Advanced Topics in R: Rejection Sampling

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Outline

1. Theory
2. Practice

Theory: Purpose

Often it is necessary to produce samples from arbitrary distributions. Some reasons for doing so include:

1. Verification of analytical results.
2. Testing new models on data with known properties
3. Examining cases where new models fail

It is possible to sample directly from common distributions using built in R functions, e.g. rnorm(), rexp(), etc.

Theory: Purpose

- Samples from some other distributions can be produced using theoretical results. For example, samples from a χ^2 distribution can be obtained by sampling from a standard normal and squaring the results.
- However, it is easy to contrive distributions for which this technique is very difficult, or even impossible.
- Rejection sampling gives us a very flexible way to sample from any univariate density which we can express in closed form.

Theory: Procedure

- The procedure works by drawing samples from a known distribution (in our examples a scaled uniform) and then rejecting some proportion of them. This is called the *proposal function*.
- We reject or accept samples from this proposal function in such a way as to create samples from the density we actually want, called the *target density*.
- Let $f(x)$ be the density of a uniform random variable over a range which covers the support of the target density. Let $g(x)$ be the target density. Let M be a real number such that

$$\frac{g(x)}{Mf(x)} < 1$$

Theory: Procedure

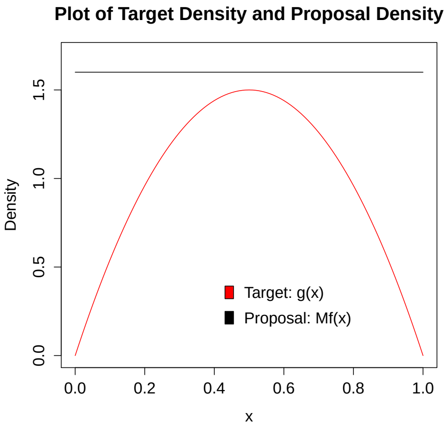

Figure: Example of a target and proposal density

1. Draw x^* from $f(x)$
2. Let $r = \frac{g(x)}{Mf(x)}$.
3. With probability r, keep the sample drawn from $f(x)$. Otherwise, discard it.
4. Repeat this procedure until you've obtained the desired number of samples from $g(x)$.

Practice: Example I, Beta Density

$$f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

$$f(x;2,2) = 6 \times x(1-x)$$

- Here, $\alpha, \beta > 0$ and $x \in [0,1]$.
- We will let $\alpha = \beta = 2$

Practice: Example II, Type IV Generalized Logistic Density

$$f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} \frac{\exp(-\beta x)}{(1+\exp(-x))^{\alpha+\beta}}$$

$$f(x;2,2) = 6 \times \frac{\exp(-2x)}{(1+\exp(-x))^4}$$

- Here, $\alpha, \beta > 0$ and $x \in \Re$.
- As before, we will let $\alpha = \beta = 2$