Question 1.

(a) The same friction force must be overcome when moving in each direction. Since the area of the retract side of the piston is smaller it will determine the minimum bore diameter. The gauge pressure/force equation is:

$$F_{retract} = (P_{\text{supply}} - \Delta P)A_{retract} - (P_{atm} + \Delta P)A_{extend}$$

$$= (P_{\text{supply}} - \Delta P)A_{retract} - \Delta P A_{extend}$$

since P_{atm} = 0 gauge.

The area of the rod is: $A_{rod} = \frac{\pi}{4} D_{rod}^2 = \frac{\pi}{4}(0.012\ m)^2 = 1.131\times10^{-4}\ m^2$

Since $A_{retract} = A_{bore} - A_{rod}$ and $A_{extend} = A_{bore}$, the equation for the area of the bore is:

$$F_{retract} = (P_{\text{supply}} - \Delta P)(A_{bore} - A_{rod}) - \Delta P A_{bore}$$

$$A_{bore} = \frac{F_{retract} + (P_{\text{supply}} - \Delta P)A_{rod}}{P_{\text{supply}} - 2\Delta P}$$

Substituting $F_{retract}$ = 300 N, $P_{supply} = 5\times10^5$ Pa gauge, and $\Delta P = 6\times10^4$ Pa gives:

$$A_{bore} = \frac{300\ N + (5\times10^5\ Pa - 6\times10^4\ Pa)(1.131\times10^{-4}\ m^2)}{5\times10^5\ Pa - (2)(6\times10^4\ Pa)}$$

$$= 9.204\times10^{-4}\ m^2$$

The minimum required bore diameter is:

$$D_{bore} = \sqrt{\frac{4A_{bore}}{\pi}} = \sqrt{\frac{4(9.204\times10^{-4}\ m^2)}{\pi}} = 3.42\times10^{-2}\ m$$

(b) The maximum volume flow rate will be required when extending, so:

$$Q = vA_{bore} = (1.5\ m/s)(9.204\times10^{-4}\ m^2) = 1.381\times10^{-3}\ m^3/s$$

To obtain the air density we must use the absolute pressure. Since we are given P_{supply} as gauge pressure we must add atmospheric pressure to it, therefore:
$P_{supply} = 5\times10^5 + 1.01\times10^5 = 6.01\times10^5$ Pa absolute. The air density is then:

$$\rho = \frac{(P_1 - \Delta P)}{R_g T} = \frac{(P_{\text{supply}} - \Delta P)}{R_g T} = \frac{(6.01\times10^5 - 6\times10^4)Pa}{(287\,J/kg^\circ K)(20+273)^\circ K} = 6.434\ kg/m^3$$

The minimum required flow coefficient is then:

$$C_V = 4.22\times10^4 Q\sqrt{\frac{\rho}{\Delta P}} = 4.22\times10^4(1.381\times10^{-3} m^3/s)\sqrt{\frac{6.434\ kg/m^3}{6\times10^4 Pa}} = 0.60$$

Question 2.

a) Based on the given information, this operating cycle has: t_{run}=0 and t_{acc}= t_{dec}=½t_{move}. The displacement for this type of motion profile is given by:

$$x_{move} = \tfrac{1}{4} a_{con} t_{move}^2$$

The equation for the max. velocity is:

$$v_{max} = \tfrac{1}{2} a_{con} t_{move}$$

Dividing these equations and solving for t_{move} gives:

$$\frac{x_{move}}{v_{\max}} = \frac{\tfrac{1}{4} a_{con} t_{move}^2}{\tfrac{1}{2} a_{con} t_{move}} = \tfrac{1}{2} t_{move}$$

$$\therefore\ t_{move} = \frac{2x_{move}}{v_{\max}} = \frac{2(0.3 \text{ m})}{0.6 \text{ m/s}} = 1 \text{ s}$$

With t_{move} known we can solve for the acceleration as follows:

$$v_{max} = \tfrac{1}{2} a_{con} t_{move}$$

$$\therefore\ a_{con} = \frac{2v_{max}}{t_{move}} = \frac{2(0.6 \text{ m/s})}{1 \text{ s}} = 1.2 \text{ m/s}^2$$

The acceleration duration is simply:

$$t_{acc} = \tfrac{1}{2} t_{move} = \tfrac{1}{2}(1 \text{ s}) = 0.5 \text{ s}$$

(b) From (3.2), the equivalent inertia of the mass driven by the timing belt is:

$$J = Mr_p^{\,2}$$

The load inertia includes this value plus the inertia of the timing belt and pulleys, as follows:

$$\begin{aligned} J_{load} &= Mr_p^{\,2} + J_{belt-and-pulleys} \\ &= (40 \text{ kg})\left(0.075 \text{ m}/2\right)^2 + 0 \\ &= 0.0562 \text{ kgm}^2 \end{aligned}$$

The optimal gear ratio is given by:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{0.0562 \text{ kgm}^2}{1.31\times10^{-4} \text{ kgm}^2}} = 20.7$$

However, the available gear ratios are 2, 4, 6 etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 20$. Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio. The motor's rated max. speed is 3400 rpm, $\therefore \omega_{rated,max} =$ 3400(2π/rev)(1 min/60 s) = 356.0 rad/s. The maximum required linear speed, $|v_{max}|$, can be obtained from part (b). Note that with a timing belt, linear velocity can be converted to angular velocity in rad/s by dividing by the pitch radius of the pulley. The max. rotational speed of the timing belt pulley is then:

$$\omega_{max} = \frac{|v_{max}|}{r_p} = \frac{0.6 \text{ m/s}}{(0.075 \text{ m}/2)} = 16.0 \text{ rad/s}$$

The corresponding motor speed is: $\omega_{motor,max} = N_r \omega_{max} = (20)(16.0 \text{ rad/s}) = 320.0 \text{ rad/s}$

Since $\omega_{motor,\,max} < \omega_{rated,max}$ this gear ratio passes the motor speed check.

