EP2A04 TUTORIAL 4

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TEACHING ASSISTANTS

- Fraser McCauley, mccaulj@mcmaster.ca
- Muhammad Munir, munirm6@mcmaster.ca

QUESTION 2.5

For the parallel-plate transmission line of Problem 2.4, the line parameters are given by $R' = 1\ \Omega/m, L' = 167\ nH/m, G' = 0$ and $C' = 172\ pF/m$. Find α, β, u_P and Z_0 at $1\ GHz$.

Givens:

$$R' = 1\ \Omega/m$$
$$L' = 167\ nH/m$$
$$G' = 0$$
$$C' = 172\ pF/m$$
$$f = 1\ GHz$$

Plan:

- Find $\gamma = \alpha + j\beta$
- Isolate real and imaginary components
- Use β and ω to obtain u_P
- Find Z_0 using the transmission line parameters

Unknowns: α, β, u_P, Z_0

Find $\gamma = \alpha + j\beta$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \sqrt{\left(1 + j(2\pi \times 10^9)(167 \times 10^{-9})\right)\left(j(2\pi \times 10^9)(172 \times 10^{-12})\right)}$$

$$\gamma = 0.016 + j33.7$$

QUESTION 2.5

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Isolate real and imaginary components

$$\alpha = \Re e\{\gamma\} = \Re e\{0.016 + j33.7\} = 0.016\frac{Np}{m}$$

$$\beta = \Im m\{\gamma\} = \Im m\{0.016 + j33.7\} = 33.7\frac{rad}{m}$$

Use β and ω to obtain u_P

$$u_P = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi(1\times 10^9)}{33.7} = 1.86\times 10^8\ m/s$$

Find Z_0 using the transmission line parameters

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$= \sqrt{\frac{1 + j(2\pi\times 10^9)(167\times 10^{-9})}{j(2\pi\times 10^9)(172\times 10^{-12})}}$$

$$= \sqrt{\frac{1 + j1049.3}{j1.081}} = \sqrt{970.7 - j0.9251}$$

$$= \sqrt{970.7e^{-j9.53\times 10^{-4}}} = 31.2e^{-j4.765\times 10^{-4}}$$

$$= (31.2 - j0.015)\ \Omega$$

QUESTION 2.6

A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\epsilon_r = 4.5$ and $\sigma = 10^{-3}\ S/m$. The conductors are made of copper. Calculate the line parameters at 1 GHz.

Givens:

$d_{inner} = 0.5\ cm$

$d_{outer} = 1\ cm$

$\epsilon_r = 4.5$

$\sigma = 10^{-3}\ \frac{S}{m}$

$f = 1\ GHz$

Hidden givens:

Copper parameters!

$\sigma_{Cu} = 5.8 \times 10^7\ S/m$

$\mu_{r-Cu} = 1$

Unknowns: R', L', G', C'

Plan:

- Compute R_S
- Use the coaxial line formulae to compute the line parameters

Compute R_S

$$R_S = \sqrt{\frac{\pi f \mu_C}{\sigma_C}} = \sqrt{\pi(1 \times 10^9)(4\pi \times 10^{-7})/(5.8 \times 10^7)}$$

$$= 8.2502 \times 10^{-3}$$

QUESTION 2.6

A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\epsilon_r = 4.5$ and $\sigma = 10^{-3}\ S/m$. The conductors are made of copper. Calculate the line parameters at 1 GHz.

