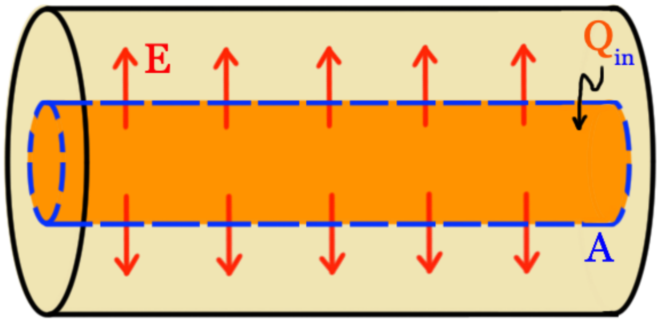
Gauss' Law

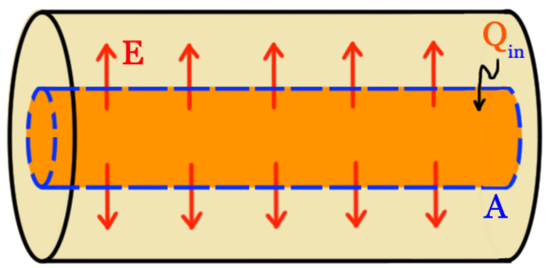
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_O}$$



“The **electric flux** through any **closed surface** is equal to the **charge within** that area”

If you choose an area A upon which the electric field E is uniform, The E can be removed from the integral and Gauss’ Law becomes..

$$\mathbf{E}\,\mathbf{A} = \frac{Q_{in}}{\epsilon_O}$$

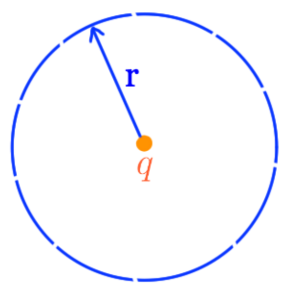


Note: The **dark orange shaded region** is charge contained within the **Gaussian surface**, the **tan shaded region** is charge not contained within the **Gaussian surface**

Spherical Symmetry

"Point Charge"

$$E(4\pi r^2) = \frac{q}{\epsilon_O}$$

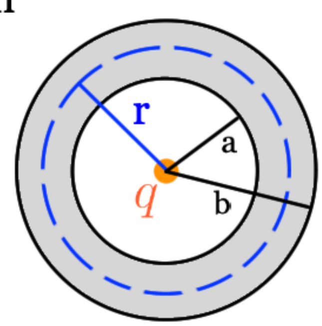


"Point charge in metal shell"

$$E(4\pi r^2) = \frac{q}{\epsilon_O} \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{q}{\epsilon_O} \quad \text{if } r > b$$

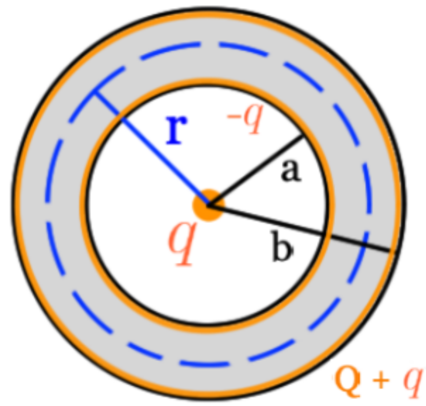


"Metal shell with net charge Q, surrounding point charge q"

$$E(4\pi r^2) = \frac{q}{\epsilon_o} \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{q+Q}{\epsilon_o} \quad \text{if } r > b$$

