perpendicular to $\hat{\mathbf{R}}_1$. The force per unit length exerted on I_3 is

$$\mathbf{F}'_{31} = \frac{\mu_0 I_1 I_3}{2\pi R_1}(\hat{\mathbf{y}} \times \hat{\mathbf{b}}_1) = -\hat{\mathbf{R}}_1 \frac{\mu_0 I_1 I_3}{2\pi R_1}.$$

Similarly, the force per unit length excited on I_3 by the field due to I_2 (which is along $-\hat{\mathbf{y}}$) is

$$\mathbf{F}'_{32} = \hat{\mathbf{R}}_2 \frac{\mu_0 I_2 I_3}{2\pi R_2}.$$

The two forces have opposite components along $\hat{\mathbf{x}}$ and equal components along $\hat{\mathbf{z}}$. Hence, with $R_1 = R_2 = \sqrt{8}$ m and $\theta = \sin^{-1}(2/\sqrt{8}) = \sin^{-1}(1/\sqrt{2}) = 45°$,

$$\begin{aligned}\mathbf{F}'_3 = \mathbf{F}'_{31} + \mathbf{F}'_{32} &= \hat{\mathbf{z}}\left(\frac{\mu_0 I_1 I_3}{2\pi R_1} + \frac{\mu_0 I_2 I_3}{2\pi R_2}\right)\sin\theta \\ &= \hat{\mathbf{z}}2\left(\frac{4\pi \times 10^{-7} \times 10 \times 20}{2\pi \times \sqrt{8}}\right) \times \frac{1}{\sqrt{2}} = \hat{\mathbf{z}}2 \times 10^{-5} \text{ N/m}.\end{aligned}$$

Problem 5.19 A square loop placed as shown in Fig. 5-44 (P5.19) has 2-m sides and carries a current $I_1 = 5$ A. If a straight, long conductor carrying a current $I_2 = 10$ A is introduced and placed just above the midpoints of two of the loop's sides, determine the net force acting on the loop.

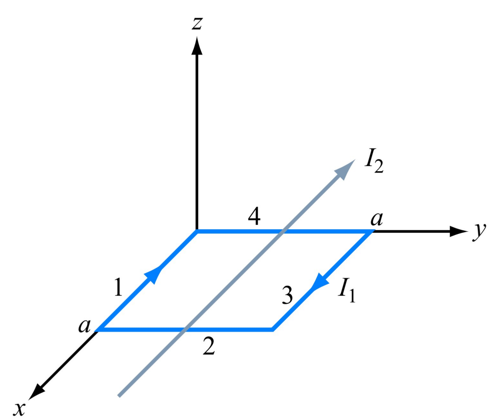

Figure P5.19: Long wire carrying current I_2, just above a square loop carrying I_1 (Problem 5.19).

Solution: Since I_2 is just barely above the loop, we can treat it as if it's in the same plane as the loop. For side 1, I_1 and I_2 are in the same direction, hence the force on side 1 is attractive. That is,

$$\mathbf{F}_1 = \hat{\mathbf{y}}\,\frac{\mu_0 I_1 I_2 a}{2\pi(a/2)} = \hat{\mathbf{y}}\,\frac{4\pi\times 10^{-7}\times 5\times 10\times 2}{2\pi\times 1} = \hat{\mathbf{y}}\,2\times 10^{-5}\text{ N}.$$

I_1 and I_2 are in opposite directions for side 3. Hence, the force on side 3 is repulsive, which means it is also along $\hat{\mathbf{y}}$. That is, $\mathbf{F}_3 = \mathbf{F}_1$.

The net forces on sides 2 and 4 are zero. Total net force on the loop is

$$\mathbf{F} = 2\mathbf{F}_1 = \hat{\mathbf{y}}\,4\times 10^{-5}\text{ N}.$$

Section 5-4: Gauss's Law for Magnetism and Ampère's Law

Problem 5.20 Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius a, and the inner and outer radii of the outer conductor are b and c, respectively.

(a) Determine the magnetic field in each of the following regions: $0 \le r \le a$, $a \le r \le b$, $b \le r \le c$, and $r \ge c$.

(b) Plot the magnitude of $\mathbf{H}$ as a function of r over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

Solution:

(a) Following the solution to Example 5-5, the magnetic field in the region $r < a$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}}\,\frac{rI}{2\pi a^2},$$

and in the region $a < r < b$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}}\,\frac{I}{2\pi r}.$$

The total area of the outer conductor is $A = \pi(c^2 - b^2)$ and the fraction of the area of the outer conductor enclosed by a circular contour centered at $r = 0$ in the region $b < r < c$ is

$$\frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} = \frac{r^2 - b^2}{c^2 - b^2}.$$

The total current enclosed by a contour of radius r is therefore

$$I_{\text{enclosed}} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = I\,\frac{c^2 - r^2}{c^2 - b^2},$$