H5

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ENGPHYS 2E04

Introduction

The circuit below was inspired by the sample lab, modelled to fit the specifications outlined in the deliverables. First, the frequency that would allow the capacitor to correct the power factor was calculated. Then, the circuit was solved analytically, digitally, and physically for the source voltage, transmission line (R3) voltage, and the current through the transmission line (which in this case is the current through the entire circuit). The digital and physical solutions involved measuring values using the Hantek and Tektronix oscilloscopes, and then performing calculations (like in the analytical solution) to yield results.

Problem Framing:

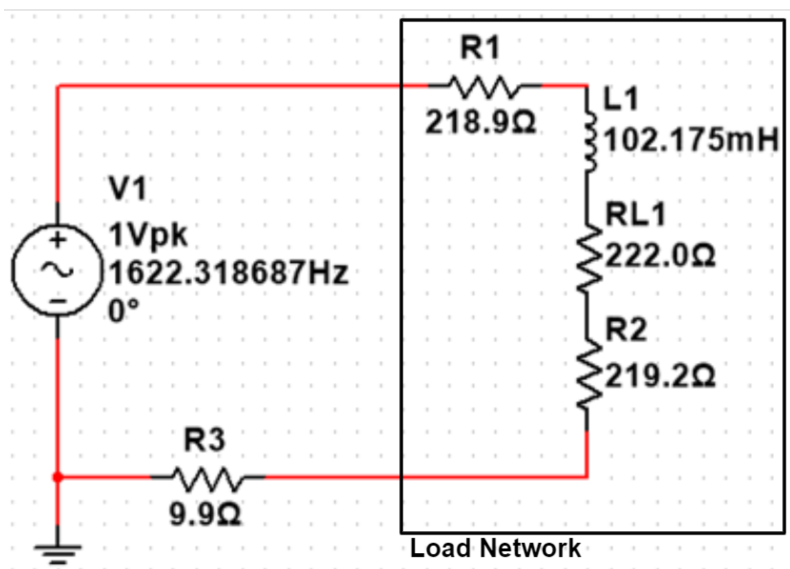


Figure 1: Circuit Diagram

The load is comprised of R1, L1, RL1, R2, all in series. The goal is to determine the voltage across and current through the transmission line resistor, the complex power, and the power factor, before and after adding a capacitor in parallel to the load network. Phenomena that occur during this lab are due to the relationship between capacitors and inductors, and their individual effect on the whole circuit. Capacitors cause current to lead voltage, while inductors cause current to lag voltage.

Analytical Solution:

First, the frequency at which the capacitor will correct the power factor was found. For power factor correction, the imaginary component of the load (including the capacitor in parallel) will be zero. So, we can solve the following equation for the frequency (values are as indicated in figure 1).

$$Im\{ZC||ZLoad\} = 0$$

$$Im\left\{\frac{1}{j\omega C + \frac{1}{j\omega L1 + R1 + R2 + RL1}}\right\} = 0$$

$$C = \frac{L1}{(R1 + R2 + RL1)^2 + ((2 * \pi * f) * L1)^2}$$

Substituting $C = 67.2 * 10^{-9}$, and solving for f:

$$f = 1622.318687 \text{ Hz}$$

Next, the power factor and complex power were found. The power factor can be defined as the ratio of average power to apparent power ($pf = S_{avg}/S_{apparent}$). The complex power can be defined as:

$$S = \frac{V_{source} * conjugate(I)}{2}$$

The current, I, can be found with Ohm's law V = IR.

$$I = \frac{V}{R}$$

$$I = \frac{V_{source}}{Z_{load} + R3} = \frac{1V}{ZL + R1 + R2 + RL1 + R3}$$

Plugging into maple, we get:

$$Iorig := \frac{Vs}{ZLoad + R3}$$

$$Iorig := 0.0004368721962 - 0.0006791102548\,\mathrm{I}$$

With this value, we can solve our formula for S. Using maple:

$$S := \frac{Vs \cdot conjugate(Iorig)}{2}$$

$$S := 0.0002184360981 + 0.0003395551274\,\mathrm{I}$$

With complex power found, we can now calculate the power factor:

$$pf = \frac{V_{avg}}{V_{reactive}} = \frac{Re(S)}{Im(S)} = \frac{2.184}{3.395} = 0.541$$

So, our power factor was calculated to be 0.541. This is a terrible power factor since there is a high amount of reactive power, so we are in dire need of correction.

Before correction, we need to find the voltage across and current through the transmission line resistor. Since all components are in series, the previously calculated current is the same as the current through the transmission line.

$$Iorig := 0.0004368721962 - 0.0006791102548\,\mathrm{I}$$

$$\mathrm{polar}(0.0008074949251, -0.9991449598)$$

Therefore, the initial current through the transmission line in phasor form is

$$\boldsymbol{I} = (0.807 * 10^{-3}\ A)e^{j*(-0.999\ rad)}$$

As for the voltage, we can find this using a voltage divider (we could also use ohm's law, but I want to find the voltage independently from the current). Using maple:

$$VR3 := Vs \cdot \left(\frac{R3}{ZLoad + R3} \right);$$

$$VR3 := 0.004325034743 - 0.006723191523\,\mathrm{I}$$

$$\mathrm{polar}(0.007994199759, -0.9991449597)$$

Therefore, the initial voltage through the transmission line in phasor form is

$$\boldsymbol{V} = (7.99 * 10^{-3}\ V)e^{j*(-0.999\ rad)}$$

After correction

Now, we can add in the capacitor (of capacitance 66.7nF as seen previously) and recalculate our acquired values. The following configuration was used:

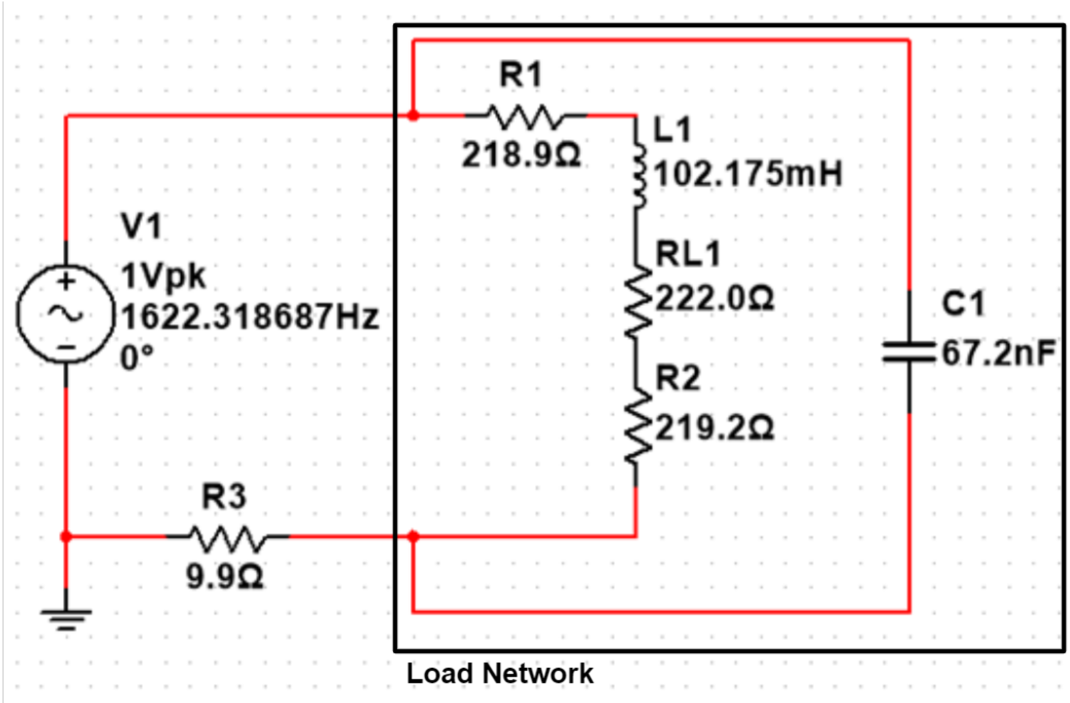


Figure 2: Circuit diagram with capacitors

Before any calculations are made, we need to redefine the load impedance.

$$Z_{Load} = \frac{1}{\frac{1}{ZC1} + \frac{1}{ZL1 + RL1 + R1 + R2}} = 2303.380259 - j(1.061112123 * 10^{-6})$$

$$\mathrm{polar}(2303.380259, -4.606760516 \times 10^{-10})$$

Observing these results, the imaginary part of the load impedance has become very small, meaning the voltage/current phase shifts caused by the load impedance will be minimal.

Finding the voltage across and current through the transmission line resistor using ohm's law:

$$new\_I := \frac{Vs}{new\_ZLoad + R3}$$

$$new\_I := 0.0004322865749 + 1.982918081 \times 10^{-13}\,\mathrm{I}$$

$$\mathrm{polar}(0.0004322865749, 4.587045252 \times 10^{-10})$$

Note that the phase shift has been greatly reduced, so it can be estimated to 0. The current is now in phase with the input voltage. Also note that the amplitude has been reduced to almost half of what it was before the correction.

