Math 4640, Numerical Analysis I
Homework 4

(1) Use the bisection algorithm to locate the smallest positive zero of the polynomial $p(x) = x^3 - 2x - 3$ correct to three significant digits.

(2) Implement the algorithm *Newton* given in the class and use it to find the roots for
  (a) $e^x - 3x^2 = 0$,
  (b) $x = 1 + 0.2\sin x$.
  (c) Find the minimizers of $G(x) = x^4 + 2x + 1$.

What is the convergence rates for the calculations? You must demonstrate how you draw your conclusion.

(3) The iteration $x_{n+1} = 2 - (1+c)x_n + cx_n^3$ will converge to $\alpha = 1$ for the value of c (provided x_0 is chosen sufficiently close to α). Find the values of c for which this is true. For what value of c will the convergence be quadratic? Any why?

(4) Which of the following iterations will converge to the indicated fixed point α (provided x_0 is sufficiently close to α)? If it does converge, give the order of convergence.
  (a) $x_{n+1} = -16 + 6x_n + \frac{12}{x_n}, \quad \alpha = 2;$
  (b) $x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2}, \quad \alpha = 3^{1/3};$
  (c) $x_{n+1} = \frac{12}{1+x_n}, \quad \alpha = 3.$

(5) Define the order n tridiagonal matrix

$$A_n = \begin{bmatrix} 2 & -1 & 0 & & \cdots & 0 \\ -1 & 2 & -1 & 0 & & \\ 0 & -1 & 2 & -1 & & \vdots \\ \vdots & & & \ddots & & \\ 0 & & \cdots & & -1 & 2 \end{bmatrix}.$$

Find a general formula for $A_n = LU$. (Hint: Consider the cases $n = 3, 4, 5$, and then guess the general pattern and verify it).

(6) The system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix},$$

and

$$\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \\ 1 \\ 2 \end{bmatrix},$$

has the solution $\vec{x} = [1, 1, 1, 1, 1, 1]^T$.

(a) Solve the system using the Jacobi iteration method (you must write your own code for it). Plot the norm of the error w.r.t. the number of iterations.

(b) Write your code to solve the system using Gauss-Seidel method. Plot the norm of the error w.r.t. the number of iterations.

(7) Use the power method to calculate the dominant eigenvalue and associated eigenvector for the following matrix

$$\begin{bmatrix} 6 & 4 & 4 & 1 \\ 4 & 6 & 1 & 4 \\ 4 & 1 & 6 & 4 \\ 1 & 4 & 4 & 6 \end{bmatrix}.$$

(8) Determine the approximate location of the eigenvalues of the following matrix

$$\begin{pmatrix} -2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & -1 & 3 \end{pmatrix}$$