7. Dealing with Uncertainty: Sensitivity Analysis

At a basic level of analysis, project parameters such as prices, interest rates, and cash flow timing are all assumed to be known with exact certainty. This makes calculations using these parameters very simple to compute. In real situations though, these parameters are almost never certain; everything in truth has some degree of uncertainty. Having these parameters be variable leads to different outputs, so it becomes important to know how sensitive the outcome is to parameter variations. The impact (or perhaps "weight") of each parameter in an analysis model is not necessarily equal, and so understanding how uncertainty affects the outcome of the evaluation is what sensitivity analysis hopes to accomplish. Sensitivity analysis can help determine whether it is worthwhile to obtain more accurate estimates, or whether some uncertainties need to be limited. In the ways that we have tool to deal with risk (chance of outcomes), sensitivity analysis helps us deal with uncertainty (impact of parameters on outcomes).

7.1. Probability Background

In the topic 6 notes, confidence interval section, we had the following calculations for confidence intervals on sample means:

Review from statistics class:
μ = population mean, or "true mean"
$\bar{x}$ = sample mean
k = critical value, or the "z-score" value
σ = standard deviation
n = number of observations in sample
α = significance level, or 1-(the desired confidence level)

The equation for calculating the confidence interval is below:

$$\mu_{1-\alpha} = (\bar{x} + k_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{x} + k_{1-\alpha/2}\frac{\sigma}{\sqrt{n}})$$

If you have all the inputs for this formula, it will return a "lower" and a "higher" value: the true mean will have a (confidence level) probability of being between these two values.
For example, you test 25 rebars to see if they meet the minimum strength requirements. The sample mean of these 25 rebars is 300 MPa. The rebar supplier quotes a 25 MPa standard deviation. With a 95% confidence level, what is the true mean of the rebar population?

$$\mu_{1-\alpha} = (\bar{x} + k_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{x} + k_{1-\alpha/2}\frac{\sigma}{\sqrt{n}})$$
$$\mu_{95\%} = \left(300\text{ MPa} + k_{0.025}\frac{25\text{ MPa}}{\sqrt{25}}; 300\text{ MPa} + k_{0.975}\frac{25\text{ MPa}}{\sqrt{25}}\right)$$
$$\mu_{95\%} = (300\text{ MPa} - 1.96 * 5\text{ MPa}; 300\text{ MPa} + 1.96 * 5\text{ MPa})$$
$$\mu_{95\%} = (290.2\text{ MPa}; 309.8\text{ MPa})$$

There is a 95% probability that the true mean is somewhere between 290.2 MPa and 309.8 MPa.

We can also calculate this with Excel. The product $k_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is also called the **confidence radius** or **margin of error**, and we can calculate it using the formula **confidence.norm(alpha, sigma, n)**
For example,

Confidence level	95%
alpha	5%
standard_dev	25
sample size	25
mean	300
Confidence Radius (Margin of Error)	9.7998	<-- =CONFIDENCE.NORM(B2, B3, B4)
Low Value	290.2002	<-- =B5 - B6
High Value	309.7998	<-- =B5 + B6

7.1.1. Where do these come from? – probability review

That is, *why* do we need a $1/\sqrt{n}$ term in the formula $\mu_{1-\alpha} = \left(\bar{x} + k_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{x} + k_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$ (when writing a range we can be 95% confident the sample mean is in)
...but if the samples themselves are normally distributed then we can be 95% confident a sample will be in the range $\left(\bar{x} + k_{\alpha/2}\sigma; \bar{x} + k_{1-\alpha/2}\sigma\right)$? (or more properly $\left(\mu + k_{\alpha/2}\sigma; \mu + k_{1-\alpha/2}\sigma\right)$)
That is, why does averaging data which has a standard deviation of σ mean the average is normally distributed with a the standard deviation of $\frac{\sigma}{\sqrt{n}}$?

7.1.1.1. Background – Expected Value, Variance, and Standard Deviation

Discrete (finite) set of events:
chance of any event happening is its probability
chance of an outcome from some set of events happening is the sum of their probabilities
Sum of Probability of all outcomes must add to 100% (there's a 100% chance *one* of the outcomes happens) (i.e., the distribution is **normalized**)

PDF = probability density function; for continuum of outcomes
chance of any one outcome *exactly* is zero.
chance of outcome being between x0 and xf is $\int_{x_0}^{x_f} f(x)\,dx$.
integral of pdf over all values must be 100% (there's a 100% chance the outcome is *something*).

Expected value of *y*: $E[y] = \sum_i y_i p_i \equiv \bar{y}$ (expected value of a parameter is the sum of the value it would have during each outcome times the probability of that outcome, also called "mean value of y" or "average value of y").
If continuous, $E[y] = \int_{-\infty}^{\infty} y(x) f(x)\,dx$
We can also find the **mean value** of the outcome number, i.e., the mean value of the probability distribution

$$\mu \equiv \bar{i} = \sum_i i p_i$$

or of the independent variable the pdf is in terms of: $\mu \equiv \bar{x} = E[x] = \int_{-\infty}^{\infty} x f(x)\,dx$

Mean value tells us the average of the outcomes we'd expect from many trials.

Variance of a distribution is the expected value of the square of the difference from the mean:

$$\mathrm{Var}(x) \equiv E\left[(x-\bar{x})^2\right]$$

However, this turns out to be the same as $E\left[x^2\right]-\bar{x}^2$, which is usually easier to calculate.

/* proof, using μ for the mean rather than $\bar{x}$, for no reason in particular

$$E\left[(x-\mu)^2\right] = \int_{-\infty}^{\infty} \left(x^2 - 2\mu x + \mu^2\right) f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - 2\mu \int_{-\infty}^{\infty} x f(x)\,dx + \mu^2 \int_{-\infty}^{\infty} f(x)\,dx$$

$$= E\left[x^2\right] - 2\mu E[x] + \mu^2$$

$$= E\left[x^2\right] - 2\mu^2 + \mu^2$$

$$= E\left[x^2\right] - \mu^2$$

Or, for a discrete distribution,

$$E\left[(i-\mu)^2\right] = \sum_i \left(i^2 - 2\mu i + \mu^2\right) p_i$$

$$= \sum_i i^2 p_i - 2\mu \sum_i i p_i + \mu^2 \sum_i p_i$$

$$= E\left[i^2\right] - \mu^2$$

*/

Standard deviation is the square root of the variance, and gives an impression of how much spread about the mean the distribution has in the same units as the distribution:

$$\sigma \equiv \sqrt{E\left[(x-\mu)^2\right]}$$

7.1.1.2. Background – Normal Distributions

Normal distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Symbols notwithstanding,

