

MECHTRON 2MD3

Data Structures and Algorithms for Mechatronics

Winter 2022

# 19 Queues

Department of Computing and Software

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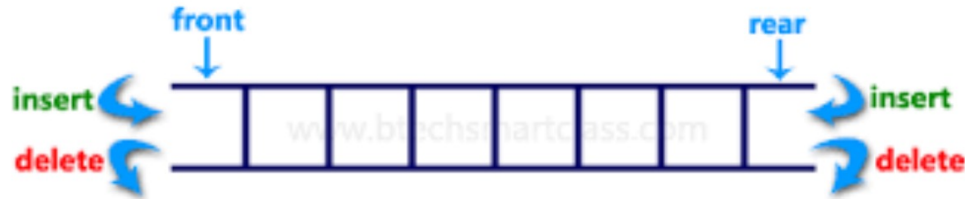
March 7, 2022

# Admin.

- One more day for the assignment 2!

# Double-Ended Queues

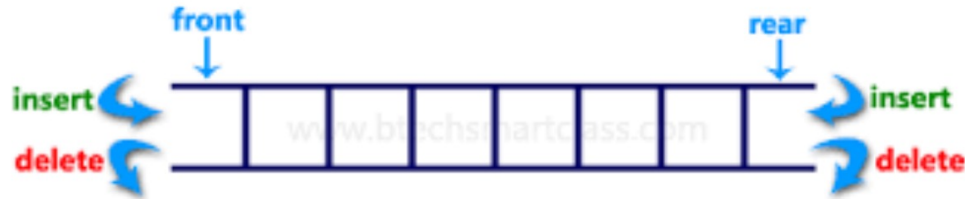
- Double-Ended Queues (sometimes pronounced like "deck")
  - supports insertion and deletion at both the front and the rear of the queue



- **insertFront(e):** Insert a new element e at the beginning of the deque.
- **insertBack(e):** Insert a new element e at the end of the deque.
- **eraseFront():** Remove the first element of the deque; an error occurs if the deque is empty.
- **eraseBack():** Remove the last element of the deque; an error occurs if the deque is empty.
- **front():** Return the first element of the deque; an error occurs if the deque is empty.
- **back():** Return the last element of the deque; an error occurs if the deque is empty.
- **size():** Return the number of elements of the deque.
- **empty():** Return true if the deque is empty and false otherwise.

# Double-Ended Queues

- Double-Ended Queues (sometimes pronounced like "deck")
  - supports insertion and deletion at both the front and the rear of the queue



- A running example:

<b>Operation</b>	<b>Output</b>	<b>D</b>
insertFront(3)	—	(3)
insertFront(5)	—	(5, 3)
front()	5	(5, 3)
eraseFront()	—	(3)
insertBack(7)	—	(3, 7)
back()	7	(3, 7)
eraseFront()	—	(7)
eraseBack()	—	()

# Implementation with Doubly Linked List



- We will use the functionalities provided by the DLL to implement `LinkedDeque`'s functions. We have seen this pattern a few times before.*

```
typedef string Elem;                                // deque element type
class LinkedDeque {                                  // deque as doubly linked list
public:
    LinkedDeque();                                    // constructor
    int size() const;                                  // number of items in the deque
    bool empty() const;                                // is the deque empty?
    const Elem& front() const throw(DequeEmpty); // the first element
    const Elem& back() const throw(DequeEmpty); // the last element
    void insertFront(const Elem& e);                    // insert new first element
    void insertBack(const Elem& e);                     // insert new last element
    void removeFront() throw(DequeEmpty);              // remove first element
    void removeBack() throw(DequeEmpty);               // remove last element
private:                                           // member data
    DLinkedList D;                                  // linked list of elements
    int n;                                           // number of elements
};
```

# Implementation with Doubly Linked List



- We will use the functionalities provided by the DLL to implement LinkedDeque's functions. We have seen this pattern a few times before.
- Performance of a deque realized by a doubly linked list.

