ENGPHYS 2A04 Tutorial 6

ELECTRICITY AND MAGNETISM

Your TAs Today

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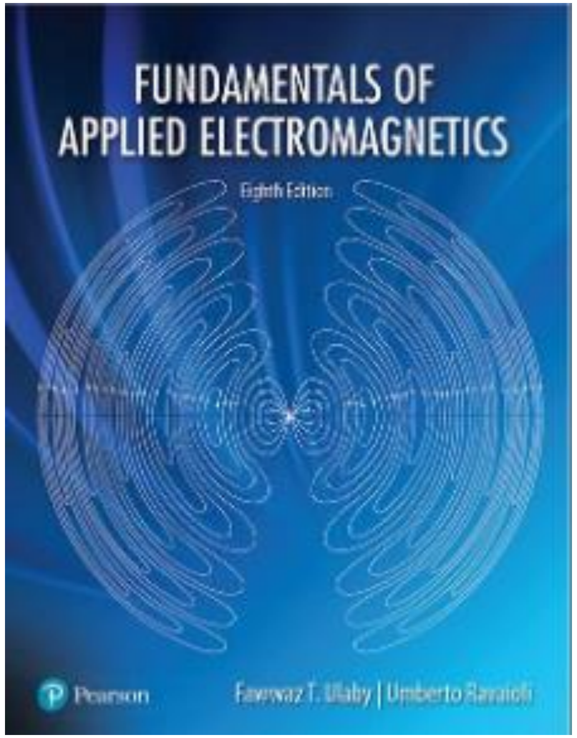
Your Textbook

Fundamentals of Applied Electromagnetics Eighth Edition

Ulaby & Ravaioli

Seventh Edition also acceptable, with some inconsistencies



Problem 4.9

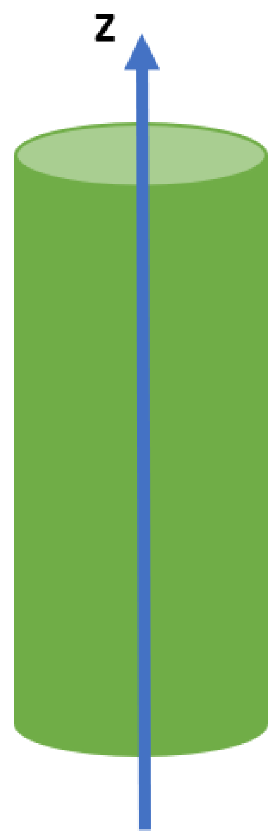
A circular beam of charge of radius *a* consists of electrons moving with a constant speed *u* along the +z direction. The beam's axis is coincident with the z-axis and the electron charge density by,

$$\rho_v = -cr^2 \ \ [c/m^3]$$

Where c is constant and *r* is the radial distance from the axis of the beam.

A) Determine the charge density per unit length.

B) Determine the current crossing the z-plane.



Problem 4.9

Solution to A)

$$\rho_v = \frac{dq}{dv} \Rightarrow \begin{matrix} dv = dl\, ds \\ \dfrac{dq}{dl} = \rho_l \end{matrix} \Rightarrow \rho_v = \frac{dq}{dv} = \frac{dq}{dl\, ds} = \frac{\rho_l}{ds}$$

$$\rho_l = \rho_v ds \to \rho_l = \iint \rho_v ds \Rightarrow ds = r dr d\emptyset \quad \text{(polar coordinates)}$$

$$\Rightarrow \quad \rho_l = \int_{r=0}^{a} \int_{\emptyset=0}^{2\pi} -cr^2 * r\, dr\, d\emptyset$$

Problem 4.9

Solution to A)

$$\rho_l = \int_{r=0}^{a}\int_{\emptyset=0}^{2\pi} -cr^2 * r \, dr \, d\emptyset = -2\pi c \frac{r^4}{4}$$

$$\rho_l = \frac{-\pi c a^4}{2} \left[\frac{C}{m}\right]$$

Problem 4.9

Solution to B)

$$\boldsymbol{I = \frac{charge}{time} = \left[\frac{Coulomb}{Second}\right] = [A]}$$

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{s} = \int_{r=0}^{a} \int_{\emptyset=0}^{2\pi} (-cur^2 \hat{\mathbf{z}}) \cdot (\hat{\mathbf{z}}) r\, dr\, d\emptyset \qquad \text{as } \boldsymbol{J} = \rho_v \boldsymbol{u} = -cr^2 \cdot u\, \hat{\mathbf{z}}$$

$$I = -2\pi cu \int_{r=0}^{a} r^3\, dr = -\frac{\pi c a^4 u}{2} = \rho_l u$$

$$Units\ for\ \rho_l u \to \left[\frac{C}{m}\right]\left[\frac{m}{s}\right] = \left[\frac{C}{s}\right] = [A]$$

Problem 4.15

Electric charge is distributed along an arc located in the x-y plane and defined by $r = 2\ cm$ and $0 \leq \emptyset \leq \frac{\pi}{4}$

If $\rho_l = 5\frac{\mu C}{m}$, find $\boldsymbol{E}$ at $(0,0,z)$ and then evaluate it at:

