Assignment 1

Monday, May 9, 2022 5:07 PM

Problem #1: In the layout of a printed circuit board for an electronic product, 14 different locations can accommodate chips.

(a) If 6 chips of different types are to be placed on the board, how many different layouts are possible?

(b) If 6 chips of the same type are to be placed on the board, how many different layouts are possible?

(c) If all of the locations are to be filled with chips, 3 of which are of one type, 6 of which are another type, and all others different, how many different layouts are possible?

a.) $n = 14$; $r = 6$

$${}_nP_r = {}_{14}P_6 = \frac{14!}{(14-6)!} = 2162160 \text{ ways}$$

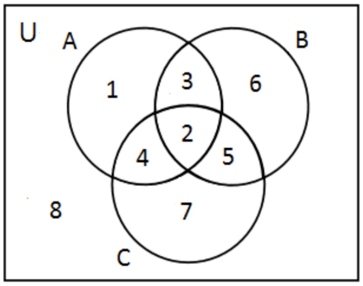
b.) $n = 14$; $r = 6$

$${}_nC_r = {}_{14}C_6 = \frac{14!}{(14-6)!\ 6!} = 3003 \text{ ways}$$

c.) $n = 14$; $a = 3$; $b = 6$

$$\frac{n!}{a!\,b!} = \frac{14!}{3!\,6!} = 20\ 180\ 160 \text{ ways}$$

Problem #2: Consider the Venn diagram given to the right. In each part, determine which of the regions correspond to the given statements.



B **(a)** $A \cup B'$

D **(b)** $(A \cap B') \cup C$

F **(c)** $A \cup (B \cap C)$

C **(d)** $A \cup B$

a.) $A \cup B'$

Regions 1, 2, 3, 4, 7, 8

b.) $(A \cap B') \cup C'$

Regions 1, 2, 4, 5, 7

c.) $A \cup (B \cap C)$

Regions 1, 2, 3, 4, 5

d.) $A \cup B$

Regions 1, 2, 3, 4, 5, 6

Problem #3: A manufacturer of front lights for automobiles tests lamps under a high-humidity, high-temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 130 lamps.

		Useful Life	
		Satisfactory	Unsatisfactory
Intensity	Satisfactory	94	9
	Unsatisfactory	12	15

Suppose that one of the above lamps is randomly selected. Let A be the event that is has a satisfactory useful life and let B be the event that it has a satisfactory intensity. Calculate the following probabilities.

(a) $P(A' \cup B)$

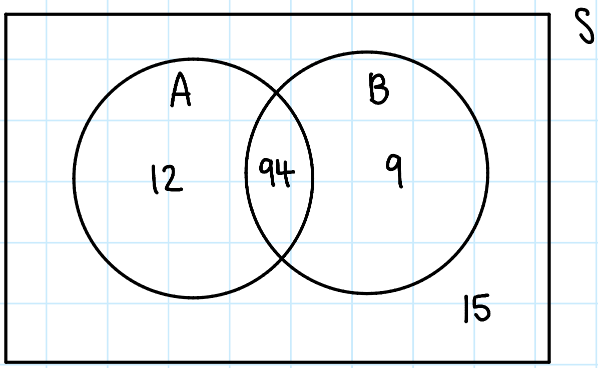
(b) $P(A' \cap B)$

(c) $P(A \cap B)$

(d) $P(A) \times P(B)$

$A = 94 + 12 = 106$　　　　$B = 94 + 9 = 103$

$A' = 9 + 15 = 24$　　　　$B' = 12 + 15 = 27$



total = 130

a.) $P(A' \cup B)$

$$P(A' \cup B) = \frac{94 + 9 + 15}{130} = \frac{118}{130}$$

b.) $P(A' \cap B)$

$$P(A' \cap B) = \frac{9}{130}$$

c.) $P(A \cap B)$

$$P(A \cap B) = \frac{94}{130}$$

d.) P(A) x P(B)

$$P(A) \times P(B) = \frac{106}{130} \times \frac{103}{130} = \frac{5459}{8450}$$

Problem #4: Suppose that a code (similar to a postal code) is of the form LDL DLD, where 'L' is an uppercase letter from A to T (i.e., 20 possible letters) and 'D' is a digit from 0 to 4. Suppose that such a code is randomly generated.

(a) Find the probability that the code has no repeated letters.

(b) Find the probability that the code either starts with an 'A' or ends with an even digit (note that 0 is even).

(c) Find the probability that the code starts with an 'A' and does not contain any 'B's.

