Prelab 4

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1. Short Answer Problems [25]:
   (a) What does a root locus plot depict? [5]
   (b) What must be done to a transfer function before it's root locus can be graphed? [5]
   (c) What is the significance of the gain K? [5]
   (d) How can a root locus plot be used to design a controller? [5]
   (e) Imagine we have a partially finished root locus plot where only the pole and zero locations have been plotted. What are the rules for completing the root locus plot using pencil and paper? (Hint! Your textbook has this information!) [5]

a) The root locust plot shows the change in behaviour of the transient response as a function of gain. Additionaly, it shows the closed loop poles of the system.

b) The polynomial numerator & denominator of the transfer function G must be factored before the root locus is graphed

c) The path of the closed-loop poles are dependent on the value of the gain

d) The root locus plot shows the system response as a function of gain. Since the poles are shown on the graph,

system response as a function of gain.
Since the poles are shown on the graph, we can determine the gain values where the system will be stable

e) i) # of branches = # of closed loop poles

ii) the root locus is symmetrical about the real axis

iii) On the real axis, for gain $K > 0$, the RL exists to the left of an odd # of open-loop poles & zeroes

iv) RL begins at poles of $G \cdot H$ & ends at the zeroes of $G \cdot H$

v) as locus approaches ∞, RL approaches asymptotes defined @ σ_a & θ_a

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeroes}}{\#\text{finite poles} - \#\text{finite zeroes}}$$

$$\theta_a = \frac{(2k+1)\pi}{\#\text{finite poles} - \#\text{finite zeroes}}, \quad k = 0, \pm 1, \pm 2, \pm 3 \ldots$$

2. Graphing Problems: For each of the following transfer functions, sketch a root locus plot using the pencil-and-paper method you outlined above. [15]
WARNING: Sketching root locus plots can be counter-intuitive! Follow the steps carefully!

   (a) $G(s) = \frac{1}{(s+5)(s+9)}$ [5]

   (b) $G(s) = \frac{(s-4)(s-7)}{(s+2)(s+5)(s+12)}$ [5]

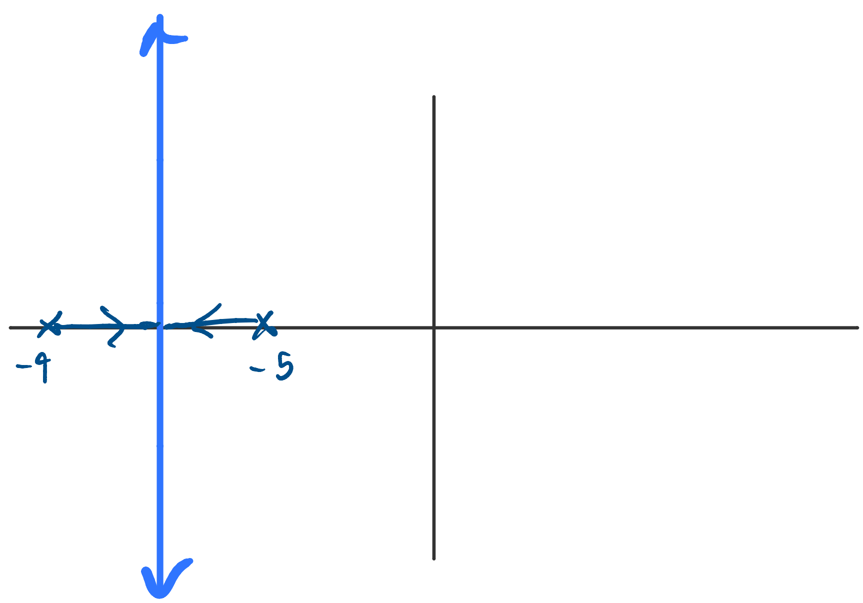
   (c) $G(s) = \frac{(s+7)}{(s+8)(s+9)(s+3)^2}$ [5]

a) poles: $s=-5$, $s=-9$

$$\sigma_a = \frac{(-5-9)-\emptyset}{2} = -7$$

$$\theta_a = (2k+1)\pi/2$$

$$\theta_a\big|_{k=0} = \frac{\pi}{2} \qquad \theta_a\big|_{k=1} = \frac{3\pi}{2} \qquad \theta_a\big|_{k=-1} = -\frac{\pi}{2}$$



b) $G(s) = \dfrac{(s-4)(s-7)}{(s+2)(s+5)(s+12)}$

poles: $s=-2, s=-5, s=-12$
zeroes: $s=4, s=7$

$$\sigma_a = \frac{(-2-5-12)-(4+7)}{3-2}$$

$$\sigma_a = -30$$

$$\theta_a = \frac{(2k+1)\pi}{3-2}$$

$\theta_a\big|_{k=0} = \pi$ $\quad\theta_a\big|_{k=1} = 3\pi$ $\quad\theta_a\big|_{k=-1} = -\pi$

breakaway/in

$$\frac{1}{\sigma-4} + \frac{1}{\sigma-7} = \frac{1}{\sigma+2} + \frac{1}{\sigma+5} + \frac{1}{\sigma+12}$$

$$\sigma = \cancel{-8.55}, -3.13, 5.18, \cancel{28.50}$$

$$G_{cl} = \frac{\left[\dfrac{K(s-4)(s-7)}{(s+2)(s+5)(s+12)}\right]}{\left[1 + \dfrac{K(s-4)(s-7)}{(s+2)(s+5)(s+12)}\right]}$$