Therefore, the new current through the transmission line in phasor form is

$$\boldsymbol{I} = (0.432 * 10^{-3}\ A)e^{j*(0\ rad)} = 0.432\ mA$$

Voltage across transmission line resistor:

$$new\_VR3 := Vs \cdot \left(\frac{R3}{R3 + new\_ZLoad} \right);$$

$$new\_VR3 := 0.004279637092 + 1.963088900 \times 10^{-12}\ \mathrm{I}$$

$$\mathrm{polar}(0.004279637092, 4.587045251 \times 10^{-10})$$

Note again that the phase is very small, so it will be estimated to zero. The voltage across the transmission line is also in phase with the current and source voltage.

Therefore, the initial voltage through the transmission line in phasor form is

$$\boldsymbol{V} = (4.28 * 10^{-3}\ V)e^{j*(0\ rad)} = 4.28\ mV$$

With current found, we can calculate the new complex power and power factor:

$$S\_new := \frac{Vs \cdot conjugate(new\_I)}{2}$$

$$S\_new := 0.0002161432874 - 9.914590405 \times 10^{-14}\ \mathrm{I}$$

$$new\_pf := \frac{\mathrm{Re}(S\_new)}{\mathrm{abs}(S\_new)}$$

$$new\_pf := 1.000000000$$

Observing these results, we can see that the reactive power has been reduced to a value that is incredibly small. The power factor has also been corrected, as it is calculated to be a perfect 1. This means that the capacitor corrected the power factor just as expected.

Multisim Solution

The following configuration was used in multisim to make measurements before correction:

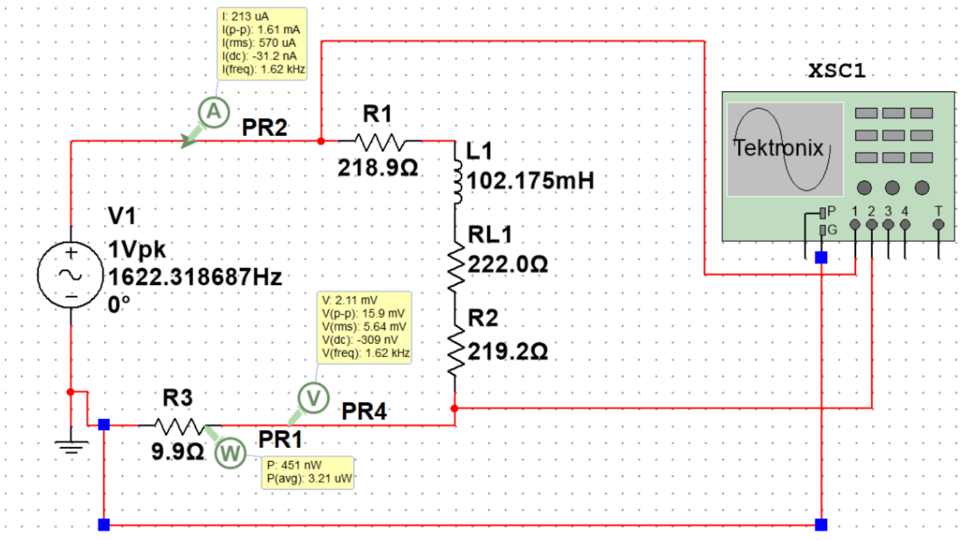
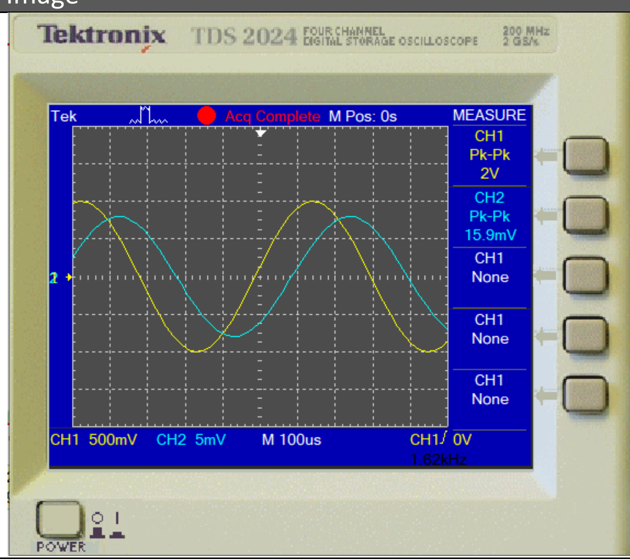


Figure 3: Multisim Circuit Configuration

To verify the calculations from the analytical solutions, the calculations will be repeated using the values measured from multisim. First, the needed values were acquired using the Tektronix.

Measurement	Image	Value
Pk-Pk voltages		$V_{source} = \frac{2V}{2} = 1V$ $**(V_{Transmission} = V_T)$ $V_T = \frac{15.9 * 10^{-3}\ V}{2}$ $= 7.95\ mV$

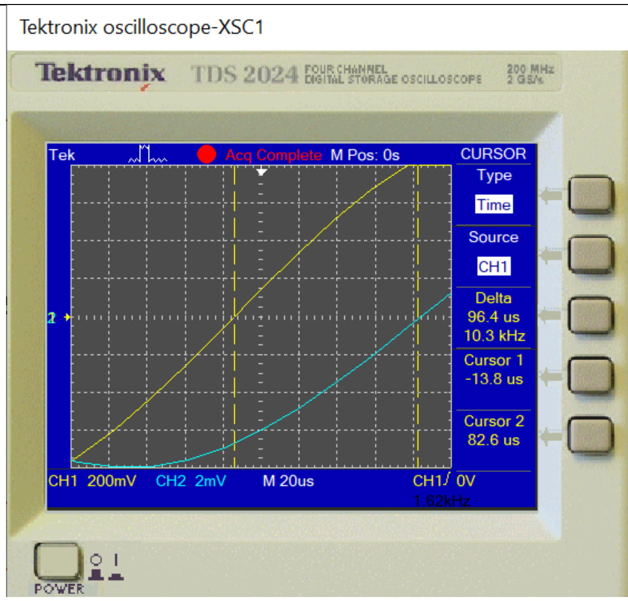
Time difference		$\Delta t = (-96.4 \pm 4)\mu s$

Table 1: Values acquired from Multisim

With the measured values, we can calculate the source voltage, current, and complex power before correction (as well as any error on each of the values). This was done using the following maple code:

Input	Output
$restart:$ $Vs := 1 : f := 1622.318687:$ $\text{omega} := 2 \cdot \text{Pi} \cdot f:$ $R1 := 218.9 : R2 := 219.2 : R3 := 9.9 : RL1 := 222.0:$ $L1 := 102.175e-3 : ZL1 := I \cdot \text{omega} \cdot L1:$ $C1 := 67.2e-9 : ZC1 := \frac{1}{I \cdot \text{omega} \cdot C1}:$ $ZLoad := ZL1 + R1 + R2 + RL1:$ "Measured Values:"; $Vsource = Vs;$ ""; $VR3 := \frac{(15.9e-3)}{2};$ $VR3\_DTime := -96.4 \cdot 10^{-6} : VR3\_DTime\_error := \frac{20e-6}{5}:$ $VR3\_DTime\_relative := \text{abs}\left(\frac{VR3\_DTime\_error}{VR3\_DTime}\right):$ $VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \text{Pi}; VR3\_Phase\_error := \text{abs}(VR3\_Phase \cdot VR3\_DTime\_relative);$ ""; $Iorig := \frac{Vs}{ZLoad + R3}$; #confirm with probe $Iamp := \text{abs}(Iorig); Iphase := \text{argument}(Iorig);$	"Measured Values:" $Vsource = 1$ $VR3 := 0.007950000000$ "Calculated" $VR3\_Phase := -0.9826369096$ $VR3\_Phase\_error := 0.04077331575$ "" $Iorig := 0.0004368721962 - 0.0006791102548\,I$ $Iamp := 0.0008074949251$ $Iphase := -0.9991449598$
$S := \frac{Vs \cdot \text{conjugate}(Iorig)}{2}$ $pf := \frac{\text{Re}(S)}{\text{abs}(S)};$	S := 0.0002184360981 + 0.0003395551274*I pf := 0.5410215999

Table 2: Maple Calculations

Summarizing the results of the above calculations:

Current through circuit (including transmission line):

$$\boldsymbol{I} = (0.807 * 10^{-3}\ A)e^{j*(-0.999\ rad)}$$

Source Voltage :

$$\boldsymbol{V} = (1\ V)e^{j*(0\ rad)} = 1\ V$$

Voltage across transmission line (note the error in the phase):

$$\boldsymbol{V} = (7.95 * 10^{-3}\ V)e^{j*((-0.993 \pm 0.04)\ rad)}$$

Complex power:

$$S\ =\ 0.218 * 10^{-3}\ +\ j * (0.340 * 10^{-3})\ \text{V·A}$$

Power factor:

$$pf\ = 0.541$$

After correction

The following configuration was used in multisim to make measurements **after** correction:

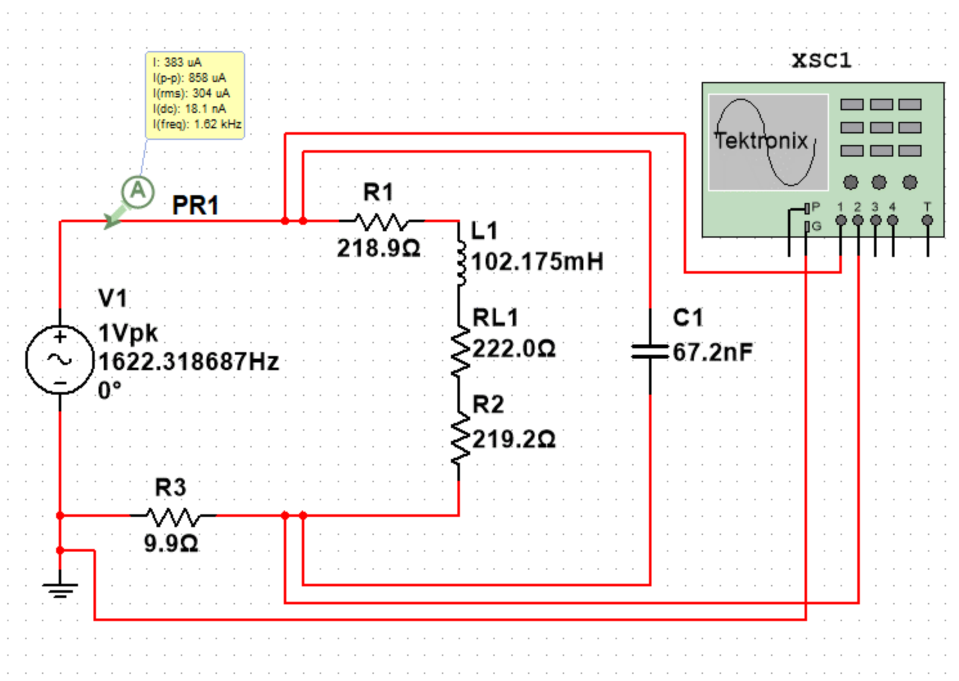
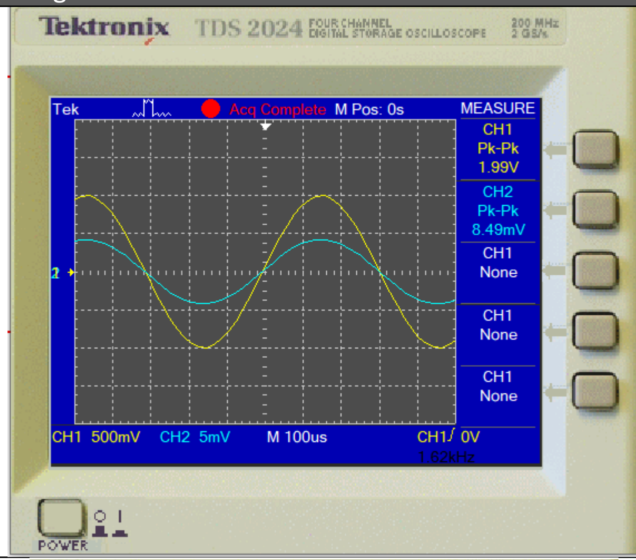


Figure 3: Multisim Circuit after correction

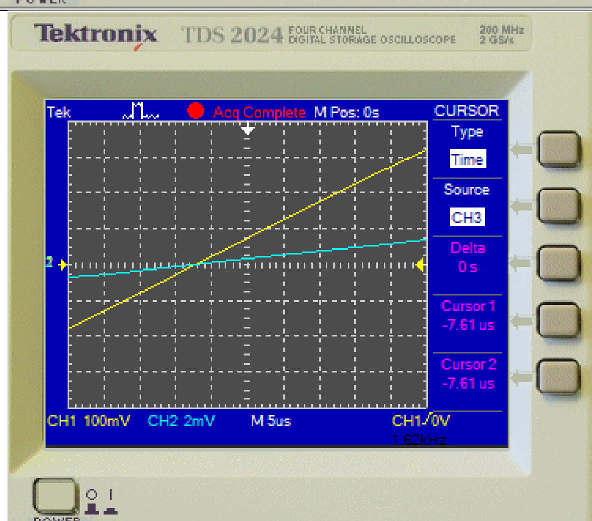
The same values found before correction were acquired again using the Tektronix.

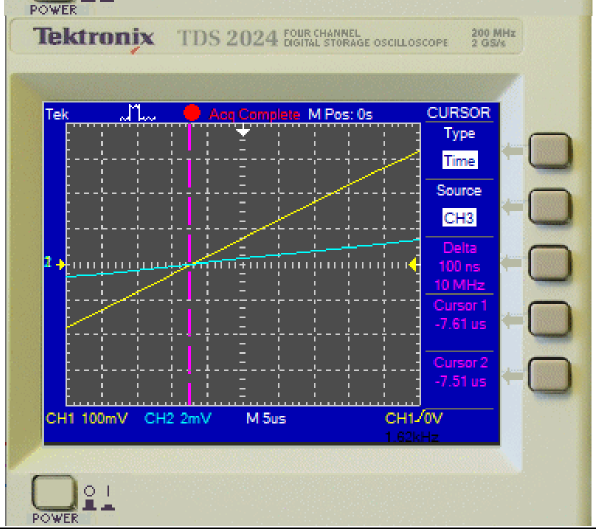
<table>
<tr><th>Measurement</th><th>Image</th><th>Value</th></tr>
<tr><td>Pk-Pk voltages</td><td>
</td><td>

$$V_{source} = \frac{2V}{2} = 1V$$

$$**(V_{Transmission} = V_T)$$

$$V_T = \frac{8.49 * 10^{-3}\ V}{2} = 4.245\ mV$$

</td></tr>
<tr><td>Time difference</td><td>


</td><td>

$$\Delta t = (0 \pm 1)\mu s$$

**Note: The cursors were placed over top of each other in the first image, causing them to disappear on the Tektronix screen. The second image shows the approximate cursor placement that was used

</td></tr>
</table>

Table 3: Values acquired from Multisim

Source voltage, current, and complex power after correction (as well as any error on each of the values) were calculated using the following maple code:

Input	Output
Vs=2/2; "Measured values after correction"; $new\_ZLoad := \dfrac{1}{\frac{1}{ZC1} + \frac{1}{(ZL1 + RL1 + R1 + R2)}};$ $Vsource = Vs;$ $VR3 := \dfrac{8.49e-3}{2};$ $VR3\_DTime := 0;$ ""; $VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \mathrm{Pi};$ $VR3\_Phase\_error := evalf(1 \cdot 10^{-6} \cdot 2 \cdot \mathrm{Pi});$ ""; $I\_new := \dfrac{Vs}{new\_ZLoad + R3}; I\_amp := \mathrm{abs}(I\_new);$ $I\_phase := \mathrm{argument}(I\_new);$	"Measured values after correction" $new\_ZLoad := 2303.380259 - 1.061112123 \times 10^{-6}\,\mathrm{I}$ $Vsource = 1$ $VR3 := 0.004245000000$ $VR3\_DTime := 0$ $VR3\_Phase := 0.$ $VR3\_Phase\_error := 6.283185308 \times 10^{-6}$ $I\_new := 0.0004322865749 + 1.982918081 \times 10^{-13}\,\mathrm{I}$ $I\_amp := 0.0004322865749$ $I\_phase := 4.587045252 \times 10^{-10}$
$S\_new := \dfrac{Vs \cdot \mathrm{conjugate}(I\_new)}{2};$ $pf\_new := \dfrac{\mathrm{Re}(S\_new)}{\mathrm{abs}(S\_new)};$	$S\_new := 0.0002161432874 - 9.914590405 \times 10^{-14}\,\mathrm{I}$ $pf\_new := 1.000000000$

Table 4: Maple calculations

Note that the phase of the current and voltage across R3 are very small. In addition, the error in the phase of VR3 is also very small. For further calculations these values will be approximated to zero.

Summarizing the results of the above calculations:

Current through circuit (including transmission line):

$$\boldsymbol{I} \approx (0.432 * 10^{-3}\ A)e^{j*(0\ rad)} = 0.432\ mA$$

Source Voltage :

$$\boldsymbol{V} \approx (1\ V)e^{j*(0\ rad)} = 1\ V$$

Voltage across transmission line:

$$\boldsymbol{V} = (4.24 * 10^{-3}\ V)e^{j*(0\ rad)} = 4.24\ mA$$

Complex power:

$$S\ =\ 0.216 * 10^{-3}\ +\ j * (-9.91 * 10^{-14})\ V \cdot A$$
$$S \approx 0.216 * 10^{-3}\ \ V \cdot A$$

Power factor:

$$pf\ = 1.00$$

Observing the results, we can see that the phase of the current and transmission line voltage are approximately zero, as well as the reactive power. Note also that the source voltage and transmission line voltage are in phase (as seen on the Tektronix and via calculations), and the current is also in phase with the source voltage. This concludes that the capacitor added to the circuit did in fact correct the power factor, and reversed the effects of the inductor on the circuit.

Physical Solution

All measurement before correction were made using the following configuration. The blue wire is the Hantek probe connected to the node between the load and transmission line (channel 2), the green wire is at the input node and is connected to the Hantek probe (channel 1).

