Force

Gravtional force $F = \frac{G*m_1*m_2}{R^2}$ (N)

Gravtional field = $\frac{G*m_1}{R^2} * -\widehat{R}$ (N/kg)

Electrical force $F = \frac{q_1*q_2}{4*\pi*\varepsilon*R^2} * \hat{R} = E * q$ (N)

Electrical field $E = \frac{q_1}{4*\pi*\varepsilon*R^2}\hat{R}$ (V/m)

$\varepsilon = 8.854 * 10^{-12}$

$\mu = 4 * \pi * 10^{-7}\ H/m$

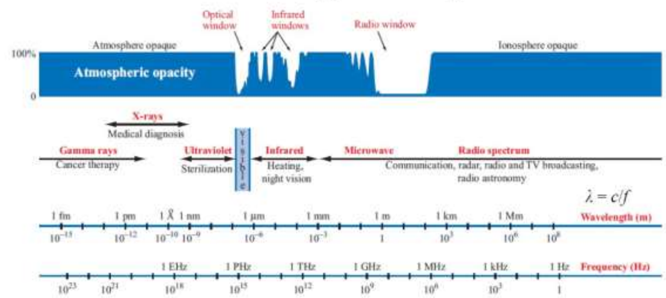
Waves

$y(x,t) = A * \cos\left(\frac{2*\pi*t}{T} - \frac{2*\pi*x}{\lambda} + \phi_0\right) \rightarrow y(x,t) = A * \cos(wt - \beta x)$

<Opposite sign positve diection> <Same sign negative direction>

$Phase\ velocity = u = \frac{\lambda}{T} = \lambda * f = \frac{w}{\beta}$

Lossy media $y(x,t) = A * e^{-\alpha x} * \cos(wt - \beta x + \phi)$



AC/DC circuit

Real battery terminal voltage $\Delta V = V_{emf} - I * r$

Kirchhoff current node law and voltage loop rule

Power $I * \Delta V = I * V_{emf} = I^2 * R = \frac{V^2}{R}$ (W)

$Average\ power = I_{rms}^2 * R$

$\sin(\theta) = \cos(90 - \theta) = \cos(\theta - 90)$ $\cos(-\theta) = \cos(\theta)$

AC voltage $\Delta V = V_{max} * \sin(wt)$ $w = 2\pi f$ $V_{rms} = \frac{V_{max}}{\sqrt{2}}$

Resistor in AC: $i = = \frac{V_{max}}{R} * \sin(wt)$ no phase difference $I_{max} = \frac{V_{max}}{R}$ $I_{rms} = \frac{I_{max}}{\sqrt{2}}$

Capacitor in AC: $I_C = w * C * \Delta V_{max} * \sin\left(wt + \frac{\pi}{2}\right)$ Current leads voltage by 90º $I_{max} = w * C * V_{max}$ $X_C = \frac{1}{w*C*j} = \frac{V_{max}}{I_{max}}$

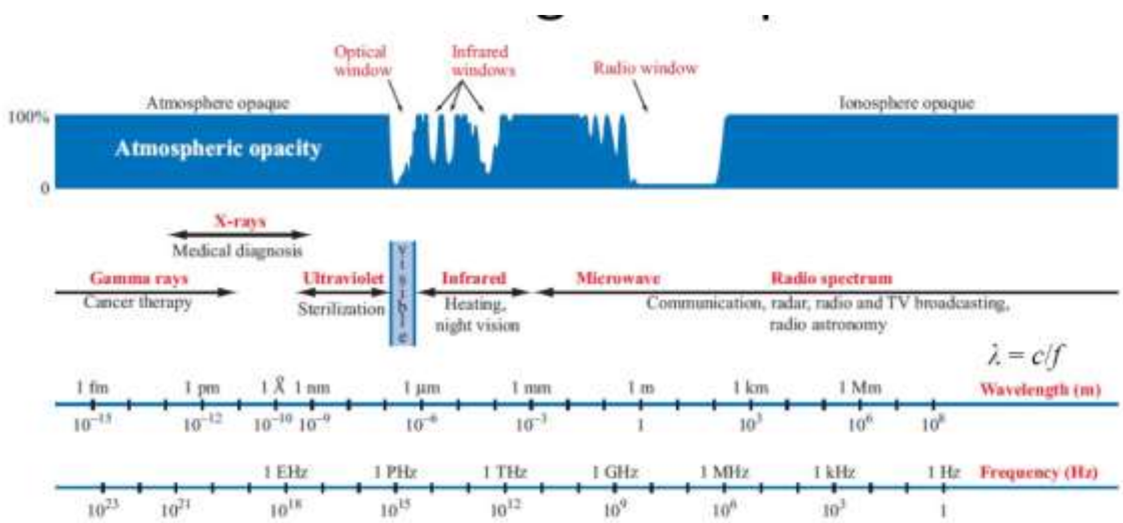
Inductor in AC: $I_L = \frac{V_{max}}{w*L} * \sin\left(wt - \frac{\pi}{2}\right)$ Current lags voltage by 90º $X_L = w * L * j$