<i>Operation</i>	<i>Time</i>
size	$O(1)$
empty	$O(1)$
front, back	$O(1)$
insertFront, insertBack	$O(1)$
eraseFront, eraseBack	$O(1)$

- The space used usage is  $O(n)$

```

// insert new first element
void LinkedDeque::insertFront(const Elem& e) {
    D.addFront(e); ←
    n++;
}

// insert new last element
void LinkedDeque::insertBack(const Elem& e) {
    D.addBack(e); ←
    n++;
}

// remove first element
void LinkedDeque::removeFront() throw(DequeEmpty) {
    if (empty())
        throw DequeEmpty("removeFront of empty deque");
    D.removeFront(); ←
    n--;
}

// remove last element
void LinkedDeque::removeBack() throw(DequeEmpty) {
    if (empty())
        throw DequeEmpty("removeBack of empty deque");
    D.removeBack(); ←
    n--;
}
    
```

# Adapter Design Pattern

- Design pattern: which describes a solution to a “typical” software design problem.
  - provides a general template for a solution that can be applied in many different situations.
  - describes the main elements of a solution in an abstract way that can be specialized for a specific problem at hand.
  - In Algorithms:
    - Recursion
    - Using Stack to solve problems
  - In Software Engineering
    - Adapter pattern
    - Iterator pattern
- You remember from previous LinkedDeque, and also Circular Linked List-based implementation of Queue

# Adapter Design Pattern

- You remember from previous LinkedDeque, and also Circular Linked List-based implementation of Queue that we took an existing data structure and **adapted** it
  - E.g. we added size **n**
  - We added operations that are meaningful for the new data structure
- For the operations, we have simply **mapped** each deque operation to the corresponding operation of DLinkedList.
- An adapter (also called a wrapper) is a data structure that translates one interface to another.
  - e.g.: In the LinkedDeque implementation:
    - deque operation **insertFront** is mapped to the corresponding operation of DLinkedList **addFront**



# Adapter Design Pattern

- Implementing a Stack using Deque:

```
typedef string Elem;           // element type
class DequeStack {           // stack as a deque
public:
    DequeStack();             // constructor
    int size() const;         // number of elements
    bool empty() const;       // is the stack empty?
    const Elem& top() const throw(StackEmpty); // the top element
    void push(const Elem& e);  // push element onto stack
    void pop() throw(StackEmpty); // pop the stack
private:
    LinkedDeque D;           // deque of elements
};
```

<i>Stack Method</i>	<i>Deque Implementation</i>
size()	size()
empty()	empty()
top()	front()
push(o)	insertFront(o)
pop()	eraseFront()

```
DequeStack::DequeStack()           // constructor
: D() { }

// number of elements
int DequeStack::size() const
{ return D.size(); }

// is the stack empty?
bool DequeStack::empty() const
{ return D.empty(); }

// the top element
const Elem& DequeStack::top() const throw(StackEmpty) {
    if (empty())
        throw StackEmpty("top of empty stack");
    return D.front();
}

// push element onto stack
void DequeStack::push(const Elem& e)
{ D.insertFront(e); }

// pop the stack
void DequeStack::pop() throw(StackEmpty)
{
    if (empty())
        throw StackEmpty("pop of empty stack");
    D.removeFront();
}
```

# Adapter Design Pattern

- Implementing a Queue using Deque:

<i><b>Queue Method</b></i>	<i><b>Deque Implementation</b></i>
size()	size()
empty()	empty()
front()	front()
enqueue( <i>e</i> )	insertBack( <i>e</i> )
dequeue()	eraseFront()

- The operations are equally efficient.
- We have used and will use this design pattern many times.

# Standard Template Library (STL)

- The Standard Template Library (STL) is a collection of classes for common data structures. In addition to the string class, which we have seen many times, it also provides data structures for the following standard containers.
  - string (String class with all operations)
  - stack (Container with last-in, first-out access)
  - queue (Container with first-in, first-out access)
  - deque (Double-ended queue)
  - vector (Resizable array)
  - list (Doubly linked list)
  - priority queue (Queue ordered by value)
  - set (Set)
  - map Associative array (dictionary)

