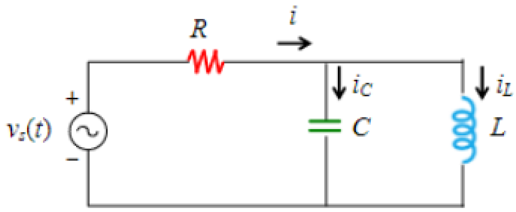
ENGPHYS 2A04 Assignment 3 Solutions

1. **AC Circuit Analysis**

The circuit shown below has the voltage source given by the equation:

$$V_s(t) = 7\cos(2 * 10^4 t - 60°)$$

If the circuit has the following values of R = 50 Ω, C = 12 μF, and L = 0.6 mH, use phasor analysis to acquire an expression for the current flowing through the inductor $i_L(t)$.



Voltage to Phasor:

$$V_s(t) = 7\cos(2 * 10^4 t - 60°)$$

$$\tilde{V}_s = 7e^{-60°j}$$

$$\omega = 2 * 10^4$$

Converting circuit elements to phasor:

$$Z_R = R = 50\ \Omega$$

$$Z_c = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -\frac{j}{(2 * 10^4)(12 * 10^{-6})} = -\frac{j}{0.24} = -4.17j\ \Omega = 4.17e^{-90°}\ \Omega$$

$$Z_L = j\omega L = j(2 * 10^4)(0.6 * 10^{-3}) = 12j\ \Omega = 12e^{90°}\ \Omega$$

Applying Kirchoff's Laws:

Junction Rule: $I = I_c + I_L$

Loop 1 (left):

$$\tilde{V}_s = IR + I_c Z_c$$

$$\tilde{V}_s = IR + (I - I_L)Z_c$$

$$\tilde{V}_s = I(R + Z_c) - I_L Z_c$$

Rearrange for current:

$$I = \frac{\tilde{V}_s + I_L Z_c}{R + Z_c}$$

Loop 2 (Right):

$$\tilde{V}_s = IR + I_L Z_L$$

Substituting *I* from previous equation:

$$\tilde{V}_s = \frac{\tilde{V}_s + I_L Z_c}{R + Z_c} R + I_L Z_L$$

$$\tilde{V}_s(R + Z_c) = \tilde{V}_s R + I_L R Z_c + I_L Z_L(R + Z_c)$$

$$\tilde{V}_s R + \tilde{V}_s Z_c = \tilde{V}_s R + I_L R Z_c + I_L R Z_L + I_L Z_L Z_c$$

$$I_L = \tilde{V}_s \frac{Z_c}{R Z_L + R Z_c + Z_L Z_c}$$

Substituting all values:

$$I_L = \frac{7e^{-60°j} 4.17e^{-90°}}{50(12j) + 50(-4.17j) + 12j(-4.17j)} = \frac{7e^{-60°j} 4.17e^{-90°}}{50.04 + 391.5j} = \frac{29.19e^{-150°}}{395e^{82.7°j}} = \frac{29.19e^{210°j}}{395e^{82.7°j}}$$

$$I_L = 0.07e^{127°j}\ \text{A} = 0.07e^{-233°j}\ \text{A}$$

Converting back to time domain:

$$i_L(t) = 0.07\cos(2 * 10^4\, t + 127°)\ \text{A}$$

$$i_L(t) = 0.07\cos(2 * 10^4\, t - 233°)\ \text{A}$$

2. **Transmission Lines**

A transmission line of length *l* is connected to a sinusoidal voltage source with a certain frequency *f*. If we assume the velocity of the wave propagates at a certain velocity *c*, in which of the following conditions shown below is it reasonable to ignore the transmission line effects within the solution of the circuit? In addition, explain the difference between a dispersive and non-dispersive transmission line.

