

## McMaster University Final Exam

Name \_\_\_\_\_  
Student Number \_\_\_\_\_

MECH ENG 4K03/6K03 ROBOTICS

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DURATION OF EXAMINATION: 2.5 HRS

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THIS EXAMINATION PAPER INCLUDES 5 PAGES (3 PAGES FOR QUESTIONS AND 2 PAGES FOR FORMULAS) AND 5 QUESTIONS.

Use of Casio FX-991 MS or MS Plus calculator.

Questions:

1. (8 points) Give the definitions of the following terms in robotics.

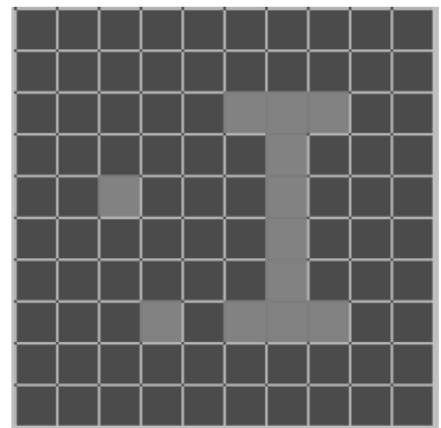
- 1) Hard automation
- 2) Flexible automation
- 3) Planar robot
- 4) Dextrous workspace

2. Short Answer Questions

1) (5 points) If the transformation matrices,  ${}^C T_D$ ,  ${}^A T_B$ ,  ${}^A T_E$  and  ${}^E T_D$ , are known, derive the transformation equation for  ${}^B T_C$  in terms of these matrices.

2) (5 points) Given the grayscale input image of the letter "I" from an optical character recognition application:

$$A = \begin{bmatrix} 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 130 & 130 & 130 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 130 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 130 & 75 & 75 & 75 \\ 75 & 75 & 75 & 130 & 75 & 130 & 130 & 130 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \\ 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 & 75 \end{bmatrix}$$



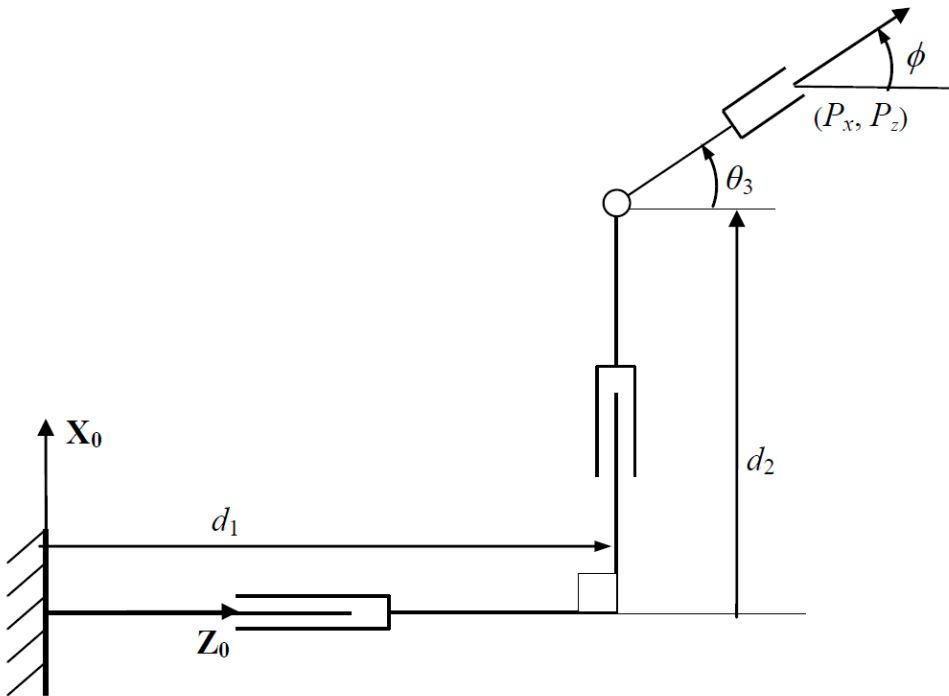
Show a sample calculation for the row=8, col=5 pixel, if we apply a Laplacian 1 filter to the input image.

3) (5 points) Which of the following matrices is a valid representation for a frame? Explain your answer.

$$A = \begin{bmatrix} 0.77 & -0.64 & 0 & 3 \\ 0.64 & 0.77 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; B = \begin{bmatrix} 0 & \sqrt{3}/2 & 0 & 3 \\ -1 & 0 & 0 & 0.5 \\ 0 & 1/2 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; C = \begin{bmatrix} 0 & \sqrt{3}/2 & -1/2 & 3 \\ -1 & 0 & 0 & 0.5 \\ 0 & 1/2 & -\sqrt{3}/2 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

