ME 4K03 Assignment #3

1.a) $P_x = d_2 S\theta_1 - d_3 C\theta_1$

$P_y = -d_2 C\theta_1 - d_3 S\theta_1$

$P_z = 0$

$$J_A = \begin{bmatrix} \frac{dP_x}{d\theta_1} & \frac{dP_x}{dd_2} & \frac{dP_x}{dd_3} \\ \frac{dP_y}{d\theta_1} & \frac{dP_y}{dd_2} & \frac{dP_y}{dd_3} \\ \frac{dP_z}{d\theta_1} & \frac{dP_z}{dd_2} & \frac{dP_z}{dd_3} \end{bmatrix}$$

$$= \begin{bmatrix} d_2 C\theta_1 + d_3 S\theta_1 & S\theta_1 & -C\theta_1 \\ d_2 S\theta_1 - d_3 C\theta_1 & -C\theta_1 & -S\theta_1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 \\ S\theta_1 & 0 & -C\theta_1 \\ & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S\theta_1 \\ -C\theta_1 \\ 0 \end{bmatrix} \quad \xi_1 = 1, \; \xi_2 = \xi_3 = 0$$

$$z_2 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 \\ S\theta_1 & 0 & -C\theta_1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -S\theta_1 & -C\theta_1 \\ 0 & C\theta_1 & -S\theta_1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -C\theta_1 \\ -S\theta_1 \\ 0 \end{bmatrix}$$

$$J_B = [\xi_1 z_0 \;\; \xi_2 z_1 \;\; \xi_3 z_2] = \left[1 * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \;\; 0 \;\; 0\right] = [1 \;\; 0 \;\; 0]$$

$$J(q) = \begin{bmatrix} d_2 C\theta_1 + d_3 S\theta_1 & S\theta_1 & -C\theta_1 \\ d_2 S\theta_1 - d_3 C\theta_1 & -C\theta_1 & -S\theta_1 \\ 1 & 0 & 0 \end{bmatrix}$$

b) $\det(J(q)) = (d_2 C\theta_1 + d_3 S\theta_1)(0-0) - S\theta_1(0 + S\theta_1)$
$- C\theta_1(0 + C\theta_1)$
$= -S^2\theta_1 - C^2\theta_1 = -(S^2\theta_1 + C^2\theta_1) = -1$

$\therefore$ there are no configurations where this robot is singular

2.a) $J_A = \begin{bmatrix} \frac{dp_y}{dd_1} & \frac{dp_y}{dd_2} & \frac{dp_y}{d\theta_3} \\ \frac{dp_z}{dd_1} & \frac{dp_z}{dd_2} & \frac{dp_z}{d\theta_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & a_3 C\theta_3 \\ 1 & 0 & a_3 S\theta_3 \end{bmatrix}$

$z_1 = z_2 = 0 \quad z_3 = 1$

$z_2 = {}^0R_2 * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$J_B = \begin{bmatrix} 0 & 0 & 1 * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$

$J(q) = \begin{bmatrix} 0 & 1 & a_3 C\theta_3 \\ 1 & 0 & a_3 S\theta_3 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\det(J(q)) = -1 * (1 - a_3 S\theta_3 * 0) = -1$

$\therefore$ there are no configurations where this robot is singular