ENGPHYS 2A04 Winter 2022 – Assignment 1 Solutions

1. The electric field of a wave traveling in space is proportional to the following equation:
$$E(x,t) = \cos(3\pi \times 10^{14} t - \pi \times 10^6 x)$$
   a) Find the direction of propagation, and the phase velocity of this wave. What is the significance of this value?
   b) Find the wavelength of this wave. What part of the electromagnetic spectrum does this fall under?

Solution:

a) *The t and x terms have opposite signs, so the wave is travelling in the positive x direction. The phase velocity can be found according to $u_P = \frac{\omega}{\beta}$:*
$$u_P = \frac{\omega}{\beta} = \frac{3\pi \times 10^{14}}{\pi \times 10^6} = 3 \times 10^8 \frac{m}{s}$$
$$\boldsymbol{u_P = 3 \times 10^8 \frac{m}{s}}$$
This is significant, because this value is approximately c, the speed of light in a vacuum.

b) *The wavelength can be found according to $\beta = \frac{2\pi}{\lambda}$:*
$$\beta = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{\beta}$$
$$\lambda = \frac{2\pi}{\pi \times 10^6} = 2 \times 10^{-6}\ m$$
$$\boldsymbol{\lambda = 2\ \mu m}$$

This wavelength, $2\mu m$, falls within the infrared part of the electromagnetic spectrum.

2. An electromagnetic wave is propagating in the z direction in a lossy medium with attenuation constant $\alpha = 0.4\ Np/m$. If the wave's electric-field amplitude is $63\ V/m$ at $z = 2m$, how far can the wave travel before its amplitude is reduced to $10\ V/m$?

Solution:

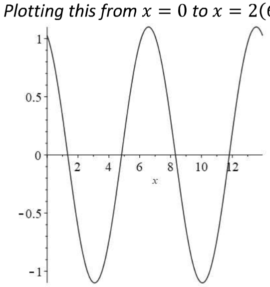
First, realize that the amplitude of the wave at a given z-value is represented by $A(z) = A_0 e^{-\alpha z}$. Since $A(2) = 63\ V/m$ is known, A_0 can be found:
$$A_0 = \frac{A(z)}{e^{-\alpha z}} = \frac{A(2)}{e^{-(0.4)(2)}} = \frac{63}{e^{-(0.4)(2)}} = 140.21\ V/m$$

With this knowledge, the amplitude function can be rearranged as follows:
$$A(z) = A_0 e^{-\alpha z}$$
$$\frac{A(z)}{A_0} = e^{-\alpha z}$$
$$ln\left(\frac{A(z)}{A_0}\right) = -\alpha z$$
$$z = \frac{-ln\left(\frac{A(z)}{A_0}\right)}{\alpha}$$
Substituting in the known values to find where $A(z) = 10\ V/m$:
$$z = \frac{-ln\left(\frac{10}{140.21}\right)}{0.4}$$
$$\therefore \boldsymbol{z = 6.6\ m}$$

3. On a windy day, the height of a wave on the lake (in meters) is described by:
$$h(x,t) = 1.1\sin(0.6t - 0.9x)$$
Determine the wavelength. Plot the height of the wave at $t = 2\ s$, over a distance of 2 wavelengths starting at $x = 0$.

Solution:

The wavelength can be found according to:
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.9}$$
$$\boldsymbol{\lambda = 6.98\ m}$$
At $t = 2\ s$, the expression simplifies to:
$$h(x,t) = 1.1\,sin(0.6(2) - 0.9x) = 1.1\,sin(1.2 - 0.9x)$$
Plotting this from $x = 0$ to $x = 2(6.98) = 13.96\ m$ yields:

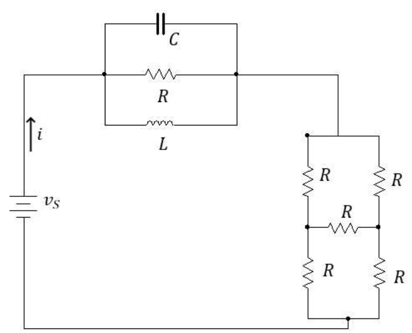


4. The electric current through a point in a circuit is given by
$$i(t) = 0.06\cos(120\pi t)$$
Elsewhere in the circuit, the voltage across some component is given by
$$v(t) = 1.2\sin(120\pi t + 30°)$$
Is this voltage lagging or leading the current? By what phase angle?

Solution:

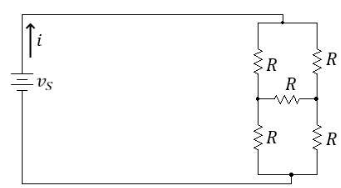
On its own, sin(x) will lag cos(x) by 90°. Adding in the 30° phase shift means that overall, ***the voltage lags the current by 60°.***

5. Find an expression for the current i in the circuit below, in terms of the source voltage v_S, the capacitance C, the inductance L, and the resistance R. Assume the voltage source is steady-state DC.

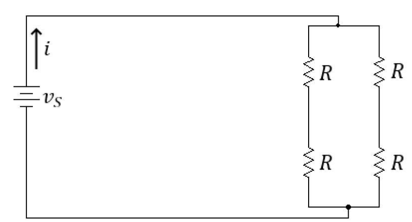


Solution:

In steady state DC, the inductor will short. This means the circuit can be simplified as follows:



By symmetry, the resistor in the center of the block will have no voltage difference across it – it can be treated as open, because no current will flow.



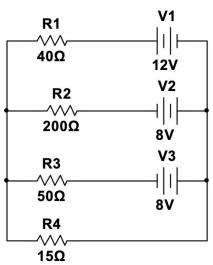
Finding the current is then trivial:

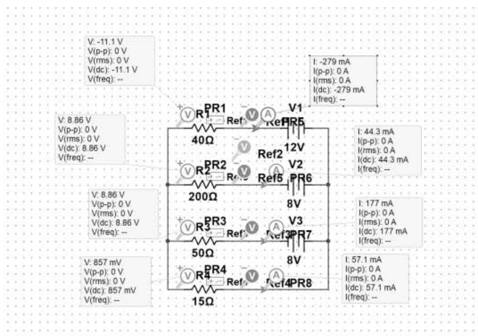
$$R_{equivalent} = \frac{1}{1/_{2R} + 1/_{2R}} = \frac{1}{2/_{2R}} = \frac{1}{1/_{R}} = R$$

$$i = \frac{v_S}{R_{equivalent}} = \frac{v_S}{R}$$

$$\therefore \boldsymbol{i = \frac{v_S}{R}}$$

1. **DC Circuit Analysis**
   5 marks total: 2 marks currents, 2 marks voltages , 1 mark for showing your work
   Using Kirchhoff's rules, (a) -
   a. Find the current across each resistor shown below
   b. And the find the potential difference across each resistor





Resistor	Current (mA)	Voltage (V)
1	-279	-11.1
2	44.3	8.86
3	177	8.86
4	57.1	0.857

Approach ①: Nodal Analysis $\quad V_A$ = Voltage at the left junction

① $I_1 + I_2 + I_3 + I_4 = 0$

$$\frac{V_A+V_1}{R_1} + \frac{V_A-V_2}{R_2} + \frac{V_A-V_3}{R_3} + \frac{V_A}{R_4}$$

$$\frac{V_A+12}{40} + \frac{V_A-8}{200} + \frac{V_A-8}{50} + \frac{V_A}{15} = 0$$

$$\frac{15(V_A+12)}{600} + \frac{3(V_A-8)}{600} + \frac{12(V_A-8)}{600} + \frac{40V_A}{600} = 0$$

$$\frac{70V_A}{600} + \frac{60}{600} = 0$$

$$V_A = 0.857$$

② Substitute V_A into current equations

$I_1 = \frac{V_A+12}{40} = 0.279$ A

$I_2 = \frac{V_A-8}{200} = -0.044$ A

$I_3 = \frac{V_A-8}{50} = -0.177$ A

$I_4 = \frac{V_A}{15} = -0.0571$ A

③ Use Ohm's Law to find the voltage across each resistor

$V_1 = I_1R_1 = 11.1$ V

$V_2 = I_2R_2 = 8.85$

$V_3 = I_3R_3 = 8.85$

$V_4 = I_4R_4 = 0.857$

2. **AC Circuit Analysis:** A certain lightbulb is rated at 90.0 W when operating at an rms voltage of 120 V.
   a. What is the peak voltage applied across the bulb? (1)
   $$V_{max} = 120\sqrt{2} = \sim 170$$
   b. What is the resistance of the bulb? (1)
   $$P = VI = \frac{V^2}{R} \rightarrow 90 = \frac{(120)^2}{R} \rightarrow R = 160\Omega$$
   c. Does a 100-W bulb have greater or less resistance than a 60.0-W bulb? Explain.

It has less resistance. 100 W light bulb has a resistance of 144 Ohms. (2)

3. **AC Circuit Analysis:** An AC source with an output rms voltage of 48.0 V at a frequency of 60.0 Hz is connected across a 11.0-mF capacitor. Find:
   a. the capacitive reactance (1)
   $$X_c = \frac{1}{wC} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60)(11 \times 10^{-3})} = 0.241\ \Omega$$
   b. the rms current, and (1)
   $$I_{rms} = \frac{V_{rms}}{X_C} = \frac{48}{241} = 200A$$
   c. the maximum current in the circuit. (1)
   $$I_{max} = \sqrt{2} I_{rms} = 280\ A$$
   d. Does the capacitor have its maximum charge when the current has its maximum value? Explain. (2)
   No. The capacitor will have minimum charge when the current is the maximum. The current and voltage are out of phase by 90˚.

4. **Phasors**:
   a. Find the phasors of the following time function
      i. $v(t) = 10\cos\left(\omega t - \frac{\pi}{4}\right)$ (V) (1)
      $$\tilde{V} = 10e^{-j\frac{\pi}{4}}$$
      ii. $v(t) = 30\sin\left(\omega t - \frac{\pi}{4}\right)$ (V) (1)
      $$\tilde{V} = 30e^{j\left(-\frac{\pi}{4}-\frac{\pi}{2}\right)} = 30e^{-j\frac{3\pi}{4}}$$
      iii. $i(x,t) = 10e^{-3x} sin(\omega t + \pi/4)\ (A)$ (1)
      $$\tilde{I} = 10e^{-3x}e^{j\left(\frac{\pi}{4}-\frac{\pi}{2}\right)} = 10e^{-3x}e^{-j\frac{\pi}{4}}$$
   b. Find the instantaneous time sinusoidal function corresponding to the following phasors
      i. $\tilde{I} = (6 + j8)$ (A) (1)
      $$\tilde{I} = 10e^{j53^\circ} \rightarrow i(t) = \cos(\omega t + 53^\circ) = \cos(\omega t + 0.927)$$
      ii. $\tilde{V} = j$ (V) (1)
      $$v(t) = cos(\omega t + \pi/2) = -\sin(\omega t)$$

5. **Bonus Question**: A source delivers an AC voltage of the form $\Delta v = 100\sin 50\pi t$, where Δv is in volts and t is in seconds, to a capacitor. The maximum current in the circuit is 0.500 A.
   a. Find for the rms voltage of the source (1)
   $$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}}$$
   $$\Delta V_{rms} = \frac{100}{\sqrt{2}}$$
   $$\Delta V_{rms} = 70.7\ V$$
   b. Find the frequency of the source (1)
   Given the general expression of a AC source:
   $$\Delta v = v_0 \sin 2\pi f t$$
   We can solve for the frequency:
   $$2\pi f = 50\pi$$
   $$f = 25\ Hz$$
   c. Find the value of the capacitance (3)
   $$X_c = \frac{1}{2\pi f C}$$
   First we must find the capacitive reactance. Given the capacitor is the sole element in the circuite, capacitance can be solved sing:
   $$X_c = \frac{\Delta V_{max}}{I_{max}}$$
   $$\frac{1}{2\pi f C} = \frac{\Delta V_{max}}{I_{max}}$$
   $$2\pi f C = \frac{I_{max}}{\Delta V_{max}}$$
   $$C = \frac{I_{max}}{\Delta V_{max}\, 2\pi f}$$
   $$C = \frac{0.5}{(100)\ 2\pi(25)}$$
   $$C = 31.8 \mu F$$

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal marks (except bonus questions).
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.

