Section 3 Actuators

3.1 Introduction

We will define an actuator as a device that acts on the plant to change it in some desired way.

For example, an electric motor applies torque to robot arm causing it to accelerate or a furnace heats the air in a room
(The furnace example is not part of Chapter 3, but is part of Chapter 4).

3.2 Mechanisms

Although mechanisms are not actuators themselves they are commonly used in mechatronic designs to transform motion or to provide a mechanical advantage.

Note that since the power output cannot be larger than the power input an increase in force is combined with a decrease in speed.

3.2.1 Four-bar Mechanisms and Cams

Figure 3.1 Kinematic diagrams for two types of four-bar mechanisms. Left: crank-rocker. Right: parallel-crank.

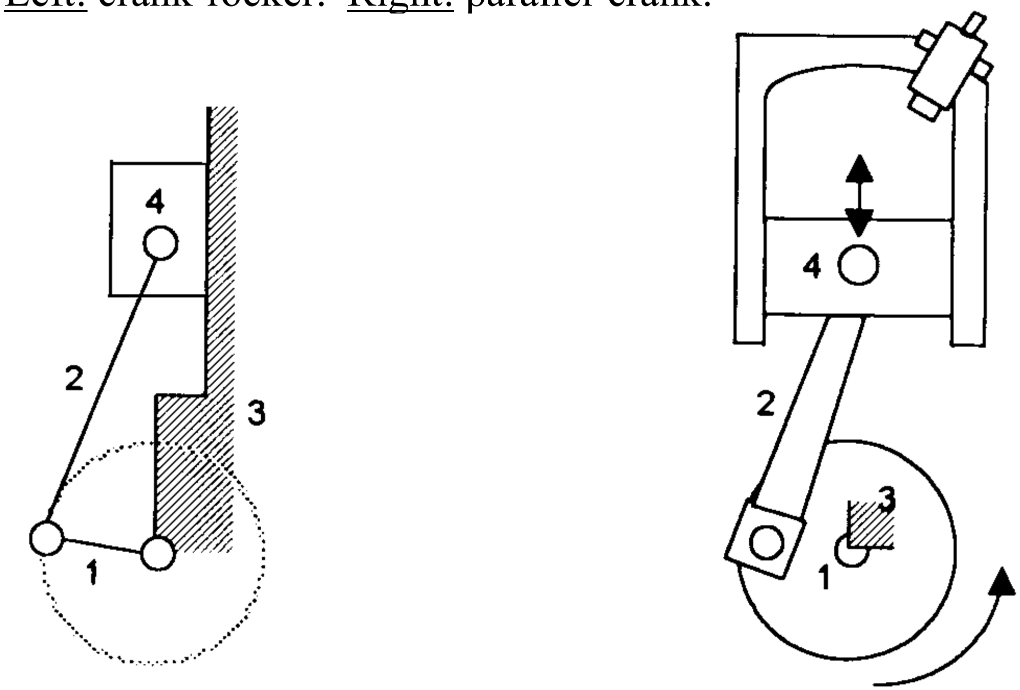

Figure 3.2 Left: Kinematic diagram for a slider-crank mechanism. Right: Application to an internal combustion engine.

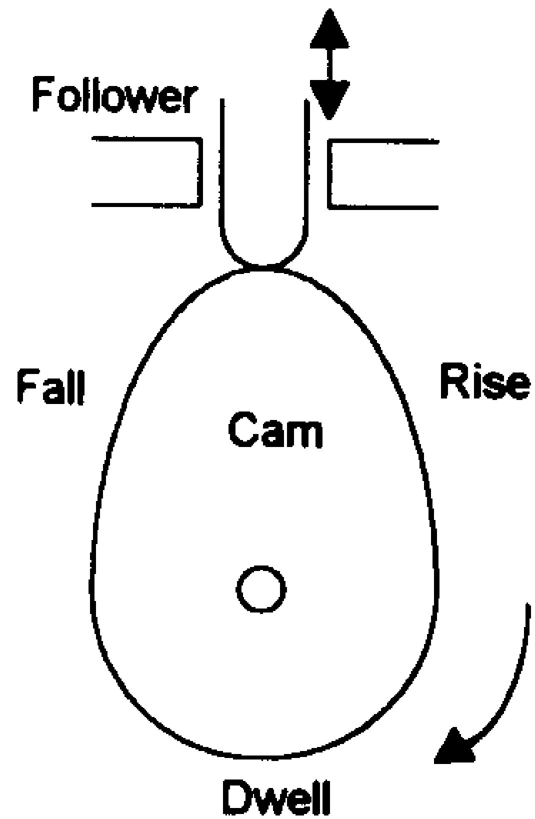

Figure 3.3 Cam and cam follower.

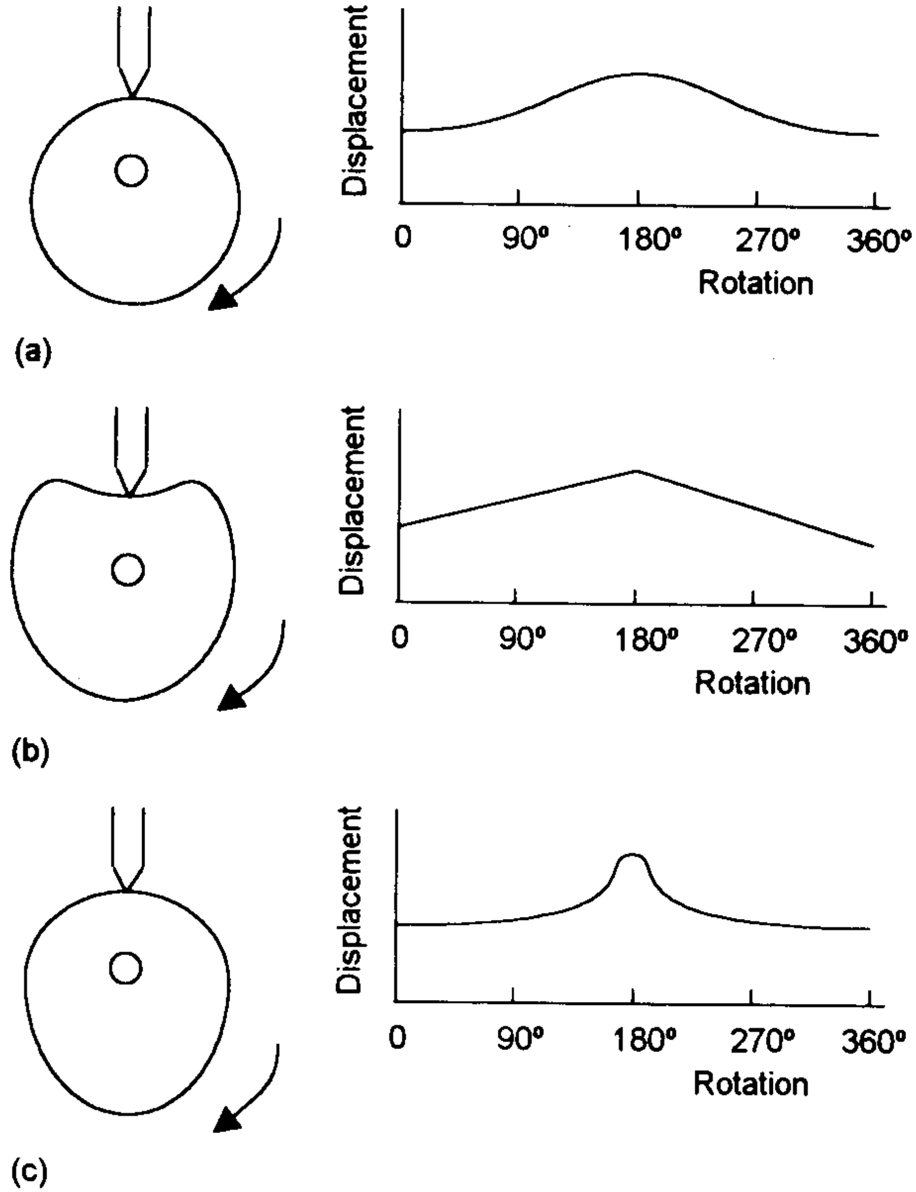

Figure 3.4 Cam examples.

Mechanisms for Conversion of Rotary to Linear and Linear to Rotary Motion

1) Lead screws and ball screws:

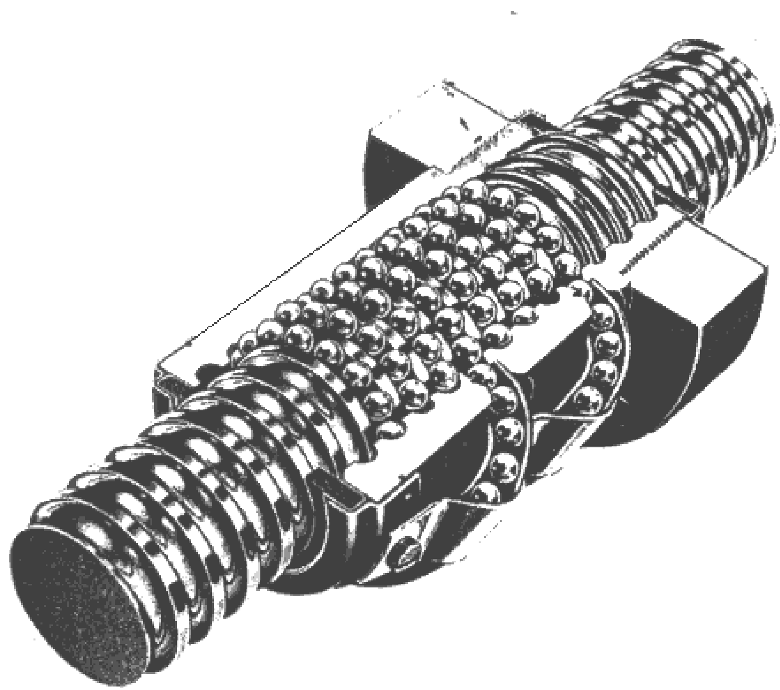

Figure 3.5 A cutaway view of a ball screw and nut.

Governing Equations

The amount of linear motion per revolution is termed the "lead", l. Assuming the torque required to accelerate the screw is negligible then:

$$\tau = \frac{Fl}{(2\pi / rev)\eta_s} \tag{3.1}$$

where τ is the input torque, F is the load, l is the lead and η_s is the efficiency of the screw.

If the friction of the screw is negligible then $\eta_s = 1$.

The equivalent inertia of a load connected to a ballscrew is:

$$J = M\left(\frac{l}{(2\pi / rev)}\right)^2 \tag{3.2}$$

where J is the equivalent rotational inertia of the mass M connected to the linear section of the screw.

Question: How will equation (3.1) change if the torque required to accelerate the screw is not negligible?

2) Rack and pinion:

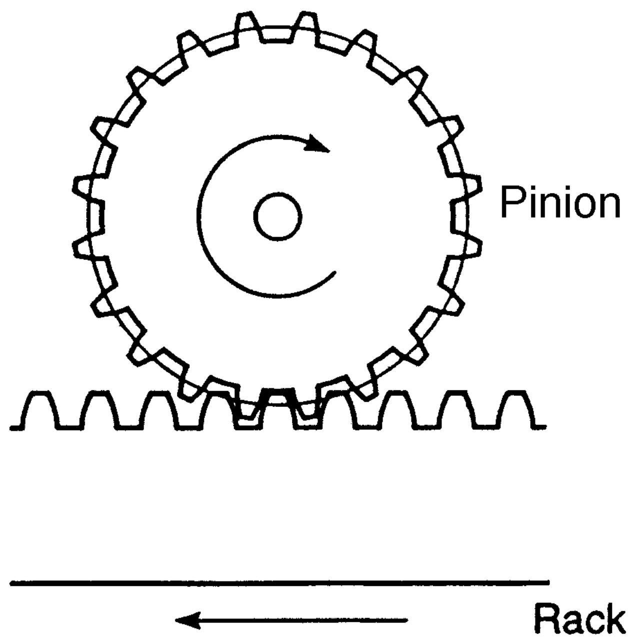

Figure 3.6 A rack and pinion mechanism [1].

Governing Equations

$$\text{Linear motion per revolution} = (2\pi/rev)r_p \qquad (3.3)$$

where r_p is the radius of the pitch circle for the pinion.

If the pinion is attached to the actuator then the force output is given by:

$$F_{out} = \frac{\tau_{in}}{r_p}\eta_{rp} \qquad (3.4)$$

where η_{rp} is the efficiency.

If the rack is attached to the actuator then the torque output is given by:

$$\tau_{out} = F_{in} r_p \eta_{rp} \qquad (3.5)$$

The equivalent inertia of a load connected to the rack is:

$$J = Mr_p^2 \tag{3.6}$$

3) Timing belt

4) Cam and cam follower

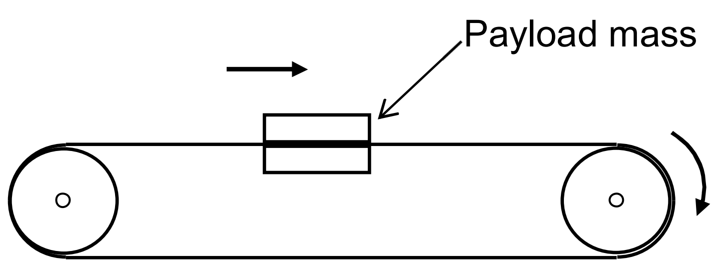

Figure 3.7 Use of a timing belt for motion conversion.

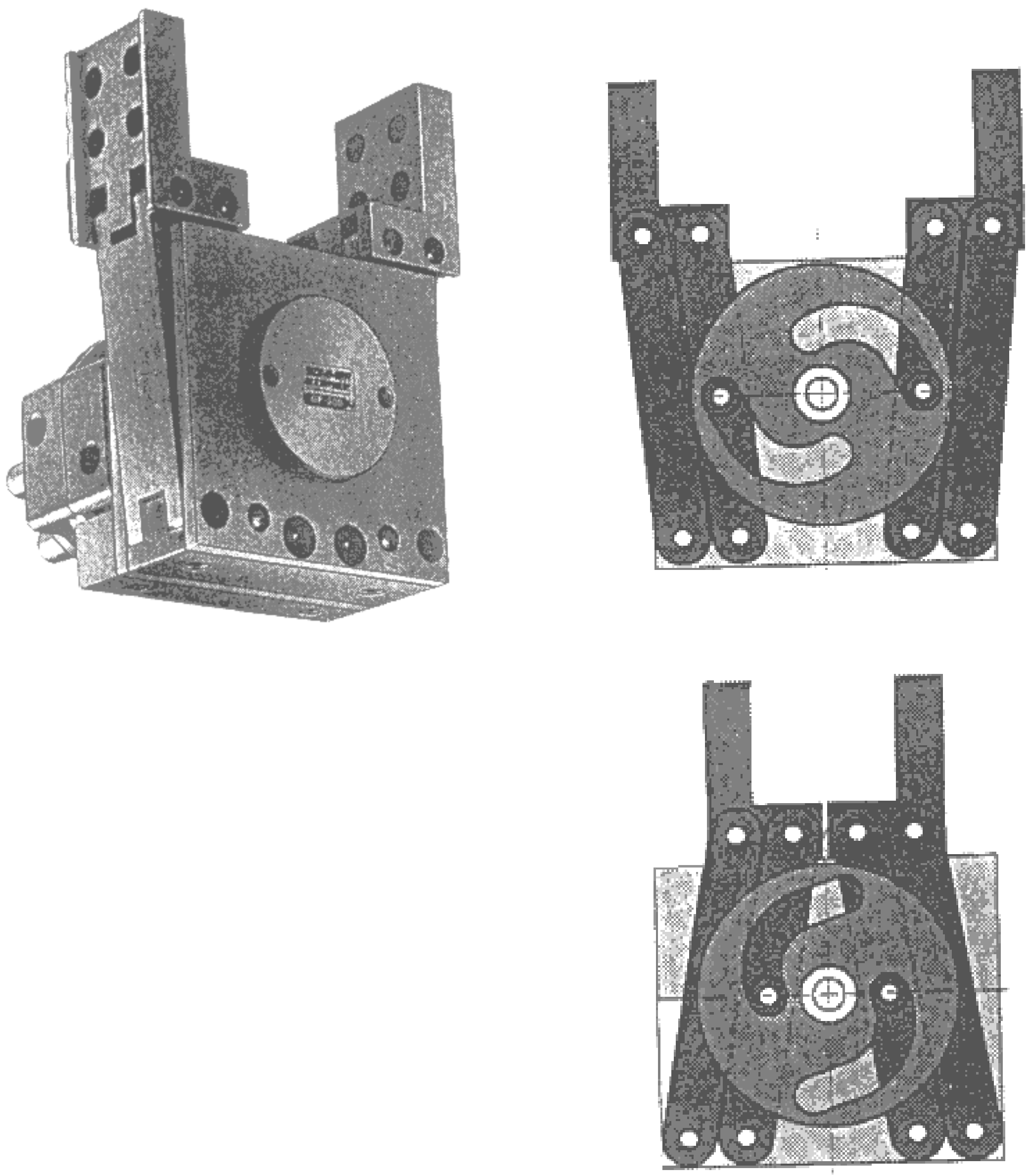

Figure 3.8 A robot gripper. The gripper is opened and closed using a positive-return cam and two double-crank mechanisms. The double-crank mechanisms keep the jaws parallel to make the gripping action more effective.

3.2.3 Mechanisms Providing Mechanical Advantage

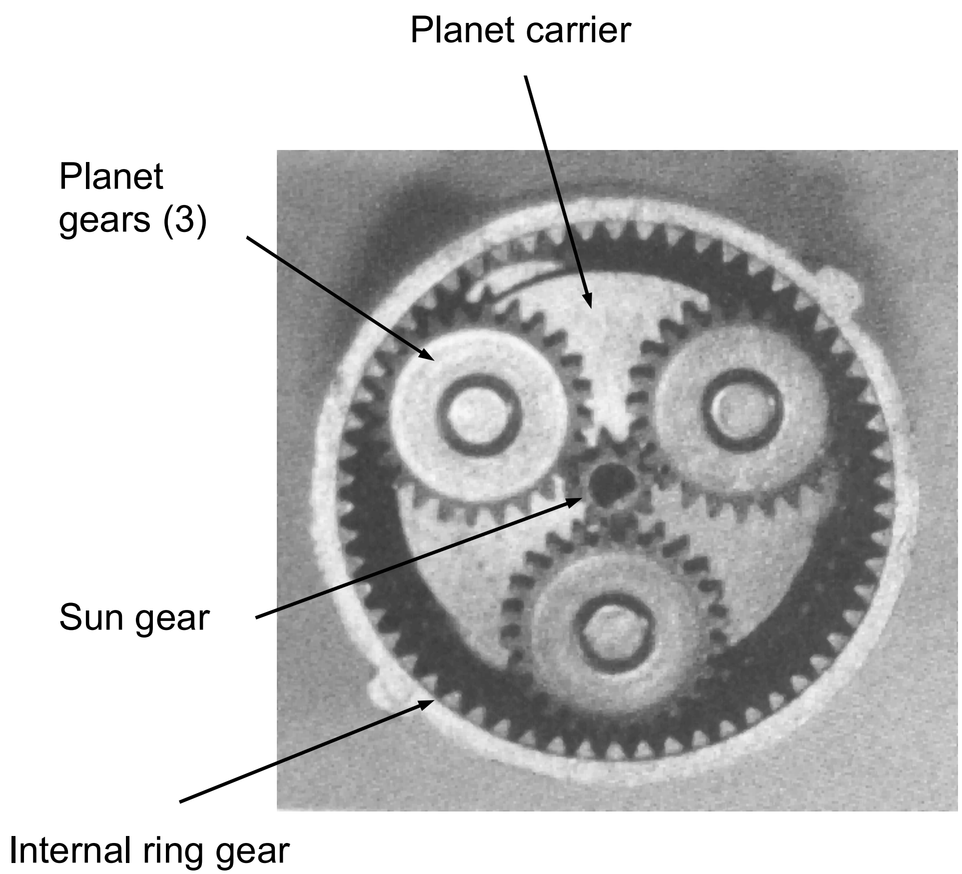

Figure 3.9 A typical planetary gearbox.