20 possible letters , 5 possible numbers

Total $= 20^3 \times 5^3$

a.)

$$\underset{20}{\underline{L}}\ \underset{5}{\underline{D}}\ \underset{19}{\underline{L}}\quad \underset{5}{\underline{D}}\ \underset{18}{\underline{L}}\ \underset{5}{\underline{D}}$$

$20 \times 18 \times 19 \times 5^3$

$$P = \frac{\cancel{20} \times 18 \times 19 \times \cancel{5^3}}{20^{\cancel{3}\,2} \times \cancel{5^3}} = \frac{171}{200}$$

b.) starts with A : $1 \times 5 \times 20 \times 5 \times 20 \times 5$

$= 20^2 \times 5^3$

ends with even digit : $20 \times 5 \times 20 \times 5 \times 20 \times 3$

$= 20^3 \times 5^2 \times 3$

both : $1 \times 5 \times 20 \times 5 \times 20 \times 3$

$= 20^2 \times 5^2 \times 3$

$$P = \frac{(20^2 \times 5^3) + (20^3 \times 5^2 \times 3) - (20^2 \times 5^2 \times 3)}{20^3 \times 5^3} = \frac{31}{50}$$

c.) $\underset{1}{\underline{L}}\ \underset{5}{\underline{D}}\ \underset{19}{\underline{L}}\quad \underset{5}{\underline{D}}\ \underset{19}{\underline{L}}\ \underset{5}{\underline{D}}$

$19^2 \times 5^3$

$$p = \frac{19^2 \times \cancel{5^3}}{20^3 \times \cancel{5^3}} = \frac{361}{8000}$$

Problem #5: A maintenance firm has gathered the following information regarding the failure mechanism for air conditioning systems.

		G	G'
		Gas Leaks	
		Yes	No
E **Evidence of**	Yes	46	18
E' **electrical failure**	No	30	16

If this is a representative sample of AC failure, find the following probabilities.

(a) That there was evidence of electrical failure, given that there was a gas leak.

(b) That there was a gas leak, given that there was evidence of electrical failure.

(c) Let A be the event that there is a gas leak and let B be the event that there is evidence of electrical failure. Find $P(B' \mid A)$.

$$\text{Total} = 46 + 18 + 30 + 16 = 110$$

a.) $P(E|G)$

$$P(G \cap E) = \frac{46}{110} \quad ; \quad P(G) = \frac{46 + 30}{110} = \frac{76}{110}$$

$$P(E|G) = \frac{P(G \cap E)}{P(G)} = \frac{46}{110} \div \frac{76}{110} = \frac{23}{38}$$

b.) $P(G|E)$

$$P(E) = \frac{46 + 18}{110} = \frac{64}{110}$$

$$P(G|E) = \frac{P(G \cap E)}{P(E)} = \frac{46}{110} \div \frac{64}{110} = \frac{23}{32}$$

c.) $P(E'|G)$

$$P(E' \cap G) = \frac{30}{110}$$

$$P(E'|G) = \frac{P(E' \cap G)}{P(G)} = \frac{30}{110} \div \frac{76}{110} = \frac{15}{38}$$

Problem #6: A production facility employs 17 workers on the day shift, 11 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to randomly select 5 of these workers for in-depth interviews.

(a) What is the probability that all 5 selected workers will be from the same shift?

(b) What is the probability that at least two different shifts will be represented among the selected workers?

(c) What is the probability that exactly 3 of the workers in the sample come from the day shift?

$$\left.\begin{array}{l} \text{Day shift} = 17 \\ \text{Swing shift} = 11 \\ \text{Graveyard shift} = 10 \end{array}\right] - \text{Total} = 17 + 11 + 10 = 38$$

a.) D : ${}_{17}C_5$

S : ${}_{11}C_5$

G : ${}_{10}C_5$

total : ${}_{38}C_5$

$$P = \frac{{}_{17}C_5 + {}_{11}C_5 + {}_{10}C_5}{{}_{38}C_5} = \frac{29}{2109}$$

b.) P = all possibilities − only one shift

$$P = 1 - \frac{29}{2109} = \frac{2080}{2109}$$

$$P = 1 - \frac{29}{2109} = \frac{2080}{2109}$$

c.) D : ${}_{17}C_3$

S + G : ${}_{21}C_2$

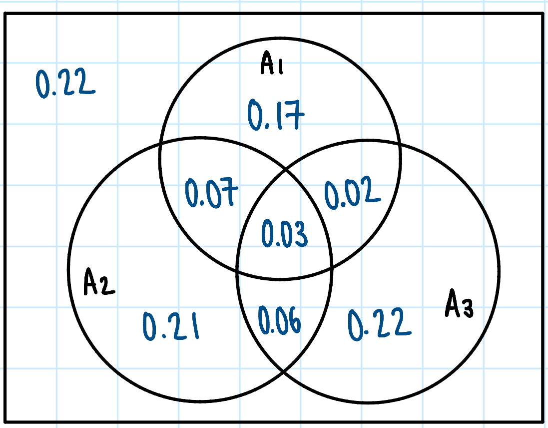
$$P = \frac{{}_{17}C_3 \times {}_{21}C_2}{{}_{38}C_5} = \frac{200}{703}$$