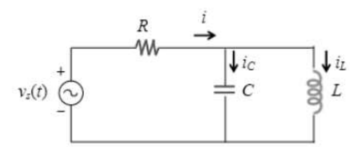
ENGPHYS 2A04 Assignment 3 Solutions

1. **AC Circuit Analysis**

The circuit shown below has the voltage source given by the equation:

$$V_S(t) = 7\cos(2 * 10^4 t - 60°)$$

If the circuit has the following values of R = 50 Ω, C = 12 μF, and L = 0.6 mH, use phasor analysis to acquire an expression for the current flowing through the inductor $i_L(t)$.



Voltage to Phasor:

$$V_S(t) = 7\cos(2 * 10^4 t - 60°)$$

$$\tilde{V}_S = 7e^{-60° j}$$

$$\omega = 2 * 10^4$$

Converting circuit elements to phasor:

$$Z_R = R = 50\ \Omega$$

$$Z_c = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -\frac{j}{(2 * 10^4)(12 * 10^{-6})} = -\frac{j}{0.24} = -4.17j\ \Omega = 4.17e^{-90°}\ \Omega$$

$$Z_L = j\omega L = j(2 * 10^4)(0.6 * 10^{-3}) = 12j\ \Omega = 12e^{90°}\ \Omega$$

Applying Kirchoff's Laws:

Junction Rule: $I = I_c + I_L$

Loop 1 (left):

$$\tilde{V}_S = IR + I_c Z_c$$

$$\tilde{V}_S = IR + (I - I_L)Z_c$$

$$\tilde{V}_S = I(R + Z_c) - I_L Z_c$$

Rearrange for current:

$$I = \frac{\tilde{V}_S + I_L Z_c}{R + Z_c}$$

Loop 2 (Right):

$$\tilde{V}_s = IR + I_L Z_L$$

Substituting *I* from previous equation:

$$\tilde{V}_s = \frac{\tilde{V}_s + I_L Z_c}{R + Z_c} R + I_L Z_L$$

$$\tilde{V}_s (R + Z_c) = \tilde{V}_s R + I_L R Z_c + I_L Z_L (R + Z_c)$$

$$\tilde{V}_s R + \tilde{V}_s Z_c = \tilde{V}_s R + I_L R Z_c + I_L R Z_L + I_L Z_L Z_c$$

$$I_L = \tilde{V}_s \frac{Z_c}{R Z_L + R Z_c + Z_L Z_c}$$

Substituting all values:

$$I_L = \frac{7e^{-60°j} 4.17e^{-90°}}{50(12j) + 50(-4.17j) + 12j(-4.17j)} = \frac{7e^{-60°j} 4.17e^{-90°}}{50.04 + 391.5j} = \frac{29.19e^{-150°}}{395e^{82.7°j}} = \frac{29.19e^{210°j}}{395e^{82.7°j}}$$

$$I_L = 0.07e^{127°j} \text{ A} = 0.07e^{-233°j} \text{ A}$$

Converting back to time domain:

$$i_L(t) = 0.07 \cos(2 * 10^4 t + 127°) \text{ A}$$

$$i_L(t) = 0.07 \cos(2 * 10^4 t - 233°) \text{ A}$$

2. **Transmission Lines**

A transmission line of length *l* is connected to a sinusoidal voltage source with a certain frequency *f*. If we assume the velocity of the wave propagates at a certain velocity *c*, in which of the following conditions shown below is it reasonable to ignore the transmission line effects within the solution of the circuit? In addition, explain the difference between a dispersive and non-dispersive transmission line.

(a) l = 5.3 cm, f = 37 kHz,
(b) l = 8.2 km, f = 475 Hz,
(c) l = 7.7 cm, f = 770 MHz,
(d) l = 2.3 mm, f =98 GHz

Using equation

$$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{lf}{c} = \frac{lf}{3 * 10^8 \frac{m}{s}}$$

If $\frac{l}{\lambda} \le 0.01$, then it is negligible.

a) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{5.3*10^{-2}\ m*37*10^3\ Hz}{3*10^8\ m/s} = 6.54 * 10^{-6}$

b) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{8.2*10^3\ m*475\ Hz}{3*10^8\ m/s} = 0.013$

c) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{7.7*10^{-2}\ m*770*10^6\ Hz}{3*10^8\ m/s} = 0.198$

d) $\frac{l}{\lambda} = \frac{lf}{3*10^8\ m/s} = \frac{2.3*10^{-3}\ m*98*10^9\ Hz}{3*10^8\ m/s} = 0.751$

Therefore, situation A would be the only situation negligible since it is the only one less than 0.01.

The difference between a dispersive and non-dispersive transmission line.

- Dispersive transmission line is one which the wave velocity is not constant as a function of the oscillating frequency f. There is distortion of the wave's shape due to different frequency components of the wave propagating at different velocities along the transmission line (degree of distortion depends on length)
- On the other hand, non-dispersive transmission lines have no distortion of the wave's shape due

3. **Transmission Lines**

A two-wire gold transmission line is embedded in an unknown dielectric material that has the following parameters: $\varepsilon_r = 2.7$ and $\sigma = 3.2 * 10^{-6}$ S/m. The wires are separate by a width of 1 cm and their radii are 1 mm each. Calculate the line parameters *R', L', G',* and *C'* at 1 GHz. (Refer to Appendix B for μ_c and σ_c of gold) Assume that $\mu_c = \mu_0$ for the dielectric material.

Values:

$$f = 1 * 10^9\ Hz$$

$$d = 2 * 10^{-3}\ m$$

$$D = 1 * 10^{-2}\ m$$

$$\varepsilon_r = 2.7$$

$$\sigma = 3.2 * 10^{-6}\ S/m$$

$$\varepsilon_0 = 8.854 * 10^{-12}\ F/m$$

For Silver:

$$\sigma_c = 4.1 * 10^7\ S/m$$

$$\mu_c = \mu_0 = 4\pi * 10^{-7}\ \text{H/m}$$

Solution:

$$R' = \frac{2}{\pi d}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{\pi * 2 * 10^{-3}}\sqrt{\frac{\pi * 1 * 10^{9} * 4\pi * 10^{-7}}{4.1 * 10^{7}}} = 3.12\ \Omega/m$$

Using:

$$\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1} = \ln\left[\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right) + \sqrt{\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right)^2 - 1} = 2.292\right.\right.$$

$$L' = \frac{\mu}{\pi}\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1} = \frac{4\pi * 10^{-7}}{\pi}\ln\left[\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right) + \sqrt{\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right)^2 - 1}\right.\right.$$

$$L' = 9.168 * 10^{-7}\ H/m$$

$$G' = \frac{\pi\sigma}{\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right.} = \frac{\pi * 3.2 * 10^{-6}}{\ln\left[\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right) + \sqrt{\left(\frac{1 * 10^{-2}}{2 * 10^{-3}}\right)^2 - 1}\right.}$$

$$G' = 4.39 * 10^{-6}\ S/m$$

$$C' = \frac{\pi\epsilon}{\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right.} = \frac{\pi\varepsilon_0\varepsilon_r}{\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right.} = \frac{\pi * 8.854 * 10^{-12} * 2.7}{2.292}$$

$$C' = 3.28 * 10^{-11}\ F/m$$

4. **Bonus Question: Reading/Research Question:**

The first transatlantic transmission line had been destroyed because high voltages had been employed to compensate for a poor, distorted signal. Briefly explain how Oliver Heaviside's work and the Heaviside condition resolved this issue, and what had been changed physically to the earlier transmission lines to improve signal quality.

- Heaviside condition is satisfied when $\frac{G}{C} = \frac{R}{L}$
  - Where G is shunt conductance
  - C is capacitance
  - R is series resistance
  - L is inductance
- Series resistance and shunt cause loss in the line
- For an ideal transmission, R = G =0
- Heaviside's findings aided in coming to solution for early transmission lines since adjusting the values of these variables in this condition could allow the engineers to come close to satisfying the condition and therefore having no loss in the signal
- Different options for changing values
  - G could be increased → but would increase loss
  - decreasing R and C would make cable bulky
  - Therefore the alternative solution of adding loading coils (load to transmission lines)
    - This would decrease voltage travelling across transmission lines
    - Decrease in voltage would be achieved by increasing inductance → leading to no signal distortion

ENGPHYS 2A04 Winter 2022 – Assignment 4 Solutions

DUE MONDAY FEBRUARY 14th, 8AM

1. Consider a coaxial air line with inner diameter 13 mm, and outer diameter 15 mm. Both conductors are made of copper (refer to your textbook for material constants).
   a. Compute the transmission line parameters for a signal at 10 kHz.

Solution:

Copper: $\mu_{Cu} = 1\mu_0 = 4\pi \times 10^{-7}\frac{H}{m}, \sigma_{Cu} = 5.8 \times 10^7 \frac{S}{m}$

$$a = \frac{13mm}{2} = 6.5mm, b = \frac{15mm}{2} = 7.5mm$$

$$R_S = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \sqrt{\frac{\pi(10000)(4\pi \times 10^{-7})}{(5.8 \times 10^7)}} = 2.609 \times 10^{-5}$$

$$\boldsymbol{R'} = \frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{2.609\times10^{-5}}{2\pi}\left(\frac{1}{6.5\times10^{-3}}+\frac{1}{7.5\times10^{-3}}\right) = \mathbf{1.19 \times 10^{-3}\ \Omega/m}$$

$$\boldsymbol{L'} = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times10^{-7}}{2\pi}\ln\left(\frac{7.5}{6.5}\right) = \mathbf{2.86 \times 10^{-8}\ H/m}$$

$\boldsymbol{G'} = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi(0)}{\ln(7.5/6.5)} = \mathbf{0\ S/m}$, *because the insulator is air which has no conductance.*

$$\boldsymbol{C'} = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{2\pi(8.854\times10^{-12})}{\ln(7.5/6.5)} = \mathbf{3.89 \times 10^{-10}\ F/m}$$

   b. Compute the attenuation constant and the phase constant of the transmission line at the specified operating frequency.

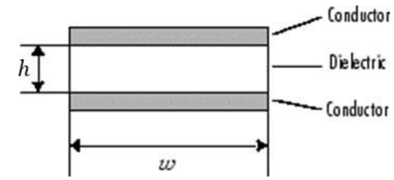
Solution:

$$\omega = 2\pi f = 2\pi(10000) = 20000\pi$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$
$$= \sqrt{\left(1.19 \times 10^{-3} + j(20000\pi)(2.86 \times 10^{-8})\right)\left(0 + j(20000\pi)(3.89 \times 10^{-10})\right)}$$
$$= \sqrt{(1.19 \times 10^{-3} + j1.80 \times 10^{-3})(j2.44 \times 10^{-5})}$$
$$= \sqrt{-4.392 \times 10^{-8} + j2.904 \times 10^{-8}} = \sqrt{5.265 \times 10^{-8}e^{j2.557}}$$
$$= (5.265 \times 10^{-8})^{0.5}e^{j2.557^{0.5}} = 2.295 \times 10^{-4}e^{j1.279} = 6.6 \times 10^{-5} + j2.2 \times 10^{-4}$$
$$\boldsymbol{\alpha = \Re e\{\gamma\} = 6.6 \times 10^{-5}\ Np/m}$$

$$\boldsymbol{\beta = \Im m\{\gamma\} = 2.2 \times 10^{-4}\ rad/m}$$

2. Calculate the phase velocity of a $3kHz$ signal travelling on a parallel-plate transmission line. Assume the conductors are gold, and assume the dielectric is air. The conductors are $5mm$ wide, separated by a $2mm$ dielectric. Does this value make sense?



