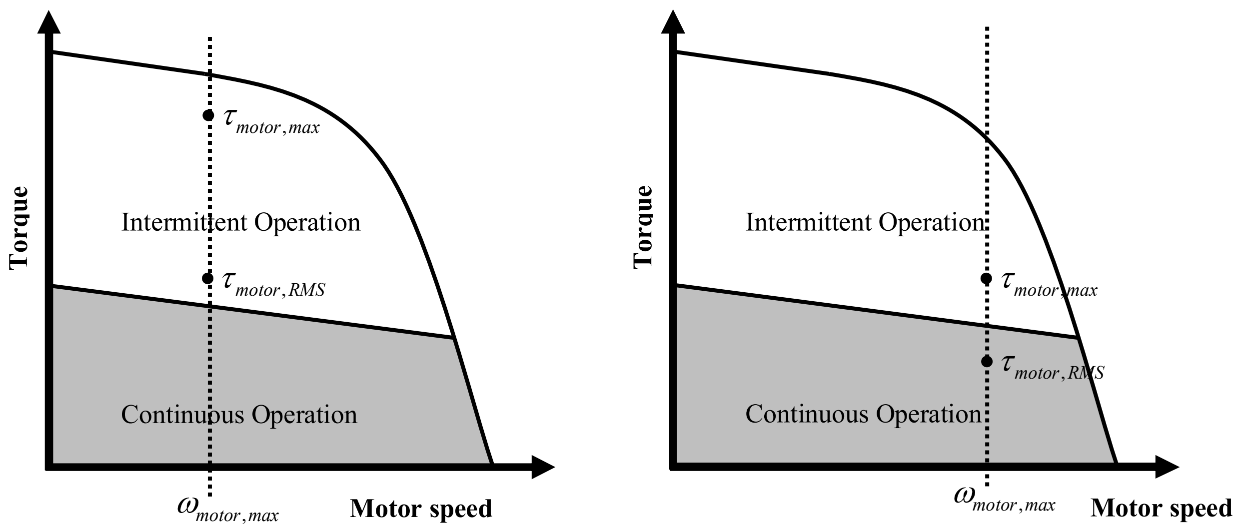


Figure 3.25b Left: Unacceptable $\tau_{motor,\,RMS}$. Right: Made acceptable by increasing N_r

In this course, to simplify the problem we will conservatively approximate the motor performance curves as shown in Figure 3.25c. These rectangular approximations make the rated torques independent of the motor speed over the range 0 to $\omega_{rated,\,max}$.

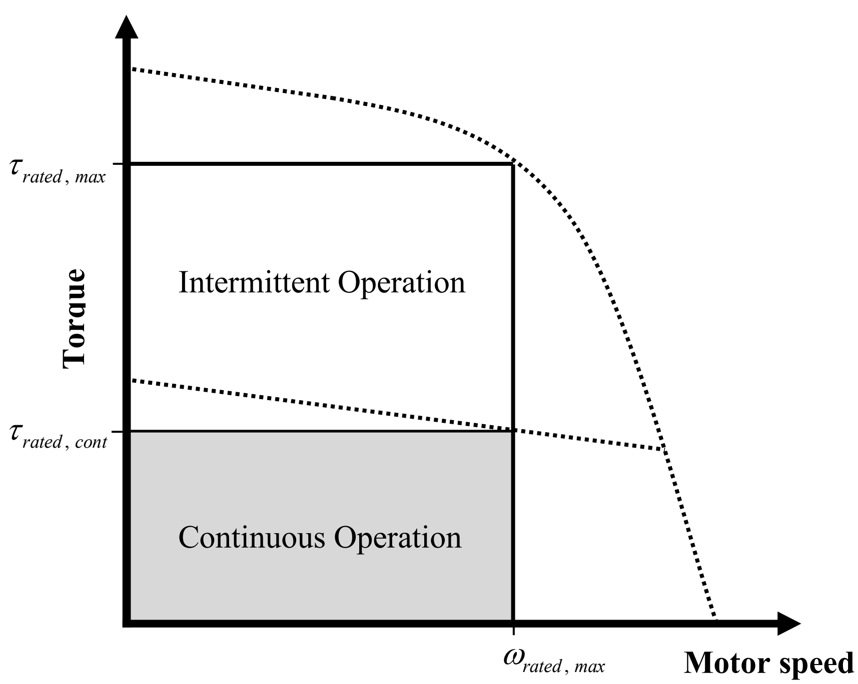


Figure 3.25c Conservative approximations of DC motor performance.

Based on the above, the selection procedure for a DC motor and gear box under dynamic loading may be summarized as follows:

1. Calculate the desired motion profiles if they are not provided.
2. Based on the load, motor data and mechanisms used, select the gear ratio which maximizes acceleration (i.e. inertia matching using (3.25)), or the closest smaller gear ratio available. The inertia ratio from (3.28) should be as close to 1 as possible, staying within the range 1 to 10.
3. Based on the velocity profile, and other mechanisms used (e.g. ball screw), find ω_{max}. Compare $\omega_{motor,\,max} = N_r \omega_{max}$ to the motor's rated maximum speed, $\omega_{rated,\,max}$. If $\omega_{motor,\,max}$ is too high, select a lower gear ratio and check $\omega_{motor,\,max}$ again. If the new value is acceptable, check that the new inertia ratio is within the range 1-10.
4. Based on the load, motor inertia, selected gear ratio, other mechanisms used, and the velocity and acceleration profiles, calculate the motor torque profile and find $\tau_{motor,\,max}$.
5. If $\tau_{motor,\,max} < \tau_{rated,\,cont}$ then proceed with step 8.
6. Compute $\tau_{motor,\,RMS}$ from the motor torque profile using (3.33). If $\tau_{motor,RMS} < \tau_{rated,\,cont}$ then proceed with step 8.
7. Select a higher gear ratio if possible, or select a more powerful motor, and return to step 2.
8. Using the method of section 3.4.4 check that the temperature rise is acceptable. If it is unacceptable then try increasing the gear ratio and return to step 2. If that is not sufficient then either try additional cooling, or select a more powerful motor and return to step 2.

Design examples are provided in the Addendum at the end of this chapter.

3.4.4 Temperature rise of the PM DC motor[2]

In this section we will study the thermal aspects of the DC motor in greater detail. The temperature of the motor will rise as long as the heat generated due to all power losses is not entirely dissipated through the motor surface. We only consider the Joule loss in the windings and neglect the friction loss. In this case the power loss, P_j, will be defined as:

$$P_j = I^2 R_{Hot} \tag{3.35}$$

[2] The material in this section was derived from "Check Temperature When Specifying Motors" by John Mazurkiewicz, Baldor Electric; and from the Maxon DC Motor catalog.

In the above equation R_{Hot} is the motor terminal resistance (or armature resistance) at the desired operating temperature (or at the maximum allowable temperature if the designer wishes to be more conservative), and *I* is the current. Note that the terminal resistance increases as the temperature rises. A catalog typically gives the terminal resistance at 25 °C. For a copper winding the resistance at the operating temperature is then given by:

$$R_{Hot} = R_{25}\left(1 + 0.00392(T_{Hot} - 25)\right) \tag{3.36}$$

where R_{25} is the terminal resistance at 25 °C and T_{Hot} is the desired operating temperature in °C. Thermal resistance R_{th}, in °C/Watt, is an indicator of how effectively the motor dissipates the generated heat and is a combination of two thermal resistances R_{th1} and R_{th2}. The first one characterizes the heat transfer from the windings to the housing while the second one is an indicator of the heat transfer from housing into the ambient.

$$R_{th} = R_{th1} + R_{th2} \tag{3.37}$$

The winding temperature of a motor may be estimated using the following equation:

$$T_w(t) = T_{initial} + \left(P_j R_{th} + T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) \tag{3.38}$$

where T_a is the ambient temperature, $T_{initial}$ is the initial winding temperature, and τ_w is the thermal time constant of the winding. τ_w is the amount of time required for a motor winding to reach 63.2% of its steady state temperature. For large motors τ_w can be several minutes while for small motors it may be only a few seconds. It is important to note that the housing (or stator) has its own time constant that is typically much larger than τ_w and is not used for calculating the winding temperature.

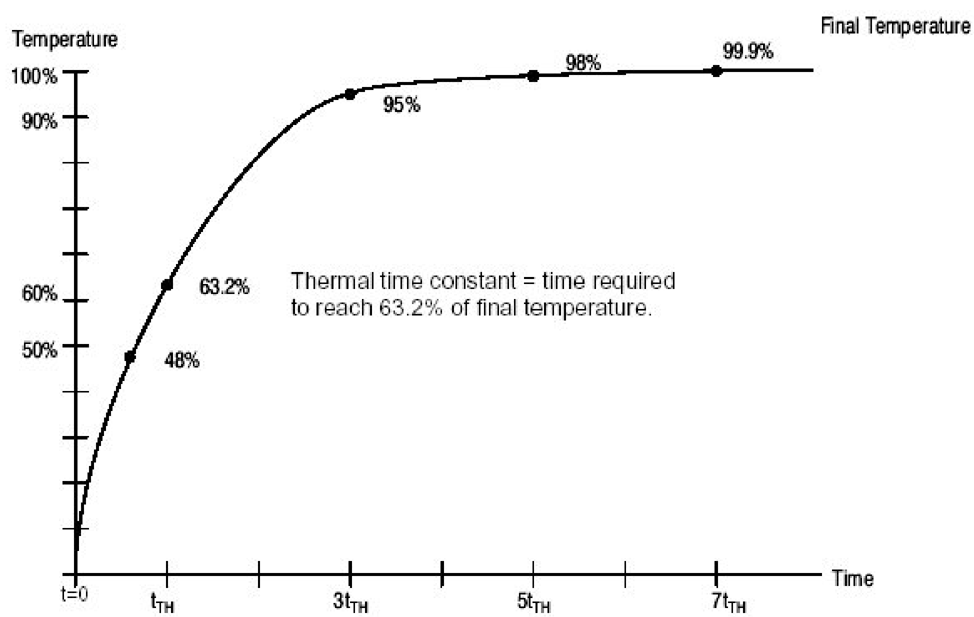


Figure 3.26 Temperature vs. time for constant motor current.