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$	
$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$	$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
$\mathbf{z} = x + jy = \lvert\mathbf{z}\rvert e^{j\theta}$	$\mathbf{z}^* = x - jy = \lvert\mathbf{z}\rvert e^{-j\theta}$
$x = \Re\mathfrak{e}(\mathbf{z}) = \lvert\mathbf{z}\rvert\cos\theta$	$\lvert\mathbf{z}\rvert = \sqrt[+]{\mathbf{z}\mathbf{z}^*} = \sqrt[+]{x^2 + y^2}$
$y = \Im\mathfrak{m}(\mathbf{z}) = \lvert\mathbf{z}\rvert\sin\theta$	$\theta = \tan^{-1}(y/x)$
$\mathbf{z}^n = \lvert\mathbf{z}\rvert^n e^{jn\theta}$	$\mathbf{z}^{1/2} = \pm\lvert\mathbf{z}\rvert^{1/2} e^{j\theta/2}$
$\mathbf{z}_1 = x_1 + jy_1$	$\mathbf{z}_2 = x_2 + jy_2$
$\mathbf{z}_1 = \mathbf{z}_2$ iff $x_1 = x_2$ and $y_1 = y_2$	$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$
$\mathbf{z}_1\mathbf{z}_2 = \lvert\mathbf{z}_1\rvert\lvert\mathbf{z}_2\rvert e^{j(\theta_1+\theta_2)}$	$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{\lvert\mathbf{z}_1\rvert}{\lvert\mathbf{z}_2\rvert} e^{j(\theta_1-\theta_2)}$
$-1 = e^{j\pi} = e^{-j\pi} = 1\angle\pm 180°$	
$j = e^{j\pi/2} = 1\angle 90°$	$-j = e^{-j\pi/2} = 1\angle -90°$
$\sqrt{j} = \pm e^{j\pi/4} = \pm\frac{(1+j)}{\sqrt{2}}$	$\sqrt{-j} = \pm e^{-j\pi/4} = \pm\frac{(1-j)}{\sqrt{2}}$

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x + \phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x + \phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0 - \pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\widetilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0 - \pi/2)}$

RLC circuit maximum frequency $w = \frac{1}{\sqrt{L*C}}$

Attenuating wave: $Ae^{-\alpha x}\cos(wt - \beta x + \phi_0) \rightarrow A * e^{-\alpha x} * e^{j(-\beta x + \phi_0)}$



Transmission lines

Lines affects $\phi = \frac{2\pi f * l}{c} = \frac{2\pi l}{\lambda}$ $\lambda = \frac{c}{f}$ $R_s = \sqrt{\frac{\pi * f * \mu}{\sigma}} \rightarrow surface\ resistance\ of\ conductor$

- When $\frac{l}{\lambda}$ is very small ignore effects $c = 3 * 10^8\ m/s$
- When $\frac{l}{\lambda} > 0.01$, need to account for phase delay and possibly reflection
- When $\frac{l}{\lambda} > 0..25$, definitely need to account for phase delay and possibly reflection

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

$L' * C' = \mu\varepsilon$

$\frac{G'}{C'} = \frac{\sigma}{\varepsilon}$

Air line: $\varepsilon = \varepsilon_0 = 8.854 * \frac{10^{-12} F}{m}$

$\mu = \mu_0 = 4 * \pi * 10^{-7}$

$\sigma = 0$

$G' = 0$

Dispersion →Distorts signals because different frequency components → Proportional to the length of the transmission line

Telegraphers equations (time domain):

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

Telegraphers equations (phasor domain):

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

Derive the *wave equations* by separating variables

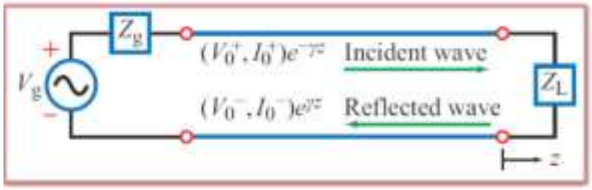
$$\frac{d^2\tilde{V}}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

$$Complex\ prepagation\ constant = \gamma = \sqrt{(R' + j * w * L')(G' + j * w * C')} = \alpha + \beta j$$

$Attenuation\ constant = \alpha = Re(\gamma)\ \ Np/m$ $\qquad$ $Phase\ constant = \beta = Im(\gamma)\quad rad/m$

$Characteristic\ Impedance = Z_0 = \frac{R'+jwL\prime}{\gamma} = \frac{\sqrt{R'+jwL\prime}}{\sqrt{G'+jwC\prime}} = \frac{V_0^+}{I_0^+} = \frac{V_0^-}{I_0^-}$ $\qquad$ $Phase\ velocity = u_p = \frac{w}{\beta} = f * \lambda$



$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{-\gamma z}$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}$$

Lossless transmission line

R' and G' are negligible → $\alpha = 0$ $\qquad$ $\beta = w * \sqrt{L' * C'} = \frac{w*\sqrt{\varepsilon_r}}{c}$ $\qquad$ $Phase\ velocity = u_p = \frac{c}{\sqrt{\varepsilon_r}}$ $\qquad$ $Z_0 = \sqrt{\frac{L'}{C'}}$

