Assignment 2

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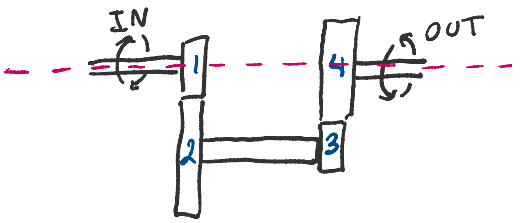
1) $T_4 = 6266.4\ N\cdot m$

$\omega_4 = 125\ RPM = \frac{25\pi}{6}\ rad/s$

$P_1 = 110\ HP = [110\ HP \times 745.7]\ W = 82027\ W$

$\omega_1 = 3000\ RPM = 100\pi\ rad/s$

$T_1 = P_1/\omega_1 = 82027/100\pi = 261.1\ N\cdot m$



★ $\omega_2 = \omega_3$

★ $D_1 + D_2 = D_3 + D_4$

$$i = i_{12} \times i_{34} = \frac{\omega_1}{\omega_2} \times \frac{\omega_3}{\omega_4} = \frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{D_2}{D_1} \times \frac{D_4}{D_3} = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

$$= \frac{\omega_1}{\omega_2} \times \frac{\omega_3}{\omega_4}$$

$$= \frac{100\pi}{\omega_2} \times \frac{\omega_3}{25\pi/6}$$

$$= \frac{100\pi}{25\pi/6} \times \frac{\omega_3}{\omega_2}$$

$= 24 \longrightarrow$ check: $\frac{T_4}{T_1} = 24 = \frac{6266.4}{261.1}$ ✓

$24 = i_{12} \times i_{34} = 4 \times 6$

$i_{12} = 4$

$4 = N_2/N_1$

$N_1 = 12$ $N_2 = 48$

$i_{34} = 6$

$6 = N_4/N_3$

$N_4 = 120$ $N_3 = 20$

$D_1 = 12/P$ (assume $P = 3$) $D_2 = 48/3$ $D_3 = 20/3$ $D_4 = 120/3$

$D_1 = 4\ in$ $D_2 = 16\ in$ $D_3 = 6.67\ in$ $D_4 = 40\ in$

2)

Face width = 4"
Pitch = 3 teeth/in
Select material & pressure angle

$$W = \frac{S \cdot FW \cdot Y}{P}\left(\frac{600}{600+V}\right) \rightarrow S = \frac{P \cdot W}{FW \cdot Y}\left(\frac{600+V}{600}\right)$$

$$T = \frac{D}{2} W \rightarrow W = \frac{2T}{D}$$

$$V = 0.262 \times D \times RPM$$

Y = from table

P = 3 FW = 4

$$S = \frac{3 \cdot 2T/D}{4 \cdot Y}\left[\frac{600 + 0.262 \cdot D \cdot RPM}{600}\right]$$

Gear 1

$T_1 = 63025(110\ HP/3000\ RPM)$
$T_1 = 2310.92$ in·lbs

$Y_1 =$ 14.5° PA: 0.21 20° PA: 0.245 [N = 12]

$D_1 = 4$ in

$$S = \frac{3 \cdot 2(2310.92)/4}{4 \cdot (Y)}\left[\frac{600 + (0.262)(4)(3000)}{600}\right]$$

$S_{14.5°} = 25750.25$ psi

$S_{20^\circ} = 22\,071.64$ psi

Gear 2

$T_2 = 4T_1 = 9243.68$ in·lbs

$RPM_2 = RPM_1/4 = 750$ RPM

$\frac{0.346-0.34}{50-45} = \frac{Y-0.34}{48-45} \rightarrow (Y_2)_{14.5} = 0.345$

$\frac{0.408-0.399}{50-45} = \frac{Y-0.399}{48-45} \rightarrow (Y_2)_{20^\circ} = 0.404$

	14.5°	20°	
$Y_2 =$	0.345	0.404	[N = 48]

