# Prac exam 2021

December 17, 2022 11:02 PM

## 1 Short Questions

*(10 points, 5 points each)*

1. **Aliasing** If we convert the pulse $x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{else} \end{cases}$ with an A/D converter it will alias. Explain why?, and make sure you compute the frequency content of this signal in your explanation.
2. **FIR Filter** What are the zeros of $H(\omega)$ for the N-point average filter for $N = 3$.

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(n-k)$$

Hint: compute the frequency response and find all zeros, how many are there ?

1) Aliasing occurs when $f_s$ is not big enough

→ Aliasing causes artifacts in the signal: unexpected outputs

$f_s < 2f_0 \longrightarrow p = 1 \quad f_0 = 1/p = 1$

$$X(\omega) = \int_0^1 e^{-i\omega t}\,dt = \frac{-1}{i\omega}\left[e^{-i\omega} - e^{0}\right]$$

$$= \frac{1 - e^{-i\omega}}{i\omega}$$

I got $f_0$ & $X(\omega)$, don't know where to go from here

2) $y(n) = \frac{1}{3}\left(x(n) + x(n-1) + x(n-2)\right)$

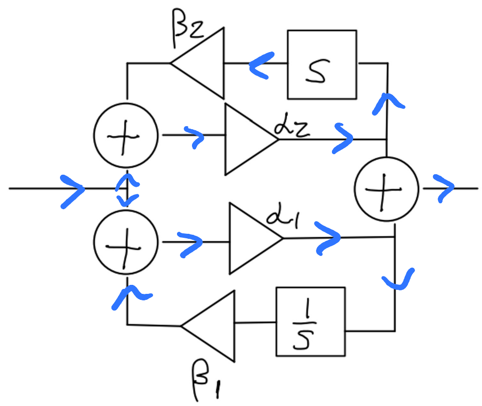
$$H(\omega)\cancel{e^{i\omega n}} = \frac{1}{3}\cancel{e^{i\omega n}} + \frac{1}{3}\cancel{e^{i\omega n}}e^{-i\omega} + \frac{1}{3}\cancel{e^{i\omega n}}e^{-i2\omega}$$

$$H(\omega) = \frac{1 + e^{-i\omega} + e^{-i2\omega}}{3}$$

Don't know where to go from here

## 2 Block Diagrams

*(20 points)* Derive the transfer function of the system described by the following block diagram:




up: $\dfrac{\alpha_2}{1 - s\beta_2\alpha_2}$ down: $\dfrac{\alpha_1}{1 - \frac{1}{s}\beta_1\alpha_1}$

$$G(s) = \frac{\alpha_2}{1 - s\beta_2\alpha_2} + \frac{\alpha_1}{1 - \frac{1}{s}\beta_1\alpha_1}$$

## 3 Stability Conditions

(*10 points*) The following questions require small computations, an explanation in English is not accepted as an answer. the answers are 2-3 step computation using the definitions.

1. Show that if $H(\omega)$ exists then the system is BIBO stable.
2. Show, for a FIR system the frequency response is always defined (meaning it is finite).

Fuck proofs

## 4 Z-Domain to State Space

*(15 points)* Given a system by its transfer function

$$G(z) = \frac{z-1}{1+z}$$

Determine the state space ($[A, B, C, D]$) representation of the system.

$Y = 1 - z^{-1}$ $Y(z^{-1} + 1) = X(1 - z^{-1})$

$Y = 1 - z^{-1}$ $\qquad Y(z^{-1}+1) = X(1-z^{-1})$

$X = z^{-1}+1$ $\qquad Yz^{-1} + Y = X - z^{-1}X$

$$y(n) = x(n) - \underbrace{x(n-1)}_{S_1} - \underbrace{y(n-1)}_{S_2}$$

$S_1(n) = x(n-1) \longrightarrow S_1(n+1) = x(n)$

$S_2(n) = y(n-1) \longrightarrow S_2(n+1) = y(n) = x(n) - S_1(n) - S_2(n)$

$$S(n+1) = \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix} S(n) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} x(n)$$

$$y(n) = \begin{pmatrix} -1 & -1 \end{pmatrix} S(n) + (1)\, x(n)$$

## 5 Step Response

*(10 points)* Compute the step output, so the output if the input is the step function as defined below, of a system with known impulse response $h(n)$.

$$step(n) = \sum_{k=0}^{\infty} \delta(n-k).$$

Hint: use linearity

$$step(n) = x(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$y(n) = \sum_{k=0}^{\infty} h(n-k)\, x(k)$$

if $k \geq 0$, $x(k) = 1$

$y(n) = \sum h(n-k)(1)$

$$\boxed{y(n) = \sum h(n-k)}$$

From tutorial

## 6 Discrete Fourier Transform

*(10 points)* We know, computing the DFT of a signal using $N = 4$ that $X_0 = 2, X_1 = 4iX_2 = 0$, the other values of $X_k$ are determined by symmetry. What was the original signal ? (give all 4 values, $x(0), \cdots, x(3)$.)