Solution:

Because of the resistances of the conductors and the dielectric, $G' = 0$. This results in:

$$\beta = \Im m\left\{\sqrt{(R' + j\omega L')(j\omega C')}\right\} = \Im m\left\{\sqrt{(j\omega R' - \omega^2 L')(C')}\right\} = \Im m\left\{\sqrt{(j\omega R'C' - \omega^2 L'C')}\right\}$$

From $L'C' = \mu\epsilon$:

$$\beta = \Im m\left\{\sqrt{j\omega R'C' - \omega^2\mu\epsilon}\right\}$$

Substituting in expressions for R' and C':

$$\beta = \Im m\left\{\sqrt{j\omega\frac{2R_S}{w}\frac{\epsilon w}{h} - \omega^2\mu\epsilon}\right\} = \Im m\left\{\sqrt{j\omega\frac{2R_S\epsilon}{h} - \omega^2\mu\epsilon}\right\} = \Im m\left\{\sqrt{j\omega\frac{2\sqrt{\pi f\mu_c/\sigma_c}\epsilon}{h} - \omega^2\mu\epsilon}\right\}$$

Inputting the known values:

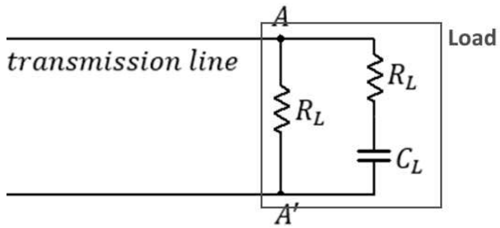
$$\beta =$$
$$= \Im m\left\{\sqrt{j\left(2\pi(3 \times 10^3)\right)\frac{2\sqrt{\pi(3 \times 10^3)(4\pi \times 10^{-7})/(4.1 \times 10^7)}8.854 \times 10^{-12}}{2 \times 10^{-3}} - \left(2\pi(3 \times 10^3)\right)^2\mu_0\epsilon_0}\right\}$$
$$= \Im m\left\{\sqrt{j(18849.6)\frac{2(1.6996 \times 10^{-5})8.854 \times 10^{-12}}{2 \times 10^{-3}} - (3.553 \times 10^8)(4\pi \times 10^{-7})(8.854 \times 10^{-12})}\right\}$$
$$= \Im m\left\{\sqrt{j2.837 \times 10^{-9} - 3.953 \times 10^{-9}}\right\} = \Im m\left\{\sqrt{10^{-9}(-3.953 + j2.837)}\right\}$$
$$= \Im m\left\{\sqrt{(4.8657 \times 10^{-9})e^{j2.5191}}\right\} = \Im m\{6.975 \times 10^{-5}e^{j1.2596}\}$$
$$= \Im m\{2.136 \times 10^{-5} + j6.64 \times 10^{-5}\} = 6.64 \times 10^{-5}\ rad/s$$

Calculating phase velocity based on this:

$$\boldsymbol{u_P} = \frac{\boldsymbol{\omega}}{\boldsymbol{\beta}} = \frac{\mathbf{2\pi f}}{\mathbf{6.64 \times 10^{-5}}} = \frac{\mathbf{2\pi(3000)}}{\mathbf{6.64 \times 10^{-5}}} = \mathbf{2.84 \times 10^8\ m/s}$$

Yes, it makes sense because it is slightly less than the speed of light in a vacuum.

3. The following load is placed at the end of a 120Ω transmission line carrying a 2MHz signal, with $R_L = 80\Omega$ and $C_L = 11nF$:



Calculate the voltage reflection coefficient at the load, and report it in polar form. Explain the meaning of this coefficient.

Solution:

Calculate the impedance of the load.

$$Z_{load} = \left(\frac{1}{R_L} + \frac{1}{R_L + Z_C}\right)^{-1} = \left(\frac{1}{R_L} + \frac{1}{R_L - \frac{j}{\omega C}}\right)^{-1} = \left(\frac{1}{R_L} + \frac{1}{R_L - \frac{j}{2\pi f C}}\right)^{-1}$$
$$= \left(\frac{1}{80} + \frac{1}{80 - \frac{j}{2\pi(2\times10^6)(11\times10^{-9})}}\right)^{-1} = \left(0.0125 + \frac{1}{80 - j7.234}\right)^{-1}$$
$$= \left(0.0125 + (0.01240 + j1.121\times10^{-3})\right)^{-1} = (0.0249 + j1.121\times10^{-3})^{-1}$$
$$= 40.1 - j1.805 = 40.1e^{-j0.0450}$$

Next, compute the normalized load impedance.

$$z_L = \frac{Z_{Load}}{Z_0} = \frac{40.1e^{-j0.0450}}{120} = 0.334e^{-j0.0450} = 0.334 - j0.0150\ \Omega$$

From here, the voltage reflection coefficient can be computed.

$$\boldsymbol{\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{0.334 - j0.0150 - 1}{0.334 - j0.0150 + 1} = \frac{-0.666 - j0.0150}{1.334 - j0.0150} = 0.5e^{-j3.1}}$$

This coefficient gives the ratio of amplitudes of the reflected and incident voltage waves.

4. A 10-metre section of a 100Ω lossless transmission line is driven by a source with $v_g(t) = 12\cos\left(2\pi\times10^6 t - \frac{\pi}{3}\right)\ (V)$, and $Z_g = 150\Omega$. The line has relative permittivity $\epsilon_r = 2.1$, and is terminated by a load with impedance $Z_L = (120 - j40)\Omega$. Express complex values in polar form. Determine:
    a. λ on the line.
    b. The reflection coefficient at the load.
    c. The input impedance.
    d. The input voltage $\widetilde{V_i}$.
    e. The time-domain input voltage $v_i(t)$.

Solution:

a) Lossless, so $u_P = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{2.1}} = 2.07\times10^8\ m/s$. *Then,*

$$\boldsymbol{\lambda = \frac{2\pi u_P}{\omega} = \frac{2\pi(2.07\times10^8)}{2\pi\times10^6} = 207\ m}$$

b) The reflection coefficient is given by:

$$\boldsymbol{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(120 - j40) - 100}{(120 - j40) + 100} = \frac{20 - j40}{220 - j40} = 0.12 - j0.16 = 0.2e^{-j0.927}\ (\Omega)}$$

c) First need to find βl*:*

$$\beta = \frac{\omega}{u_P} = \frac{2\pi\times10^6}{2.07\times10^8} = 0.00966\pi\ \left(\frac{rad}{m}\right)$$

$$\beta l = 0.03035(10) = 0.0966\pi\ (rad)$$

$$\boldsymbol{Z_i = Z_0\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} = 100\frac{(120 - j40) + j100\tan 0.3035}{100 + j(120 - j40)\tan 0.3035} = 93.62 - j38.98 = 101.4e^{-j0.395}\ \Omega}$$

d) Now that input impedance is found:

$$\boldsymbol{\widetilde{V_i} = \frac{\widetilde{V_g}Z_i}{Z_g + Z_i} = \frac{12e^{-j\frac{\pi}{3}}101.4e^{-j0.395}}{150 + 101.4e^{-j0.395}} = \frac{1216.8e^{-j1.442}}{246.7e^{-j0.1588}} = 4.9e^{-j1.28}}$$

e) Converting the above to time domain:

$$\boldsymbol{v_i(t) = \Re e\{\widetilde{V_i}e^{j\omega t}\} = \Re e\{4.9e^{-j1.28}e^{j\omega t}\} = \Re e\{4.9e^{j(\omega t - 1.28)}\}}$$
$$\boldsymbol{= 4.9\cos(2\pi\times10^6 t - 1.28)}$$

5. Using McMaster's library catalogue, Google Scholar, or some other resource, find an academic paper that discusses an application of transmission lines. In fewer than 5 sentences, summarize the paper's topic, and **explain how it is related to the transmission line theory covered in class.** Cite your source using IEEE, APA, or some comparable format.

6. BONUS: In addition to not dissipating power, a lossless line has two important features: (1) it is dispersionless (u_P independent of frequency), and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties to satisfy the condition

$$R'C' = L'G' \qquad (distortionless\ line)$$

Such a line is called a *distortionless* line, because despite the fact that it is not lossless, it nonetheless possesses the previously mentioned features of the lossless line. Show that for a distortionless line:

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}$$
$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ &= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\ &= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\ &= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.\end{aligned}$$

Hence,

$$\alpha = \mathfrak{Re}(\gamma) = R'\sqrt{\frac{C'}{L'}}, \qquad \beta = \mathfrak{Im}(\gamma) = \omega\sqrt{L'C'}, \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal points, except for bonus questions.
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.

1. **Vector Algebra**
   a. Determine whether the vector $\boldsymbol{C}$ is perpendicular to both $\boldsymbol{A}$ and $\boldsymbol{B}$ given:

   $$\boldsymbol{A} = \boldsymbol{4\hat{x} + 5\hat{y}}, \qquad \boldsymbol{B} = \boldsymbol{7\hat{x} + 6\hat{y} + 8\,\hat{z}}, \qquad \boldsymbol{C} = \boldsymbol{\hat{x} + 5\hat{z}}$$

   Show your work (2) and clearly state whether $\boldsymbol{C}$ is perpendicular to $\boldsymbol{A}$ and $\boldsymbol{B}$

   $$\boldsymbol{A}\cdot\boldsymbol{C} = 4(1) + 5(0) + 0\,(5) = 4$$
   $$\boldsymbol{B}\cdot\boldsymbol{C} = 7(1) + 6(0) + 8\,(5) = 47$$

   $\boldsymbol{A}$ and $\boldsymbol{B}$ and are not perpendicular since the dot product $\boldsymbol{A}\cdot\boldsymbol{B} \neq \boldsymbol{0}$
   $\boldsymbol{A}$ and $\boldsymbol{C}$ and are not perpendicular since the dot product $\boldsymbol{A}\cdot\boldsymbol{C} \neq \boldsymbol{0}$

   b. Find a vector $\boldsymbol{P}$ whose magnitude is 12 and whose direction is perpendicular to both vectors $\boldsymbol{Q}$ and $\boldsymbol{S}$, given: (3)

   $$\boldsymbol{Q} = 5\hat{\boldsymbol{x}} + 3\hat{\boldsymbol{y}}, \qquad \boldsymbol{S} = 20\hat{\boldsymbol{y}} - \hat{\boldsymbol{z}}.$$

   The vector $\boldsymbol{P}$ is represented by:

   $$\boldsymbol{P} = \frac{\boldsymbol{12\,(Q \times S)}}{|\boldsymbol{Q} \times \boldsymbol{S}|}$$

   First find the vector orthogonal to $\boldsymbol{Q}$ and $\boldsymbol{S}$.

   $$\boldsymbol{Q} \times \boldsymbol{S} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{z} \\ 5 & 3 & 0 \\ 0 & 20 & -1 \end{vmatrix}$$
   $$= [(3)(-1) + (20)(0)]\hat{\boldsymbol{x}} - [(5)(-1) + (0)(0)]\hat{\boldsymbol{y}} + [(5)(20) + (0)(3)]\hat{\boldsymbol{z}}$$
   $$= -3\hat{\boldsymbol{x}} + 5\hat{\boldsymbol{y}} + 100\hat{\boldsymbol{z}}$$

   Find the magnitude of the perpendicular vector

   $$|\boldsymbol{Q} \times \boldsymbol{S}| = \sqrt{(-3)^2 + (5)^2 + (100)^2} = \sqrt{10034} \approx 100.17$$

   Substitute to find $\boldsymbol{P}$.