$D_2 = 16$ in

$$S = \frac{3 \cdot 2(9243.68)/16}{4 \cdot (Y)} \left[\frac{600 + 0.262(16)(750)}{600}\right]$$

$S_{14.5} = 15\,674.07$ psi

$S_{20} = 13\,385.03$ psi

Gear 3

$T_3 = 62664$ N·m$/6 = 13865.58$ in·lbs

$RPM_3 = 6 \cdot RPM_4 = 750$ RPM

$D_3 = 6.67$ in

	14.5°	20°	
$Y_3 =$	0.283	0.82	[N = 20]

$$S_3 = \frac{3 \cdot 2(13865.58)/6.67}{4Y_3} \left(\frac{600 + 0.262(6.67)(750)}{600}\right)$$

$S_{14.5^\circ} = 35\,092.66$ psi

$S_{20^\circ} = 31\,035.07$ psi

$S_{20^\circ} = 31\,035.07$ psi

Gear 4

$T_4 = 55\,462.313$ in·lbs

$RPM_4 = 125$ RPM

$D_4 = 40$ in

$Y_4 = \underset{0.371}{\overset{14.5^\circ}{}} \quad \underset{0.451}{\overset{20^\circ}{}} \qquad [N = 120]$

$\frac{0.375 - 0.368}{150 - 100} = \frac{Y - 0.368}{120 - 100} \rightarrow (Y_4)_{14.5^\circ} = 0.371$

$\frac{0.458 - 0.446}{150 - 100} = \frac{Y - 0.446}{120 - 100} = (Y_4)_{20^\circ} = 0.451$

$$S_4 = \frac{3 \cdot 2(55\,462.313)/40}{4 \cdot Y_4}\left(\frac{600 + 0.262(40)(125)}{600}\right)$$

$S_{14.5} = 17\,845.86$ psi

$S_{20} = 14\,680.39$ psi

With pressure angle 20°,

$S_3 > S_1 > S_4 > S_2$

$S_3 = 31\,035.07$ psi

∴ material should be Steel .40 Carbon Alloy (Heat Treated) ($S = 40\,000$ psi) if all 4 gears are to use the same material (from table in slides)

3) $SF = W_{max}/W_{actual}$ $\qquad W_{max} = \frac{S \cdot FW \cdot Y}{P}$ $\qquad W_{actual} = 2T/D$

3) $SF = W_{max}/W_{actual}$ $\quad W_{max} = \frac{S \cdot FW \cdot Y}{P}$ $\quad W_{actual} = dI/D$

$$SF = \frac{S_{material} \cdot FW \cdot Y \cdot D}{2 \cdot P \cdot T}\left[\frac{600}{600 + 9.262 \cdot D \cdot RPM}\right]$$

$$SF = \frac{(40000)(4)(Y)(D)}{2 \cdot 3 \cdot T}\left[\frac{600}{600 + 9.262 \cdot D \cdot RPM}\right]$$

$$SF = \frac{80000 \cdot Y \cdot D}{3 \cdot T}\left[\frac{600}{600 + 9.262 \cdot D \cdot RPM}\right]$$

$(SF)_1 = 1.812$ $\quad (SF)_3 = 1.289$

$(SF)_2 = 2.988$ $\quad (SF)_4 = 2.725$

The weakest gear is gear 3, with a safety factor of 1.289. Ideally, we'd like the safety factor to be high as possible, but the value of 1.289 is acceptable since it is > 1.

I think my geartrain is of reasonable size considering the largest gear is 40 inches in diameter with a torque output of over 55000 lb·in. The torque relative to the gear size is quite large. If we take the smallest gear available to be ~3 inches, we would need to connect it

to be ~3 inches, we would need to connect it to a gear of diameter 72 inches to achieve $i = 24$. Since we are achieving this ratio with the largest gear being closer to half of 72", I'd say the gear train is reasonably sized.