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MECHTRON 3DX4 Tutorial Quiz 6 L02: Time Response

1. **Time Response** (10 marks)

   Consider the translational mechanical system in Figure 1

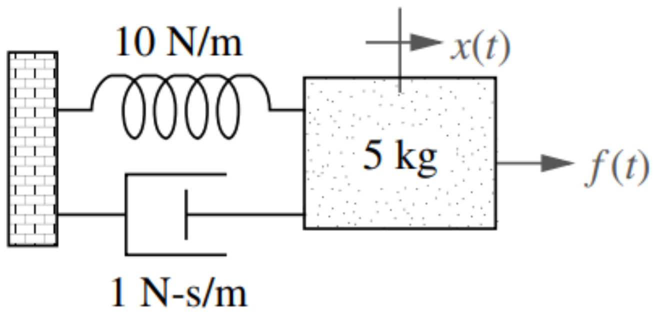
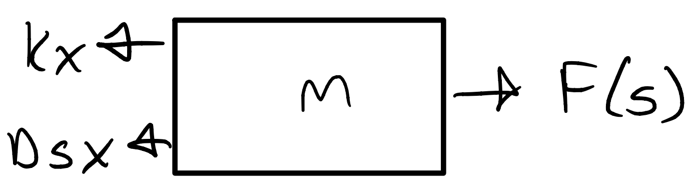


Figure 1: Translational System

   a) (3 marks) Find the transfer function $G(s) = \frac{X(s)}{F(s)}$.

   b) (7 marks) Assuming a unit step as the input, calculate ζ, ω_n, %OS, T_s, T_p, T_r, and c_{final}.

Relevant Equations can be found on page 2!



$$Ms^2 X = F(s) - Kx - DsX$$

$$F(s) = X(Ms^2 + Ds + K)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Ds + K}$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Ds + K}$$

$$= \frac{1}{5s^2 + s + 10}$$

$$= \frac{1/5}{s^2 + \frac{1}{5}s + 2} = \frac{Kb}{s^2 + as + b}$$

$$= K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$b = 2$

$1/5 = Kb$

$1/5 = K2$

$K = 1/10$

$\omega_n = \sqrt{b} = \sqrt{2}$

$1/5 = 2\zeta\omega_n$

$1/5 = 2\zeta\sqrt{2}$

$\zeta = \frac{1}{10\sqrt{2}}$

$$\boxed{\begin{aligned} \zeta &= \frac{1}{10\sqrt{2}} \\ \omega_n &= \sqrt{2} \end{aligned}}$$

$$\%OS = e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)} \times 100$$

$$= e^{-\left(\frac{1}{10\sqrt{2}}\pi \sqrt{1-\left(\frac{1}{10\sqrt{2}}\right)^2}\right)} \times 100$$

$$\%OS = e^{-0.221588} \times 100$$

$$\boxed{\%OS = 80.12\%}$$

Assuming unit is seconds

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\frac{1}{10\cancel{\sqrt{2}}}\cdot\cancel{\sqrt{2}}} = 40$$

$$\boxed{T_s = 40s}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{2}\sqrt{1-\left(\frac{1}{10\sqrt{2}}\right)^2}}$$

$$\boxed{T_p = 2.227s}$$

Assuming unit of seconds

Damping Ratio

$$\zeta = \frac{1}{10\sqrt{2}} \approx 0.0707$$

$$\frac{1.203 - 1.104}{0.2 - 0.1} = \frac{1.203 - T_{R\,norm}}{0.2 - 0.0707}$$

$$T_{R\,norm} = 1.075$$

$$T_{R\,norm} = T_R\,\omega_n$$

$$T_R = \frac{1.075}{\sqrt{2}}$$

$$\boxed{T_R = 0.76\ s}$$

Assuming Unit of Seconds

$$C(s) = \frac{1/5}{s\left(s^2 + \frac{1}{5}s + 2\right)}$$

$$c(t) = \frac{1}{10} - \frac{e^{-t/10}\left(\cos\left(\frac{\sqrt{199}}{10}t\right) + \sqrt{199}\,\sin\left(\frac{\sqrt{199}}{10}t\right)\right)}{10}$$

$$\lim_{t \to \infty} c(t) = \frac{1}{10}$$

$e^{-t/10} \to 0$

$$\boxed{c_{final} = 1/10}$$

Some relevant equations:

Peak Time:

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

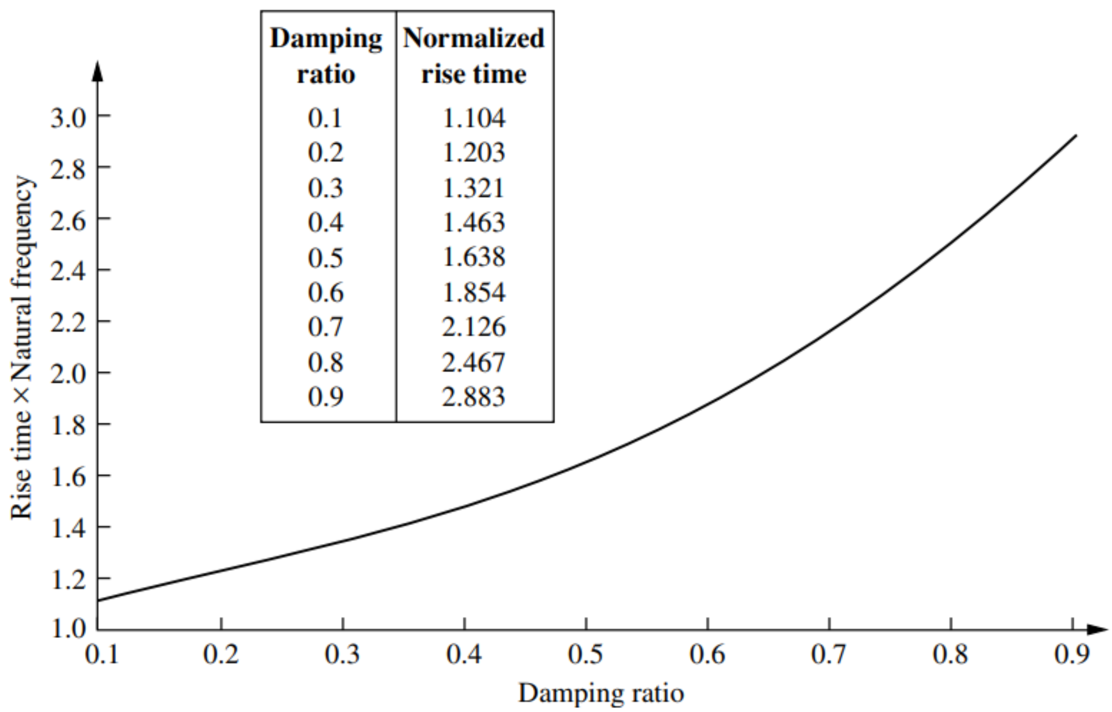
Percent Overshoot:

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

Settling Time:

$$T_s = \frac{4}{\zeta\omega_n}$$

Rise Time:



Damping ratio	Normalized rise time
0.1	1.104
0.2	1.203
0.3	1.321
0.4	1.463
0.5	1.638
0.6	1.854
0.7	2.126
0.8	2.467
0.9	2.883