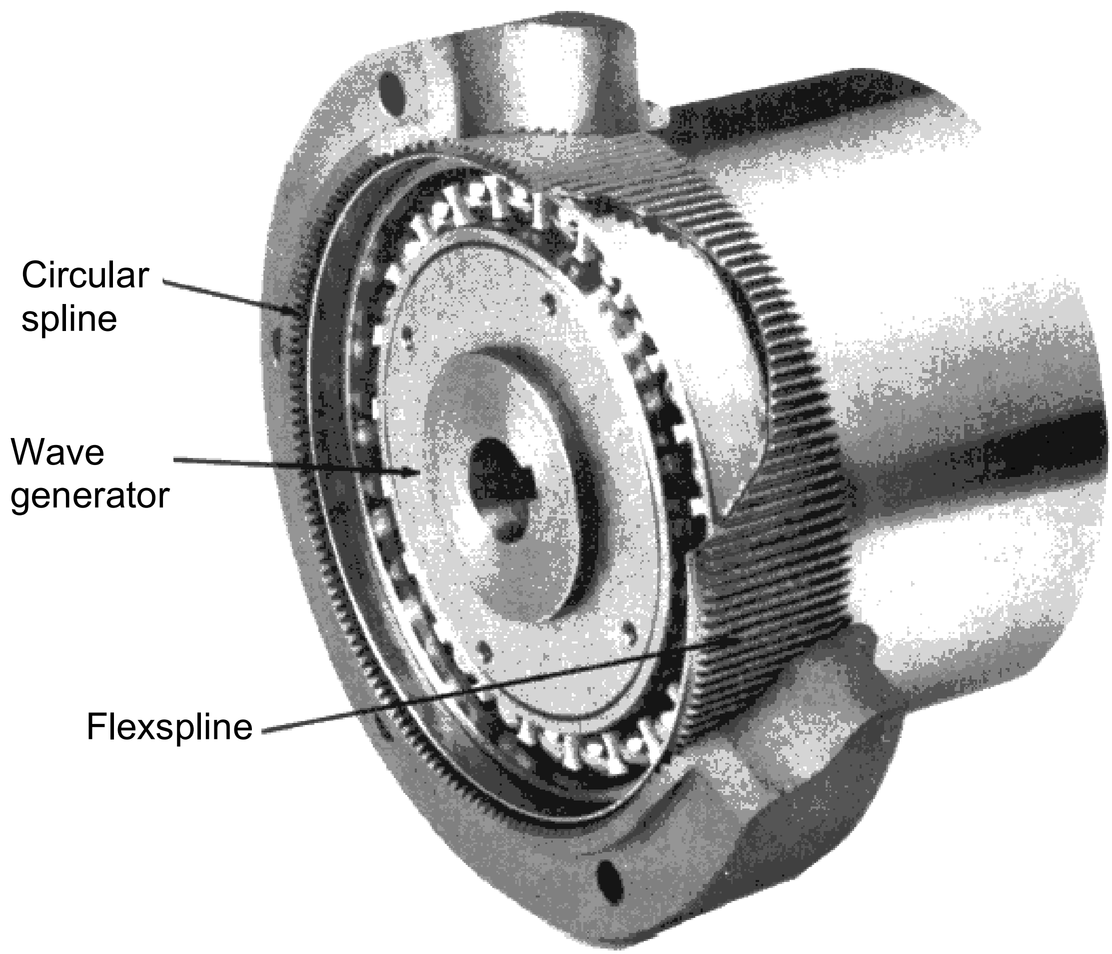

Figure 3.10 Cutaway view of a harmonic drive.

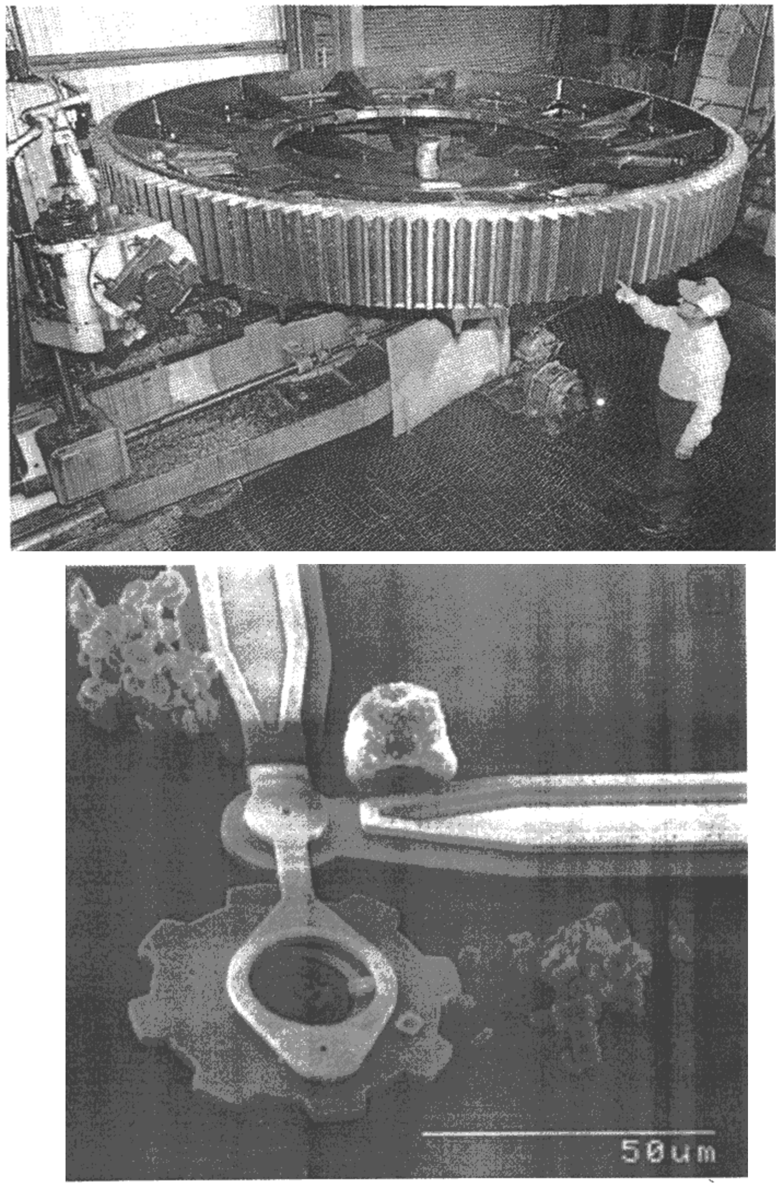

Figure 3.11 Gears come in many sizes. Top: a roughly 5 m diameter gear being machined after casting. Bottom: a roughly 5×10^{-5} m diameter gear that has been etched from silicon as part of a MEMS device (note: the item at the centre of the picture is a grain of pollen).

Gearbox Governing Equations

$$\omega_{out} = \frac{1}{N_r}\omega_{in} \tag{3.7}$$

$$\dot{\omega}_{out} = \frac{1}{N_r}\dot{\omega}_{in} \quad \text{and} \tag{3.8}$$

$$\tau_{out} = N_r \tau_{in} \eta_g \tag{3.9}$$

where N_r is the gear ratio, ω is the angular velocity, $\dot{\omega}$ is the angular acceleration, τ is the torque, and η_g is the efficiency.

In many situations η_g is not a constant, e.g. it may vary with ω and/or due to changes in gear lubrication.

For the remainder of this section, we assume that the gearbox and coupling inertia is negligible relative to the load/motor, and that the gearbox is 100% efficient (i.e. $\eta_g = 1$).

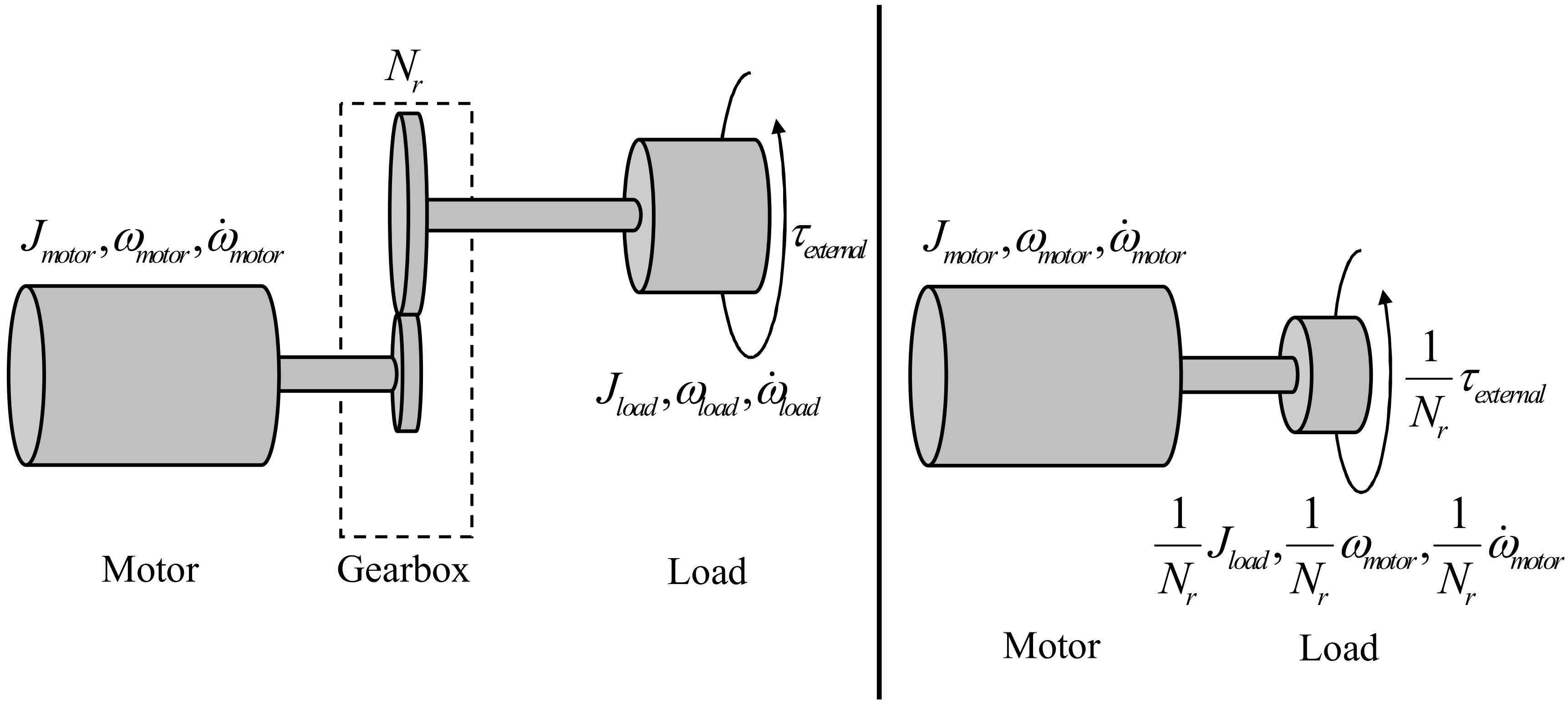

Figure 3.12 Left: Schematic of a motor connected to a load via a gearbox.
Right: Load as reflected to the motor.

The reflected torque is the combination of all inertial loads and non-inertial (e.g. friction or gravity) loads, as seen by the motor.

$$\begin{aligned}\tau_{reflected} &= \frac{1}{N_r} J_{load}\dot{\omega}_{load} + \frac{1}{N_r}\tau_{external} \\ &= \frac{1}{N_r} J_{load}\left(\frac{\dot{\omega}_{motor}}{N_r}\right) + \frac{1}{N_r}\tau_{external} \\ &= \frac{1}{N_r^{\,2}} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external}\end{aligned} \qquad (3.10)$$

The load inertial torque includes a squared gear ratio, because both the inertia and the angular acceleration are written relative to the motor.

The total torque required of the motor is then:

$$\tau_{motor} = J_{motor}\dot{\omega}_{motor} + \tau_{reflected}$$
$$= J_{motor}\dot{\omega}_{motor} + \frac{1}{N_r^{\ 2}} J_{load}\dot{\omega}_{motor} + \frac{1}{N_r}\tau_{external} \qquad (3.11)$$

3.3 Electrical Actuators

1) Solenoids

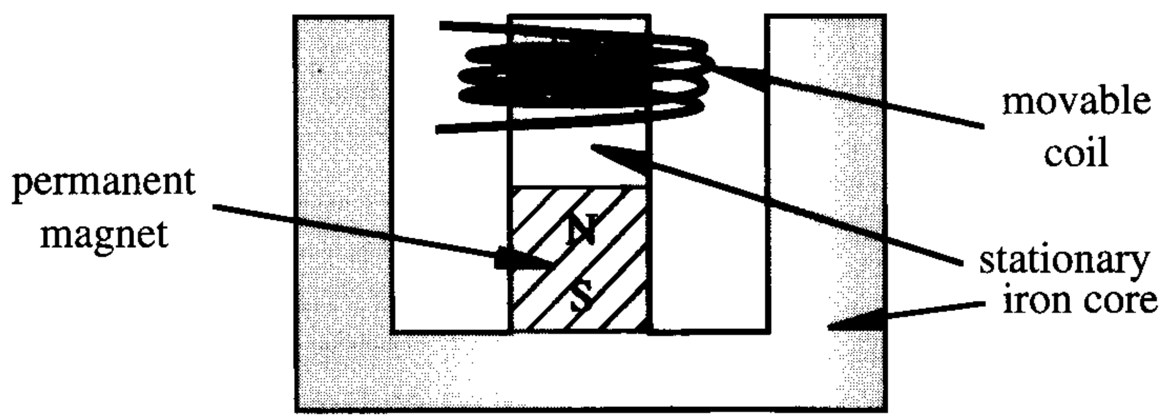

Figure 3.12

2) Voice coils

3) DC Motors

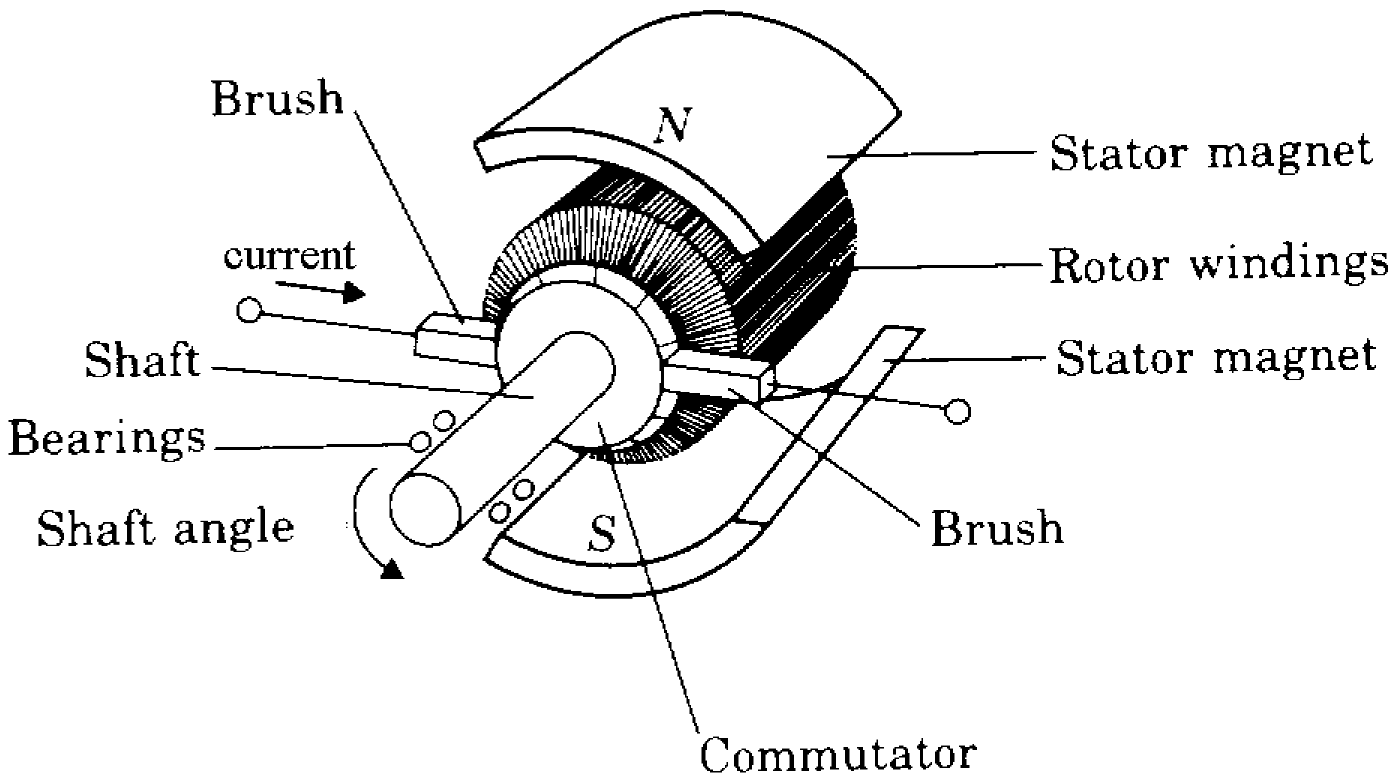

Figure 3.13 The components of a PM DC brush motor.

3) DC Motors (continued)

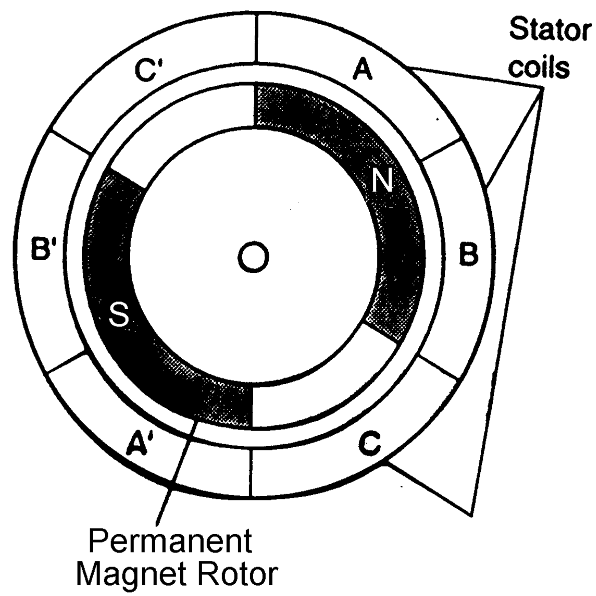

Figure 3.14 Basic structure of a PM DC brushless motor

Mathematical Modelling of PM DC Motors

The electrical elements are modelled by the equation:

$$V_a = K_b\omega + L_a\frac{di_a}{dt} + R_a i_a \tag{3.13}$$

where: $V_a \equiv$ armature voltage, $K_b \equiv$ back EMF constant, $\omega \equiv$ angular velocity $L_a \equiv$ armature inductance, $R_a \equiv$ armature resistance and $i_a \equiv$ armature current. The mechanical elements are modelled by the equation:

$$J\frac{d\omega}{dt} = K_t i_a - K_d\omega - \tau_{load} \tag{3.14}$$

where: $J \equiv$ moment of inertia of the motor, $\tau_{load} \equiv$ torque due to the load, $K_t \equiv$ torque constant and $K_d \equiv$ damping constant (due to viscous friction).

Example 3.1

Say we need to accelerate a 50 kg mass at 20 m/s^2 using a linear actuator consisting of a DC motor and a ball screw with a 15 mm lead.

