ENGPHYS 2A04 Winter 2022 – Assignment 11

Due Monday April 11, 8AM

1. A stationary conducting loop with an internal resistance of 8 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 10 A flows through it. What will the current be if the loop is opened to create a small gap and a 5 Ω resistor is connected across its open ends?

   The V_{emf} is independent of the resistance which is in the loop. Therefore, when the loop is intact and the internal resistance is only 8 Ω.

   $$V_{\mathrm{emf}} = 10A \times 8\Omega = 80\ V$$

   (2 marks)
   When the small gap is created, the total resistance in the loop is infinite and the current flow is zero. With a 5-Ω resistor in the gap,

   $$\begin{aligned} I &= \frac{V_{\mathrm{emf}}}{5\Omega + 8\Omega} \\ &= \frac{80V}{13\Omega} \\ &= 6.15\ (\mathrm{A}) \end{aligned}$$

   (1 for showing work, 1 for the final answer)

2. A rectangular conducting loop 10 cm × 16 cm with a small air gap in one of its sides is spinning at 7200 revolutions per minute. If the field $\boldsymbol{B}$ is normal to the loop axis and its magnitude is $3.5 \times\ 10^{-6}$ T, what is the peak voltage induced across the air gap?

   $$\begin{aligned} \omega &= \frac{\frac{2\pi \mathrm{rad}}{\mathrm{cycle}} \times \frac{7200 \mathrm{cycles}}{\mathrm{min}}}{60 \mathrm{s/min}} \\ &= 240\pi \frac{\mathrm{rads}}{\mathrm{s}} \\ A &= 0.1\ \mathrm{m}\ \times 0.16\ \mathrm{m} \\ &= 1.6 \times 10^{-2} \mathrm{m}^2 \end{aligned}$$

   (2 marks)
   Recall the V_{emf} is given by:

   $$V_{\mathrm{emf}} = A\omega B_0 \sin \omega t$$

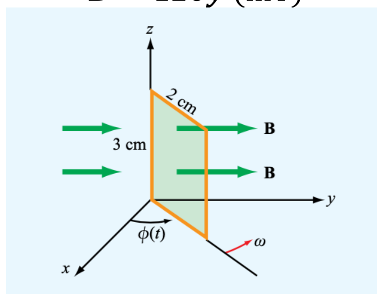
   So the peak voltage is given by:

   $$V_{peak} = A\omega B_0$$
   $$= (1.6 \times 10^{-2} \mathrm{m}^2)(240\pi \frac{\mathrm{rads}}{\mathrm{s}})(3.5 \times\ 10^{-6}\ \mathrm{T})$$
   $$42.2 \mu\mathrm{V}$$

   (2 marks)

3. The rectangular conducting loop shown in the figure below rotates at 1,200 revolutions per minute in a uniform magnetic flux density given by:

$$\boldsymbol{B} = 120\hat{\boldsymbol{y}}\ (\mathrm{mT})$$



Determine the current induced in the loop if its internal resistance is 250 mΩ.

$$\Phi = \int_S \mathbf{B}\cdot d\boldsymbol{S}$$
$$= 120\times10^{-3}\hat{\boldsymbol{y}}\cdot(2\times10^{-2})(3\times10^{-2})\cos\phi(\mathrm{t})$$
$$= 7.2\times10^{-5}\cos\phi(\mathrm{t})$$

(1)

$$\phi(\mathrm{t}) = \omega t$$
$$= \frac{2\pi\ \mathrm{rad}\times1.2\times10^{3}}{60\mathrm{s}}$$
$$= 40\pi\frac{\mathrm{rad}}{\mathrm{s}}$$

(1)

$$V_{\mathrm{emf}} = -\frac{\mathrm{d}\Phi}{\mathrm{dt}}$$
$$= 7.2\times10^{-5}\times40\pi\sin40\pi t$$
$$= 9.05\times10^{-3}\sin40\pi t\ \ (\mathrm{V})$$

(1)

$$I_{\mathrm{ind}} = \frac{V_{\mathrm{emf}}}{R_{internal}}$$
$$= \frac{9.05\times10^{-3}\sin40\pi t}{0.25}$$
$$= 3.62\ \times10^{-2}\sin40\pi t\ \ (\mathrm{A})$$

(1)

4. The transformer shown below consists of a long wire coincident with the z axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x–y plane and centered at the origin. The toroidal core uses iron material with relative permeability μr, around which 500 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure.

a. Develop an expression for V_{emf}

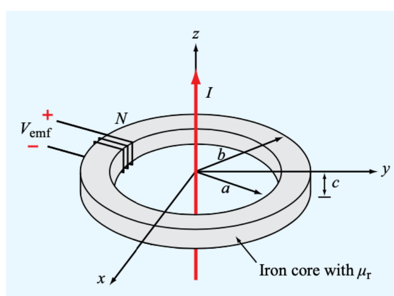
$$\begin{aligned}\Phi &= \int_S \mathbf{B} \cdot d\boldsymbol{S} \\ &= \int_a^b \frac{\mu I}{2\pi r} \hat{\mathbf{x}} \cdot c\hat{\mathbf{x}}\, dr \\ &= \frac{\mu c I}{2\pi} \ln(r)\Big|_a^b \\ &= \frac{\mu c I}{2\pi} ln\left(\frac{b}{a}\right)\end{aligned}$$

(1)

$$\begin{aligned}V_{emf} &= -N\frac{d\Phi}{dt} \\ &= -\frac{\mu c N}{2\pi} ln\left(\frac{b}{a}\right)\frac{dI}{dt} \\ &= -\frac{\mu c N \omega I_0}{2\pi} ln\left(\frac{b}{a}\right)\sin(\omega t)\ \ (\mathrm{V})\end{aligned}$$

(2)

b. Calculate V_{emf} for $f = 60$ Hz, $\mu_r = 4500$, $a = 10\ cm$, $b = 12\ cm$, $c = 3\ cm$, and $I_0 = 60\ A$



Given:
$f = 60$ Hz, $\mu_r = 4500$, $a = 10\ cm$, $b = 12\ cm$, $c = 3\ cm$, and $I_0 = 60\ A$, we can find the V_{emf} by substantiating into the expression derived above.

$$\begin{aligned}V_{\mathrm{emf}} &= -\frac{\mu c N \omega I_0}{2\pi} ln\left(\frac{b}{a}\right) \sin(\omega t) \\ &= -\frac{(4500 \cdot 4\pi \times 10^{-7})(3 \times 10^{-2})(500)(60 \cdot 2\pi)(60)}{2\pi} ln\left(\frac{0.12}{0.1}\right) \sin(60 \cdot 2\pi t) \\ &= 55.67 \sin 377t\end{aligned}$$

(1)

5. **Bonus.** Complete the self-reflection survey. [Survey Link here]

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal marks (except bonus questions).
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.