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2A04 A1

1. coax cable

$d_{inner} = 13\,mm \rightarrow a = 6.5\,mm$

$d_{outer} = 15\,mm \rightarrow b = 7.5\,mm$

copper

$\mu_c = \mu_0 = 4\pi \times 10^{-7}\ H/m$

$\sigma = 5.8 \times 10^{7}$ $\mu = \mu_0$
↑
air line

a) $f = 10\,kHz$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{6.5} + \frac{1}{7.5}\right) = \cancel{\sqrt{\pi f 4\pi \times 10^{-7}}}$$

$$R_s = \sqrt{\frac{\pi(10000)4\pi\times10^{-7}}{5.8\times10^{7}}}$$

$$R_s \doteq 2.60895 \times 10^{-5}$$

$$R' = \frac{2.609\times10^{-5}}{2\pi}\left(\frac{1}{6.5\times10^{-3}} + \frac{1}{7.5\times10^{-3}}\right)$$

$\cancel{R' = 1.44 \times 10^{-5}\ \Omega}$ $\boxed{R' = 1.19 \times 10^{-3}\ \Omega/m}$

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{7.5\times10^{-3}}{6.5\times10^{-3}}\right)$$

$$L' = \boxed{2.86 \times 10^{-8}\ H/m}$$

$$G' = 2\pi\sigma/\ln(7.5/6.5) = \varnothing\ S/m$$

$$C' = \frac{2\pi\, 8.854\times10^{-12}}{\ln(7.5/6.5)} = \boxed{3.89 \times 10^{-10}\ F/m}$$

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5) $\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$

$= \sqrt{[(1.19\times10^{-3} + j(20000\pi)(2.86\times10^{-8}))}$

$\times (0 + j(20000\pi)(3.89\times10^{-10}))]$

$= \sqrt{(-4.39\times10^{-8} + j\,2.91\times10^{-8})}$

$= 6.62\times10^{-5} + j\,2.20\times10^{-4}$

Attenuation constant

$\alpha = \mathrm{Re}(\gamma) = \boxed{6.62\times10^{-5}\ \text{Np/m}}$

Phase constant $\beta = \mathrm{Im}(\gamma)$

$\boxed{\beta = 2.20\times10^{-4}\ \text{rad/m}}$

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2. $f = 3000\ Hz$

dielectric = air
conductor = gold
$w = 5\,mm$
$h = 2\,mm$

$$u_p = \omega/\beta = f\lambda$$

$$\beta = Im(\gamma)$$

$$\beta = 1/\sqrt{L'C'}$$

$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = c/\sqrt{\varepsilon_r}$$

ε_r for air $= 1.0006$

$$u_p = \frac{3\times10^8}{\sqrt{1.0006}} = 2.9991\times10^8$$

$$\boxed{\doteq 3.0\times10^8\ m/s}$$

This value makes sense since we are measuring the ~~wave~~ phase velocity through air dielectric & metal conductors. Since these mediums are not insulators, [illegible] $\varepsilon_r \doteq 1$ because the waves are permitted to move easily through the medium. Because the wave is allowed to move at maximum speed, the phase velocity approaches the speed of light.

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3. $Z_0 = 120\,\Omega$ $\quad R_L = 80\,\Omega$

$f = 2\,MHz$ $\quad C_L = 11\,nF$

$Z_L = R_L \parallel (R_L + C_L)$

$Z_L = \left(80^{-1} + \left[80 + \left(\frac{1}{j\omega(11\times10^{-9})}\right)\right]^{-1}\right)^{-1}$

$\omega = 2\pi f = 4\pi\times10^{6}\ Hz$

$Z_L = \left(80^{-1} + \left(80 + \left(1/j\,4\pi\times10^{6}\cdot 11\times10^{-9}\right)\right)^{-1}\right)^{-1}$

$Z_L = (40.08 - 1.805j)\ \Omega$

$\Gamma = (Z_L - Z_0)/(Z_L + Z_0)$

$\Gamma = [(40.08 - j1.805) - 120]/[(40.08 - j1.805) + 120]$

$\Gamma = -0.499 - j0.0169$

$r = \sqrt{(0.499)^2 + (0.0169)^2}$

$r = 0.499$

$\Phi = \tan^{-1}(-0.0169/-0.499)$

$\Phi = 1.94° \leftrightarrow \Phi = -178.06°$

$\therefore\ \Gamma = (0.203,\ 1.94°)$

(magnitude, phase)

3. The voltage reflection coefficient describes the amount of voltage that is reflected from an incident wave travelling through a transmission line when the wave arrives at a discontinuity such as a change in medium.

