Time value of money

$$F = P + I$$

Simple interest

$$F = P(1 + r * t), r\ is\ annual\ interest\ rate$$

Compound interest

$$F = P(1 + i)^n$$

Regular deposit

$$F = A\sum_{k=0}^{n-1}(i + i)^k = A * \frac{(1 + i)^n - 1}{(1 + i) - 1}$$

Loan payment

$$F = P(1 + i)^n = A * \frac{(1 + i_e)^n - 1}{i_e}$$

$$A = \frac{P(1 + i)^n * i}{(1 + i)^n - 1} \rightarrow monthly\ payment$$

$$A = \frac{P\left(1 + \frac{r}{m}\right)^n * \frac{r}{m}}{\left(1 + \frac{r}{m}\right)^n - 1} = \frac{P(1 + i_e)^n * \frac{r}{m}}{(1 + i_e)^n - 1}$$

$$P = A * \frac{(1 + i)^n - 1}{i * (1 + i)^n}$$

Nominal interest rates (yearly rate it would match if you paid off the interest after each compounding period)

$$i\ (interest\ rate\ of\ compounding) = \frac{r \rightarrow nominal\ interest\ rate}{m \rightarrow periods\ per\ year}$$

Effective interest rate, i_e

$$1 + i_e = \left(1 + \frac{r}{m}\right)^m$$

Convert future value to present value

$$F = P(1 + i_e)^n$$

Discount rate

$$PV = \frac{CF_t}{(1 + r_d)^t}$$

Net present value

$$NPV = CF_0 + \sum_{t=1}^{N}\frac{CF_t}{(1 + r)^t}$$

With inflation rate r_i

$$NPV = CF_0 + \sum_{t=1}^{N}\frac{CF_t * (1 + r_i)^{t-1}}{(1 + r)^t}$$

Internal rate of return (IRR)

$$CF_0 + \sum_{i=1}^{N}\frac{CF_t}{(1 + r_{IRR})^t} = 0$$

Inflation → convert current dollars (C) to real dollars (R)

$$R = \frac{C}{(1 + r_i)^n}, r_i\ is\ inflation\ rate$$

Project management

Quality comes from an appropriate combination of time and cost appropriate for scope

Project:

More temporary and specific than a business or area of responsibility

Projects end when they meet their objectives

Usually have more narrowly defined objectives

Larger and more dynamic than a task

Natural planning model

Purpose & Principles | envision the outcome | generate ideas| determine next

Life cycle

Initiation | Planning | Execution | Monitoring and control | Closure

Optimization

An optimization model can have only one true optimal value, it may have

Unique optimal solution | Several alternative solutions yielding the same optimal | No optimal solutions

A constraint g(x) ≤ 0 is said to be:

Active (binding) at some point x* if g(x*) = 0

Inactive at some point x* if g(x*) < 0

Boundary point: At least one inequality constraint is satisfied as an equality at a given point

Interior point: No inequalities are active

Extreme point: If every line segment in the feasible region containing it also has that point as an endpoint

Slack variable: how far is the solution from the constraint (s^-) ≤

Use s^+ for ≥

Supply and demand

y-axis (price), x-axis (quantity)

Demand curve: quantity increases as price decreases (controlled by buyers)

- Shift to the right: same price triggers greater demand
- Shift to the left: same price triggers lower demand

Supply curve: quantity increases as price increases (controlled by sellers)

- Shift to right: suppliers are sending more into the market
  - Maybe a new competitor with technical innovations
  - Supply side has broadly improved its efficiency and generally can supply for lower price due to better efficiency.
  - Lower cost of production

Law of supply: All things being equal, Q_s of a good will increases if the price of a good increases and vice versa.

Price elasticity: How the quantity of supply or demand of a good responds to change in its price

$$E_d = \frac{|P * \Delta Q_d|}{|Q_d * \Delta P|} = \left|\frac{\%\ change\ in\ Q_d}{\%\ change\ in\ P}\right|$$

$$E_s = \frac{|P * \Delta Q_s|}{|Q_s * \Delta P|} = \left|\frac{\%\ change\ in\ Q_s}{\%\ change\ in\ P}\right|$$

$E = \infty$ → perfectly elastic $E > 1$ → elastic $E = 1$ → Unitary

$E < 1$ → Inelastic $E = 0$ → perfectly inelastic

Sensitivity Analysis

$$NV = R - I - L$$

Sensitivity analysis visualization

1. Spider plot
2. Tornado plot

To show how the model is affected by an input parameter

Larger slope, more sensitive parameter

Stochastic models

Closer to reality and higher flexibility of valid problem situations

Use random variables under different conditions as inputs and forecasts the probability of different outcomes

$$E[NV] = \sum_i NV_i * pr(i)$$

Monte Carlo simulations (perform simulation many times and assess the frequencies of different outcome)

Decide on a probability distribution for important variables

Simulation: attempt to duplicate the features, appearances, and characteristics of a real system, usually via a computerized model

Confidence interval

μ: population mean $\bar{x}$: sample mean k: critical value (z score)

σ: standard deviation n: sample number α : significance level

$$\mu_{1-\alpha} = (\bar{x} + k_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}; \bar{x} + k_{1-\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}})$$

Confidence level	k or z score
99.7%	2.968
99%	2.575
98%	2.33
97%	2.17
95%	1.96
90%	1.645
85%	1.440

Random variable

A quantity known only in terms of a probability

Deterministic model

Require parameters to be known with certainty (no variability)

Easier to determine optimum inputs

Risk

Normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Risk matrix

Tool to quickly access potential risks

Used to identify, access, and prioritize risk and their risk management plans

Validity: The degree to which inferences drawn from the model hold real meaning for the system

Tractability: The degree to which the model permits convenient analysis

Objective function: A function of the decision variables x for which we want to find the minimum or maximum

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Every optimal to an LP will be at a boundary point of its feasible region

If an LP has a unique optimum, it must occur at an extreme point of feasible region

Every local optimum for an LP is also a global optimum because all LPs are CONVEX