Use the coaxial line formulae to compute the line parameters

$$R' = \frac{R_S}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{R_S}{2\pi}\left(\frac{2}{d_{inner}} + \frac{2}{d_{outer}}\right) = \frac{8.2502\times10^{-3}}{2\pi}\left(\frac{2}{0.5\times10^{-2}} + \frac{2}{1\times10^{-2}}\right) = 0.788\ \Omega/m$$

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{\mu}{2\pi}\ln\left(\frac{d_{outer}}{d_{inner}}\right) = \frac{(4\pi\times10^{-7})}{2\pi}\ln\left(\frac{1}{0.5}\right) = 1.39\times10^{-7}\ H/m$$

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi\times10^{-3}}{\ln(1/0.5)} = 9.06\times10^{-3}\ S/m$$

$$C' = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{2\pi\epsilon_r\epsilon_0}{\ln(b/a)} = \frac{2\pi(4.5)(8.854\times10^{-12})}{\ln(1/0.5)} = 3.61\times10^{-10}\ F/m$$

QUESTION 2.18

Polyethylene with $\epsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with a characteristic impedance of 50Ω. The radius of the inner conductor is $1.2\ mm$. (A) What is the radius of the outer conductor? (B) What is the phase velocity of the line?

Plan:
- Write characteristic impedance equation using line parameters
- Solve for b
- Compute the phase velocity

Givens:

$$\epsilon_r = 2.25$$
$$Z_0 = 50\ \Omega$$
$$a = 1.2\ mm$$

Hidden givens:

Lossless! So…

$$R' = G' = 0$$

Write characteristic impedance equation using line parameters

Since R' and G' are 0, the equation simplifies to:

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)}{\left(\frac{2\pi\epsilon}{\ln(b/a)}\right)}} = \sqrt{\frac{\mu}{4\pi^2\epsilon}\left(\ln\left(\frac{b}{a}\right)\right)^2} = \sqrt{\frac{\mu}{4\pi^2\epsilon}}\ln\left(\frac{b}{a}\right)$$

QUESTION 2.18

Polyethylene with $\epsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with a characteristic impedance of 50Ω. The radius of the inner conductor is $1.2\ mm$. (A) What is the radius of the outer conductor? (B) What is the phase velocity of the line?

Solve for b

$$Z_0 = \sqrt{\frac{\mu}{4\pi^2\epsilon}} \ln\left(\frac{b}{a}\right)$$

$$\frac{Z_0}{\sqrt{\frac{\mu}{4\pi^2\epsilon}}} = \ln\left(\frac{b}{a}\right)$$

$$b = a * \exp\left(\frac{Z_0}{\sqrt{\frac{\mu}{4\pi^2\epsilon}}}\right)$$

$$b = (1.2 \times 10^{-3}) * \exp\left(\frac{50}{\sqrt{\frac{4\pi \times 10^{-7}}{4\pi^2(2.25)(8.854 \times 10^{-12})}}}\right)$$

$$b = (1.2 \times 10^{-3}) * \exp\left(\frac{50}{\sqrt{\frac{10^5}{\pi(2.25)(8.854)}}}\right)$$

$$b = 4.2\ mm$$

Compute the phase velocity

Lossless line, so…

$$u_P = \frac{c}{\sqrt{\epsilon_r}}$$

$$u_P = \frac{3 \times 10^8}{\sqrt{2.25}}$$

$$u_P = 2 \times 10^8\ m/s$$

QUESTION 2.27

At an operating frequency of $300\ MHz$, a lossless $50\ \Omega$ air-spaced transmission line $2.5\ m$ in length is terminated with an impedance $Z_L = (40 + j20)\ \Omega$. Find the input impedance.

Givens

$f = 300\ MHz$

$Z_0 = 50\ \Omega$

$l = 2.5\ m$

$Z_L = (40 + j20)\ \Omega$

Unknowns: Z_i

Plan:

- Calculate the phase constant of the line
- Plug into the input impedance formula

Calculate the phase constant of the line

Since the line is air filled, $u_P = c$. *Therefore…*

$$\beta = \frac{\omega}{c} = \frac{2\pi(300 \times 10^6)}{3 \times 10^8} = 2\pi\ rad/s$$