# Standard Template Library (STL)

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- string:

```
#include <string>
using std::string;
// ...
string s = "to be";
string t = "not " + s;
string u = s + " or " + t;
if (s > t)
    cout << u;
```

```
// t = "not to be"
// u = "to be or not to be"
// true: "to be" > "not to be"
// outputs "to be or not to be"
```

```
string s = "John";
int i = s.size();
char c = s[3];
s += " Smith";
```

```
// s = "John"
// i = 4
// c = 'n'
// now s = "John Smith"
```

s.find(p)	Return the index of first occurrence of string <i>p</i> in <i>s</i>
s.find(p, i)	Return the index of first occurrence of string <i>p</i> in <i>s</i> on or after position <i>i</i>
s.substr(i,m)	Return the substring starting at position <i>i</i> of <i>s</i> and consisting of <i>m</i> characters
s.insert(i, p)	Insert string <i>p</i> just prior to index <i>i</i> in <i>s</i>
s.erase(i, m)	Remove the substring of length <i>m</i> starting at index <i>i</i>
s.replace(i, m, p)	Replace the substring of length <i>m</i> starting at index <i>i</i> with <i>p</i>
getline(is, s)	Read a single line from the input stream <i>is</i> and store the result in <i>s</i>

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  - vector:

```
#include <vector>
using namespace std;           // make std accessible

vector<int> scores(100);        // 100 integer scores
vector<char> buffer(500);       // buffer of 500 characters
vector<Passenger> passenList(20); // list of 20 Passengers
```

# Standard Template Library (STL)

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  - stack:

```
#include <stack>
using std::stack;           // make stack accessible
stack<int> myStack;         // a stack of integers
```

**size()**: Return the number of elements in the stack.  
**empty()**: Return true if the stack is empty and false otherwise.  
**push(*e*)**: Push *e* onto the top of the stack.  
**pop()**: Pop the element at the top of the stack.  
**top()**: Return a reference to the element at the top of the stack.

# Standard Template Library (STL)

- The Standard Template Library (STL) is a collection of classes for common data structures. In addition to the string class, which we have seen many times, it also provides data structures for the following standard containers.
  - queue:

```
#include <queue>
using std::queue;           // make queue accessible
queue<float> myQueue;       // a queue of floats
```

**size():** Return the number of elements in the queue.

**empty():** Return true if the queue is empty and false otherwise.

**push(*e*):** Enqueue *e* at the rear of the queue.

**pop():** Dequeue the element at the front of the queue.

**front():** Return a reference to the element at the queue's front.

**back():** Return a reference to the element at the queue's rear.



# Standard Template Library (STL)

- The Standard Template Library (STL) is a collection of classes for common data structures. In addition to the string class, which we have seen many times, it also provides data structures for the following standard containers.
  - deque:

```
#include <deque>
using std::deque;           // make deque accessible
deque<string> myDeque;      // a deque of strings
```

**size():** Return the number of elements in the deque.

**empty():** Return true if the deque is empty and false otherwise.

**push\_front(*e*):** Insert *e* at the beginning the deque.

**push\_back(*e*):** Insert *e* at the end of the deque.

**pop\_front():** Remove the first element of the deque.

**pop\_back():** Remove the last element of the deque.

**front():** Return a reference to the deque's first element.

**back():** Return a reference to the deque's last element.



# List and Sequence Containers

- Vector (also called Array List)
  - Access each element using a notion of index in  $[0, n-1]$
  - Index of element  $e$ : the number of elements that are before  $e$
  - Typically we use the “index” (e.g., `[ ]`)
  - A more general ADT than “array”
- List
  - Not using an index to access, but use a node to access
  - Insert a new element  $e$  before some “position”  $p$
  - A more general ADT than “linked list”
- Sequence
  - Can access an element as vector and list (using both index and position)

# The Vector ADT

- The Vector or Array List ADT extends the notion of array by storing a sequence of objects
- An element can be accessed, inserted or removed by specifying its index (number of elements preceding it)
- An exception is thrown if an incorrect index is given (e.g., a negative index)
- Main methods:
  - $\text{at}(i)$ : Return the element of  $V$  with index  $i$ ; an error condition occurs if  $i$  is out of range.
  - $\text{set}(i, e)$ : Replace the element at index  $i$  with  $e$ ; an error condition occurs if  $i$  is out of range.
  - $\text{insert}(i, e)$ : Insert a new element  $e$  into  $V$  to have index  $i$ ; an error condition occurs if  $i$  is out of range.
  - $\text{erase}(i)$ : Remove from  $V$  the element at index  $i$ ; an error condition occurs if  $i$  is out of range.