(a) Assuming that friction and the moment of inertia of the ball screw are negligible, what motor torque is required?

(b) If the maximum velocity is 2 m/s, what is the maximum motor rpm?

Solution

(a) From Newton's second law:

$$F = ma = (50kg)(20m/s^2) = 1000N$$

From equation (3.1), where η_s=1 (based on the given assumptions):

$$\tau = \frac{Fl}{(2\pi/rev)\eta_s} = \frac{(1000\text{ N})(15\text{ mm/rev})(1\text{ m}/1000\text{ mm})}{(2\pi/rev)1} = 2.4\text{ Nm}$$

(b) For a ball screw, the linear velocity equals the product of the angular velocity and the lead:

$$v = \dot{\theta}\, l$$

So we have:

$$\dot{\theta} = \frac{v}{l} = \frac{(2\text{ m/s})(60\text{ s/min})}{0.015\text{ m/rev}} = 8000\text{ rev/min or rpm}$$

Example 3.2

We are analysing an actuator consisting of a PM DC motor driving a ball screw. The ball screw has a lead of 2 mm. Assume the friction and moment of inertia of the ball screw may be neglected. We wish to drive a load of 500 N at a speed of 0.25 m/s. The motor parameters are ... (please see the notes)

(a) What armature current and armature voltage will be required?
(b) The maximum continuous voltage rating is 48 V and the maximum continuous current rating is 3.6 A for this motor. Assuming the load must be driven for an extended period of time, are we within these ratings?

Solution

From the given information:

$$\tau_{load} = \frac{Fl}{(2\pi / rev)\eta_s} = \frac{(500N)(2mm / rev)(1m / 1000mm)}{(2\pi / rev)1} = 0.159 Nm$$

$$\omega = \frac{v}{l} = \frac{(0.25\ m/s)(60\ s/min)}{0.002\ m/rev} = 7500\ \text{rpm}$$

Since the speed and load are constant, from equations (3.13) and (3.14), V_a and i_a will be constant. Since the speed is constant $\frac{d\omega}{dt} = 0$ and equation (3.14) gives:

$$0 = K_t i_a - K_d \omega - \tau_{load}$$

$$K_t i_a = K_d \omega + \tau_{load}$$

$$i_a = \left(K_d \omega + \tau_{load}\right) / K_t$$

= ((3.5 × 10^{-5} Nm/rpm)(7500 rpm) + 0.159 Nm) / (63.7 × 10^{-3} Nm/A)

= 6.62 A

Since i_a is constant $\frac{di_a}{dt} = 0$ and equation (3.13) is reduced to:

$$V_a = K_b \omega + R_a i_a$$

= (0.007 V/rpm)(7500 rpm) + (1.16 Ohm)(6.62 A) = 60.2 V

(b) Based on our answer to (a), we will exceed the continuous current and voltage ratings. Therefore we must either reduce the load and/or speed, or select a more powerful motor.

Position More Information about the Position Control of PM DC Motors

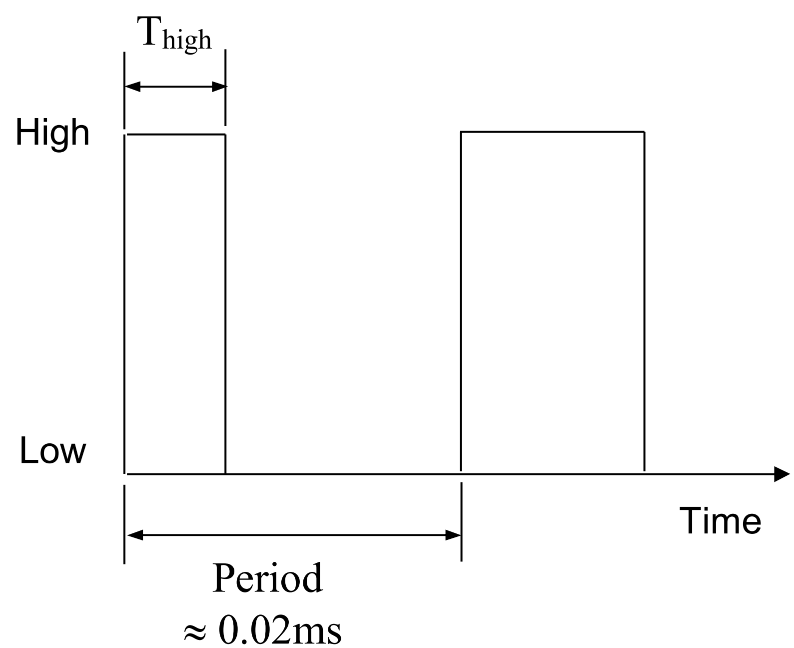

Figure 3.15 Pulse-width-modulation.

$$V_{out} = V_{average} + High\ frequency\ signal$$

$$= \frac{T_{high}V_{high} + (Period - T_{high})V_{low}}{Period} + High\ frequency\ signal \qquad (3.15)$$

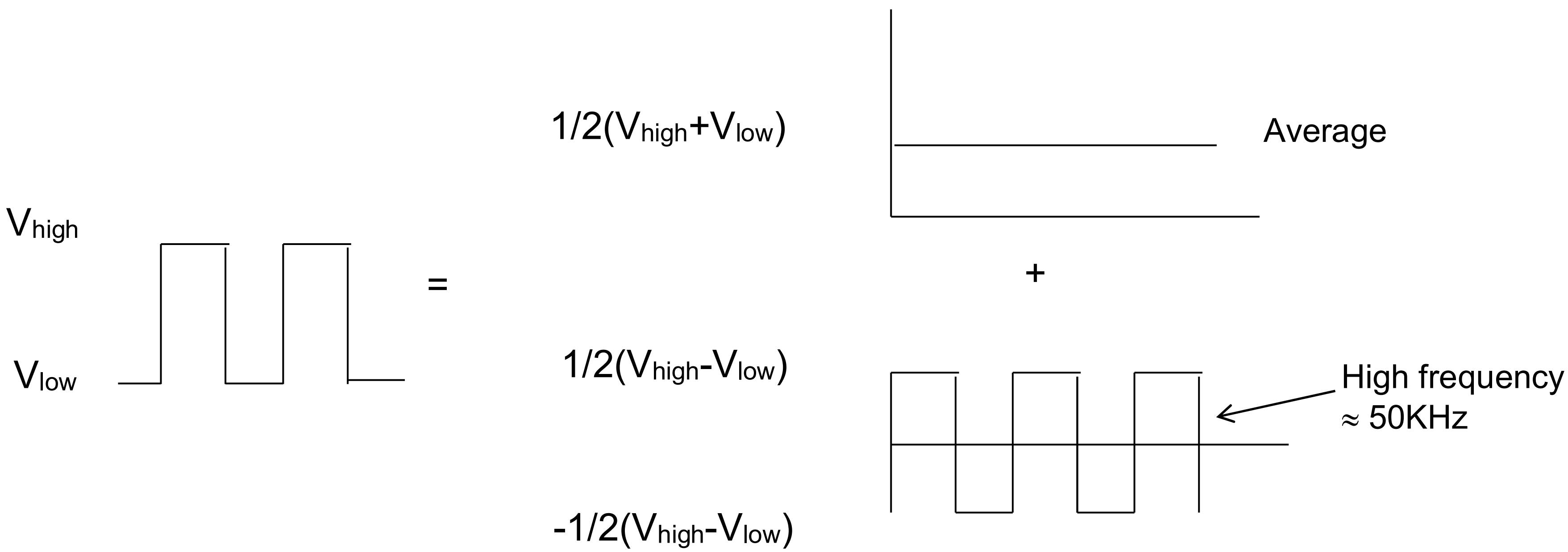

Figure 3.16 PWM example.

4) Linear Motors

A linear motor is a DC, AC, or stepper motor whose stator has been "unwrapped" into a linear form.

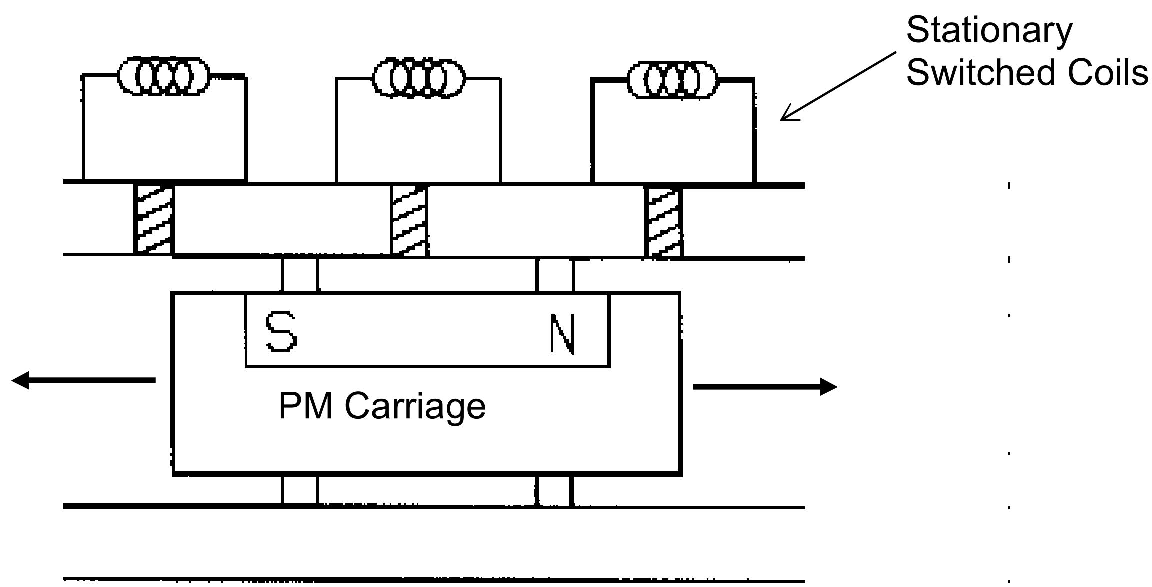

Figure 3.17 One form of a linear motor.

Question: What type of linear motor is shown in the figure?

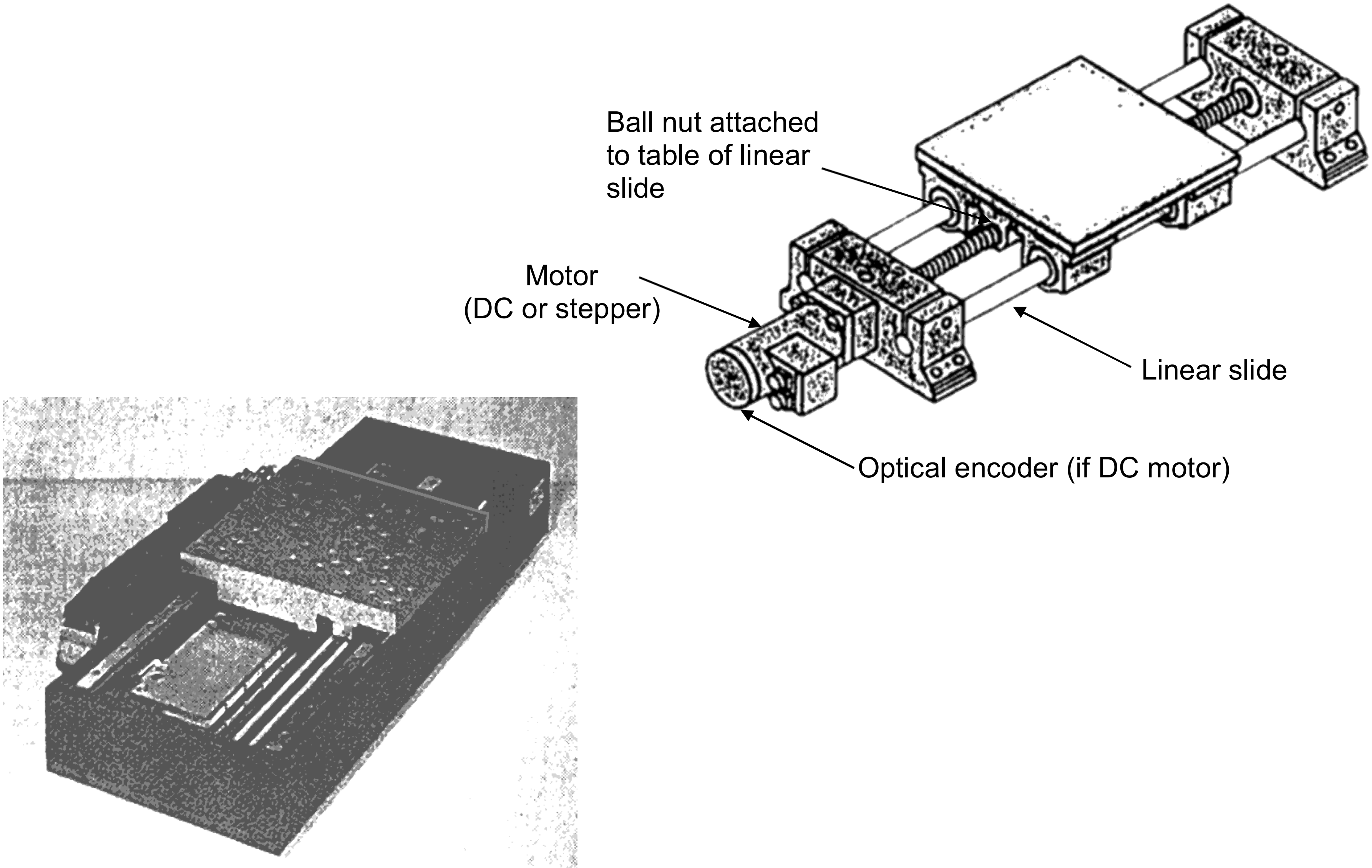

Figure 3.18 Comparison of linear actuators. Top: A linear actuator consisting a motor, encoder, ball screw and linear slide. Bottom: When a linear motor is used, the motor, encoder and linear slide are closely integrated, resulting in a more compact actuator.

5) Stepper Motors (also known as Stepping Motors)

PM Stepper Motors

i) Full stepping mode

ii) Half stepping mode

ii) Microstepping

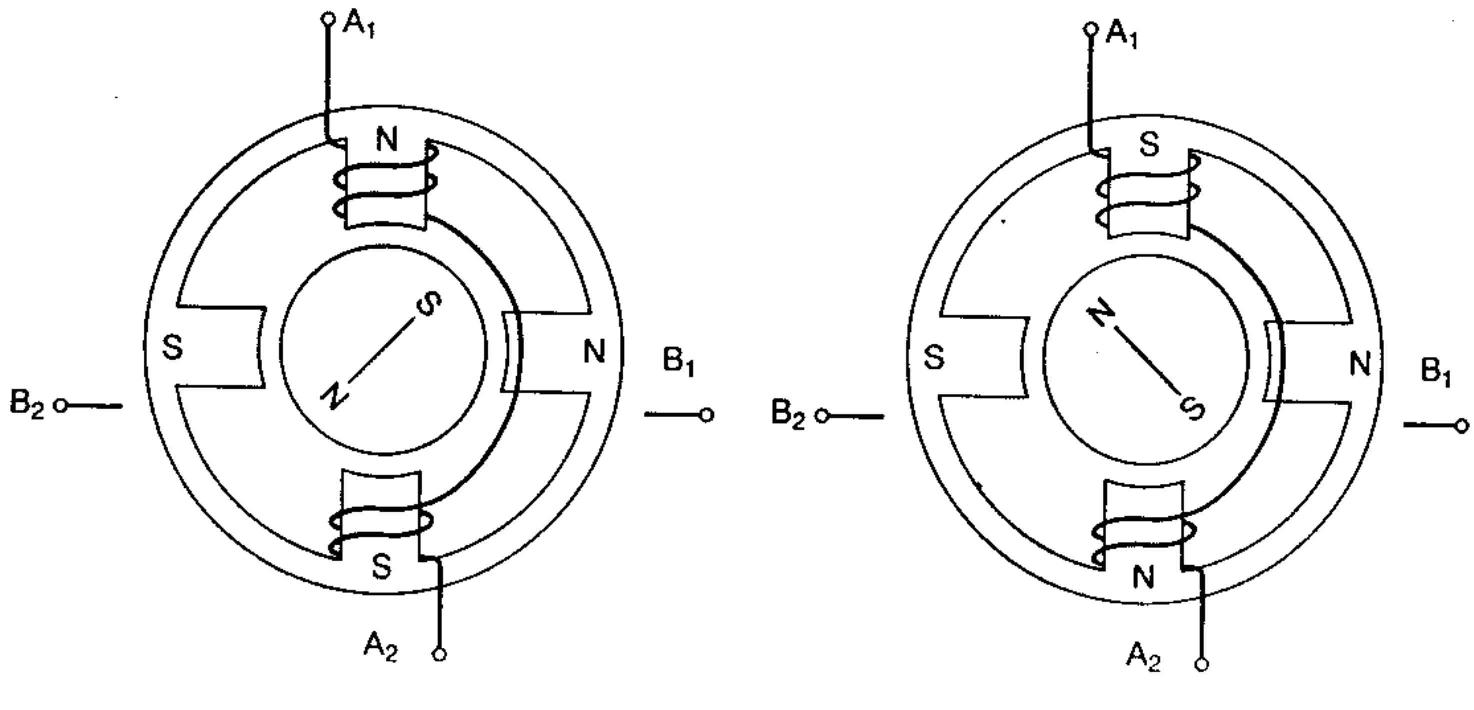

Figure 3.19 Example of full stepping mode. Left: Starting position. Centre: Reversing the polarity of one pair of poles causes the rotor to rotate clockwise by a full step.

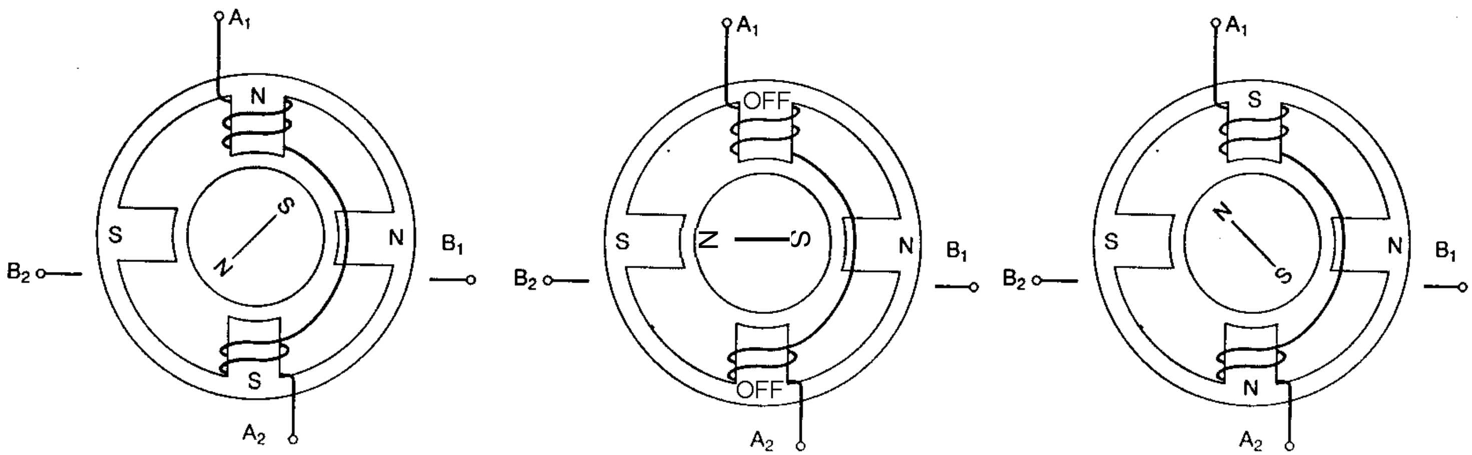

Figure 3.20 Example of half stepping mode. Left: Starting position. Centre: Shutting off a pair of poles causes the rotor to move 1/2 step clockwise. Right: Re-energising the poles with reversed polarity causes the rotor to rotate clockwise another 1/2 step.