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})\,(h/w)$

$$\lambda = \frac{c}{f * \sqrt{\varepsilon_r}}$$

$$Normalized\ load\ impedance = z_L = \frac{Z_L}{Z_0}$$

$$voltage\ reflection\ coefficient = \Gamma = \frac{z_L - 1}{z_L + 1} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^+}{V_0^-} = -\frac{I_0^+}{I_0^-} = |\Gamma| * e^{j\theta_r}$$

$$Wave\ impdeance = Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)} \rightarrow ratio\ of\ total\ voltage\ to\ total\ current$$

$Wave\ impedance\ Z(d) = Z_0 * \left[\frac{1+\Gamma_d}{1-\Gamma_d}\right], \Gamma_d = \Gamma * e^{-j*2\beta d}$ $\qquad$ $Input\ impedance = Z_{in} = Z_0 * [\frac{z_L + j*\tan(\beta l)}{1+j*z_L*\tan(\beta l)}]$

$$Forward\ voltage = V_0^+ = \left(\frac{\tilde{V}_g * Z_{in}}{Z_g + Z_{in}}\right) * \left[\frac{1}{e^{j\beta l} + \Gamma * e^{-j\beta l}}\right] \rightarrow this\ part\ may\ not\ be\ used$$

Full final equation for phasor voltage and phasor current on the lossless line:

$$\tilde{V}(z) = |V_0^+|e^{j\phi^+}[e^{-j\beta z} + |\Gamma|e^{j\theta_r}e^{j\beta z}]$$

$$\tilde{I}(z) = \frac{|V_0^+|e^{j\phi^+}}{|Z_0|e^{j\phi_z}}[e^{-j\beta z} - |\Gamma|e^{j\theta_r}e^{j\beta z}]$$

phasor solutions

Full final equation for instantaneous voltage and current on the line:

$$v(z) = \mathrm{Re}\{\tilde{V}(z)e^{j\omega t}\} = |V_0^+|\{[\cos(\omega t - \beta z + \phi^+)] + |\Gamma|[\cos(\omega t + \beta z + \theta_r + \phi^-)]\}$$

$$i(z) = \mathrm{Re}\{\tilde{I}(z)e^{j\omega t}\} = \frac{|V_0^+|}{|Z_0|}\{\cos(\omega t - \beta z + \phi^+ - \phi_z) + |\Gamma|\cos(\omega t + \beta z + \phi^- - \phi_z)\}$$

time domain solutions

$$\boxed{V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left[\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right]}$$

Vector analysis

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z$ co $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z$ si $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi$ si $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi$ co $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\|\mathbf{A}\| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

- Cylindrical <r, ϕ, z>
- Spherical <R, θ, ϕ>
- Gradient (∇)
  - $\nabla = \frac{\partial}{\partial x}\,\hat{x} + \frac{\partial}{\partial y}\,\hat{y} + \frac{\partial}{\partial z}\,\hat{z}$
- Divergence ($\nabla \cdot \vec{A}$)
  - $\nabla \cdot \vec{A} = div\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
- Curl ($\nabla \times \vec{A}$)
  - $\nabla \times \vec{A} = curl\ \vec{A} = (\frac{\partial}{\partial x}\,\hat{x} + \frac{\partial}{\partial y}\,\hat{y} + \frac{\partial}{\partial z}\,\hat{z}) \times (Ax\hat{x} + Ay\hat{y} + Az\hat{z})$
- Stroke's theorem
  - $\int \nabla \times \vec{B} \cdot ds = \oint \vec{B} \cdot \vec{dl}$
- Divergence theorem
  - $\int_V \nabla \cdot \vec{E} dV = \int_S \vec{E} \cdot \vec{ds}$

Gradient of cylindrical coordinate → $\frac{\partial A}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial A}{\partial \phi}\hat{\phi} + \frac{\partial A}{\partial z}\,\hat{z}$

Gradient of Spherical coordinate → $\frac{\partial A}{\partial R}\,\hat{R} + \frac{1}{R} * \frac{\partial A}{\partial \theta}\,\hat{\theta} + \frac{1}{Rsin\theta}\frac{\partial A}{\partial \phi}\,\hat{\phi}$

Coulomb's Law (Find electric field given charge)

$\vec{E}(R) = \frac{q}{4\pi\varepsilon * R^2}\,\hat{R} = \frac{q}{4\pi\varepsilon * |R|^3}\,\vec{R}\ \ (V/m) \rightarrow$
$electric\ field\ at\ point\ P\ due\ to\ single\ charge$