# Array-based Implementation of Vector

- Use an array  $A$  of size  $N$
- A variable  $n$  keeps track of the size of the array list (number of elements stored)
- Operation **at( $i$ )** is implemented in  $O(1)$  time by returning  $A[i]$
- Operation **set( $i, o$ )** is implemented in  $O(1)$  time by performing  $A[i] = o$



**at( $i$ ):** Return the element of  $V$  with index  $i$ ; an error condition occurs if  $i$  is out of range.

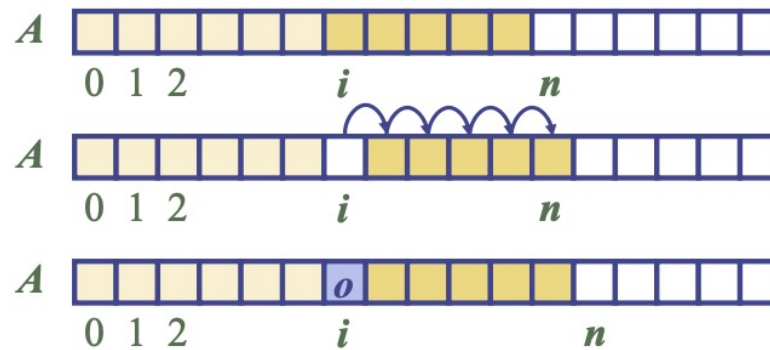
**set( $i, e$ ):** Replace the element at index  $i$  with  $e$ ; an error condition occurs if  $i$  is out of range.

**insert( $i, e$ ):** Insert a new element  $e$  into  $V$  to have index  $i$ ; an error condition occurs if  $i$  is out of range.

**erase( $i$ ):** Remove from  $V$  the element at index  $i$ ; an error condition occurs if  $i$  is out of range.

# Array-based Implementation of Vector - Insertion

- In operation  $\text{insert}(i, o)$ , we need to make room for the new element by shifting forward the  $n - i$  elements  $A[i], \dots, A[n - 1]$ 
  - In the worst case ( $i = 0$ ), this takes  $O(n)$  time



$\text{at}(i)$ : Return the element of  $V$  with index  $i$ ; an error condition occurs if  $i$  is out of range.

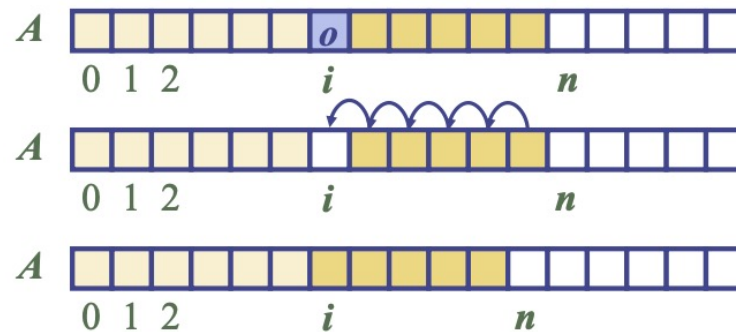
$\text{set}(i, e)$ : Replace the element at index  $i$  with  $e$ ; an error condition occurs if  $i$  is out of range.

$\text{insert}(i, e)$ : Insert a new element  $e$  into  $V$  to have index  $i$ ; an error condition occurs if  $i$  is out of range.

$\text{erase}(i)$ : Remove from  $V$  the element at index  $i$ ; an error condition occurs if  $i$  is out of range.

# Array-based Implementation of Vector - Removal

- In operation `erase(i)`, we need to fill the hole left by the removed element by shifting backward the  $n - i - 1$  elements  $A[i + 1], \dots, A[n - 1]$ 
  - In the worst case ( $i = 0$ ), this takes  $O(n)$  time



`at(i)`: Return the element of  $V$  with index  $i$ ; an error condition occurs if  $i$  is out of range.

`set(i, e)`: Replace the element at index  $i$  with  $e$ ; an error condition occurs if  $i$  is out of range.

