ENG PHYS 2A04 Tutorial 5

Electricity and Magnetism

Your TAs today

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Chapter 3

Problem 3.17 – Question

Find a vector **G** whose magnitude is 4 and whose direction is perpendicaular to both vectors **E** and **F**, where:

$\mathbf{E} = \hat{x} + \hat{y}2 - \hat{z}2$ and $\mathbf{F} = \hat{y}3 - \hat{z}6$

Problem 3.17 – Question

Find a vector **G** whose magnitude is 4 and whose direction is perpendicaular to both vectors **E** and **F**, where:

$\mathbf{E} = \hat{x} + \hat{y}2 - \hat{z}2$ and $\mathbf{F} = \hat{y}3 - \hat{z}6$

Eq. 3.1: $\boldsymbol{G} = G\hat{\boldsymbol{g}}$

Eq. 3.22: vector normal to the plane containing **E** and **F**: **E×F**

Eq. 3.22: unit vector: $\dfrac{\mathbf{E}\times\mathbf{F}}{|\mathbf{E}\times\mathbf{F}|}$

Problem 3.17 – Question

Find a vector **G** whose magnitude is 4 and whose direction is perpendicular to both vectors **E** and **F**, where:

$\mathbf{E} = \hat{x} + \hat{y}2 - \hat{z}2$ and $\mathbf{F} = \hat{y}3 - \hat{z}6$

$$\mathbf{G} = \pm 4 \frac{\mathbf{E} \times \mathbf{F}}{|\mathbf{E} \times \mathbf{F}|}$$

Problem 3.17 – Question

Find a vector **G** whose magnetude is 4 and whose direction is perpendicaular to both vectors **E** and **F**, where:

$\mathbf{E} = \hat{x} + \hat{y}2 - \hat{z}2$ and $\mathbf{F} = \hat{y}3 - \hat{z}6$

Eq.3.28: $$\boldsymbol{E}\times\boldsymbol{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \boldsymbol{E_x} & \boldsymbol{E_y} & \boldsymbol{E_z} \\ \boldsymbol{F_x} & \boldsymbol{F_y} & \boldsymbol{F_z} \end{vmatrix}$$

$$= \left(\boldsymbol{E_y}\,\boldsymbol{F_z} - \boldsymbol{E_z}\,\boldsymbol{F_y}\right)\hat{x} + \left(\boldsymbol{E_z}\boldsymbol{F_x} - \boldsymbol{E_x}\boldsymbol{F_z}\right)\hat{y} + \left(\boldsymbol{E_x}\boldsymbol{F_y} - \boldsymbol{E_y}\boldsymbol{F_x}\right)\hat{z}$$

$$= -\hat{x}6 + \hat{y}6 + \hat{z}3$$

Problem 3.17 – Question

Find a vector **G** whose magnetude is 4 and whose direction is perpendicaular to both vectors **E** and **F**, where:

$\mathbf{E} = \hat{x} + \hat{y}2 - \hat{z}2$ and $\mathbf{F} = \hat{y}3 - \hat{z}6$

Eq. 3.4: $|\mathbf{E}\times\mathbf{F}| = +\sqrt{(-6)^2 + (6)^2 + (3)^2} = 9$

$$\mathbf{G} = \pm 4\frac{\mathbf{E}\times\mathbf{F}}{|\mathbf{E}\times\mathbf{F}|} = \pm 4\frac{(-\hat{x}6 + \hat{y}6 + \hat{z}3)}{9} = \pm(-\hat{x}\frac{8}{3} + \hat{y}\frac{8}{3} + \hat{z}\frac{4}{3})$$

Problem 3.26 – Question

Find the volume described by the following, Also sketch the outline of the volume:

$$2 \leq r \leq 5; \frac{\pi}{2} \leq \varphi \leq \pi; 0 \leq z \leq 2$$

Recongnize that this is a cylindrical integration

Problem 3.26 – Question

Find the volume described by the following, Also sketch the outline of the volume:

$$2 \leq r \leq 5; \frac{\pi}{2} \leq \varphi \leq \pi; 0 \leq z \leq 2$$

Table 3–1: $V = \int_{z=0}^{2} \int_{\varphi=\pi/2}^{\pi} \int_{r=2}^{5} r dr d\varphi dz$

Problem 3.26 – Question

Find the volume described by the following, Also sketch the outline of the volume:

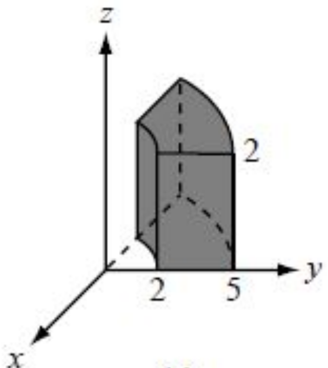
$$2 \leq r \leq 5; \frac{\pi}{2} \leq \varphi \leq \pi; 0 \leq z \leq 2$$

$$V = \left(\frac{1}{2} r^2 \Big|_{r=2}^{5}\right)\left(\varphi \Big|_{\varphi=\pi/2}^{\pi}\right)\left(z \Big|_{z=0}^{2}\right) = \frac{21\pi}{2}$$

Problem 3.26 – Question

Find the volume described by the following, Also sketch the outline of the volume:

$$2 \le r \le 5; \frac{\pi}{2} \le \varphi \le \pi; 0 \le z \le 2$$



(a)