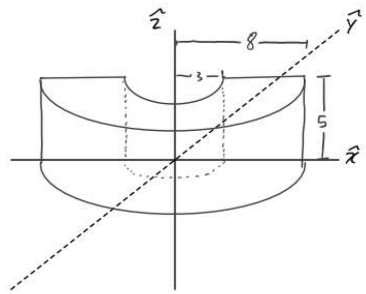
   $$\boldsymbol{P} = \frac{12}{\sqrt{10034}}(-3\hat{\boldsymbol{x}} + 5\hat{\boldsymbol{y}} + 100\hat{\boldsymbol{z}}) \approx 0.12(-3\hat{\boldsymbol{x}} - 5\hat{\boldsymbol{y}} + 100\hat{\boldsymbol{z}})$$

2. **Coordinate Systems**

   a. Provide a sketch (1) and find the volume described by (2)

$$3 \le r \le 8;\ \pi \le \varphi \le 2\pi;\ 0 \le z \le 5$$

$$Ans: \frac{275\pi}{2}\ units^3$$



$$V = \int dV$$
$$= \iiint r\,dr\,d\theta\,dz$$
$$= \int_{z=0}^{5} \int_{\phi=\pi}^{2\pi} \int_{r=3}^{8} r\,dr\,d\phi\,dz$$
$$= \int_0^5 \int_\pi^{2\pi} \left[\tfrac{1}{2}r^2\right]_3^8 d\theta\,dz$$
$$= \int_0^5 \int_\pi^{2\pi} \left[\tfrac{1}{2}(8^2 - 3^2)\right) d\theta\,dz$$
$$= \frac{55}{2} \int_0^5 \int_\pi^{2\pi} d\theta\,dz$$
$$= \frac{55}{2} \int_0^5 [2\pi - \pi]\,dz$$
$$= \frac{55}{2} \int_0^5 (\pi)\,dz$$
$$= \frac{55\pi}{2} \int_0^5 dz$$
$$= \frac{55\pi}{2}(5-0)$$
$$= \frac{275}{2}\pi$$

   b. The surface area described by: (2)

$$0 \le R \le 2;\ 180 \le \theta \le 270^\circ;\ 45^\circ \le \varphi \le 90^\circ$$

$$Ans: \pi\ units^2$$

Convert Degrees to radians

$180^\circ = \pi$ rad

$270^\circ = \frac{3\pi}{2}$ rad

$45^\circ = \frac{\pi}{4}$ rad

$90^\circ = \frac{\pi}{2}$ rad

$$S = \int_{\phi=\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\theta=\pi}^{\frac{3\pi}{2}} R^2 \sin\theta\,d\theta\,d\phi \Big|_{R=2}$$
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\pi}^{\frac{3\pi}{2}} (2)^2 \sin\theta\,d\theta\,d\phi$$
$$= 4 \int_{\frac{\pi}{4}}^{\pi/2} \int_\pi^{\frac{3\pi}{2}} \sin\theta\,d\theta\,d\phi$$
$$= 4 \int_{\frac{\pi}{4}}^{\pi/2} \left[-\cos\theta\right]_{\theta=\pi}^{\frac{3\pi}{2}} d\phi$$
$$= 4 \int_{\frac{\pi}{4}}^{\pi/2} \left[-\cos(\tfrac{3\pi}{2}) - (-\cos\pi)\right] d\theta$$
$$= 4 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [0 + \cos\pi]\,d\theta$$
$$= 4 \left[\cos\pi\right]_{\pi/4}^{\pi/2}$$
$$= 4 \left[(-1)\right]_{\pi/4}^{\pi/2}$$
$$= 4\left[-\tfrac{\pi}{2} + \tfrac{\pi}{4}\right]$$
$$= 4\left[-\tfrac{\pi}{4}\right]$$
$$= -\pi$$

Correction *. The bounds should of θ are $0 \le \theta \le 180^\circ$. So instead the above coordinates should have been:

$$0 \le R \le 2;\ 90 \le \theta \le 180^\circ;\ 45^\circ \le \varphi \le 90^\circ$$

$$= 4\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[-\cos(\pi) - \left(-\cos\left(\frac{\pi}{2}\right)\right)\right] d\theta$$
$$= 4\int_{\frac{\pi}{4}}^{\pi/2} \left[-\cos(\pi) + 0\right] d\theta$$
$$= 4\left[-\cos \pi\right]_{\pi/4}^{\pi/2}$$
$$= 4\left[(1)\right]_{\pi/4}^{\pi/2}$$
$$= 4\left[\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{4}\right)\right]$$
$$= 4\left[\frac{\pi}{4}\right]$$
$$= \pi$$

3. **Gradient** Find the gradient of the following scalar functions.

Recall the gradient is given by $\nabla f(x, y, z) = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$. Find the partial derivative with respect to each component.

a. $\boldsymbol{M} = 10/(x^2 + z^2)$ (1)
$$\nabla\boldsymbol{M} = -\frac{20x}{(x^2 + z^2)^2}\hat{\boldsymbol{x}} - \frac{20z}{(x^2 + z^2)^2}\hat{\boldsymbol{z}}$$

b. $\boldsymbol{A} = xy^3z^2$(1)
$$\nabla\boldsymbol{A} = y^3z^2\hat{\boldsymbol{x}} + 3xy^2z^2\hat{\boldsymbol{y}} + 2xy^3z\hat{\boldsymbol{z}}$$

c. $\boldsymbol{T} = e^R \sin\theta$ (1)
$$\nabla\boldsymbol{T} = \frac{\partial T}{\partial R}\hat{\boldsymbol{R}} + \frac{1}{R}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}$$
$$\nabla\boldsymbol{T} = e^R\sin(\theta)\,\hat{\boldsymbol{R}} + \frac{e^R\cos\theta}{R}\hat{\boldsymbol{\theta}}$$

d. $\boldsymbol{H} = R^3\cos^2\theta$ (1)
$$\nabla\boldsymbol{H} = \frac{\partial H}{\partial R}\hat{\boldsymbol{R}} + \frac{1}{R}\frac{\partial H}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial H}{\partial \phi}\hat{\boldsymbol{\phi}}$$
$$\nabla\boldsymbol{H} = 3R^2\cdot cos^2(\theta)\hat{\boldsymbol{R}} - 2R^2\cdot cos(\theta)\cdot sin(\theta)\hat{\boldsymbol{\theta}}$$

e. $\boldsymbol{S} = xy^2 - z^2$(1)
$$\nabla\boldsymbol{S} = y^2\hat{\boldsymbol{x}} + 2xy\hat{\boldsymbol{y}} - 2z\hat{\boldsymbol{z}}$$

4. **Divergence.** The Divergence theorem states the surface integral of a vector field over a closed surface, the flux through the surface, is equal to the volume integral of the divergence over the region within the surface. Given the Divergence theorem states:

$$\int_v \nabla \cdot \boldsymbol{E}\, d\boldsymbol{v} = \oint_S \boldsymbol{E} \cdot d\boldsymbol{s}$$

The vector field V is given by:

$$\boldsymbol{V} = x^2\hat{\boldsymbol{x}} + y^3\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$$

Verify the divergence theorem by computing

a. The total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each, parallel to the cartesian axes (2)

The closed surface has 6 sides:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{s} = F_{top} + F_{bottom} + F_{right} + F_{left} + F_{front} + F_{back}$$

We must find the flux through each side. The calculation of the flux through each side is shown on the next page.

$$\oint \boldsymbol{E} \cdot d\boldsymbol{s} = F_{top} + F_{bottom} + F_{right} + F_{left} + F_{front} + F_{back}$$
$$= (4) + (4) + (4) + (4) + (4) + (-4)$$
$$= 16$$

b. The integral of $\nabla \cdot \boldsymbol{V}$ over the cubes volume. (2)

$$\iiint \nabla \cdot \vec{E}\, d\vec{v} = \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} \nabla \cdot (x^2\hat{x} + y^3\hat{y} + z\hat{z})\, dz\, dy\, dx$$

$$= \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} (3y^2 + 2x + 1)\, dz\, dy\, dx$$

Note: integral bounds are omitted for simplicity

$$= \iiint 3y^2\, dz\, dy\, dx + \iiint 2x\, dz\, dy\, dx + \iiint dz\, dy\, dx$$
$$= (x\, y^3 z + x^2 yz + xyz)\Big|_{z=-1}^{1}\Big|_{y=-1}^{1}\Big|_{x=-1}^{1}$$
$$= xyz\,(y^2 + x + 1)\Big|_{z=-1}^{1}\Big|_{y=-1}^{1}\Big|_{x=-1}^{1}$$
$$= 16$$

c. Does the theorem hold true? (1)

Yes, the theorem holds since the surface integral of a vector field equals the volume integral of the divergence.

$$F_{top} = \int_{x=-1}^{1}\int_{y=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})\Big|_{z=1} \cdot (\hat{z}\, dy\, dx)$$
$$= \int_{x=-1}^{1}\int_{y=-1}^{1} (z|_{z=1})\, dy\, dx$$
$$= \int_{x=-1}^{1}\int_{y=-1}^{1} dy\, dx$$
$$= [x]_{-1}^{1} \int_{y=-1}^{1} dy$$
$$= (1-(-1))[y]_{-1}^{1}$$
$$= 2\,(1-(-1))$$
$$= 4$$

$$F_{bottom} = \int_{x=-1}^{1}\int_{y=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{z=-1} \cdot (-\hat{z}\, dy\, dx)$$
$$= \int_{x=-1}^{1}\int_{y=-1}^{1} -z|_{z=-1}\, dy\, dx$$
$$= \int_{x=-1}^{1}\int_{y=-1}^{1} -(-1)\, dy\, dx$$
$$= [x]_{-1}^{1}[y]_{-1}^{1}$$
$$= 4$$

$$F_{right} = \int_{x=-1}^{1}\int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{y=1} \cdot (\hat{y}\, dz\, dx)$$
$$= \int_{x=-1}^{1}\int_{z=-1}^{1} y^3|_{y=1}\, dz\, dx$$
$$= \int_{x=-1}^{1}\int_{z=-1}^{1} (1)^3\, dz\, dx$$
$$= 4$$

$$F_{left} = \int_{x=-1}^{1}\int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{y=-1} \cdot -\hat{y}\, dz\, dx$$
$$= \int_{x=-1}^{1}\int_{z=-1}^{1} -y^3|_{y=-1}\, dz\, dx$$
$$= \int_{x=-1}^{1}\int_{z=-1}^{1} -(-1)^3\, dx\, dz$$
$$= 4$$

$$F_{front} = \int_{y=-1}^{1}\int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{x=1} \cdot \hat{x}\, dz\, dy$$

$$= \int_{y=-1}^{1} \int_{z=-1}^{1} x^2|_{x=1}\, dz\, dy$$
$$= \int_{y=-1}^{1} \int_{z=-1}^{1} (1)^2\, dz\, dy$$
$$= 4$$

$$F_{back} = \int_{y=-1}^{1} \int_{z=-1}^{1} (x^2\hat{x} + y^3\hat{y} + z\hat{z})|_{x=-1} \cdot -\hat{x}\, dz\, dy$$
$$= \int_{y=-1}^{1} \int_{z=-1}^{1} -x^2|_{x=-1}\, dz\, dy$$
$$= \int_{y=-1}^{1} \int_{z=-1}^{1} -(-1)^2\, dz\, dy$$
$$= -4$$

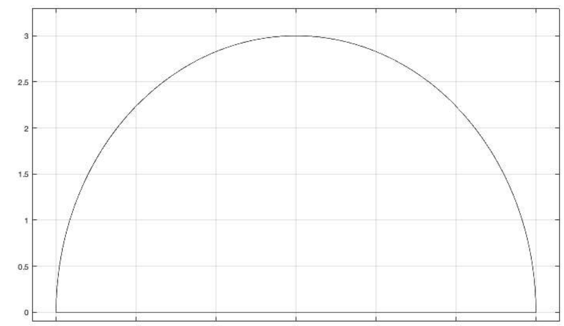
5. **Curl.** Stokes's theorem is a powerful equation that allows the conversion of a surface integral of the curl of a vector over an open surface S into a line integral, such as in the calculation of current through a closed magnetic field loop. Given that Stokes's theorem states:

$$\int_S (\nabla \times \boldsymbol{B}) \cdot d\boldsymbol{s} = \oint_C \boldsymbol{B} \cdot d\boldsymbol{l}$$