Next, we should check the max. required motor and RMS motor torques. Given that the positive direction means upwards, the torques are obtained for the 3 periods of the operating cycle are as follows:

1. **Accelerate:**

   The timing belt converts the torque input to a force output so the relevant equation is (3.4):

$$F_{out} = \frac{\tau_{in}}{r_p} \eta_{rp}$$

   Since the motion is horizontal the only external force is due to friction. Since the coefficient of friction is 0.02, and there is no viscous friction, the friction force is:

$$F_{friction} = \mu M g = (0.02)(40 \text{ kg})(9.81 \text{ N/kg}) = 7.85 \text{ N}$$

   We can then solve for the torque input required to overcome the friction force as follows:

$$\begin{aligned} \tau_{external} &= \frac{F_{out} r_p}{\eta_{rp}} \\ &= F_{friction} r_p / \eta_{rp} = (7.85 \text{ N})(0.075 \text{ m}/2)/1 = 0.294 \text{ Nm} \end{aligned}$$

   The required motor angular acceleration and torque are then:

$$\dot{\omega}_{motor} = N_r \left(\frac{a_{load}}{r_p} \right) = 20 \left(\frac{1.2 \text{ m/s}^2}{0.075 \text{ m}/2} \right) = 640 \text{ rad/s}^2$$

$$\begin{aligned} \tau_{motor,1} &= J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} \\ &= (1.31\times10^{-4} \text{ kgm}^2)(640 \text{ rad/s}^2) + \frac{1}{20^2}(0.0562 \text{ kgm}^2)(640 \text{ rad/s}^2) + \frac{1}{20}(0.294 \text{ Nm}) \\ &= 0.189 \text{ Nm} \end{aligned}$$

2. **Decelerate:**

   Since the mass is still being moved in the positive direction: $\tau_{external} = 0.294 \text{ Nm}$

$$\dot{\omega}_{motor} = N_r \left(\frac{a_{load}}{r_p} \right) = 20 \left(\frac{-1.2 \text{ m/s}^2}{0.075 \text{ m}/2} \right) = -640 \text{ rad/s}^2$$

$$\tau_{motor,2} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\ \text{kgm}^2)(-640\ \text{rad/s}^2) + \frac{1}{20^2}(0.0562\ \text{kgm}^2)(-640\ \text{rad/s}^2) + \frac{1}{20}(0.294\ \text{Nm})$$

$$= -0.159\ \text{Nm}$$

Note: The friction force helps to decelerate the mass so the required motor torque is less than during the acceleration period.

3. **Idle:**

Since the required velocity is zero, and there is no gravity force acting on the mass, the friction force will be zero. Since the required acceleration is also zero, the required motor torque is:

$$\tau_{motor,3} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = 0$$

From the above results, the max. required torque is: $\tau_{motor,max} = \tau_{motor,1} = 0.189\ \text{Nm}$. This is well below the motor's rated max. torque of 15.5 Nm. Since $\tau_{motor,\,max}$ is also well below the motor's rated max. continuous torque of 0.8 Nm there is no need to calculate $\tau_{motor,\,RMS}$.

Finally, the motor's operating temperature should be compared with its temperature limit. Since $\tau_{motor,\,max}$ is much less than the motor's rated max. continuous torque this check is redundant, but is included for completeness. It should be noted that some applications (e.g. machines operating close to people) require a much lower operating temperature limit than the 150 °C limit of this motor. The question states that the friction of the motor is negligible, therefore $K_d \approx 0$ and the torque output of the motor is simply $K_t I$. Given the low value of the max. required torque, as a simplifying and conservative approach (i.e. $I_{max} > I_{RMS}$) this max. value will be used rather than the RMS value. Then:

$$I_{max} = \frac{\tau_{motor,max}}{K_t} = \frac{0.189\ \text{Nm}}{0.2\ \text{Nm/A}} = 0.943\ \text{A}$$

The power loss is:

$$P_j = I_{max}^2 R_{Hot} = (0.943\ \text{A})^2(1.33\ \text{ohm}) = 1.18\ \text{W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 25\ °\text{C} + (1.18\ \text{W})(3.15\ °\text{C/W}) = 28.7\ °\text{C}$$

Even using this conservative calculation, the predicted winding temperature is much less than the temperature limit.

Therefore this motor, gearbox and timing belt combination has passed all of the tests and is acceptable.

Question 3.

a)

With quadrature counting:

counts/rev = 4(number of pulse/rev) = 4(500 pulse/rev) = 2000 counts/rev, and therefore:

$$Linear\ resolution = \frac{l}{\text{number of counts/rev}} = \frac{0.02\ \text{m/rev}}{2000\ \text{counts/rev}} = 1.0 \times 10^{-5}\ \text{m}$$

b) The operating cycle consists of 8 periods. The desired velocity and acceleration over each period can be found as follows:

1. Accelerate (moving in positive direction):

From the given information and the acceleration time equation, the acceleration is:

$$t_{acc} = \frac{v_{max} - v_i}{a_{acc}}$$

$$\therefore\ a_{acc} = \frac{v_{max} - v_i}{t_{acc}} = \frac{(0.35 - 0)\ \text{m/s}}{0.05\ \text{s}} = 7\ \text{m/s}^2$$

Since the acceleration is constant, and the motion starts from rest, the velocity and acceleration equations are simply:

For $0 \le t \le 0.05\,\text{s}$:

$$v(t) = a_{acc}t = (7\ \text{m/s}^2)t \quad \text{and}$$

$$a(t) = a_{acc} = 7\ \text{m/s}^2$$

2. Constant velocity (moving in positive direction):

For $0.05\,\text{s} < t \le 1.2\,\text{s}$:

$$v(t) = 0.35\ \text{m/s} \quad \text{and}$$

$$a(t) = 0$$

3. Decelerate (moving in positive direction):

From the given information and the deceleration time equation, the deceleration is:

$$t_{dec} = \frac{-v_{max}}{a_{dec}}$$

$$\therefore\ a_{dec} = \frac{-v_{max}}{t_{acc}} = \frac{-0.35\ \text{m/s}}{0.05\ \text{s}} = -7\ \text{m/s}^2$$