If the time t is greater than $7\tau_w$ then the equation may be simplified to:

$$T_w = T_a + P_j R_{th} \tag{3.39}$$

If the motor is turned off when $T_w > T_a$ then the winding temperature falls according to the following equation:

$$T_w(t) = T_{initial} + \left(T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) \tag{3.40}$$

Example 3.4

The thermal resistance of a motor is 1.4°C/Watt, and the maximum allowable winding temperature is 150°C. The ambient temperature is 30°C. The terminal resistance of a motor at the maximum allowable temperature is 3.33Ω.

(a) A motor under continuous use is pulling a constant current of 4.1A. Is the motor working in the safe temperature limit?

(b) If the motor has an operating cycle as follows then check the safety of the motor.
- Accelerates for 0.2s pulling a current of 10A
- Runs for 0.2s pulling a current of 1.5A
- Decelerates for 0.2s pulling a current of 8.5A
- Idles for 0.8s before a new cycle starts

Solution

(a) The terminal resistance for the hot winding is given as R_{Hot}=3.33Ω.
Using equations (3.35) and (3.39):

$$P_j = I^2 R_{Hot} = 4.1^2 \times 3.33 = 55.9\ Watts$$

$$T_w = T_a + P_j R_{th} = 30 + (55.9 \times 1.4) = 108.3^{\circ} C$$

So the motor winding temperature will be well within the design limit.

(b) For this part we should calculate the RMS current over the operating cycle.

The RMS value for the current is calculated using the following equation:

$$I_{RMS} = \sqrt{\sum_{i=1}^{n} I_i^2 t_i \Big/ \sum_{i=1}^{n} t_i} \tag{3.41}$$

Here the RMS current is…

$$
\begin{aligned}
I_{RMS} &= \sqrt{\sum_{i=1}^{n} I_i^2 t_i \Big/ \sum_{i=1}^{n} t_i} \\
&= \sqrt{\frac{I_{Acc}{}^2 \times t_{Acc} + I_{Run}{}^2 \times t_{Run} + I_{Dec}{}^2 \times t_{Dec}}{t_{Acc} + t_{Run} + t_{Dec} + t_{Idle}}} \\
&= \sqrt{\frac{10^2 \times 0.2 + 1.5^2 \times 0.2 + 8.5^2 \times 0.2}{0.2 + 0.2 + 0.2 + 0.8}} \\
&= 4.99 \text{ A}
\end{aligned}
$$

Substituting I_{RMS} in (3.35), and using (3.39) we have:

$$P_j = I_{RMS}^2 R_{Hot} = 4.99^2 \times 3.33 = 82.9 \text{ Watts}$$

$$T_w = T_a + P_j R_{th} = 30 + (82.9 \times 1.4) = 146.1\ ^\circ\text{C}$$

The result is close to the maximum the motor winding. A shorter “On” time could result operating above the safe winding temperature, while a longer “Off” time helps the motor operate cooler. Larger motors typically have smaller thermal resistance resulting lower operating temperature.

Example 3.5

Consider a motor operating for 1 minute dissipating 200 Watts, then turned off for 3.5 minutes. Assuming the motor starts at the ambient temperature, has a thermal resistance of 0.54 °C/Watt and a winding thermal time constant of 30 seconds, draw the temperature curve of the motor. The ambient temperature is 32°C.

Solution

Using equation (3.38) the winding temperature at the end of 1 minute (or 60 s) will be:

$$T_w = T_{initial} + \left(P_j R_{th} + T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) = 32 + (200 \times 0.54 + 32 - 32)\left(1 - e^{\frac{-60}{30}}\right) = 125.4\ ^\circ\text{C}$$

Note that in 1 minute which is 2 times the thermal time constant, the temperature increase is 86.5% of its steady state value. The continuous temperature would be 144°C if the motor was not turned off.

The temperature starts falling according to equation (3.40) when the motor is turned off. The temperature of the motor after turning off for 3.5 minutes (or 3.5 x 60 = 210 s) drops to very close to ambient temperature.

$$T_w = T_{initial} + \left(T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) = 125.4 + (32 - 125.4)\left(1 - e^{\frac{-210}{30}}\right) = 32.1\ ^\circ\text{C}$$

Figure 3.28 shows the temperature changes versus time.

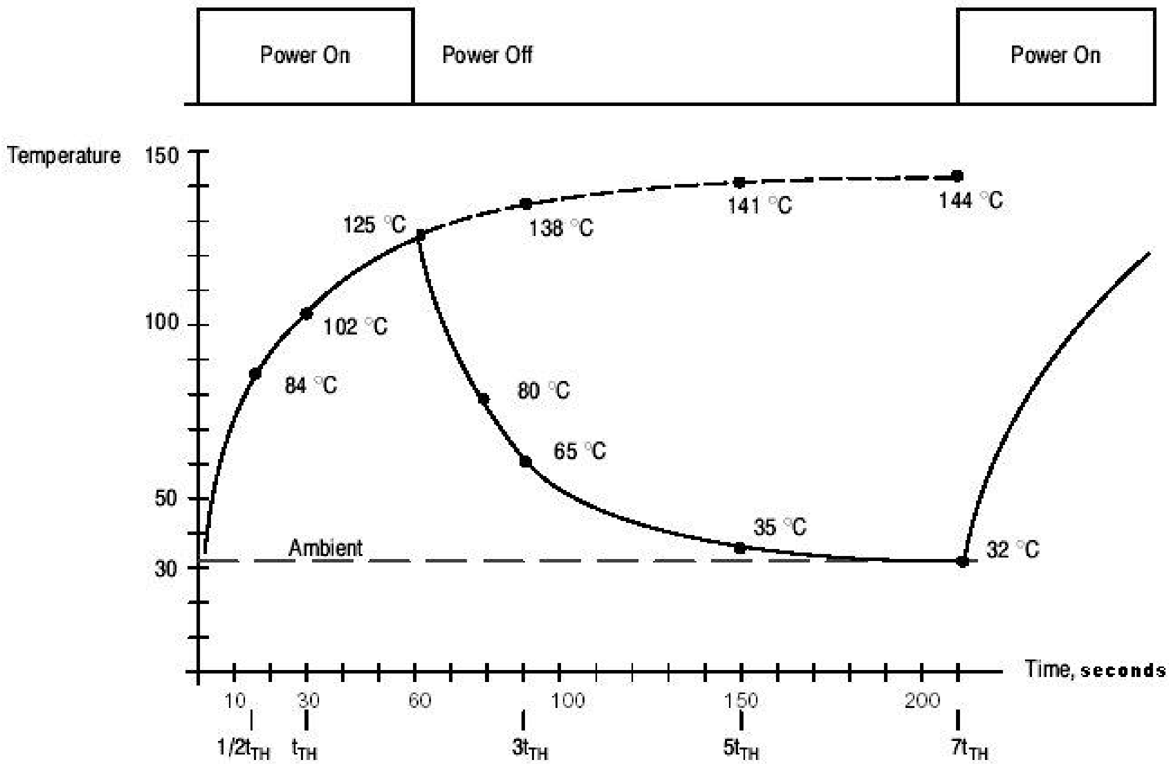


Figure 3.28 Temperature changes vs. time for example 3.4.

3.5 Pneumatic and Hydraulic Actuators

While the majority of actuators used in mechatronic systems combine electrical and mechanical elements, some applications are better suited by pneumatic and/or hydraulic actuators.

Advantages

- Can provide much higher ratios of (force or torque) to (mass and size) than electromechanical actuators.
- Less expensive than electromechanical when accurate control is not required.
- Hydraulic actuators are mechanically stiffer than electrical actuators. This allows them to hold their position without requiring compensation from a control system.
- Many companies have a factory wide high pressure air supply (a readily available pneumatic power source).

Disadvantages

- With pneumatic actuators the compressibility of air makes continuous position control very difficult.
- Hydraulic actuators are more expensive than pneumatic actuators.

- Hydraulic oil is toxic and flammable.
- Hydraulic actuators operate at high pressures (between 1000 and 5000 psi).
- Pneumatic and hydraulic power supplies are much larger and more complex than electrical ones.

Hydraulic Power Supply

The power supply for a typical hydraulic system is shown in Figure 3.29. This power supply is a “closed system” since the oil is returned to the tank or “sump”. The pump supplies oil at a particular pressure and flow rate. The non-return valve prevents oil from being driven back into the pump. The pressure relief valve is used to release the pressure if it exceeds a safe level. Finally, the accumulator acts to smooth out any short term pressure variations. Older accumulator designs use a spring-loaded piston. Newer designs use either a piston charged with high pressure gas on one side or a gas filled rubber bladder as shown in Figure 3.30.

Question: What electrical component is analogous to the accumulator?

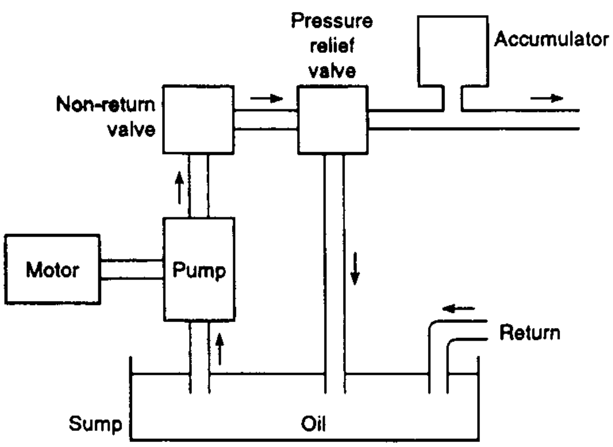


Figure 3.29 Hydraulic power supply.

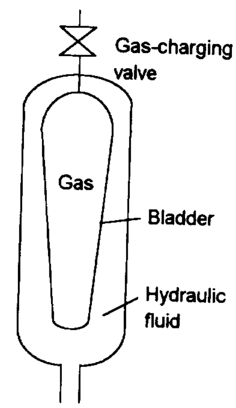


Figure 3.30 One type of hydraulic accumulator.