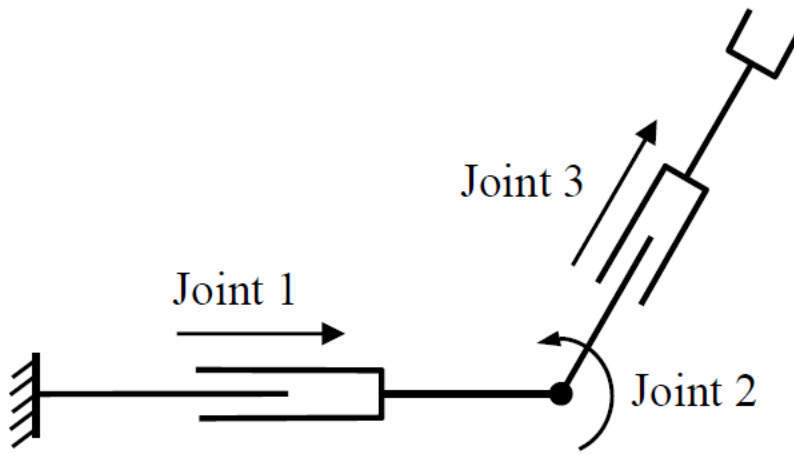
3. (25 points) For the PPR planar robot shown in the following Figure:

- Assign the frames using the D-H method.
- Determine the D-H parameters and put them in the standard table form. Identify the joint variables.
- Draw a diagram of the robot that properly shows the D-H frames, the joint variables, and any  $d$  or  $a$  parameters that are non-zero. ( $d_1$  and  $d_2$  are joint variables indicated in the figure;  $a_3$  is the link length after the 3<sup>rd</sup> joint)
- Calculate the  $A$  matrices and  ${}^0T_3$
- Its joint variables are  $d_1$ ,  $d_2$ , and  $\theta_3$ . Its end-effector position and orientation are given by  $P_x$ ,  $P_z$  and  $\phi$ .  
Derive its inverse kinematics.

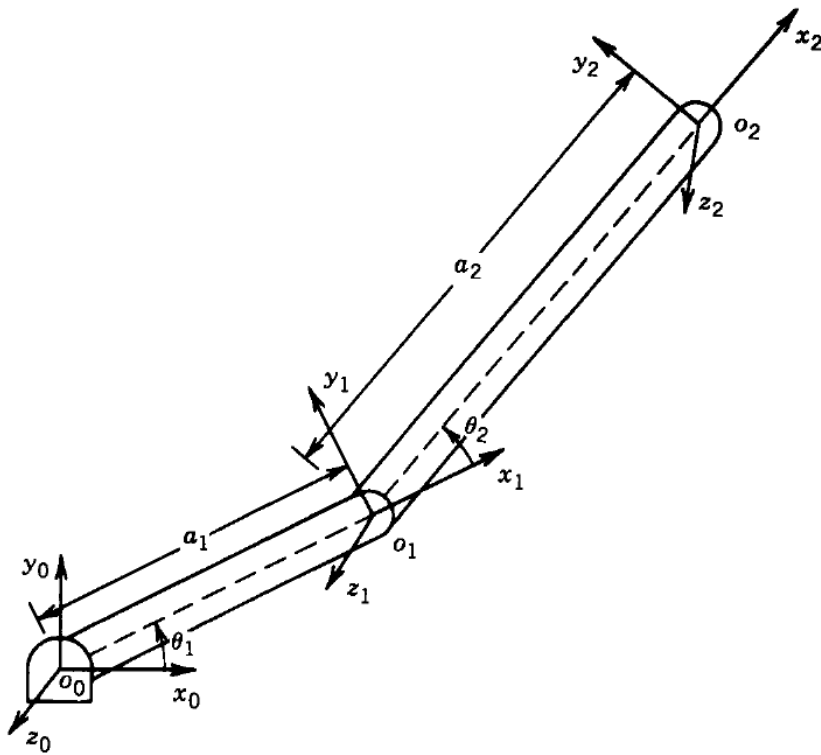


4. (25 points) For the planar PRP robot shown in the following Figure:

- Derive the 3x3 manipulator Jacobian matrix. (The form used for calculating the linear velocity and angular velocity of the tool).
- Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.
- Draw the robot in a singular configuration and indicate which degree (s) of freedom have been lost.



5. (27 points) For the RR planar robot in the following figure, if  $a_1 = 0.4\text{m}$  and  $a_2 = 0.3\text{m}$ ;
- (a) Assuming the robot operates in the horizontal plane, calculate the joint torques such that the static force at the end-effector is  $F_x = 20\text{N}$  and  $F_y = -15\text{N}$  for the configuration  $\theta_1 = 35^\circ$  and  $\theta_2 = -75^\circ$
- (b) Assuming the robot operates in the horizontal plane, calculate the static force applied by the end effector when  $\tau_1 = 10\text{Nm}$ ,  $\tau_2 = 5\text{Nm}$ ,  $\theta_1 = 35^\circ$  and  $\theta_2 = -75^\circ$ .



