Question 6

For continuity, cubic splines require, in this case, that

$p(0^+) = p(0^-)$ so

$a = 1$, since $p(x^-) = x^3 + 1$

$p(x^+) = a + bx + cx^2 + dx^3$

Now $p(x) = 1 + bx + cx^2 + dx^3$, $x \in [0,1]$

Additionally, the derivative must be continuous. Thus,

$p'(0^+) = p'(0^-)$

$p'(x^-) = 3x^2$

$p'(x^+) = b + 2cx + 3dx^2$

Thus, $b = 0$

And further, $p''(0^+) = p''(0^-)$

$p''(x^-) = 6x$

$p''(x^+) = 2c + 6dx$

Thus, $c = 0$.

Hence $p(x) = 1 + dx^3$ $x \in [0,1]$.

Therefore, the values $(a,b,c,d)$ are $(1,0,0,d)$ where $d \in \mathbb{R}$.

$\boxed{Z}$