VR Stepper Motors

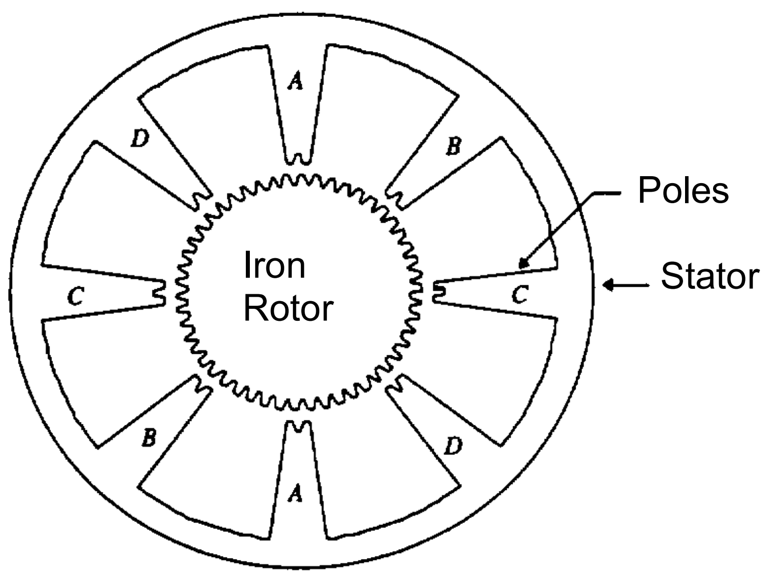

Figure 3.21 A VR stepper motor.

Hybrid Stepper Motors

Note: Full stepping, half stepping and microstepping may be used with all three types of stepper motors.

Control of Stepper Motors

3.4 Motor/Gearbox Selection and Optimization

With many mechatronic systems an electric motor is combined with a gearbox (also called a "gearhead").

In the following subsections we will look at methods for effectively selecting the motor and gearbox.

3.4.1 Maximizing Energy Efficiency

Maximizing energy efficiency is desirable for reducing cost, and for improving sustainability.

In battery powered mechatronic systems using PM DC motors, such as mobile robots, it is particularly important.

Under steady-state operating conditions, $\frac{d\omega}{dt}=0$ and $\frac{di_a}{dt}=0$, so (3.13) and (3.14) simplify to:

$$V_a = K_b\omega + R_a i_a \text{ and} \tag{3.16}$$

$$\tau_{load} = K_t i_a - K_d\omega \tag{3.17}$$

If we assume that the motor's friction is negligible (i.e. $K_d \approx 0$), then (3.17) can be further simplified to:

$$\tau_{load} = K_t i_a \tag{3.18}$$

The motor efficiency is given by:

$$\begin{aligned}\eta_{motor} &= \frac{\text{mechanical power output}}{\text{electrical power input}} \\ &= \frac{\tau_{load}\,\omega}{V_a i_a} \\ &= \frac{\left(K_t i_a\right)\omega}{\left(K_b\omega + i_a R_a\right) i_a} \\ &= \frac{K_t i_a \omega}{K_b i_a \omega + i_a^2 R_a}\end{aligned} \tag{3.19}$$

$\therefore$ Operating at maximum speed will provide the maximum efficiency.

However, if $K_d \neq 0$ the maximum efficiency will occur at a lower speed.

When the motor is connected is to a gearbox the problem gets more complicated since:

- The efficiency of a gearbox, η_g, is not usually constant (as previously discussed)

- Gearbox and motor are usually selected at the same time. The motor/gearbox combination must satisfy the speed and torque requirements of the application before efficiency can be considered.
- Only a limited number of motor/gearbox combinations will satisfy those requirements.
- Larger N_r lowers η_g, but their relationship cannot be effectively modelled.
- Operating a motor at a higher speed will increase η_{motor} make it more efficient, but this will require a gearbox with a larger N_r and therefore a lower η_g, to meet the requirements of the application.
- Best efficiency ($\eta_{motor+gearbox} = \eta_{motor}\eta_g$) will occur when the motor runs at an intermediate speed.
- If sufficient information is available from the motor and gearbox suppliers then this optimal combination of components and steady-state motor speed can still be determined by numerically analyzing all of the combinations, but it cannot be solved using equations.
- When the motor operates under dynamic conditions the efficiency analysis becomes much more complex and is beyond the scope of this course.

3.4.2 Optimal Gear Ratio for Maximum Load Acceleration and Maximum Power Transfer

Many mechatronic systems require moving a mass rapidly from one stationary position to another stationary position, e.g. robots, CNC machines, and DVD players. This requires an actuator capable of large acceleration (and deceleration).

In this section we will look at maximizing the acceleration of a motor plus gearbox.

We will consider is a motor connected to a load through a gearbox, with the following assumptions:

- The load is inertial only,
- The gears have negligible inertia, and
- Friction is negligible.

From these assumptions and the equations provided in section 3.2.3, we may obtain this equation for the load acceleration:

$$\dot{\omega}_{load} = \frac{\tau_{motor}}{N_r J_{motor} + \left(\frac{1}{N_r} J_{load} \right)} \tag{3.23}$$

By choosing N_r to minimize the denominator of (3.23), the load acceleration will be maximized. An analysis of the first and second derivatives of the denominator gives the solution:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} \tag{3.25}$$

It can also be shown that in the case of combined inertial and constant torque loads, gives:

$$\dot{\omega}_{load} = \frac{\tau_{motor} - \frac{1}{N_r} \tau_{external}}{N_r J_{motor} + \left(\frac{1}{N_r} J_{load} \right)} \tag{3.27}$$

Analysis of the first and second derivatives (of the entire fraction) yields the same equation for optimal N_r.

Equation 3.25 will give an exact value for the gear ratio (e.g. 3.77), though in reality only a finite number of specific gear ratios are available.

The closest smaller available gear ratio should be selected.

A general rule of thumb is to keep the inertia ratio, $Ratio_J = \dfrac{J_{load} / N_r^{\,2}}{J_{motor}}$ as close to 1 as possible, staying within the range 1 to 10. This is known as “inertia matching”.

An inertia ratio larger than 10 can produce an unstable system due to the interaction of the small mechanical flexibility between the motor and load, their inertias and the position control loop.

Note that $Ratio_J = 1$ will also maximize the mechanical power transmitted to the load which is relevant for steady state operation (i.e. constant speed).

3.4.3 Motor/Gearbox Selection for Dynamic Loads

Many mechatronic applications involve dynamic operation in which velocities, accelerations and torques are required to change magnitude, direction and duration.

The position, velocity and acceleration profiles for a simple operating cycle are shown in Figure 3.24. This cycle has four periods: constant acceleration ($0 \le t < 0.25\,\text{s}$), constant velocity ($0.25 \le t < 0.75\,\text{s}$), constant deceleration ($0.75 \le t < 1\,\text{s}$) and idle.

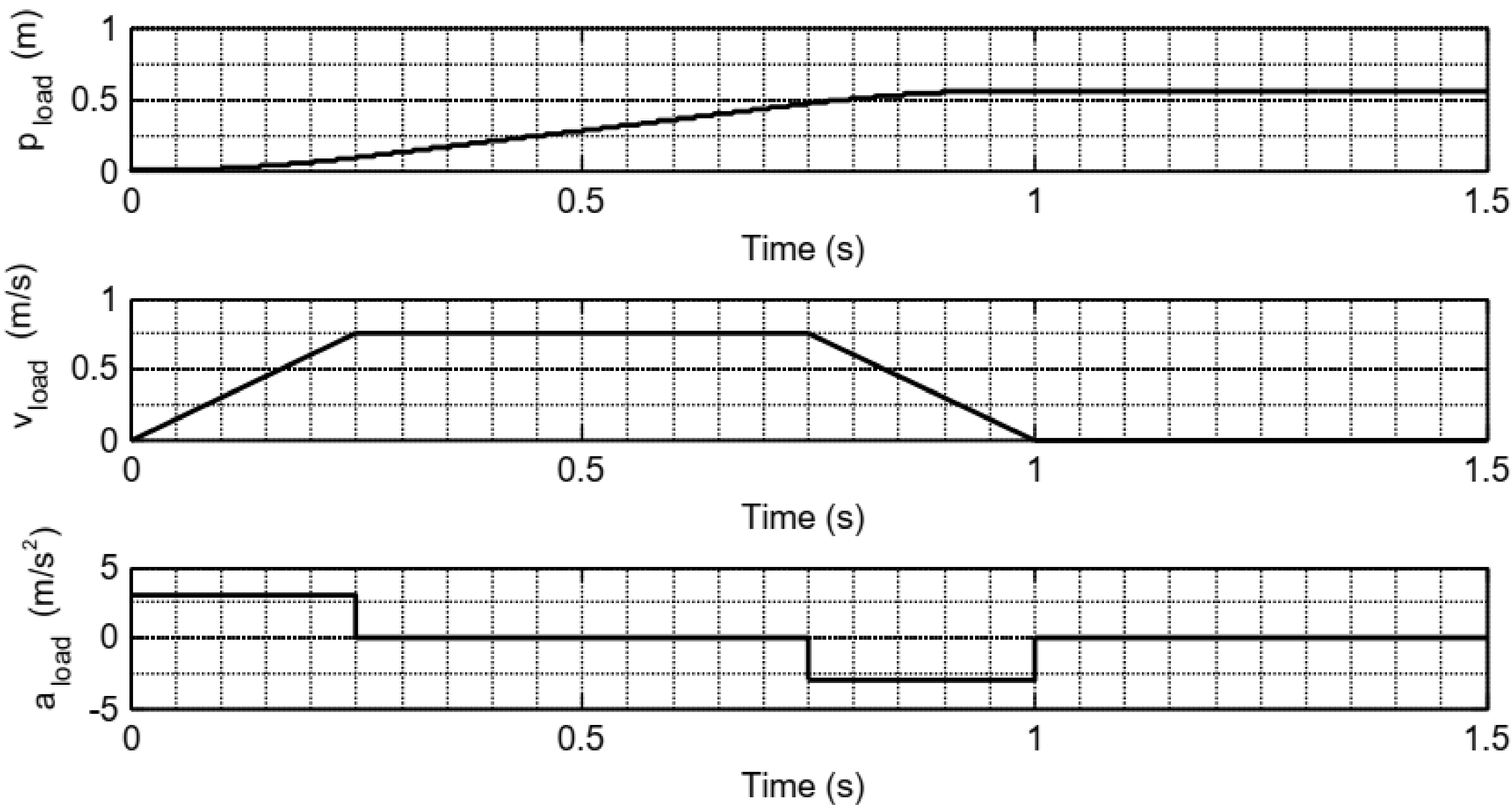

Figure 3.24 Example of position, velocity and acceleration profiles for an operating cycle with four periods.

The next step is to calculate the required motor torque and velocity for each period using standard physics equations, and the relevant equations from subsections 3.2.2 and 3.2.3.

To check that the motor is suitable for the application we need to calculate the maximum required motor torque, $\tau_{motor,\,max}$, and the maximum required motor speed, $\omega_{motor,\,max}$.

It also may be necessary to calculate the RMS motor torque, $\tau_{motor,\,RMS}$.

For a motor torque profile that is a piecewise constant function[1], the RMS value may be calculated using:

$$\tau_{motor,RMS} = \sqrt{\sum_{i=1}^{n} \tau_{motor,i}^{2} t_i \Big/ \sum_{i=1}^{n} t_i} \qquad (3.33)$$

[1] The acceleration profile shown in Figure 3.24 is an example of a piecewise constant function. Finding the RMS value of more complex motor torque profiles is beyond the scope of this course.

where $\tau_{motor,\,i}$ is the torque for the i^{th} period, t_i is the duration of the i^{th} period and n is the number of periods.

A motor can operate at its maximum rated torque, $\tau_{rated,\,max}$, intermittently (i.e. for short periods of time followed by periods operating at a lower torque).

Motors can also produce a smaller torque, $\tau_{rated,\,cont}$, continuously. If the motor is operated above its continuous torque rating for too long it will be thermally damaged.

A motor is capable of producing different intermittent and continuous torques based upon its operating speed. These are characterized by performance curves, such as the one shown in Figure 3.25a for a DC motor.

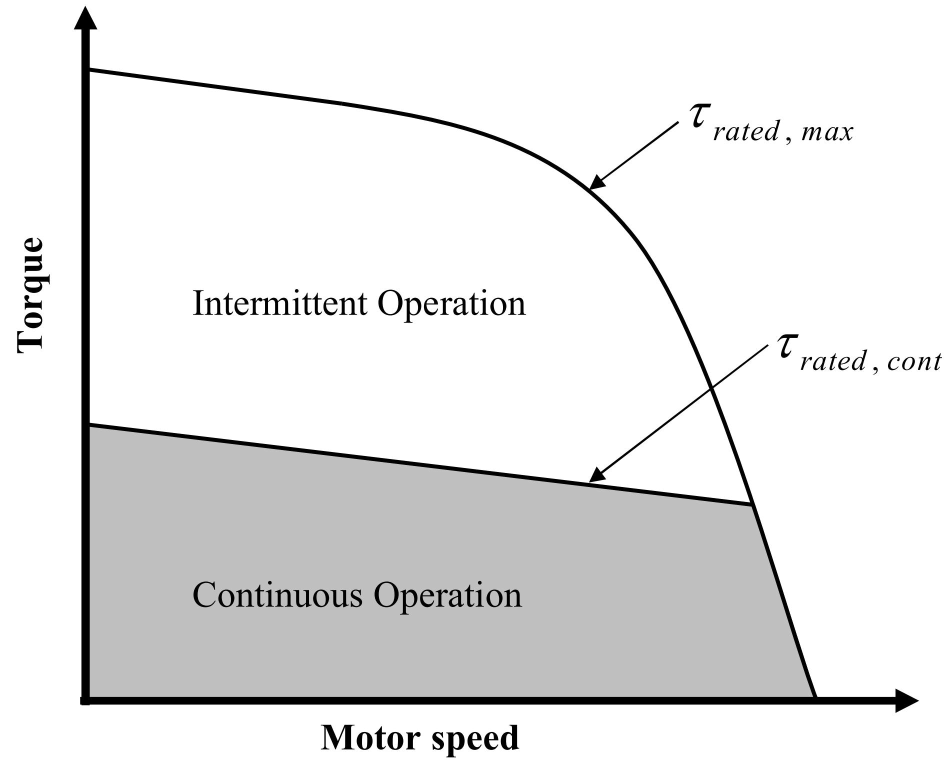

Figure 3.25a Typical DC motor performance curve

For a DC motor, if the point $\left(\omega_{motor,\,max}, \tau_{motor,\,RMS}\right)$ lies within the region of continuous operation, and the point $\left(\omega_{motor,\,max}, \tau_{motor,\,max}\right)$ is below the boundary of intermittent operation, then the motor/gearbox combination is acceptable.

Note that the torques are checked at $\omega_{motor,\,max}$ to be conservative.

If one of the points lies outside its acceptable region then the design is unacceptable.

It may be possible to shift unacceptable point(s) into the acceptable region(s) by changing the gear ratio (or other mechanism parameter(s)) used.

An example is shown in Figure 3.25b. In this example, $\tau_{motor,\,RMS}$ is too large (on the left). Increasing N_r decreases the required torques, while increasing $\omega_{motor,\,max}$, resulting in the acceptable design shown on the right.

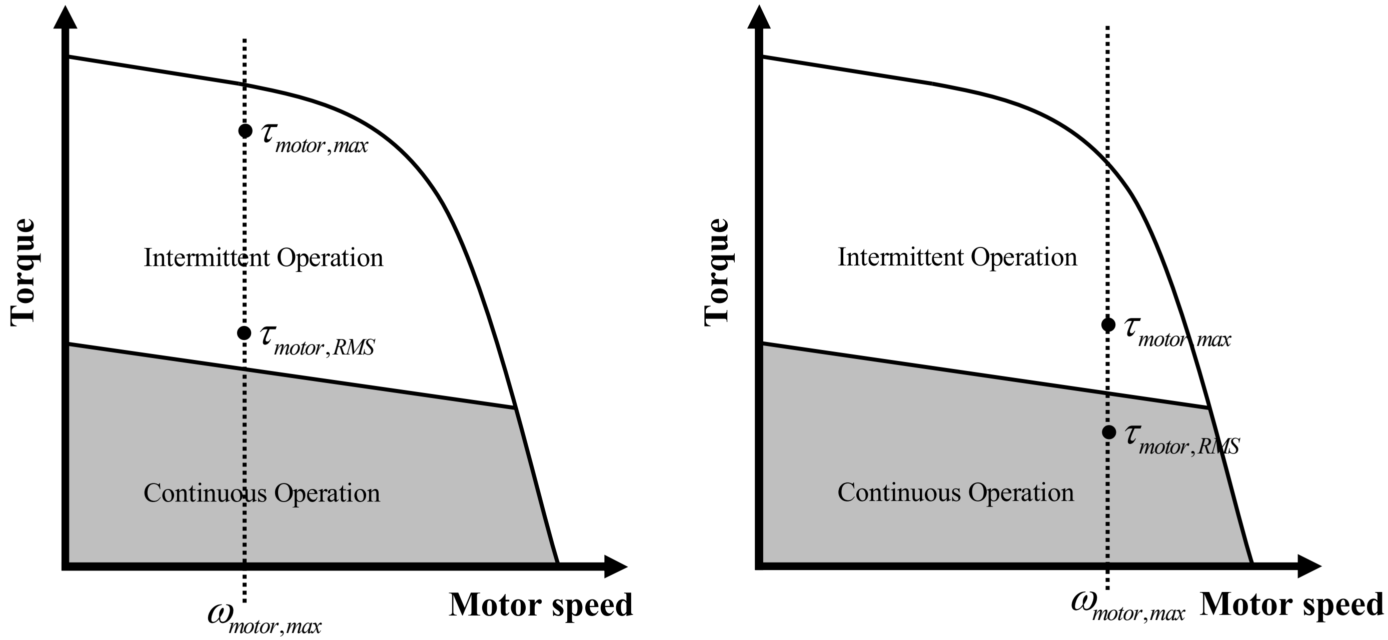

Figure 3.25b Left: Unacceptable $\tau_{motor,\ RMS}$. Right: Made

In this course, to simplify the problem we will conservatively approximate the motor performance curves as shown in Figure 3.25c. These rectangular approximations make the rated torques independent of the motor speed over the range 0 to $\omega_{rated,\,max}$.