Verify Stokes's theorem for the vector field:

$$\boldsymbol{B} = r\cos\phi\,\hat{\boldsymbol{r}} + \sin\phi\ \hat{\boldsymbol{\phi}}$$

a. By evaluating $\oint_C \boldsymbol{B} \cdot d\boldsymbol{l}$ over the semicircular contour shown below (2)
b. By evaluating $\int_S (\nabla \times \mathbf{B}) \cdot d\boldsymbol{s}$; over the semicircular contour shown below (3)



Solution:
Part a)

$$\oint_C \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{L_1} \boldsymbol{B} \cdot d\boldsymbol{l} + \int_{L_2} \boldsymbol{B} \cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{B} \cdot d\boldsymbol{l}$$

$$\boldsymbol{B} \cdot d\boldsymbol{l} = (r cos\phi\,\hat{\boldsymbol{r}} + \sin\phi\ \hat{\boldsymbol{\phi}}) \cdot (dr\,\hat{\boldsymbol{r}} + r d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\boldsymbol{z}}) = r\cos\phi\, dr + r\sin\phi\, d\phi$$

$$\int_{L_1} \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{(r=0)}^{3} r\cos\phi\, dr\Big|_{\phi=0,\ z=0} + \int_{(\phi=0)}^{0} r\sin\phi\ d\phi\Big|_{z=0}$$
$$= \frac{1}{2}r^2\Big|_{r=0}^{3} + 0$$
$$= \frac{9}{2}$$

$$\int_{L_2} \boldsymbol{B} \cdot d\boldsymbol{l} = \int_{(r=3)}^{3} r\cos\phi\, dr\Big|_{z=0} + \int_{(\phi=0)}^{\pi} r\sin\phi\ d\phi\Big|_{z=0}$$
$$= 0 + (-3\cos\phi)|_{\phi=0}^{\pi}$$
$$= 6$$

$$\int_{L_3} \boldsymbol{B}\cdot d\boldsymbol{l} = \int_{(r=3)}^{2} r\cos\phi\, dr\bigg|_{\phi=\pi,\ z=0} + \int_{(\phi=\pi)}^{\pi} r\sin\phi\ d\phi\bigg|_{z=0}$$

$$= -\frac{1}{2}r^2\bigg|_{r=3}^{0} + 0$$

$$= \frac{9}{2}$$

$$\oint_C \boldsymbol{B}\cdot d\boldsymbol{l} = \int_{L_1} \boldsymbol{B}\cdot d\boldsymbol{l} + \int_{L_2} \boldsymbol{B}\cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{B}\cdot d\boldsymbol{l}$$

$$\oint_C \boldsymbol{B}\cdot d\boldsymbol{l} = \frac{9}{2} + 6 + \frac{9}{2}$$

$$= 15$$

Part b)

$$\nabla\times\mathbf{B} = \nabla\times(r\cos\phi\ \hat{\mathbf{r}} + \sin\phi\,\hat{\boldsymbol{\phi}})$$

$$= \left(\frac{1}{\mathrm{r}}\frac{\partial}{\partial\phi}0 - \frac{\partial}{\partial z}(\sin\phi)\right)\hat{\mathbf{r}} + \left(\frac{\partial}{\partial z}(r\cos\phi) - \frac{\partial}{\partial r}0\right)\hat{\boldsymbol{\phi}}$$

$$+\frac{1}{r}\left(\frac{\partial}{\partial r}(r\sin\phi) - \frac{\partial}{\partial\phi}(r\cos\phi)\right)\hat{\mathbf{z}}$$

$$= (0)\hat{\mathbf{r}} + (0)\hat{\boldsymbol{\phi}} + \frac{1}{r}(\sin\phi + r\sin\phi)\hat{\mathbf{z}}$$

$$= \frac{1}{r}(\sin\phi + r\sin\phi)\hat{\mathbf{z}}$$

$$= \sin\phi\left(1+\frac{1}{\mathrm{r}}\right)\hat{\mathbf{z}}$$

$$\int_S (\nabla\times\mathbf{B})\cdot d\boldsymbol{s} = \int_{\phi=0}^{\pi}\int_{r=0}^{3} \sin\phi\left(1+\frac{1}{\mathrm{r}}\right)\hat{\mathbf{z}}\cdot r\,dr\,d\phi\,\hat{\mathbf{z}}$$

$$= \int_{\phi=0}^{\pi}\int_{r=0}^{3} \sin\phi\left(1+\frac{1}{\mathrm{r}}\right)\hat{\mathbf{z}}\cdot r\,dr\,d\phi\,\hat{\mathbf{z}}$$

$$= \int_{\phi=0}^{\pi}\int_{r=0}^{3} \sin\phi\,(\mathrm{r}+1)\,dr\,d\phi$$

$$= \int_{\phi=0}^{\pi} \sin\phi\left[\frac{1}{2}\mathrm{r}^2 + r\right]_{r=0}^{3} d\phi$$

$$= \int_{\phi=0}^{\pi} \sin\phi\left[\left(\frac{9}{2}\right) + 3\right] d\phi$$

$$= 7.5\,[-\cos\phi]_{\phi=0}^{\pi}$$

$$= 7.5\,[(-\cos\pi - (-\cos 0)]$$

$$= 7.5\,[2]$$

$$= 15$$

Therefore, the theorem holds true.

6. **Bonus Question**: Answer one of the following questions. Clearly state whether a, b, or c is being answered.
   a. Find the values for $\boldsymbol{V} = ax^2\hat{\boldsymbol{x}} + by^3\hat{\boldsymbol{y}} + c\hat{\boldsymbol{z}}$ where the divergence at $P = (6,4,7)$ is equal to $\nabla\cdot\mathbf{V} = 10$ (2)

$$\nabla\cdot\mathbf{V} = \frac{\partial\mathbf{V}}{\partial\mathrm{x}} + \frac{\partial\mathbf{V}}{\partial\mathrm{y}} + \frac{\partial\mathbf{V}}{\partial\mathrm{z}}$$

$$10 = 2ax + 3by^2$$

Substitute $P = (6,4,7)$

$$10 = 2(6)a + 3(4)^2b + 0(7)$$

$$10 = 12a + 48b$$

Provide any three values for (a,b,c) to satisfy the above equation. For example

$$a = 1, b = -\frac{2}{48}, c = any\ value$$

   b. Describe $\boldsymbol{A}$ in cylindrical coordinates and evaluate it at

$$P = (2,\pi,\pi/4)$$

$\boldsymbol{A} = \sin^2\theta\, cos\varphi\hat{\boldsymbol{R}} + \cos\theta\,\hat{\boldsymbol{\theta}}\ - sin\varphi\ \hat{\boldsymbol{\varphi}}$ (2)

$\boxed{\text{Bonus Q}}$

$\vec{A} = \sin^2\theta \cos\varphi \hat{R} + \cos\theta \hat{\theta} - \sin\varphi \hat{\varphi}$

Describe A in cylindrical coordinates

Evaluate at $P = (2, \pi, \pi/4)$

$\varphi = \phi$

Relations

Spherical to Cylindrical Coordinates

$r = R\sin\theta$

$\phi = \phi$

$z = R\cos\theta$

Vector Conversion

$A_r = A_R \sin\theta + A_\theta \cos\theta$

$A_\phi = A_\phi$

$A_z = A_R \cos\theta - A_\theta \sin\theta$

$\vec{A}_{spherical} = \sin^2\theta \cos\phi \hat{R} - \sin\phi \hat{\phi} + \cos\theta \hat{\theta}$

$\vec{A}_{cylindrical} = A_r \hat{r} + A_\phi \hat{\phi} + A_z \hat{z}$

$= [A_R \sin\theta + A_\theta \cos\theta]\hat{r} + A_\phi \hat{\phi} + [A_R \cos\theta - A_\theta \sin\theta]\hat{z}$

$= [(\sin^2\theta \cos\phi)\sin\theta + (-\sin\phi)(\cos\theta)]\hat{r}$

$+ [-\sin\phi]\ \phi$

$+ [\sin^2\theta \cos\phi \cos\theta - \cos\theta \sin\theta]\hat{z}$

$= [\sin^3\theta \cos\phi - \sin\phi \cos\theta]\hat{r} - \sin\phi \hat{\phi}$

$+ [\sin^2\theta \cos\phi \cos\theta - \cos\theta \sin\theta]\hat{z}$

Substitute $P_{spherical} = (2, \pi, \frac{\pi}{4})$

$P = 2\hat{R} + \pi\hat{\theta} + \frac{\pi}{4}\hat{\phi}$

$A_{cylindrical} = [\sin^3(\pi)\cos(\frac{\pi}{4}) - \sin(\frac{\pi}{4})\cos(\pi)]\hat{r}$

$-(\sin\frac{\pi}{4})\hat{\phi}$

$+[\sin^2(\pi)\cos(\frac{\pi}{4})\cos(\pi) - \cos(\pi)\sin(\pi)]\hat{z}$

$= \hat{r} - \frac{1}{\sqrt{2}}\hat{\phi}$

c. Convert the following coordinates

i. From cartesian to cylindrical and spherical coordinates: $P_1 = (5,10,15)$ (2)

Cylindrical:

$r = \sqrt{125}$
$\phi = 63.43^\circ = 1.1\, rads$
$z = 15$

Spherical:

$R = \sqrt{350}$
$\theta = 36.69^\circ = 0.64\, rads$
$\phi = 63.43^\circ = 1.1\, rads$

ii. From cylindrical to spherical and cartesian: $P_2 = \left(1, \frac{\pi}{2}, -1\right)$ (2)

Spherical:

$R = \sqrt{2}$
$\phi = 90^\circ = \frac{\pi}{2}$
$\theta = -45^\circ = \frac{\pi}{2}\, rads$

Cartesian:

$x = 0$
$y = 1$
$z = -1$

iii. From spherical to cylindrical: $P_3 = (4, \pi, \pi)$ (1)

Cylindrical:

$r = 0$
$\phi = 180^\circ = \pi$
$z = -4$

ASSIGNMENT SUBMISSION INSTRUCTIONS

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ENGPHYS 2A04 Assignment 6 Solutions

1. **Charge and Current Distributions**

a)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}\sin^2\phi \, r\, dr\, d\phi$$

$$= \frac{\rho_{s0} r^2}{2}\Big|_0^a \int_0^{2\pi}\left(\frac{1-\cos 2\phi}{2}\right) d\phi$$

$$= \frac{\rho_{s0} a^2}{4}\left(\phi - \frac{\sin 2\phi}{2}\right)\Big|_0^{2\pi}$$

$$= \frac{\pi a^2}{2}\rho_{s0}$$

b)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0} e^{-r} r\, dr\, d\phi$$

$$= 2\pi\rho_{s0}\int_0^a re^{-r} dr$$

$$= 2\pi\rho_{s0}[-re^{-r} - e^{-r}]_0^a$$

$$= 2\pi\rho_{s0}[1 - e^{-a}(1+a)]$$

c)

$$Q = \int \rho_s\, ds = \int_{r=0}^{a}\int_{\phi=0}^{2\pi}\rho_{s0}\cos\phi\, r\, dr\, d\phi$$

$$= \frac{\rho_{s0} r^2}{2}\Big|_0^a \sin\phi\,\Big|_0^{2\pi}$$

$$= 0$$

d)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi}\rho_{s0} e^{-r}\sin^2\phi\, r\, dr\, d\phi$$

$$= \rho_{s0}\int_{r=0}^{a} re^{-r} dr \int_{\phi=0}^{2\pi}\sin^2\phi\, d\phi$$

$$= \rho_{s0}[1 - e^{-a}(1+a)]\cdot\pi$$

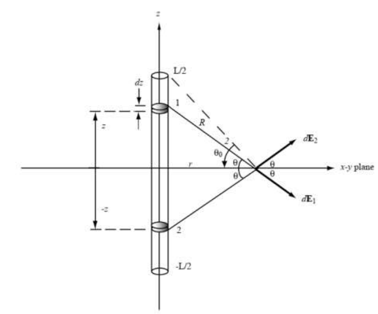
$$= \pi\rho_{s0}[1 - e^{-a}(1+a)]$$

2. **Coulomb's Law**

$$d\mathbf{E} = d\mathbf{E_1} + d\mathbf{E_2} = \hat{\mathbf{r}}\frac{2\rho_l\cos\theta\, dz}{4\pi\varepsilon_0 R^2} = \hat{\mathbf{r}}\frac{\rho_l\cos\theta\, dz}{2\pi\varepsilon_0 R^2}$$

