Assignment 3

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1. For the RPP planar robot shown in Fig. 2.23 in lecture note.

a) Using the method of chapter 3, derive the 3x3 manipulator Jacobian matrix.

b) Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.

where,

$$A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & -s\theta_1 & -c\theta_1 & d_2 s\theta_1 - d_3 c\theta_1 \\ 0 & c\theta_1 & -s\theta_1 & -d_2 c\theta_1 - d_3 s\theta_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) $p_x = d_2 s\theta_1 - d_3 c\theta_1$

$$\frac{dp_x}{d\theta_1} = d_2 c\theta_1 + d_3 s\theta_1 \qquad \frac{dp_x}{dd_2} = s\theta_1 \qquad \frac{dp_x}{d_3} = -c\theta_1$$

$p_y = -d_2 c\theta_1 - d_3 s\theta_1$

$$\frac{dp_y}{d\theta_1} = d_2 s\theta_1 - d_3 c\theta_1 \qquad \frac{dp_y}{dd_2} = -c\theta_1 \qquad \frac{dp_y}{dd_3} = -s\theta_1$$

$$J_A(q) = \begin{bmatrix} d_2 c\theta_1 + d_3 s\theta_1 & s\theta_1 & -c\theta_1 \\ d_2 s\theta_1 - d_3 c\theta_1 & -c\theta_1 & -s\theta_1 \end{bmatrix}$$

$$J_B(q) = [\ \underbrace{\xi z_0}_{\text{revolute}} \quad \underbrace{\xi z_1 \quad \xi z_2}_{\text{prismatic}}\ ]$$

$$J_B(q) = [\ 1\, z_0 \quad 0\, z_1 \quad 0\, z_2\ ]$$

$\therefore$ Robot is planar $\therefore$ $V_z = 0$, $\omega_x = 0$, $\omega_y = 0$

$$J(q) = \begin{bmatrix} d_2 c\theta_1 + d_3 s\theta_1 & s\theta_1 & -c\theta_1 \\ d_2 s\theta_1 - d_3 c\theta_1 & -c\theta_1 & -s\theta_1 \\ 1 & 0 & 0 \end{bmatrix}$$

b) $\det(J) = J_{11}(J_{33} J_{22} - J_{22} J_{23})$

b) $\det(J) = J_{11}(J_{33}J_{22} - J_{22}J_{23})$
$- J_{21}(J_{33}J_{12} - J_{32}J_{13})$
$+ J_{31}(J_{23}J_{12} - J_{22}J_{13})$
$= J_{11}(\varnothing(-c\theta_1) - (-c\theta_1)\varnothing)$
$- J_{21}(\varnothing(s\theta_1) - \varnothing(1))$
$+ \cancel{J_{31}}1((-s\theta_1)(s\theta_1) - (-c\theta_1)(-c\theta_1))$

$= -(\sin^2\theta_1 + \cos^2\theta_1)$
$= -1$
$\therefore \det(J) \neq \varnothing$, there is no singularity

2. For the PPR robot shown in Fig. 2.26:

a) Using the method of chapter 3, derive the 3x3 manipulator Jacobian matrix.

b) Determine the singular configuration(s) for this robot by examining the determinant of the Jacobian.

where

$$^{n}T_{n+1} = A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C(180^\circ) & -S(180^\circ)C(90^\circ) & S(180^\circ)S(90^\circ) & (0)C(180^\circ) \\ S(180^\circ) & C(180^\circ)C(90^\circ) & -C(180^\circ)S(90^\circ) & (0)S(180^\circ) \\ 0 & S(90^\circ) & C(90^\circ) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C(-90^\circ) & -S(-90^\circ)C(90^\circ) & S(-90^\circ)S(90^\circ) & (0)C(-90^\circ) \\ S(-90^\circ) & C(-90^\circ)C(90^\circ) & -C(-90^\circ)S(90^\circ) & (0)S(-90^\circ) \\ 0 & S(90^\circ) & C(90^\circ) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3C(0^\circ) & S\theta_3S(0^\circ) & a_3C\theta_3 \\ S\theta_3 & C\theta_3C(0^\circ) & -C\theta_3S(0^\circ) & a_3S\theta_3 \\ 0 & S(0^\circ) & C(0^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = {}^{0}T_1 * {}^{1}T_2 * {}^{2}T_3 = A_1 * A_2 * A_3$$

$$^{0}T_3 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ S\theta_3 & C\theta_3 & 0 & d_2 + a_3S\theta_3 \\ -C\theta_3 & S\theta_3 & 0 & d_1 - a_3C\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Note: this planar robot is defined in y-z plane, so the 3*3 Jacobian Matrix should be related to V_y, V_z, ω_x)

a) $^{0}T_3 \rightarrow P_x = \varnothing$
$P_y = d_2 + a_3 s\theta_3$
$P_z = d_1 - a_3 s\theta_3$

$dP_x = \varnothing$

$\dfrac{dP_y}{dd_1} = \varnothing \qquad \dfrac{dP_y}{dd_2} = 1 \qquad \dfrac{dP_y}{d\theta_3} = a_3 c\theta_3$

$\dfrac{dP_z}{dd_1} = 1 \qquad \dfrac{dP_z}{dd_2} = \varnothing \qquad \dfrac{dP_z}{d\theta_3} = -a_3 c\theta_3$

$J_B(q) = [\xi_1 b_1,\ \xi_2 b_2,\ \xi_3 b_3]$
$= [\varnothing\ ;\ \varnothing;\ 1\ ]$

$$O_B(q) = [\ \text{[illegible]}\ ]$$

$$= [\,\emptyset,\ \emptyset,\ 1\,]$$

$$J(q) = \begin{bmatrix} 0 & 1 & a_3 C\theta_3 \\ 1 & 0 & a_3 S\theta_3 \\ 0 & 0 & 1 \end{bmatrix}$$

b)

$$\det(J) = J_{11}(J_{33}J_{22} - J_{32}J_{23}) - J_{21}(J_{33}J_{12} - J_{32}J_{13}) + J_{31}(J_{23}J_{12} - J_{22}J_{13})$$

$$= \emptyset(J_{33}J_{22} - J_{32}J_{23}) - 1((1)(1) - (\emptyset)(a_3 C\theta_3)) + \emptyset(J_{23}J_{12} - J_{22}J_{13})$$

$$= \emptyset - 1(1) + \emptyset$$

$$= -1$$

$\therefore \det(J) \neq \emptyset \ \therefore$ no singularities