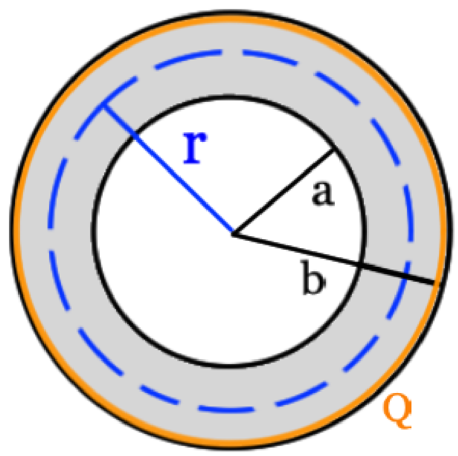


"Metal shell with net charge Q"

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

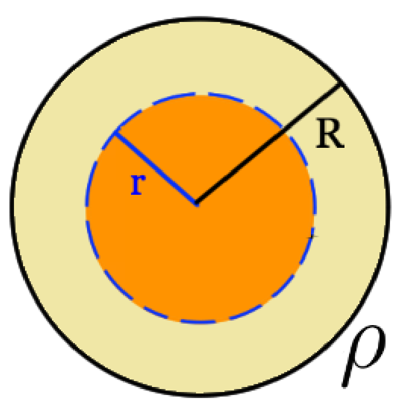
$$E(4\pi r^2) = \frac{Q}{\epsilon_o} \quad \text{if } r > b$$



"Insulating charged sphere of charge density ρ"

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_o} \quad \text{if } r < R$$

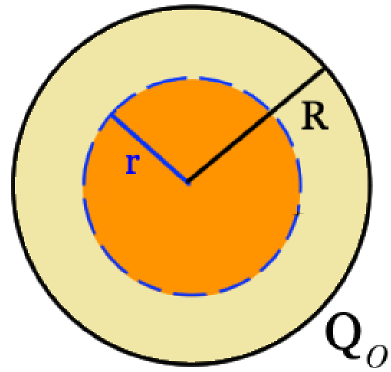
$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi R^3}{\epsilon_o} \quad \text{if } r > R$$



"Insulating charged sphere of total charge Q_o"

$$E(4\pi r^2) = \frac{\frac{Q_o}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3}{\epsilon_o} \quad \text{if } r < R$$

$$E(4\pi r^2) = \frac{Q_o}{\epsilon_o} \quad \text{if } r > R$$

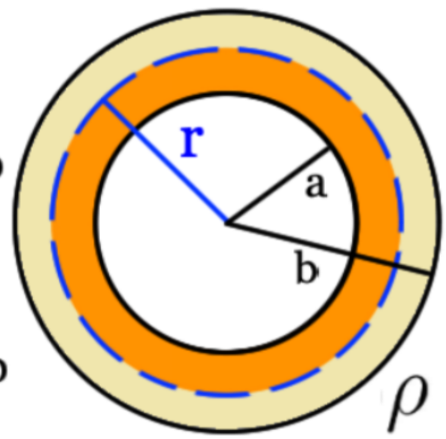


"Insulating charged shell given charge density ρ"

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3)}{\epsilon_o} \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3)}{\epsilon_o} \quad \text{if } r > b$$

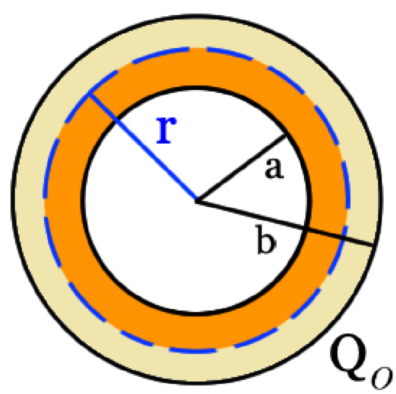


"Insulating charged shell given total charge Q_o"

$$E(4\pi r^2) = 0 \quad \text{if } r < a$$

$$E(4\pi r^2) = \frac{\frac{Q_o}{\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3} \cdot (\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3)}{\epsilon_o} \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{Q_o}{\epsilon_o} \quad \text{if } r > b$$

