Therefore, the field $d\mathbf{H}$ due to the current I_x is

$$d\mathbf{H} = \frac{\hat{\mathbf{x}}z + \hat{\mathbf{z}}x}{(x^2+z^2)^{1/2}} \frac{I_x}{2\pi R} = \frac{(\hat{\mathbf{x}}z + \hat{\mathbf{z}}x)J_s\,dx}{2\pi(x^2+z^2)},$$

and the total field is

$$\begin{aligned}
\mathbf{H}(0,0,z) &= \int_{x=0}^{w} (\hat{\mathbf{x}}z + \hat{\mathbf{z}}x) \frac{J_s\,dx}{2\pi(x^2+z^2)} \\
&= \frac{J_s}{2\pi} \int_{x=0}^{w} (\hat{\mathbf{x}}z + \hat{\mathbf{z}}x) \frac{dx}{x^2+z^2} \\
&= \frac{J_s}{2\pi} \left(\hat{\mathbf{x}}z \int_{x=0}^{w} \frac{dx}{x^2+z^2} + \hat{\mathbf{z}} \int_{x=0}^{w} \frac{x\,dx}{x^2+z^2} \right) \\
&= \frac{J_s}{2\pi} \left(\hat{\mathbf{x}}z \left(\frac{1}{z} \tan^{-1}\left(\frac{x}{z}\right)\right)\Big|_{x=0}^{w} + \hat{\mathbf{z}} \left(\tfrac{1}{2}\ln(x^2+z^2)\right)\Big|_{x=0}^{w} \right) \\
&= \frac{5}{2\pi} \left[\hat{\mathbf{x}} 2\pi \tan^{-1}\left(\frac{w}{z}\right) + \hat{\mathbf{z}}\tfrac{1}{2}(\ln(w^2+z^2) - \ln(0+z^2)) \right] \quad \text{for } z \neq 0, \\
&= \frac{5}{2\pi} \left[\hat{\mathbf{x}} 2\pi \tan^{-1}\left(\frac{w}{z}\right) + \hat{\mathbf{z}}\tfrac{1}{2}\ln\left(\frac{w^2+z^2}{z^2}\right) \right] \quad \text{(A/m)} \quad \text{for } z \neq 0.
\end{aligned}$$

An alternative approach is to employ Eq. (5.24a) directly.

Problem 5.11 An infinitely long wire carrying a 25-A current in the positive x-direction is placed along the x-axis in the vicinity of a 20-turn circular loop located in the x–y plane as shown in Fig. 5-37 (P5.11(a)). If the magnetic field at the center of the loop is zero, what is the direction and magnitude of the current flowing in the loop?

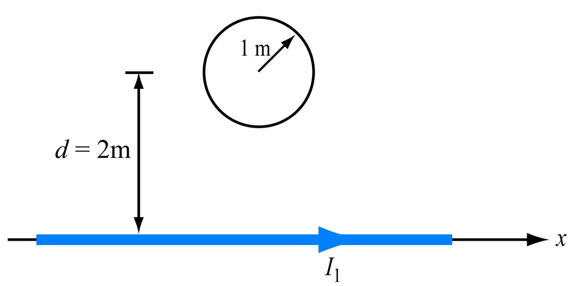

Figure P5.11: (a) Circular loop next to a linear current (Problem 5.11).

Solution: From Eq. (5.30), the magnetic flux density at the center of the loop due to

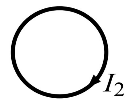

Figure P5.11: (b) Direction of I_2.

the wire is

$$\mathbf{B}_1 = \hat{\mathbf{z}}\,\frac{\mu_0}{2\pi d}\,I_1$$

where $\hat{\mathbf{z}}$ is out of the page. Since the net field is zero at the center of the loop, I_2 must be clockwise, as seen from above, in order to oppose I_1. The field due to I_2 is, from Eq. (5.35),

$$\mathbf{B} = \mu_0 \mathbf{H} = -\hat{\mathbf{z}}\,\frac{\mu_0 N I_2}{2a}\,.$$

Equating the magnitudes of the two fields, we obtain the result

$$\frac{NI_2}{2a} = \frac{I_1}{2\pi d}\,,$$

or

$$I_2 = \frac{2aI_1}{2\pi N d} = \frac{1\times 25}{\pi \times 20 \times 2} = 0.2 \text{ A}.$$

Problem 5.12 Two infinitely long, parallel wires carry 6-A currents in opposite directions. Determine the magnetic flux density at point P in Fig. 5-38 (P5.12).

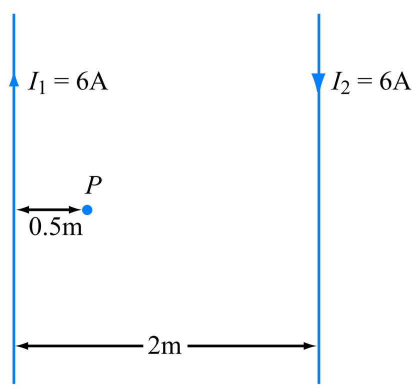

Figure P5.12: Arrangement for Problem 5.12.

Solution:

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I_1}{2\pi(0.5)} + \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I_2}{2\pi(1.5)} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0}{\pi}\,(6+2) = \hat{\boldsymbol{\phi}}\,\frac{8\mu_0}{\pi} \quad \text{(T)}.$$