Problem 3.34 – Question

Transform the following vectors into cylindrical coordinates and then evaluate them at the indicated points:

a) $\mathbf{A} = \hat{\boldsymbol{x}}(x + y)$ at $P_1 = (1,2,3)$

b) $\mathbf{C} = \hat{\boldsymbol{x}}\frac{y^2}{(x^2+y^2)} - \hat{\boldsymbol{y}}\frac{x^2}{(x^2+y^2)} + \hat{\boldsymbol{z}}4$ at $P_2 = (1,-1,2)$

Problem 3.34 – Question

$\mathbf{A} = \hat{\boldsymbol{x}}(x + y)$ at $P_1 = (1,2,3)$

Convert to cylindrical coordinates

Table 3-2: $r = +\sqrt{x^2 + y^2} = +\sqrt{1^2 + 2^2} = \sqrt{5},$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{1}\right) = 63.4°$$

$$z = z = 3$$

$$P_1 = \left(\sqrt{5}, 63.4°, 3\right)$$

Problem 3.34 – Question

$\mathbf{A} = \hat{\boldsymbol{x}}(x + y)$ at $P_1 = (1,2,3)$

Convert to cylindrical coordinates

Table 3-2: $\boldsymbol{A} = \hat{\boldsymbol{r}}\mathrm{A}_r + \hat{\boldsymbol{\varphi}}\mathrm{A}_\varphi + \hat{\mathbf{z}}\mathrm{A}_z$

$$\boldsymbol{x} = \boldsymbol{r}\cos\varphi;\ \boldsymbol{y} = \boldsymbol{r}\sin\varphi$$

$$\mathrm{A}_r = \mathrm{A}_x\cos\varphi + \mathrm{A}_y\sin\varphi = (\boldsymbol{x} + \boldsymbol{y})\cos\varphi = (\boldsymbol{r}\cos\varphi + \boldsymbol{r}\sin\varphi)\cos\varphi$$

$$\mathrm{A}_\varphi = -\mathrm{A}_x\sin\varphi + \mathrm{A}_y\cos\varphi = -(\boldsymbol{x} + \boldsymbol{y})\sin\varphi = -(\boldsymbol{r}\cos\varphi + \boldsymbol{r}\sin\varphi)\sin\varphi$$

$$\mathrm{A}_z = 0$$

$$\mathbf{A} = \hat{\boldsymbol{r}}r\cos\varphi\,(\cos\varphi + \sin\varphi) - \hat{\boldsymbol{\varphi}}r\sin\varphi\,(\cos\varphi + \sin\varphi)$$

Problem 3.34 – Question

$\mathbf{A} = \hat{\boldsymbol{x}}(x + y)$ at $P_1 = (1,2,3)$

$$\mathbf{A} = \hat{\boldsymbol{r}} r \cos\varphi\,(\cos\varphi + \sin\varphi) - \hat{\boldsymbol{\varphi}} r \sin\varphi\,(\cos\varphi + \sin\varphi)$$

$$\boldsymbol{P_1} = \left(\sqrt{5}, 63.4°, 3\right)$$

$$\mathbf{A}(\boldsymbol{P_1}) = \hat{\boldsymbol{r}}1.34 - \hat{\boldsymbol{\varphi}}2.68$$

Problem 3.34 – Question

$\mathbf{C} = \hat{\boldsymbol{x}} \frac{y^2}{(x^2+y^2)} - \hat{\boldsymbol{y}} \frac{x^2}{(x^2+y^2)} + \hat{\mathbf{z}}4$ at $P_2 = (1,-1,2)$

Convert to cylindrical coordinates

Table 3-2: $r = +\sqrt{x^2 + y^2} = +\sqrt{1^2 + (-1)^2} = \sqrt{2},$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = -45°$$

$$z = z = 2$$

$$P_2 = \left(\sqrt{2}, -45°, 2\right)$$

Problem 3.34 – Question

$\mathbf{C} = \hat{\boldsymbol{x}} \frac{y^2}{(x^2+y^2)} - \hat{\boldsymbol{y}} \frac{x^2}{(x^2+y^2)} + \hat{\mathbf{z}}4$ at $P_2 = (1,-1,2)$

Table 3-2: $\boldsymbol{C} = \hat{\boldsymbol{r}}C_r + \hat{\boldsymbol{\varphi}}C_\varphi + \hat{\mathbf{z}}C_z$

$$\boldsymbol{x} = \boldsymbol{r} \cos\varphi; \boldsymbol{y} = \boldsymbol{r} \sin\varphi$$

$$C_r = \sin\varphi \cos\varphi \,(\sin\varphi - \cos\varphi)$$

$$C_\varphi = -(\sin^3\varphi + \cos^3\varphi)$$

$$C_z = 4$$

$$C = \hat{\boldsymbol{r}} \sin\varphi \cos\varphi \,(\sin\varphi - \cos\varphi) - \hat{\boldsymbol{\varphi}}(\sin^3\varphi + \cos^3\varphi) + \hat{\mathbf{z}}4$$

Problem 3.34 – Question

$\mathbf{C} = \hat{\boldsymbol{x}}\frac{y^2}{(x^2+y^2)} - \hat{\boldsymbol{y}}\frac{x^2}{(x^2+y^2)} + \hat{\boldsymbol{z}}4$ at $P_2 = (1,-1,2)$

$$\mathrm{C} = \hat{\boldsymbol{r}}\sin\varphi\cos\varphi\,(\sin\varphi - \cos\varphi) - \hat{\boldsymbol{\varphi}}(\sin^3\varphi + \cos^3\varphi) + \hat{\boldsymbol{z}}4$$

$$P_2 = \left(\sqrt{2}, -45°, 2\right)$$

$$\mathbf{C}(\boldsymbol{P_2}) = \hat{\boldsymbol{r}}0.707 - \hat{\boldsymbol{\varphi}}4$$

