A3

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Q1) $\int_0^{\pi/2} e^{2x}\sin(x)\,dx$

a)

$$E_{trap} = -\frac{f''(\mu)}{12}(b-a)h^2$$

$a = \varnothing, \quad b = \pi/2$

$h = (\pi/2)/n$

$$E_{trap} = -\frac{1}{12}f''(\mu)(b-a)\left(\frac{b-a}{n}\right)^2$$

$$= -\frac{1}{12}f''(\mu)(b-a)^3/n^2$$

$$= -\frac{1}{12}f''(\mu)(\pi/2)^3/n^2$$

$$f''(x) = 3e^{2x}\sin(x) + 4e^{2x}\cos(x)$$

$$\max_{\varnothing \le x \le \pi/2} |f''(x)| = |f''(\pi/2)| = 3e^{\pi}$$

$$tol \ge -\frac{1}{12}f''(\mu)\left(\frac{\pi}{2}\right)^3/n^2$$

$$n^2 \ge \frac{1}{12}(-f''(\mu))\left(\frac{\pi}{2}\right)^3/tol$$

to maximize n, maximize $|-f''(\mu)|$

$$n \ge \sqrt{\frac{\max|-f''(\mu)|\left(\frac{\pi}{2}\right)^3}{12\cdot tol}}$$

$$n \geq \sqrt{\frac{\quad}{12 \cdot tol}}$$

$$n \geq \sqrt{\frac{|f''(\pi/2)|\left(\frac{\pi}{2}\right)^3}{12 \cdot tol}}$$

$$n \geq \sqrt{\frac{e^{\pi}(\pi/2)^3}{4 \cdot tol}}$$

$$n \geq \sqrt{\frac{e^{\pi}\pi^3}{32 \cdot tol}}$$

b)

$$f^{(4)}(x) = -7e^{2x}\sin(x) + 24e^{2x}\cos(x)$$

$$\max_{0 \leq x \leq \pi/2} |f^{(4)}(x)| = |f^{(4)}(\pi/2)| = 7e^{\pi}$$

$$E = -\frac{1}{180} f^{(4)}(\xi)(b-a)h^4$$

$$tol \geq -\frac{1}{180} f^{(4)}(\xi)(b-a)h^4$$

$$tol \geq -\frac{1}{180} f^{(4)}(\xi)(\pi/2)^5/n^4$$

$$n^4 \geq \frac{1}{180}\left(-f^{(4)}(\xi)\right)(\pi/2)^5/tol$$

to $\max(n) \rightarrow \max|-f^{(4)}(\xi)|$

$$n \geq \left[\frac{\max|-f^{(4)}(\xi)|\left(\frac{\pi}{2}\right)^5}{180 \cdot tol}\right]^{\frac{1}{4}}$$

$$\left[\quad 180 \cdot tol \quad \right]$$

$$n \geq \left[\frac{7e^{\pi}\left(\frac{\pi}{2}\right)^5}{180 \cdot tol}\right]^{\frac{1}{4}}$$

