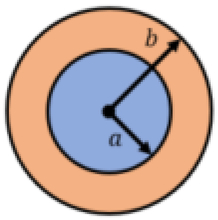
ENGPHYS 2A04 Winter 2022 – Assignment 7 Solutions

DUE MONDAY MARCH 14th, 8AM

1. The coaxial cable shown below is $10m$ long, and has a $5V$ potential applied across it. The inner cylinder is made of silicon and the outer cylinder is made of carbon. If the outer shell is to be twice the diameter of the inner cylinder, find the radii a and b for which a current of $0.2A$ would be expected.



Solution

The resistance needs to be $R = \frac{V}{I} = \frac{5}{0.2} = 25\Omega$.

The actual resistance of this will follow

$$R = \frac{l}{\sigma_1 A_1 + \sigma_2 A_2} = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2(b^2 - a^2)\right)}$$

From the geometry requirement added in the problem $b = 2a$, so this expression can be simplified:

$$R = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2((2a)^2 - a^2)\right)} = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2(4a^2 - a^2)\right)} = \frac{l}{\pi(\sigma_1 a^2 + 3\sigma_2 a^2)}$$
$$= \frac{l}{\pi(\sigma_1 + 3\sigma_2)a^2}$$

So, the inner radius can be found as:

$$a = \sqrt{\frac{l}{\pi(\sigma_1 + 3\sigma_2)R}}$$

Substituting in the known values for the length $l = 10m$, the resistance $R = 25\Omega$ and the conductances of silicon and carbon $\sigma_1 = 4.4 \times 10^{-4} S/m$, $\sigma_2 = 3 \times 10^4 S/m$:

$$a = \sqrt{\frac{10}{\pi\left(4.4 \times 10^{-4} + 3(3 \times 10^4)\right)25}} = \sqrt{\frac{10}{7068583.5}} = 1.2mm$$

Therefore, the radii must be $a = 1.2mm$, and $b = 2a = 2.4mm$.

2. A uniform sheet of charge with $\rho_{S1} = 1\frac{nC}{m^2}$ lies on the $z = 0$ plane, and a second sheet with $\rho_{S2} = -1\frac{nC}{m^2}$ occupies the $z = 5m$ plane. Find the scalar potential V_{AB} between the points $A(0,0,5m)$ and $B(0,-1m,2m)$. Explain what this scalar potential means in physical terms, in the context of this problem.

Solution

These are infinite planes parallel to the z-plane, so the electric field will be directed in the z-direction. The total electric field will be the sum of those caused by each of the plates:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 = \hat{\boldsymbol{z}}\frac{\rho_{s1}}{2\varepsilon_0} - \hat{\boldsymbol{z}}\frac{\rho_{s2}}{2\varepsilon_0} = \hat{\boldsymbol{z}}\frac{\rho_{s1} - \rho_{s2}}{2\varepsilon_0}$$

The potential between two points only depends on z, and will follow the equation:

$$V_{AB} = -\int_0^5 \boldsymbol{E} \cdot d\boldsymbol{z} = -\int_{z_B}^{z_A} \hat{\boldsymbol{z}}\frac{\rho_{s1} - \rho_{s2}}{2\varepsilon_0} \cdot \hat{\boldsymbol{z}}dz$$

$$V_{AB} = -\frac{\rho_{s1} - \rho_{s2}}{2\varepsilon_0}\int_{z_B}^{z_A} dz = -\frac{\rho_{s1} - \rho_{s2}}{2\varepsilon_0} z\Big|_{z_B}^{z_A} = -\frac{\rho_{s1} - \rho_{s2}}{2\varepsilon_0}(z_A - z_B)$$

Substituting in the known values:

$$\boldsymbol{V_{AB} = -\frac{1 \times 10^{-9} - (-1 \times 10^{-9})}{2\varepsilon_0}(5 - 2) = -3\frac{1 \times 10^{-9}}{8.85 \times 10^{-12}} = -339V}$$

This means that to move one coulomb of charge from point A to point B, it would require 339J of energy.

3. A cable with a uniform square cross-section has a length of $200m$, and there is an $8V$ potential applied across it. If the current density through it is determined to be $1.4 \times 10^6\ A/m^2$, find the conductivity of its material. Name a material that could be used here.

   Solution

   $\vec{J} = \sigma\vec{E}$, so $\sigma = \frac{J}{E}$. Electric field is related to voltage through the length:

$$E = \frac{V}{l}$$

$$\therefore \sigma = \frac{Jl}{V} = \frac{1.4 \times 10^6(200)}{8}$$

$\boldsymbol{\sigma = 3.5 \times 10^7 \frac{S}{m}}$**, one possible material is aluminium.**

4. In class, we discussed the concept of *dielectric breakdown* using the example of lightning. Research another example of dielectric breakdown. Explain your example in 5 sentences or fewer, and as always, cite your sources. Make sure you capture the following:

   - What is the mechanism or mechanisms by which breakdown occurs?
   - Is it ever desirable? What are potential consequences (or benefits)?
   - What factors contribute to the likelihood of this breakdown occurring?

5. **BONUS:** Complete the mid-semester survey!

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal points.
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.