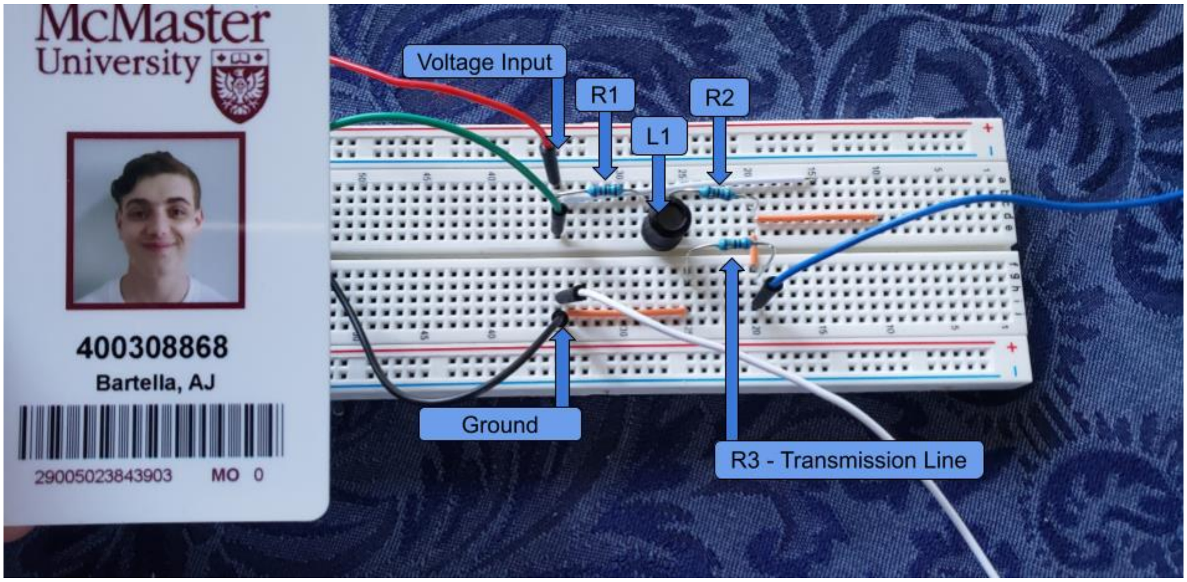


Figure 5: Physical circuit configuration

For the physical solution, the same process as the multisim solution was followed.

Values acquired from Hantek and Breadboard.

Measurement	Image	Value
Pk-Pk voltages + time difference		Voltages: $V_{source} = \frac{2V}{2} = 1V \pm 5\%$ $V_{source} = (1 \pm 0.05)V$ $V_T = \frac{20.4 * 10^{-3}\ V}{2}$ $V_T = 10.2\ mV \pm 5\%$ $V_T = (10.2 \pm 0.51)mV$ $V_T = (10.2 \pm 0.5)mV$ Time: $\Delta t = (-96.00\ )\mu s$ $\Delta t_{error} = \left(96 * 0.05 + \frac{100}{5}\right)\mu s$ $\Delta t_{error} = (24.8)\mu s \approx 25\mu s$ $\Delta t = (-96.00 \pm 25)\mu s$

Frequency		$f = 1.62 * 10^3 Hz \pm 5\%$ $f = (1.62 * 10^3 \pm 81)Hz$ $f = (1.62 \pm 0.08)kHz$

Table 5: Values acquired from Hantek

With the above values, source voltage, current, and complex power were calculated:

Input	Output
$restart:$ $Vs := 1 : f := 1.62\mathrm{e}{+}03 :$ $\mathrm{omega} := 2 \cdot \mathrm{Pi} \cdot f :$ $R1 := 218.9 : R2 := 219.2 : R3 := 9.9 : RL1 := 222.0 :$ $L1 := 102.175\mathrm{e}{-}3 : ZL1 := I \cdot \mathrm{omega} \cdot L1 :$ $C1 := 67.2\mathrm{e}{-}9 : ZC1 := \frac{1}{I \cdot \mathrm{omega} \cdot C1} :$ $ZLoad := ZL1 + R1 + R2 + RL1 :$ "Before capacitor"; $Vsource = Vs; Vs\_error := 0.05;$ ""; $VR3 := \frac{(20.4\mathrm{e}{-}3)}{2}; VR3\_error := VR3 \cdot 0.05;$ $VR3\_DTime := -96.0 \cdot 10^{-6} :$ $VR3\_DTime\_error := \frac{100\mathrm{e}{-}6}{5} + 0.05 \cdot VR3\_DTime :$ $VR3\_DTime\_relative := \mathrm{abs}\left(\frac{VR3\_DTime\_error}{VR3\_DTime}\right) :$ $VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \mathrm{Pi};$ $VR3\_Phase\_error := \mathrm{abs}(VR3\_Phase \cdot VR3\_DTime\_relative);$ ""; $Iorig := \frac{Vs}{ZLoad + R3}; Iorig\_error := 0.05;$ #from Vs $Iamp := \mathrm{abs}(Iorig); I\_amp\_error := Vs\_error \cdot Iamp;$ $Iphase := \mathrm{argument}(Iorig);$ $I\_phase\_error := Vs\_error \cdot \mathrm{abs}(Iphase);$	"Before capacitor" $Vsource = 1$ $Vs\_error := 0.05$ "" $VR3 := 0.01020000000$ $VR3\_error := 0.0005100000000$ $VR3\_Phase := -0.9771609792$ $VR3\_Phase\_error := 0.1547171550$ "" $Iorig := 0.0004377566175 - 0.0006795124941\,I$ $Iamp := 0.0008083118741$ $I\_amp\_error := 0.00004041559370$ $Iphase := -0.9984939909$ $I\_phase\_error := 0.04992469954$
$S := \frac{Vs \cdot \mathrm{conjugate}(Iorig)}{2};$ $S\_error := Iorig\_error + Vs\_error;$ #add relative uncertainties $pf\_approx := \frac{\mathrm{Re}(S)}{\mathrm{abs}(S)};$	$S := 0.0002188783088 + 0.0003397562470\,I$ $S\_error := 0.10$ $pf\_approx := 0.5415689558$

Table 6: Maple Calculations for Physical

Summarizing the results of the above calculations:

Current through circuit (including transmission line):

$$\boldsymbol{I} = [(0.808 \pm 0.04) * 10^{-3}\ A]e^{j*((-0.998 \pm 0.05)\ rad)}$$

Source Voltage :

$$\boldsymbol{V} = ((1 \pm 0.05)\ V)e^{j*(0\ rad)} = (1 \pm 0.05)\ V$$

Voltage across transmission line (note the error in the phase):

$$\boldsymbol{V} = ((10.2\ \pm 0.5) * 10^{-3}\ V)e^{j*((-0.977\ \pm\ 0.15)\ rad)}$$

Complex power:

$$S\ = (0.219 * 10^{-3}\ +\ j * (0.340 * 10^{-3})\ V \cdot A) \pm 10\%$$

Power factor (approximate):

$$pf\ = 0.541$$

After correction

The following configuration was used to make measurements **after** correction:

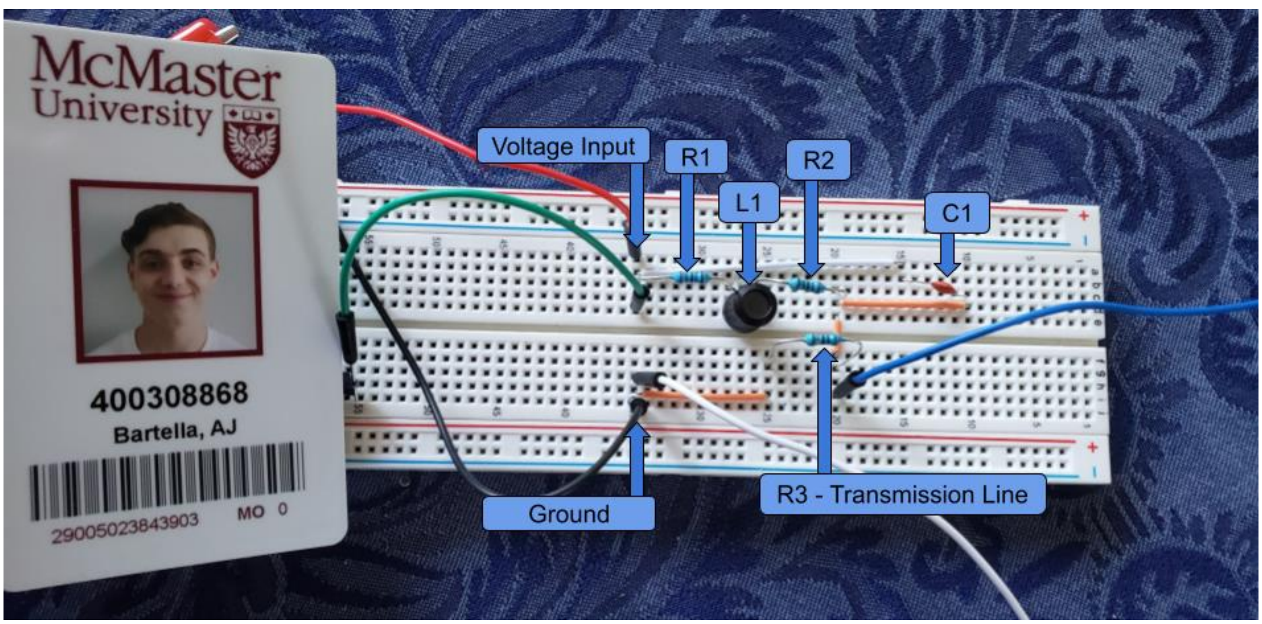


Figure 6: Physical Circuit after correction

The same values found before correction were acquired again using the Hantek. Note: the cursor positions are difficult to see in the first image: they are present, look for a thin pink line down the middle of the y-axis.

