ENGPHYS 2A04 Winter 2022 – Assignment 8

Due Monday MONDAY March 21, 8AM

1. **Boundary Conditions.** If $E = 200\,\hat{\boldsymbol{R}}$ (V/m) at the surface of a 10-cm conducting sphere centered at the origin, what is the total charge Q on the sphere's surface?

Solution:

From Table 4-3, $\hat{\boldsymbol{n}} \cdot (\boldsymbol{D}_1 - \boldsymbol{D}_2) = \boldsymbol{\rho}_s$. $\boldsymbol{E}_2$ inside the sphere is zero, since we assume it is a perfect conductor. Hence, for a sphere with surface area $S = 4\pi a^2$,

$$\begin{aligned} D_{1R} &= \rho_s \\ E_{1R} &= \frac{\rho_s}{\varepsilon_0} \\ &= \frac{Q}{S\varepsilon_0} \\ Q &= E_R S\,\varepsilon_0 \\ &= (200)4\pi(0.1)^2\varepsilon_0 \\ &= 8\pi\varepsilon_0 \end{aligned}$$

2. **Boundary Conditions.**
   a. Find $\boldsymbol{E}_1$ given,

$$E_2 = 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} + 3\,\hat{\boldsymbol{z}}$$

   $\varepsilon_1 = 3\varepsilon_0,\ \varepsilon_2 = 16\varepsilon_0$, and the boundary has a surface charge density $\rho_s = 6.25 \times 10^{-11}$ (C/m2).

   b. What angle does E2 make with the z axis?

Solution:

Recall that $\boldsymbol{E}_{1t} = \boldsymbol{E}_{2t}$ for any 2 media. Hence,

$$\begin{aligned} \boldsymbol{E}_{1t} &= \boldsymbol{E}_{2t} \\ &= 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} \end{aligned}$$

Recall $(\boldsymbol{D_1} - \boldsymbol{D_2}) \cdot \hat{\boldsymbol{n}} = \rho_s$ (from Table 4.3). Hence, $\varepsilon_1(\boldsymbol{E}_1 \cdot \hat{\boldsymbol{n}}) - \varepsilon_2(\boldsymbol{E}_1 \cdot \hat{\boldsymbol{n}}) = \rho_s$

$$\begin{aligned} \boldsymbol{E}_{1z} &= \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} \\ &= \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} \\ &= \frac{6.25 \times 10^{-11}}{3\varepsilon_0} + \frac{16\varepsilon_0(3)}{3\varepsilon_0} \\ &= \frac{6.25 \times 10^{-11}}{3\varepsilon_0} + 16 \\ &= 18.35\ (V/m) \end{aligned}$$

$$E_1 = 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} + 18.35\,\hat{\boldsymbol{z}}$$

Finding the angle $\boldsymbol{E}_2$ makes with the z-axis can be found by:

$$\boldsymbol{E}_2 \cdot \hat{\boldsymbol{z}} = |\boldsymbol{E}_2| \cos\theta$$

$$3 = \sqrt{5^2 + 7^2 + 3^2}\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{83}}\right)$$

$$= 70.8^{\circ}$$

$$= 1.24 \text{ rad}$$

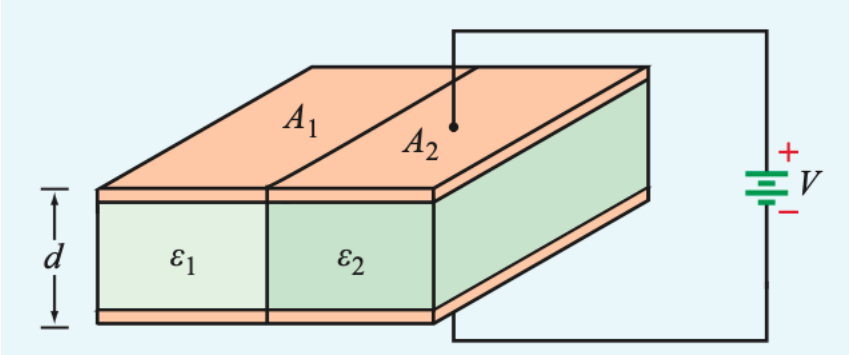
3. **Capacitance.** Given the two parallel, conducting plates separated by a distance d illustrated in the diagram below. The space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2.

   Given:

   $$C = C_1 + C_2$$

   where

   $$C_1 = \frac{\varepsilon_1 A_1}{d}, \qquad C_2 = \frac{\varepsilon_2 A_2}{d}$$



Find the following:

a. Find the electric fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the two dielectric layers.
b. Calculate the energy stored in each section.
c. Draw a circuit diagram of the above the dielectric section

Solutions

a. Find the electric fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the two dielectric layers.

$$E_1 = E_2 = \frac{V}{d}$$

b. Find an expression the energy stored in each section (1) and the total energy

$$W_{e_1} = \frac{1}{2}CV^2$$

$$C_1 = \frac{\varepsilon_1 A_1}{d}$$

$$W_{e_1} = \frac{1}{2}\frac{V^2 \varepsilon_1 A_1}{d}$$

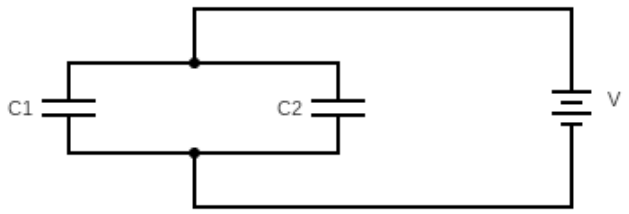
$$W_{e_2} = \frac{1}{2} C V^2$$

$$C_2 = \frac{\varepsilon_2 A_2}{d}$$

$$W_e = W_{e_1} + W_{e_2}$$

$$= \frac{1}{2}\frac{V^2}{d}(\varepsilon_1 A_1 + \varepsilon_2 A_2)$$

c. Draw a circuit diagram of the above the dielectric section



4. **Capacitance.** An electron with charge $Q_e = -1.6 \times 10^{-19} C$ and mass $m_e = 9.1 \times 10^{-31}$ kg is injected at a point adjacent to the negatively charged plate in the region between the plates of an air-filled, parallel-plate capacitor with separation of 50 cm and rectangular plates each 50 cm^2 in area (Fig. P4.54). If the voltage across the capacitor is 80 V, find the following:
   a. The force acting on the electron,
   b. The acceleration of the electron,

Solutions

a. The force acting on the electron,

$$F = Q_e E$$
$$= Q_e \frac{V}{d}$$
$$= -1.6 \times 10^{-19} \frac{80}{0.5}$$
$$= -2.56 \times 10^{-17}\ N$$

The force is directed from the negatively charged plate towards the positively charged plate.

b. The acceleration of the electron,

$$a = \frac{F}{m}$$
$$= \frac{-2.56 \times 10^{-17}}{9.1 \times 10^{-31}}$$
$$= 2.81 \times 10^{13}\ \mathrm{m/s^2}$$

c. The time it takes the electron to reach the positively charged plate, assuming that it starts from rest.

$$t = \sqrt{\frac{2d}{a}}$$
$$= \sqrt{\frac{2(0.5)}{2.81 \times 10^{13}}}$$
$$= 1.89 \times 10^{-7}\ \mathrm{s}$$

Bonus. In no more than 100 words, explain how a super capacitor functions. What are the advantages and disadvantages of a super capacitor compared to a traditional battery?

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal marks (except bonus questions).
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.