Our integration variable is z, but it will be easier to integrate over the variable θ from $\theta = 0$ to

$$\theta_0 = \sin^{-1}\frac{L/2}{\sqrt{r^2 + (L/2)^2}}$$



Here, with $R = r/\cos\theta$, and $z = r\tan\theta$ and $dz = r\sec^2\theta\, d\theta$, we have

$$\mathbf{E} = \int_{z=0}^{L/2} d\mathbf{E} = \int_{\theta=0}^{\theta_0} d\mathbf{E} = \int_0^{\theta_0}\hat{r}\frac{\rho_l}{2\pi\varepsilon_0}\frac{\cos^3\theta}{r^2} r\sec^2\theta\, d\theta$$

$$= \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\int_0^{\theta_0}\cos\theta\, d\theta$$

$$= \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\sin\theta_0$$

$$= \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}\frac{L/2}{\sqrt{r^2 + (L/2)^2}}$$

For $L \gg r$,

$$\frac{L/2}{\sqrt{r^2 + (L/2)^2}} \approx 1$$

$$\mathbf{E} = \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r} \text{ (infinite line of charge)}$$

3. **Gauss's Law**

Symmetry of the spherical shape indicates that D is radially oriented.

$$\mathbf{D} = \hat{\mathbf{R}} D_r$$

Gauss's Law at any radius or R.

$$\oint_S \mathbf{D} \cdot ds = Q$$

$$\oint_S \hat{\mathbf{R}} D_r \cdot \hat{\mathbf{R}} ds = Q$$

$$4\pi R^2 D_R = Q$$

$$D_R = \frac{Q}{4\pi R^2}$$

For R < a, there is no charge in the cavity (hollow). Therefore, Q =0.

$$D_R = 0, R \le a$$

For $a \le R \le b$,

$$Q = \int_{R=a}^{R} \rho_v dV = \int_{R=a}^{R} -\frac{\rho_{v_0}}{R^2} \cdot 4\pi R^2 \, dR = -4\pi \rho_{v_0}(R-a)$$

Therefore,

$$D_R = \frac{-4\pi \rho_{v_0}(R-a)}{4\pi R^2}, \quad a \le R \le b$$

$$D_R = -\frac{\rho_{v_0}(R-a)}{R^2}, \quad a \le R \le b$$

For $R \ge b$,

$$Q = \int_{R=a}^{b} \rho_v dV = \int_{R=a}^{b} -\frac{\rho_{v_0}}{R^2} \cdot 4\pi R^2 \, dR = -4\pi \rho_{v_0}(b-a)$$

$$D_R = \frac{-4\pi \rho_{v_0}(b-a)}{4\pi R^2}, \qquad R \ge b$$

$$D_R = -\frac{\rho_{v_0}(b-a)}{R^2}, \quad a \le R \le b$$

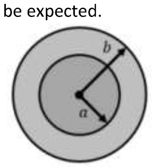
4. **Electric Scalar Potential**

- Defined as the voltage difference between two points in a circuit
  - Represents the amount of work or potential energy required to move a unit of charge from <u>one point to another</u> (voltage difference)
  - More accurate representation of voltage
    - Voltage – amount of potential energy between two points in a circuit
    - Voltage represents the "difference" in potential and therefore is more accurate for electric scalar potential
- Is scalar because of:
  - Path independence in a conservative electric field
  - Work is only present when there is work being done against the electric field, if the work done is going with the field, there is no work done
  - Therefore, work is only dependent whether it is along or going against the electric field since field is conservative it is <u>not</u> dependent on path
  - Electric potential which is a calculation of work done will therefore also be a scalar quantity not dependent on direction

ENGPHYS 2A04 Winter 2022 – Assignment 7 Solutions

DUE MONDAY MARCH 14th, 8AM

1. The coaxial cable shown below is $10m$ long, and has a $5V$ potential applied across it. The inner cylinder is made of silicon and the outer cylinder is made of carbon. If the outer shell is to be twice the diameter of the inner cylinder, find the radii a and b for which a current of $0.2A$ would be expected.



Solution

The resistance needs to be $R = \frac{V}{I} = \frac{5}{0.2} = 25\Omega$.

The actual resistance of this will follow

$$R = \frac{l}{\sigma_1 A_1 + \sigma_2 A_2} = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2(b^2 - a^2)\right)}$$

From the geometry requirement added in the problem $b = 2a$, so this expression can be simplified:

$$R = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2((2a)^2 - a^2)\right)} = \frac{l}{\pi\left(\sigma_1 a^2 + \sigma_2(4a^2 - a^2)\right)} = \frac{l}{\pi(\sigma_1 a^2 + 3\sigma_2 a^2)}$$
$$= \frac{l}{\pi(\sigma_1 + 3\sigma_2)a^2}$$

So, the inner radius can be found as:

$$a = \sqrt{\frac{l}{\pi(\sigma_1 + 3\sigma_2)R}}$$

Substituting in the known values for the length $l = 10m$, the resistance $R = 25\Omega$ and the conductances of silicon and carbon $\sigma_1 = 4.4 \times 10^{-4} S/m$, $\sigma_2 = 3 \times 10^4 S/m$:

$$a = \sqrt{\frac{10}{\pi\left(4.4 \times 10^{-4} + 3(3 \times 10^4)\right)25}} = \sqrt{\frac{10}{7068583.5}} = 1.2mm$$

Therefore, the radii must be $a = 1.2mm$, and $b = 2a = 2.4mm$.

2. A uniform sheet of charge with $\rho_{S1} = 1\frac{nC}{m^2}$ lies on the $z = 0$ plane, and a second sheet with $\rho_{S2} = -1\frac{nC}{m^2}$ occupies the $z = 5m$ plane. Find the scalar potential V_{AB} between the points $A(0,0,5m)$ and $B(0,-1m,2m)$. Explain what this scalar potential means in physical terms, in the context of this problem.

Solution

These are infinite planes parallel to the z-plane, so the electric field will be directed in the z-direction. The total electric field will be the sum of those caused by each of the plates:

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = \hat{\boldsymbol{z}}\frac{\rho_{S1}}{2\varepsilon_0} - \hat{\boldsymbol{z}}\frac{\rho_{S2}}{2\varepsilon_0} = \hat{\boldsymbol{z}}\frac{\rho_{S1} - \rho_{S2}}{2\varepsilon_0}$$

The potential between two points only depends on z, and will follow the equation:

$$V_{AB} = -\int_0^5 \boldsymbol{E} \cdot d\boldsymbol{z} = -\int_{z_B}^{z_A} \hat{\boldsymbol{z}}\frac{\rho_{S1} - \rho_{S2}}{2\varepsilon_0} \cdot \hat{\boldsymbol{z}}dz$$

$$V_{AB} = -\frac{\rho_{S1} - \rho_{S2}}{2\varepsilon_0}\int_{z_B}^{z_A} dz = -\frac{\rho_{S1} - \rho_{S2}}{2\varepsilon_0} z\Big|_{z_B}^{z_A} = -\frac{\rho_{S1} - \rho_{S2}}{2\varepsilon_0}(z_A - z_B)$$

Substituting in the known values:

$$\boldsymbol{V_{AB} = -\frac{1 \times 10^{-9} - (-1 \times 10^{-9})}{2\varepsilon_0}(5 - 2) = -3\frac{1 \times 10^{-9}}{8.85 \times 10^{-12}} = -339V}$$

This means that to move one coulomb of charge from point A to point B, it would require 339J of energy.

3. A cable with a uniform square cross-section has a length of $200m$, and there is an $8V$ potential applied across it. If the current density through it is determined to be $1.4 \times 10^6\ A/m^2$, find the conductivity of its material. Name a material that could be used here.

Solution

$\vec{J} = \sigma\vec{E}$, so $\sigma = \frac{J}{E}$. Electric field is related to voltage through the length:

$$E = \frac{V}{l}$$

$$\therefore \sigma = \frac{Jl}{V} = \frac{1.4 \times 10^6(200)}{8}$$

$\boldsymbol{\sigma = 3.5 \times 10^7 \frac{S}{m}}$**, one possible material is aluminium.**

4. In class, we discussed the concept of *dielectric breakdown* using the example of lightning. Research another example of dielectric breakdown. Explain your example in 5 sentences or fewer, and as always, cite your sources. Make sure you capture the following:

- What is the mechanism or mechanisms by which breakdown occurs?
- Is it ever desirable? What are potential consequences (or benefits)?
- What factors contribute to the likelihood of this breakdown occurring?

5. **BONUS:** Complete the mid-semester survey!

ASSIGNMENT SUBMISSION INSTRUCTIONS

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- Show all your work for full marks.
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ENGPHYS 2A04 Winter 2022 – Assignment 8

Due Monday MONDAY March 21, 8AM

1. **Boundary Conditions.** If $E = 200\,\hat{\boldsymbol{R}}$ (V/m) at the surface of a 10-cm conducting sphere centered at the origin, what is the total charge Q on the sphere's surface?

Solution:
From Table 4-3, $\hat{\boldsymbol{n}} \cdot (\boldsymbol{D}_1 - \boldsymbol{D}_2) = \boldsymbol{\rho}_s$. $\boldsymbol{E}_2$ inside the sphere is zero, since we assume it is a perfect conductor. Hence, for a sphere with surface area $S = 4\pi a^2$,

$$\begin{aligned} D_{1R} &= \rho_s \\ E_{1R} &= \frac{\rho_s}{\varepsilon_0} \\ &= \frac{Q}{S\varepsilon_0} \\ Q &= E_R S\,\varepsilon_0 \\ &= (200)4\pi(0.1)^2\varepsilon_0 \\ &= 8\pi\varepsilon_0 \end{aligned}$$

2. **Boundary Conditions.**
   a. Find $\boldsymbol{E}_1$ given,
   $$E_2 = 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} + 3\,\hat{\boldsymbol{z}}$$
   $\varepsilon_1 = 3\varepsilon_0$, $\varepsilon_2 = 16\varepsilon_0$, and the boundary has a surface charge density $\rho_s = 6.25 \times 10^{-11}$ (C/m2).

   b. What angle does E2 make with the z axis?

   Solution:
   Recall that $\boldsymbol{E}_{1t} = \boldsymbol{E}_{2t}$ for any 2 media. Hence,
   $$\begin{aligned} \boldsymbol{E}_{1t} &= \boldsymbol{E}_{2t} \\ &= 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} \end{aligned}$$

   Recall $(\boldsymbol{D_1} - \boldsymbol{D_2}) \cdot \hat{\boldsymbol{n}} = \rho_s$ (from Table 4.3). Hence, $\varepsilon_1(\boldsymbol{E}_1 \cdot \hat{\boldsymbol{n}}) - \varepsilon_2(\boldsymbol{E}_1 \cdot \hat{\boldsymbol{n}}) = \rho_s$

   $$\begin{aligned} \boldsymbol{E}_{1z} &= \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} \\ &= \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} \\ &= \frac{6.25 \times 10^{-11}}{3\varepsilon_0} + \frac{16\varepsilon_0(3)}{3\varepsilon_0} \\ &= \frac{6.25 \times 10^{-11}}{3\varepsilon_0} + 16 \\ &= 18.35\ (V/m) \end{aligned}$$

$$E_1 = 5\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}} + 18.35\,\hat{\boldsymbol{z}}$$

Finding the angle $\boldsymbol{E}_2$ makes with the z-axis can be found by:

$$\boldsymbol{E}_2 \cdot \hat{\boldsymbol{z}} = |\boldsymbol{E}_2| \cos\theta$$

$$3 = \sqrt{5^2 + 7^2 + 3^2} \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{83}}\right)$$

$$= 70.8^\circ$$

$$= 1.24 \text{ rad}$$

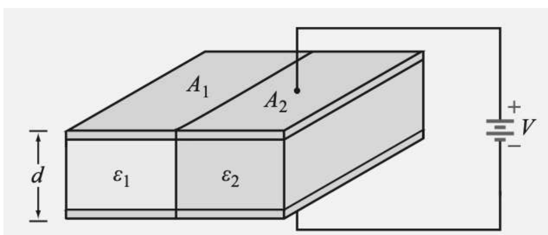
3. **Capacitance.** Given the two parallel, conducting plates separated by a distance d illustrated in the diagram below. The space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2.