1. Is this a valid pdf? (i.e., is it normalized?)
2. What's the mean?
3. What's the standard deviation?

```
> restart:
mu:=1.72: sigma:=3.875:  #random numbers
```

```
f:=1/(sigma*sqrt(2*Pi))*exp(-1/2*((x-mu)/sigma)^2);
E:= X-> int(X*f, x=-infinity..infinity);
E(1); # is it normalized?
Mean:=E(x); #expected value of x
stdev:=sqrt(E(x^2)-mu^2); #sqrt of variance
```

$$f := 0.07279865590\sqrt{2}\, e^{-0.03329864724\,(x-1.72)^2}$$

$$E := X \mapsto \int_{-\infty}^{\infty} X \cdot f \, dx$$

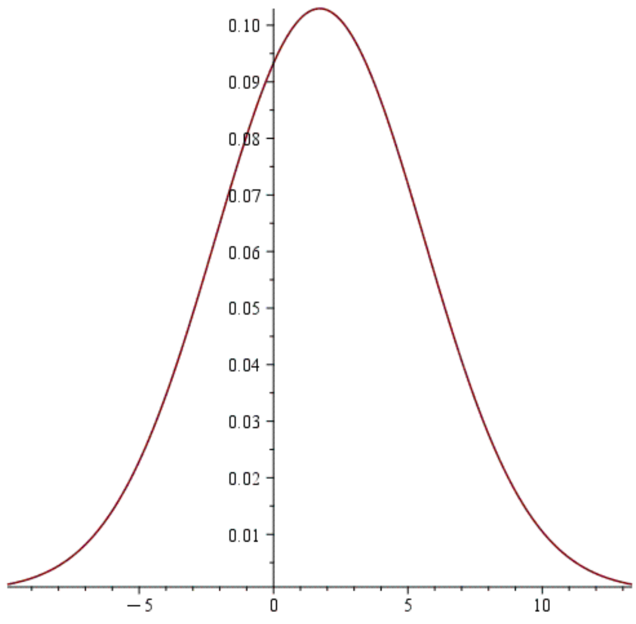
$$0.9999999995$$

$$Mean := 1.719999999$$

$$stdev := 3.874999999$$

What does it look like?

```
> plot(f, x=mu-3*sigma .. mu+3*sigma);
```



(a bell curve).

It's a PDF, so area under the curve between two x values represents probability of an outcome in that range of x values happening:

```
> ChanceOfBeingWithinSigmaOfMean:=int(f, x=mu-sigma..mu+sigma);
ChanceOfBeingWithin2SigmaOfMean:=int(f, x=mu-
2*sigma..mu+2*sigma);
ChanceOfBeingWithin2SigmaOfMean:=int(f, x=mu-
3*sigma..mu+3*sigma);
ChanceOfBeingInNinetyFivePercentConfidenceInterval:=int(f, x=mu-
1.96*sigma..mu+1.96*sigma);
```

$$ChanceOfBeingWithinSigmaOfMean := 0.6826894918$$

$$ChanceOfBeingWithin2SigmaOfMean := 0.9544997356$$

$$ChanceOfBeingWithin2SigmaOfMean := 0.9973002034$$

$$ChanceOfBeingInNinetyFivePercentConfidenceInterval := 0.9500042092$$

7.1.1.2.1. Normal Distribution Excel Functions

Without a computer algebra system, finding the integral is a pain since the integral of e^{x^2} isn't expressible in terms of elementary functions (and even with one it's a bit inconvenient for how much we're going to use this). Fortunately, there's some relevant excel functions:

`=NORM.DIST(x, mean, standard_dev, cumulative)`

If `cumulative` = 0, this tells you $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for μ = `mean`, σ = `standard_dev`, and *x* = `x`. This isn't so useful.

If `cumulative = 1`, this tells you $\int_{-\infty}^{x} f(x)\,dx$, which is very useful.

`=NORM.INV(probability, mean, standard_dev)`

Tells you the x to use so that $\int_{-\infty}^{x} f(x)\,dx = probability$, which is also very useful.

`=NORM.S.DIST(z, cumulative)`

Like `NORM.DIST` but for the "standard normal distribution" (i.e., mean = 0, stdev = 1)

`=NORM.S.INV(probability)`

Like `NORM.INV` but for the standard normal distribution.

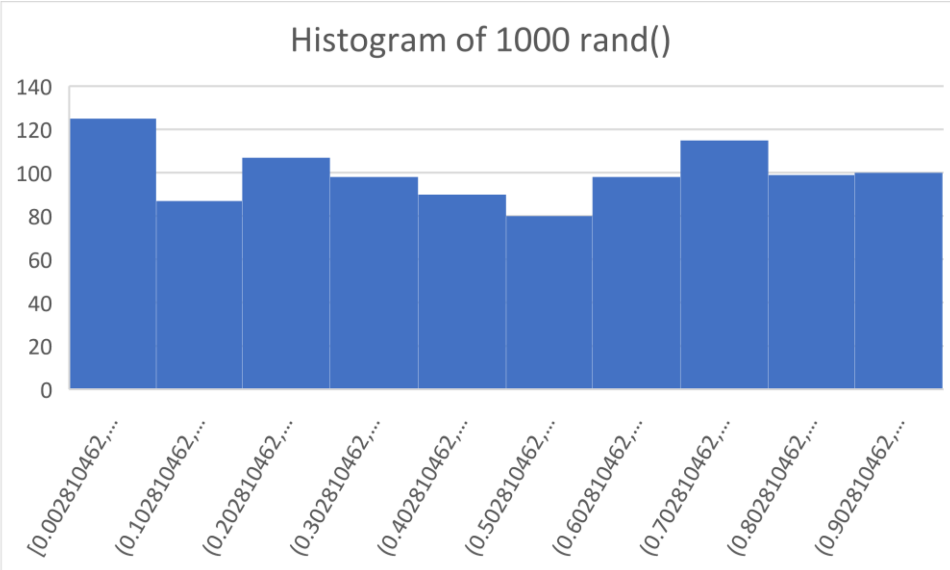
7.1.1.3. Background – Central Limit Theorem

Normal distributions are particularly common because of the ***central limit theorem***: *the sum of many independent random variables limits towards a normal distribution as the number of variables summed increases (regardless of whether those variables were themselves normally distributed).*