Plug into the input impedance formula

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

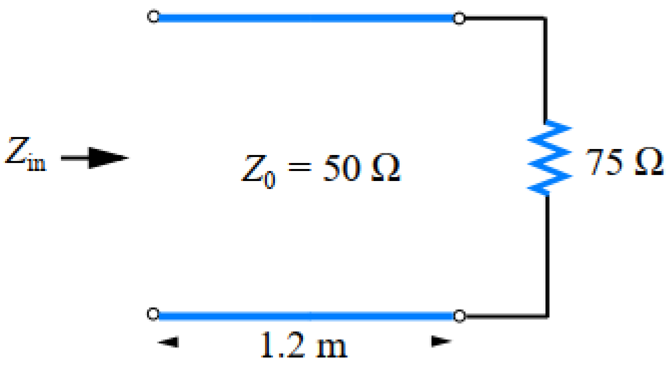
$$Z_i = 50 \frac{40 + j20 + j(50)\tan\big(2\pi(2.5)\big)}{50 + j(40 + j20)\tan\big(2\pi(2.5)\big)}$$

$$Z_i = 50 \frac{40 + j20}{50}$$

$$Z_i = (40 + j20)\ \Omega$$

QUESTION 2.35

For the lossless transmission line circuit shown in the figure, determine the equivalent series lumped-element circuit at $400\ MHz$ at the input to the line. The line has a characteristic impedance of $50\ \Omega$ and the insulating layer has $\epsilon_r = 2.25$.



Givens

$f = 400\ MHz$

$Z_0 = 50\ \Omega$

$l = 1.2\ m$

$Z_L = 75\ \Omega$

$\epsilon_r = 2.25$

Unknowns: Z_i

Plan:

- Compute phase constant of line
- Compute equivalent input impedance
- Design equivalent circuit based on impedance

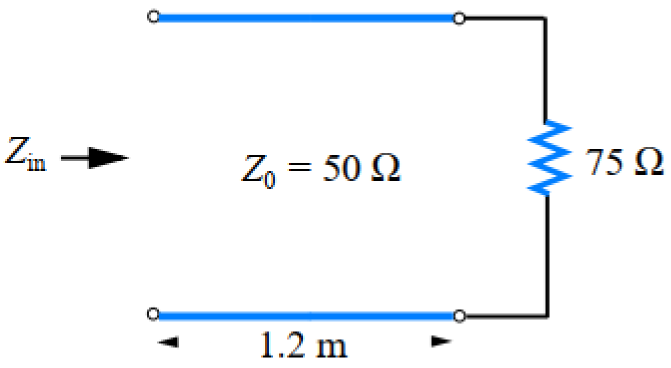
Compute phase constant of line

Lossless line, so $u_P = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{2.25}} = 2 \times 10^8\ m/s$

$$\beta = \frac{\omega}{u_P} = \frac{2\pi(400 \times 10^6)}{2 \times 10^8} = 4\pi\ rad/m$$

QUESTION 2.35

For the lossless transmission line circuit shown in the figure, determine the equivalent series lumped-element circuit at $400\ MHz$ at the input to the line. The line has a characteristic impedance of $50\ \Omega$ and the insulating layer has $\epsilon_r = 2.25$.



Compute equivalent input impedance

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

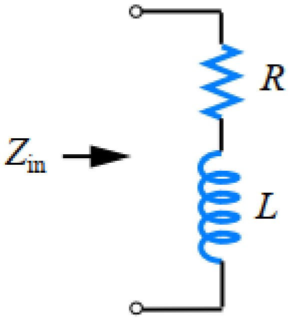
$$Z_i = 50 \frac{75 + j(50)\tan\big((4\pi)(1.2)\big)}{50 + j(75)\tan\big((4\pi)(1.2)\big)}$$

$$Z_i = 50 \frac{75 - j36.327}{50 - j54.491}$$

$$Z_i = (52.38 + j20.76)\ \Omega$$

Design equivalent circuit based on impedance

Imaginary part is positive, so if this is a series circuit, it must be a resistor and an inductor!



$$R = 52.38\ \Omega$$

$$L = \frac{20.76}{\omega} = 8.3\ nH$$

REMINDER

- Assignment 4 is out now, and is due at 8AM on February 14
- 4 calculation questions, 1 research question, 1 bonus derivation question
- Submit PDFs please!
- Good luck!