Q1

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1. **Boundary Conditions.** If $E = 200\,\hat{\boldsymbol{R}}$ (V/m) at the surface of a conducting sphere centered at the origin with a radius of 10 cm, what is the total charge Q on the sphere's surface?

$$S = 4\pi(0.1)^2 = 0.126\ m^2$$

$$(\vec{D}_1 - \vec{D}_2)\cdot\hat{n} = \rho_s\ ,\ \vec{E}_2 = 0\ \text{(inside sphere)}$$

$$D_{1R} = \rho_s \rightarrow E_{1R} = \frac{\rho_s}{\varepsilon_0} = \frac{Q}{S\varepsilon_0}$$

$$Q = E_R S \varepsilon_0$$

$$Q = (200)(0.126)(8.854\times10^{-12})$$

$$Q = 2.23\times10^{-10}\ C$$

Q2

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2. **Boundary Conditions.**
   a. Find $\boldsymbol{E}_1$ given,

$$E_2 = 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} + 3\,\hat{\boldsymbol{z}}$$

   $\varepsilon_1 = 3\varepsilon_0,\ \varepsilon_2 = 16\varepsilon_0$, and the boundary has a surface charge density: $\rho_s = 6.25 \times 10^{-11}$ (C/m2).

   b. What angle does $\boldsymbol{E}_2$ make with the z-axis?

a) $E_1 = (E_1)_t + (E_1)_z$

$(E_1)_t = (E_2)_t = 5\hat{x} + 7\hat{y}$

$\varepsilon_1(E_1 \cdot \hat{n}) - \varepsilon_2(E_2 \cdot \hat{n}) = \rho_s$

$\varepsilon_1 (E_1)_z - \varepsilon_2 (E_2)_z = \rho_s$

$$(E_1)_z = \frac{\rho_s + \varepsilon_2 (E_2)_z}{\varepsilon_1}$$

$$(E_1)_z = \frac{6.25\times10^{-11} + 16\,\varepsilon_0(3)}{3\varepsilon_0}$$

$(E_1)_z = 18.4$ (V/m)

$$E_1 = (E_1)_t + (E_1)_z$$

$$\boxed{= 5\hat{x} + 7\hat{y} + 18.4\,\hat{z} \quad (V/m)}$$

b) $$(E_2)_z = |E_2| \cos(\theta)$$

$$\angle\theta = \cos^{-1}\left(\frac{(E_2)_z}{|E_2|}\right)$$

$$= \cos^{-1}\left(3 / \sqrt{25+49+9}\right)$$

$$\boxed{\angle\theta = 70.8^\circ}$$

Q3

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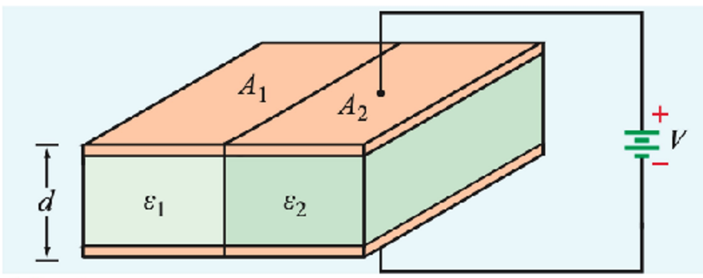
3. **Capacitance.** Given the two parallel, conducting plates separated by a distance d illustrated in the diagram below. The space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2.

Given:

$$C = C_1 + C_2$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d}, \qquad C_2 = \frac{\varepsilon_2 A_2}{d}$$



Find the following:

a. Find the electric fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the two dielectric layers.
b. Calculate the energy stored in each section.
c. Draw a circuit diagram of the above the dielectric section

$$C = C_1 + C_2 \ , \ C_1 = \frac{\varepsilon_1 A_1}{d} \ , \ C_2 = \frac{\varepsilon_2 A_2}{d}$$

a) $V = Ed$

$$E_1 = E_2 = V/d$$

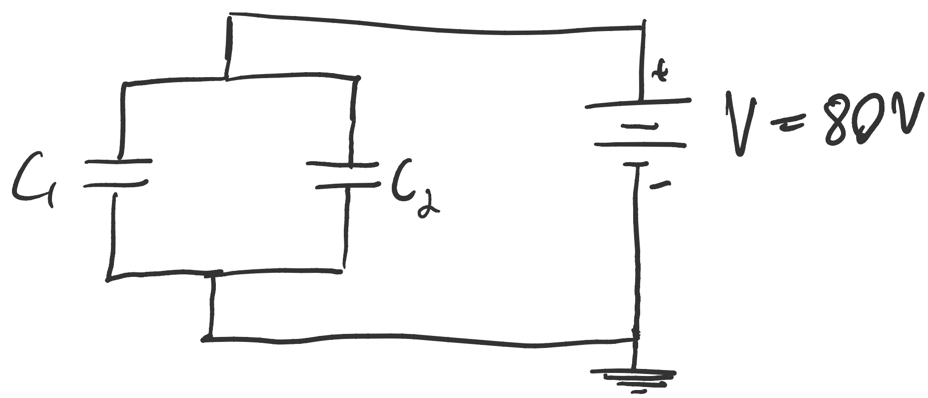
b) $W_e = \frac{1}{2}\varepsilon E^2 A d$

$$W_{e1} = \frac{1}{2} \varepsilon_1 E_1^2 A d$$

$$W_{e1} = \frac{V^2 \varepsilon_1 A}{2d}$$

$$\boxed{W_{e1} = \frac{V^2 C_1}{2}} \qquad \boxed{W_{e2} = \frac{V^2 C_2}{2}}$$

c)



Q4

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4. **Capacitance.** An electron with charge $Q_e = -1.6 \times 10^{-19} C$ and mass $m_e = 9.1 \times 10^{-31}$ kg is injected at a point adjacent to the negatively charged plate in the region between the plates of an air-filled, parallel-plate capacitor with separation of 50 cm and rectangular plates each 50 cm² in area (Fig. P4.54). If the voltage across the capacitor is 80 V, find the following:
   a. The force acting on the electron and

b. The acceleration of the electron.

$$d = 50 cm = 0.5 m$$

$$A = (0.5)^2 = 0.25 m^2$$

$$\Delta V = 80 V$$

a) $F = Q_e E = Q_e \frac{V}{d}$

$$F = (-1.6 \times 10^{-19} C)(80 V / 0.5 m)$$

$$F = -2.56 \times 10^{-17} N$$

b) $a = F/m_e = \frac{-2.56 \times 10^{-17} N}{9.1 \times 10^{-31} kg}$

$$a = 2.81 \times 10^{13} m/s^2$$

Q5

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A super capacitor is a special capacitor with very high capacitance, and that are able to store more energy than conventional capacitors. They are used for energy storage where the capacitor will be frequently charging and discharging. They are formed by two electrodes separated by a membrane but connected by an electrolyte, similar to a conventional electrolytic cell. They are able to store more energy than normal capacitors by creating a "double-layer" between the parallel plates. This is accomplished by using a porous, carbon-based dielectric which is capable of holding electrolyte, like a sponge.

References:

"Bu-209: How does a supercapacitor work?," *Battery University*, 22-Oct-2021. [Online]. Available: https://batteryuniversity.com/article/bu-209-how-does-a-supercapacitor-work. [Accessed: 20-Mar-2022].

C. Woodford, "How do supercapacitors work?," *Explain that Stuff*, 22-Jul-2020. [Online]. Available: https://www.explainthatstuff.com/how-supercapacitors-work.html. [Accessed: 20-Mar-2022].