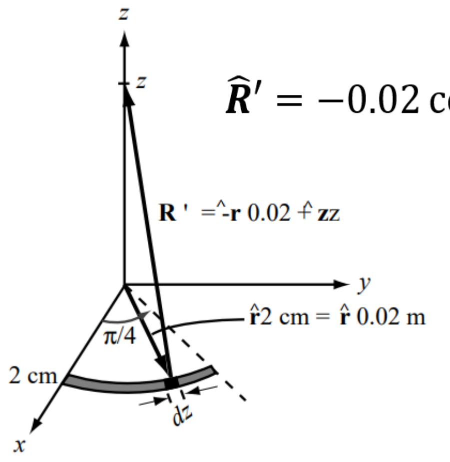
A) The origin

B) z = 5 cm

C) z = -5 cm

$$\widehat{\boldsymbol{R}}' = -0.02\cos\emptyset\hat{\boldsymbol{x}} - 0.02\sin\emptyset\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$$

$$\boldsymbol{R}' = -0.02\hat{\boldsymbol{r}} + z\hat{\boldsymbol{z}}$$



$$dl' = r\,d\emptyset = 0.02\,d\emptyset$$

Figure P4.13: Line charge along an arc.

Problem 4.15

Solution:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int_{l'} \widehat{\boldsymbol{R}}' \frac{\rho_l dl'}{R'^2} = \frac{1}{4\pi\varepsilon_0}\int_{\emptyset=0}^{\frac{\pi}{4}} \rho_l \frac{-0.02\cos\emptyset\hat{\boldsymbol{x}} - 0.02\sin\emptyset\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}}{((0.02)^2 + (z^2)^{3/2}} 0.02 d\phi$$

$$\Rightarrow \boldsymbol{E} = \frac{898.8}{((0.02)^2 + (z^2)^{3/2}}[-0.014\hat{\boldsymbol{x}} - 0.006\hat{\boldsymbol{y}} + 0.78z\hat{\boldsymbol{z}}]\left[\frac{V}{m}\right]$$

Problem 4.15

Solution

A) At origin (z = 0)

$$\boldsymbol{E} = [-1.6\hat{\boldsymbol{x}} - 0.66\hat{\boldsymbol{y}}]\ \left[\frac{MV}{m}\right]$$

B) z = 5 cm

$$\boldsymbol{E} = [-81.4\hat{\boldsymbol{x}} - 33.7\hat{\boldsymbol{y}} + 226\hat{\boldsymbol{z}}]\ \left[\frac{kV}{m}\right]$$

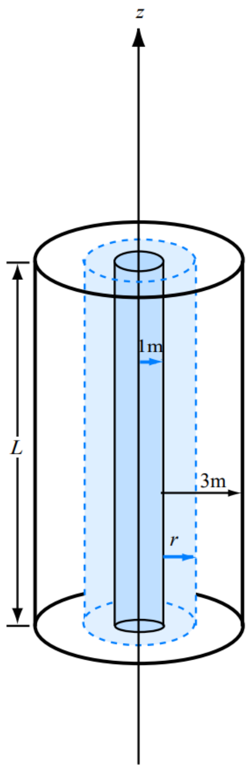
C) z = -5 cm

$$\boldsymbol{E} = [-81.4\hat{\boldsymbol{x}} - 33.7\hat{\boldsymbol{y}} - 226\hat{\boldsymbol{z}}]\ \left[\frac{kV}{m}\right]$$

Problem 4.27

An infinitely long cylindrical shell extending between r = 1 m and r = 3 m contains a uniform charge density ρ_{v_0}

Apply Gauss' law to find ***D*** in all regions.



Gauss' Law:

$$\oint_S (D_r \hat{\boldsymbol{r}}) \cdot d\boldsymbol{s} = Q$$

Ignore axial direction

Out of symmetry

$$D_r \cdot 2\pi r = Q$$

Problem 4.27

Solution:

$D_r \cdot 2\pi r = Q$

For r $< 1\, m, Q = 0, \boldsymbol{D} = 0$

For $1 \leq$ r ≤ 3 m

$Q = \rho_{v_0} \cdot \pi L(r^2 - 1)$

$D_r \cdot 2\pi r L = \rho_{v_0} \cdot \pi L(r^2 - 1)$

$$\boldsymbol{D} = D_r \hat{\boldsymbol{r}} = \frac{\rho_{v_0} \cdot \pi(r^2 - 1)}{2\pi r} \hat{\boldsymbol{r}} \quad \left[\frac{C}{m^2}\right]$$

Problem 4.27

Solution:

For r ≥ 3 m,

$$Q = \rho_{v_0} \cdot \pi(3^2 - 1^2) = 8\rho_{v_0} \cdot \pi$$

$$D_r \cdot 2\pi r L = 8\rho_{v_0} \cdot \pi L$$

$$\boldsymbol{D} = D_r \hat{\boldsymbol{r}} = \frac{4\rho_{v_0}}{r} \hat{\boldsymbol{r}} \quad \left[\frac{C}{m^2}\right]$$