2A04 Tutorial 7

March 7th, 2022

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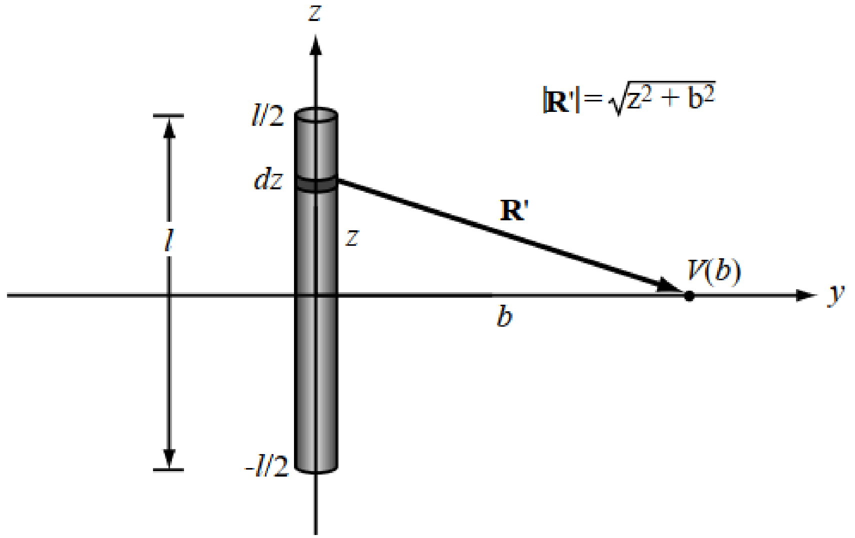
1. Problem 4.34

Find the electric potential V at a location a distance b from the origin in the $x-y$ plane due to a line charge with charge density ρ_l and of length l. The line charge is coincident with the z axis and extends from $z=-l/2$ to $z=l/2$.



For a line charge, we have the formula:

$$V=\frac{1}{4\pi\varepsilon}\int_{l'}\frac{\rho_l}{R'}dl'$$

In this specific case, we can integrate along z with the following limits:

$$V=\frac{\rho_l}{4\pi\varepsilon}\int_{-l/2}^{l/2}\frac{dz}{R'}$$

$$V=\frac{\rho_l}{4\pi\varepsilon}\int_{-l/2}^{l/2}\frac{dz}{\sqrt{z^2+b^2}}$$

$$\boldsymbol{V=\frac{\rho_l}{4\pi\varepsilon}\ln\left(\frac{\sqrt{l^2+4b^2}+l}{\sqrt{l^2+4b^2}-l}\right)}$$

$$V = \frac{\rho_\ell}{4\pi\varepsilon} \int_{-\ell/2}^{\ell/2} \frac{dz}{\sqrt{z^2+b^2}} \qquad \int \frac{1}{\sqrt{u^2+a^2}}\,du = \left|\ln\left(u+\sqrt{u^2+a^2}\right)\right| + C$$

Let $C_1 = \frac{\rho_\ell}{4\pi\varepsilon}$

$$V = C_1 \ln\left(z+\sqrt{z^2+b^2}\right)\Big|_{-\ell/2}^{\ell/2} = C_1\left(\ln\left(\ell/2+\sqrt{\ell^2/4+b^2}\right) - \ln\left(-\ell/2+\sqrt{\ell^2/4+b^2}\right)\right)$$

$$V = C_1 \ln\left(\frac{\ell/2+\sqrt{\ell^2/4+b^2}}{-\ell/2+\sqrt{\ell^2/4+b^2}}\right)$$

$$V = C_1 \ln\left(\frac{\ell+\sqrt{4(\ell^2/4+b^2)}}{-\ell+\sqrt{4(\ell^2/4+b^2)}}\right)$$

$$V = \frac{\rho_\ell}{4\pi\varepsilon} \ln\left(\frac{\sqrt{\ell^2+4b^2}+\ell}{\sqrt{\ell^2+4b^2}-\ell}\right)$$

2. Problem 4.41

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13\left(\frac{m^2}{Vs}\right), \mu_h = 0.05\left(\frac{m^2}{Vs}\right), N_e = 1.5 \times 10^{16} electrons/m^3$, and $N_h = N_e$, find the following:

a. The conductivity of silicon,

b. The current I flowing in the bar,

c. The drift velocities $\overrightarrow{u_e}$ and $\overrightarrow{u_h}$,

d. The resistance of the bar, and

e. The power dissipated in the bar.

Givens

$r = 4mm$

$l = 8cm$

$V = 5V$

$\mu_e = 0.13\left(\frac{m^2}{Vs}\right)$

$\mu_h = 0.05\left(\frac{m^2}{Vs}\right)$

$N_e = N_h = 1.5 \times 10^{16} electrons/m^3$

What does each parameter mean???