## Formulas

$$\text{Rot}(X, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\theta & -S\theta & 0 \\ 0 & S\theta & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

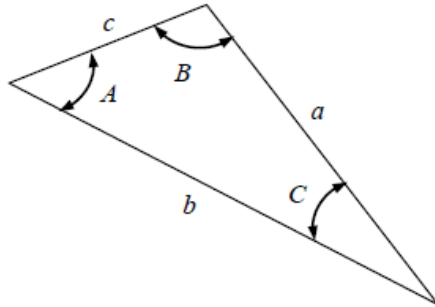
$$\text{Rot}(Y, \theta) = \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(Z, \theta) = \begin{bmatrix} C\theta & -S\theta & 0 & 0 \\ S\theta & C\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Trans}(a, b, c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\vec{P} \bullet \vec{n} \\ o_x & o_y & o_z & -\vec{P} \bullet \vec{o} \\ a_x & a_y & a_z & -\vec{P} \bullet \vec{a} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{n+1} = {}^nT_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\cos A = (b^2 + c^2 - a^2) / (2bc)$$

$$S\theta_1 C\theta_2 + C\theta_1 S\theta_2 = S(\theta_1 + \theta_2) = S\theta_{12}$$

$$C\theta_1 C\theta_2 - S\theta_1 S\theta_2 = C(\theta_1 + \theta_2) = C\theta_{12}$$

if  $a = \sin \theta$  and  $b = \cos \theta$  then  $\theta = \text{atan2}(a, b)$

$$J(q) = \begin{bmatrix} \frac{\partial p_x(q)}{\partial q_1} & \frac{\partial p_x(q)}{\partial q_2} \\ \frac{\partial p_y(q)}{\partial q_1} & \frac{\partial p_y(q)}{\partial q_2} \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \frac{\partial p_x(q)}{\partial q_1} & \frac{\partial p_x(q)}{\partial q_2} & \frac{\partial p_x(q)}{\partial q_3} \\ \frac{\partial p_y(q)}{\partial q_1} & \frac{\partial p_y(q)}{\partial q_2} & \frac{\partial p_y(q)}{\partial q_3} \\ \zeta_1 t_1 & \zeta_2 t_2 & \zeta_3 t_3 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \frac{\partial p_x(q)}{\partial q_1} & \frac{\partial p_x(q)}{\partial q_2} & \frac{\partial p_x(q)}{\partial q_3} \\ \frac{\partial p_y(q)}{\partial q_1} & \frac{\partial p_y(q)}{\partial q_2} & \frac{\partial p_y(q)}{\partial q_3} \\ \frac{\partial p_z(q)}{\partial q_1} & \frac{\partial p_z(q)}{\partial q_2} & \frac{\partial p_z(q)}{\partial q_3} \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \frac{\partial p_x(q)}{\partial q_1} & \frac{\partial p_x(q)}{\partial q_2} & \dots & \frac{\partial p_x(q)}{\partial q_n} \\ \frac{\partial p_y(q)}{\partial q_1} & \frac{\partial p_y(q)}{\partial q_2} & \dots & \frac{\partial p_y(q)}{\partial q_n} \\ \frac{\partial p_z(q)}{\partial q_1} & \frac{\partial p_z(q)}{\partial q_2} & \dots & \frac{\partial p_z(q)}{\partial q_n} \\ \hline \zeta_1 z_0(q) & \zeta_2 z_1(q) & \dots & \zeta_n z_{n-1}(q) \end{bmatrix}$$

$$z_i = {}^0R_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{where } {}^0R_i = \prod_{k=1}^i {}^{k-1}R_k$$

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ then}$$

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\det(J) = j_{11}(j_{33}j_{22} - j_{32}j_{23}) - j_{21}(j_{33}j_{12} - j_{32}j_{13}) + j_{31}(j_{23}j_{12} - j_{22}j_{13})$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J(q) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\tau = J(q)^T F$$

$$F_i = \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i}$$

$$\tau_i = \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i}$$

$$K_j = \frac{1}{2}m_j v_{\varphi}^2 + \frac{1}{2}I_j \omega_j^2$$

$$P_j = -m_j G^T p_{\varphi}$$

$$\dot{\theta}_{\max} = \frac{\theta_h - \theta_b}{t_h - t_b} = \ddot{\theta}_d t_b$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}_d^2 t_f^2 - 4\ddot{\theta}_d(\theta_f - \theta_i)}}{2|\ddot{\theta}_d|}$$

$$\theta(t) = \theta_i + \frac{1}{2}\ddot{\theta}_d t^2, \quad \dot{\theta}(t) = \ddot{\theta}_d t,$$

$$\text{and } \ddot{\theta}(t) = \ddot{\theta}_d$$

$$\theta(t) = \theta_i + \frac{1}{2}\ddot{\theta}_d t_b^2 + \ddot{\theta}_d t_b(t - t_b), \quad \dot{\theta}(t) = \ddot{\theta}_d t_b,$$

$$\text{and } \ddot{\theta}(t) = 0$$

$$\theta(t) = \theta_f - \frac{1}{2}\ddot{\theta}_d (t_f - t)^2, \quad \dot{\theta}(t) = \ddot{\theta}_d (t_f - t),$$

$$\text{and } \ddot{\theta}(t) = -\ddot{\theta}_d$$

**The End**

$$\text{Gaussian } M = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \text{ Mean } M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{Lap1 } M = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}, \text{ Lap2 } M = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\text{Sobel } M_h = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } M_v = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Prewitt } M_h = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } M_v = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$F = A + c(A - F_{smooth})$$