**Test 2, 2022** **Sample Correct Student Answers**

**Note: Question 1 is similar to parts of practice problem 3, and examples from the chapter.**

8 **1.** A machine should be designed to move a 300 kg mass at a maximum speed of 0.8 m/s using a motor plus gearbox plus ball screw. One revolution of the ball screw moves the load 0.01 m. The motor's moment of inertia is $1.5 \times 10^{-4}$ kgm$^2$. The ball screw's moment of inertia is $3 \times 10^{-5}$ kgm$^2$. You may assume that the friction of the gears and ball screw, as well as the inertias of the gears, are negligible. **The available gear ratios are 0.5, 1, 1.5, etc.**

a) Assuming the gear ratio is <u>not</u> limited by the motor's maximum speed, determine the most optimal gear ratio from the set of available ratios.

b) Show that your answer to (a) satisfies the rule of thumb for the inertia ratio.

c) If the gear ratio is limited by the motor's maximum speed of 800 rad/s, determine the most optimal ratio from the set of available ratios.

d) Show that your answer to (c) satisfies the rule of thumb for the inertia ratio.

a) $J_{load} = m(\ell \text{ rev}/2\pi)^2 + J_{screw}$

$= 300\text{kg}(0.01 \frac{m}{rev} \text{ rev}/2\pi)^2 + 3\times10^{-5} \text{ kgm}^2$

$J_{load} = 7.8991 \times 10^{-4} \text{ kg}\cdot\text{m}^2$

$J_{motor} = 1.5\times10^{-4} \text{ kg m}^2$

$N_{r,opt} = \sqrt{\dfrac{J_{load}}{J_{motor}}} = \sqrt{\dfrac{7.8991\times10^{-4} \text{ kgm}^2}{1.5\times10^{-4} \text{ kgm}^2}} = 2.29$

$\boxed{N_r = 2}$

b) $\text{Ratio}_J = \dfrac{J_{load}/N_r^2}{J_{motor}} = \dfrac{7.8991\times10^{-4} \text{ kgm}^2 / (2)^2}{1.5\times10^{-4} \text{ kg m}^2} = 1.3165$

$\boxed{\text{Ratio}_J = 1.3165}$ between 1 and 10 so OK

c) $V_{max} = 0.8$ m/s

$\omega_{motor,max} = \dfrac{V_{max}}{\ell(\text{rev}/2\pi)} = \dfrac{0.8 \text{ m/s}}{0.01 \text{ m/rev } (\text{rev}/2\pi)}$

$\omega_{motor,max} = 502.6548$ rad/s

$\omega_{rated,max} = 800$ rad/s

$N_r = \dfrac{\omega_{rated,max}}{\omega_{motor,max}} = \dfrac{800 \text{ rad/s}}{502.6548 \text{ rad/s}} = 1.5915$

$\boxed{N_r = 1.5}$

d) $\text{Ratio}_J = \dfrac{J_{load}/N_r^2}{J_{motor}} = \dfrac{7.8991\times10^{-4} \text{ kgm}^2/(1.5)^2}{1.5\times10^{-4} \text{ kgm}^2} = 2.3405$

$\boxed{\text{Ratio}_J = 2.3405}$ between 1 and 10 so OK

**Note: Question 2 is similar to parts of practice problem 3, and examples from the chapter.**

8 **2.** A linear actuator consisting of a motor, gearbox, and rack and pinion moves a load vertically. <u>A positive motor velocity moves the load upwards</u>. The pinion has a 0.015 m pitch radius. The motor's moment of inertia is $8 \times 10^{-5}$ kgm$^2$ **<u>and the gear ratio is 2.5</u>**. If the load has a mass of 250 kg and is subject to a 800 N friction force, **determine the motor torque required for the load to accelerate at 1.4 m/s$^2$ while it is moving downwards.** The friction and inertias of the gears can be neglected.