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$N = 4$

$X_0 = 2$

$X_1 = 4i$

$X_2 = 0$

$X_3 = ?$

$w_0 = \frac{2\pi}{N} = \boxed{\frac{\pi}{2}}$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i w_0 n k}$$

$$x(n) = \frac{1}{4}\left( X_0 e^{\cancel{i w_0 n (0)}} + X_1 e^{i w_0 n(1)} + X_2 e^{i w_0 n(2)} + X_3 e^{i w_0 n(3)} \right)$$

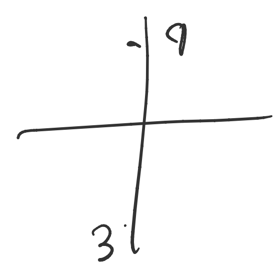
$$= \frac{1}{4}\left(2 + 4i e^{\frac{i\pi}{2} n} + X_3 e^{\frac{3i\pi}{2} n}\right)$$

$X_{-k} = X_{N-k}$ $\quad X_{-1} = \overline{X_1}$ $\quad X_{-3} = X_3$

$X_{-1} = X_{4-1}$

$X_m = \overline{X}_{N-m}$

$X_3 = \overline{X}_{4-3} = \overline{X}_1 = -4i$



$$x(n) = \frac{1}{4}\left(2 + 4i e^{\frac{i\pi n}{2}} - 4i e^{\frac{3i\pi n}{2}}\right)$$

$$x(0) = \frac{1}{4}(2 + 4i - 4i) = \frac{1}{2}$$

$$x(1) = \frac{1}{4}\left(2 + 4i(+i) + 4i(+i)\right)$$

$$= \frac{1}{4}(2 - 4 - 4)$$

$$= -3/2$$

$$x(2) = \frac{1}{4}\left(2 + 4i(-1) - 4i(-1)\right)$$

$$= \frac{1}{4}(2) = \frac{1}{2}$$

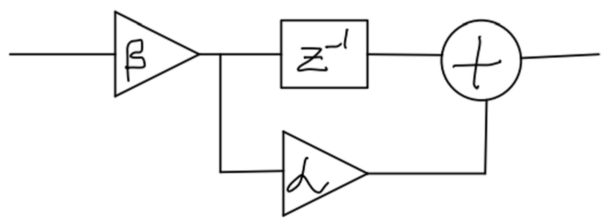
$$= \frac{1}{4}(2) = \frac{1}{2}$$

$$x(3) = \frac{1}{4}(2 + 4i(-i) - 4i(i))$$

$$= \frac{1}{4}(2 + 4 + 4) = 2$$

## 7 Filter Design

*(10 points)* Select the parameters $\alpha$ and $\beta$ such that the system given in the following diagram is a low pass filter so $H(\pi) = 0$ and a gain of $1$ at DC.




$$H(\pi) = \emptyset$$
$$H(\emptyset) = 1$$

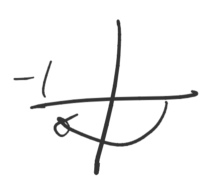
$$G = \beta(z^{-1} + \alpha)$$

$$Y/X = G = \beta z^{-1} + \alpha\beta$$

$$y(n) = X\beta z^{-1} + \alpha X\beta$$

$$y(n) = \alpha x(n) + \beta x(n-1)$$

$$\tilde{H}(\omega) = \alpha\beta + \beta e^{-i\omega}$$




$$\tilde{H}(\pi) = \alpha\beta + \beta e^{-i\pi}$$

$$\tilde{H}(\pi) = \alpha\beta + \beta(-1)$$

$$\emptyset = \alpha\beta - \beta$$

$$\alpha = 1$$

$$\tilde{h}(0) = \alpha\beta + \beta(1)$$

$$1 = \alpha\beta + \beta$$

$$1 = (1)\beta + \beta = 2\beta$$

$$\boxed{\alpha = 1} \quad \boxed{\beta = \frac{1}{2}}$$

## 8 Fourier Transform

*(15 points)*

- *(10 points)* Compute the CTFT of the signal $x(t) = 1 + \cos(t)$. You need to compute each step, the computation is key. There are at least two ways to do this.
- *(5 points)* Explain how your result is related to the Fourier series of the same signal ?

$$x(t) = 1 + \cos(t) \qquad \cos(t) = \frac{1}{2}e^{it} + \frac{1}{2}e^{-it}$$

$$X(\omega) = \int e^{-i\omega t} + \int \cos(t)e^{-i\omega t}$$

$$= \int e^{-i\omega t}dt + \frac{1}{2}\int e^{it(1-\omega)} + \frac{1}{2}\int e^{-it(\omega+1)}$$

$$= \int_{-\infty}^{\infty} \underbrace{e^{i\omega_0 t}}\,\underbrace{e^{-i\omega t}}\,dt$$

$$= \int e^{it(\omega_0-\omega)}dt$$

$$= 2\pi\delta(\omega_0 - \omega)$$

$$X(\omega) = \boxed{2\pi\delta(-\omega)} + \pi\delta(1-\omega) + \pi\delta(1+\omega)$$

?: Is this allowed?