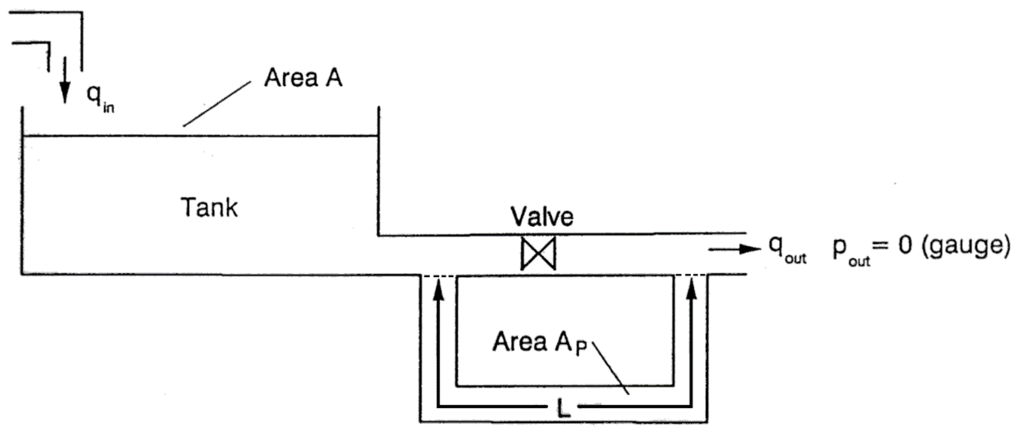
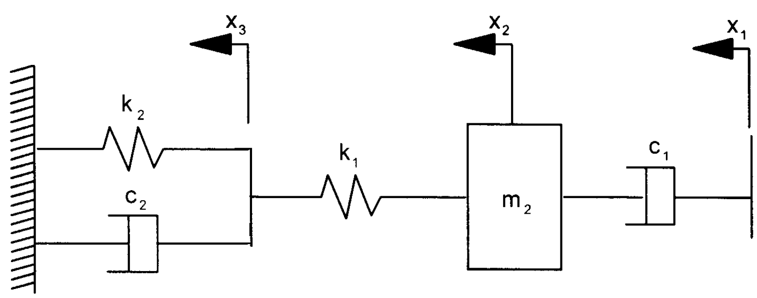
1. For the hydraulic system shown in the schematic diagram below, determine the Laplace transfer function between the input volumetric flow rate, q_{in}, and the output flow rate, q_{out}. The output pressure, p_{out}, is atmospheric (i.e. zero gauge). You may assume that the length of pipe L is the only significant one in the system, and that the valve may be modelled by a linear building block. The fluid has a density D, and may be assumed to be incompressible.



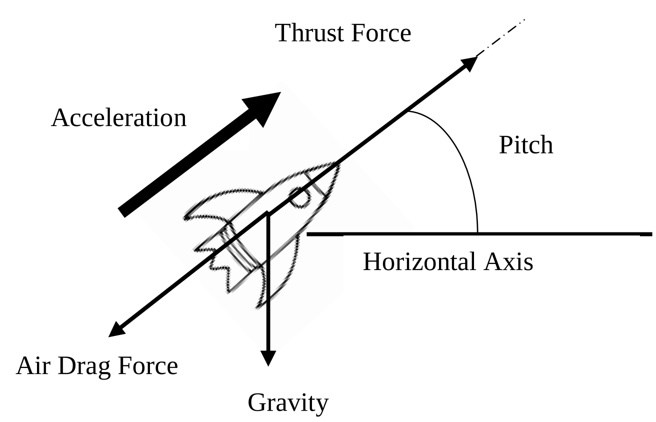
2. For the mechanical system shown below, determine the Laplace transfer function between the displacements x_1 and x_3. Write your final answer in the form $\frac{X_3(s)}{X_1(s)} = \ldots$.



3. This question involves genuine rocket science. A rocket is propelled by its engine's thrust force. This force must overcome gravity and the drag force due to the air. Neglecting the change in mass due to burnt fuel, the acceleration of a rocket is modeled by the nonlinear equation:

$$a = \frac{T}{m} - g\sin\theta - \frac{C_d v^2}{m} \qquad (1)$$

where T is the thrust force, m is the mass, θ is the angle of pitch, C_d is the drag coefficient, and v is the velocity.



a) If T, θ, and v are changing (and the other parameters are constant) derive the linearized form of this equation for the operating point (T_o, θ_o, v_o).