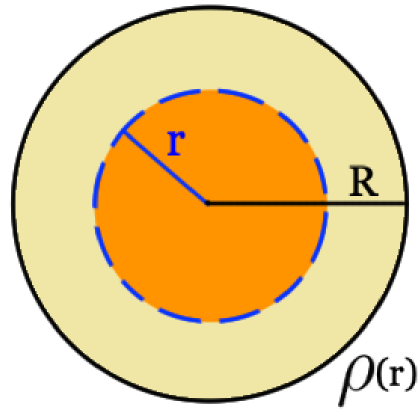


"Insulating sphere with charge density ρ(r) "

$$E(4\pi r^2) = \frac{\int_o^r \rho(r) 4\pi r^2 dr}{\epsilon_o} \quad \text{r < R}$$

$$E(4\pi r^2) = \frac{\int_o^R \rho(r) 4\pi r^2 dr}{\epsilon_o} \quad \text{r > R}$$

$$\boxed{dV_{sphere} = d(\frac{4}{3}\pi r^3) = 4\pi r^2 dr}$$

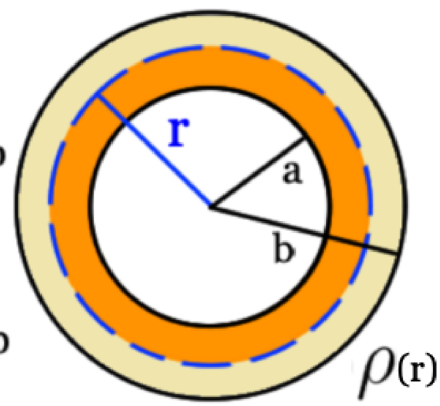


"Insulating charged shell given charge density ρ(r)"

$$E(4\pi r^2) = 0 \quad \text{if r < a}$$

$$E(4\pi r^2) = \frac{\int_a^r \rho(r) 4\pi r^2 dr}{\epsilon_o} \quad \text{if a < r < b}$$

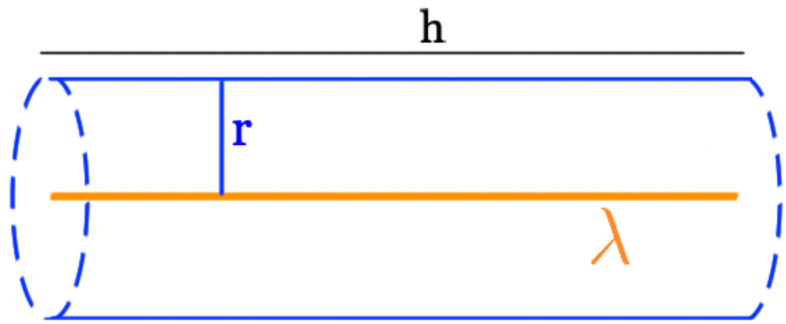
$$E(4\pi r^2) = \frac{\int_a^b \rho(r) 4\pi r^2 dr}{\epsilon_o} \quad \text{if r > b}$$



Cylindrical Symmetry

"Line of charge"

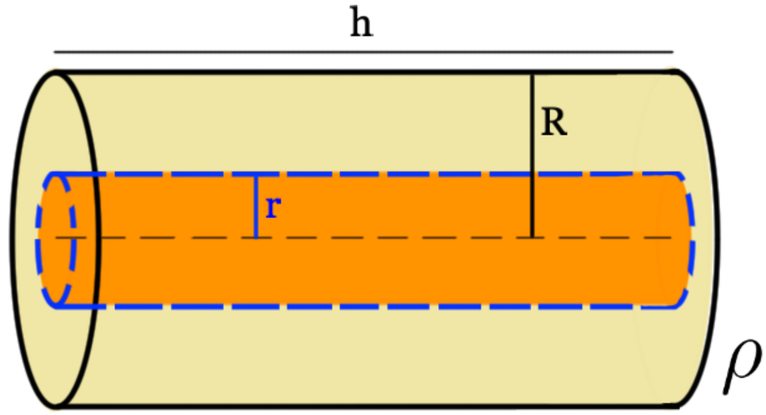
$$E(2\pi rh) = \frac{\lambda h}{\epsilon_o} \quad \text{for all r}$$



