Lecture 6 – Sensitivity Analysis

Question 1

Confidence Interval Setup

A plot of land is in the shape of a right-angle triangle, with side lengths *a* and *b*. Several measurements were taken of the plot lengths, as shown below. The hypotenuse side length *c* was not measured.

Measurement #	*a*, m	*b*, m
1	90.5	131.4
2	89.9	130.6
3	90.2	130.3
4	90.1	130.1
5	89.6	130.2
6	90.6	

Using the sample values in the table above, what is the estimated area of the triangular plot?

The estimated area is the average of the area of each measurement attempt, or the sum of (a_i * b_i).

$$\bar{A} = \frac{\sum(a_i \, * \, b_i)}{n}$$

$$= \frac{(90.5\text{ m} \, * \, 131.4\text{ m} \, + \, 89.9\text{ m} * 130.6\text{ m} \, + \, 90.2\text{ m} \, * \, 130.3\text{ m} \, + \, 90.1\text{ m} \, * \, 130.1\text{ m} \, + 89.6\text{ m} \, * \, 130.2\text{ m})}{5}$$

$$= 5877.4\text{ m}$$

Question 1.1

Confidence Interval

From the samples presented, the variance of *a* is calculated to be 0.113 m and the variance of *b* is 0.277 m. Knowing these two standard deviations, we can calculate the standard deviation of $\bar{A}$ as 4.456 m^2. Using this standard deviation of the area, calculate the 97% confidence interval of the true area of the triangular plot.

$$\mu_{1-\alpha} = (\bar{A} + k_{\alpha/2}\frac{\sigma_{\bar{A}}}{\sqrt{n}}; \bar{A} + k_{1-\alpha/2}\frac{\sigma_{\bar{A}}}{\sqrt{n}})$$

$$\mu_{97\%} = (5877.4\text{ m}^2 + k_{0.015}\frac{4.456\ \text{ m}^2}{\sqrt{5}}; 5877.4\text{ m} + k_{0.985}\frac{4.456\ \text{ m}^2}{\sqrt{5}})$$

$$\mu_{97\%} = (5877.4\text{ m}^2 - 2.17\frac{4.456\ \text{ m}^2}{\sqrt{5}}; 5877.4\text{ m}^2 + 2.17\frac{4.456\ \text{ m}^2}{\sqrt{5}})$$

$$\mu_{97\%} = (5877.4\text{ m}^2 - 4.32\text{ m}^2; 5877.4\text{ m}^2 + 4.32\text{ m}^2)$$

$$\mu_{97\%} = (5873.1\text{ m}^2, 5881.7\text{ m}^2)$$

Note that in a test, the critical value *k* should be put on your formula sheet. Common critical values would be the ones needed for calculating confidence intervals of 90%, 95%, 97%, 98%, 99%, and 99.7%.

Question 2

Deterministic Models

Basem is making a device to help engineering students succeed in university. The initial cost to start production is $10,000. The parts needed to create the device costs $45-55/device and it takes Basem 1 hour to make one device. Basem values his time at $30/hour. After surveying some students, Basem expects that he can sell each device for $150. At this price, Basem estimates that he can sell 900-1100 devices in the first year.

Assuming the uncertainties in parts cost and number of devices sold are uniform in the given range, what is the expected NV of the business for Basem after 1 year?

A) $48,500
B) $57,500
C) $60,000
D) $67,000
E) $72,500

C) Since the probability is uniform for the parts cost between $45-$55, the expected parts cost is $50. Similarly, the expected sale is 1000 devices. Therefore $E[NV] = 1000 \times \$150 - (\$50 + \$30) \times 1000 - \$10{,}000 = \$60{,}000$

Question 2.1

What is the NV for the worst-case scenario?

A) $45,000
B) $48,500
C) $57,500
D) $60,000
E) $67,000

B) $NV = 900 \times \$150 - (\$55 + \$30) \times 900 - \$10{,}000 = \$48{,}500$

Question 2.2

What is the NV for the best-case scenario?

A) $48,500
B) $57,500
C) $60,000
D) $67,000
E) $72,500

E) $NV = 1100 \times \$150 - (\$45 + \$30) \times 1100 - \$10{,}000 = \$72{,}500$

Question 2.3

Is the net value of the business more sensitive to the cost of parts or to the number of devices sold?