2. Problem 4.41a

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13\ \left(\frac{m^2}{Vs}\right), \mu_h = 0.05\ \left(\frac{m^2}{Vs}\right), N_e = 1.5 \times 10^{16} electrons/m^3$, and $N_h = N_e$, find the following:

The conductivity of silicon

Givens

$r = 4mm$

$l = 8cm$

$V = 5V$

$\mu_e = 0.13\ \left(\frac{m^2}{Vs}\right)$

$\mu_h = 0.05\ \left(\frac{m^2}{Vs}\right)$

$N_e = N_h = 1.5 \times 10^{16} electrons/m^3$

How can we calculate conductivity?

$$\sigma = (N_e\mu_e + N_h\mu_h)e\ \ (\frac{S}{m})$$

$$\sigma = \left(1.5 \times 10^{16}(0.13 + 0.05)\right)(1.602 \times 10^{-19})$$

$$\boldsymbol{\sigma = 4.33 \times 10^{-4}\ (\frac{S}{m})}$$

2. Problem 4.41a

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13 \left(\frac{m^2}{Vs}\right), \mu_h = 0.05 \left(\frac{m^2}{Vs}\right), N_e = 1.5 \times 10^{16} electrons/m^3$, and $N_h = N_e$, find the following:

The conductivity of silicon

$$\boldsymbol{\sigma = 4.33 \times 10^{-4} \left(\frac{S}{m}\right)}$$

Givens

$r = 4mm$

$l = 8cm$

$V = 5V$

$\mu_e = 0.13 \left(\frac{m^2}{Vs}\right)$

$\mu_h = 0.05 \left(\frac{m^2}{Vs}\right)$

$N_e = N_h = 1.5 \times 10^{16} electrons/m^3$

Table B-2 Conductivity σ of Some Common Materials[a]

Material	Conductivity σ (S/m)	Material	Conductivity σ (S/m)
Conductors		**Semiconductors**	
Silver	6.2×10^7	Pure germanium	2.2
Copper	5.8×10^7	Pure silicon	4.4×10^{-4}
Gold	4.1×10^7	**Insulators**	
Aluminum	3.5×10^7	Wet soil	$\sim 10^{-2}$
Tungsten	1.8×10^7	Fresh water	$\sim 10^{-3}$
Zinc	1.7×10^7	Distilled water	$\sim 10^{-4}$
Brass	1.5×10^7	Dry soil	$\sim 10^{-4}$
Iron	10^7	Glass	10^{-12}
Bronze	10^7	Hard rubber	10^{-15}
Tin	9×10^6	Paraffin	10^{-15}
Lead	5×10^6	Mica	10^{-15}
Mercury	10^6	Fused quartz	10^{-17}
Carbon	3×10^4	Wax	10^{-17}
Seawater	4		
Animal body (average)	0.3 (poor cond.)		

[a]These are low-frequency values at room temperature (20 °C).

2. Problem 4.41b

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13\ \left(\frac{m^2}{Vs}\right)$, $\mu_h = 0.05\ \left(\frac{m^2}{Vs}\right)$, $N_e = 1.5 \times 10^{16} electrons/m^3$, and $N_h = N_e$, find the following:

The current I flowing in the bar

Givens

$r = 4mm$

$l = 8cm$

$V = 5V$

$\mu_e = 0.13\ \left(\frac{m^2}{Vs}\right)$

$\mu_h = 0.05\ \left(\frac{m^2}{Vs}\right)$

$N_e = N_h = 1.5 \times 10^{16} electrons/m^3$

How can we compute the current?

$$I = JA$$

$$I = \sigma EA$$

$$I = \sigma \frac{V}{l} \pi r^2$$

$$I = (4.33 \times 10^{-4}) \frac{5}{0.08} \pi 0.004^2$$

$$\boldsymbol{I = 1.36 \mu A}$$

2. Problem 4.41c

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13 \left(\frac{m^2}{Vs}\right), \mu_h = 0.05 \left(\frac{m^2}{Vs}\right), N_e = 1.5 \times 10^{16} electrons/m^3$, and $N_h = N_e$, find the following:

The drift velocities $\overrightarrow{u_e}$ and $\overrightarrow{u_h}$

Givens

$r = 4mm$

$l = 8cm$

$V = 5V$

$\mu_e = 0.13 \left(\frac{m^2}{Vs}\right)$

$\mu_h = 0.05 \left(\frac{m^2}{Vs}\right)$

$N_e = N_h = 1.5 \times 10^{16} electrons/m^3$

From the definition of the mobility:

$$\overrightarrow{u_e} = -\mu_e \vec{E}, \quad \overrightarrow{u_h} = \mu_h \vec{E}$$

$$\overrightarrow{u_e} = -\mu_e E \frac{\vec{E}}{E}, \quad \overrightarrow{u_h} = \mu_h E \frac{\vec{E}}{E}$$

What is the fraction at the end there? Why do we need it?

$$\overrightarrow{u_e} = -\mu_e \frac{V}{l} \widehat{\boldsymbol{E}}, \quad \overrightarrow{u_h} = \mu_h \frac{V}{l} \widehat{\boldsymbol{E}}$$

$$\overrightarrow{u_e} = -0.13 \frac{5}{0.08} \widehat{\boldsymbol{E}}, \quad \overrightarrow{u_h} = 0.05 \frac{5}{0.08} \widehat{\boldsymbol{E}}$$

$$\boldsymbol{\overrightarrow{u_e} = -8.125 \widehat{E}}, \quad \boldsymbol{\overrightarrow{u_h} = 3.125 \widehat{E}}$$

2. Problem 4.41d,e

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13 \left(\frac{m^2}{Vs}\right), \mu_h = 0.05 \left(\frac{m^2}{Vs}\right), N_e = 1.5 \times 10^{16} electrons/m^3$, and $N_h = N_e$, find the following:

Resistance of, and power dissipated across, the bar.

