

Quiz 7

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Name: Alex Bartella

Student #: 400308868

MECHTRON 3DX4 Tutorial Quiz 7 L02: Modelling in the time domain

1. State-space representation (10 marks)

For the system shown in Figure 1, a torque is applied at $\theta_1(t)$. Find a state-space representation for the system. Assume the output is $\theta_2(t)$

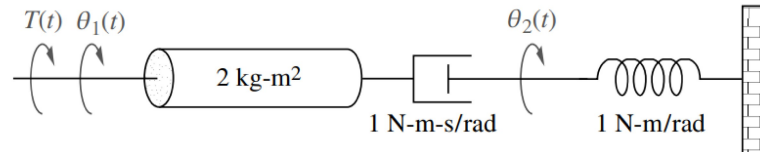
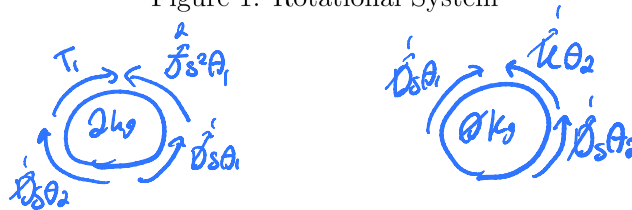


Figure 1: Rotational System



$$2s^2\theta_1 + s\theta_1 - s\theta_2 = T_1 \quad -s\theta_1 + \theta_2 + s\theta_2 = 0$$

$$2\frac{d^2\theta_1}{dt^2} + \frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} = T_1(t) \quad -\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} + \theta_2 = 0$$

$$2\frac{d\dot{\theta}_1}{dt} + \dot{\theta}_1 - \dot{\theta}_2 = T_1(t) \quad -\dot{\theta}_1 + \dot{\theta}_2 + \theta_2 = 0$$

$$\frac{d\dot{\theta}_1}{dt} = -\frac{1}{2}\dot{\theta}_1 + \frac{1}{2}\dot{\theta}_2 + \frac{1}{2}T_1(t) \quad \dot{\theta}_2 = \dot{\theta}_1 - \theta_2$$

$$\dot{\Theta} = \begin{bmatrix} \dot{\theta}_1 \\ \frac{d\dot{\theta}_1}{dt} \\ \dot{\theta}_2 \\ \frac{d\dot{\theta}_2}{dt} \end{bmatrix} \leftrightarrow \Theta = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} \quad \dot{\Theta} = A\Theta + B T(t)$$

$$\dot{\Theta} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} T(t)$$

$$y(t) = \theta_2(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix}$$