(a) *l* = 5.3 cm, *f* = 37 kHz,
(b) *l* = 8.2 km, *f* = 475 Hz,
(c) *l* = 7.7 cm, *f* = *770* MHz,
(d) *l* = 2.3 mm, *f* =98 GHz

Using equation

$$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{lf}{c} = \frac{lf}{3 * 10^8 \frac{m}{s}}$$

If $\frac{l}{\lambda} \leq 0.01$, then it is negligible.

a) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{5.3*10^{-2}\ m*37*10^3\ Hz}{3*10^8\ m/s} = 6.54 * 10^{-6}$

b) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{8.2*10^3\ m*475\ Hz}{3*10^8\ m/s} = 0.013$

c) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{7.7*10^{-2}\ m*770*10^6\ Hz}{3*10^8\ m/s} = 0.198$

d) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{2.3*10^{-3}\ m*98*10^9\ Hz}{3*10^8\ m/s} = 0.751$

Therefore, situation A would be the only situation negligible since it is the only one less than 0.01.

The difference between a dispersive and non-dispersive transmission line.

- Dispersive transmission line is one which the wave velocity is not constant as a function of the oscillating frequency f. There is distortion of the wave's shape due to different frequency components of the wave propagating at different velocities along the transmission line (degree of distortion depends on length)
- On the other hand, non-dispersive transmission lines have no distortion of the wave's shape due

3. **Transmission Lines**

A two-wire gold transmission line is embedded in an unknown dielectric material that has the following parameters: $\varepsilon_r = 2.7$ and $\sigma = 3.2 * 10^{-6}$ S/m. The wires are separate by a width of 1 cm and their radii are 1 mm each. Calculate the line parameters *R', L', G',* and *C'* at 1 GHz. (Refer to Appendix B for μ_c and σ_c of gold) Assume that $\mu_c = \mu_0$ for the dielectric material.

Values:

$$f = 1 * 10^9\ Hz$$

$$d = 2 * 10^{-3}\ m$$

$$D = 1 * 10^{-2}\ m$$

$$\varepsilon_r = 2.7$$

$$\sigma = 3.2 * 10^{-6}\ S/m$$

$$\varepsilon_0 = 8.854 * 10^{-12}\ F/m$$

For Silver:

$$\sigma_c = 4.1 * 10^7\ S/m$$

$$\mu_c = \mu_0 = 4\pi * 10^{-7}\ \text{H/m}$$

Solution:

$$R' = \frac{2}{\pi d}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{\pi * 2 * 10^{-3}}\sqrt{\frac{\pi * 1 * 10^9 * 4\pi * 10^{-7}}{4.1 * 10^7}} = 3.12\ \Omega/m$$

Using:

$$\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1} = \ln\left[\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right) + \sqrt{\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right)^2 - 1} = 2.292\right.\right.$$

$$L' = \frac{\mu}{\pi}\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1} = \frac{4\pi * 10^{-7}}{\pi}\ln\left[\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right) + \sqrt{\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right)^2 - 1}\right.\right.$$

$$L' = 9.168 * 10^{-7}\ H/m$$

$$G' = \frac{\pi\sigma}{\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right.} = \frac{\pi * 3.2 * 10^{-6}}{\ln\left[\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right) + \sqrt{\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right)^2 - 1}\right.}$$

$$G' = 4.39 * 10^{-6}\ S/m$$

$$C' = \frac{\pi\epsilon}{\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right.} = \frac{\pi\varepsilon_0\varepsilon_r}{\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right.} = \frac{\pi * 8.854 * 10^{-12} * 2.7}{2.292}$$

$$C' = 3.28 * 10^{-11}\ F/m$$

4. **Bonus Question: Reading/Research Question:**

The first transatlantic transmission line had been destroyed because high voltages had been employed to compensate for a poor, distorted signal. Briefly explain how Oliver Heaviside's work and the Heaviside condition resolved this issue, and what had been changed physically to the earlier transmission lines to improve signal quality.

- Heaviside condition is satisfied when $\frac{G}{C} = \frac{R}{L}$
  - Where G is shunt conductance
  - C is capacitance
  - R is series resistance
  - L is inductance

- Series resistance and shunt cause loss in the line
- For an ideal transmission, R = G =0
- Heaviside's findings aided in coming to solution for early transmission lines since adjusting the values of these variables in this condition could allow the engineers to come close to satisfying the condition and therefore having no loss in the signal
- Different options for changing values
    - G could be increased → but would increase loss
    - decreasing R and C would make cable bulky
    - Therefore the alternative solution of adding loading coils (load to transmission lines)
        - This would decrease voltage travelling across transmission lines
        - Decrease in voltage would be achieved by increasing inductance → leading to no signal distortion