4HO3 Test 2 solution 2020

10 **1. a)** A load is to be moved at 2 m/s using a rack and pinion. If the motor speed is 1200 rpm, determine the pinion's required pitch radius.

Answer to a):

$$l = (2\pi/rev)\, r_p$$

$$v = l\omega$$

$$v = (2\pi/rev)\, r_p\, \omega$$

$$\therefore r_p = \frac{v}{(2\pi/rev)\,\omega} = \frac{2\ m/s}{(2\pi/rev)\left(1200\ \frac{rev}{min}\right)\left(\frac{1\ min}{60\ s}\right)} = 0.0159\ m$$

b) For the same rack and pinion, if a force of 400 N is required to move the load, and the rack and pinion's efficiency is 0.8, determine the required motor torque.

Answer to b):

$$F_{out} = \frac{\tau_{in}}{r_p}\eta_r$$

$$\therefore \tau_{in} = \frac{F_{out}\, r_p}{\eta_{rp}} = \frac{(400\ N)(0.0159\ m)}{0.8} = 7.95\ Nm$$

c) Now, the load is to be moved at 0.5 m/s using a lead screw. If the screw has a lead of 0.005 m/rev, determine the required motor speed in rpm.

Answer to c):

$$v = l\omega$$

$$\therefore \omega = \frac{v}{l} = \frac{(0.5\ m/s)\left(\frac{60\ s}{1\ min}\right)}{0.005\ m/rev} = 6000\ rpm$$

d) For the same lead screw, if a force of 400 N is required to move the load and the screw's efficiency is 0.6, determine the required motor torque.

Answer to d):

$$\tau = \frac{Fl}{(2\pi/rev)\,\eta_s} = \frac{(400\ N)(0.005\ m/rev)}{(2\pi/rev)\ 0.6} = 0.53\ Nm$$

9 **2.** A single rod pneumatic cylinder will be used to move a mass horizontally in both directions at a speed of 0.2 m/s. The cylinder must overcome a friction force of 2000 N during its motion. The inertia force is relatively small and can be neglected. The rod's cross-sectional area is 9×10^{-4} m^2, the supply pressure is 6×10^5 Pa gauge and the air temperature is 25 °C. If a pressure drop of 4×10^4 Pa across the valve is desired, determine:

a) The minimum bore cross-sectional area required.

b) The minimum valve flow coefficient required.

Assume that the pressure drop across the valve is the same for the return flow as for the intake flow and that the air is returned to the atmosphere.

a) Use retract dir'n to get min. bore area with single rod cylinder

$$F_{ret} = P_{ret} A_{ret} - P_{ext} A_{ext}$$

0 gauge

$$F_{ret} = (P_{supply} - \Delta P)(A_{bore} - A_{rod}) - (\cancel{P_{Atm}} + \Delta P) A_{bore}$$

$$\therefore A_{bore} = \frac{F_{ret} + (P_{supply} - \Delta P) A_{rod}}{P_{supply} - 2\Delta P}$$

$$= \frac{2000\,N + (6 \times 10^5\,Pa - 4 \times 10^4\,Pa)(9 \times 10^{-4}\,m^2)}{6 \times 10^5\,Pa - 2(4 \times 10^4\,Pa)}$$

$$= 0.0048\ m^2$$

b) Use extend dir'n to get min C_v with single rod cylinder

$$\rho = \frac{P_1 - \Delta P}{R_g T} = \frac{(6 \times 10^5\,Pa + 1.01 \times 10^5\,Pa) - 4 \times 10^4\,Pa}{(287\,J/kgK)(25 + 273)K}$$

$$= 7.73\ kg/m^3$$

$$Q_{max} = V_{max} A_{bore} = (0.2\,m/s)(0.0048\,m^2) = 9.63 \times 10^{-4}\,m^3/s$$

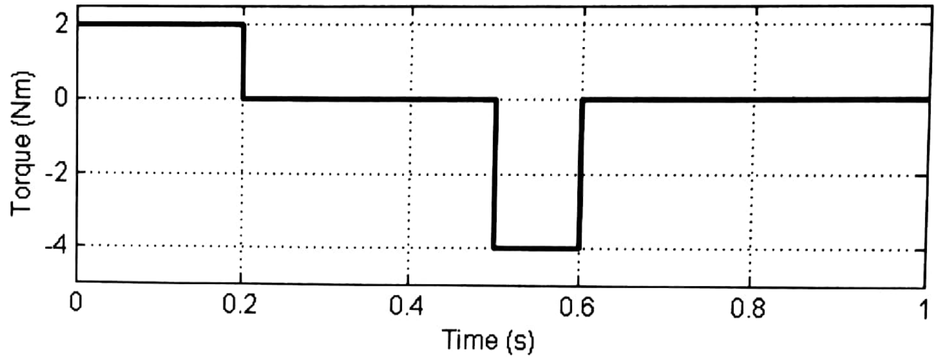
$$C_v = (4.22 \times 10^4\,m^{-2})\, Q_{max} \sqrt{\frac{\rho}{\Delta P}}$$

