ENGPHYS 2A04 TUTORIAL 9

ELECTRICITY & MAGNETISM

Your TAs Today

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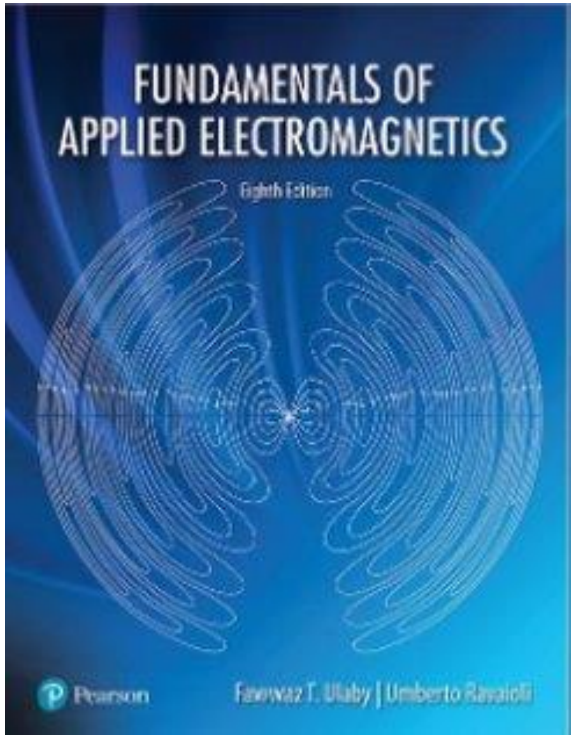
Your Textbook

Fundamentals of Applied Electromagnetics Eighth Edition

Ulaby & Ravaioli

Seventh Edition also acceptable, with some inconsistencies



Problem 5.1

An electron with a speed of 8×10^6 m/s is projected along the positive x direction into a medium containing a uniform magnetic flux density $\mathbf{B} = (\hat{\mathbf{x}}4 - \hat{\mathbf{z}}3)T$

Given that e = $1.6*10^{19}$ C and the mass of an electron is me = $9.1*10^{-31}$ kg, determine the initial acceleration vector of the electron (at the moment it is projected into the medium).

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Known Values:

$$Electron\ speed: u = 8 * 10^6 \frac{m}{s}$$

$$Magnetic\ Flux\ Density: \mathbf{B} = (\hat{\boldsymbol{x}}4 - \hat{\mathbf{z}}3)T$$

$$Elementary\ Charge: e = 1.6 * 10^{19}\ C$$

$$Electron\ Mass: m_e = 9.1 * 10^{-31}\ kg$$

Problem 5.1

Particle of a charge *q* moving with velocity **u** in a magnetic field experiences magnetic force $\mathbf{F}_m$ given by:

$$Electron\ speed: u = 8 * 10^6 \frac{m}{s}$$

$$Magnetic\ Flux\ Density: \mathbf{B} = (\hat{\boldsymbol{x}}4 - \hat{\boldsymbol{z}}3)T$$

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$$Electron\ Mass: m_e = 9.1 * 10^{-31}\ kg$$

$$\mathbf{F}_\mathrm{m} = q\mathbf{u} \times \mathbf{B} \quad \text{(N)}$$

Use Newton's Second Law: F = m * a

Rearrange equation and substitute **F** for equation above.

Problem 5.1

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$$\boldsymbol{a} = \frac{\mathbf{F}_m}{m_e} = \frac{q\mathbf{u} \times \mathbf{B}}{m_e}$$

Assuming q = -e

$$= \frac{-1.6 * 10^{-19}}{9.1 * 10^{-31}} (\hat{\boldsymbol{x}}8 * 10^6) \times (\hat{\boldsymbol{x}}4 - \hat{\mathbf{z}}3)$$