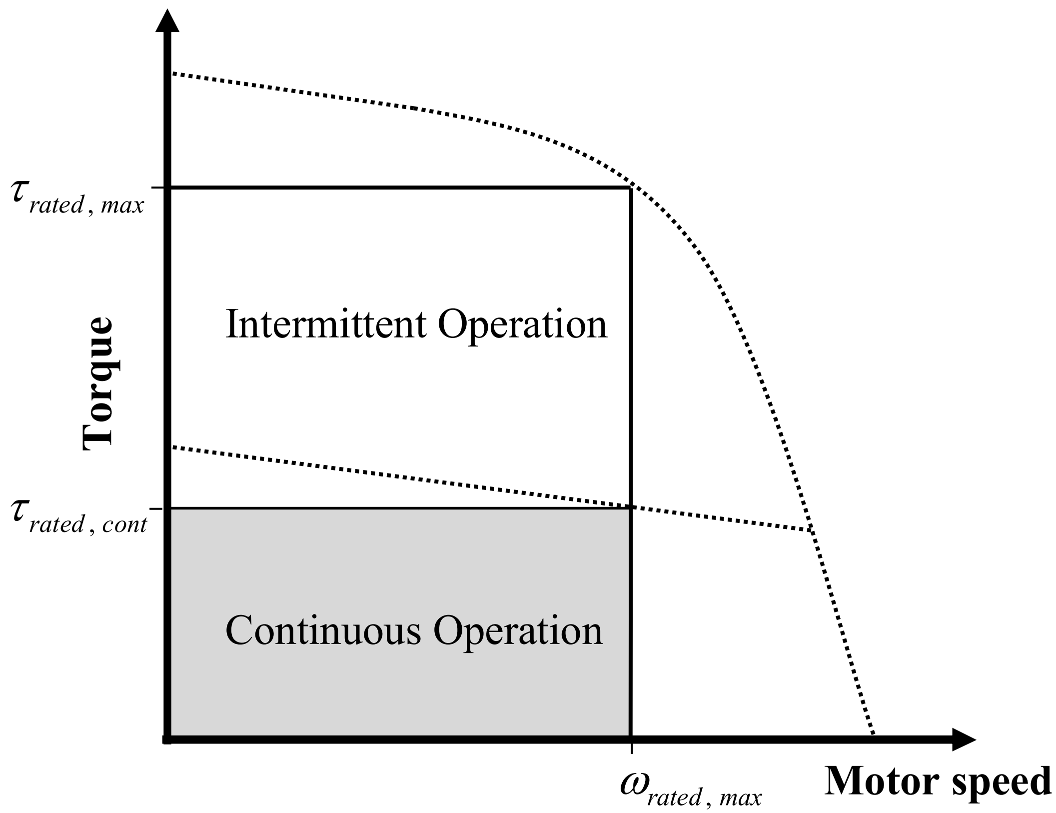

Figure 3.25c Conservative approximations of DC motor performance.

Based on the above, the selection procedure for a DC motor and gear box under dynamic loading may be summarized as follows:

1. Calculate the desired motion profiles if they are not provided.

2. Based on the load, motor data and mechanisms used, select the gear ratio which maximizes acceleration (i.e. inertia matching using (3.25)), or the closest smaller gear ratio available. The inertia ratio from (3.28) should be as close to 1 as possible, staying within the range 1 to 10.

3. Based on the velocity profile, and other mechanisms used (e.g. ball screw), find ω_{max}. Compare $\omega_{motor,\,max} = N_r \omega_{max}$ to the motor's rated maximum speed, $\omega_{rated,\,max}$. If $\omega_{motor,\,max}$ is too high, select a lower gear ratio and check $\omega_{motor,\,max}$ again. If the new value is acceptable, check that the new inertia ratio is within the range 1-10.

4. Based on the load, motor inertia, selected gear ratio, other mechanisms used, and the velocity and acceleration profiles, calculate the motor torque profile and find $\tau_{motor,\,max}$.

5. If $\tau_{motor,\,max} < \tau_{rated,\,cont}$ then proceed with step 8.

6. Compute $\tau_{motor,\,RMS}$ from the motor torque profile using (3.33). If $\tau_{motor,RMS} < \tau_{rated,\,cont}$ then proceed with step 8.

7. Select a higher gear ratio if possible, or select a more powerful motor, and return to step 2.

8. Using the method of section 3.4.4 check that the temperature rise is acceptable. If it is unacceptable then try increasing the gear ratio and return to step 2. If that is not sufficient then either try additional cooling, or select a more powerful motor and return to step 2.

Design examples are provided in the Addendum at the end of this chapter.

3.4.4 Temperature rise of the PM DC motor

The life of a motor also depends on its operating temperature.

The temperature of the motor will rise as long as the heat generated due to all power losses is not entirely dissipated through the motor surface.

Neglecting the friction loss, the power loss is:

$$P_j = I^2 R_{Hot} \tag{3.35}$$

where: R_{Hot} is the motor terminal resistance (or armature resistance) at the desired operating temperature (or at the maximum allowable temperature if the designer wishes to be more conservative), and I is the current.

Note that the terminal resistance increases as the temperature rises. A catalog typically gives the terminal resistance at 25 °C. For a copper winding the resistance at the operating temperature is then given by:

$$R_{Hot} = R_{25}\left(1 + 0.00392(T_{Hot} - 25)\right) \tag{3.36}$$

where R_{25} is the terminal resistance at 25 °C and T_{Hot} is the desired operating temperature in °C.

Thermal resistance R_{th}, in °C/Watt, is an indicator of how effectively the motor dissipates the generated heat and is a combination of two thermal resistances:

$$R_{th} = R_{th1} + R_{th2} \tag{3.37}$$

where

R_{th1} characterizes the heat transfer from the windings to the housing
R_{th2} characterizes the heat transfer from housing into the ambient.

The winding temperature of a motor may be estimated using the following basic equation:

$$T_w(t) = T_{initial} + \left(P_j R_{th} + T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) \qquad (3.38)$$

where T_a is the ambient temperature, $T_{initial}$ is the initial winding temperature, and τ_w is the thermal time constant of the winding.

τ_w is the amount of time required for a motor winding to reach 63.2% of its steady state temperature. For large motors τ_w can be several minutes while for small motors it may be only a few seconds.

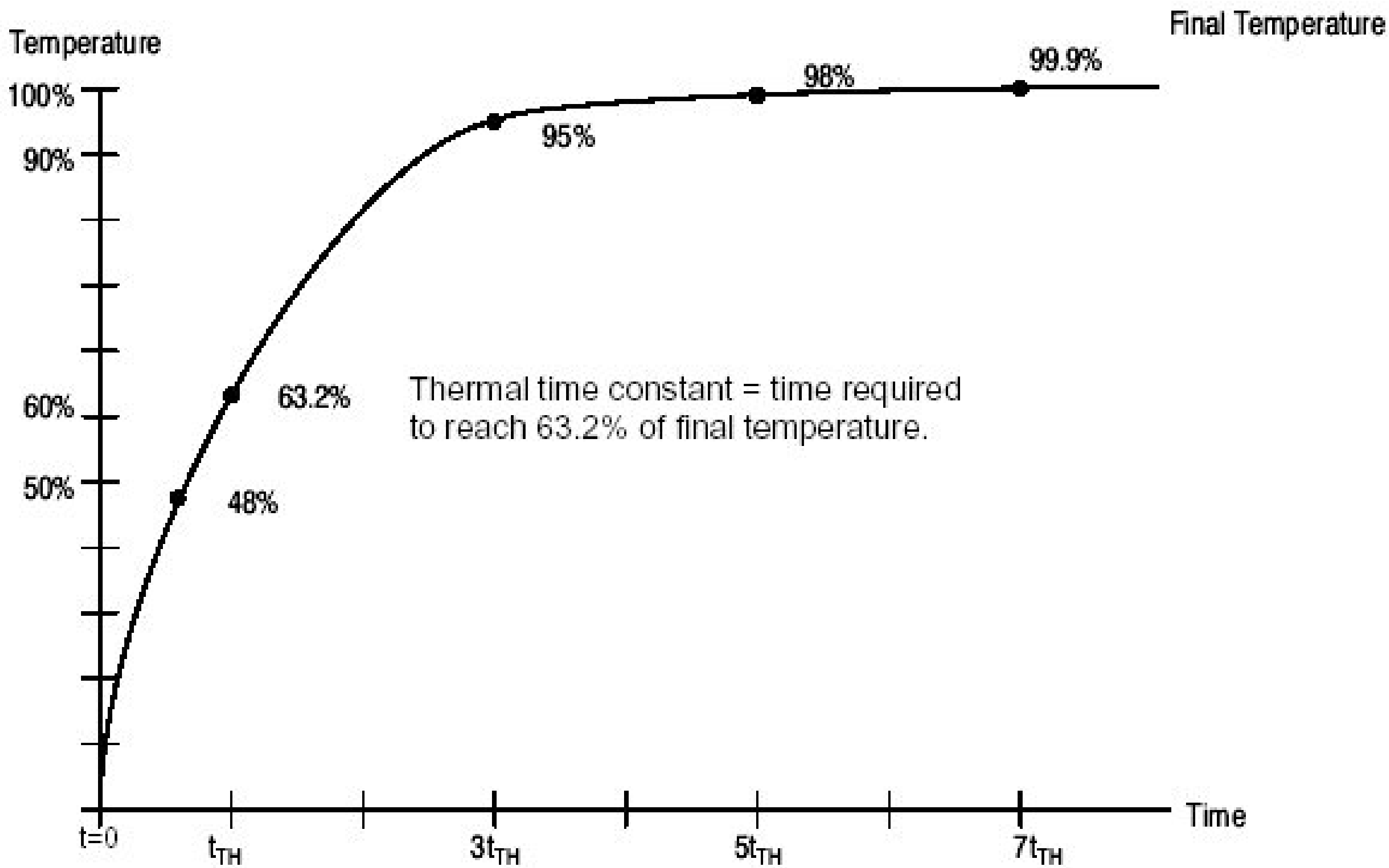

Figure 3.26 Temperature vs. time for constant motor current.

If the time t is greater than $7\tau_w$ then the equation may be simplified to:

$$T_w = T_a + P_j R_{th} \tag{3.39}$$

If the motor is turned off when $T_w > T_a$ then the winding temperature falls according to the following equation:

$$T_w(t) = T_{initial} + \left(T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) \tag{3.40}$$

Example 3.4

The thermal resistance of a motor is 1.4°C/Watt, and the maximum allowable winding temperature is 150°C. The ambient temperature is 30°C. The terminal resistance of a motor at the maximum allowable temperature is 3.33Ω.

(a) A motor under continuous use is pulling a constant current of 4.1A. Is the motor working in the safe temperature limit?

(b) If the motor has an operating cycle as follows then check the safety of the motor.

- Accelerates for 0.2s pulling a current of 10A
- Runs for 0.2s pulling a current of 1.5A
- Decelerates for 0.2s pulling a current of 8.5A
- Idles for 0.8s before a new cycle starts

Solution

(a) The terminal resistance for the hot winding is given as $R_{Hot}=3.33\Omega$.
Using equations (3.35) and (3.39):

$$P_j = I^2 R_{Hot} = 4.1^2 \times 3.33 = 55.9\,Watts$$

$$T_w = T_a + P_j R_{th} = 30 + (55.9 \times 1.4) = 108.3^{\circ} C$$

So the motor winding temperature will be well within the design limit.

(b) For this part we should calculate the RMS current over the operating cycle.

The RMS value for the current is calculated using the following equation:

$$I_{RMS} = \sqrt{\sum_{i=1}^{n} I_i^2 t_i \Big/ \sum_{i=1}^{n} t_i} \tag{3.41}$$

Here the RMS current is:

$$\begin{aligned} I_{RMS} &= \sqrt{\sum_{i=1}^{n} I_i^2 t_i \Big/ \sum_{i=1}^{n} t_i} \\ &= \sqrt{\frac{I_{Acc}{}^2 \times t_{Acc} + I_{Run}{}^2 \times t_{Run} + I_{Dec}{}^2 \times t_{Dec}}{t_{Acc} + t_{Run} + t_{Dec} + t_{Idle}}} \\ &= \sqrt{\frac{10^2 \times 0.2 + 1.5^2 \times 0.2 + 8.5^2 \times 0.2}{0.2 + 0.2 + 0.2 + 0.8}} \\ &= 4.99 \text{ A} \end{aligned}$$

Substituting I_{RMS} in (3.35), and using (3.39) we have:

$$P_j = I^2 R_{Hot} = 4.99^2 \times 3.33 = 82.9\,Watts$$
$$T_w = T_a + P_j R_{th} = 30 + (82.9 \times 1.4) = 146.1^{\circ} C$$

The result is close to the maximum the motor winding. A shorter "On" time could result operating above the safe winding temperature, while a longer "Off" time helps the motor operate cooler. Larger motors typically have smaller thermal resistance resulting lower operating temperature.

Example 3.5

Consider a motor operating for 1 minute dissipating 200 Watts, then turned off for 3.5 minutes. Assuming the motor has a thermal resistance of 0.54 °C/Watt and a winding thermal time constant of 30 seconds, draw the temperature curve of the motor. The ambient temperature is 32°C.

Solution

Using equation (3.38) the winding temperature at the end of 1 minute will be:

$$T_w = T_{initial} + \left(P_j R_{th} + T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) = 32 + (200 \times 0.54 + 32 - 32)\left(1 - e^{\frac{-60}{30}}\right) = 125^{\circ}C$$

Note that in 1 minute which is 2 times the thermal time constant, the temperature increase is 86.5% of its steady state value. The continuous temperature would be 144°C if the motor was not turned off.

The temperature starts falling according to equation (3.40) when the motor is turned off. The temperature of the motor after turning off for 3.5 minutes returns to ambient temperature.

$$T_w = T_{initial} + \left(T_a - T_{initial}\right)\left(1 - e^{\frac{-t}{\tau_w}}\right) = 125 + (32 - 125)\left(1 - e^{\frac{-210}{30}}\right) = 32^{\circ}C$$

Figure 3.28 shows the temperature changes versus time.

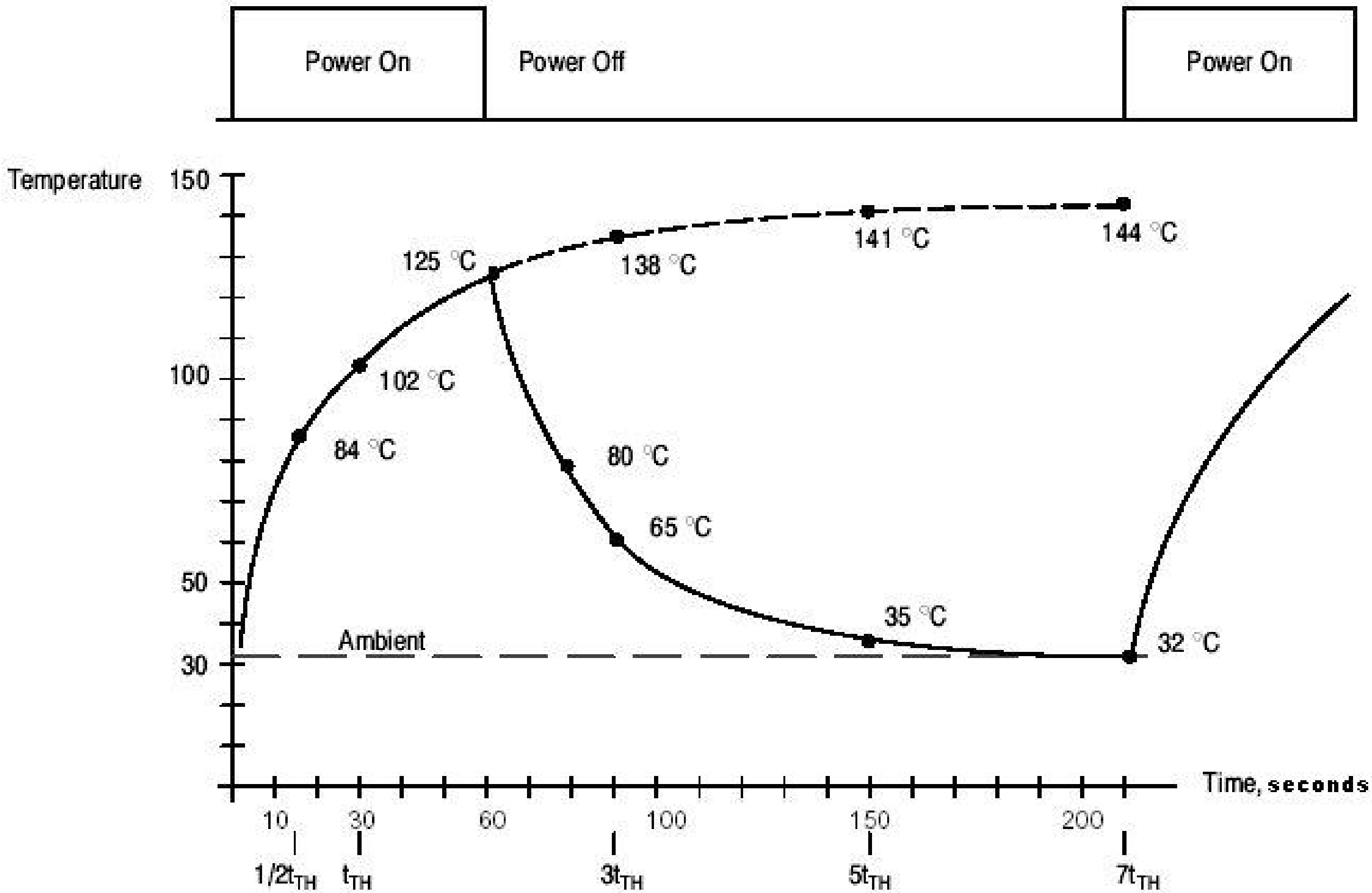

Figure 3.28 Temperature changes vs. time for example 3.4.

3.5 Pneumatic and Hydraulic Actuators

Advantages

Disadvantages

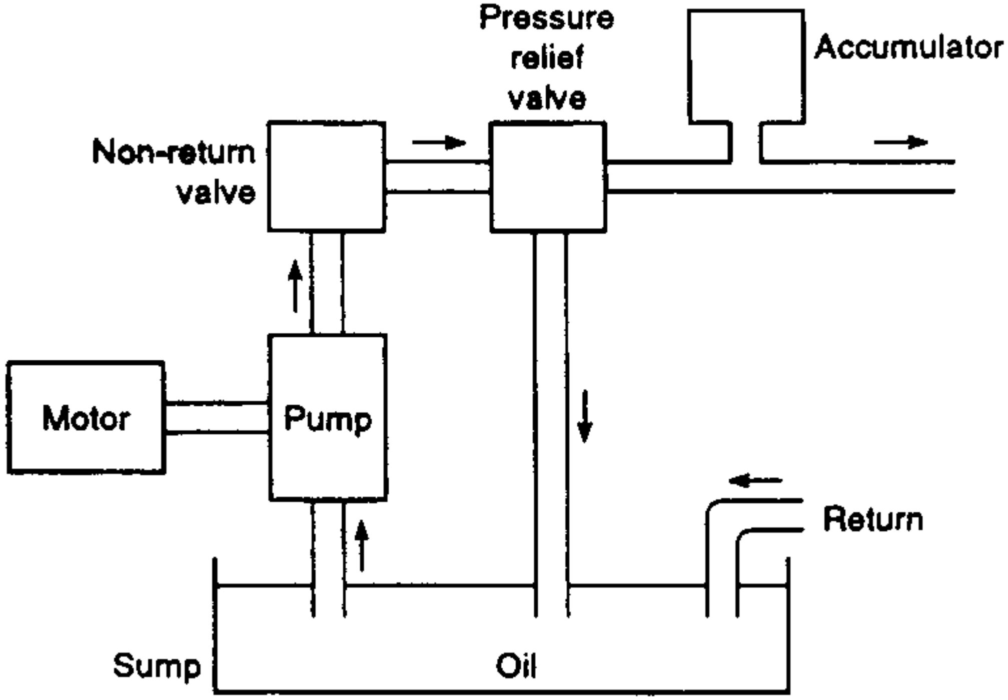

Figure 3.29 Hydraulic power supply.

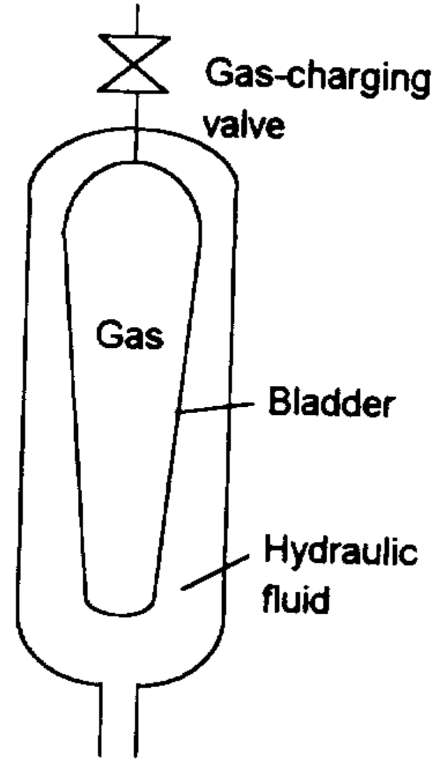

Figure 3.30 One type of hydraulic accumulator.

