

Due: 12pm, 25th November, 2016

Dropbox #10 in JHE 307

1. A planar RRR robot has the manipulator Jacobian:

$$J(q) = \begin{bmatrix} -a_1 S\theta_1 - a_2 S\theta_{12} - a_3 S\theta_{123} & -a_2 S\theta_{12} - a_3 S\theta_{123} & -a_3 S\theta_{123} \\ a_1 C\theta_1 + a_2 C\theta_{12} + a_3 C\theta_{123} & a_2 C\theta_{12} + a_3 C\theta_{123} & a_3 C\theta_{123} \\ 1 & 1 & 1 \end{bmatrix}$$

and the vector of gravity terms (gravity acts in the  $-Y_0$  direction):

$$G(q) = \begin{bmatrix} (\frac{1}{2}m_1 + m_2 + m_3)ga_1C\theta_1 + (\frac{1}{2}m_2 + m_3)ga_2C\theta_{12} + \frac{1}{2}m_3ga_3C\theta_{123} \\ (\frac{1}{2}m_2 + m_3)ga_2C\theta_{12} + \frac{1}{2}m_3ga_3C\theta_{123} \\ \frac{1}{2}m_3ga_3C\theta_{123} \end{bmatrix}$$

The link lengths and masses are:  $a_1=0.5$  m,  $a_2=0.5$  m,  $a_3=0.1$  m,  $m_1=10$  kg,  $m_2=10$  kg and  $m_3=2$  kg.

(a) If the arm is holding a 5 kg payload, when  $\theta_1 = 45^\circ$ ,  $\theta_2 = -75^\circ$  and  $\theta_3 = 30^\circ$ , calculate the required joint torques.

(b) The robot's control unit can only control the torques with a resolution of 0.1 Nm. We are interested in using the robot to apply a precise force in the  $X_0$  direction (*e.g.* to insert a component without damaging it). We can use configuration A:  $\theta_1 = 45^\circ$ ,  $\theta_2 = -75^\circ$  and  $\theta_3 = 30^\circ$ , or configuration B:  $\theta_1 = 45^\circ$ ,  $\theta_2 = -5^\circ$  and  $\theta_3 = -40^\circ$ . Note that both configurations align the tool with  $X_0$  (*i.e.*  $\phi = 0$ ). Determine which configuration will provide the most precise force output.