Due: Nov 3rd, 2016

Dropbox #10 in JHE 307

1. For the RPP planar robot shown in Fig. 2.23 in lecture note.

a) Using the method of chapter 3, derive the 3x3 manipulator Jacobian matrix.

b) Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.

where,

$$A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & -s\theta_1 & -c\theta_1 & d_2 s\theta_1 - d_3 c\theta_1 \\ 0 & c\theta_1 & -s\theta_1 & -d_2 c\theta_1 - d_3 s\theta_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. For the PPR robot shown in Fig. 2.26:

a) Using the method of chapter 3, derive the 3x3 manipulator Jacobian matrix.

b) Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.

where

$$^{n}T_{n+1} = A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C(180^\circ) & -S(180^\circ)C(90^\circ) & S(180^\circ)S(90^\circ) & (0)C(180^\circ) \\ S(180^\circ) & C(180^\circ)C(90^\circ) & -C(180^\circ)S(90^\circ) & (0)S(180^\circ) \\ 0 & S(90^\circ) & C(90^\circ) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C(-90^\circ) & -S(-90^\circ)C(90^\circ) & S(-90^\circ)S(90^\circ) & (0)C(-90^\circ) \\ S(-90^\circ) & C(-90^\circ)C(90^\circ) & -C(-90^\circ)S(90^\circ) & (0)S(-90^\circ) \\ 0 & S(90^\circ) & C(90^\circ) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3C(0^\circ) & S\theta_3S(0^\circ) & a_3C\theta_3 \\ S\theta_3 & C\theta_3C(0^\circ) & -C\theta_3S(0^\circ) & a_3S\theta_3 \\ 0 & S(0^\circ) & C(0^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = {}^{0}T_1 * {}^{1}T_2 * {}^{2}T_3 = A_1 * A_2 * A_3$$

$$^{0}T_3 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ S\theta_3 & C\theta_3 & 0 & d_2 + a_3S\theta_3 \\ -C\theta_3 & S\theta_3 & 0 & d_1 - a_3C\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Note: this planar robot is defined in y-z plane, so the 3*3 Jacobian Matrix should be related to V_y, V_z, ω_x)