ME 4K03

Assignment #4

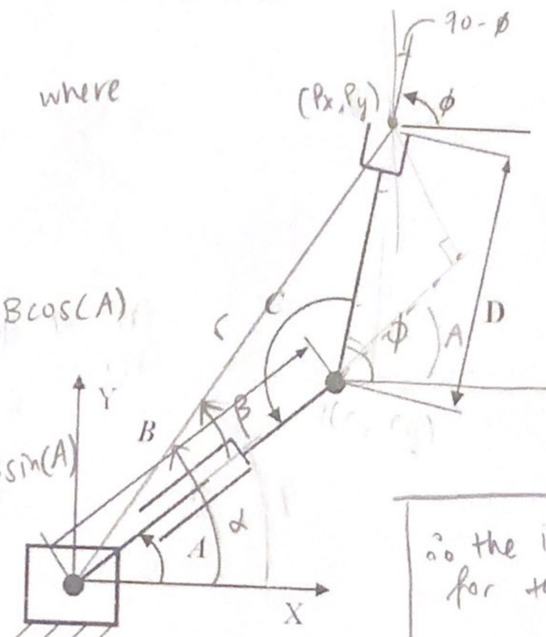
Due: 11:59pm, 19 November, 2021

1. A planar RPR robot is shown below. Its joint variables are A, B and C. Length D is a constant. Its end-effector position and orientation are given by P_x, P_y and ϕ. Derive the inverse kinematics equations for this robot.

$$\cos(C) = \frac{(B^2 + D^2 - r^2)}{2BD} \quad \text{where}$$

$$r = \sqrt{P_x^2 + P_y^2}$$

$$(\phi - A) + C = 180^\circ$$



$$B_x = P_x - D\sin(90 - \phi) = B\cos(A)$$
$$= P_x - D\cos(\phi)$$

$$B_y = P_y - D\cos(90 - \phi) = B\sin(A)$$
$$= P_y - D\sin(\phi)$$

$$B = \sqrt{B_x^2 + B_y^2}$$

$$\cos(A) = \frac{P_x - D\cos(\phi)}{B}$$

$$\sin(A) = \frac{P_y - D\sin(\phi)}{B}$$

$$A = \operatorname{atan2}\left(\frac{P_y - D\sin\phi}{B}, \frac{P_x - D\cos\phi}{B}\right)$$

∴ the inverse kinematic equations for this robot are:

$$B = \sqrt{(P_x - D\cos(\phi))^2 + (P_y - D\sin(\phi))^2}$$

$$A = \operatorname{atan2}\left(\frac{P_y - D\sin\phi}{B}, \frac{P_x - D\cos\phi}{B}\right)$$

$$C = \cos^{-1}\left(\frac{B^2 + D^2 - P_x^2 - P_y^2}{2BD}\right)$$

$$(\phi - A) + C = 180^\circ$$