$$= (4.22 \times 10^4\,m^{-2})(9.63 \times 10^{-4}\,m^3/s)\sqrt{\frac{7.73\,kg/m^3}{4 \times 10^4\,Pa}}$$

$$= 0.56$$

9 **3.** A motion cycle requires a motor to produce the torque profile shown below. The ambient temperature is 30 °C and the motor's parameters are given in the table. Determine the temperature the motor windings will reach if the motion cycle is repeated continuously.

Torque constant (Nm/A)	0.4
Resistance at max. temp. (ohm)	1.2
Total thermal resistance (°C/W)	2



$$\tau_{RMS} = \sqrt{\sum_{i=1}^{n} \tau^2 t_i \Big/ \sum_{i=1}^{n} t_i}$$

$$= \sqrt{\frac{(2\ Nm)^2 (0.2s) + (-4\ Nm)^2 (0.1s)}{0.2s + 0.3s + 0.1s + 0.4s}} = 1.55\ Nm$$

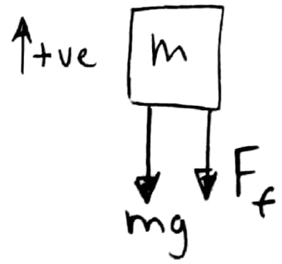
$$I_{RMS} = \frac{\tau_{RMS}}{K_t} = \frac{1.55\ Nm}{0.4\ Nm/A} = 3.88\ A$$

$$P_J = I_{RMS}^2 R_{Hot} = (3.88\ A)^2 (1.2\ ohm) = 18\ W$$

$$T_W = T_a + P_J R_{th}$$
$$= 30°C + (18\ W)(2°C/W)$$
$$= 66\ °C$$

Name:______________________________ Student #:______________

6 **4.** A linear actuator consisting of a motor, gearbox and ball screw moves a load vertically. A positive motor velocity moves the load upwards. The screw's moment of inertia is 1.5×10^{-4} kgm^2, the motor's moment of inertia is 3×10^{-5} kgm^2 and the gear ratio is 2.5. The screw's lead is 0.002 m/rev. If the load has a mass of 500 kg and is subject to a 800 N friction force, **determine the motor torque required for the load to accelerate upwards at 1.2 m/s^2**. The friction of the gears and ball screw can be neglected.



$$F_{ext} = mg + F_f$$
$$= (500\ \text{kg})(9.81\ \text{N/kg}) + 800\ \text{N}$$
$$= 5705\ \text{N}$$

$$\tau_{ext} = \frac{F_{ext}\,\ell}{2\pi/\text{rev}} = \frac{(5705\ \text{N})(0.002\ \text{m/rev})}{2\pi/\text{rev}} = 1.82\ \text{Nm}$$

$$J_{load} = m\left(\frac{\ell}{2\pi}\right)^2 + J_{screw}$$
$$= (500\ \text{kg})\left(\frac{0.002\ \text{m/rev}}{2\pi/\text{rev}}\right)^2 + 1.5\times10^{-4}\ \text{kgm}^2 = 2.01\times10^{-4}\ \text{kgm}^2$$

$$\dot{\omega}_{load} = \frac{a}{\ell} = \frac{1.2\ \text{m/s}^2}{0.002\ \text{m/rev}} = (600\ \text{rev/s}^2)\left(\frac{2\pi}{\text{rev}}\right) = 3.77\times10^3\ \frac{\text{rad}}{\text{s}^2}$$

$$\dot{\omega}_{motor} = N_r\dot{\omega}_{load} = (2.5)(3.77\times10^3\ \text{rad/s}^2) = 9.42\times10^3\ \text{rad/s}^2$$

$$\tau_{motor} = \left(J_{motor} + \frac{1}{N_r^2}J_{load}\right)\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{ext}$$
$$= \left(3\times10^{-5}\ \text{kgm}^2 + \left(\frac{1}{2.5^2}\right)(2.01\times10^{-4}\ \text{kgm}^2)\right)9.42\times10^3\ \text{rad/s}^2$$
$$+ \left(\frac{1}{2.5}\right)(1.82\ \text{Nm})$$

$$= 1.3\ \text{Nm}$$