ENGPHYS 2A04 Assignment 9 Solutions

1. **Magnetic Forces and Torques**

The acceleration vector of a free particle is the net force vector divided by the particle mass. Neglecting gravity, use equation shown:

$$a = \frac{F_m}{m_e} = \frac{q\mathbf{u}\times\mathbf{B}}{m_e}$$

Using values:

Electron speed: u = $4 * 10^6\ m/s$

Elementary charge: $e = 1.6 * 10^{19}\ C$

Electron mass: $m_e = 9.1 * 10^{-31}\ kg$

Assuming q = -e.

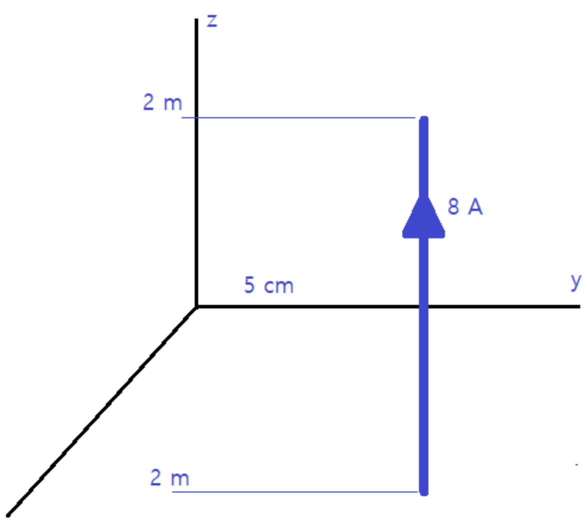
$$a = \frac{F_m}{m_e} = \frac{q\mathbf{u}\times\mathbf{B}}{m_e} = \frac{-1.6 * 10^{19}}{9.1 * 10^{-31}}(\hat{\boldsymbol{x}}7 - \hat{\boldsymbol{z}}4)$$

$$\overline{a}\times\overline{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4000000 & 0 & 0 \\ 7 & 0 & -4 \end{vmatrix} = \boldsymbol{i}(0\cdot(-4) - 0\cdot 0) - \boldsymbol{j}(4000000\cdot(-4) - 0\cdot 7) + \boldsymbol{k}(4000000\cdot 0 - 0\cdot 7) = \boldsymbol{i}(0 - 0) - \boldsymbol{j}(-16000000 - 0) + \boldsymbol{k}(0 - 0) = \{0; 16000000; 0\}$$

$$= \frac{-1.6 * 10^{19}}{9.1 * 10^{-31}} * (\hat{\boldsymbol{y}}1.6 * 10\text{^}7)$$

$$= -\hat{\boldsymbol{y}}2.81 * 10^{18}\ m/s^2$$

2. **Magnetic Forces and Torques**



a. Magnetic force:

$$\mathbf{F} = I\boldsymbol{l} \times \boldsymbol{B}$$
$$= 8\hat{\boldsymbol{z}}4 \times [\hat{\boldsymbol{r}}0.3\cos\phi$$
$$= \widehat{\boldsymbol{\phi}}9.6\cos\phi$$

At $\phi = \frac{\pi}{2}$, $\widehat{\boldsymbol{\phi}} = -\hat{\boldsymbol{x}}$, Hence,

$$\mathbf{F} = -\hat{\mathbf{x}}\mathbf{9.6}\cos\left(\frac{\pi}{2}\right) = 0 \quad \text{T}$$

b. Work:

$$W = \int_{\phi=0}^{2\pi} \mathbf{F} \cdot d\mathbf{l} = \int_{0}^{2\pi} \widehat{\boldsymbol{\phi}}[2\cos\phi] \cdot (-\widehat{\boldsymbol{\phi}}) r\, d\phi \,|_{r=5\ cm}$$
$$= -2r\int_{0}^{2\pi} \cos\phi\, d\phi \,|_{r=5\ cm}$$
$$= -10 \times 10^{-2}\, [\sin\ \phi]_{0}^{2\pi}$$
$$= 0$$

The force is in the $+\widehat{\boldsymbol{\phi}}$ direction, which means that rotating it in the $-\widehat{\boldsymbol{\phi}}$ direction would require work. However, force varies as $\cos\phi$ which means it is positive when $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$ and negative over the second half of the circle.

Therefore work is provided by force between $\phi = \frac{\pi}{2}$ and $\phi = \frac{-\pi}{2}$ (when rotated in the $-\widehat{\boldsymbol{\phi}}$ direction), and work is supplied for second half of rotation, resulting in net work of zero.

c. Force maximum

Force must be maximum when $\cos\phi = 1$, or $\phi = 0$.

3. **Biot-Savart Law**

The magnetic flux density at center of loop due to the wire is

$$\mathbf{B_1} = \hat{\mathbf{z}}\frac{\mu_0 N I_2}{2\pi d} I_1$$

The field due to I_2 is

$$\mathbf{B} = \mu_0\mathbf{H} = -\hat{\mathbf{z}}\frac{\mu_0 N I_2}{2r}$$

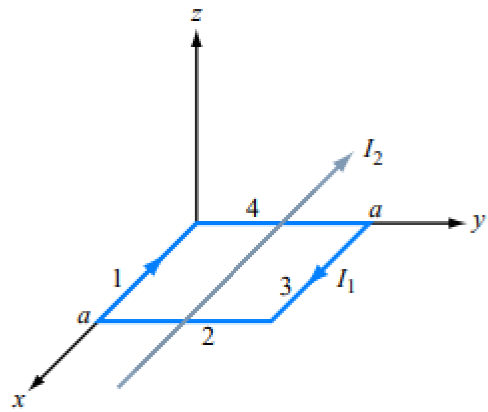
Equating the two

$$\frac{NI_2}{2r} = \frac{I_1}{2\pi d}$$

$$I_2 = \frac{rI_1}{2\pi N d} = \frac{2 * 30}{\pi * 40 * 2} = 0.239 \text{ A}$$

Counterclockwise direction to oppose current.

4. **Biot-Savart Law**



Treat I_2 in the same plane as shown loop.

For segment (as labelled above), I_1 and I_2 are in the same direction (force on side 1 is attractive).

$$\mathbf{F_1} = \frac{\hat{\mathbf{y}}(\mu_0 I_1 I_2 a)}{2\pi\left(\frac{a}{2}\right)} = \hat{\mathbf{y}}\frac{4\pi * 10^{-7} * 7 * 15 * 4}{2\pi * 2} = \hat{\mathbf{y}}\, 4.2 * 10^{-5}\ \mathrm{N}$$

I_1 and I_2 are in opposite directions for side 3. The force on side 3 is repulsive (also along $\hat{\mathbf{y}}$). $\mathbf{F_3} = \mathbf{F_1}$.

The net forces on sides 2 and 4 are zero. Total force is.

$$2\mathbf{F_1} = \hat{\mathbf{y}} 8.4 * 10^{-5}\ \ \mathrm{N}$$

5. **Bonus:**

- Ampere's Law
  - Used to calculate magnetic field and used in magnetism
  - Measure magnetic field due to current (line)
- Gauss's Law
  - used to calculate electric field and used in electrostatics
  - electric field by certain charge configuration (surface)