Problem 3.36-e – Question

Find the gradiant of the following scalr function:

$$\boldsymbol{S} = 4x^2e^{-z} + y^3$$

Problem 3.36-e – Solution

Find the gradiant of the following scalr function:

$$\boldsymbol{S} = 4x^2 e^{-z} + y^3$$

$$\textbf{AS, } \nabla = \widehat{\boldsymbol{x}}\frac{\partial}{\partial x} + \widehat{\boldsymbol{y}}\frac{\partial}{\partial y} + \hat{\boldsymbol{z}}\frac{\partial}{\partial z}$$

Problem 3.36-e – Solution

So

$$\nabla \boldsymbol{S} = \hat{\boldsymbol{x}}\frac{\partial S}{\partial x} + \hat{\boldsymbol{y}}\frac{\partial S}{\partial y} + \hat{\boldsymbol{z}}\frac{\partial S}{\partial z}$$

Putting values of function S

$$\nabla \boldsymbol{S} = \hat{\boldsymbol{x}}\frac{\partial(4x^2e^{-z} + y^3)}{\partial x} + \hat{\boldsymbol{y}}\frac{\partial(4x^2e^{-z} + y^3)}{\partial y} + \hat{\boldsymbol{z}}\frac{\partial(4x^2e^{-z} + y^3)}{\partial z}$$

$$\boxed{\nabla \boldsymbol{S} = \hat{\boldsymbol{x}}8xe^{-z} + \hat{\boldsymbol{y}}3y^2 - \hat{\boldsymbol{z}}4x^2e^{-z}}$$

Problem 3.48 – Question

For the vector field $\boldsymbol{E} = \hat{\boldsymbol{x}}xz - \hat{\boldsymbol{y}}yz^2 - \hat{\mathbf{z}}xy$, verify the divergence theorem by computing

a) The total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes.

b) The integral of $\nabla \cdot \boldsymbol{E}$ over the cube's volume.

Divergence Theorem!

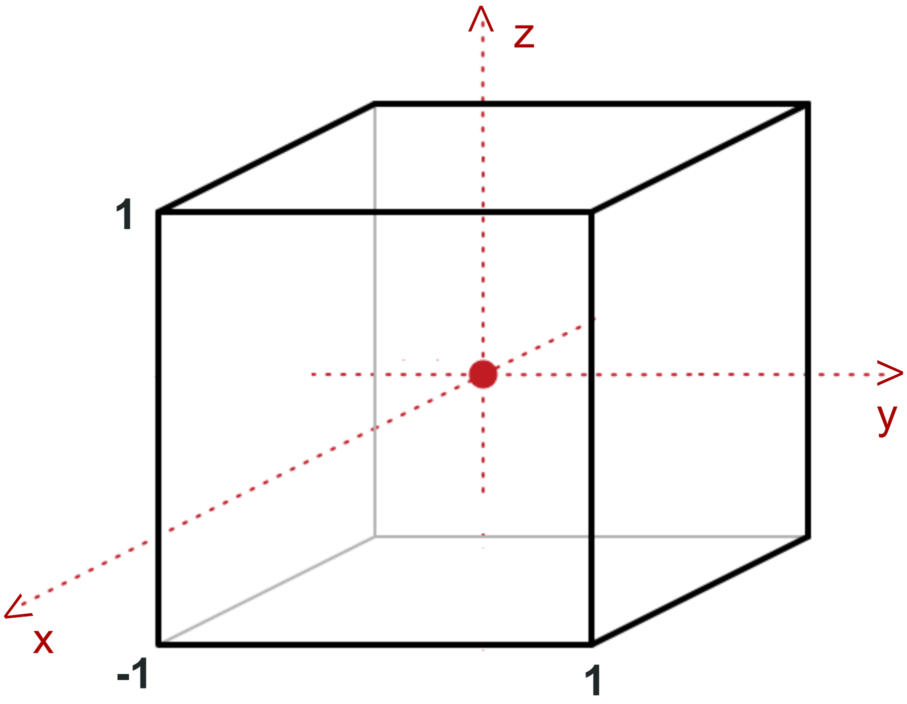
$$\oint_S \vec{E} \cdot d\vec{s} = \int_{\boldsymbol{V}} (\boldsymbol{\nabla} \cdot \boldsymbol{E}) dV$$