Pneumatic Power Supply

Pneumatic and hydraulic power supplies are similar. The important differences are:
i) Pneumatic power supplies operate at much lower pressures (≈100 psi vs. ≈2000 psi) and the components are designed accordingly.
ii) Pneumatic powers supplies are not closed systems. The return air is simply exhausted into the atmosphere. This saves the cost of installing the hoses and/or pipes needed for the return lines.
iii) A pneumatic accumulator is simply a tank. The compression of the air inside the tank provides the needed temporary energy storage and release.

A typical pneumatic power supply is shown in Figure 3.31.

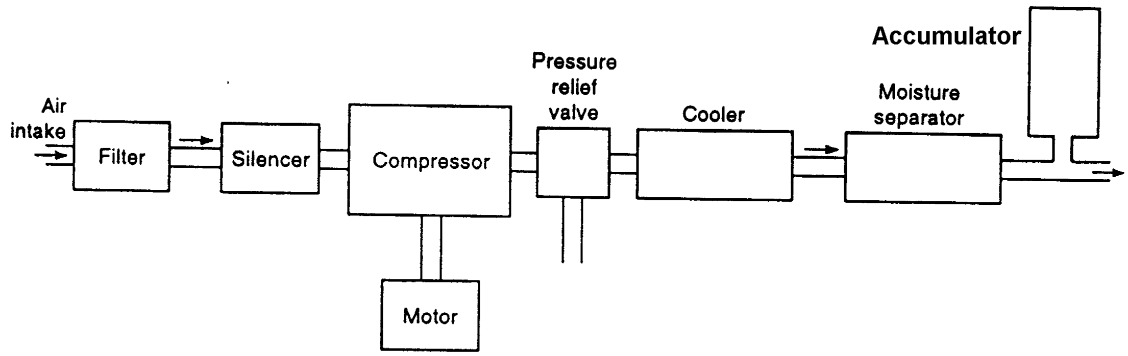


Figure 3.31 Pneumatic power supply.

Pneumatic and Hydraulic Cylinders

These are by far the most common type of hydraulic and pneumatic actuator. The principles and form are the same except hydraulic cylinders must be designed for the higher pressures involved.

There are two main types of cylinders (shown in Figure 3.32):

-Double acting.

-Single acting with spring return.

Double acting cylinders have two bi-directional ports. Since the pressure can be applied to both sides of the piston loads may be driven in both directions (i.e. extend and retract). It is also possible to continuously control the position and force output by controlling the pressures on each side of the piston.

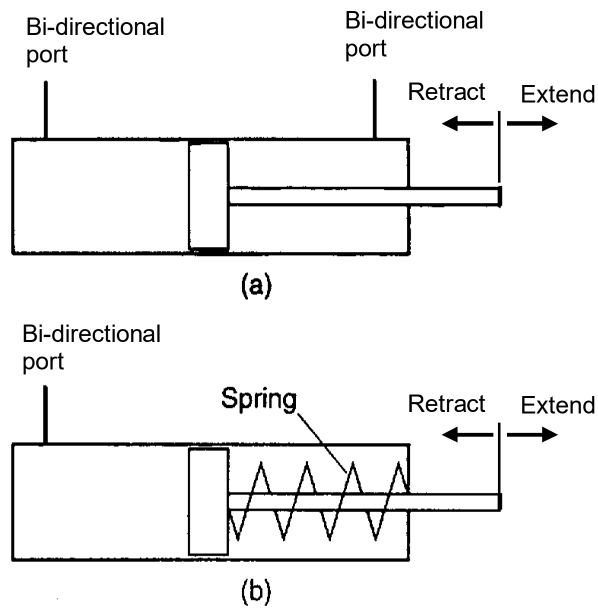


Figure 3.32 (a) Double acting cylinder.
(b) Single acting cylinder.

Single acting cylinders require only a single hose and a simpler valve than needed for double acting cylinders. They have the disadvantages that they can only supply a unidirectional force (*i.e.* they can only drive a load in the extend direction). They are also not suitable for position or force control applications.

Basic Control of Pneumatic and Hydraulic Double Cylinders

Assuming the friction of the seals is negligible, the output force in the extend direction equals:

$$F_{extend} = P_{extend} A_{extend} - P_{retract} A_{retract} \tag{3.42}$$

where P_{extend} is the gauge pressure on the extend side of the piston, A_{extend} is the cross-sectional area of the extend side of the piston, $P_{retract}$ is the gauge pressure on the retract side of the piston and $A_{retract}$ is the cross-sectional area of the retract side of the piston.

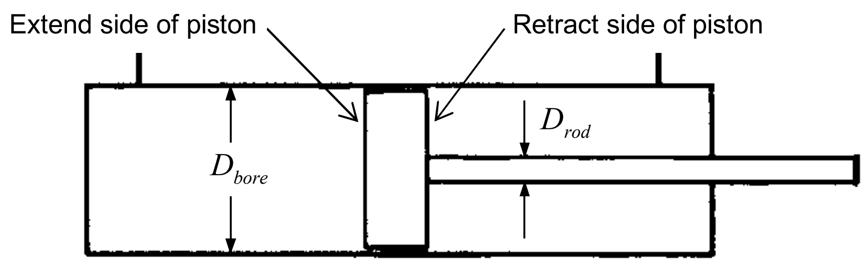
The cross-sectional areas depend on whether the cylinder is a "single rod", "double rod" or "rodless" type.

For a single rod cylinder:

$$A_{extend} = \frac{\pi D_{bore}^2}{4} \text{ and} \tag{3.43}$$

$$A_{retract} = \frac{\pi (D_{bore}^2 - D_{rod}^2)}{4} \tag{3.44}$$

where D_{bore} is the diameter of the bore and D_{rod} is the diameter of the rod. This is illustrated below.



With a rodless cylinder the cross-sectional areas are equal and are both given by equation (3.43). With a double rod cylinder the cross-sectional areas are also equal and are given by equation (3.44).

The output velocity is given by:

$$v = \frac{Q}{A} \tag{3.45}$$

where Q is the volume flow rate and A is appropriate value of the cross-sectional area.

Note that pneumatic cylinders are better suited for higher speeds and lower forces while hydraulic cylinders are better suited for higher forces and lower speeds.

Valves are used to control the flow rates and pressures.

Example 3.6

Determine the pressure and volume flow rate required for a pneumatic or hydraulic single rod double acting cylinder with a 50 mm bore diameter and a 10 mm rod diameter to drive a 500 N load at 10 m/s in both directions. Assume the return side is at atmospheric pressure (0 gauge = 101 kPa absolute).

Solution

Extend side area $A_{extend} = \frac{\pi}{4} D_{bore}^2 = \frac{\pi}{4}(0.050m)^2 = 0.00196\, m^2$

Retract side area $A_{retract} = \frac{\pi}{4}(D_{bore}^2 - D_{rod}^2) = \frac{\pi}{4}((0.050m)^2 - (0.010m)^2) = 0.00188\ m^2$

Pressures:

For motion in the extend direction, since the retract side is at atmospheric pressure (so $P_{retract}$ = 0 gauge) we have:

$$\begin{aligned} F_{extend} &= P_{extend} A_{extend} - P_{retract} A_{retract} \\ &= P_{extend} A_{extend} = 500\, N \end{aligned}$$

$$\begin{aligned} P_{extend} &= (500\, N)/(A_{extend}) \\ &= (500 N)/(0.00196\, m^2) \\ &= 2.55 \times 10^5 \text{ Pa gauge} \end{aligned}$$

For motion in the retract direction, since the extend side is at atmospheric pressure (so P_{extend} = 0 gauge) we have:

$$\begin{aligned} F_{retract} &= P_{retract} A_{retract} - P_{extend} A_{extend} \\ &= P_{retract} A_{retract} = 500\, N \end{aligned}$$

$$\begin{aligned} P_{retract} &= (500\, N)/(A_{retract}) \\ &= (500 N)/(0.00188\, m^2) \\ &= 2.66 \times 10^5 \text{ Pa gauge} \end{aligned}$$

Volume Flow Rates:

The volume flow rates (from (3.45)) are:

$$Q_{extend} = vA_{extend} = (10\, m/s)(0.00196\, m^2) = 0.0196\, m^3/s \quad \text{and}$$

$$Q_{retract} = vA_{retract} = (10\, m/s)(0.00188\, m^2) = 0.0188\, m^3/s\ .$$

So the max. P required is 2.66×10^5 Pa gauge and the max. Q required is 0.0196 m^3.

Control Valves

With pneumatic and hydraulic systems, control valves are used to direct the flow, and/or to change the flow rate, and/or to change the pressure.

Valves that can only be either open or closed are known as “finite position” and are typically actuated by a solenoid. When the solenoid is off, a spring is typically used to return the valve to its default or “normal” state.

Valve Design

The most common valve design is termed a “spool valve”. Its moving part has a spool-like shape. The basic design of a 5 port, 2 position, spool valve is shown in Figure 3.35. In position 1 the flow takes place from ports P to B and from ports A to R1. In position 2 the flow takes place from ports P to A and from ports B to R2. The actuation force can come from the various sources (*e.g.* solenoid, spring, push button, etc.). For fast switching the valve should be designed so that the spool’s mass is small and the actuation force is large. An example of a pneumatic spool valve will be shown in class.

