

ENGPYHS 2A04 Winter 2022 – Assignment 2

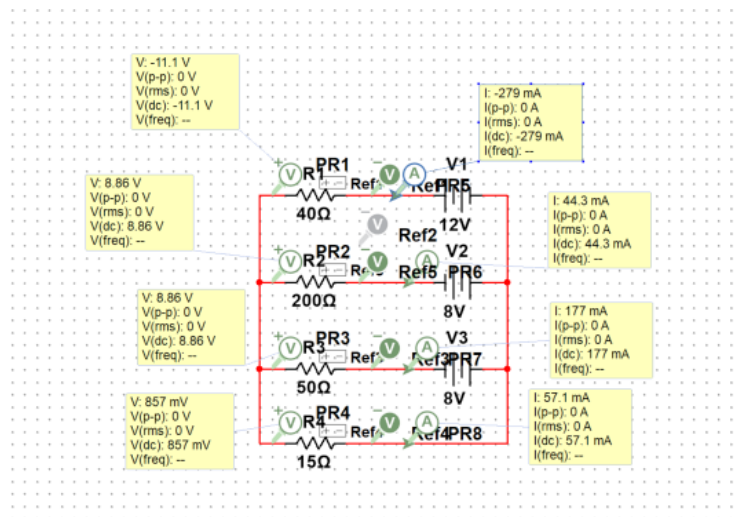
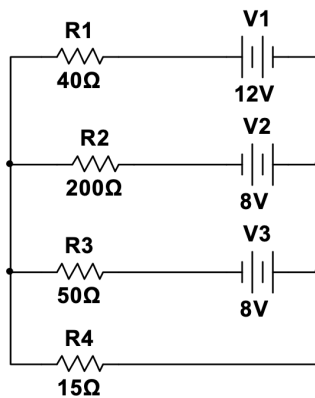
DUE MONDAY JANUARY 31, 8AM

1. DC Circuit Analysis

5 marks total: 2 marks currents, 2 marks voltages , 1 mark for showing your work

Using Kirchhoff's rules, (a) -

- Find the current across each resistor shown below
- And the find the potential difference across each resistor



Resistor	Current (mA)	Voltage (V)
1	-279	-11.1
2	44.3	8.86
3	177	8.86
4	57.1	0.857

Approach (1): Nodal Analysis

V_A = Voltage at the left junction

$$\textcircled{1} \quad I_1 + I_2 + I_3 + I_4 = 0$$

$$\frac{V_A + 12}{R_1} + \frac{V_A - 8}{R_2} + \frac{V_A - 8}{R_3} + \frac{V_A}{R_4} = 0$$

$$\frac{V_A + 12}{40} + \frac{V_A - 8}{200} + \frac{V_A - 8}{50} + \frac{V_A}{15} = 0$$

$$\frac{15(V_A + 12)}{600} + \frac{3(V_A - 8)}{600} + \frac{12(V_A - 8)}{600} + \frac{40V_A}{600} = 0$$

$$\frac{70V_A}{600} + \frac{60}{600} = 0$$

$$V_A = 0.857$$

$\textcircled{2}$ Substitute V_A into current equations

$$I_1 = \frac{V_A + 12}{40} = 0.278 \text{ A}$$

$$I_2 = \frac{V_A - 8}{200} = -0.044 \text{ A}$$

$$I_3 = \frac{V_A - 8}{50} = -0.177 \text{ A}$$

$$I_4 = \frac{V_A}{15} = -0.0571 \text{ A}$$

$\textcircled{3}$ Use Ohm's Law to find the voltage across each resistor

$$V_1 = I_1 R_1 = 11.1 \text{ V}$$

$$V_2 = I_2 R_2 = 8.85$$

$$V_3 = I_3 R_3 = 8.85$$

$$V_4 = I_4 R_4 = 0.857$$

2. **AC Circuit Analysis:** A certain lightbulb is rated at 90.0 W when operating at an rms voltage of 120 V.