Problem #7: A certain system can experience three different types of defects. Let A_i ($i = 1,2,3$) denote the event that the system has a defect of type i. Suppose that

$P(A_1) = 0.29$, $P(A_2) = 0.37$, $P(A_3) = 0.33$,
$P(A_1 \cup A_2) = 0.56$, $P(A_1 \cup A_3) = 0.57$,
$P(A_2 \cup A_3) = 0.61$, $P(A_1 \cap A_2 \cap A_3) = 0.03$

(a) Find the probability that the system has exactly 2 of the 3 types of defects.

(b) Find the probability that the system has a type 1 defect given that it does not have a type 2 defect and does not have a type 3 defect.



$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$\Rightarrow 0.56 = 0.29 + 0.37 - P(A_1 \cap A_2)$

$\Rightarrow P(A_1 \cap A_2) = 0.10$

$P(A_2 \cup A_3) = P(A_2) + P(A_3) - P(A_2 \cap A_3)$

$\Rightarrow 0.61 = 0.37 + 0.33 - P(A_2 \cap A_3)$

$\Rightarrow P(A_2 \cap A_3) = 0.09$

$P(A_1 \cup A_3) = P(A_1) + P(A_3) - P(A_1 \cap A_3)$

$\Rightarrow 0.57 = 0.29 + 0.33 - P(A_1 \cap A_3)$

$\Rightarrow P(A_1 \cap A_3) = 0.05$

a.) exactly 2 of 3

$P = 0.07 + 0.02 + 0.06 = 0.15$

b.) $P(A_1 \mid (A_2' \cap A_3'))$

$P(A_2' \cap A_3') = 0.17 + 0.22 = 0.39$

$P(A_1 \cap (A_2' \cap A_3')) = 0.17$

$$P(A_1 \mid (A_2' \cap A_3')) = \frac{P(A_1 \cap (A_2' \cap A_3'))}{P(A_2' \cap A_3')} = \frac{0.17}{0.39} = \frac{17}{39}$$

Problem #8: Heart failures are due to either natural occurrences (90%) or outside factors (10%). Outside factors are related to induced substances (77%) or foreign objects (23%). Natural occurrences are caused by arterial blockage (55%), disease (27%), and infection (18%).

(a) Determine the probability that a faliure is due to an induced substance.

(b) Determine the probability that a failure is due to disease or infection.

a.) $P = P(\text{out}) \cdot P(\text{ind. sub.}) = (10\%) \cdot (77\%) = 7.7\% = 0.077$

b.) $P = P(\text{nat}) \cdot P(\text{dis}) + P(\text{nat}) \cdot P(\text{inf})$

$= (90\%)(27\%) + (90\%)(18\%)$

$= 40.5\% = 0.405$

Problem #9: A lot of 100 semiconductor chips contains 21 that are defective.

(a) Two are selected, one at a time and without replacement from the lot. Determine the probability that the second one is defective.

(b) Three are selected, one at a time and without replacement. Find the probability that the first one is defective and the third one is not defective.

79 good, 21 defective

79 good, 21 defective

a.) 99 chips after first draw

A: if 1st chip in not defective

$$P(A) = \frac{79}{100} \times \frac{21}{99} = \frac{553}{3300}$$

B: if 1st chip is defective

$$P(B) = \frac{21}{100} \times \frac{20}{99} = \frac{7}{165}$$

$$p = P(A) + P(B) = \frac{553}{3300} + \frac{7}{165} = \frac{21}{100}$$

b.) 99 after first draw, 98 after second draw

C: if 2nd is not defective

$$P(C) = \frac{21}{100} \times \frac{79}{99} \times \frac{78}{98} = \frac{1027}{7700}$$

D: if 2nd is defective

$$P(D) = \frac{21}{100} \times \frac{20}{99} \times \frac{79}{98} = \frac{79}{2310}$$

$$p = P(C) + P(D) = \frac{1027}{7700} + \frac{79}{2310} = \frac{553}{3300}$$

Problem #10: Suppose that an operating room needs to schedule 2 knee, 4 hip, and 6 shoulder surgeries. Assume that all schedules are equally likely.

(a) Find the probability that all of the knee surgeries are completed first.

(b) Find the probability that the schedule begins with a hip surgery, given that all of the shoulder surgeries are last.

2 Knee, 4 hip, 6 shoulder $\Rightarrow$ 12 total

a.) All Knee first

$$P = \frac{2}{12} \times \frac{1}{11} = \frac{1}{66}$$

b.) Hip first given all shoulders last

$$P = \frac{4}{12-6} = \frac{4}{6} = \frac{2}{3}$$