MATH 4803 Computational Neuroscience (Fall 2024)
Problem set 1

DUE at 11:59 pm Tuesday, September 10

Submit electronically via Canvas by uploading a file. You are welcome to work together, but should submit your own work. You can utilize needed parts of the code posted on Canvas.

1. **Nernst Potentials**
   The Nernst equation computes the reversal potential E_A for ion A: $E_A = \frac{k_B T}{z_A q_e} ln\left(\frac{[A_{out}]}{[A_{in}]}\right)$. Note that T denotes absolute temperature in Kelvin (K), $k_B = 1.39 \times 10^{-23}$ JK^{-1} is the Boltzmann constant, $q_e = 1.60 \times 10^{-19}$ C is the fundamental electronic charge, and z_A is the charge of an ion A.

ion	charge	internal concentration	external concentration	Nernst potential (mV)
X	-2	60 mM	100 mM	
Y	+2	180 mM	35 mM	
Z	-1	175 mM	40 mM	
Chloride	-1	10 mM	120 mM	-66.4 mV

   (a) Calculate the Nernst potentials for the hypothetical ions in the table above. Assume a temperature of 310 Kelvin. Note that the Nernst equation returns a value in units of Volts. Convert your answer to millivolts and round to one decimal place. (Hint: Try calculating the Nernst potential for Chloride to check your answer against the Nernst potential provided.)

   (b) Hypothetical ion Z has the same charge as Chloride, yet a very different Nernst potential that is positive rather than negative. Give a explanation about what is causing them to have such different Nernst potentials.

2. **Analytically and numerically solving an ODE**
   Consider the following ODE:
$$\frac{dy}{dt} + 2ty = t, \quad y(0) = y_0$$

   (a) Find the closed form solution of the ODE by using an integrating factor.

   (b) What happens to $y(t)$ as $t \to \infty$?

   (c) Implement the forward Euler method to numerically approximate the solution. Use the initial time $t_0 = 0$ and the final time $t_n = 3$. Test two different time steps, $\Delta t = 0.3$ and $\Delta t = 0.1$. For each of the two time step, numerically solve the ODE for the initial condition at $y_0 = -1$ and $y_0 = 1$. Plot the Euler approximations on top of the exact solution.

   (d) Comment on the behavior of $y(t)$. What do you observe for the two different initial conditions?

   (e) Comment on the goodness of the numerical approximations with the two different time steps.

3. **The F-I Curve of the Leaky Integrate-and-Fire Neuron**
   The leaky integrate-and-fire neuron is modeled with the following equation:
$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_x$$
$$V(t^+) > V_{th} \to \text{spike at } t^+ \text{ , and } V(t^-) \leftarrow V_{reset}$$

where t^+ and t^- denote right after and right before time t. Assume the leak potential $E_L = -70$(mV), membrane time constant $\tau_m = 10$(ms), spike threshold $V_{th} = -50$ (mV), and the reset potential $V_{reset} = -65$ (mV).

(a) **Finding the threshold input** What is the minimum amount of input current needed to make the model eventually spike? We will call this threshold input current I_{th}. You should give an closed-form solution. (Hint: Recall the steady state solution of the leaky integrator model without spiking mechanisms, V_∞, can be expressed with the input current I_x. When the steady state solution is below V_{th}, the voltage will reach the steady state solution before it can reach the spiking threshold.)

(b) **Solve LIF model numerically** Implement LIF model numerically with the Euler Method by using $\Delta t = 0.1$ (ms) starting from $t = 0$ to a maximum time of $t_{max} = 2000$ (ms). Set the initial value of the membrane potential to the leak potential E_L. Simulate current injection just below and just above the threshold input current found in the previous part. Show the plots of V *vs* t for these two input currents. Explain why you get these results.

(c) **Approximating the f-I curve** Plot the f-I curve by performing the following steps.

i. Generate a vector of input currents ranging from just below the threshold input current I_{th} to some input current amount well above I_{th}. You can vary the currents by some set step size ΔI.

ii. For each input current value in the vector, numerically simulate the voltage $V(t)$ for sufficiently long, at least for $t_{max} = 1000$ (ms). The initial part of the $V(t)$ may be transients, so use the later parts of the spike train to compute the inter-spike-interval, T_{ISI}. You can compute this by detecting peaks of the spikes.

iii. Using the measured T_{ISI}, compute the firing rate r_{ISI} for each input current value in the vector of the range of input currents. Plot the firing rate r_{ISI} against input current amounts I_x.

(d) **The analytical solution for the f-I curve** In class, we have derived (or will derive) the analytical expression for the firing rate r_{ISI} for the LIF model. Compute these values for all input currents and plot this analytical solution on top of the numerically obtained f-I curve in part (c).