$$\left[\, 1 + \frac{\quad}{(s+2)(s+5)(s+12)} \right]$$

$$= \frac{ks^2 + 11ks + 28k}{s^3 + (19+k)s^2 + (94-11k)s + (120+28k)}$$

s^3	1	$94-11k$	$\varnothing$
s^2	$19+k$	$120+28k$	$\varnothing$
s	$\frac{-11k^2+143k-1666}{k+19}$	$\varnothing$	$\varnothing$
$s^\varnothing$	$28k+120$	$\varnothing$	$\varnothing$

set "s" row $=\varnothing$, solve for k

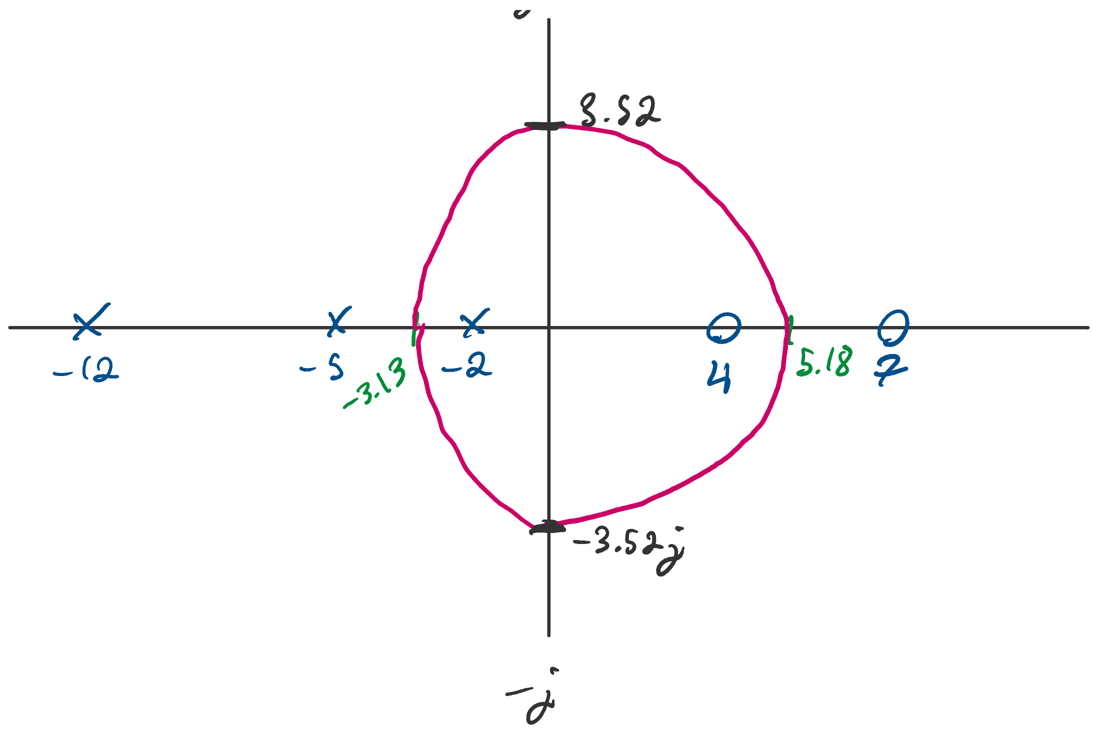
$$\varnothing = -11k^2 + 143k - 1666$$
$$k = 7.4178$$

set "s^1" row $= \varnothing$, solve for s

$$\varnothing = s^2(k+19) + 28k + 120 \quad , \text{ let } k = 7.4178$$

$$\underline{s = \pm 3.522 j}$$

j
1



c) $G(s) = \dfrac{s+7}{(s+8)(s+9)(s+3)^2}$

poles: $s = -7$
zeroes: $s = -3$ repeated, -8, -9

$$\sigma_a = \frac{(-3-3-8-9)-(-7)}{4-1}$$

$$= -5.\dot{3}$$

$$\theta_a = \frac{(2k-1)\pi}{4-1}$$

$$\theta_\alpha\big|_{K=0} = \frac{\pi}{3} \qquad \theta_\alpha\big|_{K=1} = \pi \qquad \theta_\alpha\big|_{K=-1} = -\pi/3$$

$$G_{cl} = \frac{\left[\dfrac{K(s+7)}{(s+8)(s+9)(s+3)^2}\right]}{\left[1 + \dfrac{K(s+7)}{(s+8)(s+9)(s+3)^2}\right]}$$

$$= \frac{K(s+7)}{s^4 + 23s^3 + 183s^2 + (K+585)s + 7K + 648}$$

s^4	1	183	$7K+648$
s^3	23	$K+585$	$\emptyset$
s^2	$\frac{3624}{23} - \frac{K}{23}$	$\emptyset$	$\emptyset$
s^1	$\frac{K^2+664K+1777248}{K-3624}$	$\emptyset$	$\emptyset$
s^0	$7K+648$	$\emptyset$	$\emptyset$

set s^1 row $= 0$

$$0 = \frac{K^2 + 664K + 1777248}{K - 3624} \rightarrow K = 1041.9$$

set s^2 row $= 0$

$$s^2\left(\frac{3624}{23} - \frac{1041.9}{23}\right) + 7(1041.9) + 688 = 0$$

$$s = \pm 8.4103j$$