Problem 3.48 – Solution

<u>Part (a):</u>

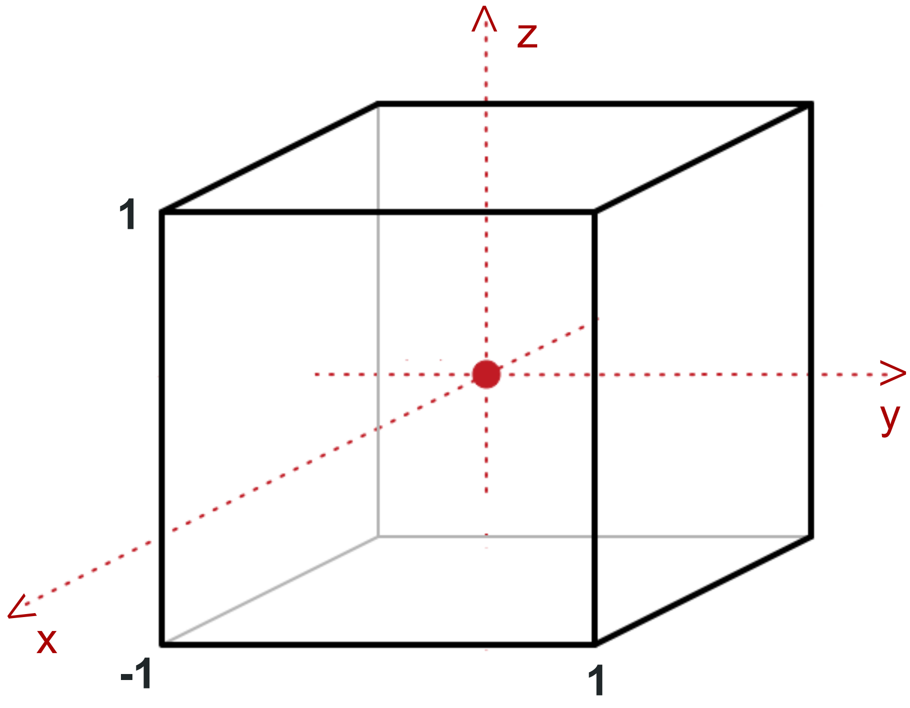
Goal → Calculate total outward flux

Given → $\boldsymbol{E} = \hat{\boldsymbol{x}}xz - \hat{\boldsymbol{y}}yz^2 - \hat{\boldsymbol{z}}xy$



Problem 3.48 – Solution

Part (a):



1) How to calculate the total flux?

Flux through each face → Add them all

2) Flux: $F = \oint_S \vec{E} \cdot d\vec{s}$

$$F_{Total} = F_{top} + F_{bottom} + F_{right} + F_{left} + F_{front} + F_{back}$$

Problem 3.48 – Solution

Part (a):

$$F_{top} = \int \left(\vec{E}\big|_{z=1} \cdot (\hat{z}\,dx\,dy) \right)$$

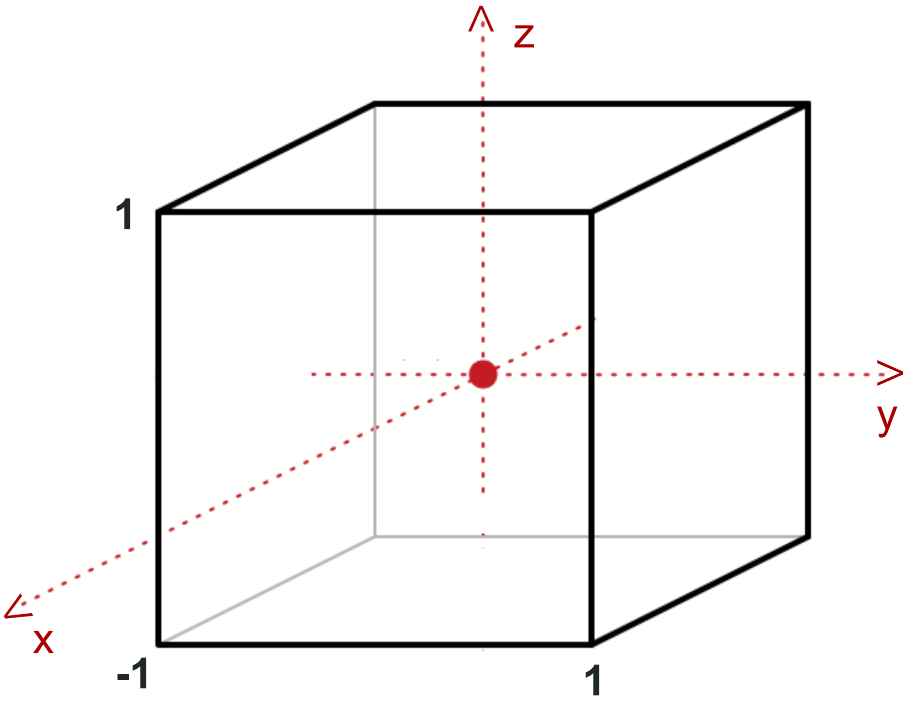
$$F_{bottom} = \int \left(\vec{E}\big|_{z=-1} \cdot (-\hat{z}\,dx\,dy) \right)$$

$$F_{right} = \int \left(\vec{E}\,\Big|_{y=1} \cdot (\hat{y}\,dx\,dz) \right)$$

$$F_{left} = \int \left(\vec{E}\,\Big|_{y=-1} \cdot (-\hat{y}\,dx\,dz) \right)$$

$$F_{front} = \int \left(\vec{E}\big|_{x=1} \cdot (\hat{x}\,dy\,dz) \right)$$

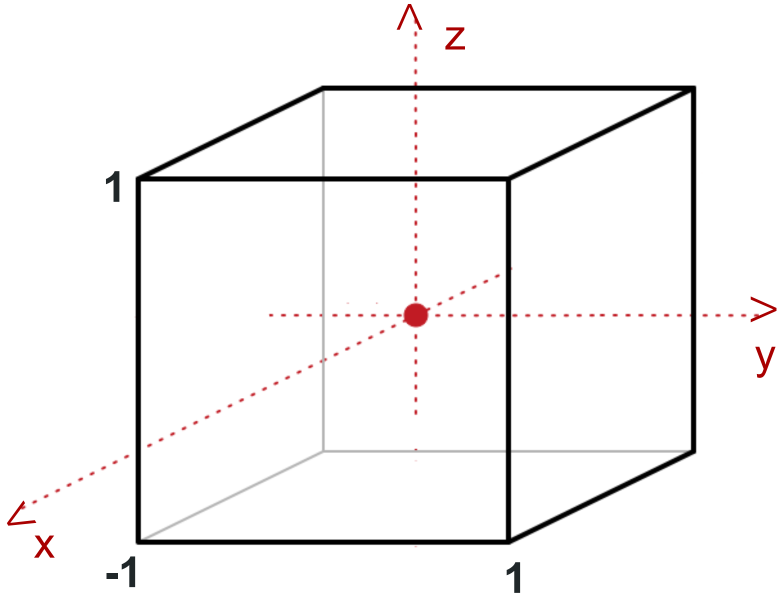
$$F_{back} = \int \left(\vec{E}\big|_{x=-1} \cdot (-\hat{x}\,dy\,dz) \right)$$