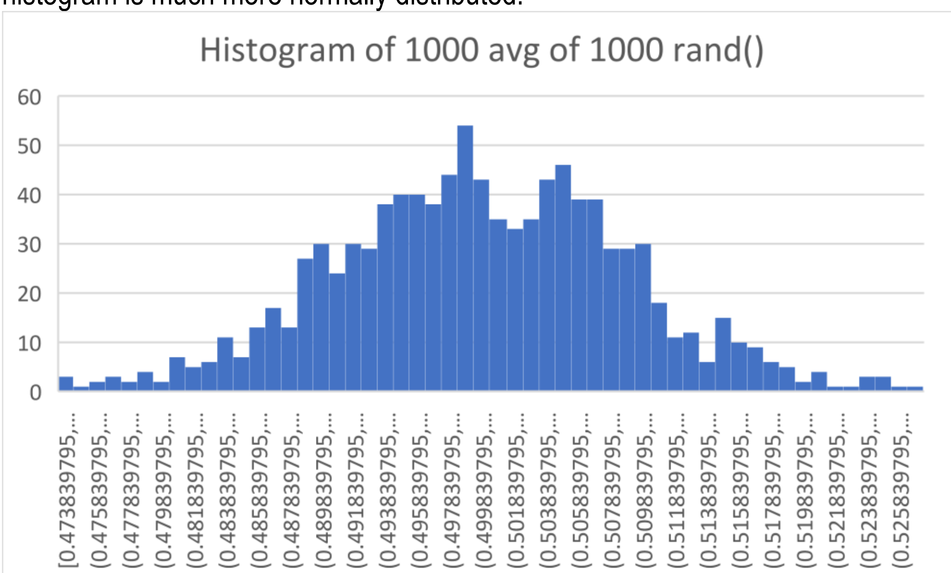
e.g., find out the shape of the distribution of

1. Excel's `rand()` function.
2. The average value of 1000 cells of Excel's rand function.

We can set up a spreadsheet with 1000 cells of rand() and make a histogram – it's more or less flat, which makes sense since rand() is uniform:



If we use *What If?* analysis to find the average of this 1000 times and plot a histogram of the averages, *that* histogram is much more normally distributed:



The reason this happens is because calculation of the average is $\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$ and each y_i is a random variable that's independent from every other one, so the central limit theorem pushes $\bar{y}$ towards itself being normally distributed.

This Monte Carlo simulation also lets us check whether the standard deviation of the means is indeed $\frac{1}{\sqrt{n}}$ times the standard deviation of the samples. And it turns out that if you find the standard deviation of the 1000 cells of rand() and the 1000 cells of averages of 1000 cells of rand() (i.e., with Excel's `=STDEV()` function) then indeed the standard deviation of the average is roughly 1/sqrt(1000) times the standard deviation of the rand() function samples themselves:

1002							
1003	0.284043	=STDEV(A2:A1001)		0.009097	=STDEV(D2:D1001)		
1004				0.008982	=A1003/SQRT(1000)		
1005							

The reason this occurs is as follows:

1. Variance of a sum [of independent random variables] is the sum of their variances
2. Variance of (y/n) (where n is constant) is Var(y)/n^2
3. Therefore, variance of average of n samples is variance of each sample / n
4. Therefore, SD of average of n samples is SD of each sample / sqrt(n)

To see that Variance of a sum is the sum of the variance, consider finding the variance of a sum of a sample of two random variables (y_1 and y_2):

$$\mathrm{Var}(y_1+y_2)=E\left[\left((y_1+y_2)-\overline{(y_1+y_2)}\right)^2\right]$$

$$=E\left[(y_1+y_2-\bar{y}_1-\bar{y}_2)^2\right]$$

$$=E\left[\left((y_1-\bar{y}_1)+(y_2-\bar{y}_2)\right)^2\right]$$

$$=E\left[(y_1-\bar{y}_1)^2+(y_2-\bar{y}_2)^2+2(y_1-\bar{y}_1)(y_2-\bar{y}_2)\right]$$

$$=\mathrm{Var}(y_1)+\mathrm{Var}(y_2)+2\mathrm{Cov}(y_1,y_2)$$

The covariance between the variables $\mathrm{Cov}(y_1,y_2)\equiv E\left[(y_1-\bar{y}_1)(y_2-\bar{y}_2)\right]$ will be 0 unless there's some correlation between the samples (i.e., for some reason y_2 tends to be higher than its mean whenever y_1 is higher than its mean, or when y_1 is lower than its mean). If the samples are independent, they'll have 0 covariance, and therefore $\boxed{\mathrm{Var}(y_1+y_2)=\mathrm{Var}(y_1)+\mathrm{Var}(y_2)}$

To see that Variance of (y/n) for constant n is the variance of y / n^2, write:

$$E\left[\left(\frac{y}{n}-\overline{\left(\frac{y}{n}\right)}\right)^2\right]=E\left[\left(\frac{y}{n}-\frac{\bar{y}}{n}\right)^2\right]=E\left[\frac{1}{n^2}(y-\bar{y})^2\right]=\frac{1}{n^2}E\left[(y-\bar{y})^2\right]$$

Therefore, the variance of an n-sample *average* is

$$E\left[\left(\left(\frac{1}{n}\sum_{i=1}^{n}y_i\right)-\overline{\left(\frac{1}{n}\sum_{i=1}^{n}y_i\right)}\right)^2\right]$$
$$=\frac{1}{n^2}E\left[\left(\sum_{i=1}^{n}\left(y_i-\bar{y}_i\right)\right)^2\right]$$
$$=\frac{1}{n^2}E\left[\sum_i \mathrm{Var}\left(y_i\right)+\sum_{i,j;i>j}2\mathrm{Cov}\left(y_i,y_j\right)\right]$$
$$=\frac{1}{n^2}E\left[\sum_i \sigma^2+\sum_{i,j;i>j}2\cdot 0\right]$$
$$=\frac{1}{n^2}E\left[n\sigma^2\right]$$
$$=\frac{\sigma^2}{n}$$

where σ^2 is the variance of the samples we're averaging.

And finally, this means the standard deviation of the n-sample average is $\sigma_{\bar{y}}=\sqrt{\frac{\sigma^2}{n}}=\frac{\sigma}{\sqrt{n}}$

THAT is why we use the following formula for confidence interval on a mean:

$$\mu_{1-\alpha}=\left(\bar{x}+k_{\alpha/2}\frac{\sigma}{\sqrt{n}};\bar{x}+k_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

Note that $k_{\alpha/2}$ is `=NORM.S.INV(α/2)`
Try it yourself! Compare `=STDEV()` for the rand() function column and the average of it using data table.

7.2. Sensitivity graphs

Sensitivity graphs are used to assess the effect of one-at-a-time changes in key parameter values of a project on an economic performance measure. A sensitivity graph will begin with a control where estimated or expected values for the parameters are used. Then, parameters are varied above and below the control values – one parameter at a time, while holding other parameters fixed – to develop the data for a sensitivity graph. This allows a visual representation of how each individual parameter has an impact on performance measurement.

