Example 3.12

A load is to be rotated by half a revolution, pause for 0.4 s, and then returned to its original position. After the return motion the load should be kept stationary for 0.6 s before the cycle restarts. Each motion should consist of a period of acceleration followed by an equal period of deceleration, and be completed in 1 second. The load has a moment of inertia of 0.025 kgm^2. The ambient temperature is 25 °C. The motor parameters are: moment of inertia=1.10×10^{-5} kgm^2, max. torque =1.5 Nm, max. continuous torque = 0.45 Nm, max. speed = 7000 rpm, temperature limit = 100 °C, torque constant = 0.1 Nm/A, resistance at temperature limit = 2.15 ohm, and total thermal resistance = 1.43 °C/W. A gear box will be attached between the motor and the load. The available gear ratios are: 5, 10, 15, etc. You may assume the torque ratings are independent of the speed and that the friction torques are negligible.

a) List the periods that make up the motion profile in order

b) Determine the best gear ratio for this application using the method of section 3.4.3. Check that the resulting motor speed, torques and temperature are within their rated values. Remember to keep the inertia ratio between 1 and 10.

c) Repeat part (b) for a load inertia of 0.25 kgm^2.

Solution

a) If the first motion is forwards, then periods are:

1. Accelerate forwards,
2. Decelerate forwards,
3. Dwell 0.4 s,
4. Accelerate backwards,
5. Decelerate backwards and
6. Dwell 0.6 s.

Sketch acc'n and vel'y vs. time on the board (ask first)

b) The optimal gear ratio is:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{0.025\ \text{kgm}^2}{1.10\times10^{-5}\ \text{kgm}^2}} = 47.7$$

However the available gear ratios are 5, 10, 15, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 45$.

Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio.

The motor's rated max. speed is:

$$\omega_{rated,max} = 7000(2\pi/\text{rev})(1\text{ min}/60\text{ s}) = 733\text{ rad/s}$$

From the given information, the load should be moved π radians in 1 second (i.e. t_{move}=1 s), with t_{run}=0 and t_{acc}= t_{dec}= 0.5 s.

Load acceleration is: $\alpha_{con} = 4\theta_{move}/t_{move}^2 = 4(\pi)/(1\text{ s})^2 = 12.57\text{ rad/s}^2$

Max. load velocity is: $\omega_{\max} = \frac{1}{2}\alpha_{con}t_{move} = \frac{1}{2}(12.57\text{ rad/s}^2)(1\text{ s}) = 6.28\text{ rad/s}$

Max. required motor speed is: $\omega_{motor,max} = N_r\omega_{\max} = (45)(6.28\text{ rad/s}) = 283\text{ rad/s}$

Since $\omega_{motor,max} < \omega_{rated,max}$ this gear ratio passes the motor speed check.

To check the torques we must first calculate the motor torque profile. In this example there are no torques due to friction or gravity so $\tau_{external} = 0$. It's obvious that the max. required torque is needed during the acceleration period, therefore:

$$\tau_{motor,\max} = \left(J_{motor}N_r + \frac{1}{N_r}J_{load}\right)\alpha_{load} = \left((1.10\times10^{-5}\text{ kgm}^2)(45) + \frac{1}{45}(0.025\text{ kgm}^2)\right)(12.57\text{ rad/s}^2) = 0.0132\text{ Nm}$$

<u>Optional:</u> For periods 2 & 4: $\tau_{motor,2} = \tau_{motor,4} = -0.0132\text{ Nm}$. For idle periods: $\tau_{motor,3} = \tau_{motor,6} = 0$.

For the given motor: $\tau_{rated,cont} = 0.45\text{ Nm}$. So we have $\tau_{motor,max} < \tau_{rated,cont}$ and there is no need to calculate $\tau_{motor,RMS}$.

Finally, since $\tau_{motor,max} < \tau_{rated,cont}$ its operating temperature will not exceed its limit. <u>Therefore this motor and gearbox combination has passed all of the tests and is acceptable.</u>

a) For a load inertia of 0.25 kgm^2 the optimal gear ratio is:

$$N_{r,opt} = \sqrt{\frac{J_{load}}{J_{motor}}} = \sqrt{\frac{0.25\text{ kgm}^2}{1.10\times10^{-5}\text{ kgm}^2}} = 150.8$$

However the available gear ratios are 5, 10, 15, etc. The closest smaller gear ratio should be chosen to make the inertia ratio slightly larger than 1. Therefore the best choice is: $N_r = 150$.

Next, we must check if the motor will operate properly with this gear ratio. Since N_r is very close to $N_{r,opt}$ it is not necessary to check the inertia ratio. From part (a), $\omega_{\max} = 6.28\text{ rad/s}$ so the max. required motor speed is:

$$\omega_{motor,max} = N_r\omega_{\max} = (150)(6.28\text{ rad/s}) = 942\text{ rad/s}$$

Since $\omega_{motor,max} > \omega_{rated,max}$ <u>this gear ratio fails the motor speed check</u>.

The upper limit on the gear ratio is:

$$N_{r,max} = \frac{\omega_{rated,\,max}}{\omega_{\max}} = \frac{733\ \text{rad/s}}{6.28\ \text{rad/s}} = 116.7$$

We should round down to the nearest available ratio to keep $\omega_{motor,max} \le \omega_{rated,\ max}$. Therefore our new choice for the best ratio is: $N_r = 115$. Now we need to check the inertia ratio. It is:

$$Ratio_J = \frac{J_{load} / N_r^{\,2}}{J_{motor}} = \frac{(0.25\ \text{kgm}^2)/(115^2)}{(1.10\times 10^{-5}\ \text{kgm}^2)} = 1.72$$

Since it is within the range 1 to 10 it is acceptable.

Due to the change in gear ratio and load inertia the new max. required torque is:

$$\begin{aligned}\tau_{motor,\max} &= \left(J_{motor} N_r + \frac{1}{N_r} J_{load} \right) \alpha_{load} \\ &= \left((1.10\times 10^{-5}\ \text{kgm}^2)(115) + \frac{1}{115}(0.25\ \text{kgm}^2) \right)(12.57\ \text{rad/s}^2) = 0.0432\ \text{Nm}\end{aligned}$$

This torque is well below the motor’s rated max. torque of 1.5 Nm. Since the max. required torque is also below its rated max. continuous torque there is no need to calculate $\tau_{motor,\,RMS}$.

Finally, the motor’s operating temperature should be compared with its temperature limit. However, since $\tau_{motor,\,max}$ is less than its rated max. continuous torque its operating temperature will not exceed its limit.

Therefore this motor and gearbox combination has passed all of the tests and is acceptable.