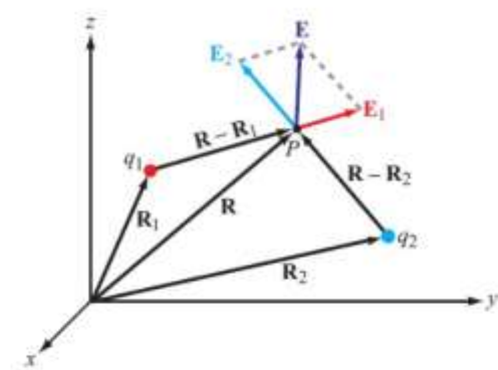
$\vec{F} = q' * \vec{E}\ (N) \rightarrow electric\ force\ on\ a\ charge\ placed\ at\ P$

$$\vec{E}(\vec{R}) = \frac{1}{4\pi\varepsilon}\sum_{i=0}^{N}\frac{q_i * (\vec{R} - \vec{R_i})}{|\vec{R} - \vec{R_i}|^3}\ \ (V/m)$$

$$Q = \int_{v'} \rho_v(\vec{r'})dV' \rightarrow total\ charge\ in\ a\ volume, \rho_v\ is\ charge\ density$$

$$\vec{E}(\vec{R}) = \int_{v'}\frac{\rho_v(\vec{R'})(\vec{R} - \vec{R'})}{4\pi\varepsilon\,|\vec{R} - \vec{R'}|^3}\,dV' \rightarrow field\ at\ point\ P$$

Infinite Plane (Disk) of charge $\vec{E} = \pm\,\hat{z}\frac{\rho_v}{2\varepsilon}$ Infinite line of charge $E = \frac{\rho_v}{2\pi\varepsilon r}\,\hat{r}$



$$\vec{E}(\vec{R}) = \int_{v'}\frac{\rho_v(\vec{R'})(\vec{R} - \vec{R'})}{4\pi\varepsilon|\vec{R} - \vec{R'}|^3}dV'$$
In a volume

$$\vec{E}(\vec{R}) = \int_{s'}\frac{\rho_s(\vec{R'})(\vec{R} - \vec{R'})}{4\pi\varepsilon|\vec{R} - \vec{R'}|^3}ds'$$
Over a surface

$$\vec{E}(\vec{R}) = \int_{l'}\frac{\rho_l(\vec{R'})(\vec{R} - \vec{R'})}{4\pi\varepsilon|\vec{R} - \vec{R'}|^3}dl'$$
On a line

Gauss's Law (Find charge given a field)

$\vec{D} = \varepsilon\vec{E}\ (C/m^2)$ $\qquad \varepsilon = \varepsilon_r * 8.854 * 10^{-12}$ $\qquad \vec{D} = \hat{R}\frac{q}{4*\pi*R^2}$

$\oint_S \vec{D}\cdot ds' = q = \int_{v'} \rho_v\, dV' \rightarrow determine\ electric\ flux\ density\ D$ $\qquad \nabla\cdot\vec{D} = \rho_v(x,y,z) \rightarrow differential\ form$

Electric Potential

$V = -\int_{l'} \vec{E}\cdot\widehat{dl'}$ $\qquad V_{21} = V_2 - V_1 = -\int_{P1}^{P2}\vec{E}\cdot\widehat{dl'} \rightarrow potential\ difference\ between\ P1\ and\ P2$

$\oint \vec{E}\cdot\widehat{dl'} = 0 \rightarrow for\ any\ closed\ path$ $\qquad V = -\int_{\infty}^{P}\vec{E}\cdot\widehat{dl'} \rightarrow zero\ reference\ at\ infinity\ (free\ space\ and\ material\ media)$

$V\ (\overrightarrow{R}) = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i}{|\vec{R} - \vec{R_i}|}$ $\qquad \vec{E} = \frac{V}{L}$ $\qquad \vec{E} = -\nabla V$

$V(\vec{R}) = \int_{v'}\frac{\rho_v(\vec{R}')}{4\pi\varepsilon|\vec{R}-\vec{R}'|}dV'$ In a volume

$V(\vec{R}) = \int_{s'}\frac{\rho_s(\vec{R}')}{4\pi\varepsilon|\vec{R}-\vec{R}'|}ds'$ Over a surface

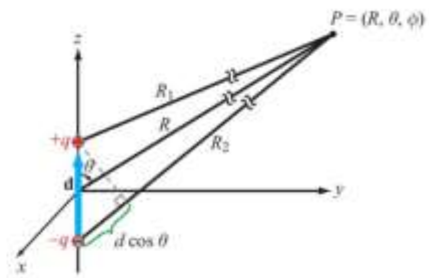
$V(\vec{R}) = \int_{l'}\frac{\rho_l(\vec{R}')}{4\pi\varepsilon|\vec{R}-\vec{R}'|}dl'$ On a line

Dielectrics

- An electric dipole consists of 2-point charges of equal magnitude but opposite polarity
    - Applications: Dielectrics, molecular bonds, antennas

$\vec{p} = q * \vec{d} \rightarrow dipole\ moment$ $\qquad V\ (\overrightarrow{R}) = \frac{\vec{p}\cdot\hat{R}}{4\pi\varepsilon_0|\vec{R} - \vec{R_i}|^2} = \frac{q*d*cos\theta}{4\pi\varepsilon_0 * R^2}$

$\vec{E}(\overrightarrow{R}) = \frac{qd}{4\pi\varepsilon_0|\vec{R} - \vec{R_i}|^3}\left(\hat{R}\ 2 * cos\theta + \hat{\theta}\ sin\theta\right)\ V/m$ only when R>>d



