ME4K03 ROBOTICS

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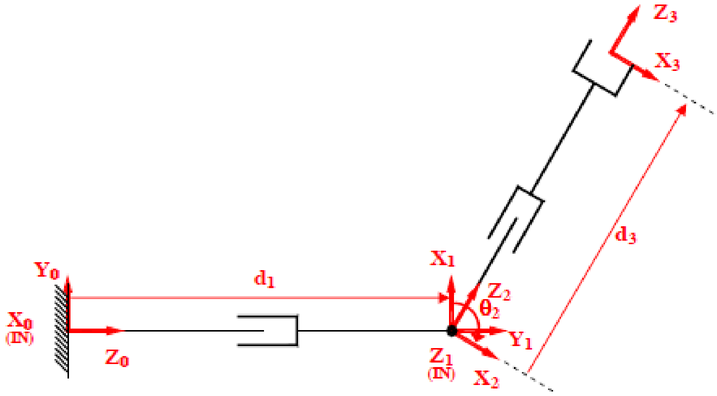
Solutions for Assignment #2

1] a) Frames 0 to 3 are shown on the following illustration:



b) D-H parameters are defined in the following table. The joint variables are shaded.

n+1	θ	d	a	α
1	90°	d_1	0	90°
2	θ_2	0	0	90°
3	0	d_3	0	0

c) All details including frames, joint variables, and non-zero *d* or *a* parameters are shown in the above illustration.

d) *A* matrices and *0T3* are calculated as follows:

$$
{}^{n}T_{n+1} = A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
A_1 = \begin{bmatrix} C(90^\circ) & -S(90^\circ)C(90^\circ) & S(90^\circ)S(90^\circ) & (0)C(90^\circ) \\ S(90^\circ) & C(90^\circ)C(90^\circ) & -C(90^\circ)S(90^\circ) & (0)S(90^\circ) \\ 0 & S(90^\circ) & C(90^\circ) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2C(90^\circ) & S\theta_2S(90^\circ) & 0 \\ S\theta_2 & C\theta_2C(90^\circ) & -C\theta_2S(90^\circ) & 0 \\ 0 & S(90^\circ) & C(90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 & 0 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

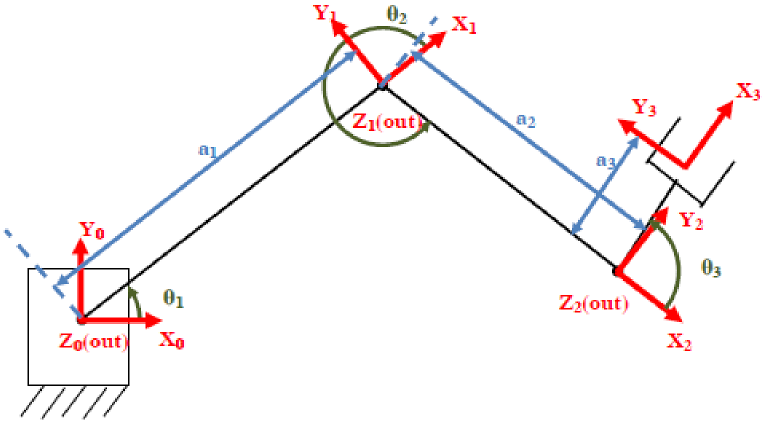
$$
A_3 = \begin{bmatrix} C(0) & -S(0)C(0) & S(0)S(0) & (0)C(0) \\ S(0) & C(0)C(0) & -C(0)S(0) & (0)S(0) \\ 0 & S(0) & C(0) & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$${}^{0}T_3 = {}^{0}T_1 * {}^{1}T_2 * {}^{2}T_3 = A_1 * A_2 * A_3$$

$${}^{0}T_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & 0 & S\theta_2 & 0 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ C\theta_2 & 0 & S\theta_2 & 0 \\ S\theta_2 & 0 & -C\theta_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ C\theta_2 & 0 & S\theta_2 & d_3 S\theta_2 \\ S\theta_2 & 0 & -C\theta_2 & d_1 - d_3 C\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2] a) Frames 0 to 3 are shown on the following illustration:



b)

n+1	θ	d	a	α
1	θ_1	0	a_1	0°
2	θ_2	0	a_2	0°
3	θ_3	0	a_3	0°

c) All details including frames, joint variables, and non-zero *d* or *a* parameters are shown in the previous illustration.

d)

$$ {}^{n}T_{n+1} = A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ A_1 = \begin{bmatrix} C(\theta_1) & -S(\theta_1)C(0) & S(\theta_1)S(0) & (a_1)C(\theta_1) \\ S(\theta_1) & C(\theta_1)C(0) & -C(\theta_1)S(0) & (a_1)S(\theta_1) \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ A_2 = \begin{bmatrix} C(\theta_2) & -S(\theta_2)C(0) & S(\theta_2)S(0) & (a_2)C(\theta_2) \\ S(\theta_2) & C(\theta_2)C(0) & -C(\theta_2)S(0) & (a_2)S(\theta_2) \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ A_3 = \begin{bmatrix} C(\theta_3) & -S(\theta_3)C(0) & S(\theta_3)S(0) & (a_3)C(\theta_3) \\ S(\theta_3) & C(\theta_3)C(0) & -C(\theta_3)S(0) & (a_3)S(\theta_3) \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_3 = {}^{0}T_1 * {}^{1}T_2 * {}^{2}T_3 = A_1 * A_2 * A_3 $$

$$ {}^{0}T_3 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_3 = \begin{bmatrix} C\theta_1C\theta_2 - S\theta_1S\theta_2 & -C\theta_1S\theta_2 - S\theta_1C\theta_2 & 0 & C\theta_1a_2C\theta_2 - S\theta_1a_2S\theta_2 + a_1C\theta_1 \\ S\theta_1C\theta_2 + C\theta_1S\theta_2 & -S\theta_1S\theta_2 + C\theta_1C\theta_2 & 0 & S\theta_1a_2C\theta_2 + C\theta_1a_2S\theta_2 + a_1S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_3 = \begin{bmatrix} C(\theta_1+\theta_2+\theta_3) & -S(\theta_1+\theta_2+\theta_3) & 0 & a_3C(\theta_1+\theta_2+\theta_3)+a_2C(\theta_1+\theta_2)+a_1C(\theta_1) \\ S(\theta_1+\theta_2+\theta_3) & C(\theta_1+\theta_2+\theta_3) & 0 & a_3S(\theta_1+\theta_2+\theta_3)+a_2S(\theta_1+\theta_2)+a_1S(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3] The D-H parameters are given in the table below.

n+1	θ	d	a	α
1	$-\pi/2$	0	-1	$\pi/2$
2	$\pm\pi$	-0.7	0.25	$\pm\pi$
3	0	0	-0.5	$\pi/2$
4	$\pi/4$	0	2.121	$\pi/2$