Problem 3.48 – Solution

Part (a):

$$\left.\begin{aligned} F_{top} &= 0 \\ F_{bottom} &= 0 \\ F_{right} &= -\frac{4}{3} \\ F_{left} &= -\frac{4}{3} \\ F_{front} &= 0 \\ F_{back} &= 0 \end{aligned}\right\} \quad \boxed{\boldsymbol{F_{Total} = -\frac{8}{3}}}$$



Problem 3.48 – Solution

Part (a):

An Idea how to calculate F_{right} and Fleft

$$\boldsymbol{F_{right}} = \int_{x=1}^{1}\int_{z=-1}^{1} \hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy \Big|_{y=1} \cdot (\hat{y}\, dx\, dz)$$

$$\boldsymbol{F_{right}} = -\int_{x=1}^{1}\int_{z=-1}^{1} z^2 dzdx$$

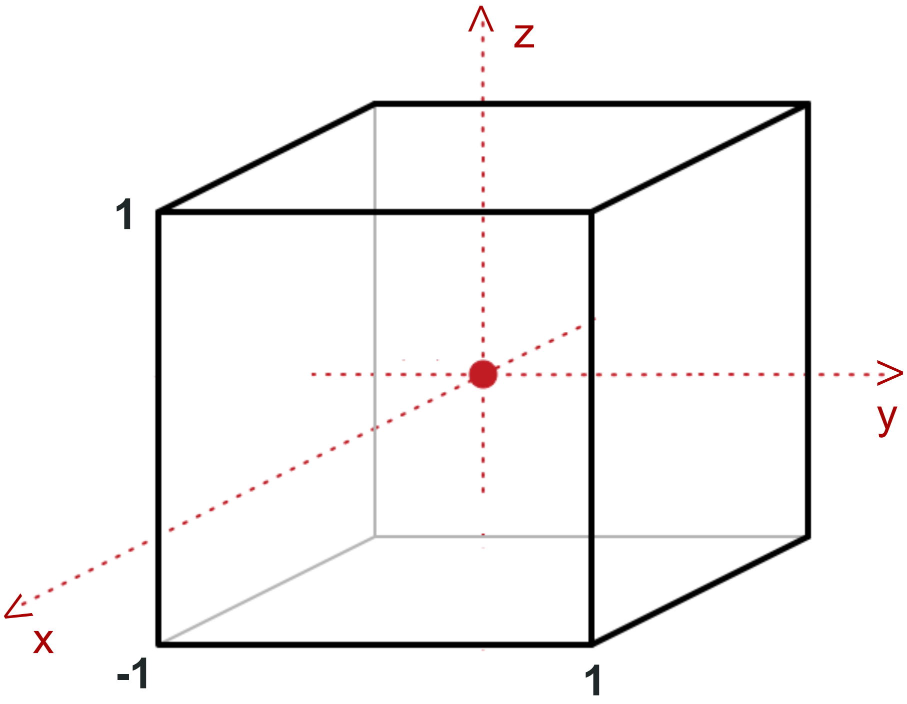
$$\boldsymbol{F_{right}} = -\left(\left(\frac{xz^3}{3}\right)\Big|_{z=-1}^{1}\right)\Big|_{x=-1}^{1}\Bigg) = -\frac{4}{3}$$

Problem 3.48 – Solution

Part=b

The integral of $\nabla \cdot \boldsymbol{E}$ over the cube's volume.

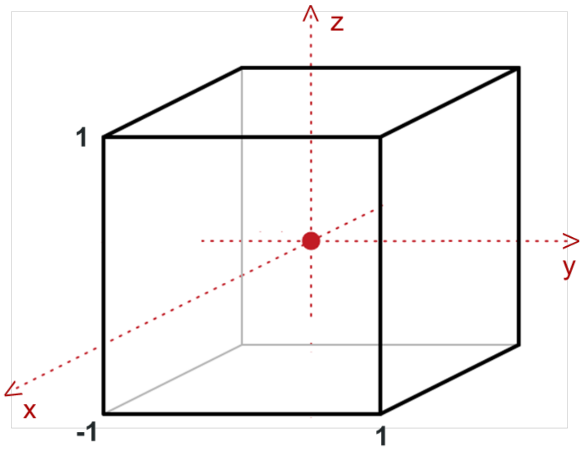
$$\nabla \cdot \boldsymbol{E} = \frac{\partial Ex}{\partial x} + \frac{\partial Ey}{\partial y} + \frac{\partial Ez}{\partial z}$$



Problem 3.48 – Solution

Part=b

$$\nabla \cdot \boldsymbol{E} = \frac{\partial Ex}{\partial x} + \frac{\partial Ey}{\partial y} + \frac{\partial Ez}{\partial z}$$