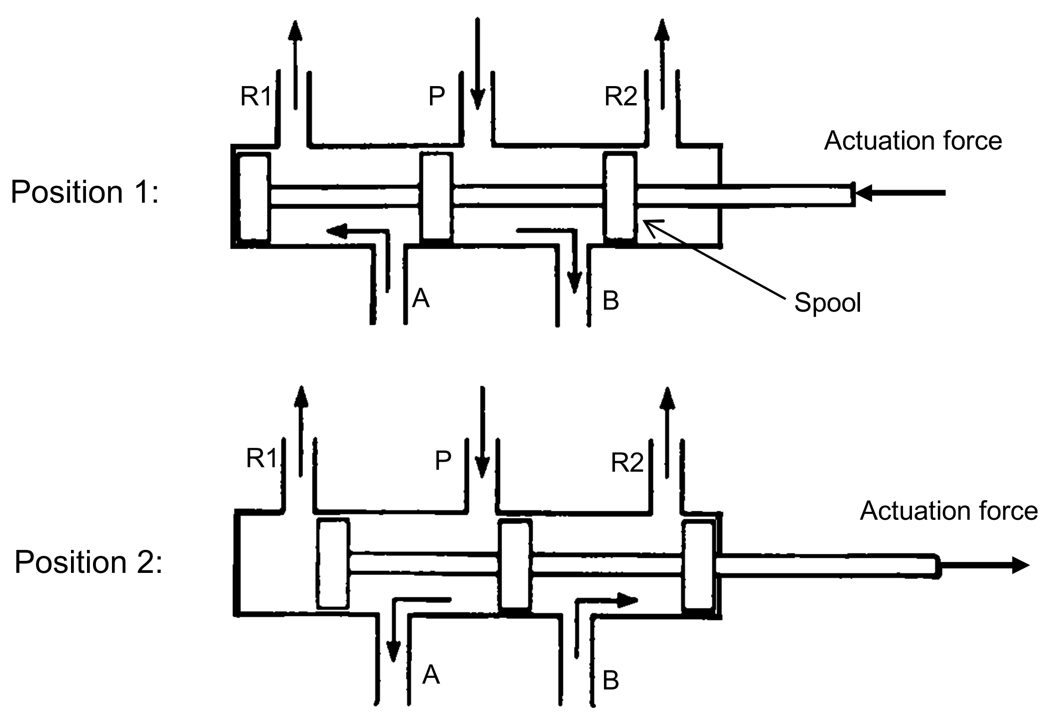


Figure 3.35 Cross-section view of a 5 port, 2 position, spool valve.

Proportional and Servo Valves

With finite position valves the orifice through which the air or hydraulic fluid passes is either set to zero (fully closed) or its maximum value (fully open). This is fine if we only want to fully extend or fully retract the cylinder. If we want to control the position or speed of the cylinder then we require a valve whose orifice area is continuously adjustable. When the orifice is made smaller the flow rate and cylinder speed are reduced. The opposite happens when the orifice is increased. There are two types of these "infinite position" valves. A "proportional valve" allows the orifice area to be adjusted in proportion to an input voltage. A "servo valve" has a built in closed-loop control system for the orifice area that gives it a faster and more accurate response than a proportional valve. Naturally, this higher performance is matched with a higher cost. A high quality hydraulic servo valve costs about $2,000.

Control Valve Sizing

The flow requirements of the pneumatic/hydraulic actuator are one of the main determinants of the valve selection (others include the valve's pressure rating and response time). The standard specification is the flow coefficient, C_v. A larger valve will have a larger flow coefficient but will also be heavier, bulkier and usually more expensive than a smaller valve. As mechatronics engineers we are interested in the smallest valve that meets our actuator requirements. This selection process is called control valve sizing. We will assume that the valve is the source of the greatest flow restriction within the circuit as this is normally the case. With this assumption the valve sizing equation in SI units is:

$$C_V = 4.22 \times 10^4 Q \sqrt{\frac{\rho}{\Delta P}} \qquad (3.46)$$

or

$$Q = 2.37 \times 10^{-5} C_V \sqrt{\frac{\Delta P}{\rho}} \qquad (3.47)$$

where Q is the required volume flow rate in m^3/s, ΔP is the pressure drop across the valve in Pa and ρ is the density of the fluid at the outlet pressure in kg/m^3.

In this course the fluid will either be air or hydraulic oil. With hydraulic oil ρ can be assumed to be constant (*i.e.* not a function of temperature and pressure changes). With air the density is given by:

$$\rho = \frac{P_2}{R_g T} = \frac{P_1 - \Delta P}{R_g T} \qquad (3.48)$$

where P_2 is the absolute outlet pressure in Pa, R_g is the gas constant = $287\ J/kg°K$, T is the air temperature in Kelvin $= {}^\circ C + 273$, P_1 is the absolute inlet pressure in Pa.

If the problem is defined in imperial units we will have to convert them to SI to use equations (3.46)-(3.48). Some useful conversion factors are:

$$1\, psi = 6895\, Pa$$
$$1\, in^3 = 1.635\times 10^{-5}\, m^3$$
absolute pressure = gauge pressure +14.7psi

We know from equation (3.45) that Q is proportional to the piston velocity. Examining equation (3.46) more closely we can see that the C_v is linked to both Q and the pressure inside the cylinder. Therefore it also affects the load carrying capacity and acceleration. This will be demonstrated in the examples that follow.

Example 3.9

We are designing a pneumatic system. We want to extend a 1 inch bore single rod pneumatic cylinder 12 inches in 0.1 seconds (not including the time for the valve to operate). The absolute supply pressure is 100 psi and a 5 psi pressure drop across the valve is acceptable. The air temperature is 20 °C. Find the minimum required C_v assuming the time to reach a constant velocity is negligible.

Solution

Due to the assumption that the time to reach constant velocity is negligible we have:

$$Q = \frac{Volume}{Time} = \frac{(\frac{\pi}{4}D_{bore}^2)(L)}{Time} = \frac{\frac{\pi}{4}(1in)^2(12in)}{0.1s} = (94.2in^3/s)(1.635\times 10^{-5}m^3/in^3) = 1.54\times 10^{-3}m^3/s$$

From equation (3.48) we have:

$$\rho = \frac{P_2}{R_g T} = \frac{(P_1 - \Delta P)}{R_g T} = \frac{(100-5)psi\cdot 6895Pa/psi}{(287J/kg°K)(20+273)°K} = 7.79\ kg/m^3$$

From the given information:

$$\Delta P = 5psi\cdot 6895Pa/psi = 3.45\times 10^4\ Pa$$

The minimum required valve flow coefficient is then (equation 3.46):

$$C_V = 4.22\times 10^4 Q\sqrt{\frac{\rho}{\Delta P}} = 4.22\times 10^4(1.54\times 10^{-3}m^3/s)\sqrt{\frac{7.79kg/m^3}{3.45\times 10^4\ Pa}} = 0.98$$

Note that C_v is a unitless quantity.

Example 3.10

We are designing a hydraulic system. We want to use a single rod hydraulic cylinder to drive a 10,000 kg mass horizontally. The maximum desired acceleration is 2 m/s^2, the maximum desired velocity is 0.5 m/s and the density of the hydraulic oil is 900 kg/m^3. If the supply pressure is 1.5×10^7 Pa absolute (≈2,200 psi) and a 2000 kPa pressure drop across the valve is acceptable then determine an appropriate bore size for the cylinder and the minimum required flow coefficient. Assume that the pressure drop across the valve is the same for the return flow as for the intake flow and that the sump is open to the atmosphere. Also assume the available bore sizes for the cylinder only come in 5 mm increments, and that the area of its rod can be neglected.

Solution to example 3.10 will be presented in-class

Example 3.11

We are designing a hydraulic system for moving a 2,000 kg payload mass vertically. A single rod cylinder will be used, mounted above the payload. The bore diameter is 100 mm and the rod diameter is 40 mm. The desired acceleration is 1 m/s^2 upwards and the desired maximum velocity is 0.1 m/s. If the supply pressure is 7×10^6 Pa gauge and the density of the oil is 900 kg/m^3 then determine the minimum valve flow coefficient required.

Solution to example 3.11 will be presented in-class

Applications of Pneumatic Systems

- Automated manufacturing systems for placing, feeding and rejecting parts.
- Pick and place robots

Applications of Hydraulic Systems

- Presses for forging and extrusion
- Farm equipment
- Power steering in cars
- Heavy construction equipment such as excavators
- Large robots

Discussion Topic 1:

What is the best type of actuator to use with a robotic gripper?

3.6 Emerging Actuators

Most of the actuators we have studied so far have been commonly used for more than 40 years. Their performance continues to gradually improve due to the development of new materials and new designs, but the underlying physics remains the same. For example, a DC motor is inherently a high speed, low torque actuator. Recently, the demand for new and higher performance actuators has significantly increased in a variety of fields such as robotics, medical devices, security, astronomy, optics, and precision machining. The emerging actuators are mostly designed for a particular application based on the physics of the actuator materials. Some are well suited for miniaturization possibilities. In this section we will cover three of the most useful types: piezoelectric, shape memory alloy, and ultrasonic.

Piezoelectric Actuators

When used in sensors, the piezoelectric effect allows certain materials (typically ceramics) to generate electric charge in proportion to applied force. This effect is reversible; meaning an applied electric charge parallel to the direction of polarization of the crystal causes expansion and force output.

Piezoelectric actuators are capable of moving large masses over microscopic distances, or light loads at high frequencies. The displacement of a piezoelectric actuator is a function of applied electric field strength, the length of the actuator, the properties of the piezoelectric crystal, and the external load. To achieve useful displacements piezoelectric actuators must be made of several layers, with each layer less than 1 mm thick. An example is shown in Figure 3.37.

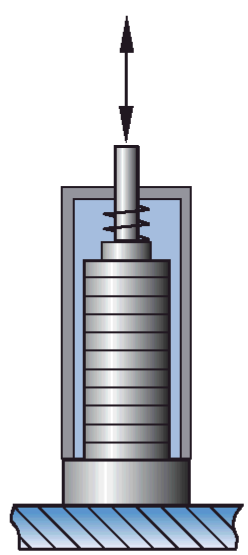


Figure 3.37 Piezoelectric actuator built from many layers. Note that the spring provides a preload force that is necessary for bidirectional motion.

Advantages:
- Better energy efficiency than traditional actuators.
- Suitable for miniaturization.
- Capable of nanometre resolution.
- Capable of response times < 1 ms
- Capable of large output forces (e.g. 4500 N for 25 mm diameter actuator).
- Large stiffness when pushing.
- No wear.

Disadvantages:
- Very small range of motion. The maximum displacement is about 0.2% of the actuator length.
- Low pulling force unless preloading is used. Inadequate preloading can cause premature failure since the ceramic material is weak in tension.
- Large displacements require large voltages (e.g. 500V or more).
- Sensitive to temperature changes.
- Requires complex form of closed-loop control to achieve low frequency displacements.
- Subject to fatigue failure.

Applications: Inkjet printer heads (to control flow of ink), piezoelectric valves, vibratory feeders, optical alignment, precision assembly and precision machining.