Types of dipoles in matter

- Permanent
    - Molecule having atoms with different electronegativity
    - Polar molecule → water
- Instantaneous
    - Electrons happen to concentrate in one place
- Induced
    - A permanent dipole or applied electric field near another atom induces a dipole

In a dielectric material $\vec{D} = \varepsilon_0\vec{E} + \vec{P} \rightarrow P\ is\ the\ electric\ flux\ density\ induced\ by\ applied\ field\ E$

$\vec{P} = \varepsilon_0 * X_e * \vec{E}$ $\qquad \vec{D} = \varepsilon_0\vec{E} + \varepsilon_0 * X_e * \vec{E} = \varepsilon_0(1 + X_e)\vec{E}$ $\qquad (1 + X_e) = relative\ permittivity = \varepsilon_r$

Conductors & Resistors

- Conductors are materials in which some of the electrons are free electrons
    - Electrons can move relatively freely through the material
    - Copper, aluminum, and silver
    - Charge Carrier: A particle carrying charge that is free to move

$Total\ Current\ I = \int_S \vec{J}\cdot\overrightarrow{ds} \quad \rightarrow \quad \vec{J} = \rho_v * \vec{u}\ \left(\frac{A}{m^2}\right) \rightarrow\ u\ is\ velocity \rightarrow \rho_v = q * N, N\ is\ \#\ of\ charges\ per\ unit\ volume$

Drift velocity, u: Steady state average velocity of the electrons

Mobility μ: Accounts for the effective mass of charged particle and the average distance before stopped by colliding

$\overrightarrow{u_e} = -\mu_e\vec{E} \rightarrow drift\ velocity\ of\ electrons\ (m/s)$ $\qquad \overrightarrow{u_h} = \mu_h\vec{E} \rightarrow drift\ velocity\ of\ holes\ (m/s)$

$\vec{J} = \vec{J_e} + \vec{J_h} = \rho_e * \overrightarrow{u_e} + \rho_h * \overrightarrow{u_h} = (-\rho_e * \mu_e + \rho_h * \mu_h)\vec{E}$ $\qquad \vec{J} = \sigma\vec{E}\ (A/m^2)$

Semiconductor / dielectric $\quad \sigma = (-\rho_e * \mu_e + \rho_h * \mu_h) = -N_e q\mu_e + N_h q\mu_h\ \ (S/m)$

Conductor $\sigma = -\rho_e * \mu_e = N_e e\mu_e$ $\qquad$ For perfect dialectic: $N_e = 0, \sigma = 0, J = 0$

For perfect conductor $\mu_e = \infty, \sigma = \infty, E = D = 0$

$For\ any\ conductor\ R = \frac{V}{I} = \frac{-\int_{l'}\vec{E}\cdot\overrightarrow{dl'}}{\int_S \vec{J}\cdot\overrightarrow{ds}} = \frac{-\int_{l'}\vec{E}\cdot\overrightarrow{dl'}}{\int_S \sigma\vec{E}\cdot\overrightarrow{ds}}$ $\qquad R = \frac{l}{\sigma_1 * A_1 + \sigma_2 * A_2} \rightarrow resistance\ coaxial\ cable$

Electric Boundary Conditions

Field Component	Any Two Media	Medium 1 Dielectric ε_1	Medium 2 Conductor
Tangential E	$\mathbf{E}_{1t} = \mathbf{E}_{2t}$	$\mathbf{E}_{1t} = \mathbf{E}_{2t} = 0$	
Tangential D	$\mathbf{D}_{1t}/\varepsilon_1 = \mathbf{D}_{2t}/\varepsilon_2$	$\mathbf{D}_{1t} = \mathbf{D}_{2t} = 0$	
Normal E	$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$	$E_{1n} = \rho_s/\varepsilon_1$	$E_{2n} = 0$
Normal D	$D_{1n} - D_{2n} = \rho_s$	$D_{1n} = \rho_s$	$D_{2n} = 0$

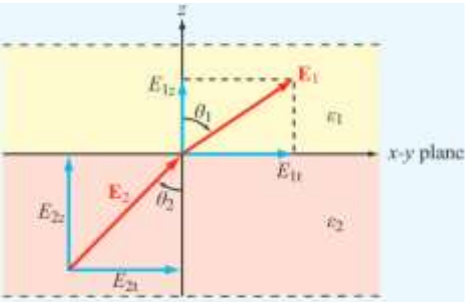
Notes: (1) ρ_s is the surface charge density at the boundary; (2) normal components of $\mathbf{E}_1$, $\mathbf{D}_1$, $\mathbf{E}_2$, and $\mathbf{D}_2$ are along $\hat{\mathbf{n}}_2$, the outward normal unit vector of medium 2.

