Question 1

Demand

Your Canadian company produces Basem's Bionic Boots, and other innovative mechatronic footwear. Prior to launching your latest revision of the product, you commission a market study to assess global market size. The research produces the following table summarizing price vs. regional quantity (Canada + select international markets).

Price		BC	Ontario	Nova Scotia	Brazil	Egypt	Malaysia	MARKET
$	**100**	2000	2500	500	3000	10000	8000	26000
$	**200**	1400	2300	300	1000	8000	5000	18000
$	**300**	700	2000	200	200	2000	2000	7100
$	**400**	0	400	100	0	1000	500	2000
$	**500**	0	0	0	0	500	0	500

Which of the following statements can be inferred from the table

A. Global market demand appears highly linear relative to the prices under consideration.
B. The Western Canadian market demand appears highly non-linear at the lowest price points.
C. The Canadian market demand appears highly linear relative to mid and lower price points.
D. North African market demand may outpace European demand at all but the highest price points.
E. The global market demand exhibits high linearity relative to the entire price range

Answer is C as the sum of the three Canadian regions when compared to each price point displays a linear progression. Also,

- A: The implied curve from "MARKET" is clearly not linear
- B: BC being Western Canada is not showing clear non-linearity in fact first look tends to imply linearity.
- D: Egypt represents North Africa but there are no European regions.
- E: if you take differences between each successive pairing in the global MARKET column you get a major drop between $200 and $300 price points. This is clearly non-linear.

Question 2

Supply

Which statement best describes why the slope of the supply curve is typically upwards-sloping?

A. Supply always increases regardless of price
B. It is natural to expect companies to "jump in" to the market if prices continue to increase
C. It is a consequence of price being the dependent variable
D. The slope of the supply curve is often orthogonal to that of the demand curve
E. It denotes strong price elasticity

The answer is B as this is the core interpretation of the supply curve. If market prices are high, suppliers are incented to take to market. Also

- A: overly broad statement and you can easily find exception circumstances. So you can't have "always"
- C: generally, price is treated as the independent variable in this course.
- D: broadly not true mathematically and there is no mention of orthogonal relationships in this course.
- E: maybe sometimes, but shallowness of supply slope is a better indicator elasticity not just positive values.

Question 3

Equilibrium

In 2010 a market assessment for a particular product determined an equilibrium point (P_0,Q_0). In 2020, the equilibrium point was assessed to be (P_0 - \$5, Q_0 + 5). Which statement describes this situation the best?

A. Supplier companies, through increase in manufacturing efficiency, have rationalized an overall lower price in the market.
B. The demand for these types of products faced a dramatic drop in popularity.
C. Inflation has been a major impediment to market growth and has suppressed prices.
D. The decrease in the slope of curves indicate significant inelasticity in price
E. The public's opinion of this type of product has settled to lower perception of value.

The answer is A. As discussed in the notes, companies can use the supply curve to express their strategy to how best to price their products to meet the demand. A vertical drop in a supply curve implies that a company is willing to supply same quantities at a lower price. But this is a basic interpretation of supply and demand. The key here is the expression "companies ... have rationalized" i.e. the supply side has broadly improved its efficiency and generally can supply for a lower price due to better efficiency. It has shifted the equilibrium.

- B: not entirely incorrect but this does not describe a shift in equilibrium state. This is more of a disturbance that will correct back to the original equilibrium and not a new equilibrium.
- C: not very relevant to this topic.
- D: not very relevant and generally decrease in slop of the supply curve denotes increased elasticity and not inelasticity.
- E: if it was simply this then supplier companies would likely just lower production according to the basic Supply Demand model but a strict vertical shift in supply curve implies the quantitites supplies remains the same.

Question 4

Supply Curve Shift

Your company produces an innovative new household device that monitors GHG emissions from gas stoves and furnaces. In its most recent annual strategic plan, company has concluded that your supply strategy will be based on a supply curve shift to the right. Which corresponds to such a shift?

A. The entire supply chain is facing pricing increases and shifting the key parameters.
B. Demand has increased due to more awareness of sustainability issues.
C. A recent engineering review identified a consolidation of several major components into a single component and the overall cost of production is significantly lower.
D. A market correction for an equilibrium point that exhibited a one time response to a global crisis.
E. Supply curves can only shift towards the right as the slope is typically positive.