$$n \geq \left[\frac{7e^{\pi}\pi^5}{5760 \cdot tol}\right]^{1/4}$$

c) $\max_{0 \leq x \leq \pi/2} |f''(x)| = |f''(\pi/2)| = 3e^{\pi}$

$$E = \frac{1}{24} f''(\xi)(b-a)h^2$$

$$= \frac{1}{24} f''(\xi)(b-a)^3/n^2$$

$$tol \geq \frac{1}{24} f''(\xi)(\pi/2)^3/n^2$$

$$n \geq \sqrt{\frac{f''(\xi)(\pi/2)^3}{24 \cdot tol}}$$

$$n \geq \sqrt{\frac{\max|f''(\xi)|(\pi/2)^3}{24 \cdot tol}}$$

$$n \geq \sqrt{\frac{e^{\pi}\left(\frac{\pi}{2}\right)^3}{8 \cdot tol}}$$

$$n \geq \sqrt{\frac{e^{\pi}\pi^3}{64 \cdot tol}}$$

$$n \geq \sqrt{\frac{e^{\pi}\pi^{3}}{64 \cdot tol}}$$

```
function error(tol)
    %trapezoidal
    n_trap = ceil(((exp(pi)*pi^3)/(32*tol))^(1/2));
    E_trap = (exp(pi)*pi^3)/(32*n_trap^2);

    %simpson
    n_s = ceil(((7*exp(pi)*pi^5)/(5760*tol))^(1/4));
    if(mod(n_s, 2)~=0)
        n_s = n_s + 1;
    end
    E_s = (7*exp(pi)*pi^5)/(5760*n_s^4);

    %midpoint
    n_mid = ceil(((exp(pi)*pi^3)/(64*tol))^(1/2));
    E_mid = (exp(pi)*pi^3)/(64*n_mid^2);

    fprintf("tol = %d\n", tol)
    fprintf("trapezoid n= %i, error=%d\n", n_trap, E_trap)
    fprintf("midpoint  n= %i, error=%d\n", n_mid, E_mid)
    fprintf("simpson   n= %i, error=%d\n", n_s, E_s)

end
```

```
>> errors(1e-4)
tol = 1.000000e-04
trapezoid n= 474, error=9.979742e-05
midpoint  n= 335, error=9.989791e-05
simpson   n= 18, error=8.198063e-05
>> errors(1e-12)
tol = 1.000000e-12
trapezoid n= 4735197, error=9.999997e-13
midpoint  n= 3348290, error=9.999997e-13
simpson   n= 1714, error=9.971443e-13
>>
```

Q 2)

$$t = \int_{v(t_0)}^{v(t)} \frac{m}{R(u)}\, du \quad , \; R(v) = -v\sqrt{v}$$

$m = 10\text{ kg}, \qquad V_0 = 10 \qquad v = 5$

$$t = \int_{10}^{5} \frac{10}{-u\sqrt{u}}\, du$$

$$= \int_{5}^{10} 10\, u^{-3/2}\, du$$

$a = 5 \quad b = 10$

$$t_{simpson} = \frac{10-5}{6}\left[f(5) + 4f\left(\frac{5+10}{2}\right) + f(10)\right]$$

$$f(5) = 10 \times 5^{-3/2} = 0.8944272$$
$$f(10) = 10 \times 10^{-3/2} = 0.3162277$$
$$f(7.5) = 10 \times 7.5^{-3/2} = 0.4868645$$

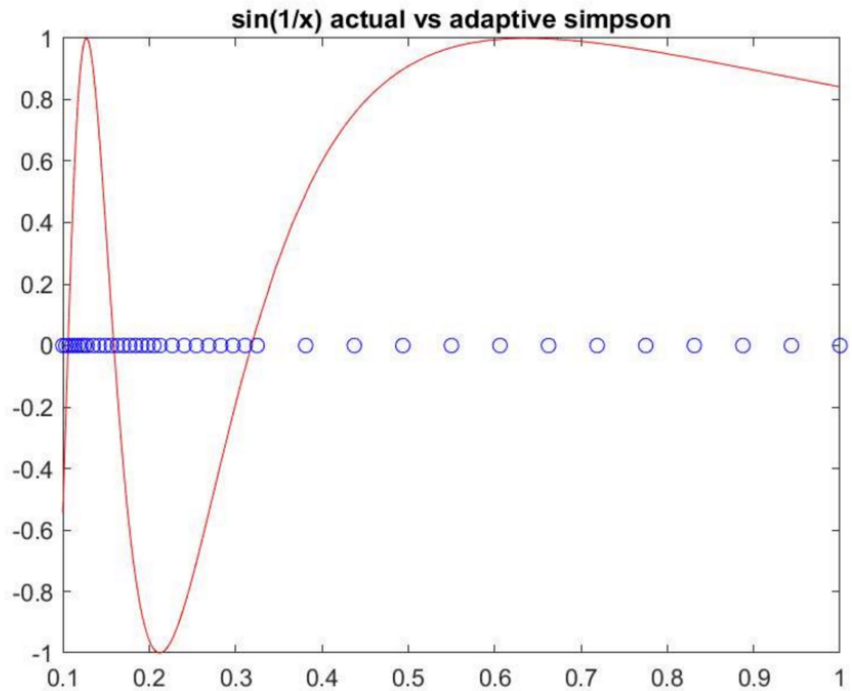
$$t_s = \frac{5}{6}\left[0.8944272 + 4 \times 0.3162277 + 0.4868645\right]$$

$$t_s = 2.2051687\,5$$

$$t_s \doteq 2.20517$$

Q3)

```
>> main_error_trapz
c=4.079766e+03   k=1.99815
c=6.131198e-02   k=2
```



Q4)