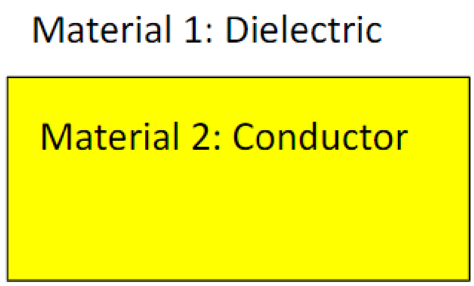
← General Boundary Conditions

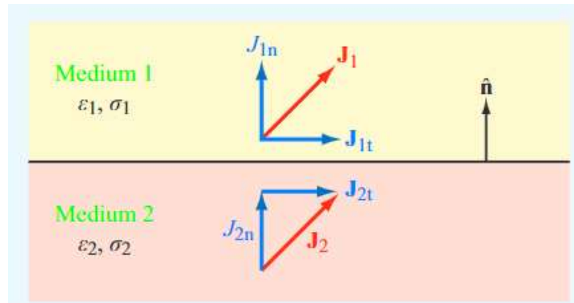
DD: $E_{1t} = E_{2t}$ & $E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n}$

DC: $E = D = 0$ $in\ perfect\ conductor$

$D_{1t} = E_{1t} = 0$ & $D_{1n} = \varepsilon_1 E_{1n} = \rho_s$







Dielectric – dielectric

$\frac{tan\theta_1}{\varepsilon_1} = \frac{tan\theta_2}{\varepsilon_2}$

Dielectric – conductor

$\overrightarrow{D_1} = \varepsilon_1 \overrightarrow{E_1} = \rho_s \hat{n}$

Conductor – Conductor

$\varepsilon_1 * \frac{J_{1n}}{\sigma_1} - \varepsilon_2 * \frac{J_{2n}}{\sigma_2} = \rho_s \rightarrow J_{1n} = J_{2n}$

Capacitor

$C = \frac{Q}{V} = \frac{\int_S \varepsilon \vec{E} \cdot \vec{ds}}{-\int_{l'} \vec{E} \cdot \vec{dl}} = \frac{\varepsilon * A}{d}$

$RC = \frac{V}{I} * \frac{Q}{V} = \frac{\varepsilon}{\sigma}$

$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{2} * \frac{Q^2}{C} = \frac{1}{2} * C * V^2 = \frac{1}{2} \varepsilon E^2 (Ad)$

$E = \frac{V}{d}$

$Q = \int_S \varepsilon \vec{E} \cdot \overrightarrow{ds}$

Magnetic Forces and torques

$Magnetic\ force = \vec{F}_m = q\vec{u} \times \vec{B}\ (N)$

$Lorentz\ force = q\vec{u} \times \vec{B} + q\vec{E}$

$dF_m = I\vec{dl} \times \vec{B} = dQ\vec{u} \times \vec{B}$

$Force\ on\ any\ closed\ current\ loop\ in\ a\ uniform\ magnetic\ field = 0$

$Magnetic\ torque\ T = \vec{m} \times \vec{B}\ (N \cdot m)$

$|T| = N * I * A * B * sin\theta$

$\vec{m} = \hat{n} N * I * A$

Biot-Savart Law

$\vec{B} = \mu_0 \vec{H}\ (T)$

$\overrightarrow{dH} = \frac{I}{4\pi R^2} * \overrightarrow{dl} \times \hat{R}\ \left(\frac{A}{m}\right)$

$\overrightarrow{dB} = \frac{I\mu_0}{4\pi R^2} * \overrightarrow{dl} \times \hat{R}\ (T)$

$$\vec{B}(\vec{R}) = \int_{V'} \frac{\mu \vec{J}(\vec{R}') \times (\vec{R} - \vec{R}')}{4\pi |\vec{R} - \vec{R}'|^3} dV' = \frac{\mu}{4\pi} \int_{V'} \frac{\vec{J} \times \hat{R}}{R^2} dV'$$

Magnetic field of a loop $B = \frac{\mu I a^2}{2(a^2+z^2)^{1.5}}\ \hat{z}$

At z = 0 $B = \mu * \frac{I}{2*a}\ \hat{z}$

At points far away $B = \frac{\mu I a^2}{2z^3}\ \hat{z}$

$$\vec{B}(\vec{R}) = \int_{S'} \frac{\mu \vec{J}_s(\vec{R}') \times (\vec{R} - \vec{R}')}{4\pi |\vec{R} - \vec{R}'|^3} ds' = \frac{\mu}{4\pi} \int_{V'} \frac{\vec{J}_s \times \hat{R}}{R^2} ds'$$

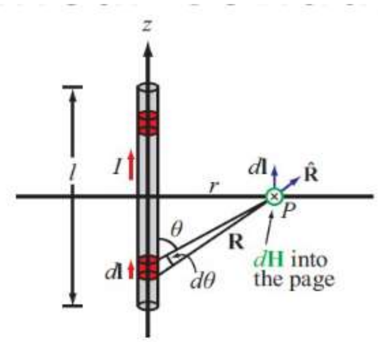
$$\vec{B}(\vec{R}) = \int_{l'} \frac{\mu I d\vec{l}' \times (\vec{R} - \vec{R}')}{4\pi |\vec{R} - \vec{R}'|^3} = \frac{\mu I}{4\pi} \int_{l'} \frac{d\vec{l}' \times \hat{R}}{R^2}$$

