Math 4640, Numerical Analysis I
Homework 1

(1) Representing numbers in different basis.
   (i) Convert the following numbers to their decimal equivalents
      (a) $(11101.10111)_2$,
      (b) $(2B3.FF)_{16}$,
      (c) $(11\ldots1)_2$ with the parentheses enclosing n 1s.
      (d) $(.101010101\ldots)$.
   (ii) Convert the following decimal numbers to their binary equivalents:
      (a) 54,
      (b) 250.025,
      (c) $\frac{1}{7}$.

(2) Compute the absolute error and relative error of following pairs of approximations.
   (a) $x_T = 10451.0023$, $x_A = 10451.001$;
   (b) $y_T = 0.451011 \times 10^4$, $y_A = 0.451010 \times 10^4$;
   (c) Compute the relative error of $x_A y_A$ and $x_A - y_A$ and comment on the number of accurate digits retained.

(3) Design algorithms to achieve the following tasks:
   (a) Convert a binary number into decimal.
   (b) Convert a decimal $x \in [0, 1)$ into binary.

   In each part, you need to give the algorithm, and write computer code (in any computer Language you prefer) to implement. You answer must contain at least 5 examples to illustrate the results of your code. You need to submit a written part, including the examples, as well as your code (code can be submitted on Canvas).

(4) The following data are taken from a polynomial of degree ≤ 5. What is the degree of the polynomial? Can you find the polynomial? If so, what is it? If not, explain why.

x	-2	-1	0	0.5	2	3
$p(x)$	-5	1	1	0.625	7	25

(5) Find the polynomial interpolation for the following data in Lagrange and Newton forms respectively. Verify that they are the same polynomial.

x	1	3/2	0	2
$f(x)$	3	13/4	3	5/3

(6) Prove the uniqueness of the interpolation polynomial in the theorem given in the lecture.

(7) Assume that $f(x) \in C^k(a, b)$, and $x_0, \cdots, x_k$ are in (a, b), show that

$$f[x_0, \cdots, x_k] = \frac{f^{(k)}(\xi)}{k!},$$

where $\xi \in (a, b)$.

(8) Find a piecewise linear interpolation $g_1(x)$ for the data

x	0	1	2	3	4
$f(x)$	1.2	1.6	0.6	-0.6	1.0