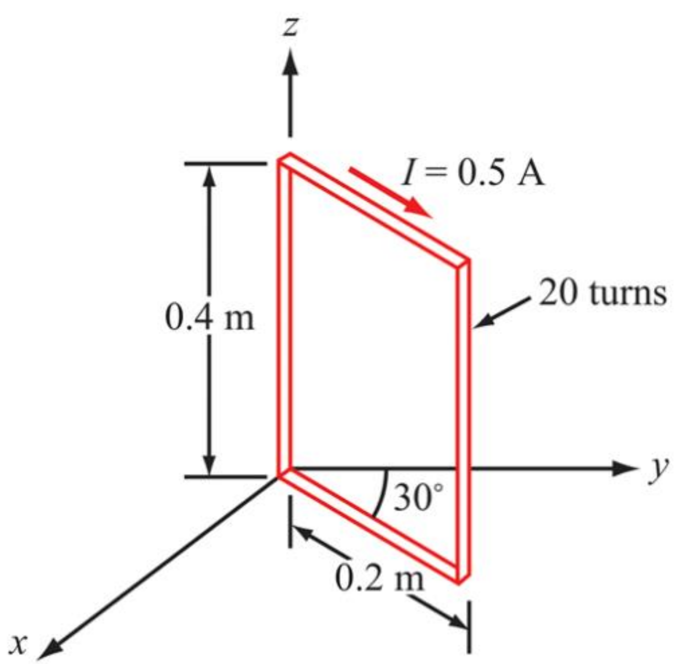
$$= -\hat{\boldsymbol{y}}4.22 * 10^{18} \quad (m/s^2)$$

$$\overline{a} \times \overline{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 8000000 & 0 & 0 \\ 4 & 0 & -3 \end{vmatrix} = \boldsymbol{i}(0\cdot(-3) - 0\cdot 0) - \boldsymbol{j}(8000000\cdot(-3) - 0\cdot 4) + \boldsymbol{k}(8000000\cdot 0 - 0\cdot 4) =$$

$$= \boldsymbol{i}(0 - 0) - \boldsymbol{j}(-24000000 - 0) + \boldsymbol{k}(0 - 0) = \{0; 24000000; 0\}$$

Problem 5.4

The rectangular loop shown in Fig. P5.4 consists of 20 closely wrapped turns and is hinged along the z axis. The plane of the loop makes an angle of 30° with the y axis, and the current in the windings is 0.5 A. What is the magnitude of the torque exerted on the loop in the presence of a uniform field $\mathbf{B} = \hat{\mathbf{y}}2.4\ \mathrm{T}$? When viewed from above, is the expected direction of rotation clockwise or counterclockwise?



(5.19)

$$\mathbf{m} = \hat{\mathbf{n}}NIA = \hat{\mathbf{n}}m \quad (\mathrm{A \cdot m^2}),$$

(5.20)

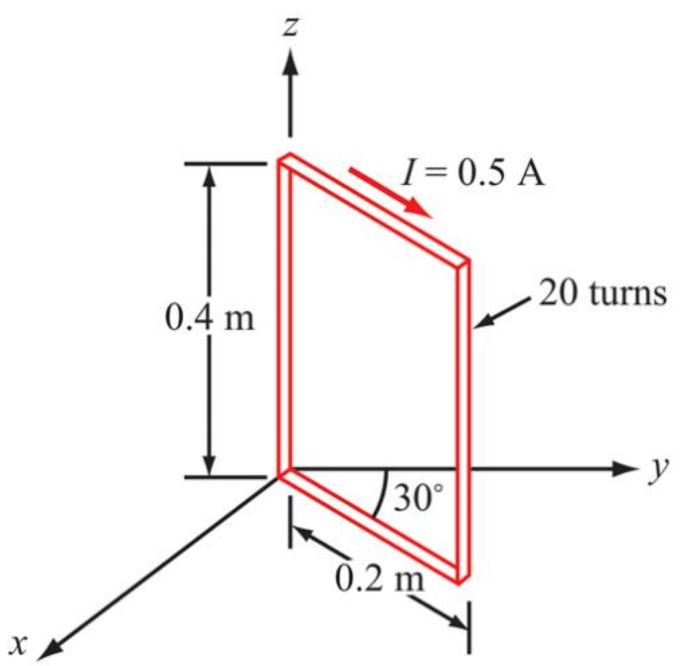
$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad (\mathrm{N \cdot m}).$$

Problem 5.4

Magnetic Moment m:

$$\boldsymbol{m} = \hat{\boldsymbol{n}}NIA = \hat{\boldsymbol{n}}m\ (A \cdot m^2)$$

where $\hat{\boldsymbol{n}}$ is the surface normal of the loop and governed by the following *right-hand rule: When the four fingers of the right-hand advance in the direction of th current* ***I****, the direction of the thumb specifies the direction of* $\hat{\boldsymbol{n}}$.