This period will start after t=1.2 s. So, for $1.2\,\text{s} < t \le 1.25\,\text{s}$:

$$v(t) = a_{dec}(t - 1.2\,\text{s}) + v_{\max} = \left(-7(t - 1.2) + 0.35\right)\ \text{m/s} \quad \text{and}$$

$$a(t) = a_{dec} = -7 \text{ m/s}^2$$

4. First idle period

For $1.25 \text{ s} < t \leq 1.75 \text{ s}$:

$$v(t) = 0 \text{ and}$$

$$a(t) = 0$$

5. Accelerate (moving in negative direction):

From the given information and the acceleration time equation, the acceleration is:

$$t_{acc} = \frac{v_{max} - v_i}{a_{acc}}$$

$$\therefore a_{acc} = \frac{v_{max} - v_i}{t_{acc}} = \frac{(-0.7 - 0) \text{ m/s}}{0.1 \text{ s}} = -7 \text{ m/s}^2$$

Since the acceleration is constant, and the motion starts from rest at 1.75 s, the velocity and acceleration equations are simply:

For $1.75 < t \leq 1.85 \text{ s}$:

$$v(t) = a_{acc}(t - 1.75) = \big((-7)(t - 1.75)\big) \text{m/s} \text{ and}$$

$$a(t) = a_{acc} = -7 \text{ m/s}^2$$

6. Constant velocity (moving in negative direction):

For $1.85 \text{ s} < t \leq 2.35 \text{ s}$:

$$v(t) = 0.35 \text{ m/s} \text{ and}$$

$$a(t) = 0$$

7. Decelerate (moving in negative direction):

From the given information and the deceleration time equation, the deceleration is:

$$t_{dec} = \frac{-v_{max}}{a_{dec}}$$

$$\therefore a_{dec} = \frac{-v_{max}}{t_{acc}} = \frac{-(-0.7 \text{ m/s})}{0.1 \text{ s}} = 7 \text{ m/s}^2$$

This period will start after t=2.35 s. So, for $2.35 \text{ s} < t \leq 2.45 \text{ s}$:

$$v(t) = a_{dec}(t - 2.35 \text{ s}) + v_{\max} = \big(7(t - 2.35) - 0.7\big) \text{ m/s} \text{ and}$$

$$a(t) = a_{dec} = 7 \text{ m/s}^2$$

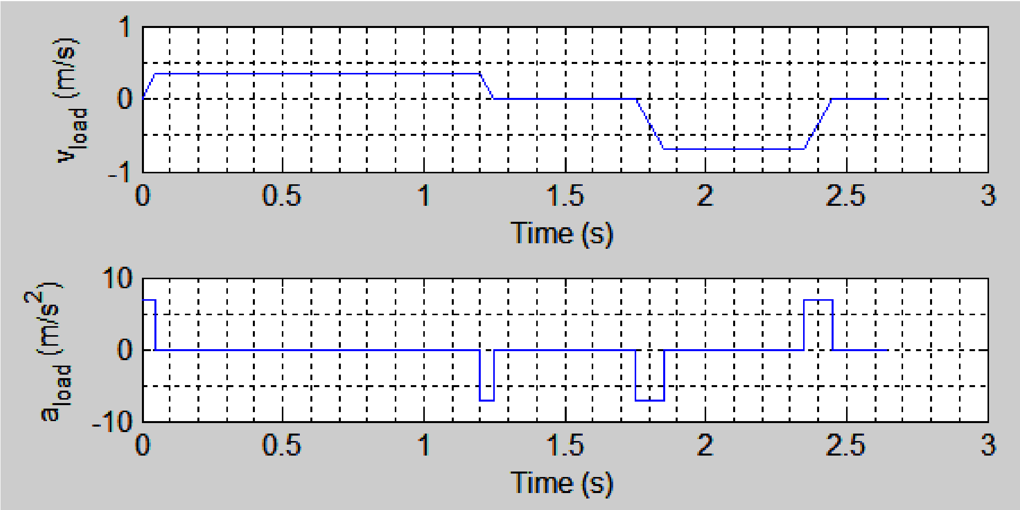
8. Second idle period

For $2.45\,\text{s} < t \le 2.65\,\text{s}$:

$$v(t) = 0 \text{ and}$$

$$a(t) = 0$$

The velocity and acceleration profiles over one operating cycle are plotted below:



c) From (3.2), the equivalent inertia of the mass driven by the ball screw is:

$$J_M = M\left(\frac{l}{(2\pi / rev)}\right)^2$$

The load inertia includes this value plus the inertia of the screw, as follows:

$$J_{load} = M\left(\frac{l}{(2\pi / rev)}\right)^2 + J_{screw}$$

$$= (20\text{ kg})\left(\frac{0.02\text{ m/rev}}{(2\pi / rev)}\right)^2 + 1.0\times10^{-4}\text{ kgm}^2$$

$$= 3.03\times10^{-4}\text{ kgm}^2$$

The optimal gear ratio is given by:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{3.03\times10^{-4}\text{ kgm}^2}{1.34\times10^{-5}\text{ kgm}^2}} = 4.76$$

However, the available gear ratios are 0.5, 1, 1.5, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 4.5$. Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio. The motor's rated max. speed is 7000 rpm, $\therefore \omega_{rated,max} =$ 7000(2π/rev)(1 min/60 s) = 733 rad/s. The maximum required linear speed, $|v_{max}|$, can be obtained from part (b). The max. rotational speed of the ball screw is then:

$$\omega_{max} = \frac{|v_{max}|}{l(\text{rev}/2\pi)} = \frac{0.7\text{ m/s}}{(0.02\text{ m/rev})(\text{rev}/2\pi)} = 220\text{ rad/s}$$

The corresponding motor speed is: $\omega_{motor,max} = N_r\omega_{max} = (4.5)(220\text{ rad/s}) = 990\text{ rad/s}$

Since $\omega_{motor,\,max} > \omega_{rated,max}$ this gear ratio fails the motor speed check. The upper limit on the gear ratio is:

$$N_{r,max} = \frac{\omega_{rated,max}}{\omega_{\max}} = \frac{733\text{ rad/s}}{220\text{ rad/s}} = 3.33$$

We should round down to the nearest available ratio to make $\omega_{motor,\,max} < \omega_{rated,max}$. Therefore our new choice for the best ratio is: $N_r = 3$. Now we need to check the inertia ratio. It is:

$$Ratio_J = \frac{J_{load}/N_r^{\,2}}{J_{motor}} = \frac{(3.03\times10^{-4}\text{ kgm}^2)/(3^2)}{(1.34\times10^{-5}\text{ kgm}^2)} = 2.51$$

Since it is within the range 1 to 10 it is acceptable.