Shape Memory Alloy Actuators

A large deformation happens in many materials when undergoing a structural phase transition as a result of temperature, stress, or electrical field. In some materials when the external source of deformation is released, the original form may be restored with the application of heat. This effect is called shape memory. The stress-strain diagrams for a typical metal and a typical shape memory alloy (SMA) are shown in the Figure 3.38.

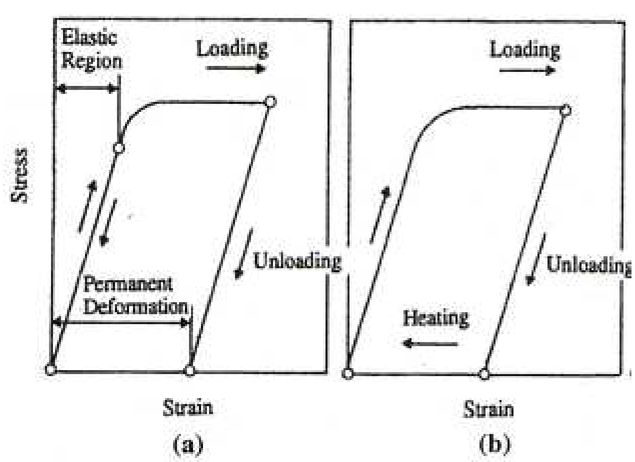


Figure 3.39 Stress vs. strain curves for: (a) typical metal, (b) shape memory alloy.

In a normal metal, when the stress exceeds the elastic region, there will be a permanent plastic deformation. With a SMA original form can be recovered by heating at an appropriate temperature.

A common alloy used in shape memory alloys is Nitinol (a Nickel- Titanium alloy). These alloys are capable of generating large forces as high as 10^8 N/m^2 during the recovery process. Figure 3.38 shows an actuator incorporating a shape memory spring. A normal spring and a shape memory spring are set in a serial configuration. At low temperatures, the SMA spring is pushed all the way to the right as a result of force from normal spring. When the SMA spring is heated up using an appropriate source (e.g. by applying an electric current), the SMA spring recovers its original shape and the shaft moves to the left. When the SMA is allowed to cool down by the ambient temperature the shaft will return to the right.

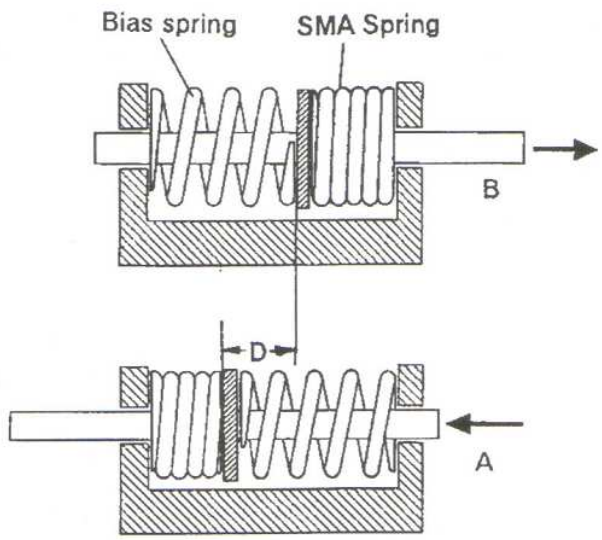


Figure 3.40 A bidirectional actuator incorporating shape memory alloy.

Advantages:
- Simple one piece design.
- Material can be shaped for the application
- Large force per unit area.
- No wear.

Disadvantages:
- Slow response speed (especially for cooling down), typically the response time is greater than 1 second.
- Small range of motion, approximately 5% of original length.
- Operating temperature is approximately 90 ºC.
- Subject to fatigue failure.
- Large hysteresis.
- Unidirectional (requires spring or second actuator to be bidirectional).
- Low energy efficiency.

Control: Typically by controlling the electric current flowing through the actuator. This allows the temperature and resulting displacement to be altered.

Applications: valves, electronic locks, safety devices.

Ultrasonic Motors

Ultrasonic motors are a special type of piezoelectric actuator. They can be linear or rotary. The principle behind the rotary type is shown in Figure 3.41. The stator is a ring that is driven by pieces of a piezoelectric ceramic material. The piezoelectric material is electrical driven to cause the stator to vibrate (at an ultrasonic frequency) in the form of a travelling wave. This wave, and the friction force between the stator and rotor, cause the rotor to rotate. A sectioned ultrasonic motor is shown in Figure 3.42. A tiny ultrasonic motor driven linear actuator sitting on top of a fingertip is shown in Figure 3.43.

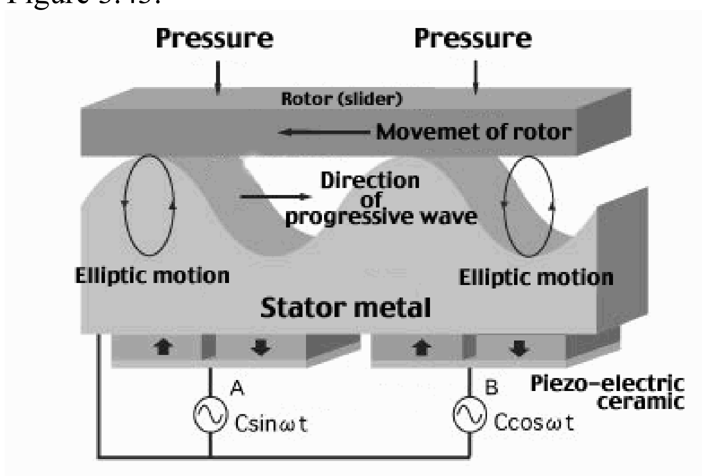


Figure 3.41 Physics of a travelling wave type ultrasonic motor (image source: Shinsei Corporation, Japan).

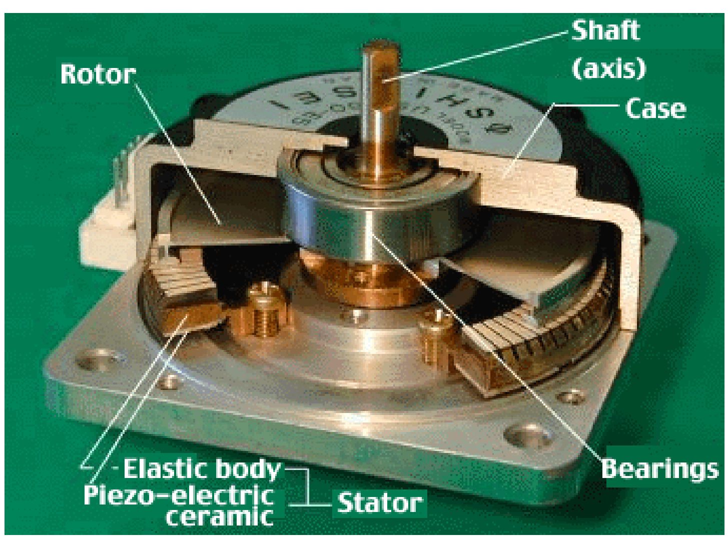


Figure 3.42 Picture of a sectioned ultrasonic motor from Shinsei Corporation, Japan.

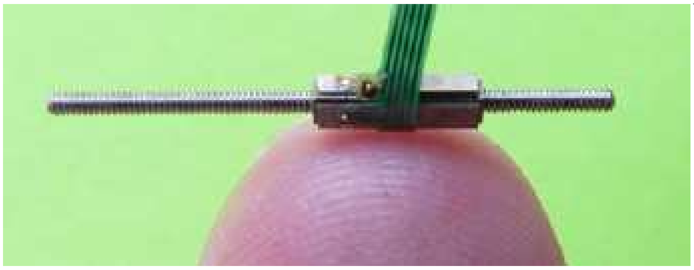


Figure 3.43 Miniature linear actuator made using an ultrasonic motor with built-in lead screw from New Scale Technologies Inc.

Advantages
- Provides high torque at low speeds.
- Smaller and lighter than a comparable DC motor and gearbox.
- Can produce large acceleration due to small rotor inertia.
- Rotor is locked when power is off (held by friction).
- Suitable for miniaturization.

Disadvantages
- Relies on friction to provide torque
- Subject to fatigue and wear.
- Driven by large voltages relative to DC motors (approximately 200 V vs. 30 V).
- Torque is not controllable.

Applications: lens motors in autofocus cameras

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Addendum:

A.1 More Information About Common Motion Profiles

It is common for desired motion profiles to be based on periods of constant acceleration. The position and velocity profiles can be derived from the acceleration profile, and vice-versa.

We often wish to move a mass from one fixed location to another fixed location. The first period of the motion will involve accelerating the mass up to the desired maximum velocity. We will term the desired acceleration a_{acc}, the desired maximum velocity v_{max}, and the desired acceleration time t_{acc}. The acceleration period may be followed by a period of constant velocity (with a=0) of duration t_{run}. This is followed by a period t_{dec} of constant deceleration to reduce the desired speed to zero. Note that deceleration refers to a reduction in the speed of the mass, so its sign is opposite to the sign of the velocity (which could be positive or negative).

An example of the resulting position, velocity and acceleration profiles is shown in Figure 3.44.