A) Cost of parts
B) Devices sold
C) Equal sensitivity

B) Using a base case of 1000 devices sold and parts cost of $50, a 10% change in devices sold will result in a 11.67% change in NV ($7000). While a 10% change in cost of parts will result in an 8.33% change in NV ($5000).

Question 2.4

After more research Basem was able to determine the probability for the cost of the parts and the number of devices that will be sold. This is summed up in the table below:

Cost of Parts per device	Probability	Device Sale	Probability
$45	30%	900	35%
$50	60%	1000	40%
$55	10%	1100	25%

What is the expected NV of the business for Basem?

A) $60,290
B) $60,000
C) $62,720
D) $67,850
E) $71,630

A) $E[NV] = (900 \times 0.35 + 1000 \times 0.4 + 1100 \times 0.25) \times \$150 - (\$45 \times 0.3 + \$50 \times 0.6 + \$55 \times 0.1) + \$30) \times (900 \times 0.35 + 1000 \times 0.4 + 1100 \times 0.25) - \$10{,}000 = \$60{,}290$

Question 3

Stochastic Models

Which of the following best describes a random variable?

1. A quantity known only in terms of a probability
2. A variable that's 42.718, which is like, so random.
3. For example, an equation variable *T* that doesn't specify what it refers to so that users will randomly substitute time period, temperature, kinetic energy, etc.

A quantity known only in terms of a probability.

Question 4

Deterministic vs. Stochastic Models

Which type of modelling is the following situation?

My mathematical model has six parameters, each of which is has quantities known by their probability. Unfortunately, my analysis of the model shows a low degree of tractability as a result.

This is stochastic modelling. Deterministic modelling would require parameters to be known with certainty (and no variability).

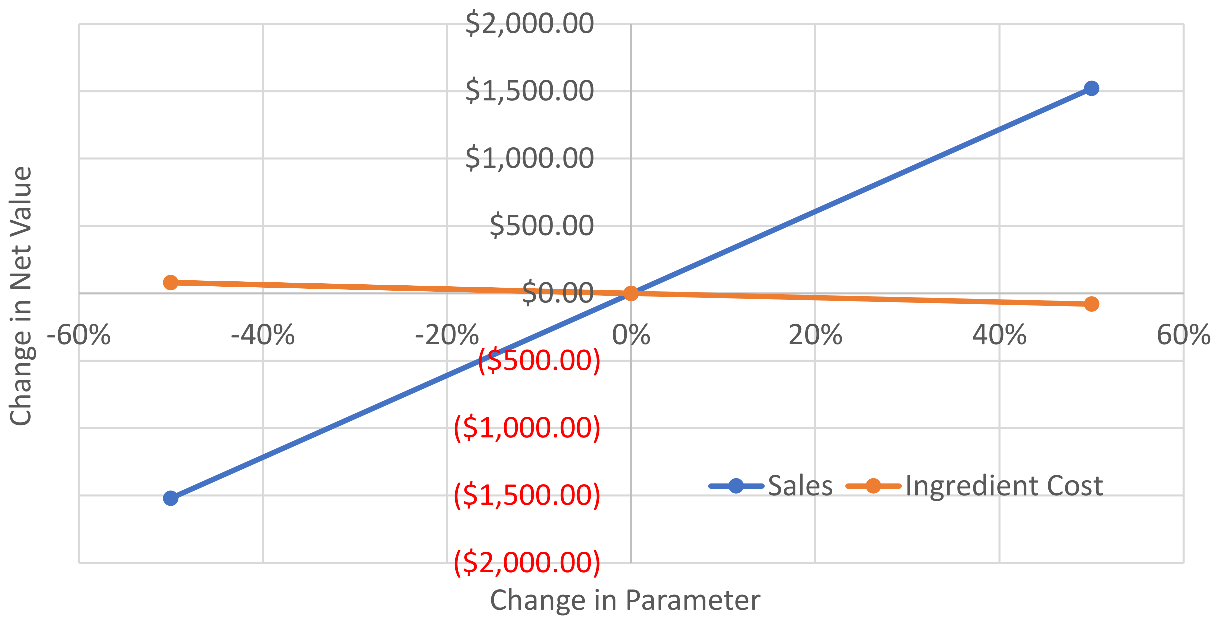
Question 5

Scenario Analysis

The spider plot below shows that the overall net value's most sensitive parameter is:

A. Ingredient Cost.
B. Sales.

C. Change in Net Value.
D. Change in Parameter.
E. $1500
F. It shows all parameters are equally sensitive.



Net value is more sensitive to a change in sales than a change in ingredients cost. This is indicated by the slope of the curves; a large change in ingredient cost has a small impact on net value, but a large change in sales has a large impact on net value. This answer would be the same if the curves sloped the opposite ways – the more sensitive parameter is the one with the larger magnitude of slope.