$$N = 20$$
$$I = 0.5\ A$$
$$A = l \times w = 0.2\ m \times 0.4\ m = 0.08\ m^2$$
$$\therefore m = NIA = (20)(0.5\ A)(0.08\ m^2) = 0.8\ A \cdot m^2$$

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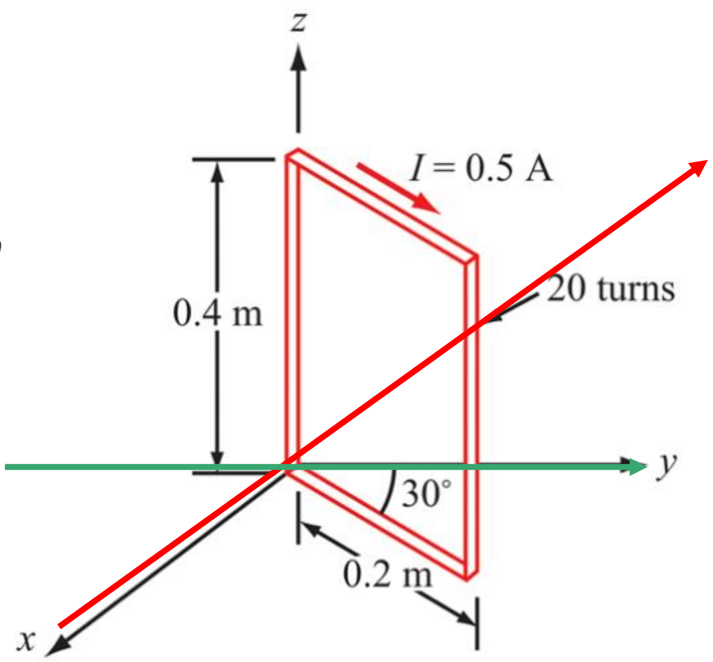

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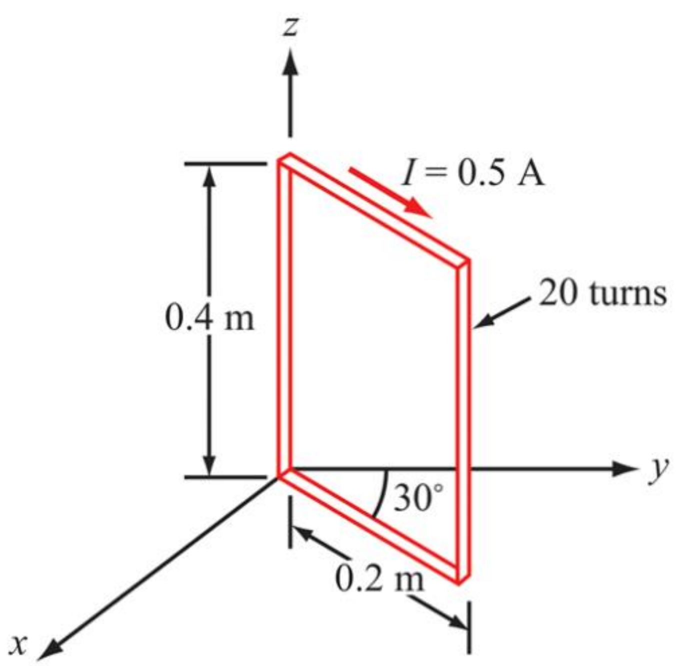