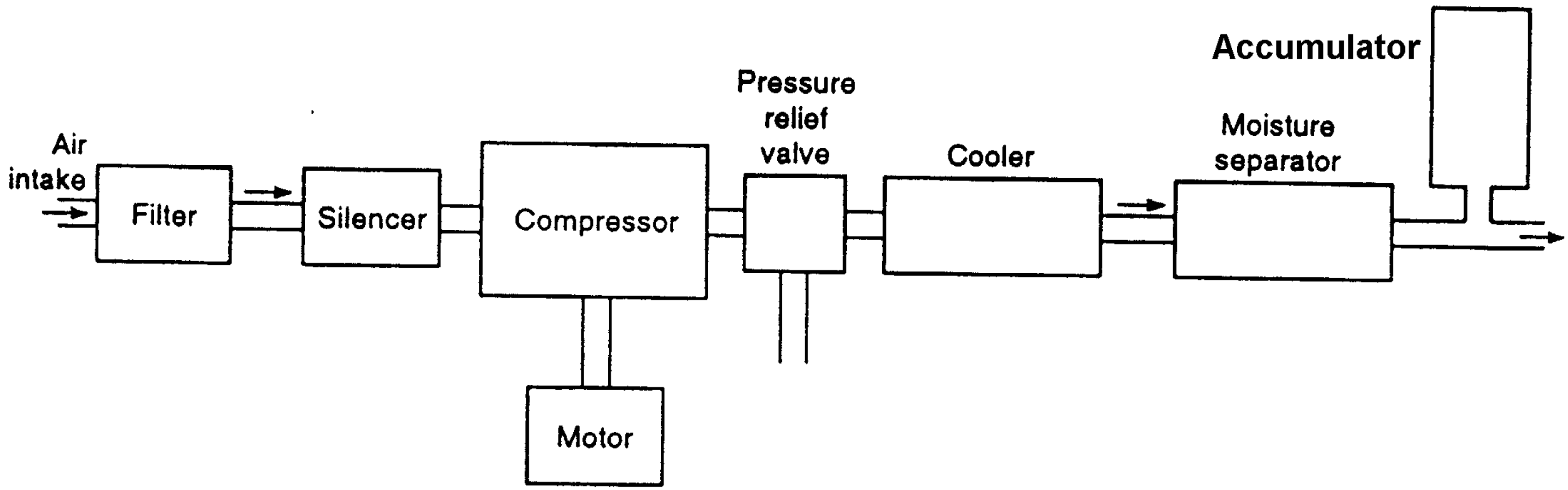

Figure 3.31 Pneumatic power supply.

Pneumatic and Hydraulic Cylinders

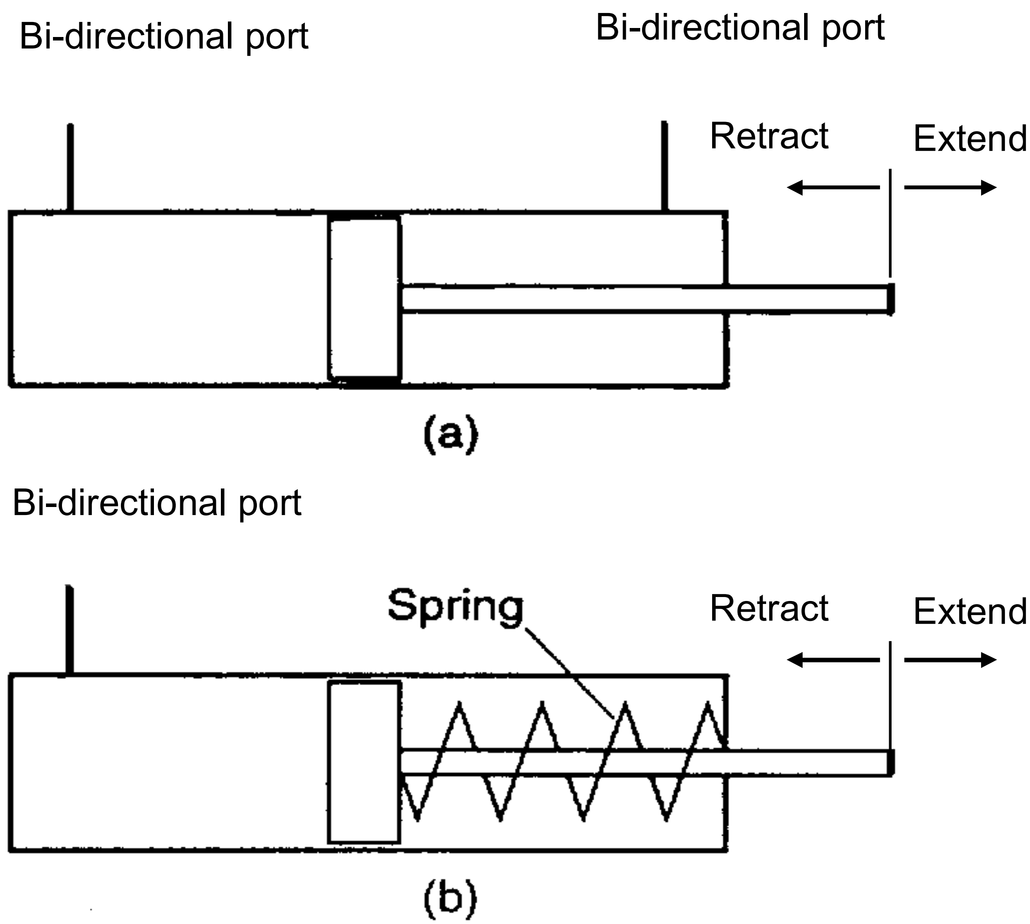

Figure 3.32 (a) Double acting cylinder.
(b) Single acting cylinder.

Basic Control of Pneumatic and Hydraulic Double Cylinders

Assuming the friction of the seals is negligible, the output force in the extend direction equals (Note: must use gauge pressures):

$$F_{extend} = P_{extend} A_{extend} - P_{retract} A_{retract} \tag{3.42}$$

The cross-sectional areas depend on whether the cylinder is a "single rod", "double rod" or "rodless" type. For a single rod cylinder:

$$A_{extend} = \frac{\pi D_{bore}^2}{4} \text{ and} \tag{3.43}$$

$$A_{retract} = \frac{\pi (D_{bore}^2 - D_{rod}^2)}{4} \tag{3.44}$$

where D_{bore} is the diameter of the bore and D_{rod} is the diameter of the rod.
With a rodless cylinder the cross-sectional areas are equal and are both given by equation (3.43). With a double rod cylinder the cross-sectional areas are also equal and are given by equation (3.44).

The output velocity is given by:

$$v = \frac{Q}{A} \tag{3.45}$$

where Q is the volume flow rate and A is appropriate value of the cross-sectional area.

Example 3.6

Determine the gauge pressure and volume flow rate required for a pneumatic or hydraulic single rod double acting cylinder with a 50 mm bore diameter and a 10 mm rod diameter to drive a 500 N load at 10 m/s in both directions. Assume the return side is at atmospheric pressure (0 gauge = 101 kPa absolute).

Solution

Extend side area: $A_{extend} = \frac{\pi}{4} D_{bore}^2 = \frac{\pi}{4}(0.050m)^2 = 0.00196\, m^2$

Retract side area: $A_{retract} = \frac{\pi}{4}(D_{bore}^2 - D_{rod}^2) = \frac{\pi}{4}((0.050m)^2 - (0.010m)^2) = 0.00188\ m^2$

Pressures:

For motion in the extend direction, assuming the retract side is at atmospheric pressure (so $P_{retract} = 0$ gauge) we have:

$$
\begin{aligned}
F_{extend} &= P_{extend} A_{extend} - P_{retract} A_{retract} \\
&= P_{extend} A_{extend} = 500\, N
\end{aligned}
$$

$$
\begin{aligned}
P_{extend} &= \left(500\, N\right) / \left(A_{extend}\right) \\
&= \left(500 N\right) / \left(0.00196\, m^2\right) \\
&= 2.55 \times 10^5 \text{ Pa gauge}
\end{aligned}
$$

For motion in the retract direction, assuming the extend side is at atmospheric pressure (so $P_{extend} = 0$ gauge) we have:

$$\begin{aligned} F_{retract} &= P_{retract} A_{retract} - P_{extend} A_{extend} \\ &= P_{retract} A_{retract} = 500\,N \end{aligned}$$

$$\begin{aligned} P_{retract} &= (500\,N) / (A_{retract}) \\ &= (500N) / (0.00188\,m^2) \\ &= 2.66 \times 10^5 \text{ Pa gauge} \end{aligned}$$

Volume Flow Rates:

The volume flow rates (from (3.45)) are:

$$Q_{extend} = vA_{extend} = (10\,m/s)(0.00196\,m^2) = 0.0196\,m^3/s$$ and

$$Q_{retract} = vA_{retract} = (10\,m/s)(0.00188\,m^2) = 0.0188\,m^3/s\,.$$

So the max. P required is 2.66×10^5 Pa gauge and the max. Q required is $0.0196\ m^3$.

Valve Design

Proportional and Servo Valves

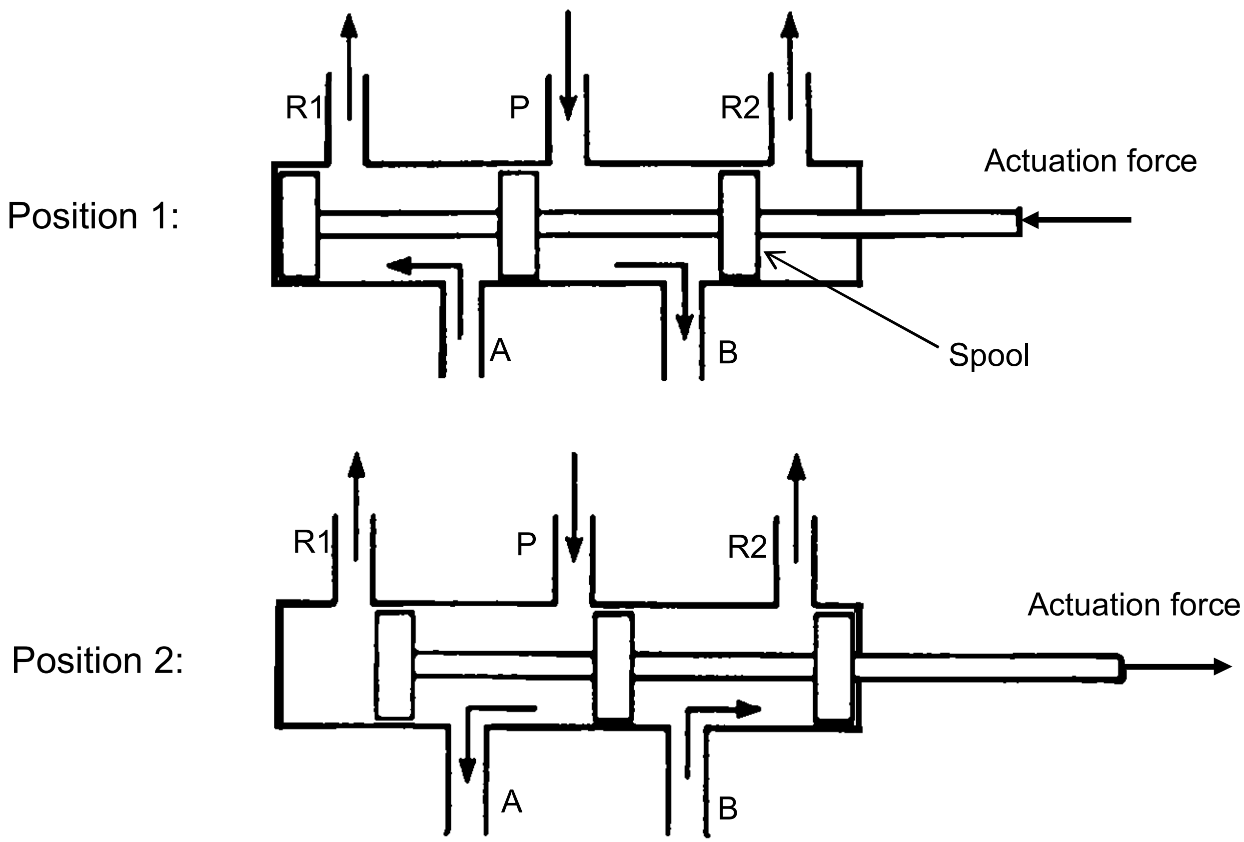

Figure 3.35 Cross-section view of a 5/2 spool valve.

Control Valve Sizing

In SI units:

$$C_V = 4.22 \times 10^4 Q \sqrt{\frac{\rho}{\Delta P}} \tag{3.46}$$

or

$$Q = 2.37 \times 10^{-5} C_V \sqrt{\frac{\Delta P}{\rho}} \tag{3.47}$$

where Q is the required volume flow rate in m^3/s, ΔP is the pressure drop across the valve in Pa and ρ is the density of the fluid at the outlet pressure in kg/ m^3.

In this course the fluid will either be air or hydraulic oil. With hydraulic oil ρ can be assumed to be constant (*i.e.* not a function of temperature and pressure changes). With air the density is given by:

$$\rho = \frac{P_2}{R_g T} = \frac{P_1 - \Delta P}{R_g T} \tag{3.48}$$

where P_2 is the absolute outlet pressure in Pa, R_g is the gas constant = 287 $J/kg°K$, T is the air temperature in Kelvin $= {}^{\circ}C + 273$ and P_1 is the absolute inlet pressure in Pa.

The choice of valve affects the max. velocity, and the max. acceleration and load capacity.

Example 3.9

We are designing a pneumatic system. We want to extend a 1 inch bore single rod pneumatic cylinder 12 inches in 0.1 seconds (not including the time for the valve to operate). The absolute supply pressure is 100 psi and a 5 psi pressure drop across the valve is acceptable. The air temperature is 20 °C. Find the minimum required C_v assuming the time to reach a constant velocity is negligible.

Solution

Due to the assumption that the time to reach constant velocity is negligable we have:

$$Q = \frac{Volume}{Time} = \frac{(\frac{\pi}{4} D_{bore}^2)(L)}{Time} = \frac{\frac{\pi}{4}(1\,in)^2(12\,in)}{0.1\,s}$$
$$= (94.2\,in^3\ /\ s)(1.635 \times 10^{-5}\,m^3\ /\ in^3) = 1.54 \times 10^{-3}\,m^3\ /\ s$$

From equation (3.12) we have:

$$\rho = \frac{P_2}{R_g T} = \frac{(P_1 - \Delta P)}{R_g T} = \frac{(100-5)\,psi \cdot 6895\,Pa\,/\,psi}{(287\,J\,/\,kg^\circ K)(20+273)^\circ K} = 7.79\ kg\,/\,m^3$$

From the given information:

$$\Delta P = 5\,psi \cdot 6895\,Pa\,/\,psi = 3.45 \times 10^4\ Pa$$

The minimum required valve flow coefficient is then (equation 3.46):

$$C_V = 4.22 \times 10^4\, Q \sqrt{\frac{\rho}{\Delta P}} = 4.22 \times 10^4 (1.54 \times 10^{-3}\, m^3\,/\,s)\sqrt{\frac{7.79 kg\,/\,m^3}{3.45 \times 10^4\ Pa}} = 0.98$$

Note that C_v is a unitless quantity.

Example 3.10

We are designing a hydraulic system. We want to use a single rod hydraulic cylinder to drive a 10,000 kg mass horizontally. The maximum desired acceleration is 2 m/s^2, the maximum desired velocity is 0.5 m/s and the density of the hydraulic oil is 900 kg/m^3. If the supply pressure is 1.5×10^7 Pa absolute (≈2,200 psi) and a 2000 kPa pressure drop across the valve is acceptable then determine an appropriate bore size for the cylinder and the minimum required flow coefficient. Assume that the pressure drop across the valve is the same for the return flow as for the intake flow and that the sump is open to the atmosphere. Also assume the available bore sizes for the cylinder only come in 5 mm increments, and that the area of its rod can be neglected.

Solution will be presented during the lecture

Relevant Equations:

pressure drop across the valve = ΔP,

$F_{extend} = P_{extend} A_{extend} - P_{retract} A_{retract}$ using gauge pressures,

$F_{retract} = P_{retract} A_{retract} - P_{extend} A_{extend}$ using gauge pressures,

$A = \frac{\pi D^2}{4}$, $v = \frac{Q}{A}$ and $C_V = 4.22 \times 10^4 Q \sqrt{\frac{\rho}{\Delta P}}$.

Example 3.11

We are designing a hydraulic system for moving a 2,000 kg payload mass vertically. A single rod cylinder will be used, mounted above the payload. The bore diameter is 100 mm and the rod diameter is 40 mm. The desired acceleration is 1 m/s^2 upwards and the desired maximum velocity is 0.1 m/s. If the supply pressure is 7×10^6 Pa gauge and the density of the oil is 900 kg/m^3 then determine the minimum valve flow coefficient required.

Solution will be presented during the lecture

Relevant Equations:

pressure drop across the valve = ΔP,

$F_{extend} = P_{extend} A_{extend} - P_{retract} A_{retract}$ using gauge pressures,

$F_{retract} = P_{retract} A_{retract} - P_{extend} A_{extend}$ using gauge pressures,

$A = \frac{\pi D^2}{4}$, $v = \frac{Q}{A}$ and $C_V = 4.22 \times 10^4 Q \sqrt{\frac{\rho}{\Delta P}}$.

Applications of Pneumatic Systems

- Automated manufacturing systems for placing, feeding and rejecting parts.
- Pick and place robots

Applications of Hydraulic Systems

- Presses for forging and extrusion
- Heavy construction equipment such as excavators
- Farm equipment
- Large robots
- Power steering in cars.

3.5 Emerging Actuators

The performance of "traditional" actuators continues to gradually improve due to the development of new materials and new designs, but the underlying physics remains the same.

For example, a DC motor is inherently a high speed, low torque actuator.

Recently, the demand for new and higher performance actuators has significantly increased in a variety of fields such as robotics, medical devices, security, astronomy, optics, and precision machining.

We will cover three of the most useful types:
- Piezoelectric,
- Shape memory alloy, and
- Ultrasonic.

Piezoelectric Actuators

These use the piezoelectric effect employed in sensors in reverse.

To achieve useful displacements piezoelectric actuators must be made of several thin layers (< 1 mm thick)

An example is shown in Figure 3.37.

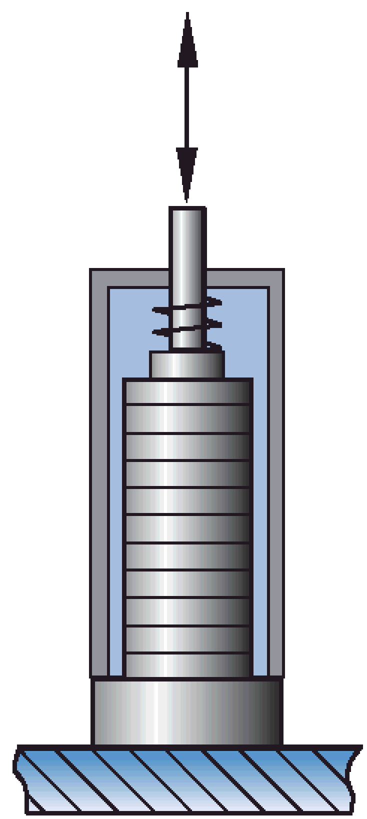

Figure 3.37 Piezoelectric actuator built from many layers. Note that the spring provides a preload force that is necessary for bidirectional motion.