On a line

$Magnetic\ field\ of\ a\ linear\ conductor\ B = \frac{\mu I a}{2\pi r\sqrt{4r^2+a^2}}\ \hat{\phi}$

R is the distance from center to point P, a is length of wire segment

For an infinity long wire $B = \frac{\mu I}{2\pi r}\ \hat{\phi}$



Ampere's law (Gauss's Law for magnetism) (net magnetic flux through a closed Gaussian surface is 0)

$\oint_C\ \vec{H}\cdot\vec{dl} = I$ $\qquad$ $\nabla\times\vec{H} = \vec{J}$

H field for long wire: $r1 \le a \rightarrow \vec{H} = \frac{r_1 * I}{2\pi a^2}\ \hat{\phi}$ $\qquad$ $r2 \ge a \rightarrow \vec{H} = \frac{I}{2\pi r}\ \hat{\phi}$

H field for toroidal coil: $r < a \rightarrow \vec{H} = 0$ $\qquad$ $r < a \rightarrow \vec{H} = 0$ $\qquad$ $a < r < b \rightarrow\ \vec{H} = -\frac{NI}{2\pi r}\ \hat{\phi}$

H field inside long solenoid: $r > a \rightarrow\ \vec{H} \approx 0$ $\qquad$ $r < a \rightarrow \vec{H} = N * \frac{I}{L}\ \hat{z}$

H field of current sheet: $z > 0 \rightarrow \vec{H} =\ -\frac{J}{2}\ \hat{y},\ J\ is\ current\ density$ $\qquad$ $z < 0 \rightarrow\ \vec{H} = \frac{J}{2}\ \hat{y}$

Magnetic vector potential & Magnetic material

$\vec{B} =\ \nabla\times\vec{A}\ (Wb/m^2)\ \ \rightarrow A\ is\ magnetic\ vector\ potential$ $\qquad$ $\overrightarrow{A(R)} =\ \int_{v'} \frac{\mu * \vec{J}\ (\vec{R'})}{4\pi|\vec{R} - \vec{R_i}|}\ dV'\ (Wb/m^2)$

Spin Magnetic Moments: $m = I * A = \frac{-e*v}{2\pi r} * \pi r^2\ =\ -\frac{e*L}{2m}$ $\qquad$ Angular momentum: $L = m * v * r$

Magnetization: $\vec{B} =\ \mu_0\vec{H}\ \ in\ free\ space$ $\qquad$ $\vec{B} =\ \mu_0\vec{H} +\ \mu_0\vec{M}\ in\ magnetic\ material$

$\vec{M} =\ X_m * \vec{H} \rightarrow\ X_m\ is\ magnetic\ susceptibility$ $\qquad$ M is sum of magnetic dipole moments in medium

$\vec{B} = \mu_0\vec{H} + \mu_0\ X_m * \vec{H} =\ \mu_0(1 + X_m)\vec{H} \rightarrow\ (1 + X_m) =\ \mu_r$

	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Primary magnetization mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis (see Fig. 5-22)
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminum, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m Typical value of μ_r	$\approx -10^{-5}$ ≈ 1	$\approx 10^{-5}$ ≈ 1	$\lvert\chi_m\rvert \gg 1$ and hysteretic $\lvert\mu_r\rvert \gg 1$ and hysteretic

Inductor

$L = \frac{Magnetic\ Flux}{Current}$ $\qquad$ $\phi_m =\ \int_S\ \vec{B}\cdot\vec{ds}\ \ (Wb) \rightarrow total\ magntic\ flux$ $\qquad$ $Self\ inductance =\ L = \frac{\phi_m}{I} = \frac{\int_S\ \vec{B}\cdot\vec{ds}}{\oint_C\ \vec{H}\cdot\vec{dl}}\ \ (H\ or\ Wb/A)$

Self-inductance in a solenoid: $\phi_m = \frac{\mu * N * I}{L} * S \rightarrow S\ is\ area\ of\ one\ loop$ $\qquad$ $L = \frac{\mu * N^2}{L} * S$ $\qquad$ $I = \frac{BL}{\mu N}$

Energy stored in solenoid: $W\ = \frac{1}{2} * L * I^2\ = \frac{1}{2} * \frac{B^2}{\mu}$ $\qquad$ Energy density $w = \frac{1}{2}\mu H^2$ $\qquad$ Total energy in any volume $W\ = \frac{1}{2}\int_V\ \ \mu H^2\ dV$

Self-inductance of toroid: $L = \frac{\mu * N^2}{2\pi r_m} * S \rightarrow\ r_m\ = \frac{a+b}{2}$ $\qquad$ $Mutual\ Inductance\ L_{12} = \frac{N_2}{I_1}\int_{S2}\overrightarrow{B_1}\cdot\overrightarrow{ds}$

$Potential\ energy\ W =\ \int_0^I i * v * dt = L\int_0^I i * di = \frac{1}{2}LI^2(J)$

Faraday's Law

- A time varying magnetic field creates transformer emf V
- A moving loop with time-varying surface area in static field B create motional emf
- A moving loop in time-varying field B is transformer emf + motional emf