Given:

$$C = C_1 + C_2$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d}, \qquad C_2 = \frac{\varepsilon_2 A_2}{d}$$



Find the following:

a. Find the electric fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the two dielectric layers.
b. Calculate the energy stored in each section.
c. Draw a circuit diagram of the above the dielectric section

Solutions

a. Find the electric fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the two dielectric layers.

$$E_1 = E_2 = \frac{V}{d}$$

b. Find an expression the energy stored in each section (1) and the total energy

$$W_{e_1} = \frac{1}{2}CV^2$$

$$C_1 = \frac{\varepsilon_1 A_1}{d}$$

$$W_{e_1} = \frac{1}{2}\frac{V^2 \varepsilon_1 A_1}{d}$$

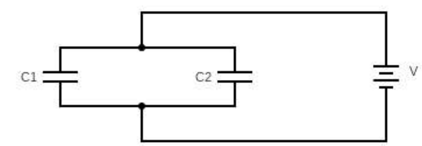
$$W_{e_2} = \frac{1}{2}CV^2$$

$$C_2 = \frac{\varepsilon_2 A_2}{d}$$

$$W_e = W_{e_1} + W_{e_2}$$

$$= \frac{1}{2}\frac{V^2}{d}(\varepsilon_1 A_1 + \varepsilon_2 A_2)$$

c. Draw a circuit diagram of the above the dielectric section



4. **Capacitance.** An electron with charge $Q_e = -1.6 \times 10^{-19} C$ and mass $m_e = 9.1 \times 10^{-31}$ kg is injected at a point adjacent to the negatively charged plate in the region between the plates of an air-filled, parallel-plate capacitor with separation of 50 cm and rectangular plates each 50 cm² in area (Fig. P4.54). If the voltage across the capacitor is 80 V, find the following:
   a. The force acting on the electron,
   b. The acceleration of the electron,

Solutions

a. The force acting on the electron,

$$F = Q_e E$$

$$= Q_e \frac{V}{d}$$

$$= -1.6 \times 10^{-19} \frac{80}{0.5}$$

$$= -2.56 \times 10^{-17}\,N$$

The force is directed from the negatively charged plate towards the positively charged plate.

b. The acceleration of the electron,

$$a = \frac{F}{m} = \frac{-2.56 \times 10^{-17}}{9.1 \times 10^{-31}} = 2.81 \times 10^{13}\ \mathrm{m/s^2}$$

c. The time it takes the electron to reach the positively charged plate, assuming that it starts from rest.

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(0.5)}{2.81 \times 10^{13}}} = 1.89 \times 10^{-7}\ \mathrm{s}$$

Bonus. In no more than 100 words, explain how a super capacitor functions. What are the advantages and disadvantages of a super capacitor compared to a traditional battery?

ASSIGNMENT SUBMISSION INSTRUCTIONS

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- Show all your work for full marks.
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ENGPHYS 2A04 Assignment 9 Solutions

1. **Magnetic Forces and Torques**

The acceleration vector of a free particle is the net force vector divided by the particle mass. Neglecting gravity, use equation shown:

$$a = \frac{F_m}{m_e} = \frac{q\mathbf{u} \times \mathbf{B}}{m_e}$$

Using values:

Electron speed: u = $4 * 10^6\ m/s$

Elementary charge: $e = 1.6 * 10^{19}\ C$

Electron mass: $m_e = 9.1 * 10^{-31}\ kg$

Assuming q = -e.

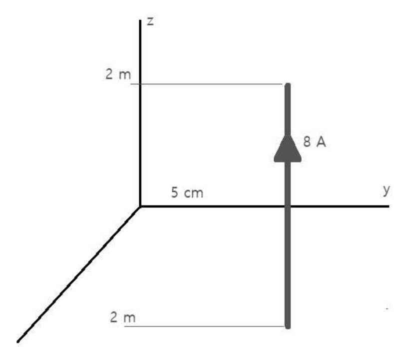
$$a = \frac{F_m}{m_e} = \frac{q\mathbf{u} \times \mathbf{B}}{m_e} = \frac{-1.6 * 10^{19}}{9.1 * 10^{-31}}(\hat{\boldsymbol{x}}7 - \hat{\boldsymbol{z}}4)$$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4000000 & 0 & 0 \\ 7 & 0 & -4 \end{vmatrix} = \boldsymbol{i}(0\cdot(-4) - 0\cdot 0) - \boldsymbol{j}(4000000\cdot(-4) - 0\cdot 7) + \boldsymbol{k}(4000000\cdot 0 - 0\cdot 7) = \boldsymbol{i}(0 - 0) - \boldsymbol{j}(-16000000 - 0) + \boldsymbol{k}(0 - 0) = \{0;\ 16000000;\ 0\}$$

$$= \frac{-1.6 * 10^{19}}{9.1 * 10^{-31}} * (\hat{\boldsymbol{y}}1.6 * 10\text{^}7)$$

$$= -\hat{\boldsymbol{y}}2.81 * 10^{18}\ m/s^2$$

2. **Magnetic Forces and Torques**



a. Magnetic force:

$$\mathbf{F} = I\boldsymbol{l} \times \boldsymbol{B}$$
$$= 8\hat{\mathbf{z}}4 \times [\hat{\mathbf{r}}0.3 \cos\phi$$
$$= \hat{\boldsymbol{\phi}}9.6 \cos\phi$$

At $\phi = \frac{\pi}{2}$, $\hat{\boldsymbol{\phi}} = -\hat{\boldsymbol{x}}$, Hence,

$$\mathbf{F} = -\hat{\mathbf{x}}\mathbf{9.6} \cos\left(\frac{\pi}{2}\right) = 0 \quad \text{T}$$

b. Work:

$$W = \int_{\phi=0}^{2\pi} \mathbf{F}\cdot d\mathbf{l} = \int_0^{2\pi} \hat{\boldsymbol{\phi}}[2\cos\phi]\cdot(-\hat{\boldsymbol{\phi}})r\,d\phi\,|_{r=5\,cm}$$
$$= -2r\int_0^{2\pi} \cos\phi\,d\phi\,|_{r=5\,cm}$$
$$= -10\times10^{-2}\,[\sin\,\phi]_0^{2\pi}$$
$$= 0$$

The force is in the $+\hat{\boldsymbol{\phi}}$ direction, which means that rotating it in the $-\hat{\boldsymbol{\phi}}$ direction would require work. However, force varies as $\cos\phi$ which means it is positive when $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$ and negative over the second half of the circle.
Therefore work is provided by force between $\phi = \frac{\pi}{2}$ and $\phi = \frac{-\pi}{2}$ (when rotated in the $-\hat{\boldsymbol{\phi}}$ direction), and work is supplied for second half of rotation, resulting in net work of zero.

c. Force maximum
Force must be maximum when $\cos\phi = 1$, or $\phi = 0$.

3. **Biot-Savart Law**
The magnetic flux density at center of loop due to the wire is

$$\mathbf{B_1} = \hat{\mathbf{z}}\frac{\mu_0 N I_2}{2\pi d} I_1$$

The field due to I_2 is

$$\mathbf{B} = \mu_0\mathbf{H} = -\hat{\mathbf{z}}\frac{\mu_0 N I_2}{2r}$$

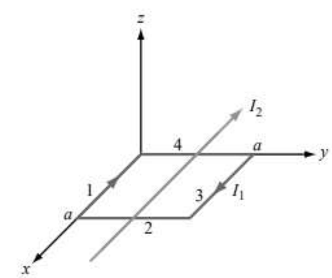
Equating the two

$$\frac{NI_2}{2r} = \frac{I_1}{2\pi d}$$

$$I_2 = \frac{rI_1}{2\pi N d} = \frac{2*30}{\pi*40*2} = 0.239 \text{ A}$$

Counterclockwise direction to oppose current.

4. **Biot-Savart Law**



Treat I_2 in the same plane as shown loop.

For segment (as labelled above), I_1 and I_2 are in the same direction (force on side 1 is attractive).

$$\mathbf{F_1} = \frac{\hat{\mathbf{y}}(\mu_0 I_1 I_2 a)}{2\pi\left(\frac{a}{2}\right)} = \hat{\mathbf{y}}\frac{4\pi*10^{-7}*7*15*4}{2\pi*2} = \hat{\mathbf{y}}\,4.2*10^{-5}\text{ N}$$

I_1 and I_2 are in opposite directions for side 3. The force on side 3 is repulsive (also along $\hat{\mathbf{y}}$). $\mathbf{F_3} = \mathbf{F_1}$.

The net forces on sides 2 and 4 are zero. Total force is.

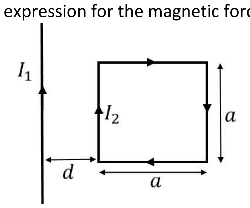
$$2\mathbf{F_1} = \hat{\mathbf{y}}8.4*10^{-5}\ \text{N}$$

5. **Bonus:**
- Ampere's Law
  - Used to calculate magnetic field and used in magnetism
  - Measure magnetic field due to current (line)
- Gauss's Law
  - used to calculate electric field and used in electrostatics
  - electric field by certain charge configuration (surface)

ENGPHYS 2A04 Winter 2022 – Assignment 10

DUE MONDAY APRIL 4th, 8AM

1. The wire on the left carries a current I_1, while the loop on the right carries a current of I_2. The loop is positioned a distance d from the wire, and has dimensions $a \times a$. Find a simplified expression for the magnetic force acting on the loop – and remember, this is a vector quantity.



