ME 4K03 MIDTERM 3

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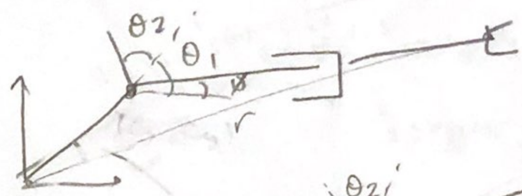
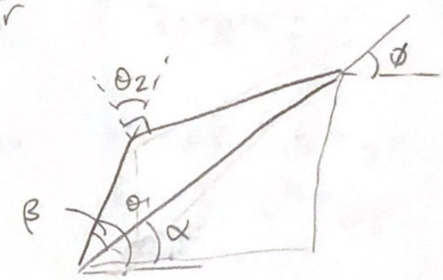
1. RRP robot.

link $1 = a_1$

$P_x = a_1 \cos\theta_1 + d_3 \cos\phi$

$P_y = a_1 \sin\theta_1 + d_3 \sin\phi$





$(\theta_1 - \phi) + \theta_2 = 90° \quad \dots (1)$

$r = \sqrt{P_x^2 + P_y^2}$

$\cos\theta_1 = \dfrac{P_x - d_3\cos\phi}{a_1}$

$\theta_1 = \text{atan2}\left(\dfrac{P_y - d_3\sin\phi}{a_1} - \dfrac{P_x - d_3\cos\phi}{a_1}\right) \quad \dots (2)$

$\sin\theta_1 = \dfrac{P_y - d_3\sin\phi}{a_1}$

$r\sin\alpha = P_y \qquad \alpha = \text{atan2}\left(\dfrac{P_y}{r}, \dfrac{P_x}{r}\right) \qquad \beta = \theta_1 - \alpha$

$r\cos\alpha = P_x$

$\cos\beta = \dfrac{r^2 + a_1^2 - d_3^2}{2ra_1} \rightarrow d_3 = \sqrt{-\cos\beta\,(2ra_1) + r^2 + a_1^2} \quad \dots (3)$

2. Newo $\rightarrow$ 1=2.

masses of the links are concentrated

$$K_2 = \frac{1}{2} m_B v_{c2}^2 + \underbrace{\frac{1}{2} I_2 w_2^2}_{=0}$$

$$= \frac{1}{2} m_B v_{c2}^2$$

$x_B = d_1 \quad \dot{x}_B = \dot{d}_1 \qquad v_{c2}^2 = \dot{x}_B^2 + \dot{y}_B^2 = \dot{d}_1^2 + \dot{d}_2^2$

$y_B = d_2 \quad \dot{y}_B = \dot{d}_2$

$$K_2 = \frac{1}{2} m_B (\dot{d}_1^2 + \dot{d}_2^2)$$

$$P_2 = -m_B G^T p_{c2}$$

$$P_2 = -m_B [0 \ -g] \begin{bmatrix} x_B \\ y_B \end{bmatrix}$$

$$P_2 = m_B g d_2$$

$K_1 = \frac{1}{2} m_A \dot{d}_1^2 \quad P_1 = -m_A [0 \ -g] \begin{bmatrix} x_A \\ y_A \end{bmatrix} \rightarrow y_A = 0 \ \therefore P_1 = 0$

$K_3 = \frac{1}{2} m_c v_{c3}^2$

$v_{c3}^2 = \dot{x}_c^2 + \dot{y}_c^2 \qquad v_{c3}^2 = (\dot{d}_1 + a_3 \dot{\theta}_3 \sin\theta_3)^2 + (\dot{d}_2 + a_3 \dot{\theta}_3 \cos\theta_3)^2$

$x_c = d_1 - a_3 \cos\theta_3 \qquad \dot{x}_c = \dot{d}_1 + a_3 \dot{\theta}_3 \sin\theta_3$

$y_c = d_2 + a_3 \sin\theta_3 \qquad \dot{y}_c = \dot{d}_2 + a_3 \dot{\theta}_3 \cos\theta_3$

$$P_3 = -m_c [0 \ -g] \begin{bmatrix} x_c \\ y_c \end{bmatrix} = m_c g (d_2 + a_3 \sin\theta_3)$$

2. cont.

$$L = K_1 + K_2 + K_3 - P_1 - P_2 - P_3$$

$$L = \frac{1}{2} m_A \dot{d}_1^2 + \frac{1}{2} m_B (\dot{d}_1^2 + \dot{d}_2^2) + \frac{1}{2} m_C \Big(\underbrace{(\dot{d}_1 + a_3 \dot{\theta}_3 \sin\theta_3)^2 + (\dot{d}_2 + a_3 \dot{\theta}_3 \cos\theta_3)^2}_{a_3^2 \dot{\theta}_3^2 + 2 \sin\theta_3 a_3 \dot{d}_1 \dot{\theta}_3 + 2\cos(\theta_3) a_3 \dot{d}_2 \dot{\theta}_3 + \dot{d}_1^2 + \dot{d}_2^2} \Big)$$

$$- m_B g d_2 - m_C g (d_2 + a_3 \sin\theta_3)$$

$$L = \frac{1}{2} m_A \dot{d}_1^2 + \frac{1}{2} m_B \dot{d}_1^2 + \frac{1}{2} m_B \dot{d}_2^2 + \frac{1}{2} m_C \big(a_3^2 \dot{\theta}_3^2 + 2 S\theta_3 a_3 \dot{d}_1 \dot{\theta}_3 + 2 C\theta_3 a_3 \dot{d}_2 \dot{\theta}_3$$

$$+ \dot{d}_1^2 + \dot{d}_2^2 \big) - m_B g d_2 - m_C g d_2 - m_C g a_3 S\theta_3$$

bc prismatic

$$F_2 = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{d}_2}\right) - \frac{\partial L}{\partial d_2}$$

$$F_2 = \frac{\partial}{\partial t}\left(m_B \dot{d}_2 + m_C C\theta_3 a_3 \dot{\theta}_3 + m_C \dot{d}_2 \right) + m_B g + m_C g$$

$$F_2 = m_B \ddot{d}_2 + m_C \ddot{d}_2 + m_C a_3 \left(-\dot{\theta}_3 S\theta_3 \dot{\theta}_3 + \ddot{\theta}_3 C\theta_3 \right) + m_B g + m_C g$$

$$F_2 = \ddot{d}_2 (m_B + m_C) + m_C a_3 \left(-\dot{\theta}_3^2 S\theta_3 + \ddot{\theta}_3 C\theta_3 \right) + g (m_B + m_C)$$