The answer is C. The interpretation of a supply curve shift to the right is that a company will produce more units given the same price. Engineering enhancements that reduce components and simplify designs can often produce lower cost of production giving a company the option to offer more at the same price which is consistent with the example in the notes. Also,

- A: Statement is overly broad.
- B: A demand increase implies a demand curve shift to the right and not the supply curve. As engineers we are often more involved in supply issues.
- D: not very consistent with the question narrative.
- E: not true. Supply curve can mathematically and conceptually shift to the right or the left.

Question 5

Supply Curve Source

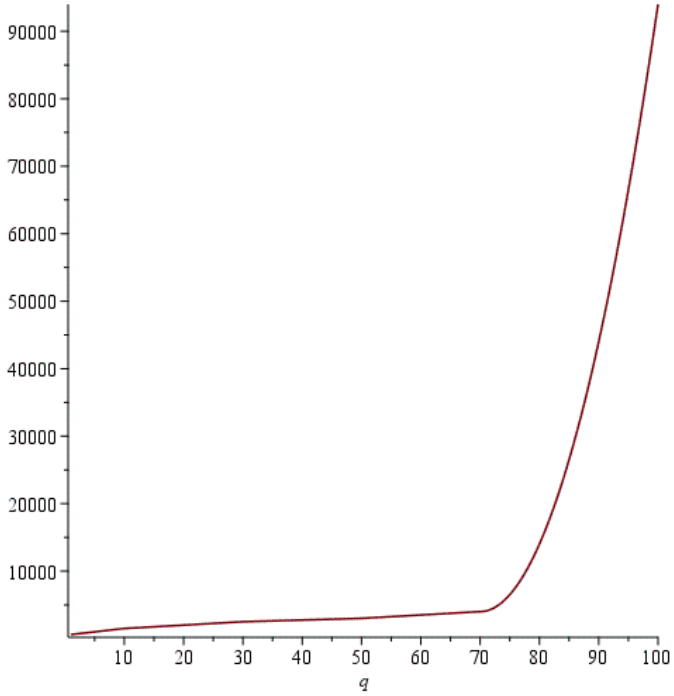
Suppose company A reasons that it would have a total cost each month to produce a given number of units as listed in the table below. At first, producing more and more units allows initial investments in renting equipment to be distributed over more units, reducing costs. However, eventually the company runs into supply chain and staffing problems associated with too much volume of this product and cost per unit begins to rise sharply.

# of units (q)	Total cost to produce this many units (C)	Average cost to produce each unit (C/q)	Slope of total cost (dC/dq) ("Marginal Cost" if quantity was continuous)
1-10	\$500 + \$100*q	\$500/q + \$100	\$100
11-30	\$1500 + \$50*(q-10)	\$1000/q + \$50	\$50
31-50	\$2500 + \$25*(q-30)	\$1750/q + \$25	\$25
51-70	\$3000 + \$50*(q-50)	\$500/q + \$50	\$50
71+	\$4000 + \$100*(q-70)^2	\$4000/q + \$100*(q-70)^2/q =\$494k/q - \$14k +\$100*q	\$200*q - \$14000