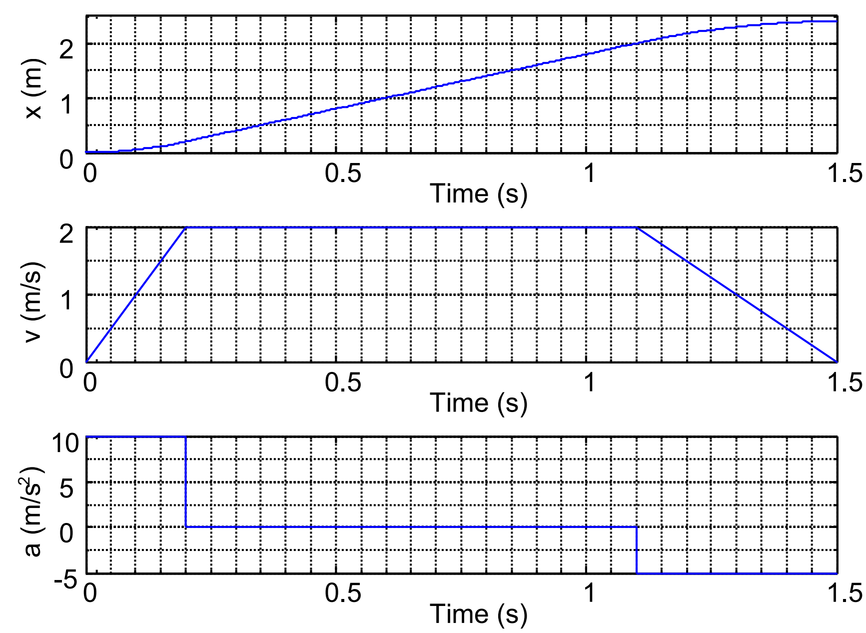

Figure 3.44 Position, velocity and acceleration profiles for the parameters:
$t_{acc} = 0.2$ s, $t_{run} = 0.9$ s, $t_{dec} = 0.4$ s, $a_{acc} = 10$ m/s^2, $a_{dec} = -5$ m/s^2 and $v_{max} = 2$ m/s.

With piecewise constant acceleration, the fastest possible motion from one fixed location to another occurs when t_{run}=0 and t_{acc}= t_{dec}= t_{con}. Since the motion starts from rest, and the period of acceleration is followed by an equal period of constant acceleration of the same magnitude, at the halfway point of the motion:

$$\tfrac{1}{2} x_{move} = \tfrac{1}{2} a_{con} \left(\tfrac{1}{2} t_{move}\right)^2 \qquad (3.49)$$

where x_{move} is the total displacement and t_{move} is the total movement time. If the acceleration magnitude and movement time are known then:

$$x_{move} = \tfrac{1}{4} a_{con} t_{move}^2 \qquad (3.50)$$

Alternatively:

$$t_{move} = \sqrt{4 x_{move} / a_{con}} \text{ and} \qquad (3.51)$$

$$a_{con} = 4 x_{move} / t_{move}^2 \qquad (3.52)$$

The maximum velocity occurs at the halfway point and is given by:

$$v_{\max} = \tfrac{1}{2} a_{con} t_{move} \qquad (3.53)$$

The position, velocity and acceleration equations for the two periods are as follows:

For $t_i \le t \le (t_i + \tfrac{1}{2} t_{move})$:

$$x(t) = \tfrac{1}{2} a_{con} (t - t_i)^2 + x_i \qquad (3.54)$$

$$v(t) = a_{con} (t - t_i) \text{ and} \qquad (3.55)$$

$$a(t) = a_{con} \qquad (3.56)$$

For $(t_i + \tfrac{1}{2} t_{move}) < t \le (t_i + t_{move})$:

$$x(t) = x_i + x_{move} - \tfrac{1}{2} a_{con} (t_i + t_{move} - t)^2 \qquad (3.57)$$

$$v(t) = a_{con} \left(t_i + t_{move} - t\right) \text{ and} \qquad (3.58)$$

$$a(t) = -a_{con} \qquad (3.59)$$

The corresponding position, velocity and acceleration profiles are shown in Figure 3.45, assuming t_i =0 and x_i =0.

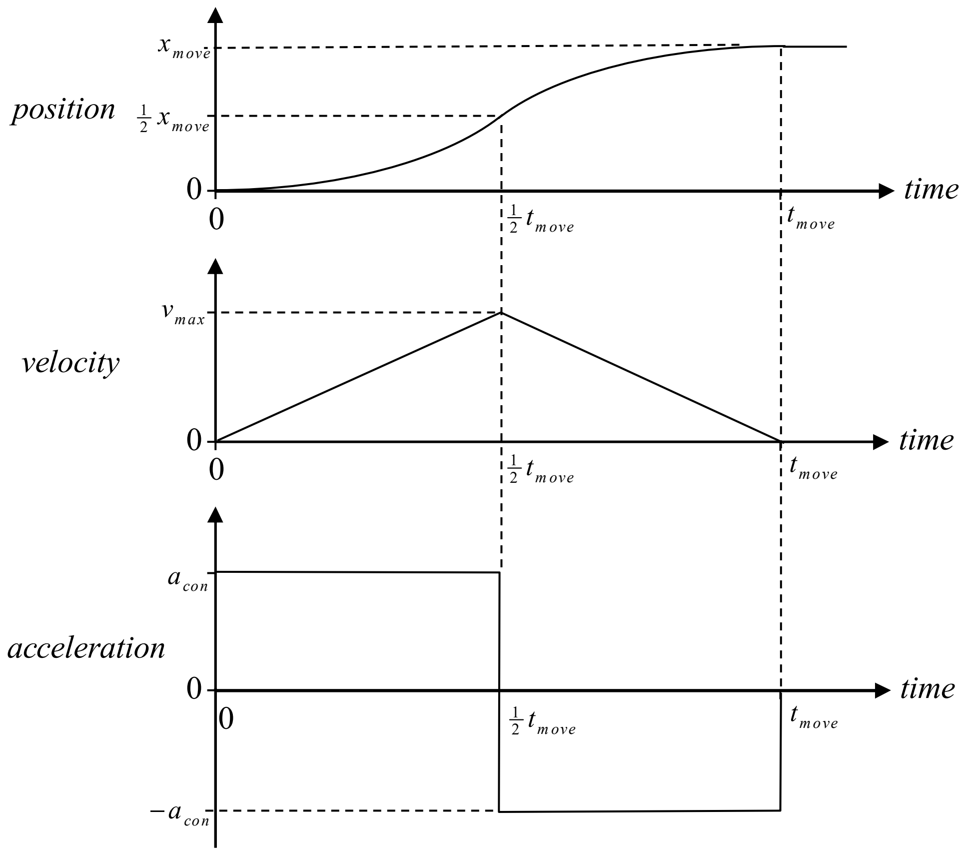

Figure 3.45 Plots of position, velocity and acceleration vs. time for a movement consisting of constant acceleration (a_{con}) followed by an equal period of constant deceleration ($-a_{con}$).

If the motion should return to its starting location then the operating cycle will typically have eight periods. Assuming the mass should be initially moved forwards, an example showing the eight periods is given in Figure 3.46.

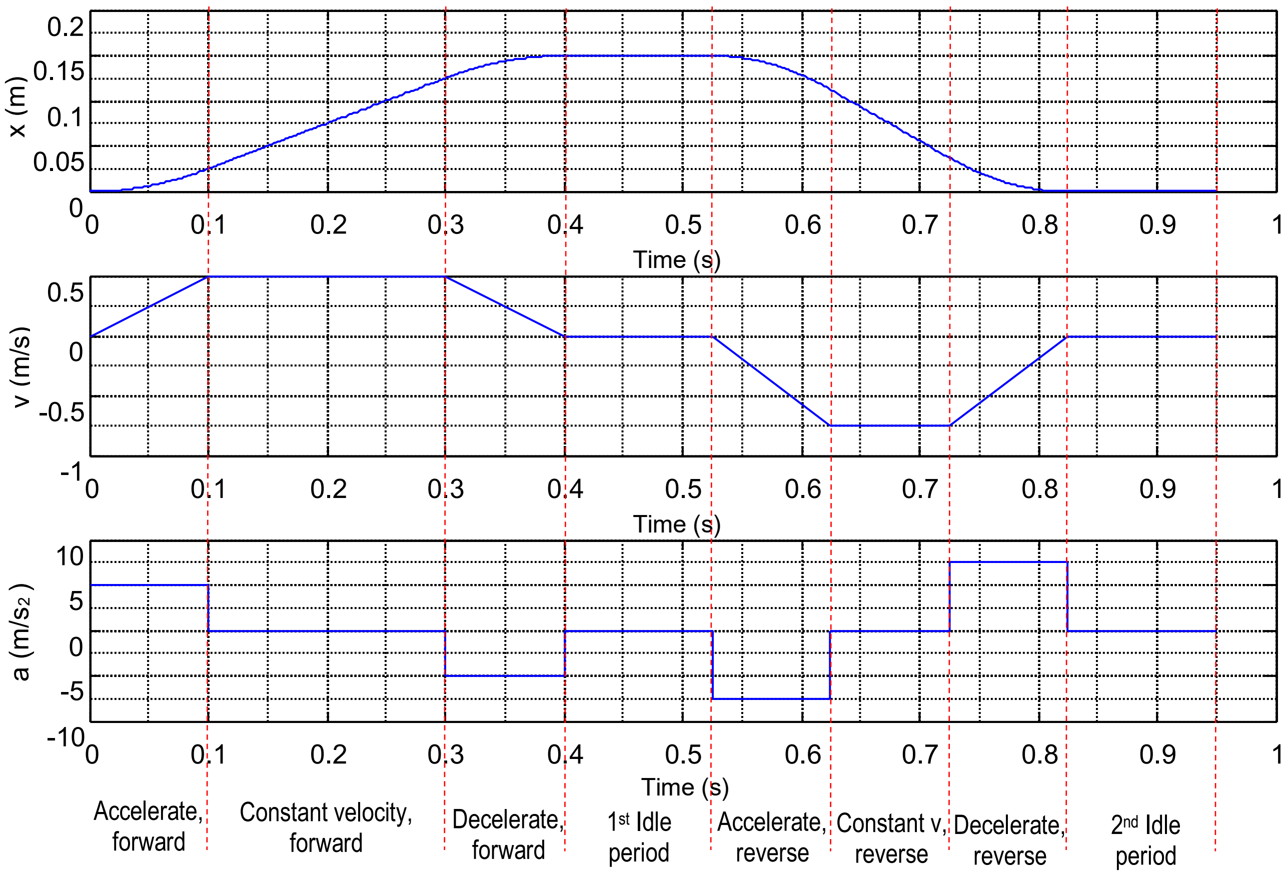

Figure 3.46 Example of position, velocity and acceleration profiles for an eight period operating cycle where the motion returns to its starting position.

The equations and examples presented above have assumed the desired motion is linear. If the desired motion is rotary then the same equations apply, except that the symbols θ, ω (or $\dot{\theta}$) and α (or $\dot{\omega}$ or $\ddot{\theta}$) are used for the angular position, velocity and acceleration, respectively.