Next, we should check the max. required motor and RMS motor torques. Given that the positive direction means upwards, the torques are obtained for the 8 periods as follows:

1. **Accelerate (moving in positive direction):**

Since friction opposes motion, it will act in the same direction as gravity when the mass is moving upwards.

$$\tau_{external} = \frac{Fl}{(2\pi/rev)\eta_s}$$

$$= \frac{(Mg + F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\big((20\text{ kg})(9.81\text{ N/kg})+10\text{ N}\big)(0.02\text{ m/rev})}{(2\pi/rev)(1)} = 0.656\text{ Nm}$$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = 6600\text{ rad/s}^2$$

$$\tau_{motor,1} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.34\times10^{-5}\text{ kgm}^2)(6600\text{ rad/s}^2) + \frac{1}{3^2}(3.03\times10^{-4}\text{ kgm}^2)(6600\text{ rad/s}^2) + \frac{1}{3}(0.656\text{ Nm})$$

$$= 0.529\text{ Nm}$$

2. **Constant velocity (moving in positive direction):**

Since the mass is still being moved upwards: $\tau_{external} = 0.656\text{ Nm}$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,2} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.656\text{ Nm}) = 0.219\text{ Nm}$$

3. **Decelerate (moving in positive direction):**

Since the mass is still being moved upwards: $\tau_{external} = 0.656$ Nm

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{-7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = -6600\text{ rad/s}^2$$

$$\begin{aligned}\tau_{motor,3} &= J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} \\ &= (1.34\times10^{-5}\text{ kgm}^2)(-6600\text{ rad/s}^2) + \frac{1}{3^2}(3.03\times10^{-4}\text{ kgm}^2)(-6600\text{ rad/s}^2) + \frac{1}{3}(0.656\text{ Nm}) \\ &= -0.092\text{ Nm}\end{aligned}$$

4. **1st Idle:**

When the velocity is zero the friction force should oppose the gravity force acting on the mass, therefore:

$$\begin{aligned}\tau_{external} &= \frac{Fl}{(2\pi/rev)\eta_s} \\ &= \frac{(Mg + F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\big((20\text{ kg})(9.81\text{ N/kg}) - 10\text{ N}\big)(0.02\text{ m/rev})}{(2\pi/rev)(1)} = 0.593\text{ Nm}\end{aligned}$$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,4} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.593\text{ Nm}) = 0.198\text{ Nm}$$

5. **Accelerate (moving in negative direction):**

The friction force will act upwards, opposing the downwards velocity. Therefore $Mg + F_{friction}$ will be same as the previous period, and: $\tau_{external} = 0.593$ Nm .

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{-7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = -6600\text{ rad/s}^2$$

$$\begin{aligned}\tau_{motor,5} &= J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} \\ &= (1.34\times10^{-5}\text{ kgm}^2)(-6600\text{ rad/s}^2) + \frac{1}{3^2}(3.03\times10^{-4}\text{ kgm}^2)(-6600\text{ rad/s}^2) + \frac{1}{3}(0.593\text{ Nm}) \\ &= -0.113\text{ Nm}\end{aligned}$$

6. **Constant velocity (moving in negative direction):**

Since the mass is still being moved downwards: $\tau_{external} = 0.593$ Nm

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,6} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.593\text{ Nm}) = 0.198\text{ Nm}$$

7. Decelerate (moving in negative direction):

Since the mass is still being moved downwards: $\tau_{external} = 0.593$ Nm

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 3\left(\frac{7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = 6600\text{ rad/s}^2$$

$$\tau_{motor,7} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$
$$= (1.34\times10^{-5}\text{ kgm}^2)(6600\text{ rad/s}^2) + \frac{1}{3^2}(3.03\times10^{-4}\text{ kgm}^2)(6600\text{ rad/s}^2) + \frac{1}{3}(0.593\text{ Nm})$$
$$= 0.508\text{ Nm}$$

8. **2nd Idle:**

The loading situation is the same as with the 1st idle period, therefore:

$$\tau_{motor,8} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{3}(0.593\text{ Nm}) = 0.198\text{ Nm}$$

From the above results, the max required torque is: $\tau_{motor,max} = \tau_{motor,1} = 0.529$ Nm. This is below the motor's rated max. torque of 2.5 Nm, but above its rated continuous torque of 0.2 Nm. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\tau_{motor,RMS} = \sqrt{\frac{\sum_{i=1}^{n}\tau_{motor,i}^2 t_i}{\sum_{i=1}^{n} t_i}}$$
$$= \sqrt{\frac{\left((0.529)^2(0.05) + (0.219)^2(1.15) + (-0.092)^2(0.05) + (0.198)^2(0.5) + (-0.113)^2(0.1) + (0.198)^2(0.5) + (0.508)^2(0.1) + (0.198)^2(0.2)\right)\text{ N}^2\text{m}^2\text{s}}{(0.05+1.15+0.05+0.5+0.1+0.5+0.1+0.2)\text{ s}}}$$
$$= 0.233\text{ Nm}$$

Since $\tau_{motor,RMS}$ exceeds the motor's continuous torque rating of 0.2 Nm, Motor A is not an acceptable choice.