"Insulating cylinder with charge density ρ"

$$E(2\pi r h) = \frac{\rho(\pi r^2 h)}{\epsilon_o} \quad r < R$$

$$E(2\pi r h) = \frac{\rho(\pi R^2 h)}{\epsilon_o} \quad r > R$$

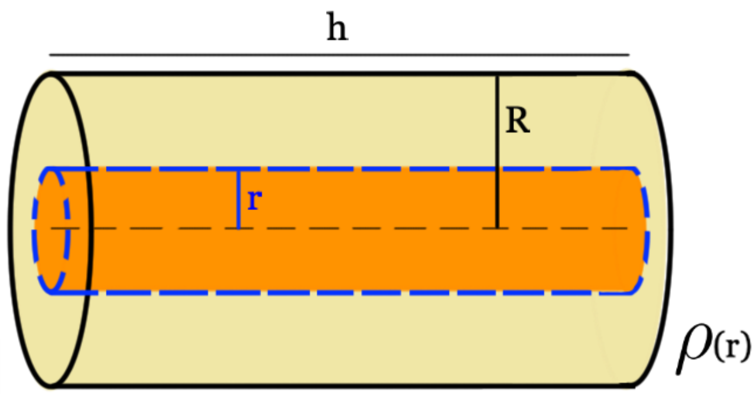


"Insulating cylinder with non-uniform charge density $\rho_{(r)}$"

$$E(2\pi r h) = \int_o^r \frac{\rho(r)2\pi r h dr}{\epsilon_o} \quad r < R$$

$$E(2\pi r h) = \int_o^R \frac{\rho(r)2\pi r h dr}{\epsilon_o} \quad r > R$$

$$\boxed{dV_{cylinder} = d(\pi r^2 h) = 2\pi r h dr}$$

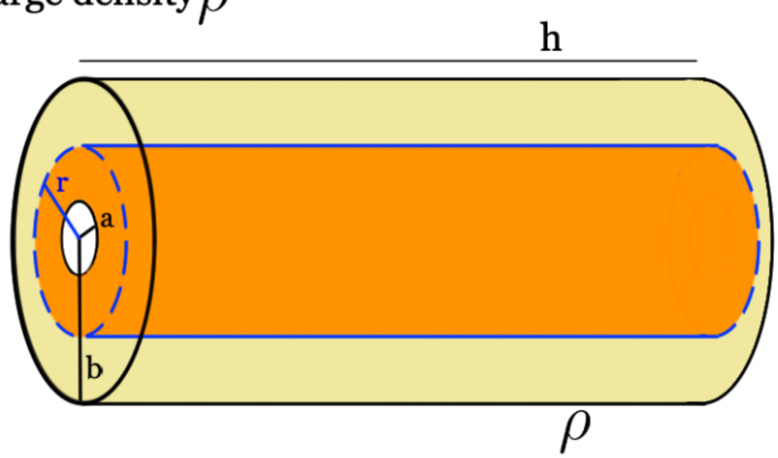


"Insulating hollow cylinder of charge density ρ"