$$\hat{\mathbf{n}} = -\hat{\mathbf{x}}\cos 30° + \hat{\boldsymbol{y}}\sin 30°$$

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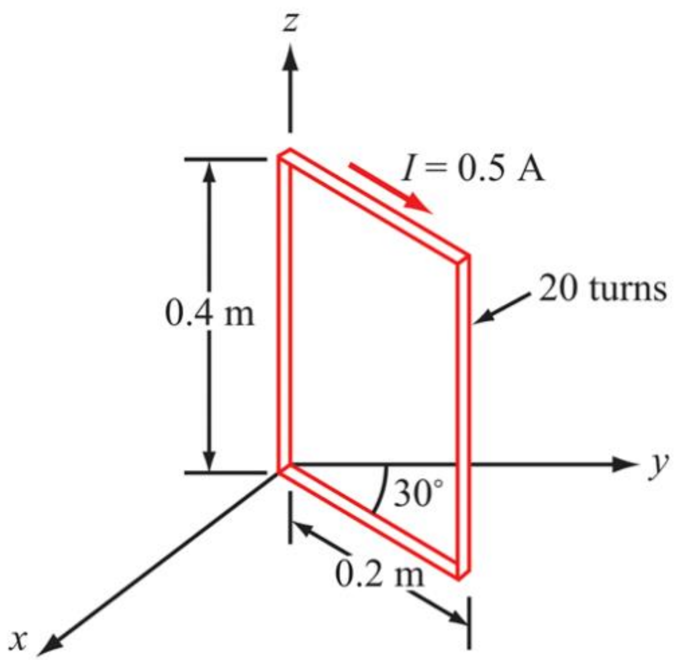


Torque is defined as:

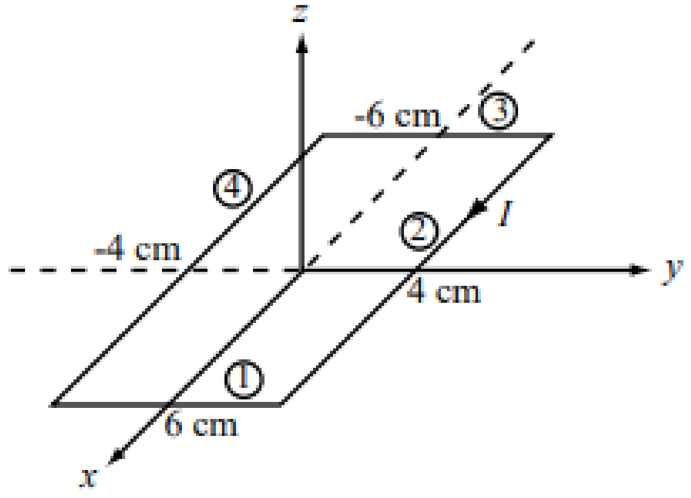
$$\mathbf{T} = \mathbf{m} \times \mathbf{B} = \hat{\mathbf{n}}m \times \mathbf{B}$$

$$= (-\hat{\mathbf{x}}\cos 30° + \hat{\boldsymbol{y}}\sin 30°)0.8 \times \hat{\boldsymbol{y}}2.4 \cong -1.66\,\hat{\boldsymbol{z}}\ N \cdot m$$

Negative torque indicates clockwise.

Problem 5.7

An 8 cm x 12 cm rectangular loop of wire is situation in the x-y plane with the center of the loop at the origin and its long sides parallel to the x-axis. The loop has a current of 50 flowing with clockwise direction (when viewed from above). Determine the magnetic field at the center of the loop.



$Biot\ Savart\ Law$: $\quad d\boldsymbol{H} = \frac{1}{4\pi}\frac{d\boldsymbol{l}\times\widehat{\boldsymbol{R}}}{R^2}\ [\frac{A}{m}]$

Where $d\boldsymbol{H} = differential\ magnetic\ field\ intensity$

$d\boldsymbol{l} = differential\ length\ vector$

$\widehat{\boldsymbol{R}} = distance\ vector\ between\ d\boldsymbol{l}\ \ and\ the\ observation\ point$

Problem 5.7

$Biot\ Savart\ Law$: $d\boldsymbol{H} = \frac{1}{4\pi}\frac{d\boldsymbol{l}\times\hat{\boldsymbol{R}}}{R^2}\left[\frac{A}{m}\right] \rightarrow \boldsymbol{H} = \int_{\boldsymbol{l}} d\boldsymbol{H}$

$\boldsymbol{B} = \mu\boldsymbol{H}$

Break conductor into 4 segments and calculate each segment's contribution to total magnetic field.