d) J_{load} is the same as in part (d). The optimal gear ratio is given by:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{3.03\times10^{-4}\text{ kgm}^2}{1.31\times10^{-4}\text{ kgm}^2}} = 1.52$$

However, the available gear ratios are 0.5, 1, 1.5, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 1.5$. Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio. The motor's rated max. speed is

3400 rpm, $\therefore \omega_{rated,max} =$ 3400(2π/rev)(1 min/60 s) = 356 rad/s. The maximum required linear speed, $|v_{max}|$, can be obtained from part (b). The max. rotational speed of the ball screw is then:

$$\omega_{max} = \frac{|v_{max}|}{l(\text{rev}/2\pi)} = \frac{0.7\text{ m/s}}{(0.02\text{ m/rev})(\text{rev}/2\pi)} = 220\text{ rad/s}$$

The corresponding motor speed is: $\omega_{motor,max} = N_r \omega_{max} = (1.5)(220\text{ rad/s}) = 330\text{ rad/s}$

Since $\omega_{motor,\,max} < \omega_{rated,max}$ this gear ratio passes the motor speed check.

Next, we should check the max. required motor and RMS motor torques. Given that the positive direction means upwards, the torques are obtained for the 8 periods as follows:

1. **Accelerate (moving in positive direction):**

   Since friction opposes motion, it will act in the same direction as gravity when the mass is moving upwards.

$$\tau_{external} = \frac{Fl}{(2\pi/rev)\eta_s}$$

$$= \frac{(Mg + F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\big((20\text{ kg})(9.81\text{ N/kg})+10\text{ N}\big)(0.02\text{ m/rev})}{(2\pi/rev)(1)} = 0.656\text{ Nm}$$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 1.5\left(\frac{7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = 3300\text{ rad/s}^2$$

$$\tau_{motor,1} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\text{ kgm}^2)(3300\text{ rad/s}^2) + \frac{1}{1.5^2}(3.03\times10^{-4}\text{ kgm}^2)(3300\text{ rad/s}^2) + \frac{1}{1.5}(0.656\text{ Nm})$$

$$= 1.314\text{ Nm}$$

2. **Constant velocity (moving in positive direction):**

   Since the mass is still being moved upwards: $\tau_{external} = 0.656\text{ Nm}$

   No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,2} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{1.5}(0.656\text{ Nm}) = 0.438\text{ Nm}$$

3. **Decelerate (moving in positive direction):**

   Since the mass is still being moved upwards: $\tau_{external} = 0.656\text{ Nm}$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 1.5\left(\frac{-7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = -3300\text{ rad/s}^2$$

$$\tau_{motor,3} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\text{ kgm}^2)(-3300\text{ rad/s}^2) + \frac{1}{1.5^2}(3.03\times10^{-4}\text{ kgm}^2)(-3300\text{ rad/s}^2) + \frac{1}{1.5}(0.656\text{ Nm})$$

$$= -0.439\text{ Nm}$$

4. **1st Idle:**

When the velocity is zero the friction force should oppose the gravity force acting on the mass, therefore:

$$\tau_{external} = \frac{Fl}{(2\pi/rev)\eta_s}$$

$$= \frac{(Mg + F_{friction})l}{(2\pi/rev)\eta_s} = \frac{\big((20\text{ kg})(9.81\text{ N/kg}) - 10\text{ N}\big)(0.02\text{ m/rev})}{(2\pi/rev)(1)} = 0.593\text{ Nm}$$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,4} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{1.5}(0.593\text{ Nm}) = 0.395\text{ Nm}$$

5. **Accelerate (moving in negative direction):**

The friction force will act upwards, opposing the downwards velocity. Therefore $Mg + F_{friction}$ will be same as the previous period, and: $\tau_{external} = 0.593\text{ Nm}$.

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 1.5\left(\frac{-7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = -3300\text{ rad/s}^2$$

$$\tau_{motor,5} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\text{ kgm}^2)(-3300\text{ rad/s}^2) + \frac{1}{1.5^2}(3.03\times10^{-4}\text{ kgm}^2)(-3300\text{ rad/s}^2) + \frac{1}{1.5}(0.593\text{ Nm})$$

$$= -0.482\text{ Nm}$$

6. **Constant velocity (moving in negative direction):**

Since the mass is still being moved downwards: $\tau_{external} = 0.593\text{ Nm}$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,6} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{1.5}(0.593\text{ Nm}) = 0.395\text{ Nm}$$

7. **Decelerate (moving in negative direction):**

Since the mass is still being moved downwards: $\tau_{external} = 0.593\text{ Nm}$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{l}\right) = 1.5\left(\frac{7\text{ m/s}^2}{(0.02\text{ m/rev})(1\text{ rev}/2\pi)}\right) = 3300\text{ rad/s}^2$$

$$\tau_{motor,7} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\text{ kgm}^2)(3300\text{ rad/s}^2) + \frac{1}{1.5^2}(3.03\times10^{-4}\text{ kgm}^2)(3300\text{ rad/s}^2) + \frac{1}{1.5}(0.593\text{ Nm})$$

$$= 1.272\text{ Nm}$$

8. **2nd Idle:**

The loading situation is the same as with the 1st idle period, therefore:

$$\tau_{motor,8} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{1.5}(0.593\text{ Nm}) = 0.395\text{ Nm}$$

From the above results, the max. required torque is: $\tau_{motor,max} = \tau_{motor,1} = 1.314$ Nm. This is below the motor's rated max. torque of 15.5 Nm, but above its rated continuous torque of 0.2 Nm. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\tau_{motor,RMS} = \sqrt{\frac{\sum_{i=1}^{n}\tau_{motor,i}^2 t_i}{\sum_{i=1}^{n} t_i}}$$

$$= \sqrt{\frac{\left((1.314)^2(0.05)+(0.438)^2(1.15)+(-0.439)^2(0.05)+(0.395)^2(0.5)+(-0.482)^2(0.1)+(0.395)^2(0.5)+(1.272)^2(0.1)+(0.395)^2(0.2)\right)\text{ N}^2\text{m}^2\text{s}}{(0.05+1.15+0.05+0.5+0.1+0.5+0.1+0.2)\text{ s}}}$$

$$= 0.510\text{ Nm}$$

Since $\tau_{motor,RMS}$ is below the motor's continuous torque rating of 0.8 Nm, Motor B passes both of the torque checks.