7.2.1.1. Green Flower Energy

Green Flower Energy (GFE) is planning to replace their current steam power plant with a new cogeneration plant. This new plant will be able to produce electrical power in addition to steam power. Under current operations, the steam plant requires a large amount of input electrical power, which GFE purchases from public utilities, which is a huge cost for GFE. GFE has designed the new plant to use wood as a fuel source and expect substantial savings from the change. To complete this replacement project, the new cogeneration plant will also require the construction and integration of a new cooling tower and generator.

In planning this project, GFE has commissioned the estimation of several costs for the project.

The estimated costs are as follows:

Initial Investment (Construction/Integration) costs: $3,000,000, one-time payment
Lifetime and Depreciation: 20 years, with $0 value remaining at the end of life.
Maintenance: $35,000 at the end of every four years for the generator, $17,000 at the end of every ten years for the cooling tower
Operating costs: $65,000/yr
Fuel costs (wood): $375,000/yr
Savings from not purchasing electricity: $1,000,000/yr
GFE's Desired **MARR**: 12%

Before we start, let's use a project manager's tool to quickly assess potential risks on the project: the risk matrix. This is a table that categorizes project risks in terms of their likelihood and the severity of their impact on various aspects of the project. Often a risk matrix is colour-coded to identify the level of risk at a glance. Risk matrices are used to identify, assess, and prioritize risks and their risk management plans. They can also be used to communicate potential project risks to stakeholders, and can help guide decision-making. For this reason, it is important that the risks are properly categorized as to not overstate or understate their likelihood/impact.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Impact</td></tr>
<tr><td>Low
• Won't be so bad (if it happens).</td><td>Medium
• Moderately bad (if it happens)</td><td>High
• Endangers the project as a whole; catastrophic (if it happens)</td></tr>
<tr><td rowspan="3">Likelihood</td><td>Low
• Happens infrequently and/or briefly.</td><td>• Not a big deal. Action/treatment may not be needed.</td><td>• Watch carefully – don't let this get worse/out of hand</td><td>• You'll need to take action to address this risk. (Shark attack, nuclear meltdown)</td></tr>
<tr><td>Medium
• Risk may repeat or reoccur daily.</td><td>• Watch carefully – don't let this get worse/out of hand.</td><td>• You'll need to take action to address this risk.</td><td>• This risk can't be ignored or avoided. A plan to deal with it is needed.</td></tr>
<tr><td>High
• A near certainty that this will occur as a</td><td>• You'll need to take action to address this risk.</td><td>• This risk can't be ignored or avoided. A plan to deal with it is needed.</td><td>• Total showstopper. Risk management plan to address these risks is</td></tr>
</table>

	nature of the project.			needed immediately.

A risk matrix isn't necessarily 3x3; a grid between 2x2 to 5x5 is common depending on the project manager's preferred differentiation between severity levels. It is also fine to have no risks in certain cells if there are no risks that fall under that specific likelihood/impact combination. The project manager is tasked with appropriately positioning the risks into the matrix grid.

A risk matrix for GFE's expansion project may look as follows:

		Impact		
		Low	Medium	High
Likelihood	Low	• Minor Personnel Turnover	• Delay in obtaining project permits/approvals	• Generator Malfunction • Cooling Tower Malfunction
	Medium		• Minor Equipment Damage • Minor safety incidents	• Disruption in Fuel Supply Chains
	High	• Minor Budget Overruns	• Unexpected Maintenance Costs • Increase in Operating Costs	• Delays in Construction Schedule • Increase in Fuel Costs • Regulatory Changes

What is the **net value** of GFE's potential replacement investment **in today's value**?

To accomplish this, we'll need to figure out the present value of all costs mentioned above. The initial construction cost happens immediately, so nothing must be done there. For the operating costs, the $65,000/yr needs to be converted to a yearly present value (a $65,000 cost in 20 years is worth less than $65,000 in today's value, as we did in previous lectures). This must be done for the rest of the costs as well.
SOLUTION: Net value calculations for replacing the power plant:

Note: The following calculations make use of a common shorthand for Time Value of Money calculations. It follows the format (Find X/Given Y, With interest rate I, Over N periods). So, (P/A,12%,20) means (Given Present Value/Find Annuity Value, With 12% interest, For 20 periods). This would expand into the formulas you were introduced to in Week 4, so the benefit of using the shorthand is just to self-confirm which formulas are needed, while keeping the equation length relatively brief.

(*P/F, i, n*) = Find present value/given future value, with given interest rate, for a certain number of periods = [*P = F * (1 + i)^-n]*

(P/A, i, n) = Find present value/given annuity value, with given interest rate, for a certain number of periods = *[P = A * [((1 + i)^n) – 1 / (i * (1 + i)^n)]]*

$$Net\ Value = Benefits - Costs$$

$$\begin{aligned}
Net\ Value = &\ Construction\ Costs + Operating\ and\ Fuel\ Costs \\
&+ Savings\ from\ not\ Purchasing\ Electricity + Maintenance\ Costs
\end{aligned}$$

$$\begin{aligned}
Net\ Value = &\ [(-\$3{,}000{,}000)] \\
&+ \left[\left((-\$65{,}000 - \$375{,}000)(P/A, 12\%, 20) + (\$1{,}000{,}000)(P/A, 12\%, 20)\right)\right] \\
&+ [(-\$17{,}000)(P/F, 12\%, 10)] \\
&+ \left[\left(-\$35{,}000(P/F, 12\%, 4) - \$35{,}000(P/F, 12\%, 8)\right.\right. \\
&\left.\left.- \$35{,}000(P/F, 12\%, 12) - \$35{,}000(P/F, 12\%, 16)\right)\right]
\end{aligned}$$

$$\begin{aligned}
Net\ Value\ = &\ (-\$3{,}000{,}000) + \left(-\$440{,}000 * \left(\frac{(1+12\%)^{20} - 1}{12\%(1+12\%)^{20}}\right)\right) + (\$1{,}000{,}000 \\
&* \left(\frac{(1+12\%)^{20} - 1}{12\%(1+12\%)^{20}}\right)) + (-\$17{,}000 * (1+12\%)^{-10}) + (-\$35{,}000 \\
&* (1+12\%)^{-4}) + (-\$35{,}000 * (1+12\%)^{-8}) + (-\$35{,}000 * (1+12\%)^{-12}) \\
&+ (-\$35{,}000 * (1+12\%)^{-16})
\end{aligned}$$

$$\begin{aligned}
Net\ Value\ = &\ (-\$3{,}000{,}000) + (-\$3{,}286{,}555.19) + (\$7{,}469{,}443.62) + (-\$5{,}473.55) \\
&+ (-\$22{,}243.13) + (-\$14{,}135.91) + (-\$8{,}983.63) + (-\$5{,}709.26)
\end{aligned}$$

$$Net\ Value = \$1{,}126{,}342.95$$

With this positive net value, the project seems to be economically viable. However, this calculation assumes that the present worth of some of these costs are exact over the lifespan of the plant, which is unlikely to be the case. For example, things like construction delays or overestimated electricity savings costs may change the values of the estimates. Additionally, GFE may feel their estimates for some values could be different compared to the final result, so they would like to understand what impact these errors may have on the final evaluation. The current estimates will be our control case, and we will generate cash flow estimates that are 5% and 10% above and below the control for each major cash flow category.

