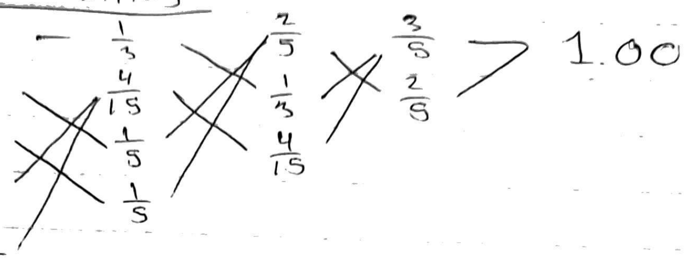
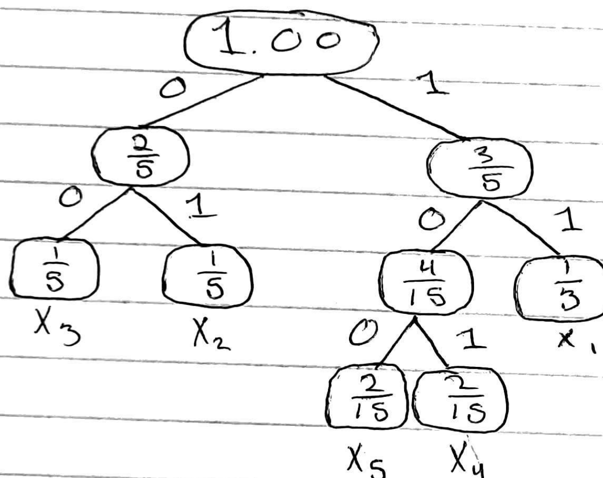
Codeword	X	Probabilities
11		$\frac{1}{3}$
01		$\frac{1}{5}$
00		$\frac{1}{5}$
101		$\frac{2}{15}$
100		$\frac{2}{15}$





This is the Huffman encoding for $\vec{p}_1 = \left(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{2}{15}, \frac{2}{15}\right)$

Its expected code length is ≈ 2.2667.

Consider the Huffman encoding we constructed for $\vec{p}_1 = (\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{2}{15}, \frac{2}{15})$. Using it for the probability distribution $\vec{p}_2 = (\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})$ we ~~achieve~~ get

Codeword	X	Probability
11	x_1	$p(x_1) = 1/5$
01	x_2	$p(x_2) = 1/5$
00	x_3	$p(x_3) = 1/5$
101	x_4	$p(x_4) = 1/5$
100	x_5	$p(x_5) = 1/5$

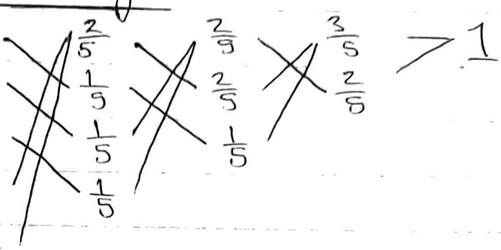
Let L_2 be the expected code length for this encoding.

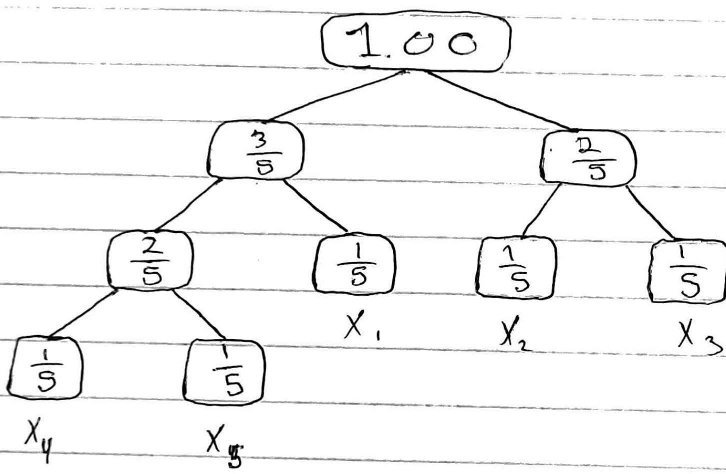
$$L_2 = 2 \cdot \frac{1}{5} + 2 \cdot \frac{1}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{5} + 3 \cdot \frac{1}{5} = 2.4$$

$$\underline{L_2 = 2.4}$$

In order to show that this is optimal, it suffices to find L_2^*, where L_2^* is the expected code length for the Huffman encoding of $\vec{p}_2$, and show that $L_2 = L_2^*$. This follows from the fact/theorem that Huffman encoding is optimal.

Codeword	X	Probability
01	x_1	$\frac{1}{5}$
10	x_2	$\frac{1}{5}$
11	x_3	$\frac{1}{5}$
000	x_4	$\frac{1}{5}$
001	x_5	$\frac{1}{5}$





The expected code length for the Huffman encoding of $\vec{p}_2$ is $L_2^* = 2.4$.

As we can see, $L_2 = L_2^*$. Thus, the encoding used before must be optimal since it has the same expected code length as the Huffman encoding, which has been shown to be optimal.

Therefore, the Huffman encoding for $\vec{p}_1$ is also an optimal encoding for $\vec{p}_2$. $\blacksquare$