Problem 4

- $f[x_0, x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{1-(-5)}{-1-(-2)} = 6$
- $f[x_1, x_2] = \dfrac{1-1}{1} = 0$
- $f[x_2, x_3] = \dfrac{0.625 - 1}{0.5} = -\frac{3}{4}$
- $f[x_3, x_4] = \dfrac{7 - 0.625}{1.5} = 4.25 = 17/4$
- $f[x_4, x_5] = \dfrac{25 - 7}{1} = 18$
- $f[x_0, x_1, x_2] = \dfrac{-6}{2} = -3$
- $f[x_1, x_2, x_3] = \dfrac{-\frac{3}{4}}{1.5} = -\frac{1}{2}$
- $f[x_2, x_3, x_4] = \dfrac{4.25 + 3/4}{2} = 2.5$
- $f[x_3, x_4, x_5] = \dfrac{18 - 4.25}{2.5} = 5.5$
- $f[x_0, x_1, x_2, x_3] = \dfrac{-\frac{1}{2} + 3}{2.5} = 1$
- $f[x_1, x_2, x_3, x_4] = \dfrac{2.5 + 1/2}{3} = 1$
- $f[x_2, x_3, x_4, x_5] = 1$
- $f[x_0, x_1, x_2, x_3, x_4] = \dfrac{1-1}{4} = 0$
- $f[x_1, x_2, x_3, x_4, x_5] = \dfrac{1-1}{4} = 0$
- $f[x_0, \ldots, x_5] = \dfrac{0-0}{5} = 0$

$$p_5(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$+ f[x_0,\dots,x_3](x-x_0)(x-x_1)(x-x_2)$$

$$+ f[x_0,\dots,x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$+ f[x_0,\dots,x_5](x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)$$

- $(x-x_0) = x+2$
- $(x-x_0)(x-x_1) = (x+2)(x+1) = x^2+3x+2$
- $(x-x_0)(x-x_1)(x-x_2) = (x^2+3x+2)(x) = x^3+3x^2+2x$
- $(x-x_0)(x-x_1)(x-x_2)(x-x_3) = (x^3+3x^2+2x)(x-1/2)$
  $= x^4+2.5x^3+0.5x^2-x$
- $(x-x_0)\cdots(x-x_4) = (x^4+2.5x^3+0.5x^2-x)(x-3)$
  $= x^5-0.5x^4-7x^3-2.5x^2+3x$

Recall: $f[x_0] = -5$, $f[x_0,x_1] = 6$, $f[x_0,x_1,x_2] = -3$, $f[x_0,x_1,x_2,x_3] = 1$, $f[x_0,\dots,x_4] = f[x_0,\dots,x_5] = 0$

Thus, $p_5(x) = -5 + 6(x+2) - 3(x^2+3x+2) + (x^3+3x^2+2x)$

$$= -5+6x+12-3x^2-9x-6+x^3+3x^2+2x$$

$$= x^3+(-3+3)x^2+(6-9+2)x+(-5+12-6)$$

$$= x^3-x+1$$

$p_5(x) = x^3-x+1$ and is a polynomial of degree 3. $\blacksquare$