Cost Category	-10%	-5%	Control	5%	10%
Initial Investment Cost	\$2,700,000	\$2,850,000	\$3,000,000	\$3,150,000	\$3,300,000
Annual Operation Costs	\$58,500	\$61,750	\$65,000	\$68,250	\$71,500
Annual Maintenance Costs, Generator	\$31,500	\$33,250	\$35,000	\$36,750	\$38,500

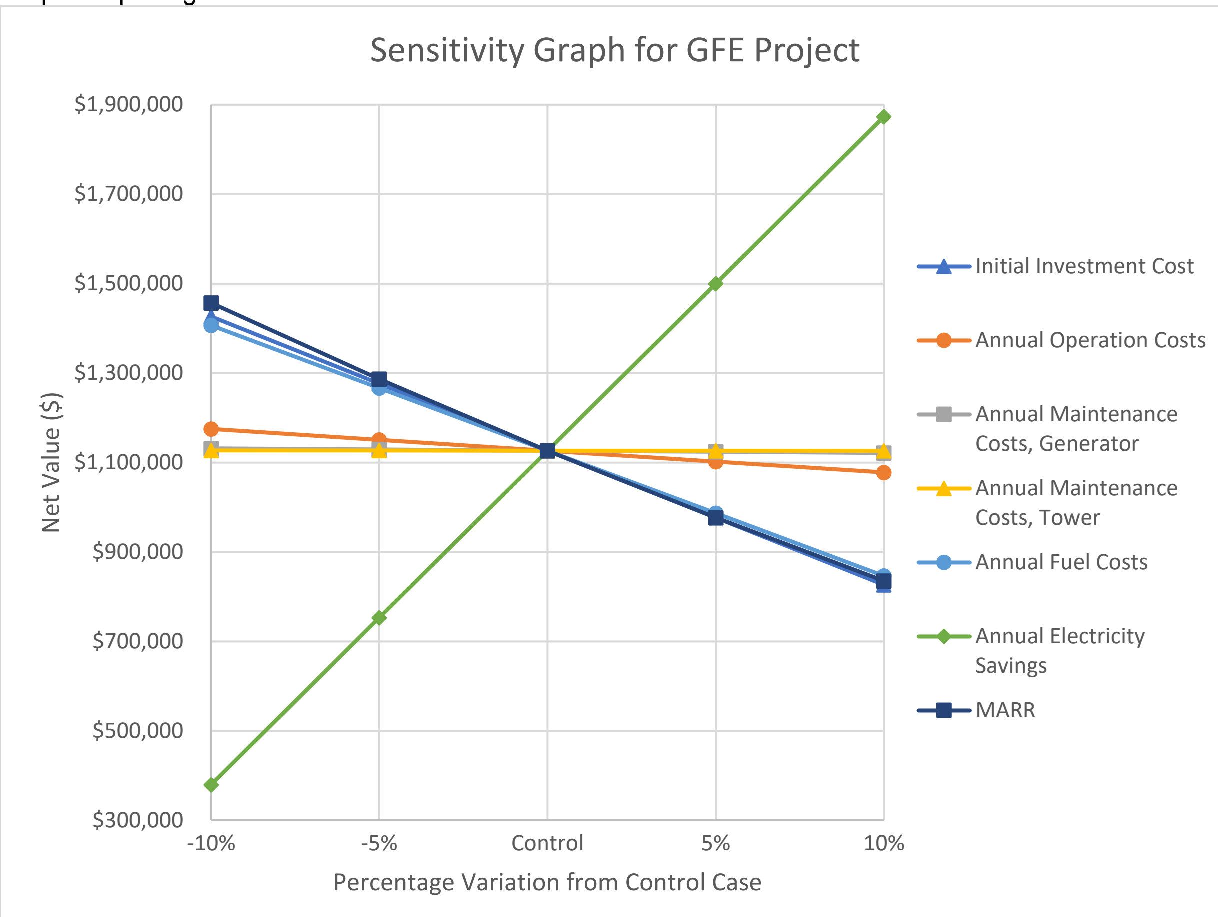
Annual Maintenance Costs, Tower	$15,300	$16,150	$17,000	$17,850	$18,700
Annual Fuel Costs	$337,500	$356,250	$375,000	$393,750	$412,500
Annual Electricity Savings	$900,000	$950,000	$1,000,000	$1,050,000	$1,100,000
MARR	10.80%	11.40%	12%	12.60%	13.20%

With this data table of cash flow estimates, we can recalculate the present net value for all 28 different combinations (7 variables x 4 non-control values each) where exactly one parameter is changed. For example, by using all the control values other than the initial investment cost, which we replace with the -10% value, the net value is recalculated exactly the same as before, but with [(-$2,700,000)] as the first term. This results in a present net value of **$1,426,343.** This can be continued (preferably not by hand) for all 28 combinations.

Cost Category	-10%	-5%	Control	5%	10%
Initial Investment Cost	$1,426,343	$1,276,343	$1,126,343	$976,343	$826,343
Annual Operation Costs	$1,174,894	$1,150,619	$1,126,343	$1,102,067	$1,077,792
Annual Maintenance Costs, Generator	$1,131,450	$1,128,897	$1,126,343	$1,123,789	$1,121,236
Annual Maintenance Costs, Tower	$1,126,890	$1,126,617	$1,126,343	$1,126,069	$1,125,796
Annual Fuel Costs	$1,406,447	$1,266,395	$1,126,343	$986,291	$846,239

Annual Electricity Savings	$379,399	$752,871	$1,126,343	$1,499,815	$1,873,287
MARR	$1,456,693	$1,286,224	$1,126,343	$976,224	$835,115

From this data chart, we can generate a sensitivity graph (spider plot) to visualize the changes. This is as simple as putting all the values on a line chart.



Here we can see how sensitive the outcome is to each parameter of the project. The incredibly steep slope of the Annual Electrical Savings line indicates the outcome is most sensitive to changes in this parameter's value. Next, the MARR, Initial Investment Cost, and Annual Fuel Costs have the steepest slopes, respectively.

What this suggests is that, with a 10% variation in any parameter, the project is still economically viable, and produces a non-negative net value. If GFE is confident in the quality of their estimates, they can move forward knowing they will generate value with the project.

There are two key drawbacks to understand and be aware of with sensitivity graphs.