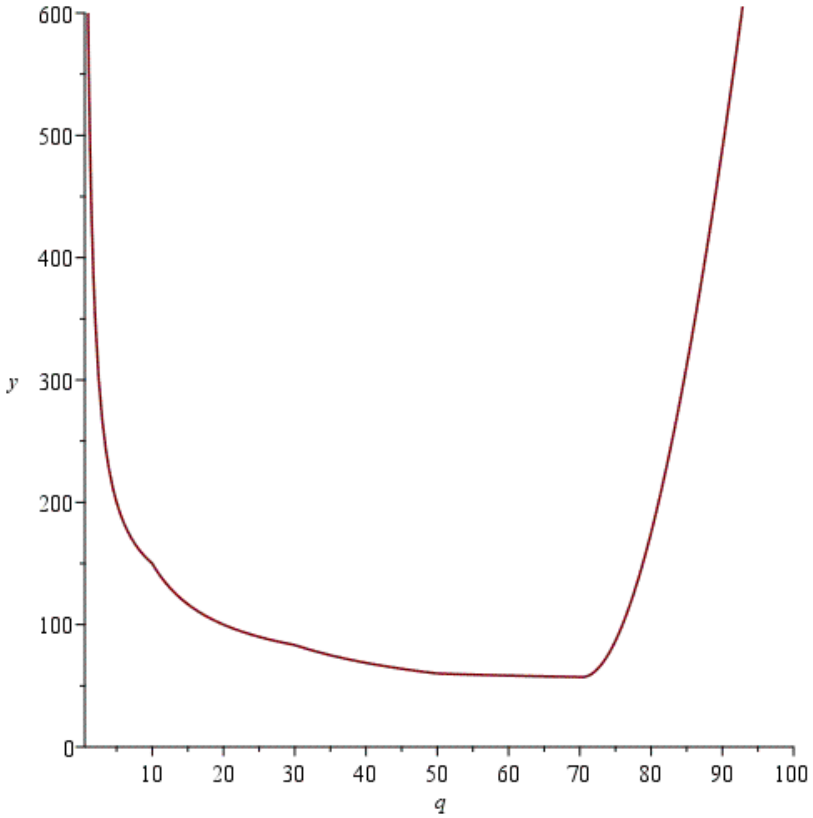
```
> restart:
C:=piecewise(q<=10, 500+100*q,
q<=30, 1500+50*(q-10),
q<=50, 2500+25*(q-30),
q<=70, 3000+50*(q-50),
q>70, 4000+100*(q-70)^2);
plot(C, q=1..100);
```

```
UnitCost:=C/q:
plot(UnitCost, q=1..100,y=0..600);
```

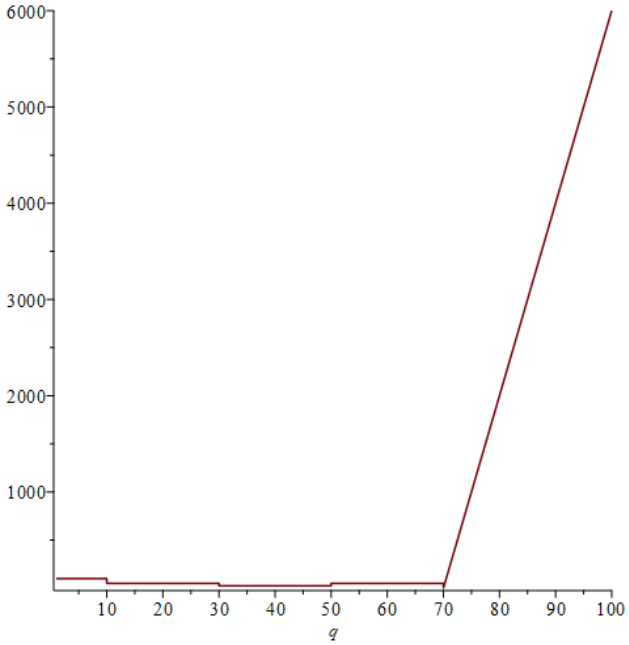
$$C := \begin{cases} 500 + 100\,q & q \le 10 \\ 1000 + 50\,q & q \le 30 \\ 1750 + 25\,q & q \le 50 \\ 500 + 50\,q & q \le 70 \\ 4000 + 100\,(q-70)^2 & 70 < q \end{cases}$$



Total cost vs. total units produced



Average cost of each unit (Total cost divided by # of units produced)



"Marginal cost" of each unit (ignoring discrete nature; i.e., derivative of total cost with respect to quantity)

If the sale price of each unit is $600, how many units should the company produce to maximize overall profit?

a) 2
b) 20
c) 73
d) 92
e) None of the above

Answer: 73. The company would not produce units if the marginal cost to produce the next one becomes more than the price they could sell them for, and the marginal cost to produce the 73rd one is $600 (actually, slope of price at 72nd one is $400, and at 74th one is $800, but more precisely, looking at total cost, we can see that the real price change from 71 to 72 is $300, while the price change from 72 to 73 is $500, and the price change from 73 to 74 is $700). Additionally, the company would only produce if the average cost per unit was lower than the sale price. This is true anywhere from 2 units to 92 units. Therefore, for a sale price of $600 they'd be willing to produce at most 73 units.