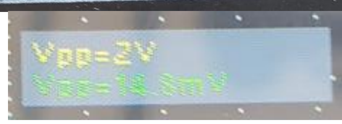
Measurement	Image	Value
Pk-Pk voltages + time difference	 	Voltages: $V_{source} = \frac{2V}{2} = 1V \pm 5\%$ $V_{source} = (1 \pm 0.05)V$ $V_T = \frac{14.8 * 10^{-3}\ V}{2}$ $V_T = 7.40\ mV \pm 5\%$ $V_T = (7.40 \pm 0.37)mV$ $V_T = (7.40 \pm 0.4)mV$ Time: $\Delta t = (0)ns$ $\Delta t_{error} = \left(0 * 0.05 + \frac{200}{5}\right)\mu s$ $\Delta t_{error} = (40)\mu s$ $\Delta t = (0 \pm 40)\mu s$
Frequency		$f = 1.62 * 10^3 Hz \pm 5\%$ $f = (1.62 * 10^3 \pm 81)Hz$ $f = (1.62 \pm 0.08)kHz$

Table 7: Values acquired from Hantek after correction

Source voltage, current, and complex power after correction were calculated using the following maple code:

Input	Output
"After Capacitor"; $ZLoad := \dfrac{1}{\dfrac{1}{ZC1} + \dfrac{1}{(ZL1 + RL1 + R1 + R2)}}$; $Vsource = \frac{2}{2}$; $Vs\_error := 0.05$; ""; $VR3 := \frac{(14.8e-3)}{2}$; $VR3\_error := VR3 \cdot 0.05$; $VR3\_DTime := 0 \cdot 10^{-6}$: $VR3\_DTime\_error := \frac{200e-6}{5}$: $VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \mathrm{Pi}$; $VR3\_Phase\_error := \mathrm{abs}(VR3\_DTime\_error \cdot f \cdot 2 \cdot \mathrm{Pi})$; ""; $Iorig := \frac{Vs}{ZLoad + R3}$; $Iorig\_error := 0.05$; #from Vs $Iamp := \mathrm{abs}(Iorig)$; $I\_amp\_error := Vs\_error \cdot Iamp$; $Iphase := \mathrm{argument}(Iorig)$; $I\_phase\_error := Vs\_error \cdot \mathrm{abs}(Iphase)$;	"After Capacitor" $ZLoad := 2298.662627 + 7.380343059\,I$ $Vsource = 1$ $Vs\_error := 0.05$ "" $VR3 := 0.007400000000$ $VR3\_error := 0.0003700000000$ $VR3\_Phase := 0.$ $VR3\_Phase\_error := 0.4071504080$ "" $Iorig := 0.0004331655412 - 1.384805531 \times 10^{-6}\,I$ $Iorig\_error := 0.05$ $Iamp := 0.0004331677548$ $I\_amp\_error := 0.00002165838774$ $Iphase := -0.003196932077$ $I\_phase\_error := 0.0001598466038$
$S := \frac{Vs \cdot conjugate(Iorig)}{2}$; $S\_error := Iorig\_error + Vs\_error$; #add relative uncertainties $pf\_approx := \frac{\mathrm{Re}(S)}{\mathrm{abs}(S)}$;	$S := 0.0002165827706 + 6.924027655 \times 10^{-7}\,I$ $S\_error := 0.10$ $pf\_approx := 0.9999948897$

Table 8: Maple calculations for physical after correction

Note: the imaginary part of ZLoad is very small compared to the real part, and the phase of the current is also very small in general.

Summarizing the results of the above calculations:

Current through circuit (including transmission line):

$$\boldsymbol{I} = [(0.433 \pm 0.02) * 10^{-3}\ A]e^{j*((-0.00320 \pm 0.0002) rad)} \approx (0.433 \pm 0.02)\ mA$$

Note: the phase is very small, so it was estimated to zero in the rightmost term.

Source Voltage:

$$\boldsymbol{V} = ((1 \pm 0.05)\ V)e^{j*(0\ rad)} = (1 \pm 0.05)\ V$$

Voltage across transmission line (note the error in the phase):

$$\boldsymbol{V} = ((7.40\ \pm 0.4) * 10^{-3}\ V)e^{j*((0 \pm 0.4))\ rad)} \approx (7.40 \pm 0.4)\ mV$$

Complex power:

$$S = (0.217 * 10^{-3} + j * (6.92 * 10^{-7})\, V \cdot A) \pm 10\%$$

$$S \approx ((0.217 \pm 0.02) * 10^{-3}\, V \cdot A)$$

Note: the reactive power is very small, so it was estimated to zero in the rightmost term.

Power factor (approximate):

$$pf = 0.999 \approx 1$$

Observing these results, we can see that the phase of the current was greatly reduced, and that the reactive power is small to the point where it can be approximated to zero. Current, source voltage, and transmission line voltage are all shown to be in phase, with phase shifts equal or approximately equal to zero. Therefore, just like in the multisim and analytical solutions, the capacitor's addition to the circuit cancelled the effects of the inductor, putting voltage and current back into phase, greatly reducing the reactive power, and correcting the power factor to approximately 1.

Analysis and Results Comparison

No correction:

	Current	Source voltage	Transmission Voltage	Complex power	Power factor
Analytical	(0.807*10^(-3) A) *e^(j*(-0.999 rad))	1 V	(7.99*10^(-3) V) *e^(j*(-0.999 rad))	0.000218+0.000340j V·A	0.541
Multisim	(0.807*10^(-3) A) *e^(j*(-0.999 rad))	1 V	(7.95*10^(-3) A) *e^(j*((-0.993 ± 0.04) rad))	0.218*10^(-3) + j*(0.340*10^(-3)) V·A	0.541
Physical	[(0.808±0.04)*10^(-3)A] *e^(j*((-0.998±0.05) rad))	(1±0.05) V	((10.2 ±0.5)*10^(-3) V) *e^(j*((-0.977 ± 0.15) rad))	(0.219*10^(-3) + j*(0.340*10^(-3)) V·A) ± 10%	0.541

After correction:

	Current	Source voltage	Transmission Voltage	Complex power	Power factor
Analytical	(0.432*10^(-3) A) e^(j*(0 rad))	1 V	(4.28*10^(-3) V) e^(j*(0 rad))	0.000216-j(9.91*10^(-14)) V·A	1.00
Multisim	(0.432*10^(-3) A) e^(j*(0 rad))	1 V	(4.24*10^(-3) V) e^(j*(0 rad))	0.216*10^(-3) + j*(-9.91*10^(-14)) V·A	1.00
Physical	[(0.433±0.02)*10^(-3) A] e^(j*((≈0)rad))	(1±0.05) V	((7.40 ±0.4)*10^(-3) V) e^(j*((0±0.4)) rad))	(0.217*10^(-3) + j*(6.92*10^(-7)) V·A)±10%	0.999

Observing the tables above, all values are matching within error except for the transmission voltages. While the difference between the analytical and multisim voltages are minimal, there are no error bounds to excuse this discrepancy. As for the physical values, they are way off: approximately 3 mA larger than expected. Overlooking that discrepancy, through all three methods the addition of the capacitor cancelled out the effect of the inductor on phase on the circuit. This is because regardless of the amplitude values and any discrepancies that may occur, the phase shift of all currents and voltages was reduced to either zero, or a number small enough to approximate it to zero after the addition of the capacitor. This also means that the voltages across and currents through the circuit were in phase after the addition of the capacitor. Finally, the power factor in each instance was improved to 1 (or approximately 1) after the addition of the capacitor. Therefore, it is evident that the addition of the capacitor in all 3 cases did in fact correct the power factor, as well as render voltage and current in phase with one another.

Error Discussion

There are many causes for error in this lab that could have caused discrepancies in digital and physical results, specifically in the measurement of the transmission voltage. Firstly, the obvious: measurements using the cursors are prone to a high degree of error. Though this is already included in error calculations, it is not to be overlooked. Another large source of error for this lab was the accuracy of the Hantek. Specifically, when measuring the voltage across the transmission resistor, the signal was **very** “fuzzy”, causing large fluctuations in the pk-pk voltage reading. To correct this next time, I can employ the use of the cursors to measure the max voltage instead, or I can find a way to measure the error of the "fuzzy-ness" and account for it in my calculations. Another significant source of error for the physical build is the estimated inductance, given from the values measured by Mr. Johnasson. All calculations involving inductance (i.e. ALL calculations in the physical portion of the lab) are jeopardized by this error. In addition, poor contact with the breadboard is definitely possible. As mentioned in previous labs, the capacitor feels as though it moves too freely inside the holes of the breadboard, possibly causing poor or inconsistent contact. Finally, the nature of the capacitor (as described by Mr. Johnasson and in previous labs) can cause discrepancies in the physically measured capacitance. The capacitors provided tend to be unstable, and tend to return different capacitance values to the Hantek depending on the position of the legs. One thing I could have done to account for this is to take the maximum and minimum capacitance readings of the Hantek and create error bars for further calculations involving capacitance. All in all, it is my hypothesis that the "fuzzy-ness" of the signal on the Hantek was the cause of my downfall in the current measurement portion.

Conclusion

The goal was to determine the voltage across and current through the transmission line resistor, as well as the source voltage, and the complex power of the circuit before and after the addition of a capacitor in parallel to the load. This was done using multisim and the Hantek to acquire values and solve mathematically like in the analytical solution. In this lab, most values were matching within error, except for the amplitudes of the current across the transmission line. Regardless of this, the phase shifts for all values aligned within error, thus verifying the addition of the capacitor in the circuit cancelling the phase shift caused by the inductor and correcting the power factor. As for the discrepancies in the amplitudes of the currents, there are many possible causes for error in this field, discussed further in the "Error" section.