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4) Lossless

$\ell = 10m$

$\omega = 2\pi \times 10^6$
$\Rightarrow f = 10^6\ Hz$

$V_g(t) = 12\cos(2\pi \times 10^6 t - \pi/3)$

$Z_g = 150\ \Omega$ $\qquad Z_0 = 100\ \Omega$

$\varepsilon_r = 2.1$

$Z_L = (120 - j40)\ \Omega$

a) $\lambda = \frac{u_p}{f} = \frac{u_p}{10^6} = \frac{2.07 \times 10^8}{10^6} = \boxed{207.02\ m}$

$u_p = c/\sqrt{\varepsilon_r} = \frac{3 \times 10^8}{\sqrt{2.1}} = 2.07 \times 10^8$

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b) $\Gamma = (Z_L - Z_0)/(Z_L + Z_0)$

$= (120 - j40 - 100)/(120 - j40 + 100)$

$= 0.12 - 0.16j$

$= (0.20, -53.13^\circ)$ in polar form

c) $Z_{in} = Z_0 \left[\frac{1+\Gamma_L}{1-\Gamma_L}\right] = Z_0 \left[\frac{z_L + j\tan(\beta l)}{1 + j z_L \tan(\beta l)}\right]$

$\Gamma_L = \Gamma e^{-j2\beta l}$

$z_L = \frac{Z_L}{Z_0} = \frac{120 - j40}{150} = 0.8 - 0.267j$ $\rightarrow$ $= 150 \left[\frac{0.8 - 0.267j + j\tan(10\beta)}{1 + j(0.8 - 0.267j)\tan(10\beta)}\right]$

$\beta = \frac{1}{\sqrt{L'C'}} = 1/\sqrt{\mu\varepsilon} = 1/\sqrt{4\pi \times 10^{-7} \cdot 2.1 \cdot 8.854 \times 10^{-12}}$

$\beta = 2.07 \times 10^{8}$ rad/m

$\boxed{Z_{in} = 134.1 + j50.43}$

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d) $\tilde{V}_i = \tilde{I}_i Z_{in}$

$$= \tilde{V}_g \left(\frac{Z_{in}}{Z_g + Z_{in}} \right)$$

$$= 12 e^{-\pi/3 j} \left(\frac{134.1 + j50.43}{150 + 134.1 + j50.43} \right)$$

$$= 12 e^{-\pi/3 j} \left(\frac{143.27 e^{j0.360 \text{ rad}}}{288.54 e^{j0.176 \text{ rad}}} \right)$$

$$= \left(5.96 e^{-j0.863 \text{ rad}} \right) \text{ V}$$

e) $r = 5.96$, $\phi = -0.863$ rad, $\cancel{f}\ \omega = 2\pi \times 10^6 \frac{\text{rad}}{\text{s}}$

$$V_i(t) = 5.96 \cos(2\pi \times 10^6 t - 0.863) \text{ V}$$

Question 5:

Gas pipelines and electrical transmission lines are sometimes constructed alongside one another to conserve space and cost. The transmission line carried an AC voltage, which is prone to inducing a current and voltage on the surrounding pipelines due to the electromagnetic fields produced by the transmission lines. This can cause issues with the pipeline, such as AC induced corrosion. The article explores solutions to this problem relevant to the course material, such as the use of an insulating material of a lower electrical permittivity, conductivity, and magnetic permeability to prevent the propagation of the electric and magnetic fields to the pipelines. The article also explores the change in geometry of the wire and the pipeline to change some of the transmission line parameters, like minimizing inductance.

Y. Zhang, C. Liao, Y. Shang, X. Zhong, and W. Cao, “Fast evaluation of lightning-induced voltages on the transmission lines above a lossy ground,” *IEEE Transactions on Electromagnetic Compatibility*, vol. 63, no. 6, May 2021.