Advantages:

- Energy efficient.
- Suitable for miniaturization.
- Nanometre resolution (**Note:** human hair diameter ≈ 100,000 nanometres).
- Response times < 1 ms
- Large output forces (e.g. 4500 N for 25 mm diameter actuator).
- Large stiffness when pushing.

Disadvantages:

- Very small range of motion, ≈ 0.2% of the actuator length.
- Low pulling force unless preloading is used.
- Require large voltages (e.g. 500V or more).
- Sensitive to temperature changes.
- Requires closed-loop to achieve low frequency displacements.

Applications: Inkjet printer heads (to control flow of ink), piezoelectric valves, vibratory feeders, optical alignment, precision assembly and precision machining.

Shape Memory Alloy Actuators

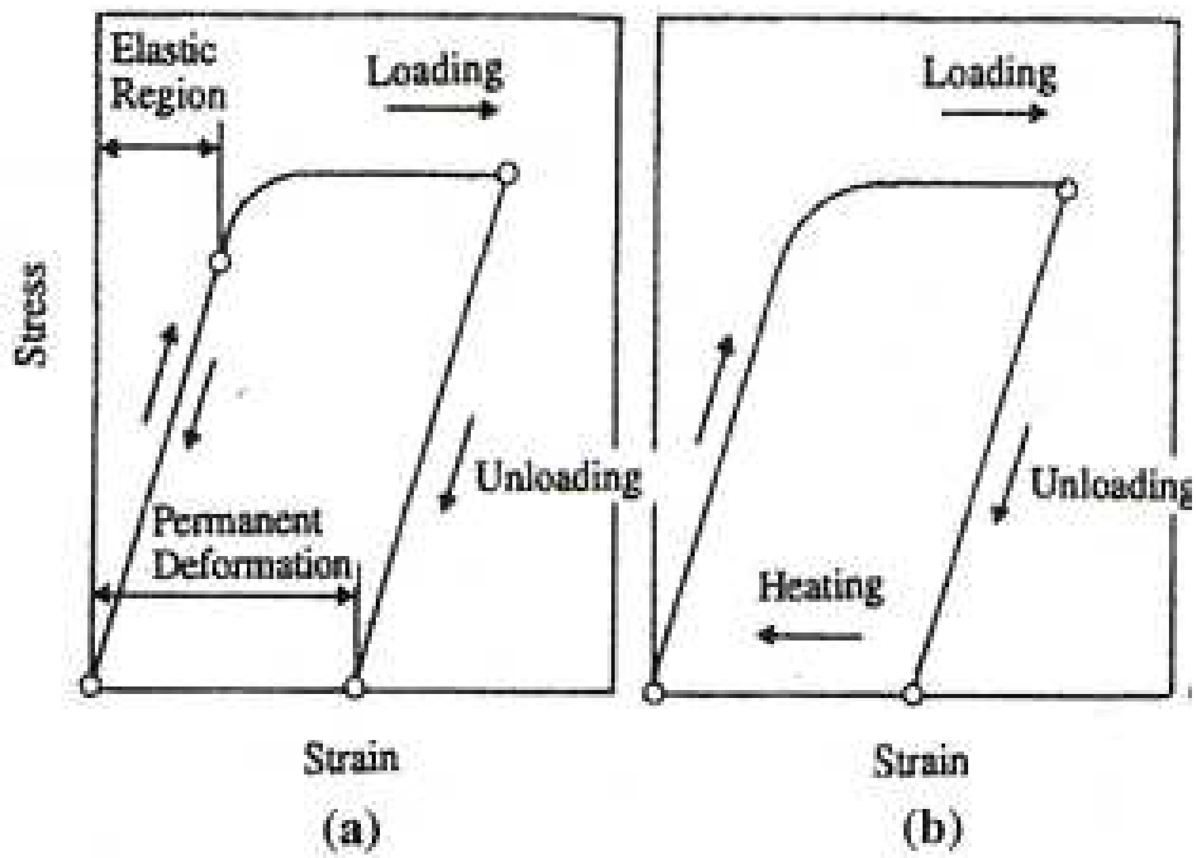

Figure 3.39 Stress vs. strain curves for:
(a) typical metal, (b) shape memory alloy (SMA).

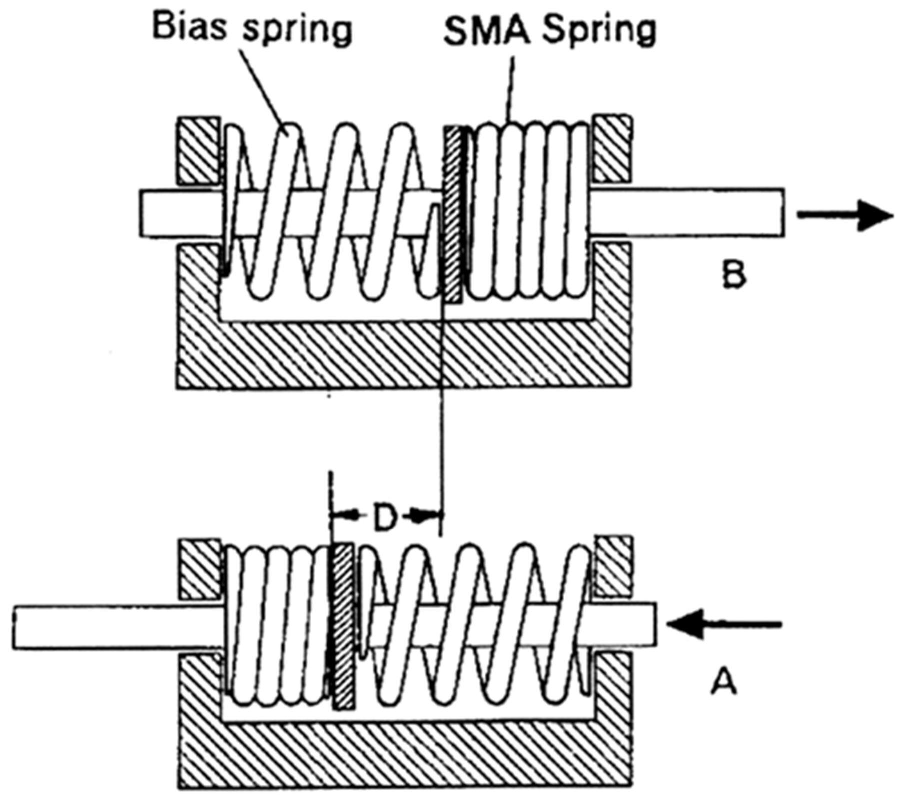

Figure 3.40 A bidirectional actuator incorporating shape memory alloy.

At low temperatures, the SMA spring is pushed all the way to the right as a result of force from normal spring.

When the SMA spring is heated up using an appropriate source (e.g. by applying an electric current), the SMA spring recovers its original shape and the shaft moves to the left.

When the SMA spring is allowed to cool down the shaft will return to the right.

Advantages:
- Simple one piece design.
- Material can be shaped for the application
- Large force per unit area
- No wear

Disadvantages:
- Slow response time, > 1 second (longer for cooling than for heating)
- Range of motion ≈ 5% of original length
- Operates at ≈ 90 ^{o}C
- Subject to fatigue failure
- Large hysteresis
- Unidirectional (requires spring or second actuator to be bidirectional)

Control: Typically use current control.

Applications: valves, electronic locks, safety devices

Ultrasonic Motors

Special type of piezoelectric actuator.

Can be linear or rotary.

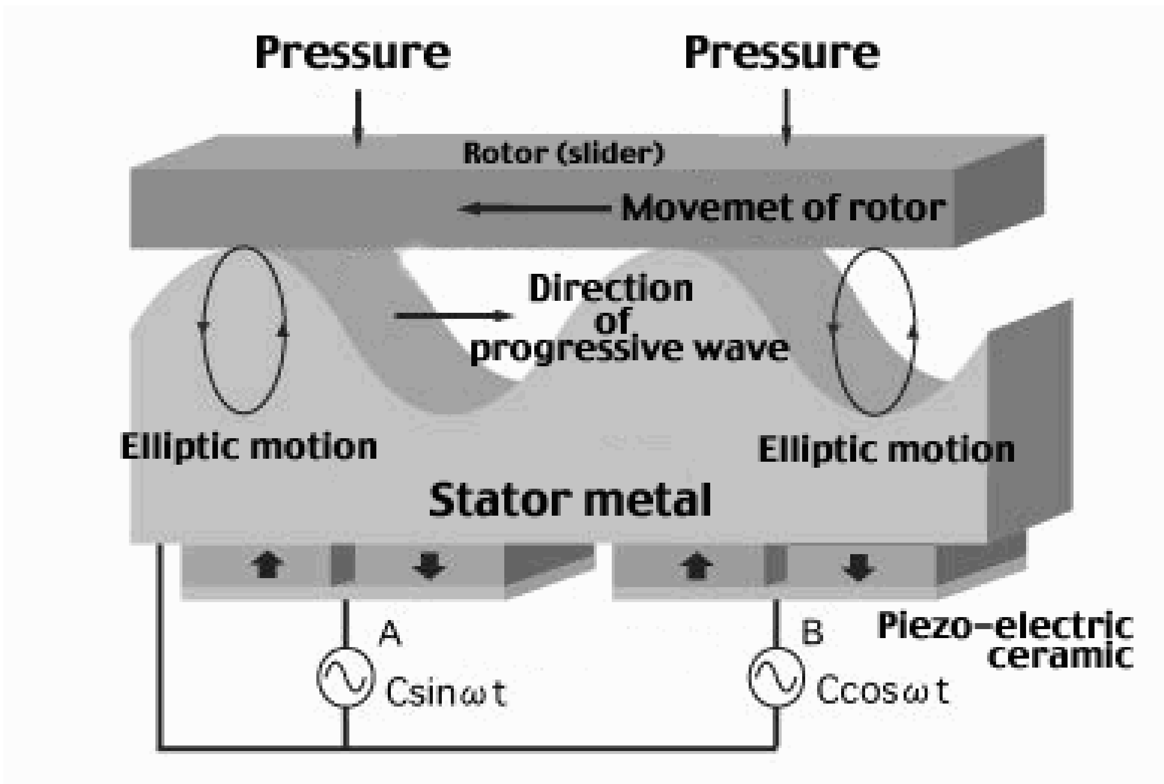

Figure 3.41 Physics of a travelling wave type ultrasonic motor (image source: Shinsei Corporation, Japan).

The piezoelectric material is electrical driven to cause the stator to vibrate in the form of a travelling wave.

This wave, and the friction force between the stator and rotor, cause the rotor to rotate.

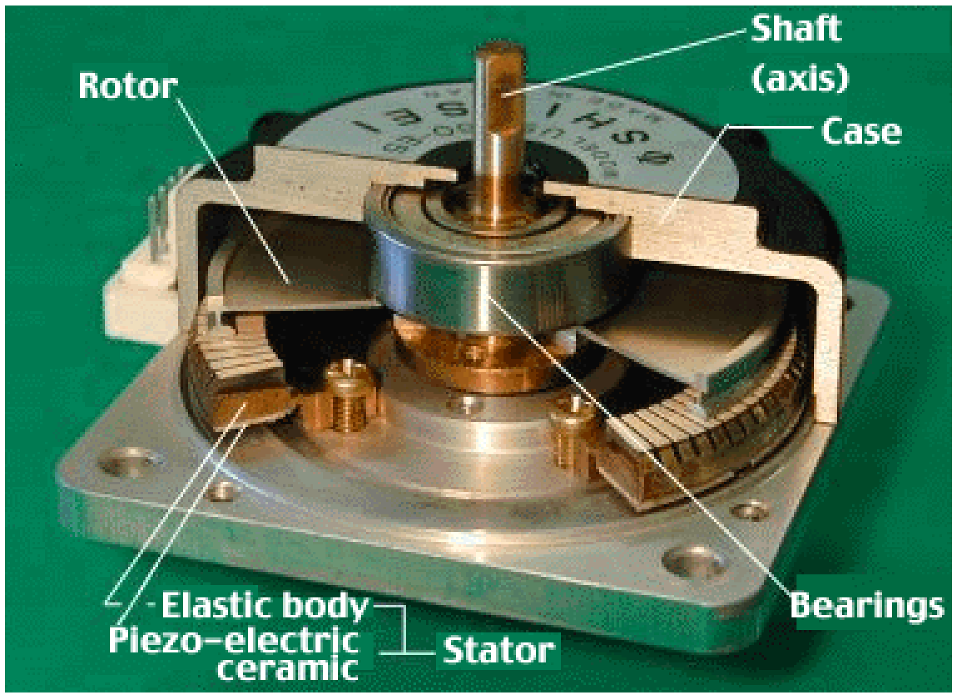

Figure 3.42 Picture of a sectioned ultrasonic motor from Shinsei Corporation, Japan.

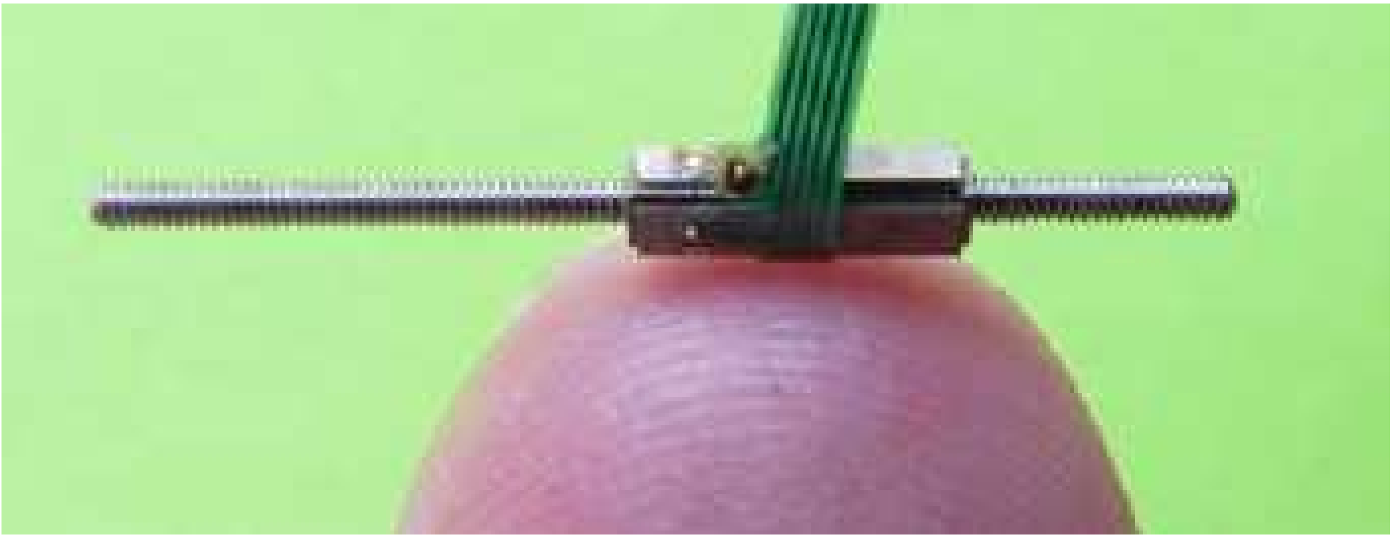

Figure 3.43 Miniature linear actuator made using an ultrasonic motor with built-in lead screw from New Scale Technologies Inc.

Advantages

- High torque at low speeds.
- Smaller and lighter than a comparable DC motor and gearbox.
- Large acceleration due to small rotor inertia.
- Rotor is locked when power is off (held by friction).
- Suitable for miniaturization.

Disadvantages

- Relies on friction to provide torque
- Subject to fatigue and wear.
- Driven by large voltages relative to DC motors (approximately 200 V vs. 30 V).

Applications: lens motor in autofocus cameras

Discussion Topic 1:

What is the best type of actuator to use with a robotic gripper?

Addendum:

A.1 More Information About Common Motion Profiles

It is common for desired (or required) motion profiles to be based on periods of constant acceleration.

The position and velocity profiles can be derived from the acceleration profile, and vice-versa.

Specific case: Moving a mass from one fixed location to another fixed location

- First motion period: accelerate up to the desired maximum velocity.
- The acceleration period may be followed by a period of constant velocity (with a=0).
- Last period: Constant deceleration.

Note: deceleration refers to a reduction in the speed, so its sign is opposite to the sign of the velocity (which could be positive or negative).

An example of the resulting position, velocity and acceleration profiles is shown in Figure 3.44.

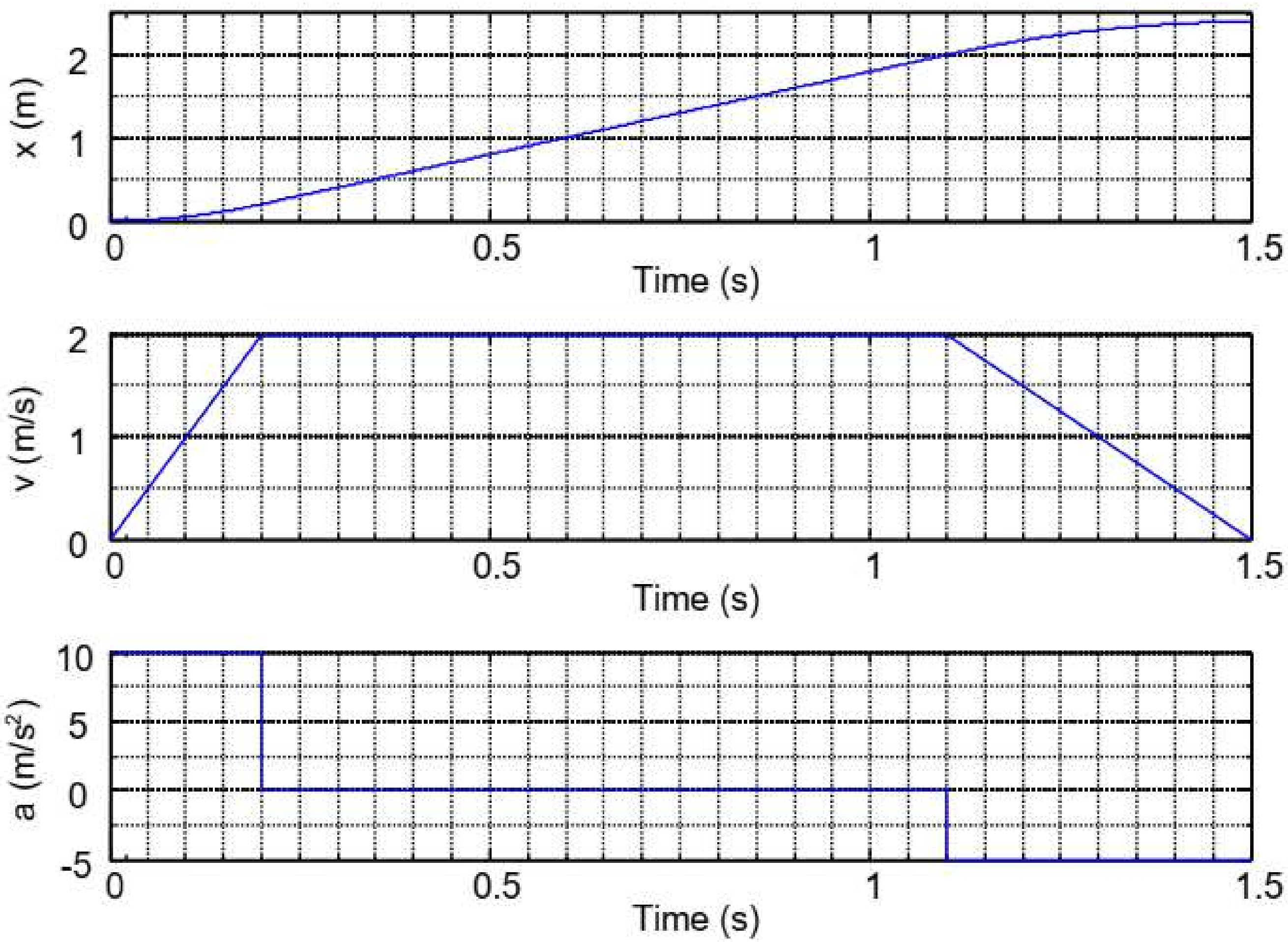

Figure 3.44 Position, velocity and acceleration profiles for the parameters: $t_{acc} = 0.2$ s, $t_{run} = 0.9$ s, $t_{dec} = 0.4$ s, $a_{acc} = 10$ m/s^2, $a_{dec} = -5$ m/s^2 and $v_{max} = 2$ m/s.

