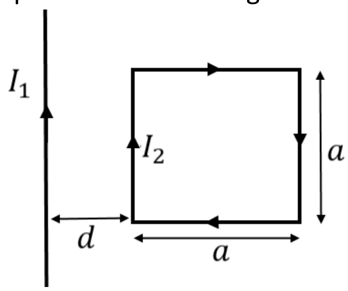
ENGPHYS 2A04 Winter 2022 – Assignment 10

DUE MONDAY APRIL 4th, 8AM

1. The wire on the left carries a current I_1, while the loop on the right carries a current of I_2. The loop is positioned a distance d from the wire, and has dimensions $a \times a$. Find a simplified expression for the magnetic force acting on the loop – and remember, this is a vector quantity.



Solution

The magnetic field strength caused by the wire is given by

$$\boldsymbol{B} = \widehat{\boldsymbol{\Phi}} \frac{\mu_0 I_1}{2\pi r}$$

Or, in the plane of the loop:

$$\boldsymbol{B} = \widehat{\boldsymbol{y}} \frac{\mu_0 I_1}{2\pi x}$$

The only sections of the loop that will experience a force are the sections parallel to the wire. For the closer segment:

$$\boldsymbol{F_{left}} = I_2 \boldsymbol{l} \times \boldsymbol{B}(x) = I_2(\widehat{\boldsymbol{z}}a) \times \left(\widehat{\boldsymbol{y}} \frac{\mu_0 I_1}{2\pi x}\right) = -\widehat{\boldsymbol{x}} \frac{\mu_0 I_1 I_2 a}{2\pi d}$$

$$\boldsymbol{F_{right}} = -I_2 \boldsymbol{l} \times \boldsymbol{B}(x) = -I_2(\widehat{\boldsymbol{z}}a) \times \left(\widehat{\boldsymbol{y}} \frac{\mu_0 I_1}{2\pi x}\right) = \widehat{\boldsymbol{x}} \frac{\mu_0 I_1 I_2 a}{2\pi (d+a)}$$

The total force acting on the loop:

$$\boldsymbol{F} = \boldsymbol{F_{left}} + \boldsymbol{F_{right}} = -\widehat{\boldsymbol{x}} \frac{\mu_0 I_1 I_2 a}{2\pi d} + \widehat{\boldsymbol{x}} \frac{\mu_0 I_1 I_2 a}{2\pi (d+a)}$$

$$\boldsymbol{F} = \widehat{\boldsymbol{x}} \frac{\mu_0 I_1 I_2 a}{2\pi} \left(\frac{1}{d+a} - \frac{1}{d}\right) = -\widehat{\boldsymbol{x}} \frac{\mu_0 I_1 I_2 a^2}{2\pi d(d+a)}$$

2. A cylindrical conductor, oriented along the *z*-axis, with a radius of a carries a current density of $\widehat{\boldsymbol{z}} J_0 e^{-kr}$. Calculate the magnetic field as a function of radial distances for distances
   a. Inside the conductor
   b. Outside the conductor

Solution

a) For $r \leq a$, Ampere's Law is

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I = \int_S \boldsymbol{J} \cdot d\boldsymbol{s}$$

$$\widehat{\boldsymbol{\varphi}} H \cdot \widehat{\boldsymbol{\varphi}} 2\pi r = \int_0^r \boldsymbol{J} \cdot d\boldsymbol{s} = \int_0^r \widehat{\boldsymbol{z}} J_0 e^{-kr'} \cdot \widehat{\boldsymbol{z}} 2\pi r' dr'$$

$$2\pi r H = 2\pi J_0 \int_0^r r' e^{-kr'} dr'$$

$$= 2\pi J_0 \left(-\frac{(kr'+1)e^{-kr'}}{k^2} \right) \Big|_{r'=0}^{r'=r}$$

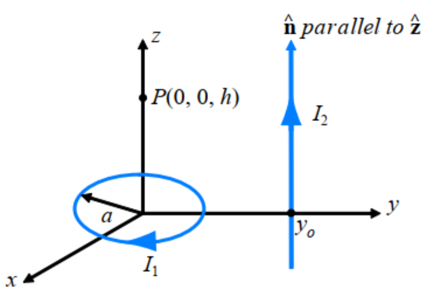
$$= 2\pi J_0 \left(-\frac{(kr+1)e^{-kr}}{k^2} + \frac{e^{-kr}}{k^2} \right)$$