$r_p = 0.015$ m

$J_{motor} = 8\times10^{-5}$ kg m$^2$

$N_r = 2.5$

$m = 250$ kg

$F_f = 800$ N

$a = -1.4$ m/s$^2$

$J_{load} = m r_p^2$

$= (250 \text{ kg})(0.015 \text{ m})^2$

$J_{load} = 0.05625$ kgm$^2$

$F = mg - F_f$

$= (250 \text{ kg})(9.81 \text{ m/s}^2) - 800\text{N}$

$F = 1652.5$ N

$\tau_{ext} = F r_p$

$= (1652.5 \text{ N})(0.015 \text{ m})$

$\tau_{ext} = 24.7875$ N·m

$\dot{\omega}_{motor} = N_r \left(\frac{a}{r_p}\right)$

$= 2.5 \left(\frac{-1.4 \text{ m/s}^2}{0.015 \text{ m}}\right)$

$\dot{\omega}_{motor} = -233.3333$ rad/s

$\tau_{motor} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{ext}$

$= (8\times10^{-5} \text{ kgm}^2)(-233.3333 \text{ rad/s}) + \frac{1}{(2.5)^2}(0.05625 \text{ kgm}^2)(-233.3333 \text{ rad/s}) + \frac{1}{2.5}(24.7875 \text{ N·m})$

$\boxed{\tau_{motor} = 7.7963 \text{ N·m}}$

**Note: Question 3 is similar to parts of practice problem 3, and examples from the chapter.**

9 **3.** A motor with a moment of inertia of $2.1 \times 10^{-5}$ kgm$^2$ is used to rotate a disk whose moment of inertia is $9.3 \times 10^{-5}$ kgm$^2$. No gearbox (or other mechanism) is used between the motor and disk. You may assume that the motor's viscous friction is negligible. The desired operating cycle consists of the following three periods of constant acceleration:

1. Starting from zero velocity, accelerate up to 100 rad/s in 0.2 s, then
2. Decelerate back to zero velocity in 0.4 s, then
3. Remain at zero velocity for 0.3 s before the cycle restarts.

Determine:

a) The motor accelerations required in the 1st and 2nd periods of this operating cycle.

b) The motor torques required in the 1st and 2nd periods of this operating cycle.

c) If the ambient temperature is 25 ˚C, motor's torque constant is 0.01 Nm/A, its armature resistance at its maximum temperature is 3.0 ohm, and its total thermal resistance is 2.0 ˚C/W, what temperature will the motor reach after this cycle has been repeated for several hours?

$J_{motor} = 2.1 \cdot 10^{-5}$ kgm$^2$ $J_{disk} = 9.3 \cdot 10^{-5}$ kg m$^2$

a) $\dot{\omega}_1 = \dfrac{100 \text{ rad/s}}{0.2 s} = 500 \text{ rad/s}^2$

$\omega_2 = \dfrac{(0-100)}{0.4} = -250 \text{ rad/s}^2$

b) $T_{external} = 0$

1: $T_{motor} = (J_{motor} + J_{disk})(\dot{\omega}_1) = (2.1 \cdot 10^{-5} + 9.3 \cdot 10^{-5})(500) = 0.057 \text{ N} \cdot \text{m}$

2: $T_{motor} = (2.1 \cdot 10^{-5} + 9.3 \cdot 10^{-5})(-250) = -0.0285 \text{ N} \cdot \text{m}$

c) $T_a = 25°C$ $K_t = 0.01 \frac{Nm}{A}$ $R_{Hot} = 3\Omega$ $R_{th} = 2.0 \text{ °C/W}$

$T_{RMS} = \sqrt{\dfrac{\sum T_i^2 \cdot t_i}{\sum t_i}} = \sqrt{\dfrac{0.057^2 \cdot 0.2 + (-0.0285)^2 \cdot 0.4}{(0.2 + 0.4 + 0.3)}} = 0.033 \text{ N} \cdot \text{m}$