Question 6

Deterministic vs. Stochastic Models

Is a deterministic or stochastic model likely closer to reality? Which is easier to determine optimum inputs for ? Which allows for a higher modelling flexibility of valid problem situations?

In reality, all parameters being known with certainty (deterministic) is unlikely. For the goal of optimization, stochastic variation is unlikely to deliver useful prescriptive results, due to its wide range of output distribution.

Stochastic, Deterministic, Stochastic

Question 7

Monte Carlo Simulations, Concept

In a Monte Carlo simulation, the simulation experimenter must:

A) Generate random numbers themselves
B) Generate the cumulative probability distributions themselves
C) Decide on a probability distribution for important variables
D) Manipulate the real system at the same time and in the same way as the simulation

C Random numbers and probability distributions can be generated by a computer (and typically are). Manipulating the real system is likely infeasible, as a key purpose to using simulation is avoiding manipulating the system to collect experimental data. However, the experimenter must decide on a probability distribution as an input for the simulation.

Question 8

Probability Distribution from Frequency

Across many days, a customer service counter records the number of customers that show up each hour and has collected the data below. What is the probability of occurrence for three customers to arrive in one hour?

Arrivals per hour	Frequency
0	4
1	7
2	11
3	16
4	19
5	13
6+	10

A) 0.20
B) 0.72
C) 0.14
D) 0.48
E) 0.25

This is a problem made easier by being able to see the possible answers. 0.48 is the cumulative probability of *3 or fewer* customers. 0.72 the cumulative probability of *3 or more* customers. 0.14 is a likely answer from calculating using the wrong column. 0.25 is plainly incorrect. 0.20 is the correct answer.

Calculating the answer uses the standard probability equation.

$$P_3 = \frac{f}{N} = \frac{16}{(4+7+11+16+19+13+10)} = \frac{16}{80} = 0.20$$

While this isn't directly present in the lecture, it is a necessary early step in the by-hand Monte Carlo simulation problems where frequency is given instead of probability.

Question 9

Simulations, Concept

[blank] is the attempt to duplicate the features, appearance, and characteristics of a real system, usually via a computerized model.

A) A random number
B) A cumulative probability distribution
C) The Monte Carlo method
D) Simulation
E) Scenario analysis

This is a definition of a simulation.

Question 10

Monte Carlo Simulation

A distribution of lead times in an inventory warehouse indicate that the lead time was:

Lead Time	Probability
1 day	20%
2 days	30%
3 days	30%
4 days	15%
5 days	5%

After a very brief Monte Carlo simulation, using the same by-hand technique as shown in lecture, the random numbers are drawn: 07, 62, 56, 40, and 03. What is the average lead time in this simulation?

A) 1.5 days
B) 2.55 days
C) 3 days
D) 3.25 days
E) None of these

Lead Time	Probability	Random Number Range
1 day	20%	01-20
2 days	30%	21-50
3 days	30%	51-80
4 days	15%	81-95
5 days	5%	96-100

Converting sample random numbers to actual outcomes/possibilities:

$07 \rightarrow 1 \text{ day}, 62 \rightarrow 3 \text{ days}, 56 \rightarrow 3 \text{ days}, 40 \rightarrow 2 \text{ days}, 03 \rightarrow 1 \text{ day}.$

Calculating average:

$$\bar{x} = \frac{\sum x}{n} = \frac{(1+3+3+1)}{4} = 2 \text{ days}.$$

Question 11

Simulation, Concepts

Which of the following statements is false?

A) Results of simulation experiments with large numbers of trials or long experimental runs will generally be better than those with fewer trials or shorter experimental runs.
B) A simulation model is designed to arrive at a single specific numerical answer to a given problem.
C) Simulation can use any probability distribution that the user defines.
D) Simulation models, because they are based on the generation of random numbers, fail to give the same solution in repeated use to any particular problem.

A simulation model is designed to *provide greater information about a problem or system.*