1. Parameter interactivity: The net value is shown to be changing as a result of the change in one and only one parameter. The possible interaction between parameters is not considered at all. When more than one parameter changes simultaneously, simply "adding" the impacts does not work; any interaction effect is more complex than this. This is a major drawback, and leaves sensitivity

graphs to be sources of greater information of how uncertainty affects the problem, rather than a rigid decision-making tool.

2. They do not encapsulate parameter variations outside the shown range. It is not necessarily true that, outside the graph range, the parameters continue to behave as linear extrapolations of themselves.

7.2.1.2. Sensitivity Analysis in Stochastic Models

Suppose the inputs to this example are randomly generated and normally distributed instead of deterministic. Now the output (net value in this case) will also have a distribution of values with its own mean and standard deviation. How does the mean of the output change based on changes to the mean and/or standard deviation of inputs? In other words, how sensitive is the output mean to changes in means and standard deviations of inputs? Also, how sensitive is the output standard deviation to these changes? How sensitive are confidence intervals on the output to these changes?

At first, you might assume the output function will have a mean which is equal to what the output function would give in response to all of the input function means (i.e., $\overline{f(x,y,z)} = f(\bar{x},\bar{y},\bar{z})$, or $E[f(x,y,z)] = f(E[x],E[y],E[z])$), but this depends on the function. For example, it is true if the function is a linear combination of the inputs like an average (as we saw when estimating the population mean from the sample mean: $\bar{y} = \overline{\frac{1}{n}\sum_{i=1}^{n} y_i} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i = \bar{y}_i$), but it isn't true for a nonlinear combination like average of the square of the values (as we saw with finding standard deviation $E\left[(y-\bar{y})^2\right] = E\left[y^2\right]-\left(E[y]\right)^2$).

With multiple independent inputs, the function only needs to be linear (i.e., have no curvature) in each one independently (i.e., $\partial f/\partial x$ is not a function of *x*; i.e., $\frac{\partial^2 f}{\partial x^2}=0$), so if $f(x,y)=xy$ then $\overline{f(x,y)} = f(\bar{x},\bar{y})$ if *x* and *y* are independent. Notice that this isn't true if x and y are dependent on each other; e.g., if $y = x$, then $f(x,y)=xy=x^2$ the function is effectively nonlinear due to the dependent variables.

Regardless of the linearity of the function, we could determine the sensitivity of the output mean and standard deviation to changes in the input means and standard deviation using a Monte Carlo simulation (i.e., follow the procedure in the Week 6 example to determine the output values for inputs drawn from one set of input means & standard deviations, then redo this for all parameters of interest to generate the spider plot).

Above [deterministic sensitivity analysis], we determined the sensitivity of the model to an input by calculating the change in the output after changing the value of one input by 5%. In stochastic sensitivity analysis, we do the same thing, except we “calculate” the outputs (either mean or standard deviation) with Monte Carlo simulations and the parameter we're determining sensitivity to is a parameter of the distribution of the function inputs (e.g., mean or standard deviation of an input, or possibly other parameters if its distribution shape is not gaussian). The change in the output mean (if any) for a 5% change in standard deviation of input y would be the change in the mean of the Monte Carlo simulated values from using the original standard deviation for input y's distribution to the mean of another Monte Carlo simulation with everything else the same but y's standard deviation increased by 5%.

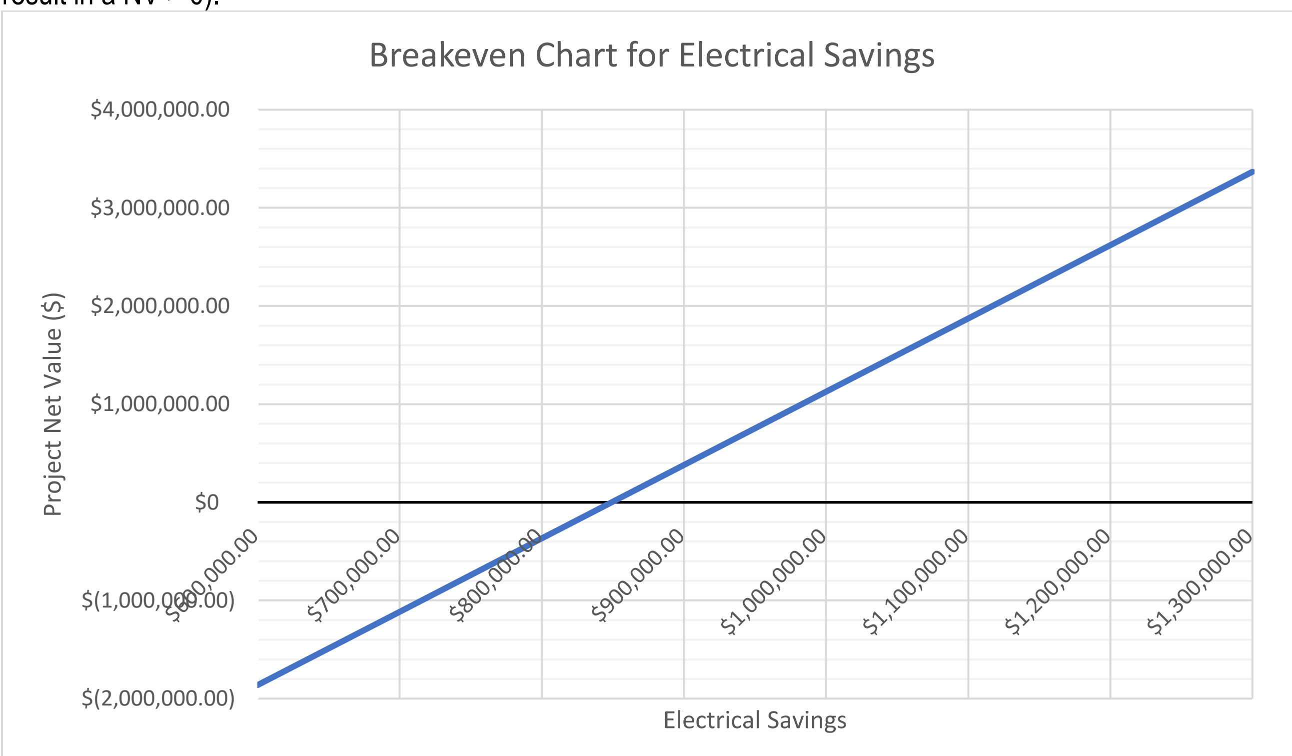
Questions:
Considering the NV function above for Green Flower Energy, does the output value depend linearly or nonlinearly on each of the inputs?
What about the stochastic sensitivity analysis from section 6.3.3.5 of the topic 6 notes – what are the "inputs" and does the output mean depend linearly or nonlinearly on each of them?