Segment 1 (blue circle):

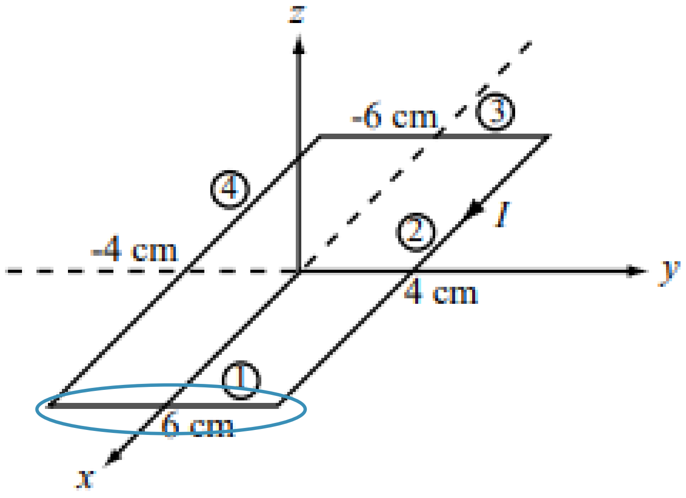
Right hand Rule gives direction of magnetic field, $\therefore$ $\boldsymbol{B_1}$ is along –z direction.

Using equation for wire of finite length.

$|\boldsymbol{B}| = \mu|\boldsymbol{H}| = \mu_0\frac{Il}{2\pi r\sqrt{4r^2+l^2}}\ [T]$

Using:

$$\mu_0 = \mu = 4\pi \times 10^{-7}\ N/A^2$$
$$I = 50\ A$$
$$r = 6\ cm$$
$$l = 8cm$$



Problem 5.7

$|\boldsymbol{B}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}}$ $[T]$

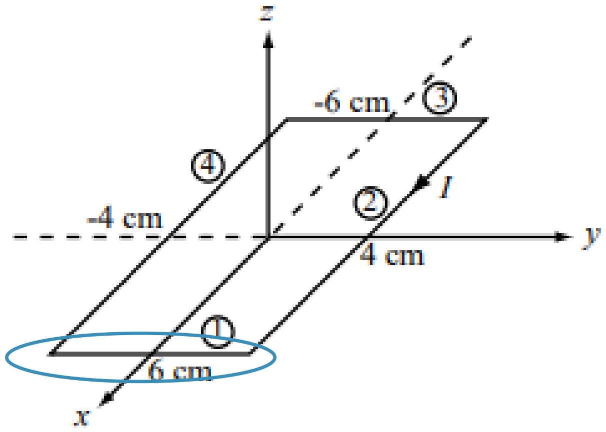
Segment 1 (blue circle):

$|\boldsymbol{B_1}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}}(-\mathbf{z})$

$$= 4\pi * 10^{-7}\ NA^{-2} \frac{50\ A\ * 0.08\ m}{2\pi(0.06\ m)\sqrt{4(0.06\ m)^2 + (0.08\ m)^2}}(-\boldsymbol{z})$$

$$= -9.24 * 10^{-5}\boldsymbol{z}\ \ [T]$$

$$\boldsymbol{B_1} = \ -9.24 * 10^{-5}\boldsymbol{z}\ [T]$$



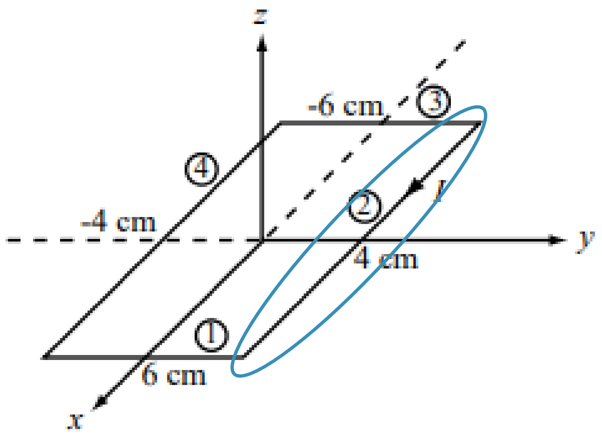
Problem 5.7

$|\boldsymbol{B}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}}\ \ [T]$

Segment 2 (blue circle):