A.2. Examples of Motor and/or Mechanism Selection

Example 3.12

A load is to be rotated by half a revolution, pause for 0.4 s, and then returned to its original position. After the return motion the load should be kept stationary for 0.6 s before the cycle restarts. Each motion should consist of a period of acceleration followed by an equal period of deceleration, and be completed in 1 second. The load has a moment of inertia of 0.025 kgm^2. The ambient temperature is 25 ºC. The motor parameters are: moment of inertia=1.10×10^{-5} kgm^2, max. torque =1.5 Nm, max. continuous torque = 0.45 Nm, max. speed = 7000 rpm, temperature limit = 100 ºC, torque constant = 0.1 Nm/A, resistance at temperature limit = 2.15 ohm, and total thermal resistance = 1.43 ºC/W. A gear box will be attached between the motor and the load. The available gear ratios are: 5, 10, 15, etc. You may assume the torque ratings are independent of the speed and that the friction torques are negligible.

a) List the periods that make up the motion profile in order.
b) Determine the best gear ratio for this application using the method of section 3.4.3. Check that the resulting motor speed, torques and temperature are within their rated values. Remember to keep the inertia ratio between 1 and 10.
c) Repeat part (b) for a load inertia of 0.25 kgm^2.

Solution to example 3.12 will be presented in-class

Example 3.13

A 25 kg mass is to be translated vertically. It is subject to a 50 N friction force. Its desired motion profiles are given in Figure 3.46, where positive indicates upwards motion. The motor to be used is the same as in example 3.12. The ambient temperature is 30 ºC. You may assume the torque ratings are independent of the speed and that the friction of the motor, gears and ball screw may be neglected.

a) The linear motion of the mass is obtained by coupling the motor to a gearbox that drives a ball screw with a 0.02 m lead. The inertia of the screw is 3.14×10^{-5} kgm^2. Determine the best gear ratio for this application using the method of section 3.4.3. The available gear ratios are 0.5, 1, 1.5, etc. Check that the resulting motor speed, torques and temperature are within their rated values.
b) The linear motion of the mass is obtained by directly coupling the motor to a ball screw. Choose the lead of the ball screw based on the method presented in section 3.4.3. The available leads are: 2 mm, 4 mm,…, 20 mm. You may assume that the inertia of the screw is

not a function of its lead, and equals 3.14×10^{-5} kgm^2. Note that, even though $N_r = 1$, the ball screw can be used to provide the necessary mechanical advantage. Check that the resulting motor speed, torques and temperature are within their rated values. Comment on the performance of this design compared to your answer from part (a).

Solution

a) From (3.2), the equivalent inertia of the mass driven by the ball screw is:

$$J_M = M\left(\frac{l}{(2\pi/rev)}\right)^2$$

The load inertia includes this value plus the inertia of the screw, as follows:

$$J_{load} = M\left(\frac{l}{(2\pi/rev)}\right)^2 + J_{screw}$$

$$= (25\text{ kg})\left(\frac{0.02\text{ m/rev}}{(2\pi/rev)}\right)^2 + 3.14\times10^{-5}\text{ kgm}^2$$

$$= 2.85\times10^{-4}\text{ kgm}^2$$

The optimal gear ratio is given by:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{2.85\times10^{-4}\text{ kgm}^2}{1.10\times10^{-5}\text{ kgm}^2}} = 5.09$$

However, the available gear ratios are 0.5, 1, 1.5, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 5$. Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio. The motor's rated max. speed is 7000 rpm, $\therefore \omega_{rated,max} =$ 7000(2π/rev)(1 min/60 s) = 733 rad/s. The maximum required linear speed, $|v_{max}|$, can be obtained from Figure 3.46. The max. rotational speed of the ball screw is then:

$$\omega_{max} = \frac{|v_{max}|}{l(\text{rev}/2\pi)} = \frac{0.75\text{ m/s}}{(0.02\text{ m/rev})(\text{rev}/2\pi)} = 236\text{ rad/s}$$

The corresponding motor speed is: $\omega_{motor,max} = N_r\omega_{max} = 1180\text{ rad/s}$

Since $\omega_{motor,max} > \omega_{rated,max}$ this gear ratio fails the motor speed check. The upper limit on the gear ratio is:

$$N_{r,max} = \frac{\omega_{motor,max}}{\omega_{\max}} = \frac{733 \text{ rad/s}}{236 \text{ rad/s}} = 3.11$$

We should round down to the nearest available ratio to keep $\omega_{motor,max} \le \omega_{rated,max}$. Therefore our new choice for the best ratio is: $N_r = 3$. Now we need to check the inertia ratio. It is:

$$Ratio_J = \frac{J_{load}/N_r^2}{J_{motor}} = \frac{(2.85\times10^{-4} \text{ kgm}^2)/(3^2)}{(1.10\times10^{-5} \text{ kgm}^2)} = 2.88$$

Since it is within the range 1 to 10 it is acceptable.

Next, we should check the maximum required motor torque and RMS motor torque. The torques are obtained for the eight periods shown in Figure 3.46 as follows:

1. **Accelerate, forward (upwards in this example):**

$$\tau_{external} = \frac{Fl}{(2\pi/rev)\eta_s}$$

$$= \frac{(Mg + F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\big((25 \text{ kg})(9.81 \text{ N/kg})+50 \text{ N}\big)(0.02 \text{ m/rev})}{(2\pi/rev)(1)} = 0.940 \text{ Nm}$$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{5 \text{ m/s}^2}{(0.02 \text{ m/rev})(1 \text{ rev}/2\pi)}\right) = 4710 \text{ rad/s}^2$$

$$\tau_{motor,1} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.1\times10^{-5} \text{ kgm}^2)(4710 \text{ rad/s}^2) + \frac{1}{3^2}(2.85\times10^{-4} \text{ kgm}^2)(4710 \text{ rad/s}^2) + \frac{1}{3}(0.940 \text{ Nm})$$

$$= 0.514 \text{ Nm}$$

2. **Constant velocity, forward:**

   Since the mass is still being moved upwards: $\tau_{external} = 0.940$ Nm

   No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,2} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.940 \text{ Nm}) = 0.313 \text{ Nm}$$

3. **Decelerate, forward:**

   Since the mass is still being moved upwards: $\tau_{external} = 0.940$ Nm

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{-5 \text{ m/s}^2}{(0.02 \text{ m/rev})(1 \text{ rev}/2\pi)}\right) = -4710 \text{ rad/s}^2$$

$$\tau_{motor,3} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.1\times10^{-5}\text{ kgm}^2)(-4710\text{ rad/s}^2) + \frac{1}{3^2}(2.85\times10^{-4}\text{ kgm}^2)(-4710\text{ rad/s}^2) + \frac{1}{3}(0.940\text{ Nm})$$

$$= 0.112\text{ Nm}$$

4. 1st Idle:

When the velocity is zero the friction force should oppose the gravity force acting on the mass, therefore:

$$\tau_{external} = \frac{Fl}{(2\pi/rev)\eta_s}$$

$$= \frac{(Mg - F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\left((25\text{ kg})(9.81\text{ N/kg}) - 50\text{ N}\right)(0.02\text{ m/rev})}{(2\pi/rev)(1)} = 0.622\text{ Nm}$$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,4} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.622\text{ Nm}) = 0.207\text{ Nm}$$

5. Accelerate, reverse (downwards in this example):

The friction force will act upwards, opposing the downwards velocity. Therefore $Mg - F_{friction}$ will be same as the previous period, and: $\tau_{external} = 0.622\text{ Nm}$.

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{-7.5\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = -7070\text{ rad/s}^2$$

$$\tau_{motor,5} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.1\times10^{-5}\text{ kgm}^2)(-7070\text{ rad/s}^2) + \frac{1}{3^2}(2.85\times10^{-4}\text{ kgm}^2)(-7070\text{ rad/s}^2) + \frac{1}{3}(0.622\text{ Nm})$$

$$= -0.094\text{ Nm}$$

6. Constant velocity, reverse:

Since the mass is still being moved downwards: $\tau_{external} = 0.622\text{ Nm}$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,6} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.622\text{ Nm}) = 0.207\text{ Nm}$$

7. Decelerate, reverse:

Since the mass is still being moved downwards: $\tau_{external} = 0.622\text{ Nm}$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{7.5\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = 7070\text{ rad/s}^2$$

$$\begin{aligned}\tau_{motor,7} &= J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} \\ &= (1.1\times10^{-5}\text{ kgm}^2)(7070\text{ rad/s}^2) + \frac{1}{3^2}(2.85\times10^{-4}\text{ kgm}^2)(7070\text{ rad/s}^2) + \frac{1}{3}(0.622\text{ Nm}) \\ &= 0.509\text{ Nm}\end{aligned}$$

8. 2nd Idle:

The loading situation is the same as with the 1st idle period, therefore:

$$\tau_{motor,8} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.622\text{ Nm}) = 0.207\text{ Nm}$$

From the above results, the max. required motor torque is: $\tau_{motor,max} = \tau_{motor,1} = 0.514\text{ Nm}$. This is below the motor's rated max. torque of 1.5 Nm, but above its rated continuous torque. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\begin{aligned}\tau_{motor,RMS} &= \sqrt{\frac{\sum_{i=1}^{n}\tau_{motor,i}^2 t_i}{\sum_{i=1}^{n} t_i}} \\ &= \sqrt{\frac{\left((0.514)^2(0.1)+(0.313)^2(0.2)+(0.112)^2(0.1)+(0.207)^2(0.125)+(-0.094)^2(0.1)+(0.207)^2(0.1)+(0.509)^2(0.1)+(0.207)^2(0.125)\right)\text{ N}^2\text{m}^2\text{s}}{(0.1+0.2+0.1+0.125+0.1+0.1+0.1+0.125)\text{ s}}} \\ &= 0.306\text{ Nm}\end{aligned}$$

Since $\tau_{motor,RMS}$ is below the motor's continuous torque rating of 0.45 Nm, neither torque limit has been exceeded.