With piecewise constant acceleration, the fastest possible motion from one fixed location to another occurs when t_{run}=0 and t_{acc}= t_{dec}= t_{con}.

The equations of motion for this case are presented in the notes.

The corresponding position, velocity and acceleration profiles are shown in Figure 3.45, assuming t_i =0 and x_i =0.

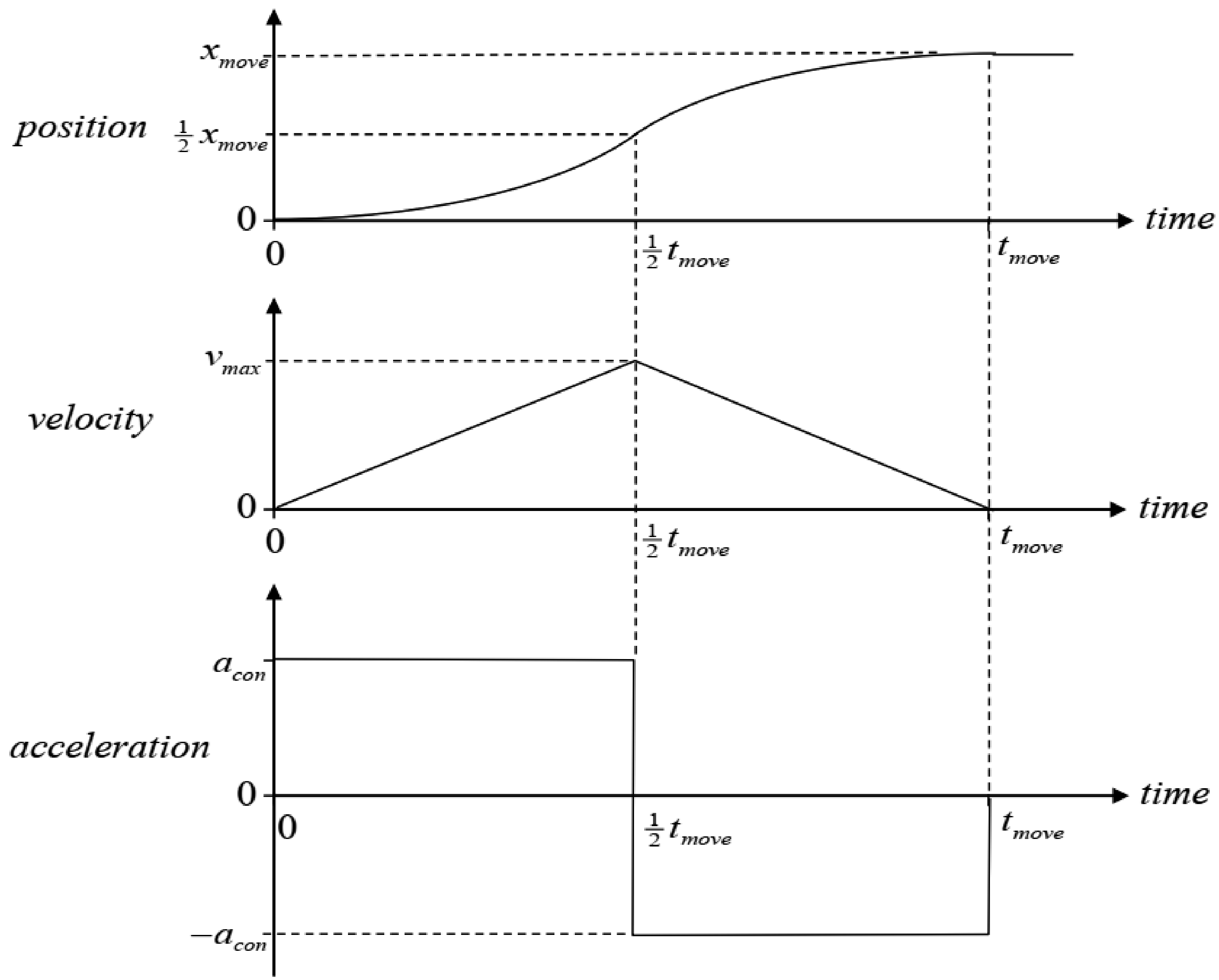

Figure 3.45 Plots of position, velocity and acceleration vs. time for a movement consisting of constant acceleration (a_{con}) followed by an equal period of constant deceleration ($-a_{con}$).

Desired Motion that Returns the Mass to its Starting Location

If the motion should return to its starting location then the operating cycle will typically have eight periods. Assuming the mass should be initially moved forwards, an example showing the eight periods is given in Figure 3.46.

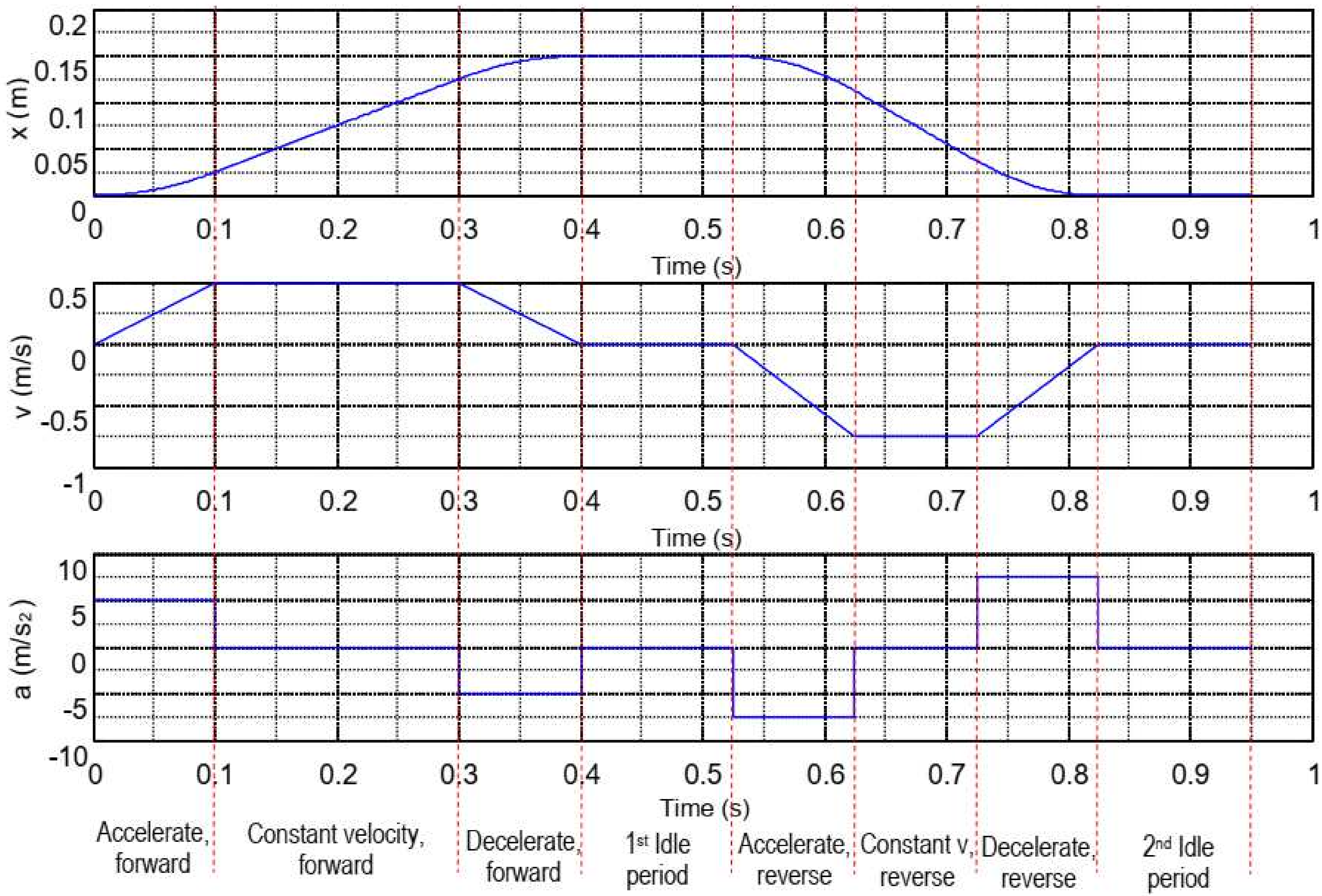

Figure 3.46 Example of position, velocity and acceleration profiles for an eight period operating cycle where the motion returns to its starting position.

If the desired motion is rotary then the same equations apply, except that the symbols θ, ω (or $\dot{\theta}$) and α (or $\dot{\omega}$ or $\ddot{\theta}$) are used for the angular position, velocity and acceleration, respectively.

A.2. Examples of Motor and/or Mechanism Selection

Example 3.12

A load is to be rotated by half a revolution, pause for 0.4 s, and then returned to its original position. After the return motion the load should be kept stationary for 0.6 s before the cycle restarts. Each motion should consist of a period of acceleration followed by an equal period of deceleration, and be completed in 1 second. The load has a moment of inertia of 0.025 kgm^2. The ambient temperature is 25 ^oC. The motor parameters are given in the notes. A gear box will be attached between the motor and the load. The available gear ratios are: 5, 10, 15, etc. You may assume the torque ratings are independent of the speed and that the friction torques are negligible.

a) List the periods that make up the motion profile in order.

b) Determine the best gear ratio for this application using the method of section 3.4.3. Check that the resulting motor speed, torques and temperature are within their rated values. Remember to keep the inertia ratio between 1 and 10.

c) Repeat part (b) for a load inertia of 0.25 kgm^2.

Solution will be presented during the lecture

Relevant equations:

$$\alpha_{con} = 4\theta_{move} / t_{move}^2 \,,\ \omega_{\max} = \tfrac{1}{2}\alpha_{con} t_{move}$$

For $0 \le t \le \tfrac{1}{2} t_{move}$: $\omega(t) = \alpha_{con} t$ and $\alpha(t) = \alpha_{con}$ (assuming t_i=0)

For $\tfrac{1}{2} t_{move} < t \le t_{move}$: $\omega(t) = -\alpha_{con}(t - \tfrac{1}{2} t_{move}) + \omega_{max}$ and $\alpha(t) = -\alpha_{con}$

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} \,,\ \omega_{motor,max} = N_r \omega_{\max} \,,\ \tau_{motor} = \left(J_{motor} N_r + \frac{1}{N_r} J_{load} \right) \alpha_{load}$$

$$Ratio_J = \frac{J_{load} / N_r^{\,2}}{J_{motor}}$$

Example 3.13

A 25 kg mass is to be translated vertically. It is subject to a 50 N friction force. Its desired motion profiles are given in Figure 3.46, where positive indicates upwards motion. The motor to be used is the same as in example 3.12. The ambient temperature is 30 ^{o}C. You may assume the torque ratings are independent of the speed and that the friction of the motor, gears and ball screw may be neglected.

a) The linear motion of the mass is obtained by coupling the motor to a gearbox that drives a ball screw with a 0.02 m lead. The inertia of the screw is 3.14×10^{-5} kgm^2. Determine the best gear ratio for this application using the method of section 3.4.3. The available gear ratios are 0.5, 1, 1.5, etc. Check that the resulting motor speed, torques and temperature are within their rated values.

b) The linear motion of the mass is obtained by directly coupling the motor to a ball screw. Choose the lead of the ball screw based on the method presented in section 3.4.3. The available leads are: 2 mm, 4 mm,…, 20 mm. You may assume that the inertia of the screw is not a function of its lead, and equals 3.14×10^{-5} kgm^2. Note that, even though $N_r = 1$, the ball screw can be used to provide the necessary mechanical advantage. Check that the resulting motor speed, torques and temperature are within their rated values. Comment on the performance of this design compared to your answer from part (a).

Solution

a) The load inertia includes the equivalent inertia of the mass driven by the ball screw, plus the inertia of the screw, as follows:

$$J_{load} = M\left(\frac{l}{(2\pi / rev)}\right)^2 + J_{screw}$$

$$= (25\text{ kg})\left(\frac{0.02\text{ m/rev}}{(2\pi / rev)}\right)^2 + 3.14\times10^{-5}\text{ kgm}^2$$

$$= 2.85\times10^{-4}\text{ kgm}^2$$

The optimal gear ratio is given by:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{2.85\times10^{-4}\ \text{kgm}^2}{1.10\times10^{-5}\ \text{kgm}^2}} = 5.09$$

However, the available gear ratios are 0.5, 1, 1.5, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 5$. Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio.

The maximum required linear speed, $|v_{max}|$, can be obtained from Figure 3.46. The max. rotational speed of the ball screw is then:

$$\omega_{max} = \frac{|v_{max}|}{l(\text{rev}/2\pi)} = \frac{0.75\ \text{m/s}}{(0.02\ \text{m/rev})(\text{rev}/2\pi)} = 236\ \text{rad/s}$$

The corresponding motor speed is: $\omega_{motor,max} = N_r\omega_{max} = 1180\ \text{rad/s}$

Since $\omega_{motor,max} > \omega_{rated,max}$ this gear ratio fails the motor speed check. The upper limit on the gear ratio is:

$$N_{r,max} = \frac{\omega_{rated,max}}{\omega_{max}} = \frac{733 \text{ rad/s}}{236 \text{ rad/s}} = 3.11$$

We should round down to the nearest available ratio to keep $\omega_{motor,max} \leq \omega_{rated,max}$. Therefore our new choice for the best ratio is: $N_r = 3$. Now we need to check the inertia ratio. It is:

$$Ratio_J = \frac{J_{load} / N_r^{\,2}}{J_{motor}} = \frac{(2.85 \times 10^{-4} \text{ kgm}^2)/(3^2)}{(1.10 \times 10^{-5} \text{ kgm}^2)} = 2.88$$

Since it is within the range 1 to 10 it is acceptable.

Next, we should check the max. required motor torque and RMS motor torque. The torques are obtained for the eight periods shown in Figure 3.46 … see the notes for the details.

From the eight periods, the max. required torque is: $\tau_{motor,max} = \tau_{motor,1} = 0.514 \text{ Nm}$.

This is below the motor's rated max. torque of 1.5 Nm, but above its rated continuous torque. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\tau_{motor,RMS} = \sqrt{\frac{\sum_{i=1}^{n} \tau_{motor,i}^{2} t_i}{\sum_{i=1}^{n} t_i}} = 0.306 \text{ Nm}$$

Since $\tau_{motor,RMS}$ is below the motor's continuous torque rating of 0.45 Nm, neither torque limit has been exceeded.

Lastly, we need to check the motor's operating temperature. The question states that the friction of the motor is negligible, therefore $K_d \approx 0$ and the torque output of the motor is simply $K_t I$. Then the RMS current is simply:

$$I_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} I_i^2 t_i}{\sum_{i=1}^{n} t_i}} = \sqrt{\frac{\sum_{i=1}^{n} \left(\tau_{motor,i} / K_t\right)^2 t_i}{\sum_{i=1}^{n} t_i}} = \frac{\tau_{RMS}}{K_t} = \frac{0.306 \text{ Nm}}{0.1 \text{ Nm/A}} = 3.06 \text{ A}$$

The power loss is:

$$P_j = I_{RMS}^2 R_{Hot} = (3.06 \text{ A})^2 (2.15 \text{ ohm}) = 20.1 \text{ W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 30\ °\text{C} + (20.1 \text{ W})(1.43\ °\text{C/W}) = 58.8\ °\text{C}$$

Since this is less than the motor's temperature limit this test has been passed.

We can conclude that this motor, gearbox and ballscrew combination has passed all of the tests and is acceptable.

b) The load inertia includes the equivalent inertia of the mass driven by the ball screw, plus the inertia of the screw, as follows:

$$J_{load} = M\left(\frac{l}{(2\pi/rev)}\right)^2 + J_{screw}$$

With $N_r = 1$ the inertia ratio is simply: $Ratio_J = \dfrac{J_{load}}{J_{motor}}$. As in Example 3.12, we should first try to make this equal to the optimal value of 1, i.e. try to make $J_{load} = J_{motor}$. With this substitution the inertia equation becomes:

$$J_{motor} = M\left(\frac{l}{(2\pi/rev)}\right)^2 + J_{screw}$$

However, since $J_{screw} > J_{motor}$ this equation has no solution. The lead from the set of available screws that gives the closest inertia ratio to 1 is 2 mm = 0.002 m. From Figure 3.46 the max. desired linear speed is 0.75 m/s. The corresponding motor speed is:

$$\omega_{motor,max} = \frac{|v_{max}|}{l} = \frac{0.75 \text{ m/s}}{(0.002 \text{ m/rev})(1 \text{ rev}/2\pi)} = 2360 \text{ rad/s}$$

This $\omega_{motor,max}$ is clearly too large, and this lead cannot be used. The lower limit on the lead is:

$$l_{min} = \frac{|v_{max}|}{\omega_{max}} = \frac{0.75 \text{ m/s}}{(733 \text{ rad/s})(1 \text{ rev} / 2\pi)} = 0.00642 \text{ m/rev}$$

Since a larger lead will reduce the required motor speed we should round this up to closest available value. The new best lead is then:

$l = 0.008$ m/rev.

Now we need to check the inertia ratio. It is:

$$Ratio_J = \frac{J_{load}}{J_{motor}} = \frac{7.19 \times 10^{-5} \text{ kgm}^2}{1.10 \times 10^{-5} \text{ kgm}^2} = 6.54$$

Since it is within the range 1 to 10 it is acceptable.

Next, we should check the max. required motor torque and RMS motor torque. The torques are obtained for the eight periods shown in Figure 3.46… see the notes for the details.

From the eight periods, the max. required torque is: $\tau_{motor,max} = \tau_{motor,7} = 0.737\ \text{Nm}$. This is below the motor's rated max. torque of 1.5 Nm, but above its rated continuous torque. This means it is necessary to check the RMS torque value. Using (3.33) we have:

$$\tau_{motor,RMS} = \sqrt{\frac{\sum_{i=1}^{n} \tau_{motor,i}^{2} t_i}{\sum_{i=1}^{n} t_i}} = 0.410\ \text{Nm}$$

Since $\tau_{motor,RMS} < \tau_{motor,cont}$ neither torque limit has been exceeded.

Lastly, we need to check the motor's operating temperature.

As before, the RMS current is simply:

$$I_{RMS} = \frac{\tau_{RMS}}{K_t} = \frac{0.410\text{Nm}}{0.1\ \text{Nm/A}} = 4.10\ \text{A}$$

The power loss is:

$$P_j = I_{RMS}^2 R_{Hot} = (4.10\text{ A})^2(2.15\text{ ohm}) = 36.0\text{ W}$$

Using this power and the given information, the corresponding winding temperature is:

$$T_w = T_a + P_j R_{th} = 30\text{ °C} + (36.1\text{ W})(1.43\text{ °C/W}) = 81.5\text{ °C}$$

Since this is less than the motor's temperature limit this test has been passed.

We can conclude that this motor and ballscrew combination has passed all of the tests and is acceptable.

Compared to the design from part (a), this design is less energy efficient as shown by the P_j values (36.1 W vs. 20.1 W).