Put

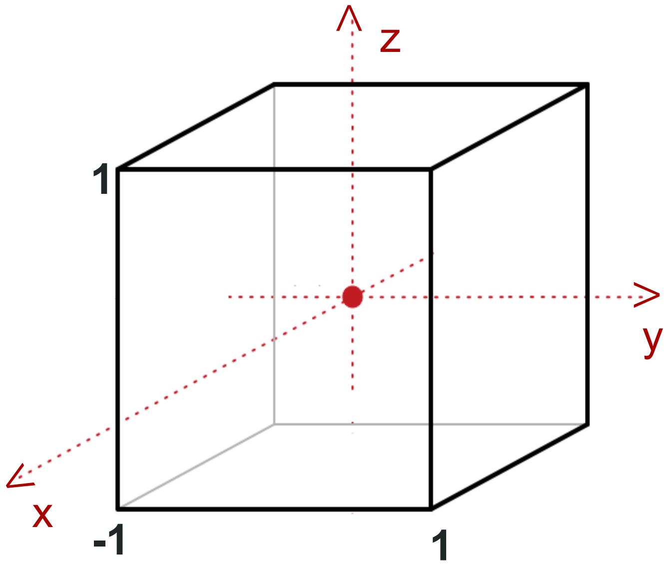
$$\boldsymbol{E_x} = (\hat{\boldsymbol{x}}xz\ )$$
$$\boldsymbol{E_y} = (-\hat{\boldsymbol{y}}yz^2$$
$$\boldsymbol{E_z} = (-\hat{\boldsymbol{z}}xy)$$

$$\nabla \cdot \boldsymbol{E} = \frac{\partial(\hat{\boldsymbol{x}}xz)}{\partial x} + \frac{\partial(-\hat{\boldsymbol{y}}yz^2)}{\partial y} + \cancel{\frac{\partial(-\hat{\boldsymbol{z}}xy)}{\partial z}}$$

Zero

Problem 3.48 – Solution

Part=b



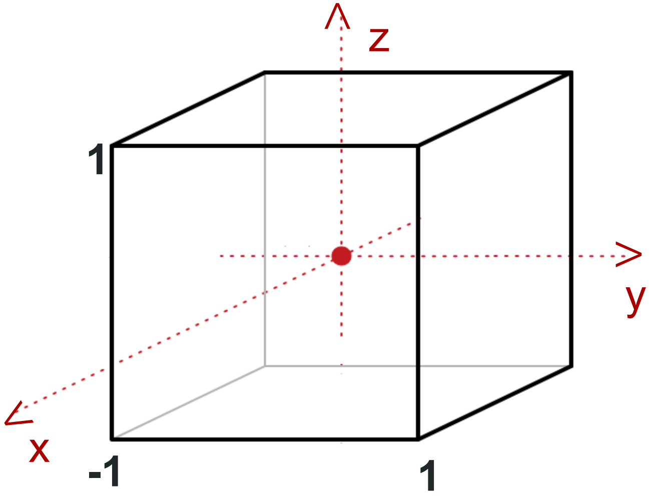
The integral of $\nabla \cdot \boldsymbol{E}$ over the cube's volume.

We get,

$$\nabla \cdot \boldsymbol{E} = -z^2 + z$$

Problem 3.48 – Solution

Part=b



$$\int\int\int(\nabla\cdot\boldsymbol{E})dV=\int_{x-1}^{1}\int_{y-1}^{1}\int_{z-1}^{1}\nabla.\left(\hat{\boldsymbol{x}}xz-\hat{\boldsymbol{y}}yz^2-\hat{\mathbf{z}}xy\right)dxdydz$$

$$\int\int\int(\nabla\cdot\boldsymbol{E})dV=\int_{x-1}^{1}\int_{y-1}^{1}\int_{z-1}^{1}\left(z-z^2\right)dxdydz$$

Problem 3.48 – Solution

Part=b

$$\int\int\int(\boldsymbol{\nabla}\cdot\boldsymbol{E})dV=\left(\left(\left(\boldsymbol{xy}\left(\frac{\boldsymbol{z}^2}{2}-\frac{\boldsymbol{z}^3}{\boldsymbol{3}}\right)\right)\Big|_{\boldsymbol{z}=-\boldsymbol{1}}^{\boldsymbol{1}}\right)\Big|_{\boldsymbol{y}=-\boldsymbol{1}}^{\boldsymbol{1}}\right)\Big|_{\boldsymbol{x}=-\boldsymbol{1}}^{\boldsymbol{1}}\Bigg)=\boxed{-\frac{\boldsymbol{8}}{\boldsymbol{3}}}$$

Divergence Theorem!

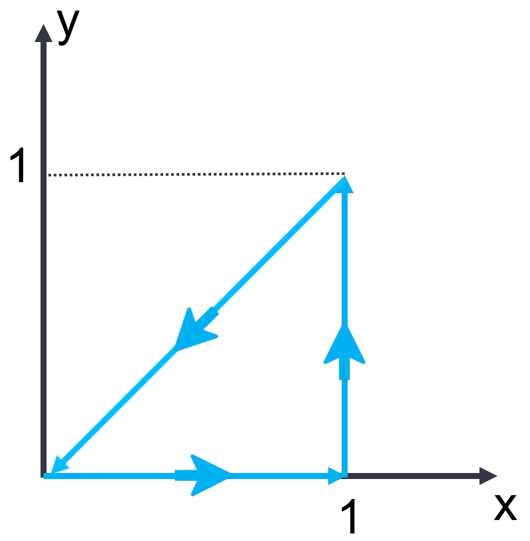
$$\oint_S \vec{E} \cdot d\vec{s} = \int_{\boldsymbol{V}} (\boldsymbol{\nabla} \cdot \boldsymbol{E})\, dV$$