I found this lab satisfying. As usual, the introduction of many new concepts was difficult to understand at first, **especially** due to there being many distinct ways of solving for parameters, such as using RMS voltages and currents. However, I stuck to the methods I understood best and (after overcoming confusion in the analytical portion), found the lab enjoyable.

During this lab I learned yet another application of capacitors and inductors. It's very interesting: I never considered the use of a capacitor or inductor to cancel each other out, since I thought why not just get rid of the inductor to achieve the same effect. But this opened my eyes to a reason why: naturally inductive or capacitive loads. As discussed by Dr. Minnick, things like vacuums have naturally inductive loads, and therefore can only have the inductive load removed with a capacitor. I also learned that I can save money on electricity by doing this in my home, although I have a feeling adding a large capacitor to my home's electrical network might be illegal.

All in all, the analytical portion of this lab left me stumped as I struggled to interpret my results, but after finishing the lab I was left with the knowledge of a bona-fide real life application of the theory learned in class.

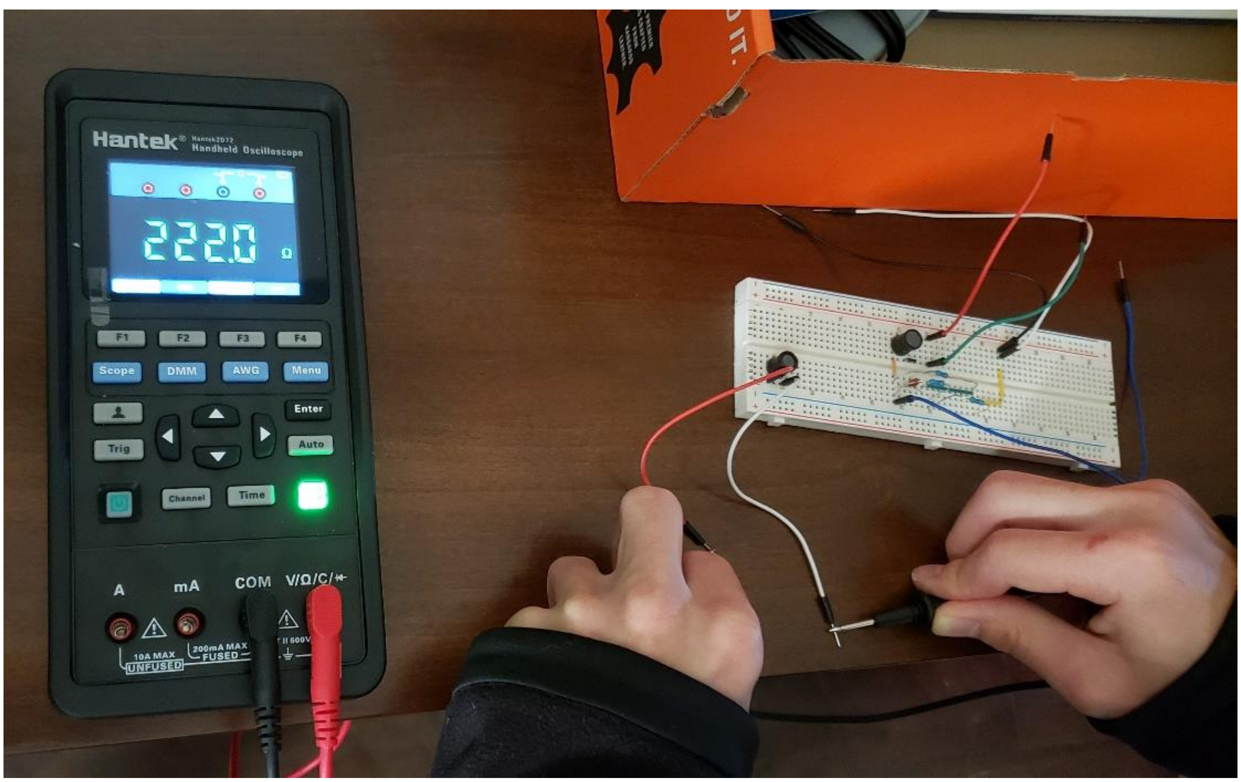
Video link https://youtu.be/F47QwRYP8z8

Appendix A: Measurement pictures

Note all measurements were taken in the same manner as in the picture below (in parallel with the component).

C1:

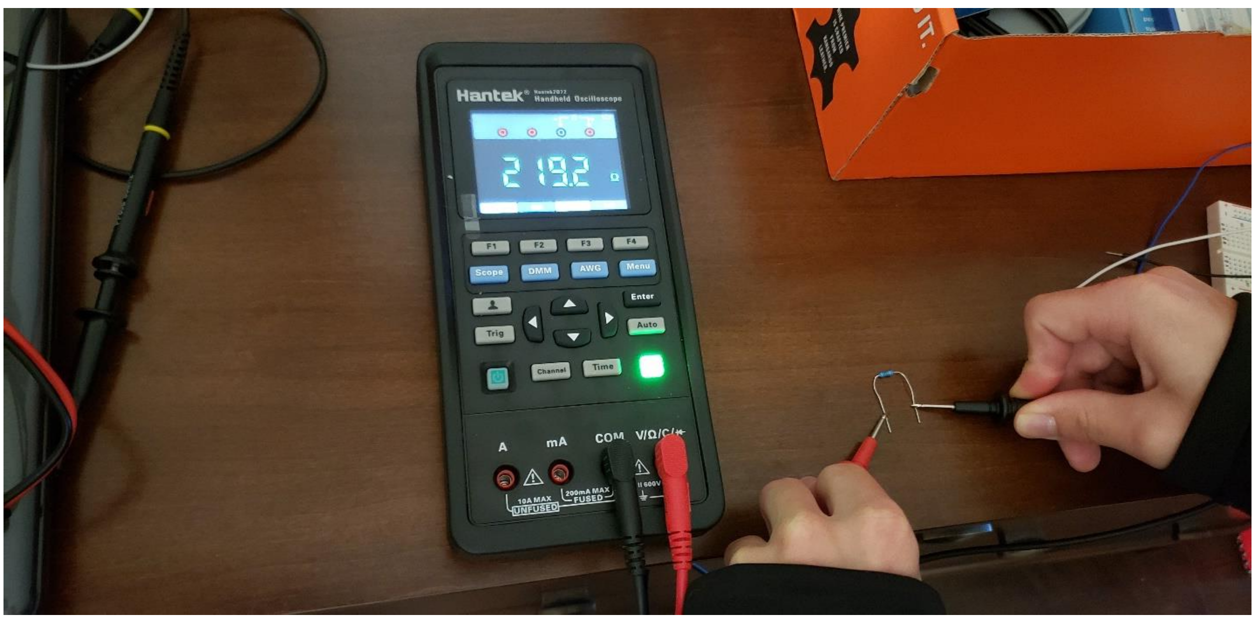
RL1:



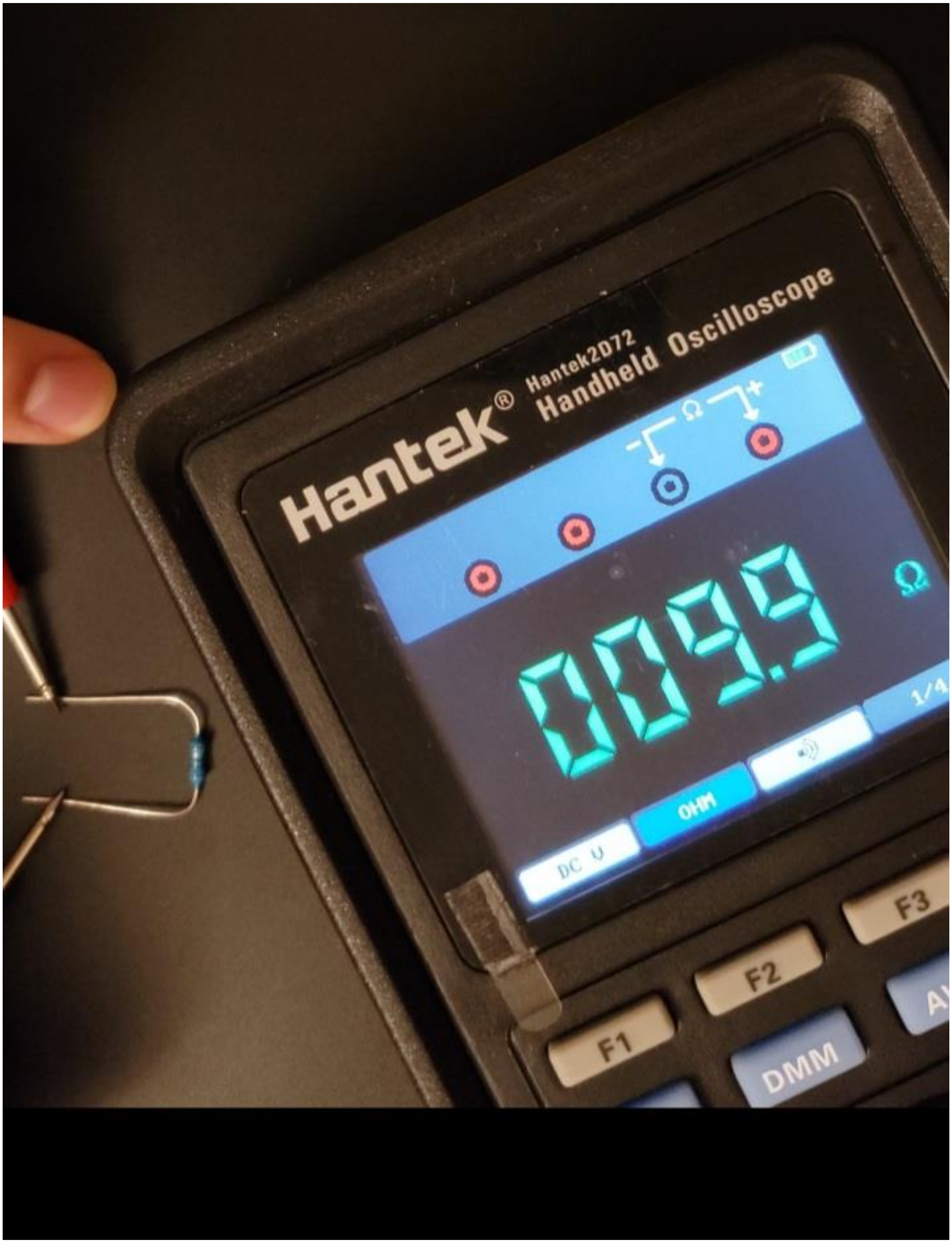
R1:


Hantek
Handheld Oscilloscope
2 18.9 Ω
F1
F2
F3
F4
Scope
DMM
AWG
Menu
Enter
Trig
Auto
Channel
Time
A
mA
COM
V/Ω/C/
10A MAX
UNFUSED
200mA MAX
FUSED

R2:


Hantek
Handheld Oscilloscope
2 19.2 Ω
F1
F2
F3
F4
Scope
DMM
AWG
Menu
Enter
Trig
Auto
Channel
Time
A
mA
COM
V/Ω/C/
10A MAX
UNFUSED
200mA MAX
FUSED

R3:


Hantek
Hantek2D72
Handheld Oscilloscope
Ω
009.9
Ω
1/4
OHM
DC V
F1
F2
F3
DMM

Appendix B: Maple code

Analytical

```
>
restart :
"Ideal frequency for power correction";
Vs := 1 :
omega := 2·Pi·f :
R1 := 218.9 : R2 := 219.2 : R3 := 9.9 : RL1 := 222.0 :
L1 := 102.175e−3 : ZL1 := I·omega·L1 :

"Im{ZC||(ZL)}=0";
```

$$C := \frac{L1}{(R1 + R2 + RL1)^2 + (\text{omega}\cdot L1)^2};\ C1 := 67.2\text{e}-9;$$

$$f := solve(C = C1, f)\,[2];$$

"Ideal frequency for power correction"

"Im{ZC||(ZL)}=0"

$$C := \frac{0.102175}{435732.01 + 0.4121440453 f^2}$$

$$C1 := 6.72 \times 10^{-8}$$

$$f := 1622.318687$$

```
>
"Currents, Voltages, Complex Power";
```

$$ZLoad := ((ZL1 + RL1) + (R1 + R2));$$

$$Iorig := \frac{Vs}{ZLoad + R3};\ \text{polar}(\%);$$

$$VLoad := Iorig\cdot ZLoad;\ \text{polar}(\%);$$

$$S := \frac{Vs\cdot conjugate(Iorig)}{2};\#complex\ power$$

$$SLoad := \frac{VLoad\cdot conjugate(Iorig)}{2}:$$

"R3 is in series with the load therefore has same current";

$$IR3 := Iorig;$$

$$VR3 := Vs\cdot\left(\frac{R3}{ZLoad + R3}\right);\ \text{polar}(\%);$$

"Currents, Voltages, Complex Power"

$$ZLoad := 660.1 + 1041.503384\,\mathrm{I}$$

$$Iorig := 0.0004368721962 - 0.0006791102548\,\mathrm{I}$$

$$\text{polar}(0.0008074949251, -0.9991449598)$$

$$VLoad := 0.9956749652 + 0.006723191524\,I$$

$$\text{polar}(0.9956976638, 0.006752293249)$$

$$S := 0.0002184360981 + 0.0003395551274\,I$$

"R3 is in series with the load therefore has same current"

$$IR3 := 0.0004368721962 - 0.0006791102548\,I$$

$$VR3 := 0.004325034743 - 0.006723191523\,I$$

$$\text{polar}(0.007994199759, -0.9991449597)$$

> "Power factor"; $pf := \dfrac{\text{Re}(S)}{\text{abs}(S)}$;

"Power factor"

$$pf := 0.5410215999$$

>

"PF correction";

$ZC1 := \dfrac{1}{I \cdot \text{omega} \cdot C1}$:

$new\_ZLoad := \dfrac{1}{\dfrac{1}{ZC1} + \dfrac{1}{(ZL1 + RL1 + R1 + R2)}}$; polar(%);

$new\_I := \dfrac{Vs}{new\_ZLoad + R3}$; polar(%);##################

$new\_VLoad := new\_I \cdot new\_ZLoad$:

$S\_new := \dfrac{Vs \cdot conjugate(new\_I)}{2}$; #complex power

$\#SLoad\_new := \dfrac{new\_VLoad \cdot conjugate(new\_I)}{2}$:

$new\_pf := \dfrac{\text{Re}(S\_new)}{\text{abs}(S\_new)}$;

"PF correction"

$$new\_ZLoad := 2303.380259 - 1.061112123 \times 10^{-6}\,I$$

$$\text{polar}(2303.380259, -4.606760516 \times 10^{-10})$$

$$new\_I := 0.0004322865749 + 1.982918081 \times 10^{-13}\,I$$

$$\text{polar}(0.0004322865749, 4.587045252 \times 10^{-10})$$

$$S\_new := 0.0002161432874 - 9.914590405 \times 10^{-14}\,I$$

$$new\_pf := 1.000000000$$

> "Transmission Line Voltage and Current";

$new\_IR3 := new\_I$;

$new\_VR3 := Vs \cdot \left(\dfrac{R3}{R3 + new\_ZLoad}\right)$; polar($new\_VR3$);

"Transmission Line Voltage and Current"

$$new\_IR3 := 0.0004322865749 + 1.982918081 \times 10^{-13}\,\mathrm{I}$$

$$new\_VR3 := 0.004279637092 + 1.963088900 \times 10^{-12}\,\mathrm{I}$$

$$\mathrm{polar}(0.004279637092, 4.587045251 \times 10^{-10})$$

\>

```
"summary";
"before correction";

VLoad "Voltage"; Amp = abs(VLoad); phase = argument(VLoad);
"Current"Iorig; Amp = abs(Iorig); phase = argument(Iorig);
S "Complex Power"; Real = abs(S); Reactive = argument(S);
pf "Power Factor";
"Transmission Line Voltage"VR3; Amp = abs(VR3); phase = argument(VR3);
"Transmission Line Current"IR3; Amp = abs(IR3); phase = argument(IR3);
"";
"After correction";
new_I "Current"; Amp = abs(new_I); phase = argument(new_I);
S_new "Complex Power"; Real = Re(S_new); Reactive = Im(S_new);
new_pf "Power Factor";
"Transmission Line Voltage"new_VR3; Amp = abs(new_VR3); phase = argument(new_VR3);
"Transmission Line Current"new_IR3; Amp = abs(new_IR3); phase = argument((new_IR3));
```

"summary"

"before correction"

$(0.9956749652 + 0.006723191524\,\mathrm{I})$ "Voltage"

$Amp = 0.9956976638$

$phase = 0.006752293249$

$(0.0004368721962 - 0.0006791102548\,\mathrm{I})$ "Current"

$Amp = 0.0008074949251$

$phase = -0.9991449598$

$(0.0002184360981 + 0.0003395551274\,\mathrm{I})$ "Complex Power"

$Real = 0.0004037474625$

$Reactive = 0.9991449598$

0.5410215999 "Power Factor"

$(0.004325034743 - 0.006723191523\,\mathrm{I})$ "Transmission Line Voltage"

$Amp = 0.007994199759$

$phase = -0.9991449597$

$(0.0004368721962 - 0.0006791102548\,\mathrm{I})$ "Transmission Line Current"

$Amp = 0.0008074949251$

$phase = -0.9991449598$

""

$$\text{"After correction"}$$

$$(0.0004322865749 + 1.982918081 \times 10^{-13}\, \mathrm{I})\ \text{"Current"}$$

$$Amp = 0.0004322865749$$

$$phase = 4.587045252 \times 10^{-10}$$

$$(0.0002161432874 - 9.914590405 \times 10^{-14}\, \mathrm{I})\ \text{"Complex Power"}$$

$$Real = 0.0002161432874$$

$$Reactive = -9.914590405 \times 10^{-14}$$

$$1.000000000\ \text{"Power Factor"}$$

$$(0.004279637092 + 1.963088900 \times 10^{-12}\, \mathrm{I})\ \text{"Transmission Line Voltage"}$$

$$Amp = 0.004279637092$$

$$phase = 4.587045251 \times 10^{-10}$$

$$(0.0004322865749 + 1.982918081 \times 10^{-13}\, \mathrm{I})\ \text{"Transmission Line Current"}$$

$$Amp = 0.0004322865749$$

$$phase = 4.587045252 \times 10^{-10}$$

Digital

\>

$restart :$

$Vs := 1 : f := 1622.318687 :$

$\text{omega} := 2 \cdot \text{Pi} \cdot f :$

$R1 := 218.9 : R2 := 219.2 : R3 := 9.9 : RL1 := 222.0 :$

$L1 := 102.175\text{e}{-3} : ZL1 := I \cdot \text{omega} \cdot L1 :$

$C1 := 67.2\text{e}{-9} : ZC1 := \dfrac{1}{I \cdot \text{omega} \cdot C1} :$

$ZLoad := ZL1 + R1 + R2 + RL1 :$

>

"Measured Values:";