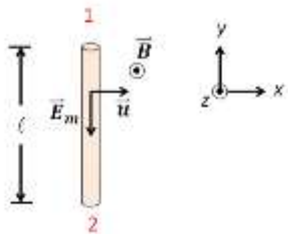
$V_{emf} = -N * \frac{d\phi_m}{dt} = -N\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}\ (V)$ $\qquad$ $Transformer\ V_{emf} = -N\int_S \frac{d\vec{B}}{dt}\cdot\vec{ds}$

$\oint_C \vec{E}\cdot\vec{dl} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$ $\qquad$ $\nabla\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$

Lenz's Law

- The current in the loop is always in a direction that opposes the change of magnetic flux that produced I

Moving Conductor in a static magnetic field



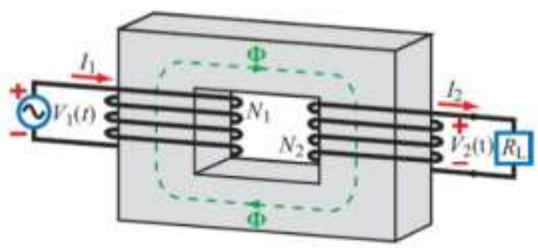
$Motional\ EMF = V_{12} = \int_2^1(\vec{u}\times\vec{B})\cdot\vec{dl}$

$\vec{u}\times\vec{B} = u\hat{x}\times B_0\hat{z} = -uB_0\hat{y}$ $\qquad$ $V_{emf} = -uB_0L$

$V_{emf} = -\int_S \frac{d\vec{B}}{dt}\cdot\vec{ds} + \int_2^1(\vec{u}\times\vec{B})\cdot\vec{dl}$

Transformers

$\frac{V_1}{V_2} = \frac{N_1}{N_2} \rightarrow turns\ ratio \rightarrow\ N_2 > N_1$ Step up traersnform; $N_2 < N_1$ "$Step\ down\ traernsform$" $\qquad$ $\frac{I_1}{I_2} = \frac{N_2}{N_1}$



$P_1 = I_1V_1 = P_2 = I_2V_2$

$R_{in} = \frac{V_1}{I_1} = (\frac{N_1}{N_2})^2 * R_L$

$Z_{in} = (\frac{N_1}{N_2})^2 * Z_L$

$V_{emf} = -\frac{d\phi_m}{dt} = A * w * B_0 * \sin(wt + C_0)$

DC Generators: Same components as AC generator, main difference is contacts to the rotating loop are made using a split ring called commutator.

Motor: Electrical to mechanical energy $\qquad$ Generators works opposite

Motors are devices into which energy is transferred by electrical transmission while energy is transferred out by work

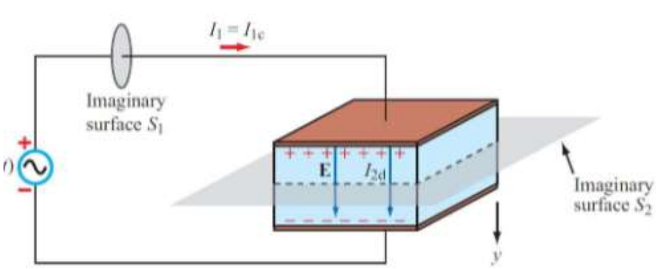
EM Motor: A current is supplied to the coil by a battery and the torque acting on the current carrying coil causes it to rotate

- Induced back emf, acts to reduce the current in the coil
- The back emf increases in magnitude as the rotational speed of coil increases
- $I = \frac{V_{app} - V_{emf}}{R}$

The displacement current

$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \rightarrow \frac{\partial\vec{D}}{\partial t} = \vec{J_d} \rightarrow displacement\ current\ density$

$\oint_C \vec{H}\cdot\vec{dl} = \int_S \vec{J}\cdot ds' + \int_S \frac{\partial\vec{D}}{\partial t}\cdot ds' = I_C(conduction\ current) + I_D(Displacement\ current) = I$



$In\ perfect\ conducting\ wire: I_1 = I_{1c} + I_{1d} = -CV_0wsin(wt)$

$In\ perfect\ conducting\ capacitor:$

$I_2 = I_{2c} + I_{2d} = -CV_0wsin(wt) = I_1$

Continuity of current flow through the circuit

- The displacement current behaves like a real current
- The displacement current accounts for polarization in the medium
- The perfect wire has infinite conductivity
    - If it has finite conductivity, then D in the wire would be non-zero and I_1 would consist of both conduction and displacement currents
- A magnetic field can be produced either by currents or by changing electric fields

$\nabla \cdot (\nabla \times \mathrm{H}) = \nabla \cdot J + \frac{\partial \nabla D}{\partial t} = 0$ $\qquad$ $Continuity\ equation: \nabla \cdot \vec{J} = \frac{-\partial \rho}{\partial t}$

$\oint_S \vec{J} \cdot ds' = 0\ for\ steady\ currents = \sum_i I_i = 0$

- An electric field can be produced either by charges or changing magnetic fields