Problem 3.52-Question

For the vector field $\boldsymbol{E} = \hat{\boldsymbol{x}}xy - \hat{\boldsymbol{y}}(x^2 + 2y^2)$, calculate

a) $\oint_C \vec{E} \cdot d\vec{l}$ around the triangular contour shown in the Figure

b) $\int_S (\boldsymbol{\nabla} \times \boldsymbol{E}) \cdot d\boldsymbol{s}$ over the area of the triangle

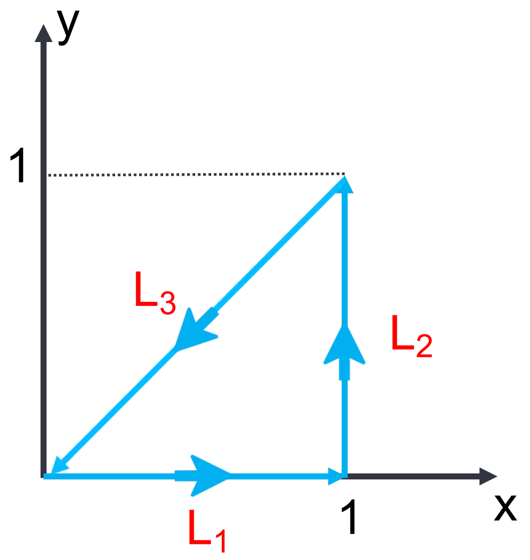


Stokes' Theorem!

$$\oint_C \vec{E} \cdot d\vec{l} = \int_{\boldsymbol{S}} (\boldsymbol{\nabla} \times \boldsymbol{E}) \cdot d\boldsymbol{s}$$

Problem 3.52

Part (a)



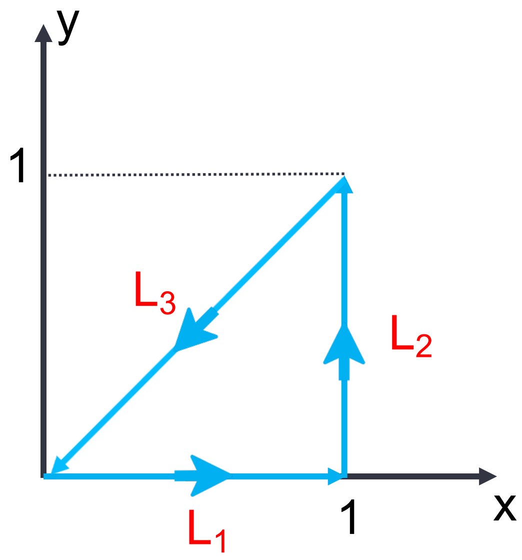
Split the countor C in 3 sections:

$$\oint_C \vec{E} \cdot d\vec{l} = \int_{L_1} \vec{E} \cdot d\overrightarrow{l_1} + \int_{L_2} \vec{E} \cdot d\overrightarrow{l_2} + \int_{L_3} \vec{E} \cdot d\overrightarrow{l_3}$$

Problem 3.52

Part (a)

$$\boldsymbol{E} = \hat{\boldsymbol{x}}xy - \hat{\boldsymbol{y}}(x^2 + 2y^2)$$



$$\int_{L_1} \vec{E} \cdot d\vec{l_1} = \int_0^1 [\hat{\boldsymbol{x}}xy - \hat{\boldsymbol{y}}(x^2 + 2y^2)]\Big|_{y=0} \cdot \hat{\boldsymbol{x}}dx$$

$$\int_{L_2} \vec{E} \cdot d\vec{l_2} = \int_1^1 [\hat{\boldsymbol{x}}xy - \hat{\boldsymbol{y}}(x^2 + 2y^2)]\Big|_{x=1} \cdot \hat{\boldsymbol{y}}dy$$

$$\int_{L_3} \vec{E} \cdot d\vec{l_3} = \int [\hat{\boldsymbol{x}}xy - \hat{\boldsymbol{y}}(x^2 + 2y^2)]\Big|_{y=x} \cdot (\hat{\boldsymbol{x}}dx + \hat{\boldsymbol{y}}dy)$$

Problem 3.52
Part (a)

$Practice\ for\ solving\ L_2$

$$L_2 = \int_{L_2} \vec{E} \cdot \overrightarrow{dl_2} = \int [\hat{\boldsymbol{x}}xy - \hat{\boldsymbol{y}}(x^2 + 2y^2).(\hat{\boldsymbol{x}}dx + \hat{\boldsymbol{y}}\,dy + \hat{\boldsymbol{z}}dz)]$$

$$L_2 = \int_{x=1}^{1}[xy)|_{\boldsymbol{z=0}}dx - \int_{y=0}^{1}(x^2 + 2y^2\,\Big|_{\boldsymbol{x=1,z=o}}^{\boldsymbol{1}}dy\int_{z=0}^{1}(0)\Big|_{\boldsymbol{x=1}}^{\boldsymbol{1}}\boldsymbol{dz}$$

$$L_2 = 0 - (y + \frac{2y^2}{3}\Big|_{\boldsymbol{y=1}}^{\boldsymbol{0}} + \boldsymbol{0} = -\boldsymbol{\frac{5}{3}}$$

$L_1\ and\ L_3\ can\ be\ solved\ by\ same\ process$

Problem 3.52

Part (a)