$\boldsymbol{B_2}$ *is along* $-z$ *direction.*

$|\boldsymbol{B_2}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}}(-\mathbf{z})$



$$= 4\pi * 10^{-7}\ NA^{-2} \frac{50\ A\ * 0.12\ m}{2\pi(0.04\ m)\sqrt{4(0.04\ m)^2 + (0.12\ m)^2}}(-\boldsymbol{z})$$

$$= -20.80 * 10^{-5}\boldsymbol{z}\ \ [T]$$

$$\boldsymbol{B_2} = -20.80 * 10^{-5}\boldsymbol{z}\ [T]$$

Problem 5.7

$|\boldsymbol{B}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}} \quad [T]$

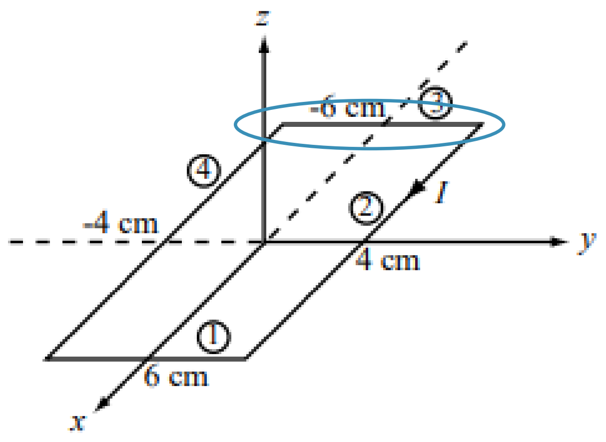
Segment 3 (blue circle):

$|\boldsymbol{B_3}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}}(-\mathbf{z})$

$$= 4\pi * 10^{-7}\ NA^{-2} \frac{50\ A\ * 0.08\ m}{2\pi(0.06\ m)\sqrt{4(0.06\ m)^2 + (0.08\ m)^2}}(-\boldsymbol{z})$$

$$= -9.24 * 10^{-5}\boldsymbol{z}\ \ [T]$$

$$\boldsymbol{B_3} = \ -9.24 * 10^{-5}\boldsymbol{z}\ [T]$$



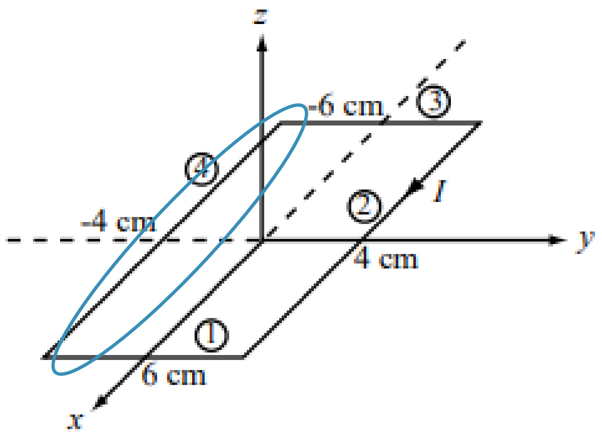
Problem 5.7

$|\boldsymbol{B}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}} \quad [T]$

Segment 4 (blue circle):

$\boldsymbol{B_4}$ *is along* $-z$ *direction.*

$|\boldsymbol{B_4}| = \mu|\boldsymbol{H}| = \mu_0 \frac{Il}{2\pi r\sqrt{4r^2+l^2}}(-\mathbf{z})$



$$= 4\pi * 10^{-7}\ NA^{-2} \frac{50\ A\ * 0.12\ m}{2\pi(0.04\ m)\sqrt{4(0.04\ m)^2 + (0.12\ m)^2}}(-\boldsymbol{z})$$

$$= -20.80 * 10^{-5}\boldsymbol{z}\ \ [T]$$

$$\boldsymbol{B_4} = -20.80 * 10^{-5}\boldsymbol{z}\ [T]$$

Problem 5.7

$\boldsymbol{B} = \boldsymbol{B_1} + \boldsymbol{B_2} + \boldsymbol{B_3} + \boldsymbol{B_4}$

$= -9.24 * 10^{-5}\boldsymbol{z} + -20.80 * 10^{-5}\boldsymbol{z} + -9.24 * 10^{-5}\boldsymbol{z} + -20.80 * 10^{-5}\boldsymbol{z}$

$\boldsymbol{B} = -0.60 * 10^{-3}\boldsymbol{z}\ [T]$