$I_{RMS} = \dfrac{T_{RMS}}{K_t} = 3.29 \text{ A}$

$P_j = I_{RMS}^2 \cdot R_{Hot} = (3.29)^2(3) = 32.4723 \text{ W}$

$T_w = T_a + P_j R_{th} = 25 + (32.4723)(2) = 89.94°C$

**Note: Question 4 is similar to practice problems 5 and 1(b).**

9 **4.** A double rod pneumatic cylinder is used to move a 5 kg mass horizontally. The mass is subject to a 120 N friction force during its motion. The bore's area is $6 \times 10^{-4}$ m$^2$, the rod's area is $7 \times 10^{-5}$ m$^2$, the supply pressure is $4\times10^5$ Pa gauge and the air temperature is 30°C. Assume that the pressure drop across the valve is the same for the return flow as for the intake flow and that the air is returned to the atmosphere. **If the desired maximum speed when ~~the~~ one rod is extending equals 1.7 m/s, and the stroke is not the limiting factor, determine:**

a) The corresponding value of the pressure drop.

b) The required flow coefficient for the valve.

$F_f = 120N \quad A_{bore} = 6\times10^{-4} m^2, \quad A_{rod} = 7\times10^{-5} m^2, \quad P_{supp} = 4\times10^5 Pa$ Gauge,

$T_{emp} = 30°C + 273 = 303 K \quad P_{sump} = 0$ Gauge.

$v_{ext} = 1.7 m/s$.

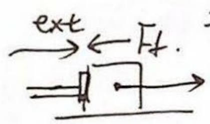
a).

$\because$ Double rod

$A_{ext} = A_{ret} = A_{bore} - A_{rod}$

$= 6\times10^{-4} - 7\times10^{-5}$

$= 5.3\times10^{-4} m^2$




$F_{ext} = F_f = 120N = P_{ext}A_{ext} - P_{ret}A_{ret}$

$P_{ext} = 4\times10^5 Pa - \Delta P$

$P_{ret} = \Delta P$

$\Rightarrow 120 = (4\times10^5 - \Delta P)(5.3\times10^{-4}) - \Delta P(5.3\times10^{-4})$

$-1.06\times10^{-3}\Delta P = -92$

$\Delta P = 8.68\times10^4 Pa$

$\therefore$ The corresponding pressure drop is $8.68\times10^4 Pa$.

b) $Q = vA$

$= 1.7 m/s \cdot 5.3\times10^{-4} m^2$

$= 9.01\times10^{-4} m^3/s$

$\Rightarrow \rho = \dfrac{P_1 - \Delta P}{R_g T}$

$= \dfrac{(4\times10^5 + 101\times10^3) - 8.68\times10^4}{287 \cdot 303}$

$= 4.76 kg/m^3$

$\therefore C_v = 4.22\times10^4 Q \sqrt{\dfrac{\rho}{\Delta P}}$

$= 4.22\times10^4 \cdot (9.01\times10^{-4}) \sqrt{\dfrac{4.76}{8.68\times10^4}}$

$= 0.282$

$\therefore$ The required flow coefficient is 0.282

**Please note:** 1) We marked this question out of 7. For example, if you could got one answer wrong, we gave you 6 out of 6.

2) The answers to this question can be found in Chapter 3.

3) None of these questions will appear again on the final exam.

6 **5. Based on the material covered in this course**, answer the following questions in the spaces provided:

a) A __________________ in a hydraulic power supply is analogous to a capacitor in an electric power supply.

b) Other than a ball screw, lead screw, or rack and pinion, list two different mechanisms for converting rotary motion into linear motion:

Answer #1: ________________________________________________________

Answer #2: ________________________________________________________

c) List one advantage and one disadvantage of a harmonic drive compared to a planetary gearbox:

. Advantage: __________________________________________________

Disadvantage: ________________________________________________

d) Other than "no wear", list one advantage of a shape memory alloy actuator:

Answer: _________________________________________________________

e) Why can brushless motors produce higher continuous torques than brush motors?

Answer: __________________________________________________________________________

__________________________________________________