- a. What is the peak voltage applied across the bulb? (1)

$$V_{\max} = 120\sqrt{2} = \sim 170$$

- b. What is the resistance of the bulb? (1)

$$P = VI = \frac{V^2}{R} \rightarrow 90 = \frac{(120)^2}{R} \rightarrow R = 160\Omega$$

- c. Does a 100-W bulb have greater or less resistance than a 60.0-W bulb? Explain.

It has less resistance. 100 W light bulb has a resistance of 144 Ohms. (2)

3. **AC Circuit Analysis:** An AC source with an output rms voltage of 48.0 V at a frequency of 60.0 Hz is connected across a 11.0-mF capacitor. Find:

- a. the capacitive reactance (1)

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60)(11 \times 10^{-3})} = 0.241 \Omega$$

- b. the rms current, and (1)

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{48}{241} = 200A$$

- c. the maximum current in the circuit. (1)

$$I_{max} = \sqrt{2}I_{rms} = 280 A$$

- d. Does the capacitor have its maximum charge when the current has its maximum value? Explain. (2)

No. The capacitor will have minimum charge when the current is the maximum. The current and voltage are out of phase by 90° .

4. Phasors:

- a. Find the phasors of the following time function

i. $v(t) = 10 \cos\left(\omega t - \frac{\pi}{4}\right) (V)$ (1)

$$\tilde{V} = 10e^{-j\frac{\pi}{4}}$$

ii. $v(t) = 30 \sin\left(\omega t - \frac{\pi}{4}\right) (V)$ (1)

$$\tilde{V} = 30e^{j\left(-\frac{\pi}{4}-\frac{\pi}{2}\right)} = 30e^{-j\frac{3\pi}{4}}$$

iii. $i(x, t) = 10e^{-3x} \sin(\omega t + \pi/4) (A)$ (1)

$$\tilde{I} = 10e^{-3x} e^{j\left(\frac{\pi}{4}-\frac{\pi}{2}\right)} = 10e^{-3x} e^{-j\frac{\pi}{4}}$$

- b. Find the instantaneous time sinusoidal function corresponding to the following phasors

i. $\tilde{I} = (6 + j8) (A)$ (1)

$$\tilde{I} = 10e^{j53^\circ} \rightarrow i(t) = \cos(\omega t + 53^\circ) = \cos(\omega t + 0.927)$$

ii. $\tilde{V} = j (V)$ (1)

$$v(t) = \cos(\omega t + \pi/2) = -\sin(\omega t)$$

5. **Bonus Question:** A source delivers an AC voltage of the form $\Delta v = 100 \sin 50\pi t$, where Δv is in volts and t is in seconds, to a capacitor. The maximum current in the circuit is 0.500 A.

- a. Find for the rms voltage of the source (1)

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}}$$

$$\Delta V_{rms} = \frac{100}{\sqrt{2}}$$

$$\Delta V_{rms} = 70.7 \text{ V}$$

- b. Find the frequency of the source (1)

Given the general expression of a AC source:

$$\Delta v = v_0 \sin 2\pi f t$$

We can solve for the frequency:

$$2\pi f = 50\pi$$

$$f = 25 \text{ Hz}$$

- c. Find the value of the capacitance (3)

$$X_c = \frac{1}{2\pi f C}$$

First we must find the capacitive reactance. Given the capacitor is the sole element in the circuite, capacitance can be solved sing:

$$X_c = \frac{\Delta V_{max}}{I_{max}}$$

$$\frac{1}{2\pi f C} = \frac{\Delta V_{max}}{I_{max}}$$

$$2\pi f C = \frac{I_{max}}{\Delta V_{max}}$$

$$C = \frac{I_{max}}{\Delta V_{max} 2\pi f}$$

$$C = \frac{0.5}{(100) 2\pi(25)}$$

$$C = 31.8 \mu F$$

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal marks (except bonus questions).
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.