7.3. Break-even Analysis

Breakeven analysis is a form of sensitivity analysis wherein the goal is to determine what performance measures reach some threshold value or "breakeven" point. It is an additional step past a sensitivity graph that enables us to quickly determine the parameter values for which a project is economically viable.

7.3.1. Example

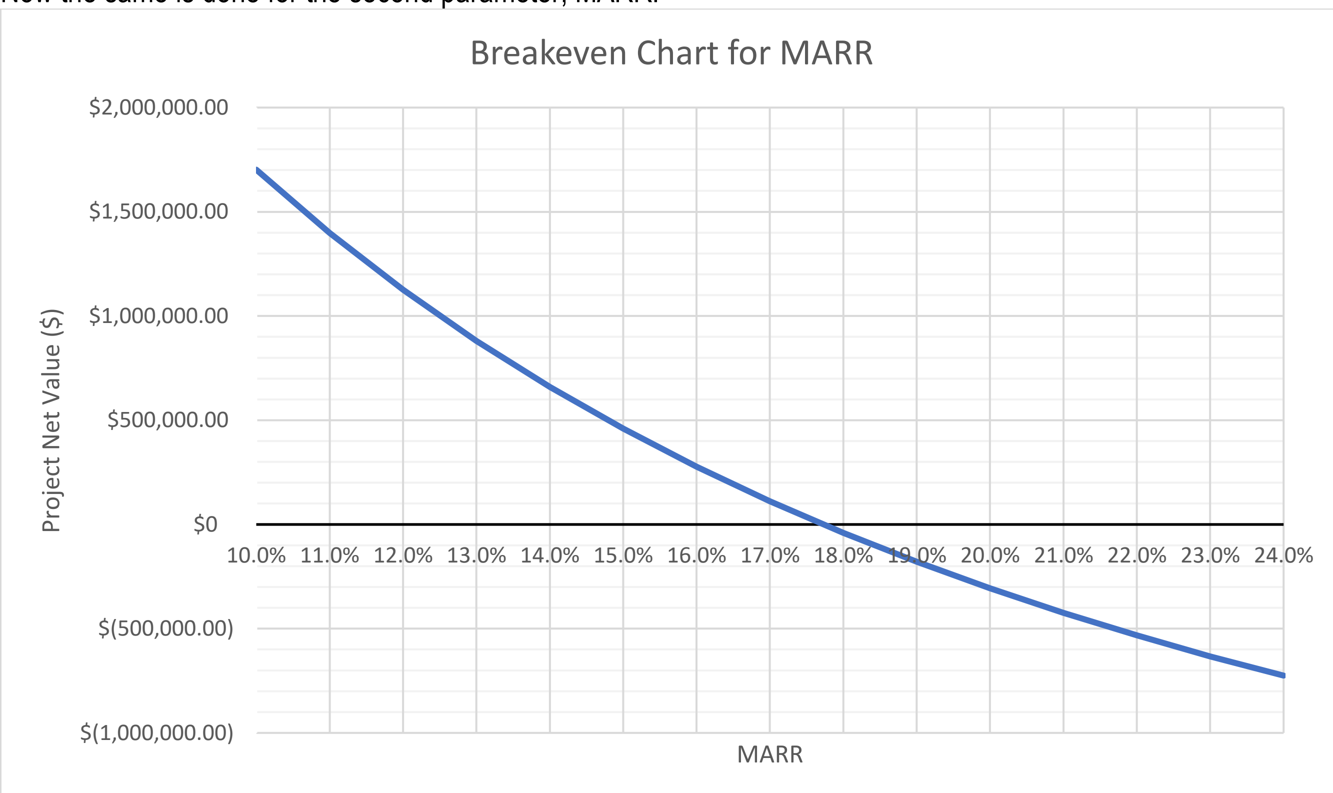
With the sensitivity graphs developed for the GFE power plant example, we can see that the most sensitive parameters are the electrical savings, MARR, initial investment costs, and annual fuel costs. From this information, GFE decides they would like further analysis in regard to these particularly sensitive parameters. A breakeven analysis is to be performed on these parameters (as in, what range of values result in a NV > 0).



From this breakeven chart, we can quickly see that an actual "Electrical Savings" of roughly $850,000 per year will result in this project being economically viable, if all the other estimates are accurate. To get the precise breakeven value (the x-intercept value) we can use the slope of the line calculation. Or, we can use the Solver tool in Excel to do the same thing. From this, we can instantly calculate that an electrical savings of $849,207 per year gives a net value of $0.00 *while still meeting the MARR set by GFE*, so this amount or more will be economically viable.

CONTROL NET VALUE		PV		NPV
Investment	$3,000,000	$ 3,000,000.00		$ 0.00
Operating	$65,000	$485,513.84		
Wood Fuel	$375,000	$2,801,041.36		
Electric Savings	$849,207	$6,343,100.67		
Tower Maint.	$17,000	$5,473.55		
Gener. Maint.	$35,000	$22,243.13		
	$ 35,000.00	$14,135.91		
	$ 35,000.00	$8,983.63	Sum Gener. Maint.	
	$ 35,000.00	$5,709.26	$51,071.93	
MARR	12.000%			

