Q1

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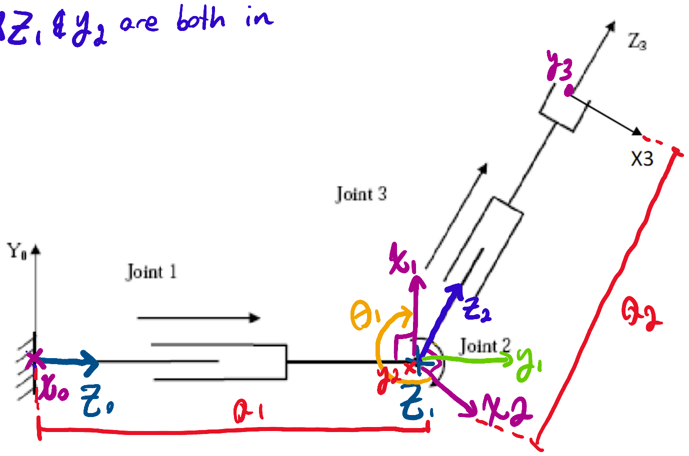
Alex Bartella, mecheng 4k03 assignment 2, 400308868

Z_1 & y_2 are both in



$n+1$	θ_{n+1}	d_{n+1}	a_{n+1}	α_{n+1}
	$x_n \parallel x_{n+1}$	x_n colinear x_{n+1}	Translate F_n until x_n & x_{n+1} are coincident	z_n coincide z_{n+1}
1	90°	a_1^*	∅	90°
2	θ_1^*	∅	∅	90°
3	∅	a_2^*	∅	∅

$$^{n}T_{n+1} = A_{n+1} = Rot(z,\theta_{n+1})\cdot Trans(0,0,d)\cdot Trans(a_{n+1},0,0)\cdot Rot(x,\alpha_{n+1})$$

$$= \begin{bmatrix} \cos\theta_{n+1} & -\sin\theta_{n+1}\cos\alpha_{n+1} & \sin\theta_{n+1}\sin\alpha_{n+1} & a_{n+1}\cos\theta_{n+1} \\ \sin\theta_{n+1} & \cos\theta_{n+1}\cos\alpha_{n+1} & -\cos\theta_{n+1}\sin\alpha_{n+1} & a_{n+1}\sin\theta_{n+1} \\ 0 & \sin\alpha_{n+1} & \cos\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{1}T_2 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ \cos(\theta_1) & 0 & \sin(\theta_1) & a_2\cdot\sin(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & a_1 - a_2\cdot\cos(\theta_1) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q2

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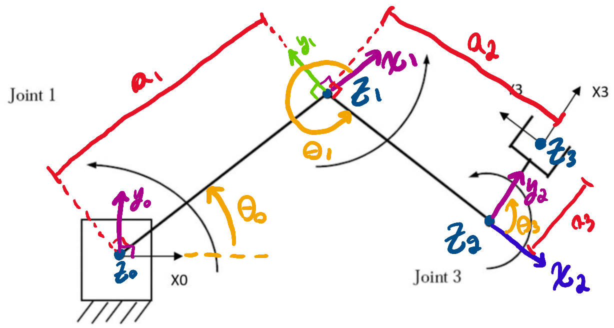

n+1	θ_{n+1}	d_{n+1}	a_{n+1}	α_{n+1}
1	θ_0^*	$\varnothing$	a_1	$\varnothing$
2	θ_1^*	$\varnothing$	a_2	$\varnothing$
3	θ_2^*	$\varnothing$	a_3	$\varnothing$

$${}^0T_1 = \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) & 0 & a_1\cdot\cos(\theta_0) \\ \sin(\theta_0) & \cos(\theta_0) & 0 & a_1\cdot\sin(\theta_0) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_2\cdot\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_2\cdot\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2T_3 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_3\cdot\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_3\cdot\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = {}^0T_1 + {}^1T_2 + {}^2T_3$$

$$= \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) & 0 & a_1\cdot\cos(\theta_0) \\ \sin(\theta_0) & \cos(\theta_0) & 0 & a_1\cdot\sin(\theta_0) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_2\cdot\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_2\cdot\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_3\cdot\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_3\cdot\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\sin(\theta_0)\cdot\sin(\theta_1) + \cos(\theta_0)\cdot\cos(\theta_1) & -\sin(\theta_0)\cdot\cos(\theta_1) - \sin(\theta_1)\cdot\cos(\theta_0) & 0 & a_1\cos(\theta_0) - a_2\cdot\sin(\theta_0)\cdot\sin(\theta_1) + a_2\cdot\cos(\theta_0)\cdot\cos(\theta_1) \\ \sin(\theta_0)\cdot\cos(\theta_1) + \sin(\theta_1)\cdot\cos(\theta_0) & -\sin(\theta_0)\cdot\sin(\theta_1) + \cos(\theta_0)\cdot\cos(\theta_1) & 0 & a_1\cdot\sin(\theta_0) + a_2\cdot\sin(\theta_0)\cdot\cos(\theta_1) + a_2\cdot\sin(\theta_1)\cdot\cos(\theta_0) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_3\cdot\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_3\cdot\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_0 + \theta_1 + \theta_2) & -\sin(\theta_0 + \theta_1 + \theta_2) & 0 & a_1\cdot\cos(\theta_0) + a_2\cdot\cos(\theta_0 + \theta_1) + a_3\cdot\cos(\theta_0 + \theta_1 + \theta_2) \\ \sin(\theta_0 + \theta_1 + \theta_2) & \cos(\theta_0 + \theta_1 + \theta_2) & 0 & a_1\cdot\sin(\theta_0) + a_2\cdot\sin(\theta_0 + \theta_1) + a_3\cdot\sin(\theta_0 + \theta_1 + \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$