# ME 4K03 MIDTERM 2

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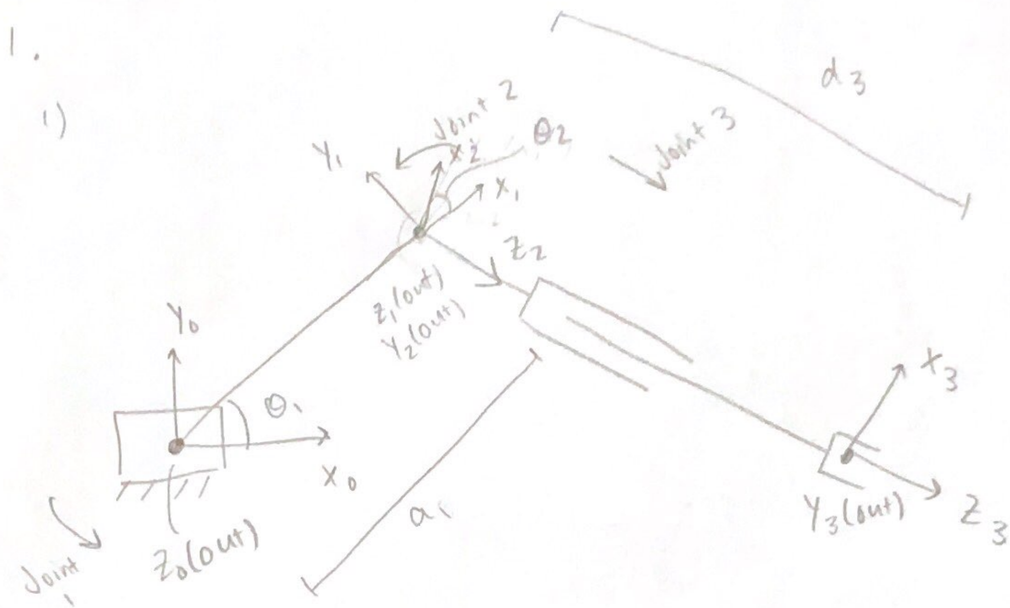
1.

1)




2)

| n+1 | $\theta_{n+1}$ | $d_{n+1}$ | $a_{n+1}$ | $\alpha_{n+1}$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | 0 | $a_1$ | $0°$ |
| 2 | $\theta_2$ | 0 | 0 | $90°$ |
| 3 | $0°$ | $d_3$ | 0 | $0°$ |

joint variables: $\theta_1, \theta_2, d_3$
fixed parameters: $a_1$

3) see diagram for (1)

4) using
$$A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 & 0 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^0T_3 = A_1A_2A_3 = \begin{bmatrix} C\theta_{12} & 0 & S\theta_{12} & d_3S\theta_{12} + a_1C\theta_1 \\ S\theta_{12} & 0 & -C\theta_{12} & a_1S\theta_1 + d_3C\theta_{12} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.

| n+1 | $\theta_{n+1}$ | $d_{n+1}$ | $a_{n+1}$ | $\alpha_{n+1}$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | 0 | $a_1$ | 0° |
| 2 | $\theta_2$ | 0 | $a_2$ | 0° |

1)

using the same method as in Q1

$a_1 = 0.4$
$a_2 = 0.3$

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0.4C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & 0.4S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0.3C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 0.3S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_2 = A_1A_2 = \begin{bmatrix} C\theta_{12} & -S\theta_{12} & 0 & 0.3C\theta_{12} + 0.4C\theta_1 \\ S\theta_{12} & C\theta_{12} & 0 & 0.3S\theta_{12} + 0.4S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = {}^0R_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2)

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \underbrace{\begin{bmatrix} -0.3S\theta_{12} - 0.4S\theta_1 & -0.3S\theta_{12} \\ 0.3C\theta_{12} + 0.4C\theta_1 & 0.3C\theta_{12} \\ 1 & 1 \end{bmatrix}}_{J(q)} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \frac{\partial P_x}{\partial \theta_1} & \frac{\partial P_x}{\partial \theta_2} \\ \frac{\partial P_y}{\partial \theta_1} & \frac{\partial P_y}{\partial \theta_2} \\ \xi_1 z_0 & \xi_2 z_1 \end{bmatrix}$$

$\xi_1 = \xi_2 = 1 \rightsquigarrow$ b/c revolute joints

3)

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -0.3S(35-75) - 0.4S(35°) & -0.3S(35-75°) \\ 0.3C(35-75) + 0.4C(35°) & 0.3C(35-75°) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 100°/s \\ -50°/s \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -13.3012 \\ 44.2567 \\ 50 \end{bmatrix}$$

$\therefore v_x = -13.3012$ m/s
$v_y = 44.2567$ m/s

extra work / rough notes

2. 2) $z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad z_1 = {}^0R_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$