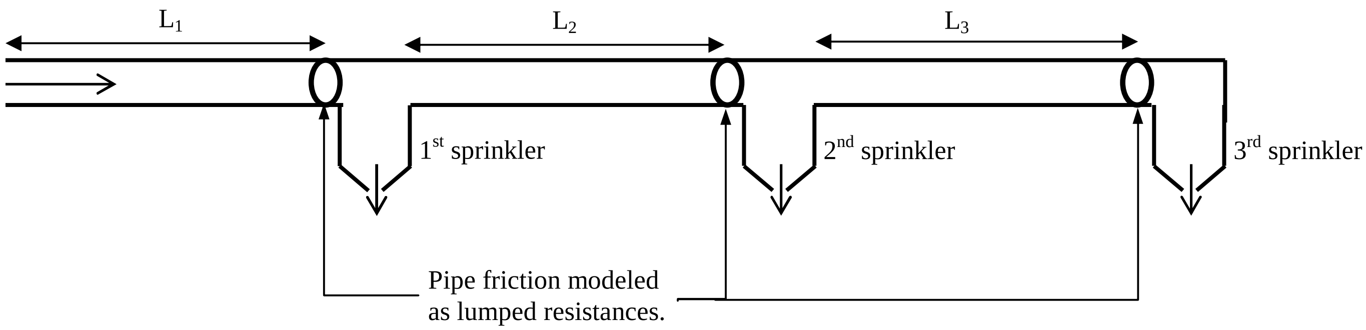
b) The second derivative is a measure of curvature. The greater the nonlinearity the greater the curvature and the larger the error of the linearized equation.

i) Derive the partial second derivatives of equation (1) with respect to θ and v.

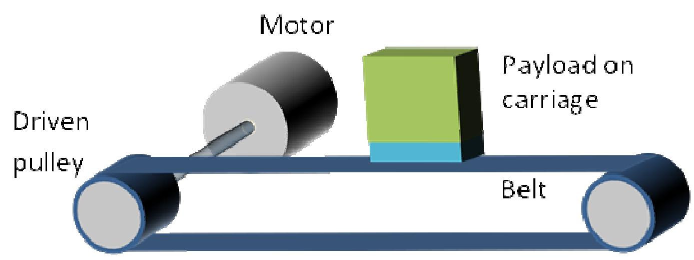
ii) Based on your answer to (i) describe how the error of the linearized equation varies with v_O.

iii) Based on your answer to (i) describe how the error of the linearized equation varies with θ_o.

4. A fire suppression system uses three sprinklers separated by long lengths of pipe. Each sprinkler head may be modelled as a hydraulic resister with the same resistance value, R_v. The pressure losses due to friction along the length of the pipe between each sprinkler cannot be neglected and should be modeled as a lumped hydraulic resistance in series with the inertance. Determine the Laplace transfer function between the volume flow rate exiting the first sprinkler and the volume flow rate exiting last sprinkler.



5. We know from Chapter 3 that a timing belt can be used to convert rotary motion to linear motion. An example of a belt driven linear actuator is shown below to the left (sold by Isel Automation). Its schematic is shown below to the right. The belt runs on two pulleys. The motor is directly coupled to the pulley which moves the belt, known as the "driven pulley". A carriage is attached to the belt to provide linear motion. It carries the payload. The masses of the pulleys are negligible. The payload and carriage may be modelled as a single mass subject to a friction force defined by equation (4.106). The belt can only pull this mass, it cannot push it. The belt is flexible and may be modelled as a massless spring. Denote the rotation angle of the driven pulley as θ, the payload's linear motion as x and the motor's torque as τ_m in your answer.



(a) Draw free body diagrams of the motor plus driven pulley; and the payload plus carriage.

(b) Derive the differential equations for the angular acceleration of the motor plus pulley; and for the linear acceleration of the payload plus carriage.

(c) Assuming friction forces $F_{dynamic}$ and F_{static} are negligible, starting with your answer to (b), derive the transfer function $X(s)/\tau_m(s)$.