Givens

$r = 4mm$

$l = 8cm$

$V = 5V$

$\mu_e = 0.13 \left(\frac{m^2}{Vs}\right)$

$\mu_h = 0.05 \left(\frac{m^2}{Vs}\right)$

$N_e = N_h = 1.5 \times 10^{16} electrons/m^3$

We already found $I = 1.36\mu A$. How can we find the resistance?

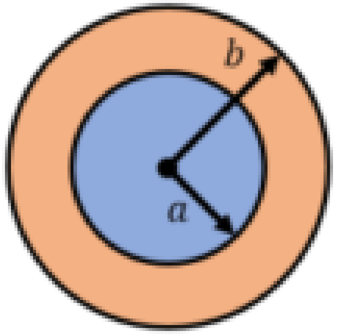
Ohm's Law: $\boldsymbol{R = \frac{V}{I} = \frac{5}{1.36\times10^{-6}} = 3.68\ M\Omega}$

With the same parameters, we can find the power dissipated:

$$\boldsymbol{P = VI = 5(1.36 \times 10^{-6}) = 6.8\ \mu W}$$

3. Problem 4.44

A coaxial resistor of length l consists of two concentric cylinders. The inner cylinder has radius a and is made of a material with conductivity σ_1, and the outer cylinder, extending between $r = a$ and $r = b$, is made of a material with conductivity σ_2. If the two ends of the resistor are capped with conducting plates, show that the resistance between the two ends is



$$R = \frac{l}{\pi(\sigma_1 a^2 + \sigma_2(b^2 - a^2))}.$$

So how can we find R_1 and R_2?

Each R will follow $R = \frac{l}{\sigma A}$ where A is the surface area.

$$\text{R}= \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{\sigma_1 A_1}{l} + \frac{\sigma_2 A_2}{l}\right)^{-1}$$ … what are the areas?

These are basically parallel resistors!

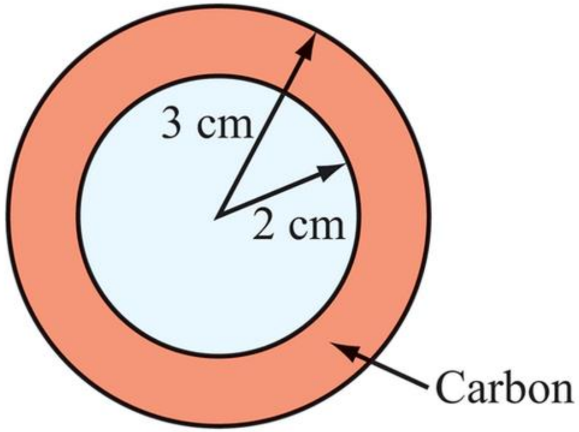
$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$R = \left(\frac{\sigma_1 \pi a^2}{l} + \frac{\sigma_2 \pi (b^2 - a^2)}{l}\right)^{-1} = \left(\frac{\sigma_1 \pi a^2 + \sigma_2 \pi (b^2 - a^2)}{l}\right)^{-1}$$

$$\boldsymbol{R = \frac{l}{\sigma_1 \pi a^2 + \sigma_2 \pi (b^2 - a^2)} = \frac{l}{\pi(\sigma_1 a^2 + \sigma_2 (b^2 - a^2))}}$$

4. Problem 4.45

Apply the result of Problem 4.44 (*that's the one we just did!*) to find the resistance of a 20 cm long hollow cylinder made of carbon with $\sigma = 3 \times 10^4\ S/m$.



$$R = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2 (b^2 - a^2)\right)}.$$

Givens

$a = 2cm$

$b = 3cm$

$\sigma = 3 \times 10^4\ S/m$

$l = 20cm$

Which σ is the given value? And what's the other value?

Cylinder is <u>hollow</u>… so take $\sigma_1 = 0$ and $\sigma_2 = 3 \times 10^4\ S/m$

$$\boldsymbol{R = \frac{0.2}{\pi\left((0)(0.02)^2 + (3 \times 10^4)(0.03^2 - 0.02^2)\right)}}$$

$$\boldsymbol{= \frac{0.2}{\pi\left((3 \times 10^4)(5 \times 10^{-4})\right)} = 4.2\ m\Omega}$$

Reminders

- Assignment 7 is out, and is due at 8AM on March 14.
- Complete the mid-semester survey for a 5% bonus on Assignment 7! Survey must be completed by Sunday March 13th.
- Have a great week!