$$E = ch^k$$

$$\ln(E) = \ln(ch^k)$$

$$\ln(E) = \ln(c) + k\ln(h)$$

solve using linear least squares

```
function [c, k] = error_trapz(f, a, b)

    F = integral(f, a, b);

    points = 10000;

    [h, ln, E, E_ln] = deal(zeros(points, 1));

    for i=1:points
        h(i) = (b-a)/(i*10);
        ln(i) = log(h(i));
    end

    for i=1:points
        x = a:h(i):b;
        t = trapz(x, f(x));
        E(i) = abs(F-t);
```

```
        x = a:h(i):b;
        t = trapz(x, f(x));
        E(i) = abs(F-t);
        E_ln(i) = log(E(i));
    end

    A = [sum(ln.^2) sum(ln); sum(ln) points];
    B = [sum(ln.*E_ln); sum(E_ln)];

    y = A\B;
    k = y(1);
    c = exp(y(2));

end
```

```
>> main_error_trapz
c=4.079766e+03  k=1.99815
c=6.131198e-02  k=2
```

Q5)

a)

```
pts = 1:1:10;
```

$\leftarrow n$

```
>> timeadd
k_row= 6.619813e-01  c_row= 6.459380e-06
k_col= 5.356396e-01  c_col= 8.370112e-06
```

b)

```
>> timeadd
k_row= 3.123068e+00  c_row= 1.240866e-10
k_col= 1.938302e+00  c_col= 2.321119e-08
```

```matlab
function [k_row, c_row, k_col, c_col] = timeadder()
    pts = 1000:500:5000;
    ln = log(pts);
    points = length(pts);

    [tR, tC] = deal(zeros(points, 1));

    for i=1:points
        a = rand(pts(i));
        b = rand(pts(i));
        tR(i) = timef(@addR, a, b);
        tC(i) = timef(@addC, a, b);
    end

    A1 = [sum(ln.^2) sum(ln); sum(ln) points];
    b1 = [sum(ln*log(tR)); sum(log(tR))];

    A2 = [sum(ln.^2) sum(ln); sum(ln) points];
    b2 = [sum(ln*log(tC)); sum(log(tC))];

    y1 = A1\b1;
    y2 = A2\b2;

    k_row = y1(1);
    c_row = exp(y1(2));

    k_col = y2(1);
    c_col = exp(y2(2));

    fprintf("k_row= %d  c_row= %d\n", k_row, c_row)
    fprintf("k_col= %d  c_col= %d\n", k_col, c_col)
end
```

Q6) $r_k = a x_k + b - y_k$

$b \simeq \emptyset$

$F(l) = k(l - l_0)$

$l_0 = 5.3$

$r_k = k(l_k - l_0) - F(l_k)$

$\Phi = \sum r_k^2 = \sum (k(l_k - l_0) - F(l_k))^2$

$$\frac{\partial \Phi}{\partial k} = \emptyset$$

$$\emptyset = \sum_{k=\emptyset}^{m} (k(l_k - l_0) - f(l_k))(l_k - l_0)$$

$$\emptyset = \sum \left[k(l_k - l_0)^2 - f(l_k)(l_k - l_0)\right]$$

$$\sum f(l_k)(l_k - l_\emptyset) = \sum k(l_k - l_0)^2$$

$$k = \frac{\sum f(l_k)(l_k - l_0)}{\sum (l_k - l_0)^2}$$

a)

$$k = \frac{(2)(7-5.3) + (4)(9.4-5.3) + (6)(12.3-5.3)}{(7-5.3)^2 + (9.4-5.3)^2 + (12.3-5.3)^2}$$

$$k = 0.889563319$$

b)

$$k = \frac{(3)(8.3-5.3) + (5)(11.3-5.3) + (8)(14.4-5.3) + (10)(15.9-5.3)}{(8.3-5.3)^2 + (11.3-5.3)^2 + (14.4-5.3)^2 + (15.9-5.3)^2}$$

$$k = 0.906857643$$

best fit:

$$\Phi(k) = \sum \left[k(l_k - l_0) - f(l_k)\right]^2$$

$$\Phi(a) = \sum_{k=0}^{6} \left[0.889563319(l_k - 5.3) - f(l_k)\right]^2$$

$$\Phi(a) = \sum_{k=0} [0.889563319(l_k - 5.3) - f'(l_k)]$$

$$\Phi(a) = 0.972095678$$

$$\Phi(b) = \sum_{k=0}^{6} [0.906857643(l_k - 5.3) - f(l_k)]^2$$

$$\Phi(b) = 0.897047695$$

$$\Phi(a) > \Phi(b)$$

$\therefore$ the data set from (a) is a better fit for the whole dataset