Lastly, we need to check the motor's operating temperature. The question states that the friction of the motor is negligible, therefore $K_d \approx 0$ and the torque output of the motor is simply $K_t I$. Then the RMS current is simply:

$$I_{RMS} = \frac{\tau_{RMS}}{K_t} = \frac{0.510\text{ Nm}}{0.2\text{ Nm/A}} = 2.55\text{ A}$$

The power loss is:

$$P_j = I_{RMS}^2 R_{Hot} = (2.55\text{ A})^2(1.33\text{ ohm}) = 8.65\text{ W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 30\text{ °C} + (8.65\text{ W})(3.15\text{ °C/W}) = 57.2\text{ °C}$$

Since this is less than the motor's temperature limit this test has been passed.

We can conclude that this motor, gearbox and ball screw combination has passed all of the tests and is acceptable.

e) From (3.2), the equivalent inertia of the mass driven by the rack and pinion is:

$$J = Mr_p^{\,2}$$

The load inertia includes this value plus the inertia of the rack and pinion, as follows:

$$J_{load} = M r_p^{\,2} + J_{rack-and-pinion}$$

$$= (20\ \text{kg})\left(0.05\ \text{m}/2\right)^2 + 0$$

$$= 0.0125\ \text{kgm}^2$$

The optimal gear ratio is given by:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{0.0125\ \text{kgm}^2}{1.31\times10^{-4}\ \text{kgm}^2}} = 9.77$$

However, the available gear ratios are 0.5, 1, 1.5, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 9.5$. Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio. The motor's rated max. speed is 3400 rpm, $\therefore \omega_{rated,max} =$ = 3400(2π/rev)(1 min/60 s) = 356.0 rad/s. The maximum required linear speed, $|v_{max}|$, can be obtained from part (b). Note that with a rack and pinion, linear velocity can be converted to angular velocity in rad/s by dividing by the pitch radius of the pinion. The max. rotational speed of the pinion is then::

$$\omega_{max} = \frac{|v_{max}|}{r_p} = \frac{0.7\ \text{m/s}}{0.025\ \text{m}} = 28.0\ \text{rad/s}$$

The corresponding motor speed is: $\omega_{motor,max} = N_r \omega_{max} = (9.5)(28.0\ \text{rad/s}) = 266.0\ \text{rad/s}$

Since $\omega_{motor,max} < \omega_{rated,max}$ this gear ratio passes the motor speed check.

Next, we should check the max. required motor and RMS motor torques. Given that the positive direction means upwards, the torques are obtained for the 8 periods as follows:

1. **Accelerate (moving in positive direction):**

   The rack and pinion converts the torque input to a force output so the relevant equation is (3.4):

$$F_{out} = \frac{\tau_{in}}{r_p}\eta_{rp}$$

   This gives:

$$\tau_{external} = \frac{F_{out} r_p}{\eta_{rp}}$$

$$= \frac{(Mg + F_{friction}) r_p}{\eta_{rp}} = \frac{\left((20\ \text{kg})(9.81\ \text{N/kg}) + 10\ \text{N}\right)(0.025\text{m})}{(1)} = 5.155\ \text{Nm}$$

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{r_p}\right) = 9.5\left(\frac{7\ \text{m/s}^2}{0.025\ \text{m}}\right) = 2660\ \text{rad/s}^2$$

$$\tau_{motor,1} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\ \text{kgm}^2)(2660\text{rad/s}^2) + \frac{1}{9.5^2}(0.0125\ \text{kgm}^2)(2660\ \text{rad/s}^2) + \frac{1}{9.5}(5.155\ \text{Nm})$$

$$= 1.260\text{Nm}$$

2. Constant velocity (moving in positive direction):

Since the mass is still being moved upwards: $\tau_{external} = 5.155$ Nm

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,2} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{9.5}(5.155\text{Nm}) = 0.543\ \text{Nm}$$

3. Decelerate (moving in positive direction):

Since the mass is still being moved upwards: $\tau_{external} = 5.155$ Nm

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{r_p}\right) = 9.5\left(\frac{-7\ \text{m/s}^2}{0.025\ \text{m}}\right) = -2660\ \text{rad/s}^2$$

$$\tau_{motor,3} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4}\ \text{kgm}^2)(-2660\ \text{rad/s}^2) + \frac{1}{9.5^2}(0.0125\ \text{kgm}^2)(-2660\ \text{rad/s}^2) + \frac{1}{9.5}(5.155\ \text{Nm})$$

$$= -0.174\ \text{Nm}$$

4. 1st Idle:

When the velocity is zero the friction force should oppose the gravity force acting on the mass, therefore:

$$\tau_{in} = \frac{F_{out} r_p}{\eta_{rp}}$$

$$= \frac{(Mg + F_{friction})r_p}{\eta_{rp}} = \frac{\big((20\ \text{kg})(9.81\ \text{N/kg}) - 10\ \text{N}\big)(0.025\text{m})}{(1)} = 4.655\ \text{Nm}$$

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,4} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{9.5}(4.655\text{Nm}) = 0.49\ \text{Nm}$$

5. Accelerate (moving in negative direction):

The friction force will act upwards, opposing the downwards velocity. Therefore $Mg + F_{friction}$ will be same as the previous period, and: $\tau_{external} = 4.655$ Nm .