Lastly, we need to check the motor's operating temperature. The question states that the friction of the motor is negligible, therefore $K_d \approx 0$ and the torque output of the motor is simply $K_t I$. Then the RMS current is simply:

$$I_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} I_i^2 t_i}{\sum_{i=1}^{n} t_i}} = \sqrt{\frac{\sum_{i=1}^{n}\left(\tau_{motor,i}/K_t\right)^2 t_i}{\sum_{i=1}^{n} t_i}} = \frac{\tau_{RMS}}{K_t} = \frac{0.306\text{ Nm}}{0.1\text{ Nm/A}} = 3.06\text{ A}$$

The power loss is:

$$P_j = I_{RMS}^2 R_{Hot} = (3.06\text{ A})^2(2.15\text{ ohm}) = 20.1\text{ W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 30\ °\text{C} + (20.1\ \text{W})(1.43\ °\text{C/W}) = 58.8\ °\text{C}$$

Since this is less than the motor's temperature limit this test has been passed.

We can conclude that this motor, gearbox and ball screw combination has passed all of the tests and is acceptable.

b) From (3.2), the equivalent inertia of the mass driven by the ball screw is:

$$J_M = M\left(\frac{l}{(2\pi / rev)}\right)^2$$

The load inertia includes this value plus the inertia of the screw, as follows:

$$J_{load} = M\left(\frac{l}{(2\pi / rev)}\right)^2 + J_{screw}$$

With $N_r = 1$ the inertia ratio is simply: $Ratio_J = \frac{J_{load}}{J_{motor}}$. As in Example 3.12, we should first try to make this equal to the optimal value of 1, i.e. try to make $J_{load} = J_{motor}$. With this substitution the inertia equation becomes:

$$J_{motor} = M\left(\frac{l}{(2\pi / rev)}\right)^2 + J_{screw}$$

However, since $J_{screw} > J_{motor}$ this equation has no solution. The lead from the set of available screws that gives the closest inertia ratio to 1 is 2 mm = 0.002 m. From Figure 3.46 the max. desired linear speed is 0.75 m/s. The corresponding motor speed is:

$$\omega_{motor,max} = \frac{|v_{max}|}{l} = \frac{0.75\ \text{m/s}}{(0.002\ \text{m/rev})(1\ \text{rev} / 2\pi)} = 2360\ \text{rad/s}$$

Since the motor's rated max. speed is 7000 rpm, $\therefore \omega_{rated,max} =$ 7000(2π/rev)(1 min/60 s) = 733 rad/s, $\omega_{motor,max}$ is clearly too large, and this lead cannot be used. The lower limit on the lead is:

$$l_{min} = \frac{|v_{max}|}{\omega_{max}} = \frac{0.75\ \text{m/s}}{(733\ \text{rad/s})(1\ \text{rev} / 2\pi)} = 0.00642\ \text{m/rev}$$

Since a larger lead will reduce the required motor speed we should round this up to closest available value. The new best lead is then: l = 0.008 m/rev. Now we need to check the inertia ratio. With this lead the load inertia is:

$$J_{load} = M\left(\frac{l}{(2\pi / rev)}\right)^2 + J_{screw}$$

$$= (25\text{ kg})\left(\frac{0.008\text{ m/rev}}{(2\pi / rev)}\right)^2 + 3.14\times10^{-5}\text{ kgm}^2 = 7.19\times10^{-5}\text{ kgm}^2$$

The inertia ratio is then:

$$Ratio_J = \frac{J_{load}}{J_{motor}} = \frac{7.19\times10^{-5}\text{ kgm}^2}{1.10\times10^{-5}\text{ kgm}^2} = 6.54$$

Since it is within the range 1 to 10 it is acceptable.

Next, we should check the max. required motor and RMS motor torques. The torques are obtained for the eight periods shown in Figure 3.46 as follows:

1. **Accelerate, forward (upwards in this example):**

$$\tau_{external} = \frac{Fl}{(2\pi / rev)\eta_s}$$

$$= \frac{(Mg + F_{friction})l}{(2\pi / rev)\eta_s} = \frac{\big((25\text{ kg})(9.81\text{ N/kg})+50\text{ N}\big)(0.008\text{ m/rev})}{(2\pi / rev)(1)} = 0.376\text{ Nm}$$

$$\dot{\omega}_{motor} = \frac{a_{load}}{l} = \frac{5\text{ m/s}^2}{(0.008\text{ m/rev})(1\text{ rev}/2\pi)} = 3930\text{ rad/s}^2$$

$$\begin{aligned}\tau_{motor,1} &= J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external} \\ &= (1.1\times10^{-5}\text{ kgm}^2)(3930\text{ rad/s}^2) + (7.19\times10^{-5}\text{ kgm}^2)(3930\text{ rad/s}^2) + 0.376\text{ Nm} \\ &= 0.702\text{ Nm}\end{aligned}$$

2. **Constant velocity, forward:**

Since the mass is still being moved upwards: $\tau_{external} = 0.376\text{ Nm}$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,2} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external} = \tau_{external} = 0.376\text{ Nm}$$

3. **Decelerate, forward:**

Since the mass is still being moved upwards: $\tau_{external} = 0.376\text{ Nm}$

$$\dot{\omega}_{motor} = \frac{a_{load}}{l} = \frac{-5\text{ m/s}^2}{(0.008\text{ m/rev})(1\text{ rev}/2\pi)} = -3930\text{ rad/s}^2$$

$$\tau_{motor,3} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external}$$
$$= (1.10\times10^{-5}\text{ kgm}^2)(-3930\text{ rad/s}^2) + (7.19\times10^{-5}\text{ kgm}^2)(-3930\text{ rad/s}^2) + 0.376\text{ Nm}$$
$$= 0.050\text{ Nm}$$

4. 1st Idle:

When the velocity is zero the friction force should oppose the gravity force acting on the mass, therefore:

$$\tau_{external} = \frac{Fl}{(2\pi/rev)\eta_s}$$
$$= \frac{(Mg + F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\big((25\text{ kg})(9.81\text{ N/kg}) - 50\text{ N}\big)(0.008\text{ m/rev})}{(2\pi/rev)(1)} = 0.249\text{ Nm}$$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,4} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external} = \tau_{external} = 0.249\text{ Nm}$$

5. Accelerate, reverse (downwards in this example):

The friction force will act upwards, opposing the downwards velocity. Therefore $Mg + F_{friction}$ will be same as the previous period, and: $\tau_{external} = 0.249\text{ Nm}$.

$$\dot{\omega}_{motor} = \frac{a_{load}}{l} = \frac{-7.5\text{ m/s}^2}{(0.008\text{ m/rev})(1\text{ rev}/2\pi)} = -5890\text{ rad/s}^2$$

$$\tau_{motor,5} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external}$$
$$= (1.10\times10^{-5}\text{ kgm}^2)(-5890\text{ rad/s}^2) + (7.19\times10^{-5}\text{ kgm}^2)(-5890\text{ rad/s}^2) + 0.249\text{ Nm}$$
$$= -0.240\text{ Nm}$$

6. Constant velocity, reverse:

Since the mass is still being moved downwards: $\tau_{external} = 0.249\text{ Nm}$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,6} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external} = \tau_{external} = 0.249\text{ Nm}$$

7. Decelerate, reverse:

Since the mass is still being moved downwards: $\tau_{external} = 0.249\text{ Nm}$

$$\dot{\omega}_{motor} = \frac{a_{load}}{l} = \frac{7.5\text{ m/s}^2}{(0.008\text{ m/rev})(1\text{ rev}/2\pi)} = 5890\text{ rad/s}^2$$

$$\tau_{motor,7} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external}$$
$$= (1.10\times10^{-5}\text{ kgm}^2)(5890\text{ rad/s}^2) + (7.19\times10^{-5}\text{ kgm}^2)(5890\text{ rad/s}^2) + 0.249\text{ Nm}$$
$$= 0.737\text{ Nm}$$

8. 2nd Idle:

The loading situation is the same as with the 1st idle period, therefore:

$$\tau_{motor,8} = J_{motor}\dot{\omega}_{motor} + J_{load}\dot{\omega}_{motor} + \tau_{external} = \tau_{external} = 0.249\text{ Nm}$$

From the above results, the max. required torque is: $\tau_{motor,max} = \tau_{motor,7} = 0.737\text{ Nm}$. This is below the motor's rated max. torque of 1.5 Nm, but above its rated continuous torque. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\tau_{motor,RMS} = \sqrt{\frac{\sum_{i=1}^{n}\tau_{motor,i}^2 t_i}{\sum_{i=1}^{n} t_i}}$$
$$= \sqrt{\frac{\left((0.702)^2(0.1)+(0.376)^2(0.2)+(0.050)^2(0.1)+(0.249)^2(0.125)+(-0.240)^2(0.1)+(0.249)^2(0.1)+(0.737)^2(0.1)+(0.249)^2(0.125)\right)\text{ N}^2\text{m}^2\text{s}}{(0.1+0.2+0.1+0.125+0.1+0.1+0.1+0.125)\text{ s}}}$$
$$= 0.410\text{ Nm}$$

Since $\tau_{motor,RMS}$ is below the motor's continuous torque rating of 0.45 Nm, neither torque limit has been exceeded.

Lastly, we need to check the motor's operating temperature. The question states that the friction of the motor is negligible, therefore $K_d \approx 0$ and the torque output of the motor is simply $K_t I$. Then the RMS current is simply:

$$I_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} I_i^2 t_i}{\sum_{i=1}^{n} t_i}} = \sqrt{\frac{\sum_{i=1}^{n}\left(\tau_{motor,i}/K_t\right)^2 t_i}{\sum_{i=1}^{n} t_i}} = \frac{\tau_{motor,RMS}}{K_t} = \frac{0.410\text{Nm}}{0.1\text{ Nm/A}} = 4.10\text{ A}$$

The power loss is:

$$P_j = I_{RMS}^2 R_{Hot} = (4.10\text{ A})^2(2.15\text{ ohm}) = 36.0\text{ W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 30\text{ °C} + (36.1\text{ W})(1.43\text{ °C/W}) = 81.5\text{ °C}$$

Since this is less than the motor's temperature limit this test has been passed.

We can conclude that this motor and ball screw combination has passed all of the tests and is acceptable. Compared to the design from part (a), this design is less energy efficient as shown by the P_j values (36.1 W vs. 20.1 W).