`insert(i, e)`: Insert a new element  $e$  into  $V$  to have index  $i$ ; an error condition occurs if  $i$  is out of range.

`erase(i)`: Remove from  $V$  the element at index  $i$ ; an error condition occurs if  $i$  is out of range.

# Array-based Implementation of Vector - Performance

- In the array-based implementation of an array list:
  - The space used by the data structure is  $O(n)$
  - `size`, `empty`, `at` and `set` run in  $O(1)$  time
  - `insert` and `erase` run in  $O(n)$  time in worst case
- If we use the array in a circular fashion, operations `insert(0, x)` and `erase(0, x)` run in  $O(1)$  time
- In an insert operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

<i><b>Operation</b></i>	<i><b>Time</b></i>
<code>size()</code>	$O(1)$
<code>empty()</code>	$O(1)$
<code>at(i)</code>	$O(1)$
<code>set(i, e)</code>	$O(1)$
<code>insert(i, e)</code>	$O(n)$
<code>erase(i)</code>	$O(n)$

# Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time  $T(n)$  needed to perform a series of  $n$  insert(o) operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of an insert operation the average time taken by an insert over the series of operations, i.e.,  $T(n)/n$

# Incremental Strategy Analysis

- We replace the array  $k = n/c$  times
- The total time  $T(n)$  of a series of  $n$  insert operations is proportional to

$$\begin{aligned} n + c + 2c + 3c + 4c + \dots + kc &= \\ n + c(1 + 2 + 3 + \dots + k) &= \\ n + ck(k + 1)/2 \end{aligned}$$

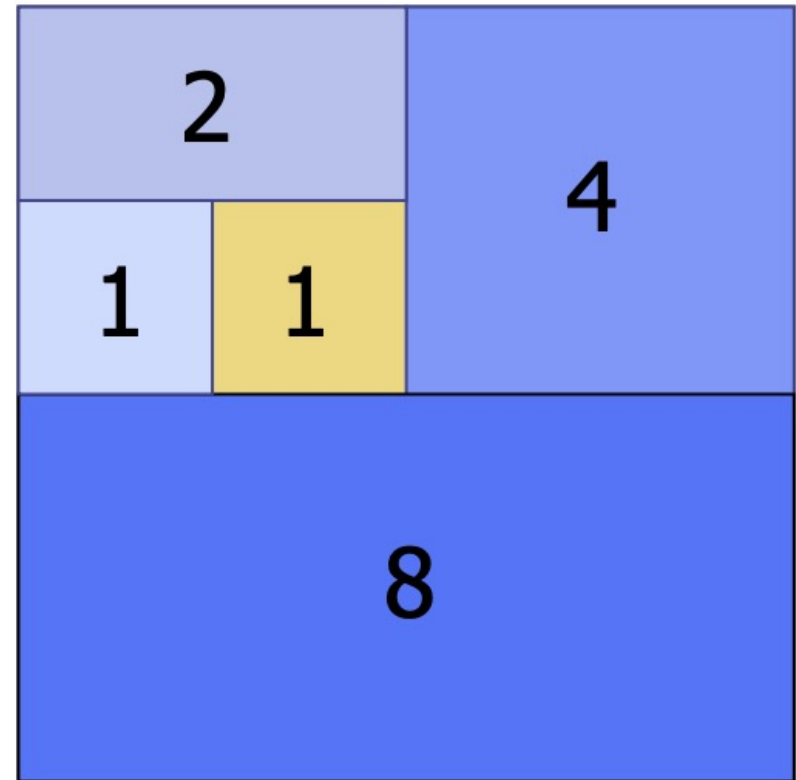
- Since  $c$  is a constant,  $T(n)$  is  $O(n + k^2)$ , i.e.,  $O(n^2)$
- The amortized time of an insert operation is  $O(n)$



# Doubling Strategy Analysis

- We replace the array  $k = \log_2 n$  times
- The total time  $T(n)$  of a series of  $n$  insert operations is proportional to
- $$n + 1 + 2 + 4 + 8 + \dots + 2^k = n + 2^{k+1} - 1 =$$
- $$3n - 1$$
- $T(n)$  is  $O(n)$
- The amortized time of an insert operation is  $O(1)$

## geometric series



# Questions?