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{r_p}\right) = 9.5\left(\frac{-7 \text{ m/s}^2}{0.025 \text{ m}}\right) = -2660 \text{ rad/s}^2$$

$$\tau_{motor,5} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4} \text{ kgm}^2)(-2660 \text{ rad/s}^2) + \frac{1}{9.5^2}(0.0125 \text{ kgm}^2)(-2660 \text{ rad/s}^2) + \frac{1}{9.5}(4.655 \text{ Nm})$$

$$= -0.227 \text{ Nm}$$

6. **Constant velocity (moving in negative direction):**

Since the mass is still being moved downwards: $\tau_{external} = 4.655$ Nm

No acceleration so: $\dot{\omega}_{motor} = 0$

$$\tau_{motor,6} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{9.5}(4.655\text{Nm}) = 0.49 \text{ Nm}$$

7. **Decelerate (moving in negative direction):**

Since the mass is still being moved downwards: $\tau_{external} = 4.655$ Nm

$$\dot{\omega}_{motor} = N_r\left(\frac{a_{load}}{r_p}\right) = 9.5\left(\frac{7 \text{ m/s}^2}{0.025 \text{ m}}\right) = 2660 \text{ rad/s}^2$$

$$\tau_{motor,7} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}$$

$$= (1.31\times10^{-4} \text{ kgm}^2)(2660 \text{ rad/s}^2) + \frac{1}{9.5^2}(0.0125 \text{ kgm}^2)(2660 \text{ rad/s}^2) + \frac{1}{9.5}(4.655 \text{ Nm})$$

$$= 1.207 \text{ Nm}$$

8. **2nd Idle:**

The loading situation is the same as with the 1st idle period, therefore:

$$\tau_{motor,8} = J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^2}J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} = \frac{1}{N_r}\tau_{external} = \frac{1}{9.5}(4.655\text{Nm}) = 0.49 \text{ Nm}$$

From the above results, the max. required torque is: $\tau_{motor,max} = \tau_{motor,1} = 1.260$ Nm . This is below the motor's rated max. torque of 15.5 Nm, but above its rated continuous torque of 0.8 Nm. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\tau_{motor,RMS} = \sqrt{\frac{\sum_{i=1}^{n} \tau_{motor,i}^2 t_i}{\sum_{i=1}^{n} t_i}}$$

$$= \sqrt{\frac{\left((1.260)^2(0.05)+(0.543)^2(1.15)+(-0.174)^2(0.05)+(0.49)^2(0.5)+(-0.227)^2(0.1)+(0.49)^2(0.5)+(1.207)^2(0.1)+(0.49)^2(0.2)\right)\ \mathrm{N^2m^2s}}{(0.05+1.15+0.05+0.5+0.1+0.5+0.1+0.2)\ \mathrm{s}}}$$

$$= 0.569\ \mathrm{Nm}$$

Since $\tau_{motor,RMS}$ is below the motor's continuous torque rating of 0.8 Nm, Motor B passes both of the torque checks.

Lastly, we need to check the motor's operating temperature. The question states that the friction of the motor is negligible, therefore $K_d \approx 0$ and the torque output of the motor is simply $K_t I$.

Then the RMS current is simply:

$$I_{RMS} = \frac{\tau_{RMS}}{K_t} = \frac{0.569\ \mathrm{Nm}}{0.2\ \mathrm{Nm/A}} = 2.845\ \mathrm{A}$$

The power loss is:

$$P_j = I_{RMS}^2 R_{Hot} = (2.845\ \mathrm{A})^2 (1.33\ \mathrm{ohm}) = 10.77\ \mathrm{W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 30\ °\mathrm{C} + (10.77\ \mathrm{W})(3.15\ °\mathrm{C/W}) = 63.9\ °\mathrm{C}$$

Since this is less than the motor's temperature limit $(150°\mathrm{C})$ this test has been passed.

We can conclude that this motor, gearbox, and rack & pinion combination has passed all of the tests and is acceptable.

f) A quick way to check whether or not part (f) can be solved by using a different pinion pitch diameter without a gearbox between the motor and pinion is to first calculate the pitch radius of the equivalent pinion. In part (f), r_p =0.025 m and N_r =9.5. The equivalent pitch radius is given by:

$$r_{p,equivalent} = \frac{r_p}{N_r} = \frac{0.025\ \mathrm{m}}{9.5} = 0.0026\ \mathrm{m}$$

Although a small pinion like this could be purchased, its teeth would be much too weak to transmit the required torque, therefore using a rack and pinion without a gearbox is not a feasible solution for this application.

Question 4.

Linear resolution = *l* / (steps/rev)

(a) Resolution for full stepping = (5 mm/rev) / (150 step/rev) = 0.033 mm

(b) Resolution for half stepping = (5 mm/rev) / (2(150 step/rev)) = 0.016 mm

(c) Resolution for microstepping = (5 mm/rev) / (250(150 step/rev)) = 0.0001 mm

(d) The ball screw's repeatability is ±0.002 mm. Now imagine what will happen if the motor moves by a full step. Given the screw's repeatability, the movement of the payload will be 0.033±0.002 mm. If the motor moves by a half step the payload will move to 0.016±0.002 mm. If the motor moves a microstep (1/250 step) then the payload will move 0.0001±0.002 mm. Clearly this last result is not meaningful since the payload may not move at all, or could even move in the negative direction if a negative force was applied to it! So the answers for full and half stepping are meaningful, but the answer for microstepping is not.

Question 5.

