In-Class Test (#3)

Name ___________________
Student Number ___________________

ROBOTICS 4K03

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DURATION OF EXAMINATION: 50 MINS

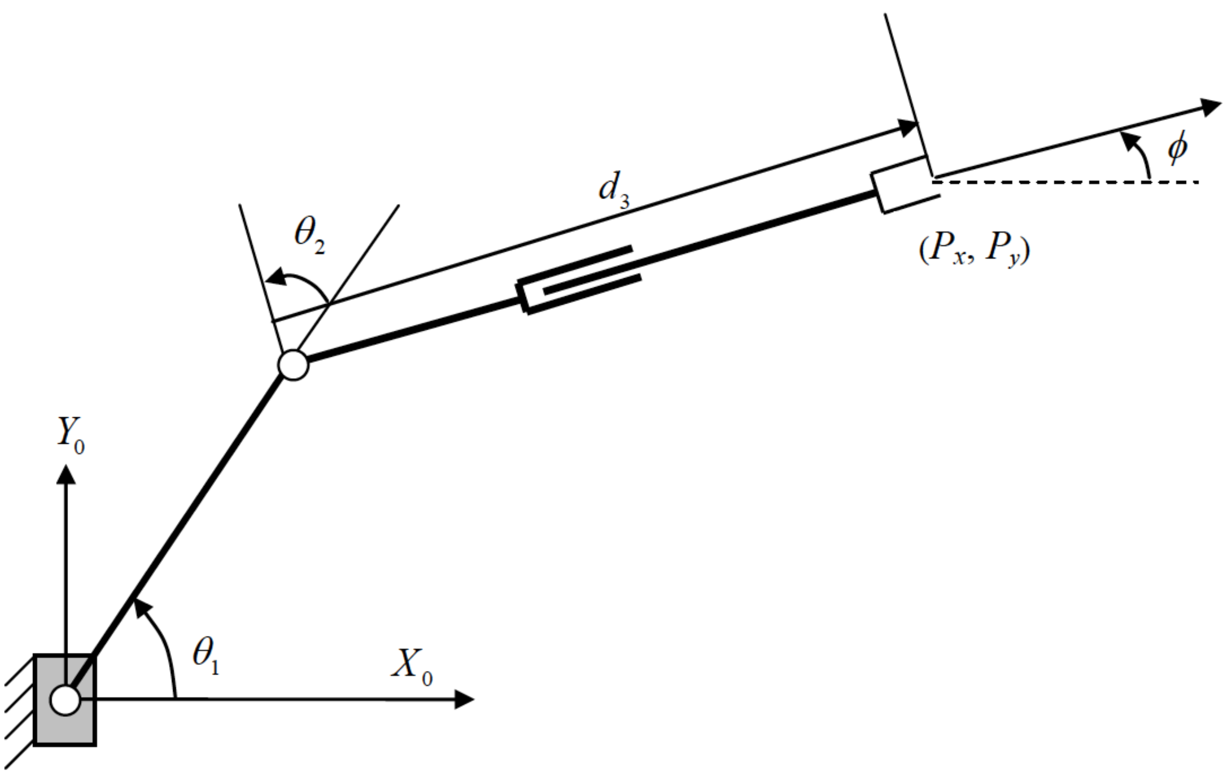
Nov. 17th, 2016

THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 2 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Use of Casio FX-991 calculator. This paper must be returned with your answers.

Questions:

1. (45 points) A RRP planar robot is shown in the following figure. Its joint variables are q_1 and q_2, and d_3. Its end-effector position and orientation are given by P_x and P_y, and f. Derive the inverse kinematics equations for this robot.



Solutions:

$\Theta_1 + \Theta_2 - 90^\circ = \phi$

$P_x = a_1 . C\Theta_1 + d_3\, C\phi$ -----> $C\Theta_1 = \frac{P_x - d_3\, C\,\phi}{a_1}$

$P_y = a_1 . S\Theta_1 + d_3 S\phi$ -----> $S\Theta_1 = \frac{P_y - d_3 S\phi}{a_1}$

$C^2\Theta_1 + S^2\,\Theta_1 = (\frac{P_x - d_3\, C\,\phi}{a_1})^2 + (\frac{P_y - d_3\, S\,\phi}{a_1})^2 = 1$

$d_3{}^2 - 2(\,P_x\, C\phi + P_y\, S\phi)]\, d_3 + P_x{}^2 + P_y{}^2 - a_1{}^2 = 0$

$$d_3 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where:

a= 1

$\mathrm{b} = -2(P_x C\emptyset + P_y S\emptyset)$

$\mathrm{c} = P_x^2 + P_y^2 - a_1^2$

$$d_3 = P_x C\emptyset + P_y S\emptyset \pm \frac{\sqrt{[2(P_x C\emptyset + P_y S\emptyset)]^2 - 4(P_x^2 + P_y^2 - a_1^2)}}{2}$$

Assume $\Delta = \left[2\left(\,P_x\, C\,\phi\; + \; P_y\, S\,\phi\right)\right]^2 - 4\,(P_x^2 + \; P_y^2 - \; a_1^2)$

If $\Delta < 0$, there will be No solution for d_3.

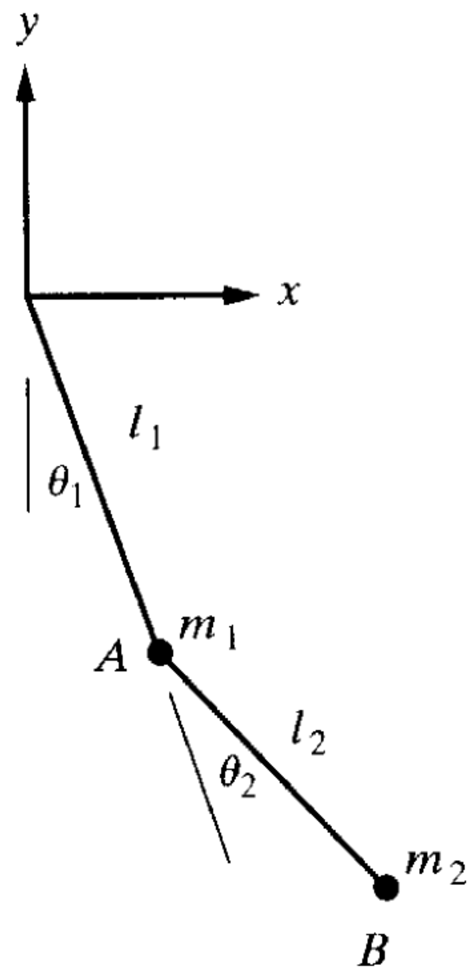
If $\Delta = 0$, there will be only ONE solution for d_3.

If $\Delta > 0$, there will be TWO solutions for d_3.

$$\Theta_1 = atan2(\frac{P_x \; - \; d_3\, C\,\phi}{a_1}, \frac{P_y \; - \; d_3 S\phi}{a_1})$$

$$\Theta_2 = 90^\circ + \varphi - \Theta_1$$

2. (55 points) The planar RR robot shown in the following figure operates in the vertical plane (i.e., gravity acts in the $-y$ direction). The masses of the links are concentrated at the end of each link and are m_1 and m_2, respectively. Derive the dynamics equations for this robot using the Lagrangian method.



Solutions:

Please refer to Example 5.1 in lecture note.