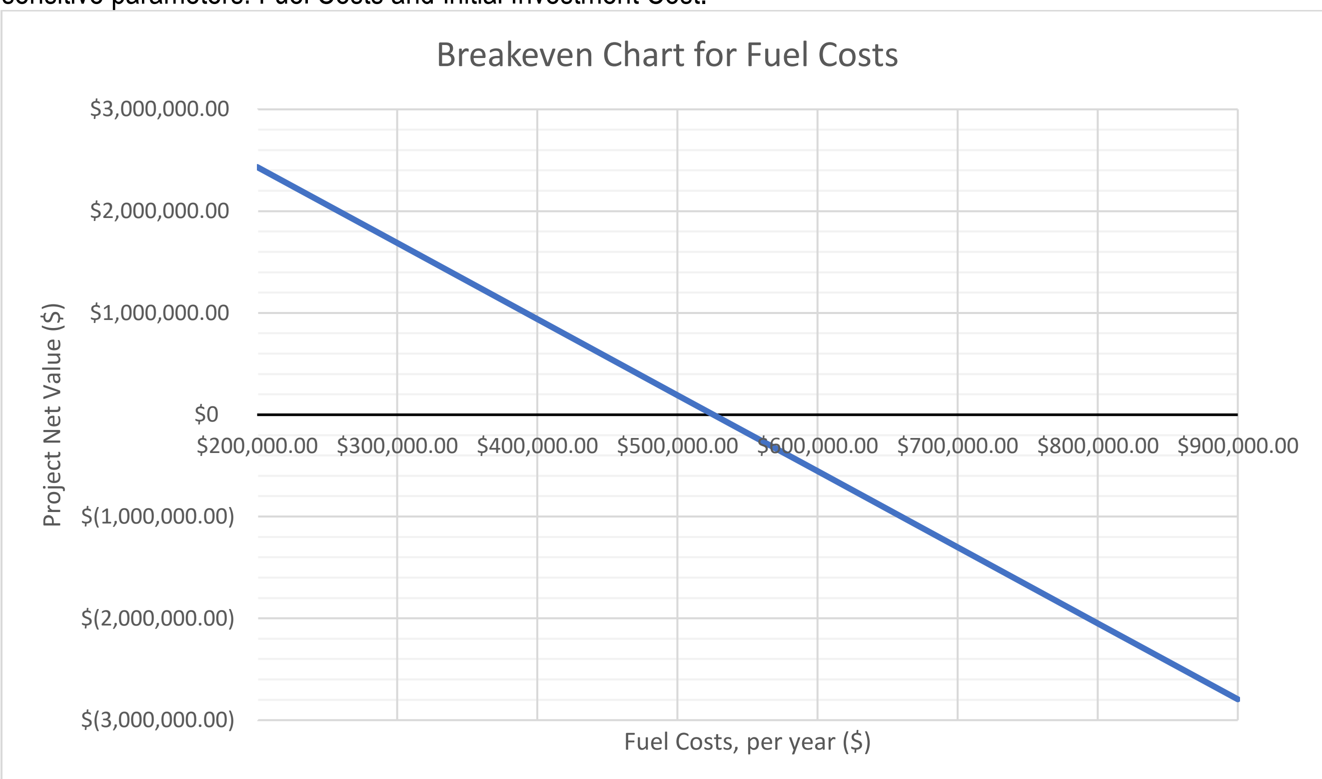
Now the same is done for the second parameter, MARR:



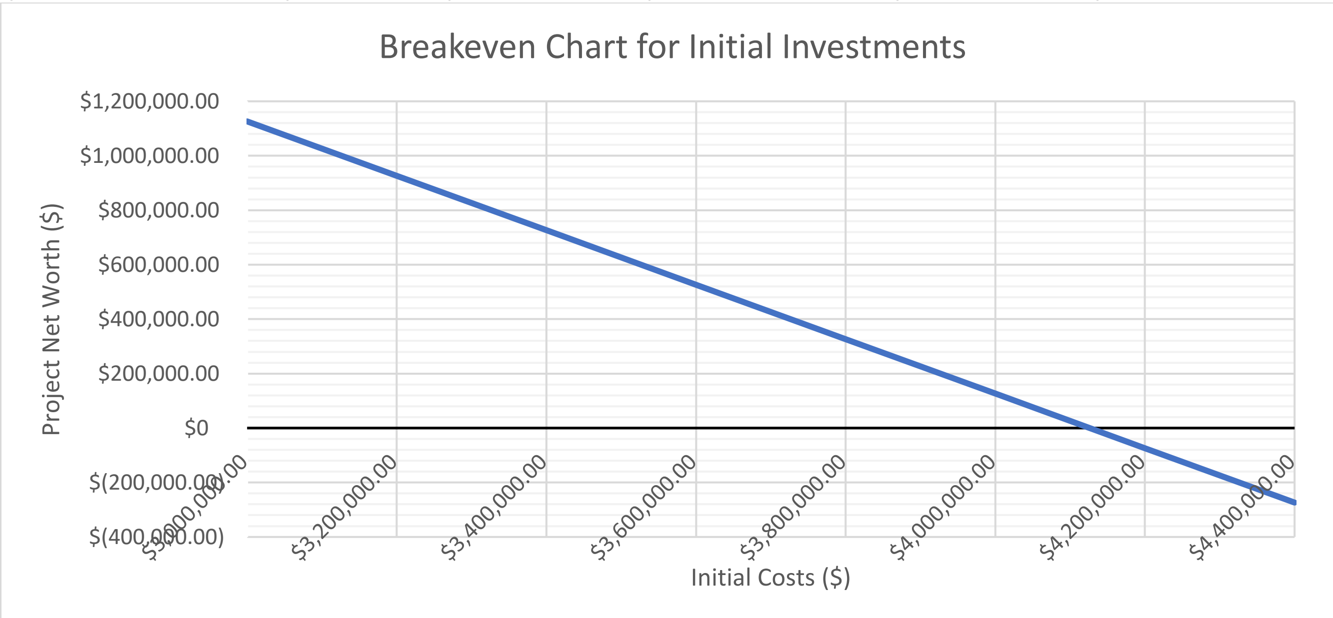
This is slightly trickier to find the breakeven value from. We can see it is somewhere between 17.5 – 18.0%, but can't quickly calculate it using the slope equation since the curve is non-linear. Excel will do it for us with the Solver tool.

CONTROL NET VALUE		PV		NPV
Investment	$3,000,000	$ 3,000,000.00		$ (0.00)
Operating	$65,000	$352,688.15		
Wood Fuel	$375,000	$2,034,739.30		
Electric Savings	$1,000,000	$5,425,971.47		
Tower Maint.	$17,000	$3,324.79		
Gener. Maint.	$35,000	$18,221.92		
	$ 35,000.00	$9,486.81		
	$ 35,000.00	$4,939.08	Sum Gener. Maint.	
	$ 35,000.00	$2,571.42	$35,219.23	
MARR	17.725%			

Again, see below the breakeven graphs as well as the exact breakeven value for the remaining two-most sensitive parameters: Fuel Costs and Initial Investment Cost.



CONTROL NET VALUE		PV		NPV
Investment	$3,000,000	$ 3,000,000.00		$ 0.00
Operating	$65,000	$485,513.84		
Wood Fuel	$525,793	$3,927,384.31		
Electric Savings	$1,000,000	$7,469,443.62		
Tower Maint.	$17,000	$5,473.55		
Gener. Maint.	$35,000	$22,243.13		
	$ 35,000.00	$14,135.91		
	$ 35,000.00	$8,983.63	Sum Gener. Maint.	
	$ 35,000.00	$5,709.26	$51,071.93	
MARR	12.000%			



CONTROL NET VALUE		PV		NPV
Investment	$4,126,343	$ 4,126,342.95		$ 0.00
Operating	$65,000	$485,513.84		
Wood Fuel	$375,000	$2,801,041.36		
Electric Savings	$1,000,000	$7,469,443.62		
Tower Maint.	$17,000	$5,473.55		
Gener. Maint.	$35,000	$22,243.13		
	$ 35,000.00	$14,135.91		
	$ 35,000.00	$8,983.63	Sum Gener. Maint.	
	$ 35,000.00	$5,709.26	$51,071.93	
MARR	12.000%			