(a) The force required to accelerate the mass is:

$$F = ma = (800\,Kg)(3\,m/s^2) = 2400\ N$$

With a double rod cylinder the areas on both sides of the piston are equal and are given by:

$$A = \frac{\pi}{4}(D_{bore}^2 - D_{rod}^2) = \frac{\pi}{4}((0.035\,m)^2 - (0.025\,m)^2) = 4.712\times10^{-4}\ m^2$$

Since the same acceleration is required in both directions, and the areas are the same we can use either the extend or retract equation to answer this question. Since the pressure drops across the valve are the same for the return flow as for the intake flow, the gauge pressure/force equation for the cylinder is:

$$\begin{aligned} F_{extend} &= P_{extend}A - P_{retract}A \\ &= (P_{\text{supply}} - \Delta P)A - (P_{sump} + \Delta P)A \\ &= (P_{\text{supply}} - 2\Delta P)A \end{aligned}$$

Note that $P_{sump} = P_{atm} = 0$ gauge, and $P_{\text{supply}} = 1.1\times10^7\ Pa - 1.01\times10^5\ Pa = 1.09\times10^7\ Pa$ gauge. We can now substitute $F_{extend} = F$ and solve for ΔP:

$$F = (P_{\text{supply}} - 2\Delta P)A$$

$$\therefore \Delta P = \frac{P_{\text{supply}}A - F}{2A}$$

$$= \frac{(1.09\times10^{7}\ Pa)(4.712\times10^{-4}\ m^{2}) - 2400\ N}{(2)(4.712\times10^{-4}\ m^{2})}$$
$$= 2.903\times10^{6}\ Pa$$

The fastest possible motion from one fixed location to another is accomplished by a period of constant acceleration, followed by a period of constant deceleration of equal magnitude and duration (please see section A.1 of Chapter 3). The equation relating the movement distance to the acceleration and movement time is:

$$x_{move} = \tfrac{1}{4} a_{con} t_{move}^{2}$$

Solving for the movement time gives:

$$t_{move} = \sqrt{\frac{4x_{move}}{a_{con}}} = \sqrt{\frac{(4)(2.5\ m)}{3\,m/s^{2}}} = 1.826\ s$$

and the maximum velocity is:

$$v_{\max} = a_{con}(\tfrac{1}{2}t_{move}) = (3\ m/s^{2})(\tfrac{1}{2})(1.826\ s) = 2.739\ m/s$$

The maximum volume flow rate is:

$$Q = Av_{\max} = (4.712\times10^{-4}\ m^{2})(2.739\ m/s) = 1.291\times10^{-3} m^{3}/s$$

The minimum valve flow coefficient is:

$$C_V = 4.22\times10^{4} Q\sqrt{\frac{\rho}{\Delta P}} = 4.22\times10^{4}(1.291\times10^{-3} m^{3}/s)\sqrt{\frac{900\ kg/m^{3}}{2.903\times10^{6} Pa}} = 0.959$$

(b) $t_{move} = 1.826\ s$ (Please see part (a) for the calculation).

Question 6.

(a) With a rodless cylinder the areas on both sides of the piston are equal to:

$$A = \frac{\pi}{4} D_{bore}^{2} = \frac{\pi}{4}(0.03\ m)^{2} = 7.07\times10^{-4}\ m^{2}$$

The output force in terms of gauge pressures is:

$$F_{output} = (P_{\text{supply}} - \Delta P)A - (P_{atm} + \Delta P)A$$
$$= P_{\text{supply}}A - 2\Delta PA$$

where $P_{supply} = 5\times10^{5} - 1.01\times10^{5} = 3.99\times10^{5}$ Pa gauge, and P_{atm} = 0 gauge.

When moving upwards at the maximum speed:

$$\Sigma F = F_{output} - mg = ma = 0$$
$$F_{output} = mg = (5\,kg)(9.81\,m/s^{2}) = 49.05\,N$$

Substituting this value into the pressure/force equation above and solving for ΔP gives:

$$F_{output} = P_{\text{supply}}A - 2A\Delta P$$

$$\Delta P = \frac{P_{\text{supply}} A - F_{output}}{2A}$$
$$= \frac{(3.99\times10^{5}\ Pa)(7.07\times10^{-4}\ m^{2}) - 49.05\ N}{(2)(7.07\times10^{-4}\ m^{2})}$$
$$= 1.65\times10^{5}\ Pa$$

To obtain the air density we must use the absolute pressure. Since we are given P_{supply} as absolute pressure we can use it directly, and the air density is:

$$\rho = \frac{(P_1 - \Delta P)}{R_g T} = \frac{(P_{\text{supply}} - \Delta P)}{R_g T} = \frac{(5\times10^{5} - 1.65\times10^{5})Pa}{(287 J/kg^{\circ}K)(20+273)^{\circ}K} = 3.98\ kg/m^{3}$$

The maximum volume flow rate is:

$$Q = 2.37\times10^{-5} C_V \sqrt{\frac{\Delta P}{\rho}} = 2.37\times10^{-5}(0.15)\sqrt{\frac{1.65\times10^{5}\ Pa}{3.98\ kg/m^{3}}} = 7.24\times10^{-4} m^{3}/s$$

and the maximum speed is:

$$v_{\max} = \frac{Q}{A} = \frac{7.24\times10^{-4}\ m^{3}/s}{7.07\times10^{-4}\ m^{2}} = 1.02\ m/s$$

(b) When moving downwards at the maximum speed:

$$\Sigma F = F_{output} + mg = ma = 0$$
$$F_{output} = -mg = -49.05\ N$$

Using the ΔP equation from part (a) gives:

$$\Delta P = \frac{P_{\text{supply}} A - F_{output}}{2A}$$
$$= \frac{(3.99\times10^{5}\ Pa)(7.07\times10^{-4}\ m^{2}) - (-49.05\ N)}{(2)(7.07\times10^{-4}\ m^{2})}$$
$$= 2.34\times10^{5}\ Pa$$

Similar to part (a), the air density is:

$$\rho = \frac{(P_{\text{supply}} - \Delta P)}{R_g T} = \frac{(5\times10^{5} - 2.34\times10^{5})Pa}{(287 J/kg^{\circ}K)(20+273)^{\circ}K} = 3.16\ kg/m^{3}$$

The maximum volume flow rate is:

$$Q = 2.37\times10^{-5} C_V \sqrt{\frac{\Delta P}{\rho}} = 2.37\times10^{-5}(0.15)\sqrt{\frac{2.34\times10^{5}\ Pa}{3.16\ kg/m^{3}}} = 9.67\times10^{-4} m^{3}/s$$

and the maximum speed is:

$$v_{\max} = \frac{Q}{A} = \frac{9.67\times10^{-4}\ m^{3}/s}{7.07\times10^{-4}\ m^{2}} = 1.37\ m/s$$