Solution

The magnetic field strength caused by the wire is given by

$$\boldsymbol{B} = \widehat{\boldsymbol{\Phi}}\frac{\mu_0 I_1}{2\pi r}$$

Or, in the plane of the loop:

$$\boldsymbol{B} = \hat{\boldsymbol{y}}\frac{\mu_0 I_1}{2\pi x}$$

The only sections of the loop that will experience a force are the sections parallel to the wire. For the closer segment:

$$\boldsymbol{F_{left}} = I_2\boldsymbol{l} \times \boldsymbol{B}(x) = I_2(\hat{\boldsymbol{z}}a) \times \left(\hat{\boldsymbol{y}}\frac{\mu_0 I_1}{2\pi x}\right) = -\hat{\boldsymbol{x}}\frac{\mu_0 I_1 I_2 a}{2\pi d}$$

$$\boldsymbol{F_{right}} = -I_2\boldsymbol{l} \times \boldsymbol{B}(x) = -I_2(\hat{\boldsymbol{z}}a) \times \left(\hat{\boldsymbol{y}}\frac{\mu_0 I_1}{2\pi x}\right) = \hat{\boldsymbol{x}}\frac{\mu_0 I_1 I_2 a}{2\pi (d+a)}$$

The total force acting on the loop:

$$\boldsymbol{F} = \boldsymbol{F_{left}} + \boldsymbol{F_{right}} = -\hat{\boldsymbol{x}}\frac{\mu_0 I_1 I_2 a}{2\pi d} + \hat{\boldsymbol{x}}\frac{\mu_0 I_1 I_2 a}{2\pi (d+a)}$$

$$\boldsymbol{F} = \hat{\boldsymbol{x}}\frac{\mu_0 I_1 I_2 a}{2\pi}\left(\frac{1}{d+a} - \frac{1}{d}\right) = -\hat{\boldsymbol{x}}\frac{\mu_0 I_1 I_2 a^2}{2\pi d(d+a)}$$

2. A cylindrical conductor, oriented along the *z*-axis, with a radius of a carries a current density of $\hat{\boldsymbol{z}}J_0 e^{-kr}$. Calculate the magnetic field as a function of radial distances for distances
   a. Inside the conductor
   b. Outside the conductor

Solution

a) For $r \le a$, Ampere's Law is

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I = \int_S \boldsymbol{J} \cdot d\boldsymbol{s}$$

$$\widehat{\boldsymbol{\varphi}}H \cdot \widehat{\boldsymbol{\varphi}}2\pi r = \int_0^r \boldsymbol{J} \cdot d\boldsymbol{s} = \int_0^r \hat{\boldsymbol{z}}J_0 e^{-kr'} \cdot \hat{\boldsymbol{z}}2\pi r' dr'$$

$$2\pi r H = 2\pi J_0 \int_0^r r' e^{-kr'} dr'$$

$$= 2\pi J_0\left(-\frac{(kr'+1)e^{-kr'}}{k^2}\right)\Big|_{r'=0}^{r'=r}$$

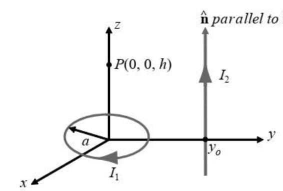
$$= 2\pi J_0\left(-\frac{(kr+1)e^{-kr}}{k^2} + \frac{e^{-kr}}{k^2}\right)$$

$$\therefore H = \frac{J_0}{r}\left(-\frac{(kr+1)e^{-kr}}{k^2} + \frac{e^{-kr}}{k^2}\right)$$

b) The same logic, but replacing *r* with *a* in Ampere's Law:

$$H = \frac{J_0}{r}\left(-\frac{(ka+1)e^{-ka}}{k^2} + \frac{e^{-ka}}{k^2}\right)$$

3. The loop centered at the origin in the figure below has a radius of 5cm, lies in the *x*-*y* plane, and carries a current of $I_1 = 8A$. A straight wire parallel to *z* intersects the point $y = 12cm$, and carries a current of $I_2 = 6A$. Calculate the magnetic field at the point P(0, 0, 10cm).



(a) The magnetic field at $P(0,0,h)$ is composed of $\mathbf{H}_1$ due to the loop and $\mathbf{H}_2$ due to the wire:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2.$$

From (5.34), with $z = h$,

$$\mathbf{H}_1 = \hat{\mathbf{z}}\frac{I_1 a^2}{2(a^2+h^2)^{3/2}} \quad \text{(A/m)}.$$

From (5.30), the field due to the wire at a distance $r = y_0$ is

$$\mathbf{H}_2 = \hat{\boldsymbol{\phi}}\frac{I_2}{2\pi y_0}$$

where $\hat{\boldsymbol{\phi}}$ is defined with respect to the coordinate system of the wire. Point P is located at an angel $\phi = -90°$ with respect to the wire coordinates. From Table 3-2,

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$
$$= \hat{\mathbf{x}} \quad (\text{at } \phi = -90°).$$

Hence,

$$\mathbf{H} = \hat{\mathbf{z}}\frac{I_1 a^2}{2(a^2+h^2)^{3/2}} + \hat{\mathbf{x}}\frac{I_2}{2\pi y_0} \quad \text{(A/m)}.$$

$$\boldsymbol{H} = \hat{\boldsymbol{z}}\frac{8(0.05^2)}{2(0.05^2+0.1^2)^{3/2}} + \hat{\boldsymbol{x}}\frac{6}{2\pi(0.12)}$$

$$\boldsymbol{H} = \hat{\boldsymbol{z}}7.16 + \hat{\boldsymbol{x}}7.96\ A/m$$

4. Consider a 5-meter long section of a coaxial transmission line, with an inner conductor radius of 3cm and an outer conductor inner radius of 8 cm. If the insulator is air and the line is carrying a DC current of 12A, how much magnetic energy is stored in the insulating medium?

Solution

The inductance per unit length of an air-filled coaxial line is

$$L' = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

Total inductance is $L = lL' = \frac{l\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{5\mu_0}{2\pi}\ln\left(\frac{0.08}{0.03}\right) = 981 \times 10^{-9}\ H$

$$W = \frac{LI^2}{2} = \frac{(981 \times 10^{-9})(12)^2}{2} = 70.6\ \mu J$$

Could arrive at the same conclusion using

$$W_{\mathrm{m}} = \frac{1}{2}\int_{\mathcal{V}} \mu_0 H^2\, d\mathcal{V}$$

5. The 'technology brief' in the textbook mentions several applications of electromagnets: magnetic relays, doorbells, loudspeakers and maglev trains. Research one of these topics, and find an academic source that discusses a challenge in this application that is related to the theory discussed in class. In 5 sentences or fewer, explain what the challenge is, how it is related to this week's material, and what some potential solutions are. Cite your source using a recognized citation format.

ASSIGNMENT SUBMISSION INSTRUCTIONS

- Each question is worth equal points.
- Show all your work for full marks.
- Clearly label your name and student number at the top of the first page of your assignment.
- All assignments should be submitted in pdf format to the assignments drop box on Avenue to Learn.
- No late assignments will be accepted. A grade of 0% will be given for late assignments. If you have completed part of the assignment, submit the portion you have completed before the deadline for partial marks.

1. A stationary conducting loop with an internal resistance of 8 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 10 A flows through it. What will the current be if the loop is opened to create a small gap and a 5 Ω resistor is connected across its open ends?

The V_{emf} is independent of the resistance which is in the loop. Therefore, when the loop is intact and the internal resistance is only 8 Ω.

$$V_{\mathrm{emf}} = 10A \times 8\Omega = 80\ V$$

(2 marks)

When the small gap is created, the total resistance in the loop is infinite and the current flow is zero. With a 5-Ω resistor in the gap,

$$I = \frac{V_{\mathrm{emf}}}{5\Omega + 8\Omega} = \frac{80V}{13\Omega} = 6.15\ (\mathrm{A})$$

(1 for showing work, 1 for the final answer)

2. A rectangular conducting loop 10 cm × 16 cm with a small air gap in one of its sides is spinning at 7200 revolutions per minute. If the field $\boldsymbol{B}$ is normal to the loop axis and its magnitude is $3.5 \times\ 10^{-6}\ \mathrm{T}$, what is the peak voltage induced across the air gap?

$$\omega = \frac{\frac{2\pi\mathrm{rad}}{\mathrm{cycle}} \times \frac{7200\mathrm{cycles}}{\mathrm{min}}}{60\mathrm{s/min}} = 240\pi\frac{\mathrm{rads}}{\mathrm{s}}$$

$$A = 0.1\ \mathrm{m}\ \times 0.16\ \mathrm{m} = 1.6 \times 10^{-2}\mathrm{m}^2$$

(2 marks)

Recall the V_{emf} is given by:

$$V_{\mathrm{emf}} = A\omega B_0 \sin \omega t$$

So the peak voltage is given by:

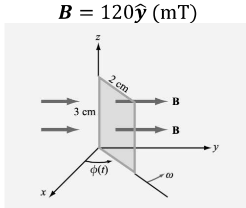
$$V_{peak} = A\omega B_0 = (1.6 \times 10^{-2}\mathrm{m}^2)(240\pi\frac{\mathrm{rads}}{\mathrm{s}})(3.5 \times\ 10^{-6}\ \mathrm{T})$$

$$42.2\mu\mathrm{V}$$

(2 marks)

3. The rectangular conducting loop shown in the figure below rotates at 1,200 revolutions per minute in a uniform magnetic flux density given by:

$$\boldsymbol{B} = 120\hat{\boldsymbol{y}}\ (\mathrm{mT})$$



Determine the current induced in the loop if its internal resistance is 250 mΩ.

$$\Phi = \int_s \mathbf{B} \cdot d\boldsymbol{S}$$
$$= 120 \times 10^{-3}\hat{\boldsymbol{y}} \cdot (2 \times 10^{-2})(3 \times 10^{-2}) \cos\phi(\mathrm{t})$$
$$= 7.2 \times 10^{-5} \cos\phi(\mathrm{t})$$

(1)

$$\phi(\mathrm{t}) = \omega t$$
$$= \frac{2\pi\ \mathrm{rad} \times 1.2 \times 10^3}{60\mathrm{s}}$$
$$= 40\pi \frac{\mathrm{rad}}{\mathrm{s}}$$

(1)

$$V_{\mathrm{emf}} = -\frac{\mathrm{d}\Phi}{\mathrm{dt}}$$
$$= 7.2 \times 10^{-5} \times 40\pi \sin 40\pi t$$
$$= 9.05 \times 10^{-3} \sin 40\pi t\ \ (\mathrm{V})$$

(1)

$$I_{\mathrm{ind}} = \frac{V_{\mathrm{emf}}}{R_{internal}}$$
$$= \frac{9.05 \times 10^{-3} \sin 40\pi t}{0.25}$$
$$= 3.62\ \times 10^{-2} \sin 40\pi t\ \ (\mathrm{A})$$

(1)

4. The transformer shown below consists of a long wire coincident with the z axis carrying a current $I\ =\ I_0\ cos\ \omega t$, coupling magnetic energy to a toroidal coil situated in the x–y plane and centered at the origin. The toroidal core uses iron material with relative permeability μr, around which 500 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure.

a. Develop an expression for V_{emf}

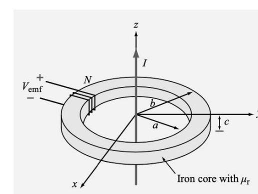
$$\Phi = \int_s \mathbf{B} \cdot d\boldsymbol{S}$$
$$= \int_a^b \frac{\mu I}{2\pi r} \hat{\mathbf{x}} \cdot c\hat{\mathbf{x}}\, dr$$
$$= \frac{\mu c I}{2\pi} \ln(r)\Big|_a^b$$
$$= \frac{\mu c I}{2\pi} ln\left(\frac{b}{a}\right)$$

(1)

$$V_{emf} = -N\frac{d\Phi}{dt}$$
$$= -\frac{\mu c N}{2\pi} ln\left(\frac{b}{a}\right)\frac{dI}{dt}$$
$$= -\frac{\mu c N \omega I_0}{2\pi} ln\left(\frac{b}{a}\right) \sin(\omega t)\ \ (\mathrm{V})$$

(2)

b. Calculate V_{emf} for $f\ =\ 60$ Hz, $\mu_r\ =\ 4500$, $a\ =\ 10\ cm$, $b\ =\ 12\ cm$, $c\ =\ 3\ cm$, and $I_0\ =\ 60\ A$



Given:
$f\ =\ 60$ Hz, $\mu_r\ =\ 4500$, $a\ =\ 10\ cm$, $b\ =\ 12\ cm$, $c\ =\ 3\ cm$, and $I_0\ =\ 60\ A$, we can find the V_{emf} by substantiating into the expression derived above.

$$V_{\mathrm{emf}} = -\frac{\mu c N \omega I_0}{2\pi} ln\left(\frac{b}{a}\right)\ \sin(\omega t)$$
$$= -\frac{(4500 \cdot 4\pi \times 10^{-7})(3 \times 10^{-2})(500)(60 \cdot 2\pi)(60)}{2\pi} ln\left(\frac{0.12}{0.1}\right)\ \sin(60 \cdot 2\pi t)$$
$$= 55.67 \sin 377t$$

(1)

5. **Bonus.** Complete the self-reflection survey. Survey Link here

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