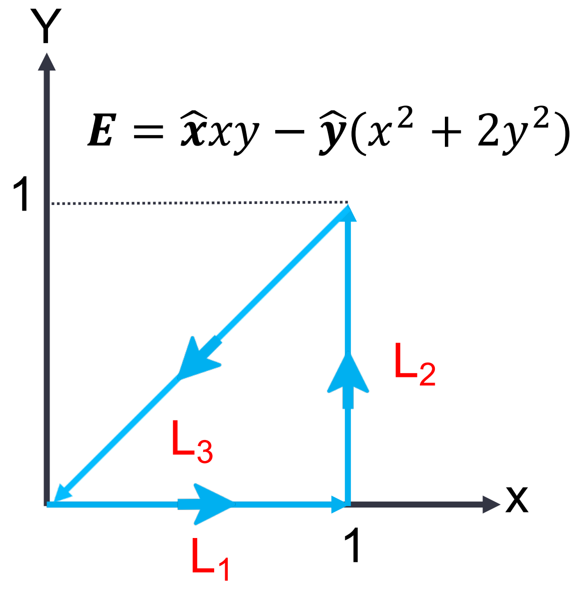
$$\int_{L_1} \vec{E} \cdot d\overrightarrow{l_1} = 0$$

$$\int_{L_2} \vec{E} \cdot d\overrightarrow{l_2} = -\frac{5}{3}$$

$$\int_{L_3} \vec{E} \cdot d\overrightarrow{l_3} = \frac{2}{3}$$

$$\oint_C \vec{E} \cdot d\vec{l} = \int_{L_1} \vec{E} \cdot d\overrightarrow{l_1} + \int_{L_2} \vec{E} \cdot d\overrightarrow{l_2} + \int_{L_3} \vec{E} \cdot d\overrightarrow{l_3}$$

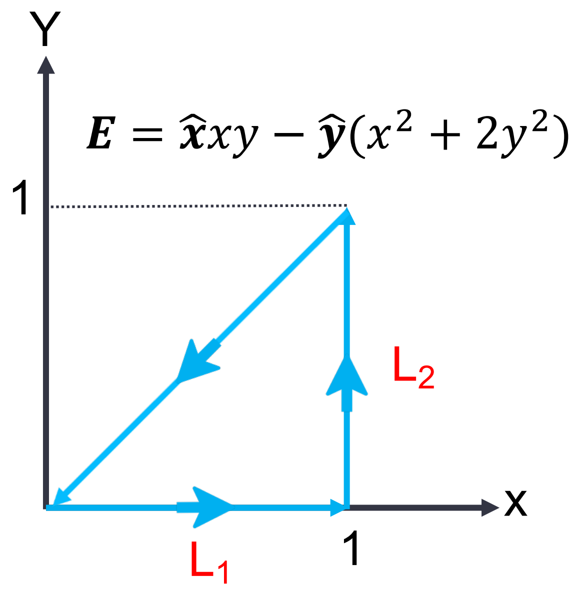
$$\boxed{\oint_C \vec{E} \cdot d\vec{l} = 0 - \frac{5}{3} + \frac{2}{3} = -1}$$



Problem 3.52

Part (b)

$$\int_S (\nabla \times \boldsymbol{E}) \cdot d\boldsymbol{s}$$



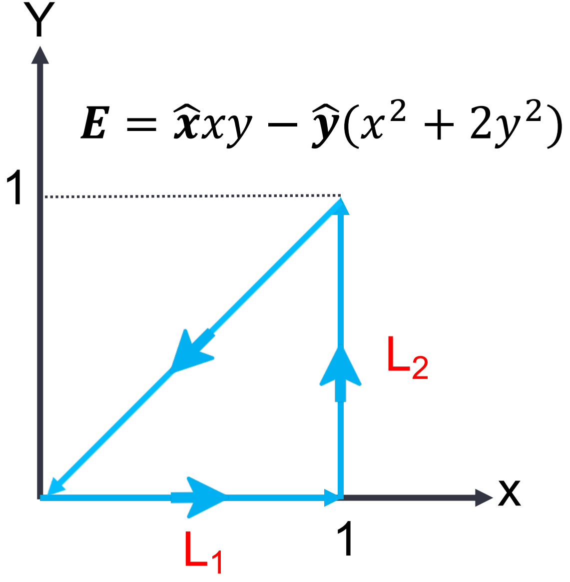
Calculate the curl:

$$(\nabla \times \boldsymbol{E}) = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -3x\hat{\boldsymbol{z}}$$

Problem 3.52

Part (b)

$$d\boldsymbol{s} = dxdy\hat{\mathbf{z}} \quad \text{and} \quad (\nabla\times\boldsymbol{E}) = -3x\hat{\mathbf{z}}$$



$$\iint (\nabla\times\boldsymbol{E})\cdot d\boldsymbol{s} = \int_{x-0}^{x=1}\int_{y=0}^{y=x} ((-3x\hat{\mathbf{z}}).(\hat{\mathbf{z}}dydx))\Big|_{z=\mathbf{0}}$$

$$\iint (\nabla\times\boldsymbol{E})\cdot d\boldsymbol{s} = -\int_{x=0}^{x=1}\int_{y=0}^{y=x} 3x\,dydx\,) = -\int_{x=0}^{1} 3x(x-0)dx = -(x^3)\Big|_{\mathbf{0}}^{\mathbf{1}} = \boxed{\mathbf{-1}}$$

Stokes' Theorem!

$$\oint_C \vec{E} \cdot d\vec{l} = \int_{\boldsymbol{S}} (\boldsymbol{\nabla} \times \boldsymbol{E}) \cdot d\boldsymbol{s}$$