Note that they'd make the same overall profit selling 72 or 73 units.

Question 5.1

If the sale price was $100, how many units should be produced to maximize profit for the company?

a) 20
b) 42
c) 71
d) 75
e) None of the above

Answer: None of the above (70). The average cost per unit is lower than $100 from about 20 units to about 75 units, but the marginal cost to produce the 71st unit is $200 (total cost for 70 units is $4000, and for 71 units is $4000+$200. For 72 units it's $4000+$400, so $300 marginal. Therefore, 70 is the most they'd produce at this sale price).

Question 5.2

At *most* how many units would the company be willing to produce without losing money if they could sell them for $80 each?

a) 35
b) 47
c) 70
d) 74
e) None of the above

Answer: 74. It's important to note that the most units the company could produce without losing money is NOT the same as the profit-maximizing number of units. Instead, 74 is the maximum number of units where the average cost per unit is still below the total revenue. The profit-maximizing number of units (70 in this case) is determined by the # of units where the average cost is at its minimum.

Question 5.3

What's the minimum unit price the company would be willing to produce *any* units, and how many would they produce at this price?

a) $600, 2
b) $100, 50
c) $57.15, 70
d) $25, 50
e) None of the above

Answer: c – from 51 until 70 the marginal cost is $50, while the average cost is falling with more and more units produced, reaching a minimum at 70 units where the average cost is $57.143. Therefore, this unit sale price is the lowest the company would be willing to produce any number of units for, and they'd be willing to produce 70 units at this price.

Question 5.4

Given this, what can you say about the supply curve for the company?

a) The supply curve follows the average cost per unit the whole time, sloping downward at first and later sloping upward
b) The supply curve follows marginal cost the whole time: it's a series of horizontal straight lines in each range until and eventually an upward sloping line with a slope of $200.
c) The supply curve is similar to the average cost plot, but only the right side of it once it starts sloping upwards
d) The supply curve is similar to the total cost plot
e) None of the above

Answer e; based on the previous examples, the supply curve for the company is 70 units at $57.14, and then follows the marginal curve from that point, rising to 71 at $100, 72 at $300, 73 at $500, etc.

Question 5.5

If the demand curve for this product is Q_D = 90*(1-P/$200) where P is the sale price, and the company is the only supplier of this product, what should they set the sale price at to maximize profit?

a) $150
b) $133.33
c) $113.33.
d) $88.89
e) $50

Solution: Profit is $P \cdot q - C$. Since the company has a monopoly, they can unilaterally set the sale price.

This demand curve linearly slopes from a high QD of 90 at P = $0 to QD = 0 at P = $200. The most revenue available is the price that maximizes the area of the box under this line, that is, P = $100 where QD = 45 and revenue is $4500. Around this point (between 31 and 50 units), the cost is $2500 + $25*(q-30). So increasing quantity by 1 (say, from 44 to 45) costs us $25 more in cost, and requires lowering the price by $200/90 and the resulting revenue by that price * the number of units:

Revenue @44 units = 44 * ($200 - $20*44/9) = $4498.

With this in mind, price should rise and quantity reduce until the drop in revenue overpowers the drop in cost.

$$R = Pq = \left(\$200 - \frac{\$200}{90} q \right) q$$

$$\frac{dR}{dq} = \$200 - \frac{\$400}{90} q$$

This equals $25 when q = 39.375, which is still in the range of 31-50. Therefore, the company should produce around 39 units to maximize profit, since 39.375 is the point where the drop in revenue from reducing quantity (by increasing sale price) matches the drop in marginal cost of $25.

Specifically, producing 39 units would mean setting the price at $200-$20*39/9 = $113.33, for a revenue of $4419.87 at a total cost of $1750+$25*q = $2725, meaning a profit of $1694.87.

If instead they produced and sold 40, they'd do so at a price of $111.11, revenue of $4444.44, cost of $2750, and profit of $1694.44.

At 41 units, profit drops to $1689.44, and at 38 units it drops to $1691.11.

Therefore, the price that maximizes profit is $113.33 (corresponding to 39 units sold).