$Vsource = Vs$;

""; $VR3 := \frac{(15.9e-3)}{2}$;

$VR3\_DTime := -96.4 \cdot 10^{-6}$: $VR3\_DTime\_error := \frac{20e-6}{5}$: $VR3\_DTime\_relative :=$ $\mathrm{abs}\left(\frac{VR3\_DTime\_error}{VR3\_DTime}\right)$:

$VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \mathrm{Pi}$; $VR3\_Phase\_error := \mathrm{abs}(VR3\_Phase \cdot VR3\_DTime\_relative)$;

""; $Iorig := \frac{Vs}{ZLoad + R3}$; #confirm with probe

$Iamp := \mathrm{abs}(Iorig)$; $Iphase := \mathrm{argument}(Iorig)$;

"";

$S := \frac{Vs \cdot conjugate(Iorig)}{2}$;

$pf := \frac{\mathrm{Re}(S)}{\mathrm{abs}(S)}$;

"Measured Values:"

$$Vsource = 1$$

""

$$VR3 := 0.007950000000$$

$$VR3\_Phase := -0.9826369096$$

$$VR3\_Phase\_error := 0.04077331575$$

""

$$Iorig := 0.0004368721962 - 0.0006791102548\,\mathrm{I}$$

$$Iamp := 0.0008074949251$$

$$Iphase := -0.9991449598$$

""

$$S := 0.0002184360981 + 0.0003395551274\,\mathrm{I}$$

$$pf := 0.5410215999$$

\>

"Measured values after correction";

$new\_ZLoad := \dfrac{1}{\dfrac{1}{ZC1} + \dfrac{1}{(ZL1 + RL1 + R1 + R2)}};$

$Vsource = Vs;$

$VR3 := \dfrac{8.49\text{e}-3}{2};$

$VR3\_DTime := 0;$ "";

$VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \mathrm{Pi};$

$VR3\_Phase\_error := evalf\left(1 \cdot 10^{-6} \cdot 2 \cdot \mathrm{Pi}\right);$

""; $I\_new := \dfrac{Vs}{new\_ZLoad + R3}; I\_amp := \mathrm{abs}(I\_new);$

$I\_phase := \mathrm{argument}(I\_new);$

$I\_phase\_approx := fnormal(0.00000001);$ #matches multi

$S\_new := \dfrac{Vs \cdot conjugate(I\_new)}{2};$

$pf\_new := \dfrac{\mathrm{Re}(S\_new)}{\mathrm{abs}(S\_new)};$

"Measured values after correction"

$$new\_ZLoad := 2303.380259 - 1.061112123 \times 10^{-6}\,\mathrm{I}$$

$$Vsource = 1$$

$$VR3 := 0.004245000000$$

$$VR3\_DTime := 0$$

""

$$VR3\_Phase := 0.$$

$$VR3\_Phase\_error := 6.283185308 \times 10^{-6}$$

""

$$I\_new := 0.0004322865749 + 1.982918081 \times 10^{-13}\,\mathrm{I}$$

$$I\_amp := 0.0004322865749$$

$$I\_phase := 4.587045252 \times 10^{-10}$$

$$I\_phase\_approx := 1. \times 10^{-8}$$

$$S\_new := 0.0002161432874 - 9.914590405 \times 10^{-14}\,\mathrm{I}$$

$$pf\_new := 1.000000000$$

Physical

\>
$restart:$
$Vs := 1 : f := 1.62e+03 :$
$\text{omega} := 2 \cdot \text{Pi} \cdot f :$
$R1 := 218.9 : R2 := 219.2 : R3 := 9.9 : RL1 := 222.0 :$
$L1 := 102.175e-3 : ZL1 := I \cdot \text{omega} \cdot L1 :$
$C1 := 67.2e-9 : ZC1 := \frac{1}{I \cdot \text{omega} \cdot C1} :$
$ZLoad := ZL1 + R1 + R2 + RL1 :$

\>
"Before capacitor";
$Vsource = Vs; Vs\_error := 0.05;$ "";
$VR3 := \frac{(20.4e-3)}{2}; VR3\_error := VR3 \cdot 0.05;$
$VR3\_DTime := -96.0 \cdot 10^{-6} :$
$VR3\_DTime\_error := \frac{100e-6}{5} + 0.05 \cdot VR3\_DTime :$
$VR3\_DTime\_relative := \text{abs}\left(\frac{VR3\_DTime\_error}{VR3\_DTime}\right) :$
$VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \text{Pi};$
$VR3\_Phase\_error := \text{abs}(VR3\_Phase \cdot VR3\_DTime\_relative);$
""; $Iorig := \frac{Vs}{ZLoad + R3}; Iorig\_error := 0.05;$ *#from Vs*
$Iamp := \text{abs}(Iorig); I\_amp\_error := Vs\_error \cdot Iamp;$
$Iphase := \text{argument}(Iorig);$
$I\_phase\_error := Vs\_error \cdot \text{abs}(Iphase);$

$S := \frac{Vs \cdot conjugate(Iorig)}{2};$
$S\_error := Iorig\_error + Vs\_error;$ *#add relative uncertainties*
$pf\_approx := \frac{\text{Re}(S)}{\text{abs}(S)};$

"Before capacitor"

$Vsource = 1$

$Vs\_error := 0.05$

""

$VR3 := 0.01020000000$

$VR3\_error := 0.0005100000000$

$VR3\_Phase := -0.9771609792$

$VR3\_Phase\_error := 0.1547171550$

""

$$Iorig := 0.0004377566175 - 0.0006795124941\,\mathrm{I}$$

$$Iorig\_error := 0.05$$

$$Iamp := 0.0008083118741$$

$$I\_amp\_error := 0.00004041559370$$

$$Iphase := -0.9984939909$$

$$I\_phase\_error := 0.04992469954$$

$$S := 0.0002188783088 + 0.0003397562470\,\mathrm{I}$$

$$S\_error := 0.10$$

$$pf\_approx := 0.5415689558$$

\>

$restart :$

$Vs := 1 : f := 1.62\mathrm{e}{+}03 :$

$\mathrm{omega} := 2 \cdot \mathrm{Pi} \cdot f :$

$R1 := 218.9 : R2 := 219.2 : R3 := 9.9 : RL1 := 222.0 :$

$L1 := 102.175\mathrm{e}{-}3 : ZL1 := I \cdot \mathrm{omega} \cdot L1 :$

$C1 := 67.2\mathrm{e}{-}9 : ZC1 := \dfrac{1}{I \cdot \mathrm{omega} \cdot C1} :$

"After Capacitor";

$ZLoad := \dfrac{1}{\dfrac{1}{ZC1} + \dfrac{1}{(ZL1 + RL1 + R1 + R2)}};$

$Vsource = \dfrac{2}{2}; Vs\_error := 0.05; "";$

$VR3 := \dfrac{(14.8\mathrm{e}{-}3)}{2}; VR3\_error := VR3 \cdot 0.05;$

$VR3\_DTime := 0 \cdot 10^{-6} :$

$VR3\_DTime\_error := \dfrac{200\mathrm{e}{-}6}{5} :$

$VR3\_Phase := VR3\_DTime \cdot f \cdot 2 \cdot \mathrm{Pi};$

$VR3\_Phase\_error := \mathrm{abs}(VR3\_DTime\_error \cdot f \cdot 2 \cdot \mathrm{Pi});$

$""; Iorig := \dfrac{Vs}{ZLoad + R3}; Iorig\_error := 0.05;$ *#from Vs*

$Iamp := \mathrm{abs}(Iorig); I\_amp\_error := Vs\_error \cdot Iamp;$

$Iphase := \mathrm{argument}(Iorig);$

$I\_phase\_error := Vs\_error \cdot \mathrm{abs}(Iphase);$

$S := \dfrac{Vs \cdot conjugate(Iorig)}{2};$

$S\_error := Iorig\_error + Vs\_error;$ *#add relative uncertainties*

$pf\_approx := \dfrac{\mathrm{Re}(S)}{\mathrm{abs}(S)};$

"After Capacitor"

$$ZLoad := 2298.662627 + 7.380343059\,I$$

$$Vsource = 1$$

$$Vs\_error := 0.05$$

""

$$VR3 := 0.007400000000$$

$$VR3\_error := 0.0003700000000$$

$$VR3\_Phase := 0.$$

$$VR3\_Phase\_error := 0.4071504080$$

""

$$Iorig := 0.0004331655412 - 1.384805531 \times 10^{-6}\,I$$

$$Iorig\_error := 0.05$$

$$Iamp := 0.0004331677548$$

$$I\_amp\_error := 0.00002165838774$$

$$Iphase := -0.003196932077$$

$$I\_phase\_error := 0.0001598466038$$

$$S := 0.0002165827706 + 6.924027655 \times 10^{-7}\,I$$

$$S\_error := 0.10$$

$$pf\_approx := 0.9999948897$$