Math 4640, Numerical Analysis I
Homework 2

(1) Given $f'(0) = -1$ and $f'(4) = 1$, find the cubic spline interpolation for the data given in the following table.

x	0	1	2	3	4
$f(x)$	1.1	0.8	0.5	0.3	0.5

(2) Let $p_3(x)$ denote the cubic polynomial interpolation f(x) at the evenly spaced points $x_j = x_0 + jh$, $j = 0, 1, 2, 3$. Assuming $f(x)$ is sufficiently differentiable, bound the error in using $p_3'(x)$ as an approximation to $f'(x)$, $x_0 \le x \le x_3$.

(3) Find a polynomial $p(x)$ of degree ≤ 2 that satisfies

$$p(x_0) = a, \quad p(x_1) = b, \quad p'(x_1) = c,$$

where a, b, c are given constants and x_0, x_1 are two different points.

(4) Write your own code to perform the algorithms *Divdif* and *Interp* given in the class. Test your code on the following data. Give a polynomial of degree 5 that interpolates the given data.

x	-2	-1	0	1	2	3
$p(x)$	w	y	1	0	2	z

where w, y and z are the last three digits of your student ID number.

(5) For $f(x) = 2/(1+x^2)$, $-5 \le x \le 5$, use your code *Divdif* and *Interp* to produce n-th degree polynomial interpolation $p_n(x)$ using $n+1$ evenly spaced nodes on $[-5, 5]$. Evaluate $p_2(x)$ at a large number of points (you decide how large), and graph the results and the error on $[-5, 5]$. Repeat the problem for $p_4(x)$.

(6) Find values of (a, b, c, d) that make the following function a cubic spline?

$$p(x) = \begin{cases} x^3 + 1 & x \in [-1, 0) \\ a + bx + cx^2 + dx^3 & x \in [0, 1] \end{cases}$$

(7) Use Taylor expansion to find the error term for central difference scheme

$$\frac{1}{2} f'(a) \approx \frac{f(a + h/2) - f(a - h/2)}{2h}.$$

(8) Derive a formula for $f'''(a)$ by differentiating

$$f(x) = p_k(x) + f[x_0, \cdots, x_k, x]\phi_k(x)$$

three times. Choose $k = 3$ and set $x_0 = a, x_1 = a - h, x_2 = a + h, x_3 = a + 2h$. Also derive the error term for this formula.