

Assignment 5

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You want to plan a joint space LSPB trajectory for a 2R robot for one motion segment. The starting point of the segment is $\theta_1(0) = -10^\circ$, $\theta_2(0) = 25^\circ$, and the end point is $\theta_1(t_f) = 20^\circ$, $\theta_2(t_f) = 100^\circ$. If the magnitude of $\ddot{\theta}_{d1}$ equals $20^\circ/s^2$, the magnitude of $\ddot{\theta}_{d2}$ equals $80^\circ/s^2$ and $t_f = 2.5$ s, determine:

a) t_b and $\dot{\theta}_{\max}$ for each joint

b) θ for each joint when $t = 1.5$ s

$$\begin{aligned}\theta_1(0) &= -10^\circ & \theta_2(0) &= 25^\circ & t_f &= 2.5\text{ s} \\ \theta_1(t_f) &= 20^\circ & \theta_2(t_f) &= 100^\circ \\ \ddot{\theta}_{d1} &= 20^\circ/s^2 & \ddot{\theta}_{d2} &= 80^\circ/s^2\end{aligned}$$

$$a) \quad t_b = \frac{\ddot{\theta}_d t_f - \sqrt{\ddot{\theta}_d^2 t_f^2 - 4\ddot{\theta}_d(\theta_f - \theta_i)}}{2\ddot{\theta}_d}$$

$$\begin{aligned}\theta_1: \quad t_b &= \frac{\ddot{\theta}_{d1} t_f - \sqrt{\ddot{\theta}_{d1}^2 t_f^2 - 4\ddot{\theta}_{d1}(\theta_1(t_f) - \theta_1(0))}}{2\ddot{\theta}_{d1}} \\ t_b &= \frac{(20^\circ/s^2)(2.5\text{ s}) - \sqrt{(20^\circ/s^2)^2(2.5\text{ s})^2 - 4(20^\circ/s^2)(20 + 10^\circ)}}{2(20^\circ/s^2)} \\ t_b &= 1\text{ s}\end{aligned}$$

$$\begin{aligned}\dot{\theta}_{\max} &= \ddot{\theta}_d t_b \\ (\dot{\theta}_1)_{\max} &= (20^\circ/s^2)(1\text{ s}) \\ &= 20^\circ/s\end{aligned}$$

Θ_2 :

$$t_b = \frac{\ddot{\Theta}_d t_f - \sqrt{\ddot{\Theta}_d^2 t_f^2 - 4\ddot{\Theta}_d(\Theta_f - \Theta_i)}}{2\ddot{\Theta}_d}$$

$$t_b = \frac{(80^\circ/s^2)(2.5) - \sqrt{80^2 \cdot 2.5^2 - 4 \cdot 80(100^\circ - 25^\circ)}}{2(80)}$$

$$t_b = 0.459 \text{ s}$$

$$\begin{aligned}(\dot{\Theta}_2)_{\max} &= \ddot{\Theta}_d \cdot t_b \\ &= (80)(0.459) \\ &= 36.75^\circ/s\end{aligned}$$

b) Θ for each joint when $t = 1.5 \text{ s}$

Θ_1 :

$$\begin{aligned}t_f - t_b &\leq t \leq t_f \\ 2.5 - 1 &\leq 1.5 \leq 2.5 \\ 1.5 &\leq 1.5 \leq 2.5 \checkmark\end{aligned}$$

$$\Theta_1(t) = \Theta_f - \frac{1}{2} \ddot{\Theta}_d (t_f - t)^2$$

$$\begin{aligned}
 \Theta_1(t) &= \Theta_f - \frac{1}{2} \ddot{\Theta}_d (t_f - t)^2 \\
 &= 20 - \frac{1}{2} (20) (2.5 - 1.5)^2 \\
 &= 10^\circ
 \end{aligned}$$

$$\Theta_2: \quad t_b \leq t \leq (t_f - t_b)$$

$$0.459 \leq 1.5 \leq 2.041$$

$$\begin{aligned}
 \Theta_1(t) &= \Theta_i + \frac{1}{2} \ddot{\Theta}_d t_b^2 + \ddot{\Theta}_d t_b (t - t_b) \\
 &= 25 + \frac{1}{2} (80) (0.459)^2 + 80 (0.459) (1.5 - 0.459)^2 \\
 &= 73.22^\circ
 \end{aligned}$$