$$\therefore H = \frac{J_0}{r} \left(-\frac{(kr+1)e^{-kr}}{k^2} + \frac{e^{-kr}}{k^2} \right)$$

b) The same logic, but replacing *r* with *a* in Ampere's Law:

$$H = \frac{J_0}{r} \left(-\frac{(ka+1)e^{-ka}}{k^2} + \frac{e^{-ka}}{k^2} \right)$$

3. The loop centered at the origin in the figure below has a radius of 5cm, lies in the *x*-*y* plane, and carries a current of $I_1 = 8A$. A straight wire parallel to *z* intersects the point $y = 12cm$, and carries a current of $I_2 = 6A$. Calculate the magnetic field at the point P(0, 0, 10cm).



(a) The magnetic field at $P(0,0,h)$ is composed of $\mathbf{H}_1$ due to the loop and $\mathbf{H}_2$ due to the wire:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2.$$

From (5.34), with $z = h$,

$$\mathbf{H}_1 = \hat{\mathbf{z}} \frac{I_1 a^2}{2(a^2+h^2)^{3/2}} \quad \text{(A/m)}.$$

From (5.30), the field due to the wire at a distance $r = y_0$ is

$$\mathbf{H}_2 = \hat{\boldsymbol{\phi}} \frac{I_2}{2\pi y_0}$$

where $\hat{\boldsymbol{\phi}}$ is defined with respect to the coordinate system of the wire. Point *P* is located at an angel $\phi = -90°$ with respect to the wire coordinates. From Table 3-2,

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$$
$$= \hat{\mathbf{x}} \qquad (\text{at } \phi = -90°).$$

Hence,

$$\mathbf{H} = \hat{\mathbf{z}} \frac{I_1 a^2}{2(a^2+h^2)^{3/2}} + \hat{\mathbf{x}} \frac{I_2}{2\pi y_0} \quad \text{(A/m)}.$$

$$\boldsymbol{H} = \hat{\boldsymbol{z}} \frac{8(0.05^2)}{2(0.05^2+0.1^2)^{3/2}} + \hat{\boldsymbol{x}} \frac{6}{2\pi(0.12)}$$

$$\boldsymbol{H} = \hat{\boldsymbol{z}} 7.16 + \hat{\boldsymbol{x}} 7.96 \ \ A/m$$

4. Consider a 5-meter long section of a coaxial transmission line, with an inner conductor radius of 3cm and an outer conductor inner radius of 8 cm. If the insulator is air and the line is carrying a DC current of 12A, how much magnetic energy is stored in the insulating medium?

   Solution

   The inductance per unit length of an air-filled coaxial line is

   $$L' = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

   Total inductance is $L = lL' = \frac{l\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{5\mu_0}{2\pi} \ln\left(\frac{0.08}{0.03}\right) = 981 \times 10^{-9}\ H$

   $$W = \frac{LI^2}{2} = \frac{(981 \times 10^{-9})(12)^2}{2} = 70.6\ \mu J$$

   Could arrive at the same conclusion using

   $$W_{\mathrm{m}} = \frac{1}{2} \int_{\mathcal{V}} \mu_0 H^2\, d\mathcal{V}$$

5. The ‘technology brief’ in the textbook mentions several applications of electromagnets: magnetic relays, doorbells, loudspeakers and maglev trains. Research one of these topics, and find an academic source that discusses a challenge in this application that is related to the theory discussed in class. In 5 sentences or fewer, explain what the challenge is, how it is related to this week’s material, and what some potential solutions are. Cite your source using a recognized citation format.

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal points.
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.