$$E(2\pi r h) = 0 \quad r < a$$

$$E(2\pi r h) = \frac{\rho(\pi r^2 h - \pi a^2 h)}{\epsilon_o} \quad b > r > a$$

$$E(2\pi r h) = \frac{\rho(\pi b^2 h - \pi a^2 h)}{\epsilon_o} \quad r > b$$

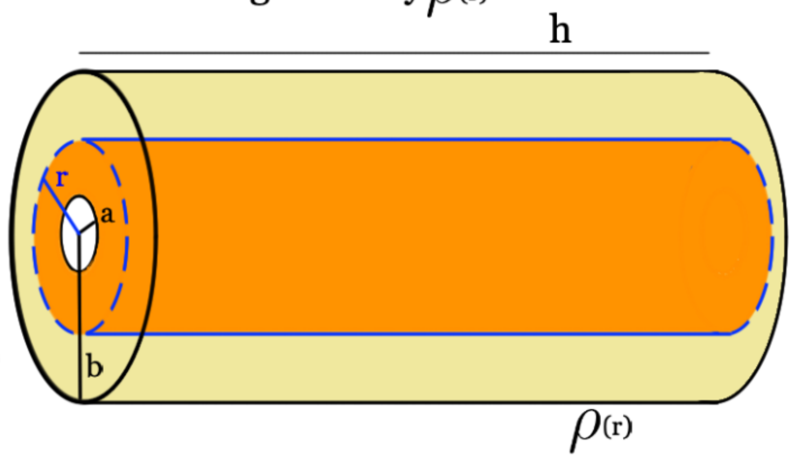


"Insulating hollow cylinder of non-uniform charge density $\rho_{(r)}$"

$$E(2\pi r h) = 0 \quad r < a$$

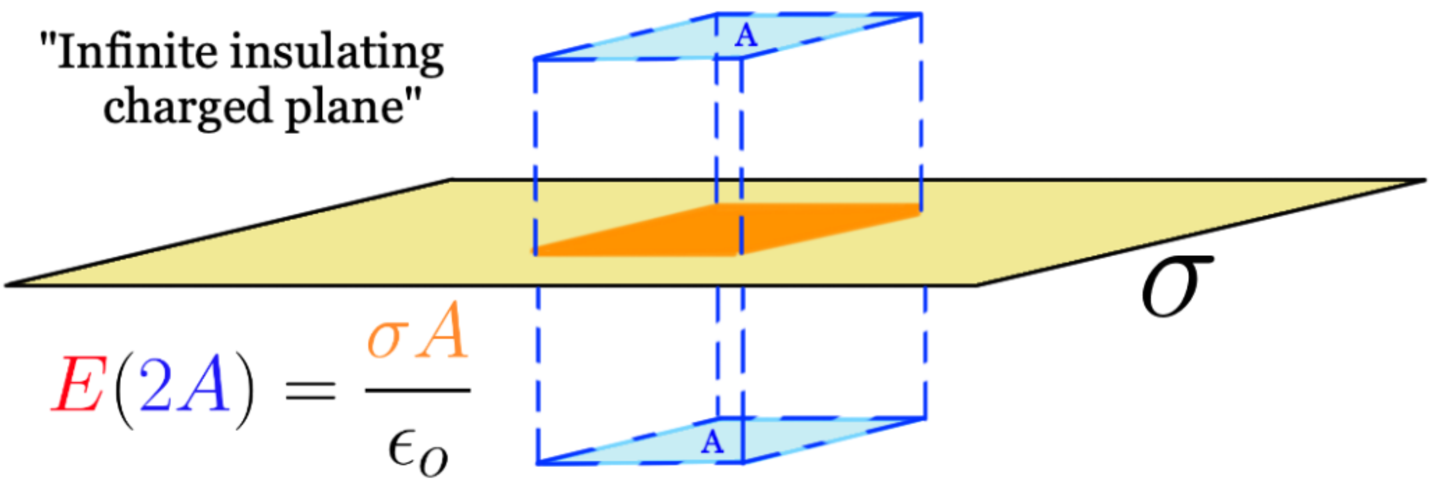
$$E(2\pi r h) = \frac{\int_a^r \rho(r) 2\pi r h dr}{\epsilon_O} \quad b > r > a$$

$$E(2\pi r h) = \frac{\int_a^b \rho(r) 2\pi r h dr}{\epsilon_O} \quad r > b$$



Planar Symmetry

"Infinite insulating charged plane"



$$E(2A) = \frac{\sigma A}{\epsilon_O}$$